

GroupRepresentationsForCAP

**Skeletal category of group
representations for CAP**

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Contents

1	Associators	3
1.1	Introduction	3
1.2	Quickstart	3
1.3	Read, Write, and Display	3
1.4	Computing associators	5
1.5	Technical functions	7
2	Semisimple Categories	10
2.1	Introduction	10
2.2	Constructors	10
2.3	Attributes	12
2.4	Operations	14
2.5	GAP Categories	16
3	Irreducible Objects	17
3.1	Introduction	17
3.2	Constructors	17
3.3	Attributes	18
3.4	Properties	20
3.5	Operations	20
4	Representation Category of Groups	23
4.1	Introduction	23
4.2	Quickstart	23
4.3	Constructors	24
	Index	26

Chapter 1

Associators

1.1 Introduction

Let G be a finite group and let $G\text{-mod}$ be a skeletal version of the monoidal category of finite dimensional complex representations of G . The purpose of these GAP methods is the computation of the associators of $G\text{-mod}$.

1.2 Quickstart

The following commands compute the associator of D_8 and write all data necessary for the reproducibility of the computation to files with the prefix "D8".

Example

```
gap> G := DihedralGroup( 8 );;  
gap> ComputeAssociator( G, true, true, false );;  
gap> path := Concatenation(  
>   PackageInfo( "GroupRepresentationsForCAP" )[1].InstallationPath,  
>   "/examples/doc/D8" );;  
gap> WriteAssociatorComputationToFiles( path );
```

1.3 Read, Write, and Display

The following intermediate steps of the associator computation can be read from/written to files.

- Irreducible representations of a finite group given by matrices (Data 1).
- Decomposition isomorphisms of tensor products into direct sums of irreducibles (Data 2).

Furthermore, the following data can be written to files.

- A database key for the AssociatorsDatabase/DatabaseKeys.g file.
- The final result, namely the associator (Data 3).

Data 1 and Data 2 involve choices and thus are subject to changes in further versions of this package. However, the process Data 2 \rightarrow Data 3 is a mathematical function and thus stable. For reproducibility, it is recommended to store all three data. To facilitate this task, use the function `WriteAssociatorComputationToFiles`.

1.3.1 WriteDatabaseKeysToFile (for IsString)

▷ WriteDatabaseKeysToFile(*s*) (operation)

Returns: nothing

The argument is a filename *s*. This operation writes the database keys computed by the last call of InitializeGroupData to the corresponding file.

1.3.2 WriteRepresentationsDataToFile (for IsString)

▷ WriteRepresentationsDataToFile(*s*) (operation)

Returns: nothing

The argument is a filename *s*. This operation writes the representations computed by the last call of InitializeGroupData to the corresponding file.

1.3.3 WriteSkeletalFunctorDataToFile (for IsString)

▷ WriteSkeletalFunctorDataToFile(*s*) (operation)

Returns: nothing

The argument is a filename *s*. This operation writes the skeletal functor data computed by the last call of SkeletalFunctorTensorData to the corresponding file.

1.3.4 WriteAssociatorDataToFile (for IsString)

▷ WriteAssociatorDataToFile(*s*) (operation)

Returns: nothing

The argument is a filename *s*. This operation writes the associator data of the initialized group to the corresponding file. You have to call AssociatorForSufficientlyManyTriples first.

1.3.5 WriteAssociatorComputationToFiles (for IsString)

▷ WriteAssociatorComputationToFiles(*s*) (operation)

Returns: nothing

Only call this function if you did a whole associator computation first (e.g. using ComputeAssociator). The argument is a string *s*. This function writes 4 files:

- *s*Key.g: A file for the database key of the associator computation.
- *s*Reps.g: A file containing the irreducible representations used for the associator computation.
- *s*Dec.g: A file for the tensor decompositions used for the associator computation.
- *s*Ass.g or *s*AssD.g: A file containing the computed associator. The suffix *D* is used if the associator was not computed for all triples.

1.3.6 ReadDatabaseKeys (for IsString)

▷ ReadDatabaseKeys(*s*) (operation)

Returns: a list

The argument is a filename *s* of a file written by WriteDatabaseKeysToFile. The output is a list [group, conductor, position of trivial character, field, category].

1.3.7 ReadRepresentationsData (for IsString, IsString)

▷ `ReadRepresentationsData(s_1 , s_2)` (operation)

Returns: a list

The arguments are a filename s_1 of a file written by `WriteDatabaseKeysToFile`, and a filename s_2 of a file written by `WriteRepresentationsDataToFile`. The output is a list [,number of irreducibles, irreducibles, representations given by images of generators, inverses of these images, vector space objects for the irreducibles].

1.3.8 ReadSkeletalFunctorData (for IsString, IsString)

▷ `ReadSkeletalFunctorData(s_1 , s_2)` (operation)

Returns: a list

The arguments are a filename s_1 of a file written by `WriteDatabaseKeysToFile`, and a filename s_2 of a file written by `WriteSkeletalFunctorDataToFile`. The output is a list [irreducibles, skeletal functor tensor data, vector space objects for the irreducibles].

1.4 Computing associators

1.4.1 InitializeGroupData (for IsGroup)

▷ `InitializeGroupData(G)` (operation)

Returns: a list

The argument is a group G . This method calls `InitializeGroupData(G , false)`.

1.4.2 InitializeGroupData (for IsGroup, IsBool)

▷ `InitializeGroupData(G , b)` (operation)

Returns: a list

The arguments are a group G and a boolean b . The output is a list [generators of G , number of irreducibles, irreducibles, representations given by images of generators, inverses of these images, vector space objects for the irreducibles]. Furthermore, this method stores the database key, which can be written using `WriteDatabaseKeysToFile`. If b is true, then the id of the group in the database key is given by its string, otherwise it is given by its id in the `SmallGroupLibrary`.

1.4.3 InitializeGroupDataDixon (for IsGroup)

▷ `InitializeGroupDataDixon(G)` (operation)

Returns: a list

The argument is a group G . This method calls `InitializeGroupDataDixon(G , false)`.

1.4.4 InitializeGroupDataDixon (for IsGroup, IsBool)

▷ `InitializeGroupDataDixon(G , b)` (operation)

Returns: a list

The arguments are a group G and a boolean b . This method does the same as `InitializeGroupData`, but uses `IrreducibleRepresentationsDixon` for affording irreducible representations.

1.4.5 InitializeGroupData (for IsGroup, IsList, IsBool)

▷ InitializeGroupData(arg1, arg2, arg3) (operation)

1.4.6 SkeletalFunctorTensorData

▷ SkeletalFunctorTensorData() (operation)

Returns: a list

There is no argument. This methods calls SkeletalFunctorTensorData with the output of the last call of InitializeGroupData or InitializeGroupDataDixon.

1.4.7 SkeletalFunctorTensorData (for IsList)

▷ SkeletalFunctorTensorData(l) (operation)

Returns: a list

The argument is a list l which is the output of InitializeGroupData, InitializeGroupDataDixon, or ReadRepresentationsData. The output is a triple $[t_1, t_2, t_3]$. t_1 is the list of all characters of G . t_2 is a list such that the (i, j) -th entry, where i, j range from 1 to the number of irreducibles, is a pair of mutual inverse morphisms $[\alpha, \alpha^{-1}]$, and α is a decomposition isomorphism $\bigoplus_{\chi \in \text{Irr}(G)} V_{\chi}^{n_{\chi}} \rightarrow V_i \otimes V_j$. t_3 is a list of vector space objects for the irreducibles.

1.4.8 AssociatorDataFromSkeletalFunctorTensorData (for IsInt, IsInt, IsInt, IsList)

▷ AssociatorDataFromSkeletalFunctorTensorData(a, b, c, l) (operation)

Returns: a list

The arguments are integers a, b, c and a list l which is the output of SkeletalFunctorTensorData. The output is a list containing homalg matrices representing the components of the associator of V_a, V_b, V_c , where the numbers correspond to the enlisting of the irreducible characters given by l .

1.4.9 AssociatorForSufficientlyManyTriples

▷ AssociatorForSufficientlyManyTriples() (operation)

Returns: a list

There is no argument. This methods calls AssociatorForSufficientlyManyTriples with the output of the last call of SkeletalFunctorTensorData and false.

1.4.10 AssociatorForSufficientlyManyTriples (for IsList, IsBool)

▷ AssociatorForSufficientlyManyTriples(l, b) (operation)

Returns: a list

The arguments are a list l which is the output of SkeletalFunctorTensorData, and a boolean b . The output is a list of lists L such that $L[a][b][c]$ contains the associator computed by AssociatorDataFromSkeletalFunctorTensorData(a,b,c). If b is true, then a, b, c ranges through all possible triples, otherwise, a, b, c are computed for so many triples such that the others can be obtained using braidings.

1.4.11 ComputeAssociator (for IsGroup, IsBool)

▷ `ComputeAssociator(G , b_1)` (operation)

Returns: a list

The arguments are a group G , and a boolean b_1 . The output is `ComputeAssociator(G , b_1 , false, true)`.

1.4.12 ComputeAssociator (for IsGroup, IsBool, IsBool)

▷ `ComputeAssociator(G , b_1 , b_2)` (operation)

Returns: a list

The arguments are a group G , and two booleans b_1, b_2 . The output is `ComputeAssociator(G , b_1 , b_2 , true)`.

1.4.13 ComputeAssociator (for IsGroup, IsBool, IsBool, IsBool)

▷ `ComputeAssociator(G , b_1 , b_2 , b_3)` (operation)

Returns: a list

The arguments are a group G , and three booleans b_1, b_2, b_3 . The output is a list l whose (a, b, c) -th entry contains a string representing the associator of the objects V_a, V_b, V_c in a skeleton of the representation category of G , where V_* are irreducible representations corresponding to the ordering of the irreducible characters $\text{Irr}(G)$. If b_1 is true, this method uses `IrreducibleRepresentationsDixon`, otherwise it uses `IrreducibleAffordingRepresentation`. If b_2 is true, the associators are computed for all possible triples a, b, c , otherwise only for sufficiently many such that the others can be reproduced using the braiding in the representation category. If b_3 is true, then the id of the group in the database key is given by its string, otherwise it is given by its id in the `SmallGroupLibrary`. This last boolean is relevant only if you want to write the computed associators to files (e.g. using `WriteAssociatorComputationToFiles`).

1.5 Technical functions

1.5.1 SetInfoLevelForAssociatorComputations (for IsInt)

▷ `SetInfoLevelForAssociatorComputations(l)` (operation)

Returns: nothing

The argument is an integer l . If $l > 0$, then the functions for computing associators provide information during the computation. This is useful in cases where the computation may take a long time.

1.5.2 DefinedOverCyclotomicField (for IsInt, IsGroupHomomorphism)

▷ `DefinedOverCyclotomicField(n , f)` (operation)

Returns: a boolean

The arguments are an integer n and a group homomorphism f whose images are matrices. The output is true if the entries of the images of f lie in a cyclotomic field generated by a primitive n -th root of unity, false otherwise.

1.5.3 GroupReperesentationByImages (for IsGroup, IsList)

▷ `GroupReperesentationByImages(G, L)` (operation)

Returns: a group homomorphism

The arguments are a group G with generators g_1, \dots, g_n and a list $L = [l_1, \dots, l_n]$. The output is the group homomorphism from G to the group generated by the elements of L , mapping g_i to l_i .

1.5.4 DiagonalizationTransformationOfBraiding (for IsVectorSpaceMorphism)

▷ `DiagonalizationTransformationOfBraiding(e)` (attribute)

Returns: an invertible endomorphism in $\text{Hom}(V, V)$

The argument is an endomorphism $e \in \text{Hom}(V, V)$ of vector spaces whose minimal polynomial divides $x^2 - 1$. The output is an invertible endomorphism t such that $t^{-1} \circ e \circ t$ is a diagonal matrix.

1.5.5 AffordAllIrreducibleRepresentations (for IsGroup)

▷ `AffordAllIrreducibleRepresentations(G)` (operation)

Returns: a list

The argument is a group G . The output is a list of all irreducible representations of G using the command `IrreducibleAffordingRepresentation`.

1.5.6 AffordAllIrreducibleRepresentationsDixon (for IsGroup)

▷ `AffordAllIrreducibleRepresentationsDixon(G)` (operation)

Returns: a list

The argument is a group G . The output is a list of all irreducible representations of G using the command `IrreducibleRepresentationsDixon`.

1.5.7 DefaultFieldForListOfRepresentations (for IsList)

▷ `DefaultFieldForListOfRepresentations(L)` (operation)

Returns: a GAP field

The argument is a list L of representations of a group G . The output is a field over which all representations are defined simultaneously.

1.5.8 RewriteMatrixInCyclotomicGenerator (for IsMatrix, IsInt)

▷ `RewriteMatrixInCyclotomicGenerator(M, n)` (operation)

Returns: a matrix

The arguments are a matrix M and an integer n . The output is a matrix N in $Q[\varepsilon]$. Substituting an n -th root of unity for ε in N yields M .

1.5.9 InternalHomToTensorProductAdjunctionMapTemp (for IsVectorSpaceObject, IsVectorSpaceObject, IsVectorSpaceMorphism)

▷ `InternalHomToTensorProductAdjunctionMapTemp(b, c, g)` (operation)

Returns: a morphism in $\text{Hom}(a \otimes b, c)$.

The arguments are objects b, c and a morphism $g : a \rightarrow \underline{\text{Hom}}(b, c)$. The output is a morphism $f : a \otimes b \rightarrow c$ corresponding to g under the tensor hom adjunction.

1.5.10 HomalgMatrixAsString (for IsHomalgMatrix)

▷ HomalgMatrixAsString(M) (operation)

Returns: a string

The argument is a homalg matrix M . The output is a string consisting of the elements of M , separated by commas.

1.5.11 DataFromSkeletalFunctorTensorDataAsStringList (for IsList)

▷ DataFromSkeletalFunctorTensorDataAsStringList(l) (operation)

Returns: a list of strings and empty entries

The argument is a list l of homalg matrices. In l , empty entries are allowed. The output is a list where each non-empty entry of l is converted to a string using HomalgMatrixAsString.

1.5.12 AsVectorSpaceMorphism (for IsHomalgMatrix)

▷ AsVectorSpaceMorphism(M) (attribute)

Returns: a vector space morphism

The argument is a homalg matrix M . The output is a vector space morphism whose underlying matrix is given by M .

1.5.13 CreateEndomorphismFromString (for IsVectorSpaceObject, IsString)

▷ CreateEndomorphismFromString(V , s) (operation)

Returns: a vector space morphism

The arguments are a vector space object V and a string s consisting of $\dim(V)^2$ elements of the ground field of V . The output is a vector space endomorphism $V \rightarrow V$ defined by s .

Chapter 2

Semisimple Categories

2.1 Introduction

Let k be a field and I be a totally ordered set. We denote the matrix category of k by $k\text{-vec}$ (see the package `LinearAlgebraForCAP`). The semisimple category $\bigoplus_{i \in I} k\text{-vec}$ associated to k and I is defined as the full subcategory of the product category $\prod_{i \in I} k\text{-vec}$ generated by those I -indexed tuples having only finitely many non-zero entries. By χ^i , we denote the object which is 1 at entry i and 0 otherwise. Thus, an arbitrary object in $\bigoplus_{i \in I} k\text{-vec}$ can be written as $\bigoplus_{i \in I} a_i \chi^i$ for non-negative numbers a_i for which only finitely many are non-zero.

2.2 Constructors

2.2.1 SemisimpleCategory (for IsFieldForHomalg, IsFunction, IsObject, IsString, Is-Bool, IsList)

▷ `SemisimpleCategory(k, m, u, s, b, L)` (operation)

Returns: a category

The arguments are:

- a homalg field k ,
- a membership function m sending any GAP object to a boolean,
- a GAP object u ,
- a string s containing a filename in the folder `"/gap/AssociatorsDatabase/"` of this package,
- a boolean b ,
- a list L containing 4 entries, where the first 3 are filters and the last one is a string.

The output is a CAP category modelling $\bigoplus_{i \in I} k\text{-vec}$, where I is the set defined by the membership function m . Note that objects in I are expected to be equipped with operations enlisted in the chapter "Irreducible Objects". Furthermore, this CAP category is a rigid symmetric closed monoidal Abelian category. Its tensor product is defined by the data of the file s , where the boolean b is true if the associator stored in s was computed for all triples, and false otherwise (cf. chapter "Associators"). Its braiding and duality comes from the additional structure required for I . Its tensor unit is modelled by

u . The three filters of the L are filters for the resulting category, its objects, and its morphisms. L_4 is the name of the resulting category.

2.2.2 SemisimpleCategory (for IsFieldForHomalg, IsFunction, IsObject, IsString, Is-Bool)

▷ SemisimpleCategory(k, m, u, s, b) (operation)

Returns: a category

The arguments are:

- a homalg field k ,
- a membership function m sending any GAP object to a boolean,
- a GAP object u ,
- a string s containing a filename in the folder "/gap/AssociatorsDatabase/" of this package,
- a boolean b .

This function calls SemisimpleCategory on the six arguments [$k, m, u, s, b, [\text{IsObject}, \text{IsObject}, \text{IsObject}, \text{automatically generated name}]$]

2.2.3 SemisimpleCategoryMorphism (for IsSemisimpleCategoryObject, IsList, IsSemisimpleCategoryObject)

▷ SemisimpleCategoryMorphism(s, L, r) (operation)

Returns: a morphism

The arguments are an object s in a semisimple category $\bigoplus_{i \in I} k\text{-vec}$, a list of pairs $L = [[\phi_1, i_1], \dots, [\phi_l, i_l]]$ where ϕ_j are morphisms in the Matrix Category $k\text{-vec}$ and $i_j \in I$, and another object r in the same semisimple category. The output is a morphism in $\bigoplus_{i \in I} k\text{-vec}$ from s to r whose i -th component is given by ϕ_i . For this morphism to be well defined, there has to be an ϕ_i for every i in the support of s and r .

2.2.4 SemisimpleCategoryMorphismSparse (for IsSemisimpleCategoryObject, IsList, IsSemisimpleCategoryObject)

▷ SemisimpleCategoryMorphismSparse(s, L, r) (operation)

Returns: a morphism

The arguments are an object s in a semisimple category $\bigoplus_{i \in I} k\text{-vec}$, a list of pairs $L = [[\phi_1, i_1], \dots, [\phi_l, i_l]]$ where ϕ_j are morphisms in the Matrix Category $k\text{-vec}$ and $i_j \in I$, and another object r in the same semisimple category. The output is a morphism in $\bigoplus_{i \in I} k\text{-vec}$ from s to r whose i_j -th component is given by ϕ_{i_j} for $j = 1, \dots, l$, and by the zero morphism otherwise.

2.2.5 ComponentInclusionMorphism (for IsSemisimpleCategoryObject, IsObject)

▷ ComponentInclusionMorphism(v, j) (operation)

Returns: a morphism

The arguments are an object $v = \bigoplus_{i \in I} a_i \chi^i$ in a semisimple category $\bigoplus_{i \in I} k\text{-vec}$, and an object $j \in I$. The output is the canonical inclusion $a_j \chi^j \hookrightarrow \bigoplus_{i \in I} a_i \chi^i$ in $\bigoplus_{i \in I} k\text{-vec}$.

2.2.6 ComponentProjectionMorphism (for IsSemisimpleCategoryObject, IsObject)

▷ `ComponentProjectionMorphism(v , j)` (operation)

Returns: a morphism

The arguments are an object $v = \bigoplus_{i \in I} a_i \chi^i$ in a semisimple category $\bigoplus_{i \in I} k\text{-vec}$, and an object $j \in I$. The output is the canonical projection $\bigoplus_{i \in I} a_i \chi^i \rightarrow a_j \chi^j$ in $\bigoplus_{i \in I} k\text{-vec}$.

2.2.7 SemisimpleCategoryObject (for IsList, IsCapCategory)

▷ `SemisimpleCategoryObject(L , C)` (operation)

Returns: an object

The arguments are a list L and a semisimple category $C = \bigoplus_{i \in I} k\text{-vec}$. The list L contains pairs $L = [[a_1, i_1], \dots, [a_l, i_l]]$ of non-negative integers a_j and objects $i_j \in I$. The output is the object in C given by $\bigoplus_{j=1}^l a_j \chi^{i_j}$.

2.2.8 SemisimpleCategoryObjectConstructorWithFlatList (for IsList, IsCapCategory)

▷ `SemisimpleCategoryObjectConstructorWithFlatList(L , C)` (operation)

Returns: an object

The arguments are a list L and a semisimple category $C = \bigoplus_{i \in I} k\text{-vec}$. The list L contains an even number of elements $L = [a_1, i_1, \dots, a_l, i_l]$ of non-negative integers a_j and objects $i_j \in I$. The output is the object in C given by $\bigoplus_{j=1}^l a_j \chi^{i_j}$.

2.3 Attributes

2.3.1 MembershipFunctionForSemisimpleCategory (for IsSemisimpleCategory)

▷ `MembershipFunctionForSemisimpleCategory(C)` (attribute)

Returns: a function

The argument is a semisimple category $C = \bigoplus_{i \in I} k\text{-vec}$. The output is its underlying membership function m for I .

2.3.2 UnderlyingCategoryForSemisimpleCategory (for IsSemisimpleCategory)

▷ `UnderlyingCategoryForSemisimpleCategory(C)` (attribute)

Returns: a category

The argument is a semisimple category $C = \bigoplus_{i \in I} k\text{-vec}$. The output is its underlying category $k\text{-vec}$.

2.3.3 UnderlyingFieldForHomalgForSemisimpleCategory (for IsSemisimpleCategory)

▷ `UnderlyingFieldForHomalgForSemisimpleCategory(C)` (attribute)

Returns: a homalg field

The argument is a semisimple category $C = \bigoplus_{i \in I} k\text{-vec}$. The output is its underlying field k .

2.3.4 GivenObjectFilterForSemisimpleCategory (for IsSemisimpleCategory)

▷ `GivenObjectFilterForSemisimpleCategory(C)` (attribute)

Returns: a filter

The argument is a semisimple category $C = \bigoplus_{i \in I} k\text{-vec}$. The output is its object filter which could be specified in the constructor of C .

2.3.5 GivenMorphismFilterForSemisimpleCategory (for IsSemisimpleCategory)

▷ `GivenMorphismFilterForSemisimpleCategory(C)` (attribute)

Returns: a filter

The argument is a semisimple category $C = \bigoplus_{i \in I} k\text{-vec}$. The output is its morphism filter which could be specified in the constructor of C .

2.3.6 SemisimpleCategoryMorphismList (for IsSemisimpleCategoryMorphism)

▷ `SemisimpleCategoryMorphismList(alpha)` (attribute)

Returns: a list

The argument is a morphism $\alpha = (\alpha_i)_{i \in I}$ in a semisimple category $\bigoplus_{i \in I} k\text{-vec}$. The output is the list of pairs $[[\alpha_{i_1}, i_1], \dots, [\alpha_{i_l}, i_l]]$ where i_j ranges through the support of the source and range of α .

2.3.7 UnderlyingFieldForHomalg (for IsSemisimpleCategoryMorphism)

▷ `UnderlyingFieldForHomalg(alpha)` (attribute)

Returns: a homalg field

The argument is a morphism $\alpha = (\alpha_i)_{i \in I}$ in a semisimple category $\bigoplus_{i \in I} k\text{-vec}$. The output is the homalg field k .

2.3.8 SemisimpleCategoryObjectList (for IsSemisimpleCategoryObject)

▷ `SemisimpleCategoryObjectList(v)` (attribute)

Returns: a list

The argument is an object $v = \bigoplus_{j=1}^l a_j \chi^{i_j}$ in a semisimple category. The output is the list $[[a_1, i_1], \dots, [a_l, i_l]]$.

2.3.9 SemisimpleCategoryObjectListWithActualObjects (for IsSemisimpleCategory-Object)

▷ `SemisimpleCategoryObjectListWithActualObjects(v)` (attribute)

Returns: a list

The argument is an object $v = \bigoplus_{j=1}^l a_j \chi^{i_j}$ in a semisimple category. The output is the list $[[a_1, \chi^{i_1}], \dots, [a_l, \chi^{i_l}]]$.

2.3.10 Support (for IsSemisimpleCategoryObject)

▷ `Support(v)` (attribute)

Returns: a list

The argument is an object $v = \bigoplus_{j=1}^l a_j \chi^{i_j}$ in a semisimple category. The output is the list $[i_1, \dots, i_l]$.

2.3.11 UnderlyingFieldForHomalg (for IsSemisimpleCategoryObject)

▷ `UnderlyingFieldForHomalg(v)` (attribute)

Returns: a homalg field

The argument is an object $v = \bigoplus_{j=1}^l a_j \chi^{i_j}$ in a semisimple category $\bigoplus_{i \in I} k\text{-vec}$. The output is the homalg field k .

2.3.12 Dimension (for IsSemisimpleCategoryObject)

▷ `Dimension(v)` (attribute)

Returns: an integer

The argument is an object $v = \bigoplus_{j=1}^l a_j \chi^{i_j}$ in a semisimple category $\bigoplus_{i \in I} k\text{-vec}$. The output is the integer $\sum_{j=1}^l a_j \cdot \dim(i_j)$. This functions works under the assumption that there is a notion of dimension on the objects in I .

2.4 Operations

2.4.1 Component (for IsSemisimpleCategoryMorphism, IsObject)

▷ `Component(alpha, i)` (operation)

Returns: a vector space morphism

The argument is a morphism $\alpha = (\alpha_i)_{i \in I}$ in a semisimple category $\bigoplus_{i \in I} k\text{-vec}$ and an object $i \in I$. The output is α_i .

2.4.2 NormalizeSemisimpleCategoryObjectList (for IsList)

▷ `NormalizeSemisimpleCategoryObjectList(L)` (operation)

Returns: a list

The argument is a list $L = [[a_1, i_1], \dots, [a_l, i_l]]$ of non-negative integers a_j and objects $i_j \in I$, where I correspond to irreducible objects in a semisimple category $\bigoplus_{i \in I} k\text{-vec}$. The output is again a list of pairs consisting of integers an elements in I , but with the following normalization:

- Each a_j is positive,
- i_j is strictly less than i_{j+1} .

2.4.3 Multiplicity (for IsSemisimpleCategoryObject, IsObject)

▷ `Multiplicity(v, i)` (operation)

Returns: an integer

The arguments are an object $v = \bigoplus_{j=1}^l a_j \chi^{i_j}$ in a semisimple category $\bigoplus_{i \in I} k\text{-vec}$, and an element $i \in I$. The output is the integer a_i .

2.4.4 Component (for IsSemisimpleCategoryObject, IsObject)

▷ `Component(v, i)` (operation)

Returns: a vector space object

The arguments are an object $v = \bigoplus_{j=1}^l a_j \chi^{i_j}$ in a semisimple category $\bigoplus_{i \in I} k\text{-vec}$, and an element $i \in I$. The output is the k -vector space object k^{a_i} in Cap's Matrix Category.

2.4.5 TestPentagonIdentity (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ TestPentagonIdentity(v_1, v_2, v_3, v_4) (operation)

Returns: a boolean

This is a debug operation. The arguments are 4 objects v_1, v_2, v_3, v_4 in a category. The output is true if the pentagon identity holds for those 4 objects, false otherwise.

2.4.6 TestPentagonIdentityForAllQuadruplesInList (for IsList)

▷ TestPentagonIdentityForAllQuadruplesInList(L) (operation)

Returns: a boolean

This is a debug operation. The argument is a list L consisting of quadruples of objects in a semisimple category $\bigoplus_{i \in I} k\text{-vec}$. The output is true if the pentagon identity holds for all those quadruples, false otherwise.

2.4.7 TestBraidingCompatability (for IsSemisimpleCategoryObject, IsSemisimpleCategoryObject, IsSemisimpleCategoryObject)

▷ TestBraidingCompatability(v_1, v_2, v_3) (operation)

Returns: a boolean

This is a debug operation. The arguments are 3 objects v_1, v_2, v_3 in a semisimple category $\bigoplus_{i \in I} k\text{-vec}$. The output is true if the braiding compatibilities with the associator hold, false otherwise.

2.4.8 TestBraidingCompatabilityForAllTriplesInList (for IsList)

▷ TestBraidingCompatabilityForAllTriplesInList(L) (operation)

Returns: a boolean

This is a debug operation. The argument is a list L consisting of triples of objects in a semisimple category $\bigoplus_{i \in I} k\text{-vec}$. The output is true if the braiding compatibilities with the associator hold for all those triples false otherwise.

2.4.9 TestZigZagIdentitiesForDual (for IsSemisimpleCategoryObject)

▷ TestZigZagIdentitiesForDual(v) (operation)

Returns: a boolean

This is a debug operation. The argument is an object v in a semisimple category $\bigoplus_{i \in I} k\text{-vec}$. The output is true if the zig zag identity for duals hold, false otherwise.

2.4.10 TestZigZagIdentitiesForDualForAllObjectsInList (for IsList)

▷ TestZigZagIdentitiesForDualForAllObjectsInList(L) (operation)

Returns: a boolean

This is a debug operation. The argument is a list L consisting of objects in a semisimple category $\bigoplus_{i \in I} k\text{-vec}$. The output is true if the zig zag identity for duals hold for all those objects, false otherwise.

2.5 GAP Categories

2.5.1 IsSemisimpleCategoryMorphism (for IsCapCategoryMorphism and IsCellOfSkeletalCategory)

▷ IsSemisimpleCategoryMorphism(*object*) (filter)

Returns: true or false

The GAP category of morphisms in a semisimple category.

2.5.2 IsSemisimpleCategoryObject (for IsCapCategoryObject and IsCellOfSkeletalCategory)

▷ IsSemisimpleCategoryObject(*object*) (filter)

Returns: true or false

The GAP category of objects in a semisimple category.

Chapter 3

Irreducible Objects

3.1 Introduction

For a semisimple category C of the form $\bigoplus_{i \in I} k\text{-vec}$ to become a rigid symmetric closed monoidal skeletal category, the set I has to be equipped with extra structure. To become a skeletal category, we need:

- a total ordering on I .

To become a monoidal category, we need:

- a function `IsYieldingIdentities`, deciding whether an object yields the identity whenever it is part of an associator triple or a braiding pair,
- functions `Multiplicity` and `*`, defining the tensor product on objects,
- a function `AssociatorFromData`, defining the tensor product on morphisms.

To become a symmetric monoidal category, we need:

- a function `ExteriorPower`.

To become a rigid symmetric monoidal category, we need:

- a function `Dual`, defining duals on objects.

In the following, two families of such sets I are introduced:

- `GIrreducibleObject`: For a group G , the set I consists of the irreducible characters of G . We call the elements in I the G -irreducible objects.
- `GZGradedIrreducibleObject`: For a group G , the set I consists of the irreducible characters of G together with a degree $n \in \mathbb{Z}$. We call the elements in I the $G - \mathbb{Z}$ -irreducible objects.

3.2 Constructors

3.2.1 `GIrreducibleObject` (for `IsCharacter`)

▷ `GIrreducibleObject(c)` (attribute)

Returns: a G -irreducible object

The argument is a character c of a group. The output is its associated G -irreducible object.

3.2.2 GZGradedIrreducibleObject (for IsInt, IsCharacter)

▷ `GZGradedIrreducibleObject(n, c)` (operation)

Returns: a $G - \mathbb{Z}$ -irreducible object

The argument is an integer n and a character c of a group. The output is their associated $G - \mathbb{Z}$ -irreducible object.

3.3 Attributes

3.3.1 UnderlyingCharacter (for IsGIrreducibleObject)

▷ `UnderlyingCharacter(i)` (attribute)

Returns: an irreducible character

The argument is a G -irreducible object i . The output is its underlying character.

3.3.2 UnderlyingGroup (for IsGIrreducibleObject)

▷ `UnderlyingGroup(i)` (attribute)

Returns: a group

The argument is a G -irreducible object i . The output is its underlying group.

3.3.3 UnderlyingCharacterTable (for IsGIrreducibleObject)

▷ `UnderlyingCharacterTable(i)` (attribute)

Returns: a character table

The argument is a G -irreducible object i . The output is the character table of its underlying group.

3.3.4 UnderlyingIrreducibleCharacters (for IsGIrreducibleObject)

▷ `UnderlyingIrreducibleCharacters(i)` (attribute)

Returns: a list

The argument is a G -irreducible object i . The output is a list consisting of the irreducible characters of its underlying group.

3.3.5 UnderlyingCharacterNumber (for IsGIrreducibleObject)

▷ `UnderlyingCharacterNumber(i)` (attribute)

Returns: an integer

The argument is a G -irreducible object i . The output is the integer n such that the n -th entry of the list of the underlying irreducible characters is the underlying irreducible character of i .

3.3.6 Dimension (for IsGIrreducibleObject)

▷ `Dimension(i)` (attribute)

Returns: an integer

The argument is a G -irreducible object i . The output is the dimension of its underlying irreducible character.

3.3.7 Dual (for IsGIrreducibleObject)

▷ `Dual(i)` (attribute)

Returns: a G -irreducible object

The argument is a G -irreducible object i . The output is the G -irreducible object associated to the dual character of c , where c is the associated character of i .

3.3.8 UnderlyingCharacter (for IsGZGradedIrreducibleObject)

▷ `UnderlyingCharacter(i)` (attribute)

Returns: an irreducible character

The argument is a $G - \mathbb{Z}$ -irreducible object i . The output is its underlying character.

3.3.9 UnderlyingDegree (for IsGZGradedIrreducibleObject)

▷ `UnderlyingDegree(i)` (attribute)

Returns: an integer

The argument is a $G - \mathbb{Z}$ -irreducible object i . The output is its underlying degree.

3.3.10 UnderlyingGroup (for IsGZGradedIrreducibleObject)

▷ `UnderlyingGroup(i)` (attribute)

Returns: a group

The argument is a $G - \mathbb{Z}$ -irreducible object i . The output is its underlying group.

3.3.11 UnderlyingCharacterTable (for IsGZGradedIrreducibleObject)

▷ `UnderlyingCharacterTable(i)` (attribute)

Returns: a character table

The argument is a $G - \mathbb{Z}$ -irreducible object i . The output is the character table of its underlying group.

3.3.12 UnderlyingIrreducibleCharacters (for IsGZGradedIrreducibleObject)

▷ `UnderlyingIrreducibleCharacters(i)` (attribute)

Returns: a list

The argument is a $G - \mathbb{Z}$ -irreducible object i . The output is a list consisting of the irreducible characters of its underlying group.

3.3.13 UnderlyingCharacterNumber (for IsGZGradedIrreducibleObject)

▷ `UnderlyingCharacterNumber(i)` (attribute)

Returns: an integer

The argument is a $G - \mathbb{Z}$ -irreducible object i . The output is the integer n such that the n -th entry of the list of the underlying irreducible characters is the underlying irreducible character of i .

3.3.14 Dimension (for IsGZGradedIrreducibleObject)

▷ `Dimension(i)` (attribute)

Returns: an integer

The argument is a $G - \mathbb{Z}$ -irreducible object i . The output is the dimension of its underlying irreducible character.

3.3.15 Dual (for IsGZGradedIrreducibleObject)

▷ `Dual(i)` (attribute)

Returns: a $G - \mathbb{Z}$ -irreducible object

The argument is a $G - \mathbb{Z}$ -irreducible object i . The output is the $G - \mathbb{Z}$ -irreducible object associated to the degree $-n$ and the dual character of c , where n is the underlying degree and c is the underlying character of i .

3.4 Properties

3.4.1 IsYieldingIdentities (for IsGIrreducibleObject)

▷ `IsYieldingIdentities(i)` (property)

Returns: a boolean

The argument is a G -irreducible object i . The output is true if the underlying character of i is the trivial one, false otherwise.

3.4.2 IsYieldingIdentities (for IsGZGradedIrreducibleObject)

▷ `IsYieldingIdentities(i)` (property)

Returns: a boolean

The argument is a $G - \mathbb{Z}$ -irreducible object i . The output is true if the underlying character of i is the trivial one, false otherwise.

3.5 Operations

3.5.1 Multiplicity (for IsGIrreducibleObject, IsGIrreducibleObject, IsGIrreducibleObject)

▷ `Multiplicity(i, j, k)` (operation)

Returns: an integer

The arguments are 3 G -irreducible objects i, j, k . Let their underlying characters be denoted by a, b, c , respectively. Then the output is the number $\langle a, b \cdot c \rangle$, i.e., the multiplicity of a in the product of characters $b \cdot c$.

3.5.2 * (for IsGIrreducibleObject, IsGIrreducibleObject)

▷ `*(i, j)` (operation)

Returns: a list

The arguments are 2 G -irreducible objects i, j with underlying irreducible characters a, b , respectively. The output is a list $L = [[n_1, k_1], \dots, [n_l, k_l]]$ consisting of positive integers n_c and G -irreducible objects k_c representing the character decomposition into irreducibles of the product $a \cdot b$.

3.5.3 AssociatorFromData (for IsGIrreducibleObject, IsGIrreducibleObject, IsGIrreducibleObject, IsList, IsFieldForHomalg, IsList)

▷ `AssociatorFromData(i, j, k, A, F, L)` (operation)

Returns: a list

The arguments are

- three G -irreducible objects i, j, k ,
- a list A containing the associator on all irreducibles as strings, e.g., the list constructed by the methods provided in this package,
- a homalg field F ,
- a list $L = [[n_1, h_1], \dots, [n_l, h_l]]$ consisting of positive integers n_c and G -irreducible objects h_c representing the character decomposition into irreducibles of the product of i, j, k .

The output is the list $[[\alpha_{h_1}, h_1], \dots, [\alpha_{h_l}, h_l]]$, where α_{h_c} is the F -vector space homomorphism representing the h_c -th component of the associator of i, j, k .

3.5.4 ExteriorPower (for IsGIrreducibleObject, IsGIrreducibleObject)

▷ `ExteriorPower(i, j)` (operation)

Returns: a list

The arguments are two G -irreducible objects i, j . The output is the empty list if i is not equal to j . Otherwise, the output is a list $L = [[n_1, k_1], \dots, [n_l, k_l]]$ consisting of positive integers n_j and G -irreducible objects k_j , corresponding to the decomposition of the second exterior power character $\wedge^2 c$ into irreducibles. Here, c is the associated character of i .

3.5.5 Multiplicity (for IsGZGradedIrreducibleObject, IsGZGradedIrreducibleObject, IsGZGradedIrreducibleObject)

▷ `Multiplicity(i, j, k)` (operation)

Returns: an integer

The arguments are 3 $G - \mathbb{Z}$ -irreducible objects i, j, k . Let their underlying characters be denoted by a, b, c , respectively, and their underlying degrees by n_i, n_j, n_k , respectively. The output is 0 if n_i is not equal to $n_j + n_k$. Otherwise, the output is the number $\langle a, b \cdot c \rangle$, i.e., the multiplicity of a in the product of characters $b \cdot c$.

Let their underlying characters be denoted by a, b , respectively, and their underlying degrees by n_i, n_j , respectively. if $n_i = n_j$ and the underlying character number of j

3.5.6 * (for IsGZGradedIrreducibleObject, IsGZGradedIrreducibleObject)

▷ `*(i, j)` (operation)

Returns: a list

The arguments are 2 $G - \mathbb{Z}$ -irreducible objects i, j with underlying irreducible characters a, b , respectively. The output is a list $L = [[n_1, k_1], \dots, [n_l, k_l]]$ consisting of positive integers n_c and G -irreducible objects k_c representing the character decomposition into irreducibles of the product $a \cdot b$. The underlying degrees of k_c are given by the sum of the underlying degrees of i and j .

3.5.7 AssociatorFromData (for IsGZGradedIrreducibleObject, IsGZGradedIrreducibleObject, IsList, IsFieldForHomalg, IsList)

▷ AssociatorFromData(i, j, k, A, F, L) (operation)

Returns: a list

The arguments are

- three $G - \mathbb{Z}$ -irreducible objects i, j, k ,
- a list A containing the associator on all irreducibles (of G -irreducible objects) as strings, e.g., the list constructed by the methods provided in this package,
- a homalg field F ,
- a list $L = [[n_1, h_1], \dots, [n_l, h_l]]$ consisting of positive integers n_c and $G - \mathbb{Z}$ -irreducible objects h_c representing the character decomposition into irreducibles of the product of i, j, k .

The output is the list $[[\alpha_{h_1}, h_1], \dots, [\alpha_{h_l}, h_l]]$, where α_{h_c} is the F -vector space homomorphism representing the h_c -th component of the associator of i, j, k .

3.5.8 ExteriorPower (for IsGZGradedIrreducibleObject, IsGZGradedIrreducibleObject)

▷ ExteriorPower(i, j) (operation)

Returns: a list

The arguments are two $G - \mathbb{Z}$ -irreducible objects i, j . The output is the empty list if the underlying characters of i and j are unequal. Otherwise, the output is a list $L = [[n_1, k_1], \dots, [n_l, k_l]]$ consisting of positive integers n_j and $G - \mathbb{Z}$ -irreducible objects k_a , corresponding to the decomposition of the second exterior power character $\wedge^2 c$ into irreducibles. Here, c is the associated character of i and j . The underlying degree of k_a is the sum of the underlying degrees of i and j .

Chapter 4

Representation Category of Groups

4.1 Introduction

For a finite group G , the following methods provide computational tools for working with $G\text{-mod}$, a skeletal version of the monoidal category of finite dimensional complex representations of G , and with $G - \mathbb{Z}\text{-mod}$, a skeletal version of the monoidal category of finite dimensional complex representations of G equipped with a degree in \mathbb{Z} .

4.2 Quickstart

The following commands construct the category $D_8\text{-mod}$, the unique object v corresponding to the irreducible character of degree 2, and perform some computations.

```
Example
gap> RepG := RepresentationCategory( 8, 3 );
The representation category of Group( [ f1, f2, f3 ] )
gap> G := UnderlyingGroupForRepresentationCategory( RepG );
<pc group of size 8 with 3 generators>
gap> StructureDescription( G );
"D8"
gap> c := First( Irr( G ), i -> Degree( i ) = 2 );
Character( CharacterTable( D8 ), [ 2, 0, 0, -2, 0 ] )
gap> v := RepresentationCategoryObject( c, RepG );
1*(x_5)
gap> Dimension( v );
2
gap> Display( AssociatorLeftToRight( v, v, v ) );
Component: (x_5)

1/2,-1/2,1/2, 1/2,
1/2,-1/2,-1/2,-1/2,
1/2,1/2, 1/2, -1/2,
1/2,1/2, -1/2,1/2

A morphism in Category of matrices over Q
-----
gap> Display( Braiding( v, v ) );
Component: (x_1)
```

```

1
A morphism in Category of matrices over Q
-----
Component: (x_2)

1
A morphism in Category of matrices over Q
-----
Component: (x_3)

1
A morphism in Category of matrices over Q
-----
Component: (x_4)

-1
A morphism in Category of matrices over Q
-----
gap> alpha := IdentityMorphism( TensorProductOnObjects( v, v ) ) + Braiding( v, v );
<A morphism in The representation category of Group( [ f1, f2, f3 ] )>
gap> CokernelObject( alpha );
1*(x_4)
gap> TensorUnit( RepG );
1*(x_1)

```

4.3 Constructors

4.3.1 RepresentationCategory (for IsGroup)

▷ RepresentationCategory(G) (attribute)

Returns: a Cap category

The argument is a group G . The output is the Cap category G -mod. This method uses String(G) as an identifier of G .

4.3.2 RepresentationCategory (for IsInt, IsInt)

▷ RepresentationCategory(o, n) (operation)

Returns: a Cap category

The arguments are 2 integers o, n . The output is the Cap category G -mod, where G is the group of order o corresponding to the SmallGroupLibrary identification number n .

4.3.3 RepresentationCategoryObject (for IsList, IsCapCategory)

▷ RepresentationCategoryObject(L, C) (operation)

Returns: an object in G -mod

There are 2 arguments. The first argument is a list $L = [[n_1, c_1], \dots, [n_l, c_l]]$ consisting of non-negative integers n_i and characters c_i of the same group. Alternatively, the first argument can simply be an irreducible character c , which will be then interpreted as giving the input $[[1, c]]$. The second argument is the Cap category $C = G\text{-mod}$. The output is the unique object in $G\text{-mod}$ having L as its character decomposition.

Index

- AffordAllIrreducibleRepresentations
 - for IsGroup, [8](#)
- AffordAllIrreducibleRepresentations-Dixon
 - for IsGroup, [8](#)
- AssociatorDataFromSkeletalFunctorTensorData
 - for IsInt, IsInt, IsInt, IsList, [6](#)
- AssociatorForSufficientlyManyTriples, [6](#)
 - for IsList, IsBool, [6](#)
- AssociatorFromData
 - for IsGIrreducibleObject, IsGIrreducibleObject, IsGIrreducibleObject, IsList, IsFieldForHomalg, IsList, [21](#)
 - for IsGZGradedIrreducibleObject, IsGZGradedIrreducibleObject, IsGZGradedIrreducibleObject, IsList, IsFieldForHomalg, IsList, [22](#)
- AsVectorSpaceMorphism
 - for IsHomalgMatrix, [9](#)
- *
 - for IsGIrreducibleObject, IsGIrreducibleObject, [20](#)
 - for IsGZGradedIrreducibleObject, IsGZGradedIrreducibleObject, [21](#)
- Component
 - for IsSemisimpleCategoryMorphism, IsObject, [14](#)
 - for IsSemisimpleCategoryObject, IsObject, [14](#)
- ComponentInclusionMorphism
 - for IsSemisimpleCategoryObject, IsObject, [11](#)
- ComponentProjectionMorphism
 - for IsSemisimpleCategoryObject, IsObject, [12](#)
- ComputeAssociator
 - for IsGroup, IsBool, [7](#)
 - for IsGroup, IsBool, IsBool, [7](#)
 - for IsGroup, IsBool, IsBool, IsBool, [7](#)
- CreateEndomorphismFromString
 - for IsVectorSpaceObject, IsString, [9](#)
- DataFromSkeletalFunctorTensorDataAsStringList
 - for IsList, [9](#)
- DefaultFieldForListOfRepresentations
 - for IsList, [8](#)
- DefinedOverCyclotomicField
 - for IsInt, IsGroupHomomorphism, [7](#)
- DiagonalizationTransformationOfBraiding
 - for IsVectorSpaceMorphism, [8](#)
- Dimension
 - for IsGIrreducibleObject, [18](#)
 - for IsGZGradedIrreducibleObject, [20](#)
 - for IsSemisimpleCategoryObject, [14](#)
- Dual
 - for IsGIrreducibleObject, [19](#)
 - for IsGZGradedIrreducibleObject, [20](#)
- ExteriorPower
 - for IsGIrreducibleObject, IsGIrreducibleObject, [21](#)
 - for IsGZGradedIrreducibleObject, IsGZGradedIrreducibleObject, [22](#)
- GIrreducibleObject
 - for IsCharacter, [17](#)
- GivenMorphismFilterForSemisimpleCategory
 - for IsSemisimpleCategory, [13](#)
- GivenObjectFilterForSemisimpleCategory
 - for IsSemisimpleCategory, [13](#)
- GroupReperesentationByImages
 - for IsGroup, IsList, [8](#)
- GZGradedIrreducibleObject

- for IsInt, IsCharacter, [18](#)
- HomalgMatrixAsString
 - for IsHomalgMatrix, [9](#)
- InitializeGroupData
 - for IsGroup, [5](#)
 - for IsGroup, IsBool, [5](#)
 - for IsGroup, IsList, IsBool, [6](#)
- InitializeGroupDataDixon
 - for IsGroup, [5](#)
 - for IsGroup, IsBool, [5](#)
- InternalHomToTensorProductAdjunction-MapTemp
 - for IsVectorSpaceObject, IsVectorSpaceObject, IsVectorSpaceMorphism, [8](#)
- IsSemisimpleCategoryMorphism
 - for IsCapCategoryMorphism and IsCellOfSkeletalCategory, [16](#)
- IsSemisimpleCategoryObject
 - for IsCapCategoryObject and IsCellOfSkeletalCategory, [16](#)
- IsYieldingIdentities
 - for IsGIRreducibleObject, [20](#)
 - for IsGZGradedIrreducibleObject, [20](#)
- MembershipFunctionForSemisimpleCategory
 - for IsSemisimpleCategory, [12](#)
- Multiplicity
 - for IsGIRreducibleObject, IsGIRreducibleObject, IsGIRreducibleObject, [20](#)
 - for IsGZGradedIrreducibleObject, IsGZGradedIrreducibleObject, IsGZGradedIrreducibleObject, [21](#)
 - for IsSemisimpleCategoryObject, IsObject, [14](#)
- NormalizeSemisimpleCategoryObjectList
 - for IsList, [14](#)
- ReadDatabaseKeys
 - for IsString, [4](#)
- ReadRepresentationsData
 - for IsString, IsString, [5](#)
- ReadSkeletalFunctorData
 - for IsString, IsString, [5](#)
- RepresentationCategory
 - for IsGroup, [24](#)
 - for IsInt, IsInt, [24](#)
- RepresentationCategoryObject
 - for IsList, IsCapCategory, [24](#)
- RewriteMatrixInCyclotomicGenerator
 - for IsMatrix, IsInt, [8](#)
- SemisimpleCategory
 - for IsFieldForHomalg, IsFunction, IsObject, IsString, IsBool, [11](#)
 - for IsFieldForHomalg, IsFunction, IsObject, IsString, IsBool, IsList, [10](#)
- SemisimpleCategoryMorphism
 - for IsSemisimpleCategoryObject, IsList, IsSemisimpleCategoryObject, [11](#)
- SemisimpleCategoryMorphismList
 - for IsSemisimpleCategoryMorphism, [13](#)
- SemisimpleCategoryMorphismSparse
 - for IsSemisimpleCategoryObject, IsList, IsSemisimpleCategoryObject, [11](#)
- SemisimpleCategoryObject
 - for IsList, IsCapCategory, [12](#)
- SemisimpleCategoryObjectConstructor-WithFlatList
 - for IsList, IsCapCategory, [12](#)
- SemisimpleCategoryObjectList
 - for IsSemisimpleCategoryObject, [13](#)
- SemisimpleCategoryObjectListWithActualObjects
 - for IsSemisimpleCategoryObject, [13](#)
- SetInfoLevelForAssociatorComputations
 - for IsInt, [7](#)
- SkeletalFunctorTensorData, [6](#)
 - for IsList, [6](#)
- Support
 - for IsSemisimpleCategoryObject, [13](#)
- TestBraidingCompatability
 - for IsSemisimpleCategoryObject, IsSemisimpleCategoryObject, IsSemisimpleCategoryObject, [15](#)
- TestBraidingCompatabilityForAllTriplesInList
 - for IsList, [15](#)
- TestPentagonIdentity
 - for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, [15](#)

- CategoryObject, [15](#)
- TestPentagonIdentityForAllQuadruples-
InList
 - for IsList, [15](#)
- TestZigZagIdentitiesForDual
 - for IsSemisimpleCategoryObject, [15](#)
- TestZigZagIdentitiesForDualForAll-
ObjectsInList
 - for IsList, [15](#)
- UnderlyingCategoryForSemisimple-
Category
 - for IsSemisimpleCategory, [12](#)
- UnderlyingCharacter
 - for IsGIrreducibleObject, [18](#)
 - for IsGZGradedIrreducibleObject, [19](#)
- UnderlyingCharacterNumber
 - for IsGIrreducibleObject, [18](#)
 - for IsGZGradedIrreducibleObject, [19](#)
- UnderlyingCharacterTable
 - for IsGIrreducibleObject, [18](#)
 - for IsGZGradedIrreducibleObject, [19](#)
- UnderlyingDegree
 - for IsGZGradedIrreducibleObject, [19](#)
- UnderlyingFieldForHomalg
 - for IsSemisimpleCategoryMorphism, [13](#)
 - for IsSemisimpleCategoryObject, [14](#)
- UnderlyingFieldForHomalgForSemisimple-
Category
 - for IsSemisimpleCategory, [12](#)
- UnderlyingGroup
 - for IsGIrreducibleObject, [18](#)
 - for IsGZGradedIrreducibleObject, [19](#)
- UnderlyingIrreducibleCharacters
 - for IsGIrreducibleObject, [18](#)
 - for IsGZGradedIrreducibleObject, [19](#)
- WriteAssociatorComputationToFiles
 - for IsString, [4](#)
- WriteAssociatorDataToFile
 - for IsString, [4](#)
- WriteDatabaseKeysToFile
 - for IsString, [4](#)
- WriteRepresentationsDataToFile
 - for IsString, [4](#)
- WriteSkeletalFunctorDataToFile
 - for IsString, [4](#)