## The homalg project and its related packages

The homalg project authors

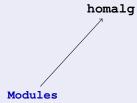
2007-2020



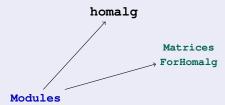
The idea: A homological algebra meta-package for computable ABELian categories

## homalg

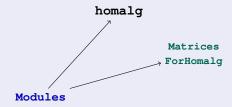
The category of finitely presented modules as the basic example of a computable ABELian category



Matrices provide the needed data structure for finitely presented modules and their morphisms

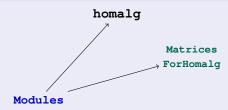


## Candidates: There are several systems that could host homalg



Maple MAGMA Macaulay2 Sage GAP SINGULAR

Candidates: There are several systems that could host homalg, each supporting certain kinds of rings





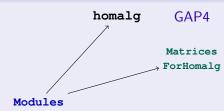






$$\begin{array}{ccc} \mathsf{GAP} & \mathsf{SINGULAR} \\ \downarrow & & \downarrow \\ \mathbb{Z} & & \mathbb{F}[x], \\ & & \mathbb{F}[x, \partial], \end{array}$$

GAP4: The best suited language for abstract mathematical programming



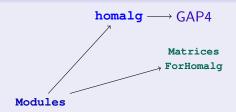






GAP SINGULAR 
$$\downarrow$$
  $\downarrow$   $\mathbb{F}[x]$ ,

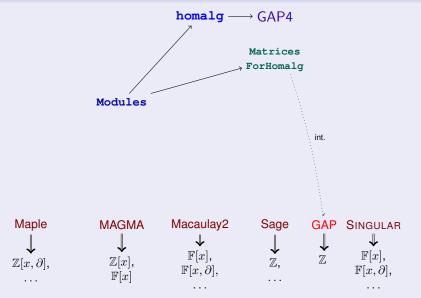
 $\mathbb{F}[x,\partial],$ 



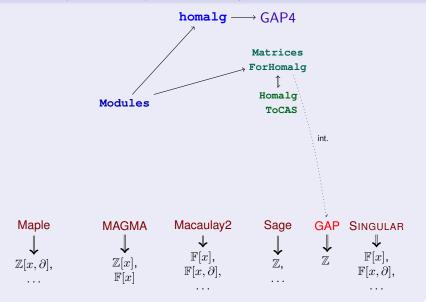


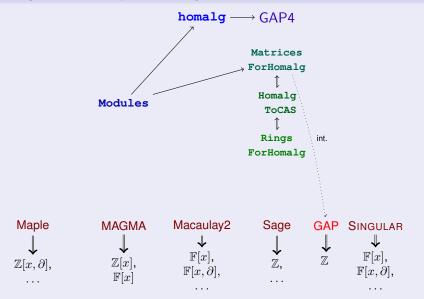
 $\bigcup_{\mathbb{Z}[x],}^{\mathbb{Z}[x]}$ 

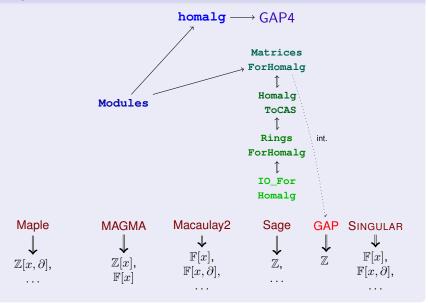
Sage ↓ ℤ,  $\begin{array}{ccc} \mathsf{GAP} & \mathsf{SINGULAR} \\ \downarrow & & \downarrow \\ \mathbb{Z} & & \mathbb{F}[x], \\ & & \mathbb{F}[x, \partial], \end{array}$ 

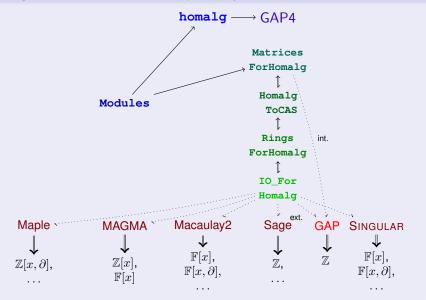


HomalgToCAS: External objects and the GAP4-representations: external rings and external matrices

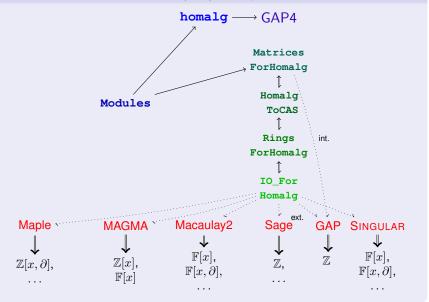


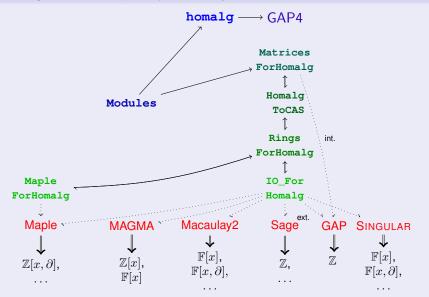




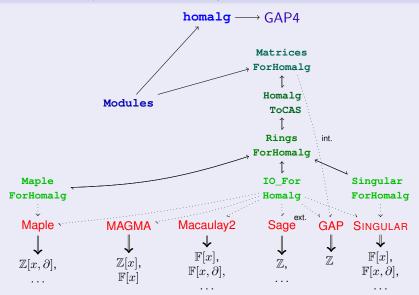


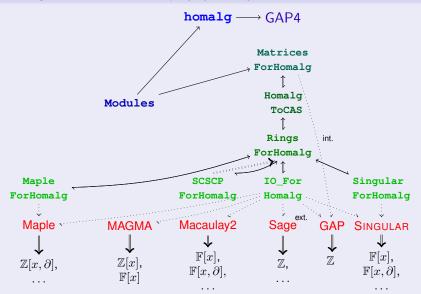
## External CASs host the matrices and GAP4 contains the higher logic $\rightarrow$ *Principle of least communication*

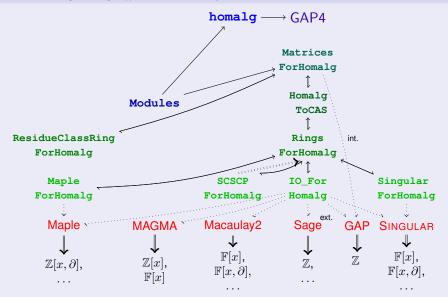




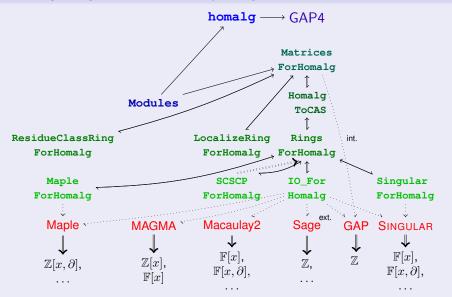
Future: Communicate with interpreters of various CASs shortcutting their command line interface.



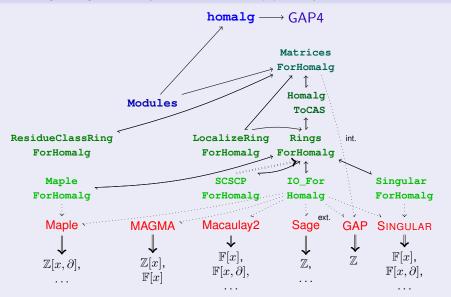


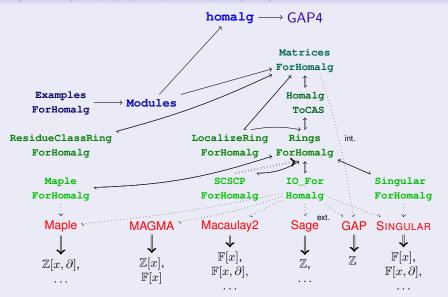


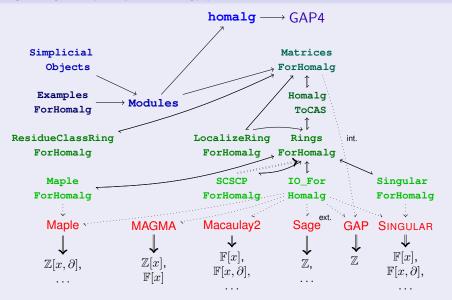
LocalizeRingForHomalg: Localizations of commutative rings in homalg at maximal ideals.

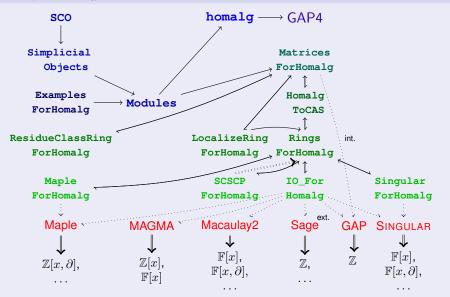


LocalizeRingForHomalg: Use MORA's algorithm in SINGULAR to localize polynomial rings at maximal ideals.





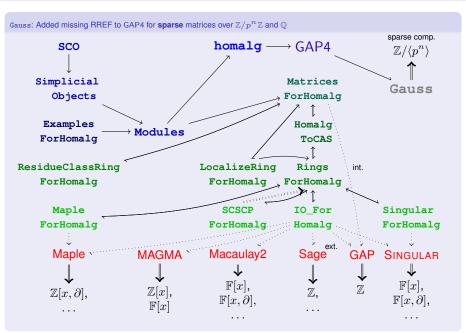


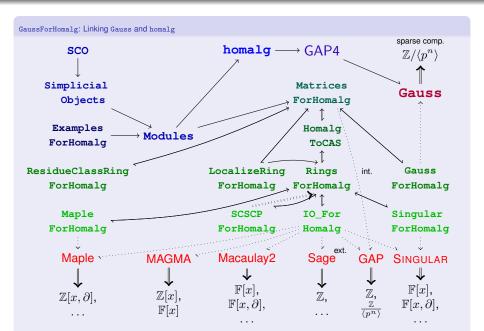


 $\mathbb{F}[x,\partial],$ 

 $\mathbb{F}[x,\partial]$ ,

 $\mathbb{F}[x]$ 





GradedRingForHomalg: Multi-graded rings serve as the data structure underlying many geometric constructions

