

Assignment: Module 11

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Disclaimer: This is my work, not that of others

Total Score: 40

1. 10
2. 10
3. 10
4. 10

1. (10 pt) Use a 5th order interpolating polynomial to estimate y at $x = 3.5$. In your answer, please show the interpolating polynomial. You can use Python library to find the interpolating polynomial.

x 0 1.8 5 6 8.2 9.2

y 2.6 16.415 5.375 3.5 2.015 2.54

When using python's polyfit, the result comes out as

5.74407918e-03, -1.73471191e-01, 1.99870979e+00, -1.04592064e+01
2.09771368e+01 2.60000000e+00

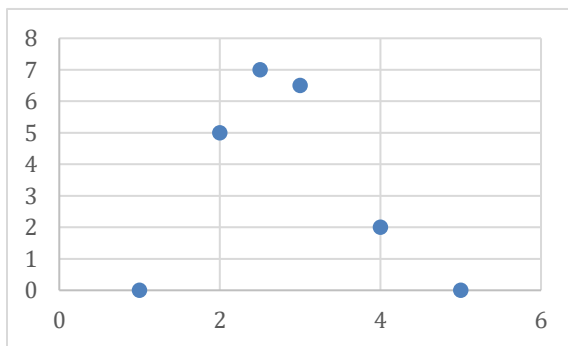
$5.74407918e-03 x^5 - 1.73471191e-01 x^4 + 1.99870979 x^3 - 0.104592064 x^2$
 $+ 20.9771368 x + 2.60000000$

2. (10 pt) Given the data:

x 1 2 2.5 3 4 5

y=f(x) 0 5 7 6.5 2 0

a. Plot the data and estimate $f(3.4)$ from the plot. (You can use excel to plot the data.)



b. Calculate $f(3.4)$ using Newton Interpolating polynomial of order 1. (Do this by hand. Also, you only need two data point for order 1 interpolation. Make sure you pick the right points)

$$7 - 5 / 3 - 3.5$$

$$= 2 / 0.5 = 4$$

$$7 + 4(x - 3.5)$$

$$f(3.4) = 7 + 4((3.4) - 3.5) = 6.6$$

c. Calculate $f(3.4)$ using Newton Interpolating polynomial of order 5. (Do this by hand. Use finite divided differences to find the coefficients for each Newton interpolating polynomial.)

1	0					
		5				
2	5		-0.66667			
		4		-2.16667		
2.5	7		-5		1.16667	
		-1		1.333333		-0.28333
3	6.5		-2.33333		0.033333	
		-4.5		1.433333		
4	2		1.25			
		-2				
5	0					

$$f(x) = 0 + 5(x-1) - 0.6667(x-1)(x-2) - 2.1666(x-1)(x-2)(x-2.5) + 1.1667(x-1)(x-2)(x-2.5)(x-3) - 0.2833(x-1)(x-2)(x-2.5)(x-3)(x-4)$$

$$f(3.4) = 4.82$$

3. (10 pt) Given the data:

x 1 2 3 5 6

y=f(x) 4.75 4 5.25 19.75 36

Estimate $f(4)$ using Lagrange polynomials of order one and three. (Do this by hand.)

$$f(x) = (x-5)/(3-5) * 5.25 + (x-3)/(5-3) * 19.75$$

$$f(4) = 2.625 + 9.875 = 12.5$$

order3 =

$$4(x-3)(x-5)(x-6)/(2-3)(2-5)(2-6) + 5.25(x-2)(x-5)(x-6)/(3-2)(3-5)(3-6) + 19.75(x-2)(x-3)(x-6)/(5-2)(5-3)(5-6) + 36(x-2)(x-3)(x-5)/(6-2)(6-3)(6-5)$$

$$4(x-3)(x-5)(x-6)/-12 + 5.25(x-2)(x-5)(x-6)/6 + 19.75(x-2)(x-3)(x-6)/-6 + 36(x-2)(x-3)(x-5)/12$$

$$f(4) = 10$$

5. (10 pt) Given the data.

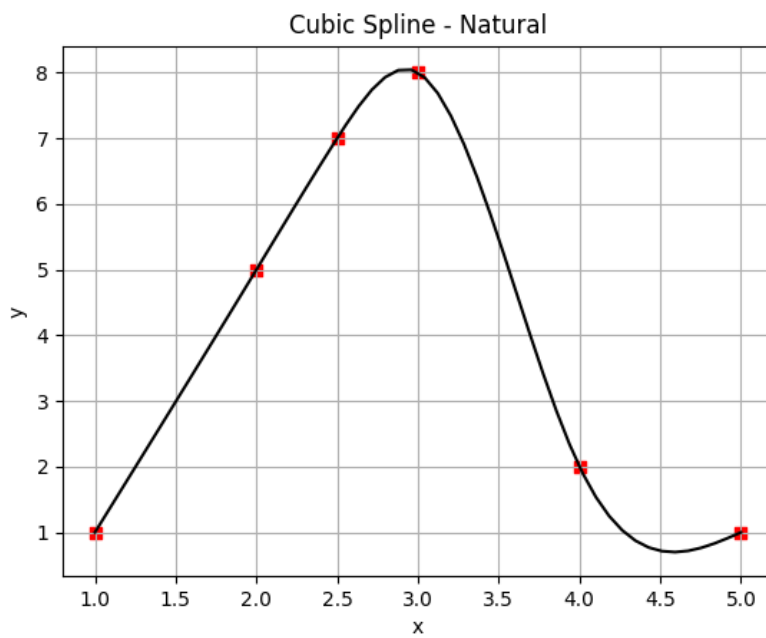
x 1 2 2.5 3 4 5

y 1 5 7 8 2 1

Fit these data with (a) cubic splines with natural end conditions, (b) cubic splines with not-a-knot end conditions. Present comparative plots of 50 equally spaced interpolation points over the domain $1 \leq x \leq 5$. You may use Python for this problem. However, make sure you understand the definition of cubic spline and the various end conditions.

A

when python is used to solve natural spline, which has errors due to round off error



B

when python is used to solve natural spline, which has less errors due to round off error

