Probability and Statistics (ECE 3710)

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HW 2

Reading: **Chapter 2** of our textbook, presentation slides and lecture notes on Canvas.

Q1. **(10 Points)** Let S be the event that a randomly selected college student has taken a statistics course, and let C be the event that the same student has taken a chemistry course. Suppose $P(S) = 0.4$, $P(C) = 0.3$, and $P(S \cap C) = 0.2$.

a. Find the probability that a student has taken statistics, chemistry, or both.

b. Find the probability that a student has taken neither statistics nor chemistry.

Q2. (10 Points) Let A and B be events with $P(A) = 0.8$ and $P(A \cap B) = 0.2$. For what value of $P(B)$ will A and B be independent?

Q3. (20 Points) A lot of 10 components contains 3 that are defective. Two components are drawn at random and tested. Let A be the event that the first component drawn is defective and let B be the event that the second component drawn is defective.

a. Find p(A).

b. Find p(B|A).

c. Find p(A ∩B).

d. Find p(A^c ∩B).

e. Find p(B).

f. Are A and B independent? Explain.

Q4. (10 Points) Computer chips often contain surface imperfections. For a certain type of computer chip, the probability mass function of the number of defects X is presented in the following table.

x	0	1	2	3	4
$p(x)$	0.4	0.3	0.15	0.10	0.05

a. Find p(X ≤ 2).

b. Find p(X > 1).

c. Find the mean μ_x.

Q5. (20 Points) A computer sends a packet of information along a channel and waits for a return signal acknowledging that the packet has been received. If no acknowledgment is received within a certain time, the packet is re-sent. Let X represent the number of times the packet is sent. Assume that the probability mass function of X is given by

$$p(x) = \begin{cases} cx & \text{for } x = 1, 2, 3, 4, \text{ or } 5 \\ 0 & \text{otherwise} \end{cases}$$

where c is a constant.

a. Find the value of the constant c so that p(x) is a probability mass function.

b. Find p(X = 2).

c. Find the mean number of times the packet is sent.

d. Find the variance of the number of times the packet is sent.

e. Find the cumulative distribution function of the lifetime.

f. Find the median lifetime.

g. Find the 60th percentile of the lifetimes.

Q6. (20 Points) A computer fan's lifetime (in years) is modeled by a continuous random variable with the following probability distribution function.

$$f(x) = \begin{cases} cx & 0 < x < 10 \\ 0 & otherwise \end{cases}$$

where c is a constant.

a. Find the value of the constant c so that f(x) is a probability distribution function.

b. Find p(X > 2).

c. Find the mean of the fan's lifetime.

d. Find the median lifetime.

e. Find the 60th percentile of the lifetime.

Q7. (10 points) If X and Y are _independent_ random variables with means $\mu_x = 9.5$ and $\mu_y = 6.8$, and standard deviations $\sigma_x = 0.4$ and $\sigma_y = 0.1$, find the means and standard deviations of the following:

a. 3X

b. Y − X

c. X + 4Y

Good luck