Assignment: Module 9 Name: Hoyoung kim
Disclaimer: This is my work, not that of others
Total Score: 55

1. 25
2. 10
3. 10
4. 10

1. The following system of equations is designed to determine the concentrations (c's in g/m3
) in a series of coupled, well-mixed tanks as a function of mass input to each tank. The right-hand side of the equations below represent these inputs in g/day.

$15c1 - 3c2 - c3 = 4000$
$-3c1 + 18c2 - 6c3 = 1200$
$-4c1 - c2 + 12c3 = 2350$

a. (5 pt) Determine the inverse of the coefficient matrix. (You can use Python to solve the inverse.)

```
import numpy as np
x = np.matrix('15, -3, -1;-3, 18, -6;-4,-1,12')
x = np.linalg.inv(x)
print(x)
```

[[0.07253886 0.01278066 0.01243523]
[0.02072539 0.06079447 0.03212435]
[0.02590674 0.00932642 0.09015544]]

b. (10 pt) Use the inverse to determine the solution. (Do this by hand.)

[[0.07253886 0.01278066 0.01243523] [4000]
[0.02072539 0.06079447 0.03212435] [1200]
[0.02590674 0.00932642 0.09015544]] [2350]

[334.7150
231.3471
326.683948]

c. (10 pt) Determine how much the rate of mass input to tank 3 must increase to induce a 10 g/m3
0.01243523 * x = 10
X = 804.16687
rise in the concentration in tank 1.

2. (10 pt) Determine $\|A\|f$, $\|A\|1$
, and $\|A\|\infty$ for
A =

8	2	−10
−9	1	3
15	−1	6

$\|A\|f$ = sqrt (EA^2) 521
$\|A\|1 = \text{max sum of each column } 32$
$\|A\|\infty = \text{max sum of each row } 22$

3. (10 pt) Solve the following system using three iterations of the Gauss-Seidel method. If necessary, rearrange the equations. Show all the steps in your solution. At the end of your computation, compute the true error of your final results. (Do this by hand.)

$7x1 - x2 = 5$ (5 + x2)/7 = x1
$3x1 + 8x2 = 11$ (11- 3x1)/8 = x2

7 1 = x1 (5 + 0)/7 = x1… x1 = 5/7. (11 - 3(5/7))/8 = 1.107142871
3 8 = x2 (5 + 1.107142871)/7 = 0.872448981571 (11- 3x1)/8 = 1.04783163191
(5 + 1.107142871)/7 = 0.863975947416. (11- 3x1)/8 = 1.05100901972

Relative error
(0.872448981571 - 0.863975947416) /0.863975947416 = 0.009807…
(1.04783163191- 1.05100901972)/ 1.05100901972 = −0.0030231…

Realistic error
By using substitution, we find that actual value of x1 is 0.8644067
By using substitution, we find that actual value of x2 is 1.0508474
(0.863975947416-0.8644067)/0.8644067 is 0.000498321662708
(1.05100901972-1.0508474)/ 1.0508474 is 0.000153799419402

4. (10 pt) Use the Gauss-Seidel method (a) without relaxation and (b) with relaxation (λ = 1.2) to solve the following set of linear equation to meet an error tolerance of εs = 5%. If necessary, rearrange the equations to achieve convergence.
$2x_1 - 6x_2 - x_3 = -38$
$-3x_1 - x_2 + 7x_3 = -34$
$-8x_1 + x_2 - 2x_3 = -20$

$-8x_1 + x_2 - 2x_3 = -20$ x1 = (-20 +2x3-x2)/-8
$2x_1 - 6x_2 - x_3 = -38$ x2 = (-38+x3-2x1)/-6
$-3x_1 - x_2 + 7x_3 = -34$ x3 = (-34+3x1+x2)/7

x1 = (-20 +2(0)-(0))/-8 = 2.5
x2 = (-38+x3-2x1)/-6

Iteration	x1	x2	x3			
0	0	0	0			
1	2.5	7.16666667	-2.7619048	100%	100%	100%
2	4.08630952	8.15575397	-1.9407596	39%	12%	42%
3	4.00465916	7.99167966	-1.9991918	2.00%	2.10%	2.90%
4	3.99875792	7.99945128	-2.0006107	0.10%	0.10%	0.10%