Final practice:

Q1. $\begin{cases} \hat{\mu}_x = \bar{X} = 101.5\,\Omega \\ \hat{\delta}^2_x = 25 \end{cases}$

~~α~~ 0.05 : Measure for p-value threshold.

$\mu_0 = 100\,\Omega$

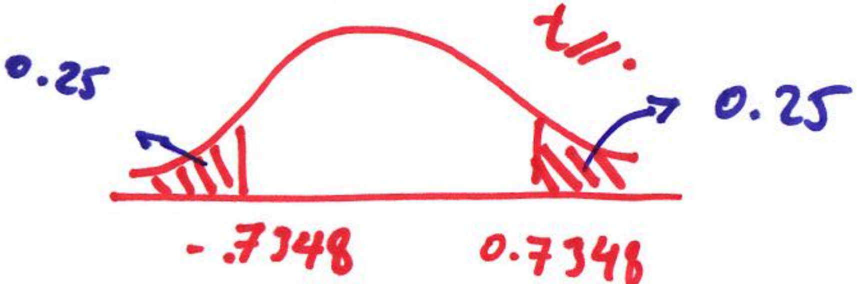
a). $n=6$, $\mathcal{H}_0 : \mu = 100$ vs. $\mathcal{H}_1 : \mu \neq 100$

$\mathcal{H}_0$: Assume $\mathcal{H}_0$ is true.

$n = 6 < 30$: small sample size $\rightarrow$ t-table. / DoF $\nu = n-1 = \underline{\underline{5}}$.

$$t = \frac{\hat{\mu}_x - \mu_0}{\hat{\delta}/\sqrt{n}} = \frac{101.5 - 100}{\sqrt{25}/\sqrt{6}} \simeq \underline{\underline{0.7348}}$$

$\nu = 5$.



P-value $= 2(0.25) = 0.5 \overset{?}{<} 0.05$

We do not have enough evidence to reject H_0.

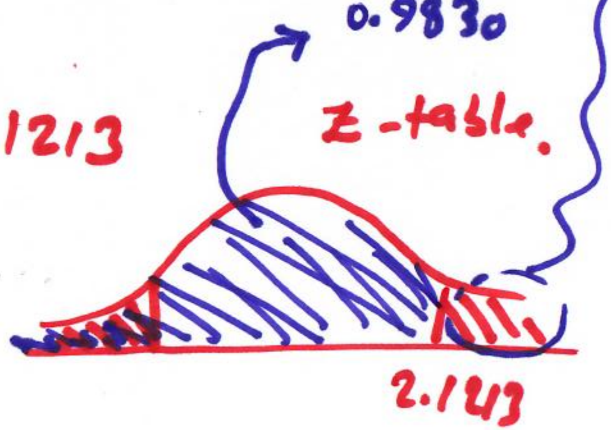
b). $n=50$, $H_0: \mu = 100$, $H_1: \mu \neq 100$

H_0 : Assume H_0 is true.

$n = 50 \geq 30$: large sample size, Z-table.

$$Z = \frac{\hat{\mu}_x - \mu_0}{\hat{\sigma}_x/\sqrt{n}} = \frac{101.5 - 100}{\sqrt{25}/\sqrt{50}} = 2.1213$$

$1 - 0.9830 = 0.017$



P-value $= 2(0.017) = 0.034 \overset{?}{<} 0.05$

We have enough evidence to reject H_0

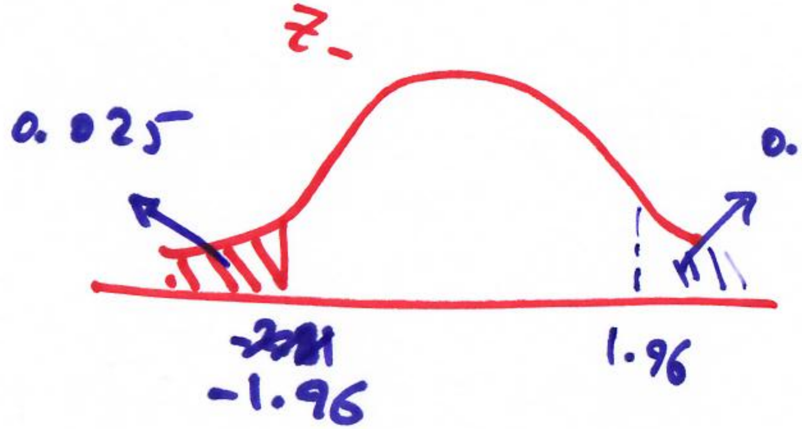
Q2. Confidence Interval.

$N = 30 \geq 30 \rightarrow$ large sample size - Z_table.

$10.2 \pm 1.9 \rightarrow \hat{\mu}_x = 10.2$

$\hat{\delta}_x = 1.9.$

a). 95% Confidence Interval

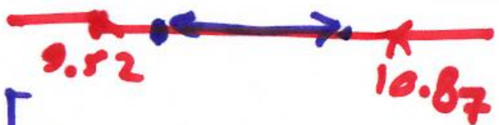


$1 - \alpha = 0.95$

$\alpha = 0.05$

$\alpha/2 = 0.025.$

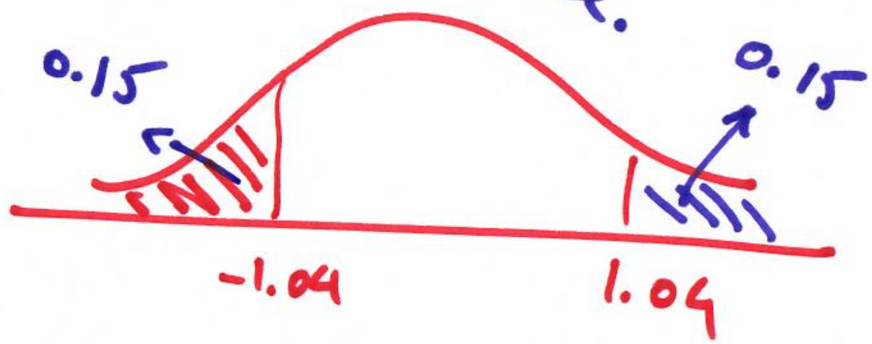
$$\hat{\mu}_x \pm Z_{\alpha/2} \, \hat{\delta}_x / \sqrt{n}.$$



$$10.2 \pm 1.96 \, \frac{1.9}{\sqrt{30}} : [9.5201 \; , \; 10.8799]$$

b). 70% Confidence Interval:

z-table.



$1-\alpha = 0.7$

$\alpha = 0.3$

$\alpha/2 = 0.15$

$$\hat{\mu}_x \pm Z_{\alpha/2}\, \hat{\sigma}_x/\sqrt{n}$$

$$10.2 \pm 1.04\, 1.9/\sqrt{30} \rightarrow [9.8392,\ 10.5608].$$

Q_3. Chapter 7.

$$x \to \boxed{} \to y$$

$$y = ax + b + e$$

$$\hat{y} = ax + b$$

x	y
26.6	81.1
26.0	93.3
27.4	87.8
21.7	82.6

a). Write down the matrix structure.

$$\begin{bmatrix} 81.1 \\ 93.3 \\ 87.8 \\ 82.6 \end{bmatrix} = \begin{bmatrix} 26.6 & 1 \\ 26.0 & 1 \\ 27.4 & 1 \\ 21.7 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} e^{(1)} \\ e^{(2)} \\ e^{(3)} \\ e^{(4)} \end{bmatrix}$$

$$\underline{y} = X_{aug} \underline{\beta} + \underline{e}$$

b). Write down the structure of the solution, & solve for a, b.

$$\underline{\beta} = \left(X_{aug}^{T} X_{aug}\right)^{-1} X_{aug}^{T} \underline{y}.$$

$$\underline{\beta} = \left(\begin{bmatrix} 26.6 & 26.0 & 27.4 & 21.7 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 26.6 & 1 \\ 26.0 & 1 \\ 27.4 & 1 \\ 21.7 & 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 26.6 & 26.0 & 27.4 & 21.7 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 81.1 \\ 93.3 \\ 87.8 \\ 82.5 \end{bmatrix}$$

$$\underline{\beta} = \begin{bmatrix} 0.7523 \\ 67.07 \end{bmatrix} \begin{matrix} \rightarrow a \\ \rightarrow b. \end{matrix}$$

c). Assume: $a = \cancel{0.8}$ 0.75, $b = \cancel{67}$ 67.07 $\rightarrow \hat{y} = \cancel{0.8}x + \cancel{67}$

$0.75x + 67.07$

$$e = \sqrt{\frac{1}{4}\sum_{n=1}^{4}(y_n - \hat{y}_n)^2}.$$

x	y	$\hat{y}$	$y - \hat{y}$
26.6	81.1	87.08	-5.98
26.0	93.3	86.6	6.67
27.4	87.8	87.7	0.11
21.7	82.6	83.4	-0.79

$$\boxed{e = 4.4975}$$

d). $a = 0.8$, $b = 67$ $\rightarrow \boxed{e = 4.6410}$

7.