Q1. (10 Points) Let S be the event that a randomly selected college student has taken a statistics course,

and let C be the event that the same student has taken a chemistry course. Suppose P(S) = 0.4, P(C) = 0.3,

and P(S∩C) = 0.2.

a. Find the probability that a student has taken statistics, chemistry, or both.  
P(S ∪ C) = 0.4 + 0.3 - 0.2 = 0.5

b. Find the probability that a student has taken neither statistics nor chemistry.

1 - P(S ∪ C) = 0.5  
Q2. (10 Points) Let A and B be events with P(A) = 0.8 and P(A∩B) = 0.2. For what value of P(B) will A

and B be independent?

0.2 = 0.8 × P(B)

0.2 / 0.8 = P(B)

= 0.25

Q3. (20 Points) A lot of 10 components contains 3 that are defective. Two components are drawn at

random and tested. Let A be the event that the first component drawn is defective and let B be the event

that the second component drawn is defective.

a. Find p(A).

3/10 = 0.3

b. Find p(B|A).

 P(A∩B)/P(A) = 2/9 = 0.22

c. Find p(A∩B).

 P(A∩B) = P(A) x P(B|A) = 0.3 x 0.22 = 0.066

d. Find p(A^c∩B).

P(Aᶜ ∩ B) = P(Aᶜ) × P(B|Aᶜ)

P(Aᶜ) = 7/10 = 0.7

P(B|Aᶜ) = 3/9 = 1/3 ≈ 0.333

Then, P(Aᶜ ∩ B) = 0.7 × 0.333 ≈ 0.233

e. Find p(B).

P(B) = P(B|A) × P(A) + P(B|Aᶜ) × P(Aᶜ)

P(B) ≈ 0.222 × 0.3 + 0.333 × 0.7 ≈ 0.0666 + 0.233 = 0.2996

f. Are A and B independent? Explain.

No, P(A) × P(B) = 0.3 × 0.2996 ≈ 0.08988

But since p(A∩B) = 0.066 they are not independent

Q4. (10 Points) Computer chips often contain surface imperfections. For a certain type of computer chip,

the probability mass function of the number of defects X is presented in the following table.

a. Find p(X ≤ 2).

P(X = 0) + P(X = 1) + P(X = 2) = 0.4 + 0.3 + 0.15 = 0.85

b. Find p(X > 1).

1 - P(X ≤ 1) = 1 - (P(X = 0) + P(X = 1)) = 1 - (0.4 + 0.3) = 1 - 0.7 = 0.3

c. Find the mean μx.

(0 × 0.4) + (1 × 0.3) + (2 × 0.15) + (3 × 0.1) + (4 × 0.05) = 0 + 0.3 + 0.3 + 0.3 + 0.2 = 1.1

Q5. (20 Points) A computer sends a packet of information along a channel and waits for a return signal

acknowledging that the packet has been received. If no acknowledgment is received within a certain time,

the packet is re-sent. Let X represent the number of times the packet is sent. Assume that the probability

mass function of X is given by

where c is a constant.

a. Find the value of the constant c so that p(x) is a probability mass function.

ΣP(x) = c(1) + c(2) + c(3) + c(4) + c(5) = 1

c(1 + 2 + 3 + 4 + 5) = 1

c(15) = 1

c = 1/15

b. Find p(X = 2).

P(X = 2) = (1/15)(2) = 2/15

c. Find the mean number of times the packet is sent.

1 × (1/15)(1) + 2 × (1/15)(2) + 3 × (1/15)(3) + 4 × (1/15)(4) + 5 × (1/15)(5)

= 3.6666

d. Find the variance of the number of times the packet is sent.

E(X^2) = Σ[x^2 × P(X = x)]

E(X^2) = 1^2 × (1/15)(1) + 2^2 × (1/15)(2) + 3^2 × (1/15)(3) + 4^2 × (1/15)(4) + 5^2 × (1/15)(5)

= 15

E(X^2) - (E(X))^2

= 15 - (55/15)^2 = 1.555

e. Find the cumulative distribution function of the lifetime.

 (1/15)(1) = 1/15

(1/15)(1) + (1/15)(2) = 3/15

(1/15)(1) + (1/15)(2) + (1/15)(3) = 6/15

(1/15)(1) + (1/15)(2) + (1/15)(3) + (1/15)(4) = 10/15

(1/15)(1) + (1/15)(2) + (1/15)(3) + (1/15)(4) + (1/15)(5) = 1

1/15 for x = 1  
3/15 for x = 2  
6/15 for x = 3  
10/15 for x = 4  
1 for x = 5  
0 otherwise

f. Find the median lifetime.

F(2) = 3/15 = 0.2 < 0.5  
F(3) = 6/15 = 0.4 < 0.5  
F(4) = 10/15 = 0.666… > 0.5

Median lifetime is 4

g. Find the 60th percentile of the lifetimes.

F(3) = 6/15 = 0.4 < 0.6  
F(4) = 10/15 = 0.666.. > 0.6

60th percentile is also at 4

Q6. (20 Points) A computer fan’s lifetime (in years) is modeled by a continuous random variable with the

following probability distribution function.

𝑓(𝑥) = 𝑐x 0 < 𝑥 < 10

0 Otherwise

where c is a constant.

a. Find the value of the constant c so that f(x) is a probability distribution function.

(c/2)(9^2) - (c/2)(1^2) = 1

40c = 1

C = 1/40

b. Find p(X > 2).

[(1/80)(9^2)] - [(1/80)(3^2)] = 0.9

P(X > 2) = 0.9

c. Find the mean of the fan’s lifetime.

(1/120)(9^3) - (1/120)(1^3) = 6.06666666667

d. Find the median lifetime.

(1/80)x^2 - (1/80)(1^2) = 0.5

(1/80)x^2 - 1/80 = 0.5

(1/80)x^2 = 9/16

x^2 = 45

x = 6.7082039325

e. Find the 60th percentile of the lifetime.

(1/80)x^2 - (1/80)(1^2) = 0.6

(1/80)x^2 = 0.6 + (1/80)

(1/80)x^2 = 0.6125

x^2 = 49

x = 7

Q7. (10 points) If X and Y are independent random variables with means μx = 9.5 and μy = 6.8, and

standard deviations σx = 0.4 and σy = 0.1, find the means and standard deviations of the following:

a. 3X

b. Y − X

c. X + 4Y