ACM-ICPC Team Reference Document

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Contents

1 Geometry

1.1 Convex Hull With Graham's Scan

```
// Takes in >= 3 points
// Returns convex hull in clockwise order
// Ignores points on the border
vector<Vec<int>> buildConvexHull(vector<Vec<int>> pts) {
    if(pts.size() <= 3) return pts;
    sort(pts.begin(), pts.end());
    stack<Vec<int>> hull;
    hull.push(pts[0]);
    auto p = pts[0];
    sort(pts.begin()+1, pts.end(), [&](Vec<int> a, Vec<int> b)
        -> bool {
        // p->a->b is a ccw turn
        int turn = sgn((a-p)^(b-a));
        //if(turn == 0) return (a-p).norm() > (b-p).norm();
        // ^ among collinear points, take the farthest one
        return turn == 1;
    });
    hull.push(pts[1]);
    FOR(i, 2, (int)pts.size()) {
        auto c = pts[i];
        if(c == hull.top()) continue;
```

```
    while(true) {
        auto a = hull.top(); hull.pop();
        auto b = hull.top();
        auto ba = a-b;
        auto ac = c-a;
        if((ba^ac) > 0) {
            hull.push(a);
            break;
        } else if((ba^ac) == 0) {
            if(ba*ac < 0) c = a;
            // ^ c is between b and a, so it shouldn't be
                added to the hull
            break;
        }
    }
    hull.push(c);
  }
  vector<Vec<int>> hullPts;
  while(!hull.empty()) {
    hullPts.pb(hull.top());
    hull.pop();
  }
  return hullPts;
}
```

1.2 Convex Hull Gift Wrapping

```
vector<Vec<int>> buildConvexHull(vector<Vec<int>>& pts)
    {
  int n = pts.size();
  sort(pts.begin(), pts.end());
  auto currP = pts[0]; // choose some extreme point to be on
      the hull

  vector<Vec<int>> hull;
  set<Vec<int>> used;
  hull.pb(pts[0]);
  used.insert(pts[0]);
  while(true) {
    auto candidate = pts[0]; // choose some point to be a
        candidate

    auto currDir = candidate-currP;
    vector<Vec<int>> toUpdate;
    FOR(i, 0, n) {
      if(currP == pts[i]) continue;
      // currently we have currP->candidate
      // we need to find point to the left of this
      auto possibleNext = pts[i];
      auto nextDir = possibleNext - currP;
      auto cross = currDir ^ nextDir;
      if(candidate == currP || cross > 0) {
        candidate = possibleNext;
        currDir = nextDir;
      } else if(cross == 0 && nextDir.norm() > currDir.
          norm()) {
        candidate = possibleNext;
        currDir = nextDir;
      }
    }
    if(used.find(candidate) != used.end()) break;
    hull.pb(candidate);
    used.insert(candidate);
    currP = candidate;
  }
  return hull;
}
```

1.3 Circle Line Intersection

```
double r, a, b, c; // ax+by+c=0, radius is at (0, 0)
// If the center is not at (0, 0), fix the constant c to translate
    everything so that center is at (0, 0)
double x0 = -a*c/(a*a+b*b), y0 = -b*c/(a*a+b*b);
if (c*c > r*r*(a*a+b*b)+eps)
  puts ("no points");
else if (abs (c*c - r*r*(a*a+b*b)) < eps) {
  puts ("1 point");
  cout << x0 << ' ' << y0 << '\n';
}
else {
  double d = r*r - c*c/(a*a+b*b);
  double mult = sqrt (d / (a*a+b*b));
  double ax, ay, bx, by;
  ax = x0 + b * mult;
  bx = x0 - b * mult;
  ay = y0 - a * mult;
  by = y0 + a * mult;
  puts ("2 points");
  cout << ax << ' ' << ay << '\n' << bx << ' ' << by
      << '\n';
}
```

1.4 Common Tangents To Two Circles

```
struct pt {
  double x, y;

  pt operator- (pt p) {
    pt res = { x-p.x, y-p.y };
    return res;
  }
};
struct circle : pt {
  double r;
};
struct line {
  double a, b, c;
};
void tangents (pt c, double r1, double r2, vector<line> & ans) {
  double r = r2 - r1;
  double z = sqr(c.x) + sqr(c.y);
  double d = z - sqr(r);
  if (d < -eps) return;
  d = sqrt (abs (d));
  line l;
  l.a = (c.x * r + c.y * d) / z;
  l.b = (c.y * r - c.x * d) / z;
  l.c = r1;
  ans.push_back (l);
}
vector<line> tangents (circle a, circle b) {
  vector<line> ans;
  for (int i=-1; i<=1; i+=2)
    for (int j=-1; j<=1; j+=2)
      tangents (b-a, a.r*i, b.r*j, ans);
  for (size_t i=0; i<ans.size(); ++i)
    ans[i].c -= ans[i].a * a.x + ans[i].b * a.y;
  return ans;
}
```

1.5 2d Vector

```
template <typename T>
struct Vec {
  T x, y;
  Vec(): x(0), y(0) {}
  Vec(T _x, T _y): x(_x), y(_y) {}
  Vec operator+(const Vec& b) {
    return Vec<T>(x+b.x, y+b.y);
  }
  Vec operator-(const Vec& b) {
    return Vec<T>(x-b.x, y-b.y);
  }
  Vec operator*(T c) {
    return Vec(x*c, y*c);
  }
  T operator*(const Vec& b) {
    return x*b.x + y*b.y;
  }
  T operator^(const Vec& b) {
    return x*b.y-y*b.x;
  }
  bool operator<(const Vec& other) const {
    if(x == other.x) return y < other.y;
    return x < other.x;
  }
  bool operator==(const Vec& other) const {
    return x==other.x && y==other.y;
  }
  bool operator!=(const Vec& other) const {
    return !(*this == other);
  }
  friend ostream& operator<<(ostream& out, const Vec& v) {
```

```
        return out << "(" << v.x << ", " << v.y << ")";
    }
    friend istream& operator>>(istream& in, Vec<T>& v) {
        return in >> v.x >> v.y;
    }
    T norm() { // squared length
        return (*this)*(*this);
    }
    ld len() {
        return sqrt(norm());
    }
    ld angle(const Vec& other) { // angle between this and
         other vector
        return acosl((*this)*other/len()/other.len());
    }
    Vec perp() {
        return Vec(-y, x);
    }
};
/* Cross product of 3d vectors: (ay*bz-az*by, az*bx-ax*bz, ax*
     by-ay*bx)
 */
```

1.6 Line

```
template <typename T>
struct Line { // expressed as two vectors
    Vec<T> start, dir;
    Line() {}
    Line(Vec<T> a, Vec<T> b): start(a), dir(b-a) {}

    Vec<ld> intersect(Line l) {
        ld t = ld((l.start-start)^l.dir)/(dir^l.dir);
        // For segment-segment intersection this should be in
             range [0, 1]
        Vec<ld> res(start.x, start.y);
        Vec<ld> dirld(dir.x, dir.y);
        return res + dirld*t;
    }
};
```

1.7 Usage Of Complex

```
typedef long long C; // could be long double
typedef complex<C> P; // represents a point or vector
#define X real()
#define Y imag()
...
P p = {4, 2}; // p.X = 4, p.Y = 2
P u = {3, 1};
P v = {2, 2};
P s = v+u; // {5, 3}
P a = {4, 2};
P b = {3, -1};
auto l = abs(b-a); // 3.16228
auto plr = polar(1.0, 0.5); // construct a vector of length 1 and
     angle 0.5 radians
v = {2, 2};
auto alpha = arg(v); // 0.463648
v *= plr; // rotates v by 0.5 radians counterclockwise. The
     length of plt must be 1 to rotate correctly.
auto beta = arg(v); // 0.963648
a = {4, 2};
b = {1, 2};
C p = (conj(a)*b).Y; // 6 <- the cross product of a and b
```

1.8 Misc

Distance from point to line.

We have a line $l(t) = \vec{a} + \vec{b}t$ and a point $\vec{p}$. The distance from this point to the line can be calculated by expressing the area of a triangle in two different ways. The final formula: $d = \frac{(\vec{p}-\vec{a})\times(\vec{p}-\vec{b})}{|\vec{b}-\vec{a}|}$

Point in polygon.

Send a ray (half-infinite line) from the points to an arbitrary direction and calculate the number of times it touches the boundary of the polygon. If the number is odd, the point is inside the polygon, otherwise it's outside.

Using cross product to test rotation direction.

Let's say we have vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$. Let's define $\vec{ab} = b - a, \vec{bc} = c - b$ and $s = sgn(\vec{ab} \times \vec{bc})$. If $s = 0$, the three points are collinear. If $s = 1$, then $\vec{bc}$ turns in the counterclockwise direction compared to the direction of $\vec{ab}$. Otherwise it turns in the clockwise direction.

Line segment intersection.

The problem: to check if line segments ab and cd intersect. There are three cases:

1. **The line segments are on the same line.** Use cross products and check if they're zero - this will tell if all points are on the same line. If so, sort the points and check if their intersection is non-empty. If it is non-empty, there are an infinite number of intersection points.
2. **The line segments have a common vertex.** Four possibilities: $a = c, a = d, b = c, b = d$.
3. **There is exactly one intersection point that is not an endpoint.** Use cross product to check if points c and d are on different sides of the line going through a and b and if the points a and b are on different sides of the line going through c and d.

Angle between vectors.

$arccos(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|})$.

Dot product properties.

If the dot product of two vectors is zero, the vectors are orthogonal. If it is positive, the angle is acute. Otherwise it is obtuse.

Lines with line equation.

Any line can be described by an equation $ax + by + c = 0$.

- Construct a line using two points A and B:
  1. Take vector from A to B and rotate it 90 degrees ($(x, y) \to (-y, x)$). This will be (a, b).
  2. Normalize this vector. Then put A (or B) into the equation and solve for c.
- Distance from point to line: put point coordinates into line equation and take absolute value. If (a, b) is not normalized, you still need to divide by $\sqrt{a^2 + b^2}$.

- Distance between two parallel lines: $|c_1 - c_2|$ (if they are not normalized, you still need to divide by $\sqrt{a^2+b^2}$).
- Project a point onto a line: compute signed distance d between line L and point P. Answer is $P - d(\vec{a,b})$.
- Build a line parallel to a given one and passing through a given point: compute the signed distance d between line and point. Answer is $ax + by + (c - d) = 0$.
- Intersect two lines: $d = a_1b_2 - a_2b_1, x = \frac{c_2b_1 - c_1b_2}{d}, y = \frac{c_1a_2 - c_2a_1}{d}$. If $abs(d) < \epsilon$, then the lines are parallel.

Half-planes.

Definition: define as line, assume a point (x, y) belongs to half plane iff $ax + by + c \geq 0$.

Intersecting with a convex polygon:

1. Start at any point and move along the polygon's traversal.
2. Alternate points and segments between consecutive points.
3. If point belongs to half-plane, add it to the answer.
4. If segment intersects the half-plane's line, add it to the answer.

Some more techniques.

- Check if point A lies on segment BC:
  1. Compute point-line distance and check if it is 0 (abs less than ϵ).
  2. $\vec{BA} \cdot \vec{BC} \geq 0$ and $\vec{CA} \cdot \vec{CB} \geq 0$.
- Compute distance between line segment and point: project point onto line formed by the segment. If this point is on the segment, then the distance between it and original point is the answer. Otherwise, take minimum of distance between point and segment endpoints.

1.9 Pick's Theorem

We are given a lattice polygon with non-zero area. Let's denote its area by S, the number of points with integer coordinates lying strictly inside the polygon by I and the number of points lying on the sides of the polygon by B. Then:

$$S = I + \frac{B}{2} - 1.$$

1.10 Number Of Lattice Points On Segment

Let's say we have a line segment from (x_1, y_1) to (x_2, y_2). Then, the number of lattice points on this segment is given by

$$gcd(x_2 - x_1, y_2 - y_1) + 1.$$

1.11 Circle Circle Intersection

Let's say that the first circle is centered at $(0, 0)$ (if it's not, we can move the origin to the center of the first circle and adjust the coordinates), and the second one is at (x_2, y_2). Then, let's construct a line $Ax + By + C = 0$, where $A = -2x_2, B = -2y_2, C = x_2^2 + y_2^2 + r_1^2 - r_2^2$. Finding the intersection between this line and the first circle will give us the answer. The only tricky case: if both circles are centered at the same point. We handle this case separately.

2 General

2.1 C++ Template

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp> // gp_hash_table
        <int, int> == hash map
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
typedef long long ll;
typedef unsigned long long ull;
typedef long double ld;
typedef pair<int, int> pii;
typedef pair<ll, ll> pll;
typedef pair<double, double> pdd;
template <typename T> using min_heap = priority_queue<T,
        vector<T>, greater<T>>;
template <typename T> using max_heap = priority_queue<T,
        vector<T>, less<T>>;
template <typename T> using ordered_set = tree<T,
        null_type, less<T>, rb_tree_tag,
        tree_order_statistics_node_update>;
template <typename K, typename V> using hashmap =
        gp_hash_table<K, V>;

template<typename A, typename B> ostream& operator<<(
        ostream& out, pair<A, B> p) { out << "(" << p.first
        << ", " << p.second << ")"; return out;}
template<typename T> ostream& operator<<(ostream& out,
        vector<T> v) { out << "["; for(auto& x : v) out << x
        << ", "; out << "]";return out;}
template<typename T> ostream& operator<<(ostream& out,
        set<T> v) { out << "{"; for(auto& x : v) out << x <<
        ", "; out << "}"; return out; }
template<typename K, typename V> ostream& operator<<(
        ostream& out, map<K, V> m) { out << "{"; for(auto& e
        : m) out << e.first << " -> " << e.second << ", "; out
        << "}"; return out; }
template<typename K, typename V> ostream& operator<<(
        ostream& out, hashmap<K, V> m) { out << "{"; for(
        auto& e : m) out << e.first << " -> " << e.second <<
        ", "; out << "}"; return out; }

#define FAST_IO ios_base::sync_with_stdio(false); cin.tie(
        NULL)
#define TESTS(t) int NUMBER_OF_TESTS; cin >>
        NUMBER_OF_TESTS; for(int t = 1; t <=
        NUMBER_OF_TESTS; t++)
#define FOR(i, begin, end) for (int i = (begin) - ((begin) > (
        end)); i != (end) - ((begin) > (end)); i += 1 - 2 * ((begin)
        > (end)))
#define sgn(a) ((a) > eps ? 1 : ((a) < -eps ? -1 : 0))
#define precise(x) fixed << setprecision(x)
#define debug(x) cerr << "> " << #x << " = " << x <<
        endl;
#define pb push_back
#define rnd(a, b) (uniform_int_distribution<int>((a), (b))(rng
        ))
#ifndef LOCAL
    #define cerr if(0)cout
    #define endl "\n"
#endif
mt19937 rng(chrono::steady_clock::now().time_since_epoch().
        count());
```

```
clock_t ___clock___;
void startTime() {___clock___ = clock();}
void timeit(string msg) {cerr << ">␣" << msg << ":␣" <<
        precise(6) << ld(clock()-___clock___)/
        CLOCKS_PER_SEC << endl;}
const ld PI = asin(1) * 2;
const ld eps = 1e-14;
const int oo = 2e9;
const ll OO = 2e18;
const ll MOD = 1000000007;
const int MAXN = 1000000;

int main() {
    FAST_IO;
    startTime();

    timeit("Finished");
    return 0;
}
```

2.2 Automatic Test

```
# Linux Bash
# gen, main and stupid have to be compiled beforehand
for((i=1;;++i)); do
    echo $i;
    ./gen $i > genIn;
    diff <(./main < genIn) <(./stupid < genIn) || break;
done

# Windows CMD
@echo off
FOR /L %%I IN (1,1,2147483647) DO (
    echo %%I
    gen.exe %%I > genIn
    main.exe < genIn > mainOut
    stupid.exe < genIn > stupidOut
    FC mainOut stupidOut || goto :eof
)
```

2.3 Python Template

```
import sys
import re
from math import ceil, log, sqrt, floor

___local_run___ = False
if ___local_run___:
    sys.stdin = open('input.txt', 'r')
    sys.stdout = open('output.txt', 'w')

def main():
    a = int(input())
    b = int(input())
    print(a*b)

main()
```

2.4 C++ Visual Studio Includes

```
#define _CRT_SECURE_NO_WARNINGS
#pragma comment(linker, "/STACK:167772160000")
#include <iostream>
#include <iomanip>
#include <fstream>
#include <cstdio>
#include <cstdlib>
#include <cassert>
#include <climits>
#include <cmath>
#include <algorithm>
#include <cstring>
#include <string>
#include <vector>
#include <list>
#include <stack>
#include <set>
#include <bitset>
#include <queue>
#include <map>
#include <sstream>
#include <functional>
#include <unordered_map>
#include <unordered_set>
#include <complex>
#include <random>
#include <chrono>
```

2.5 Compilation

```
# Simple compile
g++ -DLOCAL -O2 -o main.exe -std-c++17 -Wall -Wno-unused
        -result -Wshadow main.cpp
# Debug
g++ -DLOCAL -std=c++17 -Wshadow -Wall -o main.exe main.
        cpp -fsanitize=address -fsanitize=undefined -fuse-ld=gold
        -D_GLIBCXX_DEBUG -g
```

3 Graphs

3.1 Lowest Common Ancestor

```
int n, l; // l == logN (usually about ~20)
vector<vector<int>> adj;

int timer;
vector<int> tin, tout;
vector<vector<int>> up;

void dfs(int v, int p)
{
    tin[v] = ++timer;
    up[v][0] = p;
    // wUp[v][0] = weight[v][u]; // <- path weight sum to 2^i-th
        ancestor
    for (int i = 1; i <= l; ++i)
        up[v][i] = up[up[v][i-1]][i-1];
        // wUp[v][i] = wUp[v][i-1] + wUp[up[v][i-1]][i-1];

    for (int u : adj[v]) {
        if (u != p)
            dfs(u, v);
    }

    tout[v] = ++timer;
}

bool isAncestor(int u, int v)
{
    return tin[u] <= tin[v] && tout[v] <= tout[u];
}

int lca(int u, int v)
{
    if (isAncestor(u, v))
        return u;
    if (isAncestor(v, u))
        return v;
    for (int i = l; i >= 0; --i) {
        if (!isAncestor(up[u][i], v))
            u = up[u][i];
    }
    return up[u][0];
}

void preprocess(int root) {
    tin.resize(n);
    tout.resize(n);
    timer = 0;
    l = ceil(log2(n));
    up.assign(n, vector<int>(l + 1));
    dfs(root, root);
}
```

3.2 Shortest Paths Of Fixed Length

Define $A \bigodot B = C \iff C_{ij} = min_{p=1..n}(A_{ip} + B_{pj})$. Let G be the adjacency matrix of a graph.

Also, let $L_k = G \odot \ldots \odot G = G^{\odot k}$. Then the value $L_k[i][j]$ denotes the length of the shortest path between i and j which consists of exactly k edges.

3.3 Strongly Connected Components

```
vector < vector<int> > g, gr; // adjList and reversed adjList
vector<bool> used;
vector<int> order, component;

void dfs1 (int v) {
    used[v] = true;
    for (size_t i=0; i<g[v].size(); ++i)
        if (!used[ g[v][i] ])
            dfs1 (g[v][i]);
    order.push_back (v);
}

void dfs2 (int v) {
    used[v] = true;
    component.push_back (v);
    for (size_t i=0; i<gr[v].size(); ++i)
        if (!used[ gr[v][i] ])
            dfs2 (gr[v][i]);
}

int main() {
    int n;
    // read n
    for (;;) {
        int a, b;
        // read edge a -> b
        g[a].push_back (b);
        gr[b].push_back (a);
    }

    used.assign (n, false);
    for (int i=0; i<n; ++i)
        if (!used[i])
            dfs1 (i);
    used.assign (n, false);
    for (int i=0; i<n; ++i) {
        int v = order[n-1-i];
        if (!used[v]) {
            dfs2 (v);
            // do something with the found component
            component.clear(); // components are generated in
                toposort-order
        }
    }
}
```

3.4 Min Cut

```
init();
ll f = maxFlow(); // Ford-Fulkerson
cur_time++;
dfs(0);
set<int> cc;
for (auto e : edges) {
    if (timestamp[e.from] == cur_time && timestamp[e.to] !=
        cur_time) {
        cc.insert(e.idx);
    }
}
// (# of edges in min-cut, capacity of cut)
// [indices of edges forming the cut]
cout << cc.size() << "␣" << f << endl;
for (auto x : cc) cout << x + 1 << "␣";
```

3.5 Number Of Paths Of Fixed Length

Let G be the adjacency matrix of a graph. Then $C_k = G^k$ gives a matrix, in which the value $C_k[i][j]$ gives the number of paths between i and j of length k.

3.6 Max Flow With Dinic 2

```
struct FlowEdge {
    int v, u;
    long long cap, flow = 0;
    FlowEdge(int v, int u, long long cap) : v(v), u(u), cap(cap)
        {}
};

struct Dinic {
    const long long flow_inf = 1e18;
    vector<FlowEdge> edges;
    vector<vector<int>> adj;
    int n, m = 0;
    int s, t;
    vector<int> level, ptr;
    queue<int> q;

    Dinic(int n, int s, int t) : n(n), s(s), t(t) {
        adj.resize(n);
        level.resize(n);
        ptr.resize(n);
    }

    void add_edge(int v, int u, long long cap) {
        edges.push_back(FlowEdge(v, u, cap));
        edges.push_back(FlowEdge(u, v, 0));
        adj[v].push_back(m);
        adj[u].push_back(m + 1);
        m += 2;
    }

    bool bfs() {
        while (!q.empty()) {
            int v = q.front();
            q.pop();
            for (int id : adj[v]) {
                if (edges[id].cap - edges[id].flow < 1)
                    continue;
                if (level[edges[id].u] != -1)
                    continue;
                level[edges[id].u] = level[v] + 1;
                q.push(edges[id].u);
            }
        }
        return level[t] != -1;
    }

    long long dfs(int v, long long pushed) {
        if (pushed == 0)
            return 0;
        if (v == t)
            return pushed;
        for (int& cid = ptr[v]; cid < (int)adj[v].size(); cid++) {
            int id = adj[v][cid];
            int u = edges[id].u;
            if (level[v] + 1 != level[u] || edges[id].cap - edges[id].
                flow < 1)
                continue;
            long long tr = dfs(u, min(pushed, edges[id].cap -
                edges[id].flow));
            if (tr == 0)
                continue;
            edges[id].flow += tr;
            edges[id ^ 1].flow -= tr;
            return tr;
        }
        return 0;
    }

    long long flow() {
        long long f = 0;
        while (true) {
            fill(level.begin(), level.end(), -1);
            level[s] = 0;
            q.push(s);
            if (!bfs())
                break;
            fill(ptr.begin(), ptr.end(), 0);
            while (long long pushed = dfs(s, flow_inf)) {
                f += pushed;
            }
        }
        return f;
    }
};
```

3.7 Max Flow With Dinic

```
struct Edge {
    int f, c;
    int to;
    pii revIdx;
    int dir;
    int idx;
};

int n, m;
vector<Edge> adjList[MAX_N];
int level[MAX_N];

void addEdge(int a, int b, int c, int i, int dir) {
    int idx = adjList[a].size();
    int revIdx = adjList[b].size();
    adjList[a].pb({ 0,c,b, {b, revIdx} ,dir,i });
    adjList[b].pb({ 0,0,a, {a, idx} ,dir,i });
}

bool bfs(int s, int t) {
    FOR(i, 0, n) level[i] = -1;
    level[s] = 0;
    queue<int> Q;
    Q.push(s);
    while (!Q.empty()) {
        auto t = Q.front(); Q.pop();
        for (auto x : adjList[t]) {
            if (level[x.to] < 0 && x.f < x.c) {
                level[x.to] = level[t] + 1;
                Q.push(x.to);
            }
        }
    }
    return level[t] >= 0;
}

int send(int u, int f, int t, vector<int>& edgeIdx) {
    if (u == t) return f;
    for (; edgeIdx[u] < adjList[u].size(); edgeIdx[u]++) {
        auto& e = adjList[u][edgeIdx[u]];
        if (level[e.to] == level[u] + 1 && e.f < e.c) {
            int curr_flow = min(f, e.c - e.f);
            int next_flow = send(e.to, curr_flow, t, edgeIdx);
            if (next_flow > 0) {
                e.f += next_flow;
                adjList[e.revIdx.first][e.revIdx.second].f -=
                        next_flow;
                return next_flow;
            }
        }
    }
    return 0;
}

int maxFlow(int s, int t) {
    int f = 0;
    while (bfs(s, t)) {
        vector<int> edgeIdx(n, 0);
        while (int extra = send(s, oo, t, edgeIdx)) {
            f += extra;
        }
    }
    return f;
}

void init() {
    cin >> n >> m;
    FOR(i, 0, m) {
        int a, b, c;
        cin >> a >> b >> c;
        a--; b--;
        addEdge(a, b, c, i, 1);
        addEdge(b, a, c, i, -1);
    }
}
```

3.8 Dijkstra

```
vector<vector<pair<int, int>>> adj;
void dijkstra(int s, vector<int> & d, vector<int> & p) {
    int n = adj.size();
    d.assign(n, oo);
    p.assign(n, -1);
    d[s] = 0;
    min_heap<pii> q;
    q.push({0, s});
    while (!q.empty()) {
        int v = q.top().second;
        int d_v = q.top().first;
        q.pop();
        if (d_v != d[v]) continue;
        for (auto edge : adj[v]) {
            int to = edge.first;
            int len = edge.second;
            if (d[v] + len < d[to]) {
                d[to] = d[v] + len;
                p[to] = v;
                q.push({d[to], to});
            }
        }
    }
}
```

3.9 Bellman Ford Algorithm

```
struct Edge
{
    int a, b, cost;
};

int n, m, v; // v - starting vertex
vector<Edge> e;

/* Finds SSSP with negative edge weights.
 * Possible optimization: check if anything changed in a
       relaxation step. If not - you can break early.
 * To find a negative cycle: perform one more relaxation step. If
       anything changes - a negative cycle exists.
 */
void solve() {
    vector<int> d (n, oo);
    d[v] = 0;
    for (int i=0; i<n-1; ++i)
        for (int j=0; j<m; ++j)
            if (d[e[j].a] < oo)
                d[e[j].b] = min (d[e[j].b], d[e[j].a] + e[j].cost);
    // display d, for example, on the screen
}
```

3.10 Finding Bridges

```
int n; // number of nodes
vector<vector<int>> adj; // adjacency list of graph

vector<bool> visited;
vector<int> tin, fup;
int timer;

void processBridge(int u, int v) {
    // do something with the found bridge
}

void dfs(int v, int p = -1) {
    visited[v] = true;
    tin[v] = fup[v] = timer++;
    for (int to : adj[v]) {
        if (to == p) continue;
        if (visited[to]) {
            fup[v] = min(fup[v], tin[to]);
        } else {
            dfs(to, v);
            fup[v] = min(fup[v], fup[to]);
            if (fup[to] > tin[v])
                processBridge(v, to);
        }
    }
}

// Doesn't work with multiple edges
// But multiple edges are never bridges, so it's easy to check
void findBridges() {
    timer = 0;
    visited.assign(n, false);
    tin.assign(n, -1);
    fup.assign(n, -1);
```

```
    bridges.clear();
    FOR(i, 0, n) {
        if (!visited[i])
            dfs(i);
    }
}
```

3.11 Dfs With Timestamps

```
vector<vector<int>> adj;
vector<int> tIn, tOut, color;
int dfs_timer = 0;

void dfs(int v) {
    tIn[v] = dfs_timer++;
    color[v] = 1;
    for (int u : adj[v])
        if (color[u] == 0)
            dfs(u);
    color[v] = 2;
    tOut[v] = dfs_timer++;
}
```

3.12 Finding Articulation Points

```
int n; // number of nodes
vector<vector<int>> adj; // adjacency list of graph

vector<bool> visited;
vector<int> tin, fup;
int timer;

void processCutpoint(int v) {
    // problem-specific logic goes here
    // it can be called multiple times for the same v
}

void dfs(int v, int p = -1) {
    visited[v] = true;
    tin[v] = fup[v] = timer++;
    int children=0;
    for (int to : adj[v]) {
        if (to == p) continue;
        if (visited[to]) {
            fup[v] = min(fup[v], tin[to]);
        } else {
            dfs(to, v);
            fup[v] = min(fup[v], fup[to]);
            if (fup[to] >= tin[v] && p!=-1)
                processCutpoint(v);
            ++children;
        }
    }
    if(p == -1 && children > 1)
        processCutpoint(v);
}

void findCutpoints() {
    timer = 0;
    visited.assign(n, false);
    tin.assign(n, -1);
    fup.assign(n, -1);
    for (int i = 0; i < n; ++i) {
        if (!visited[i])
            dfs (i);
    }
}
```

3.13 Bipartite Graph

```
class BipartiteGraph {
private:
    vector<int> _left, _right;
    vector<vector<int>> _adjList;
    vector<int> _matchR, _matchL;
    vector<bool> _used;

    bool _kuhn(int v) {
        if (_used[v]) return false;
        _used[v] = true;
        FOR(i, 0, (int)_adjList[v].size()) {
            int to = _adjList[v][i] - _left.size();
            if (_matchR[to] == -1 || _kuhn(_matchR[to])) {
                _matchR[to] = v;
                _matchL[v] = to;
                return true;
            }
        }
        return false;
    }
    void _addReverseEdges() {
        FOR(i, 0, (int)_right.size()) {
            if (_matchR[i] != -1) {
                _adjList[_left.size() + i].pb(_matchR[i]);
            }
        }
    }
    void _dfs(int p) {
        if (_used[p]) return;
        _used[p] = true;
        for (auto x : _adjList[p]) {
            _dfs(x);
        }
    }
    vector<pii> _buildMM() {
        vector<pair<int, int> > res;
        FOR(i, 0, (int)_right.size()) {
            if (_matchR[i] != -1) {
                res.push_back(make_pair(_matchR[i], i));
            }
        }

        return res;
    }
public:
    void addLeft(int x) {
        _left.pb(x);
        _adjList.pb({});
        _matchL.pb(-1);
        _used.pb(false);
    }
    void addRight(int x) {
        _right.pb(x);
        _adjList.pb({});
        _matchR.pb(-1);
        _used.pb(false);
    }
    void addForwardEdge(int l, int r) {
        _adjList[l].pb(r + _left.size());
    }
    void addMatchEdge(int l, int r) {
        if(l != -1) _matchL[l] = r;
        if(r != -1) _matchR[r] = l;
    }
    // Maximum Matching
    vector<pii> mm() {
        _matchR = vector<int>(_right.size(), -1);
        _matchL = vector<int>(_left.size(), -1);
        // ^ these two can be deleted if performing MM on
            already partially matched graph
        _used = vector<bool>(_left.size() + _right.size(), false
            );


        bool path_found;
        do {
            fill(_used.begin(), _used.end(), false);
            path_found = false;
            FOR(i, 0, (int)_left.size()) {
                if (_matchL[i] < 0 && !_used[i]) {
                    path_found |= _kuhn(i);
                }
            }
        } while (path_found);

        return _buildMM();
    }

    // Minimum Edge Cover
    // Algo: Find MM, add unmatched vertices greedily.
    vector<pii> mec() {
        auto ans = mm();
        FOR(i, 0, (int)_left.size()) {
            if (_matchL[i] != -1) {
                for (auto x : _adjList[i]) {
                    int ridx = x - _left.size();
                    if (_matchR[ridx] == -1) {
                        ans.pb({ i, ridx });
```

```
                    __matchR[ridx] = i;
                }
            }
        }
    }
    FOR(i, 0, (int)__left.size()) {
        if (__matchL[i] == -1 && (int)__adjList[i].size() > 0)
            {
            int ridx = __adjList[i][0] - __left.size();
            __matchL[i] = ridx;
            ans.pb({ i, ridx });
        }
    }
    return ans;
}

// Minimum Vertex Cover
// Algo: Find MM. Run DFS from unmatched vertices from
     the left part.
// MVC is composed of unvisited LEFT and visited RIGHT
     vertices.
pair<vector<int>, vector<int>> mvc(bool runMM = true)
     {
    if (runMM) mm();
    __addReverseEdges();
    fill(__used.begin(), __used.end(), false);
    FOR(i, 0, (int)__left.size()) {
        if (__matchL[i] == -1) {
            __dfs(i);
        }
    }
    vector<int> left, right;
    FOR(i, 0, (int)__left.size()) {
        if (!__used[i]) left.pb(i);
    }
    FOR(i, 0, (int)__right.size()) {
        if (__used[i + (int)__left.size()]) right.pb(i);
    }
    return { left,right };
}

// Maximal Independant Vertex Set
// Algo: Find complement of MVC.
pair<vector<int>, vector<int>> mivs(bool runMM = true)
     {
    auto m = mvc(runMM);
    vector<bool> containsL(__left.size(), false), containsR(
         __right.size(), false);
    for (auto x : m.first) containsL[x] = true;
    for (auto x : m.second) containsR[x] = true;
    vector<int> left, right;
    FOR(i, 0, (int)__left.size()) {
        if (!containsL[i]) left.pb(i);
    }
    FOR(i, 0, (int)__right.size()) {
        if (!containsR[i]) right.pb(i);
    }
    return { left, right };
}

};
```

3.14 Max Flow With Ford Fulkerson

```
struct Edge {
    int to, next;
    ll f, c;
    int idx, dir;
    int from;
};

int n, m;
vector<Edge> edges;
vector<int> first;

void addEdge(int a, int b, ll c, int i, int dir) {
    edges.pb({ b, first[a], 0, c, i, dir, a });
    edges.pb({ a, first[b], 0, 0, i, dir, b });
    first[a] = edges.size() - 2;
    first[b] = edges.size() - 1;
}

void init() {
    cin >> n >> m;
    edges.reserve(4 * m);
    first = vector<int>(n, -1);
    FOR(i, 0, m) {
        int a, b, c;
        cin >> a >> b >> c;
        a--; b--;
        addEdge(a, b, c, i, 1);
        addEdge(b, a, c, i, -1);
    }
}

int cur_time = 0;
vector<int> timestamp;

ll dfs(int v, ll flow = OO) {
    if (v == n - 1) return flow;
    timestamp[v] = cur_time;
    for (int e = first[v]; e != -1; e = edges[e].next) {
        if (edges[e].f < edges[e].c && timestamp[edges[e].to] !=
             cur_time) {
            int pushed = dfs(edges[e].to, min(flow, edges[e].c -
                 edges[e].f));
            if (pushed > 0) {
                edges[e].f += pushed;
                edges[e ^ 1].f -= pushed;
                return pushed;
            }
        }
    }
    return 0;
}

ll maxFlow() {
    cur_time = 0;
    timestamp = vector<int>(n, 0);
    ll f = 0, add;
    while (true) {
        cur_time++;
        add = dfs(0);
        if (add > 0) {
            f += add;
        }
        else {
            break;
        }
    }
    return f;
}
```

4 Dynamic Programming

4.1 Optimizations

1. Convex Hull 1:
   - Recurrence: $dp[i] = \min_{j<i}\{dp[j] + b[j] \cdot a[i]\}$
   - Condition: $b[j] \geq b[j+1], a[i] \leq a[i+1]$
   - Complexity: $\mathcal{O}(n^2) \to \mathcal{O}(n)$
2. Convex Hull 2:
   - Recurrence: $dp[i][j] = \min_{k<j}\{dp[i-1][k] + b[k] \cdot a[j]\}$
   - Condition: $b[k] \geq b[k+1], a[j] \leq a[j+1]$
   - Complexity: $\mathcal{O}(kn^2) \to \mathcal{O}(kn)$
3. Divide and Conquer:
   - Recurrence: $dp[i][j] = \min_{k<j}\{dp[i-1][k] + C[k][j]\}$
   - Condition: $A[i][j] \leq A[i][j+1]$
   - Complexity: $\mathcal{O}(kn^2) \to \mathcal{O}(kn\log(n))$
4. Knuth:
   - Recurrence: $dp[i][j] = \min_{i<k<j}\{dp[i][k] + dp[k][j]\} + C[i][j]$
   - Condition: $A[i][j-1] \leq A[i][j] \leq A[i+1][j]$
   - Complexity: $\mathcal{O}(n^3) \to \mathcal{O}(n^2)$

Notes:

- $A[i][j]$ - the smallest k that gives the optimal answer
- $C[i][j]$ - some given cost function

4.2 Convex Hull Trick

```
/*
Let's say we have a relation:
dp[i] = min(dp[j] + h[j+1]*w[i]) for j<=i
Let's set k_j = h[j+1], x = w[i], b_j = dp[j]. We get:
dp[i] = min(b_j+k_j*x) for j<=i.
This is the same as finding a minimum point on a set of lines.
After calculating the value, we add a new line with
k_i = h[i+1] and b_i = dp[i].
*/
struct Line {
    int k;
    int b;

    int eval(int x) {
        return k*x+b;
    }

    int intX(Line& other) {
        int x = b-other.b;
        int y = other.k-k;
        int res = x/y;
        if(x%y != 0) res++;
        return res;
    }
};

struct BagOfLines {
    vector<pair<Line, int>> lines;

    void addLine(int k, int b) {
        Line current = {k, b};
        if(lines.empty()) {
            lines.pb({current, -OO});
            return;
        }
        int x = -OO;
        while(true) {
            auto line = lines.back().first;
            int from = lines.back().second;
            x = line.intX(current);
            if(x > from) break;
            lines.pop_back();
        }
        lines.pb({current, x});
    }

    int findMin(int x) {
        int lo = 0, hi = (int)lines.size()-1;
        while(lo < hi) {
            int mid = (lo+hi+1)/2;
            if(lines[mid].second <= x) {
                lo = mid;
            } else {
                hi = mid-1;
            }
        }
        return lines[lo].first.eval(x);
    }
};
```

4.3 Divide And Conquer

```
/*
Let A[i][j] be the optimal answer for using i objects to satisfy j
        first
requirements.
The recurrence is:
A[i][j] = min(A[i-1][k] + f(i, j, k)) where f is some function that
        denotes the
cost of satisfying requirements from k+1 to j using the i-th
        object.
Consider the recursive function calc(i, jmin, jmax, kmin, kmax),
         that calculates
all A[i][j] for all j in [jmin, jmax] and a given i using known A[i
        -1][*].
*/
void calc(int i, int jmin, int jmax, int kmin, int kmax) {
    if(jmin > jmax) return;
    int jmid = (jmin+jmax)/2;
    // calculate A[i][jmid] naively (for k in kmin...min(jmid,
        kmax){...})
    // let kmid be the optimal k in [kmin, kmax]
    calc(i, jmin, jmid-1, kmin, kmid);
    calc(i, jmid+1, jmax, kmid, kmax);
}

int main() {
    // set initial dp values
    FOR(i, start, k+1){
        calc(i, 0, n-1, 0, n-1);
    }
    cout << dp[k][n-1];
}
```

5 Data Structures

5.1 Trie

```
struct Trie {
    const int ALPHA = 26;
    const char BASE = 'a';
    vector<vector<int>> nextNode;
    vector<int> mark;
    int nodeCount;
    Trie() {
        nextNode = vector<vector<int>>(MAXN, vector<int>(
                ALPHA, -1));
        mark = vector<int>(MAXN, -1);
        nodeCount = 1;
    }
    void insert(const string& s, int id) {
        int curr = 0;
        FOR(i, 0, (int)s.length()) {
            int c = s[i] - BASE;
            if(nextNode[curr][c] == -1) {
                nextNode[curr][c] = nodeCount++;
            }
            curr = nextNode[curr][c];
        }
        mark[curr] = id;
    }

    bool exists(const string& s) {
        int curr = 0;
        FOR(i, 0, (int)s.length()) {
            int c = s[i] - BASE;
            if(nextNode[curr][c] == -1) return false;
            curr = nextNode[curr][c];
        }
        return mark[curr] != -1;
    }
};
```

5.2 Fenwick 2D

```
struct Fenwick2D {
    vector<vector<ll>> bit;
    int n, m;
    Fenwick2D(int _n, int _m) {
        n = _n; m = _m;
        bit = vector<vector<ll>>(n+1, vector<ll>(m+1, 0));
    }
    ll sum(int x, int y) {
        ll ret = 0;
        for (int i = x; i > 0; i -= i & (-i))
            for (int j = y; j > 0; j -= j & (-j))
                ret += bit[i][j];
        return ret;
    }
    ll sum(int x1, int y1, int x2, int y2) {
        return sum(x2, y2) - sum(x2, y1-1) - sum(x1-1, y2) +
            sum(x1-1, y1-1);
    }
    void add(int x, int y, ll delta) {
        for (int i = x; i <= n; i += i & (-i))
            for (int j = y; j <= m; j += j & (-j))
                bit[i][j] += delta;
```

```
    }
};
```

5.3 Treap

```
namespace Treap {
    struct Node {
        Node *l, *r;
        ll key, prio, size;
        Node() {}
        Node(ll key) : key(key), l(nullptr), r(nullptr), size(1) {
            prio = rand() ^ (rand() << 15);
        }
    };

    typedef Node* NodePtr;

    int sz(NodePtr n) {
        return n ? n->size : 0;
    }

    void recalc(NodePtr n) {
        if (!n) return;
        n->size = sz(n->l) + 1 + sz(n->r); // add more
              operations here as needed
    }

    void split(NodePtr tree, ll key, NodePtr& l, NodePtr& r) {
        if (!tree) {
            l = r = nullptr;
        }
        else if (key < tree->key) {
            split(tree->l, key, l, tree->l);
            r = tree;
        }
        else {
            split(tree->r, key, tree->r, r);
            l = tree;
        }
        recalc(tree);
    }

    void merge(NodePtr& tree, NodePtr l, NodePtr r) {
        if (!l || !r) {
            tree = l ? l : r;
        }
        else if (l->prio > r->prio) {
            merge(l->r, l->r, r);
            tree = l;
        }
        else {
            merge(r->l, l, r->l);
            tree = r;
        }
        recalc(tree);
    }

    void insert(NodePtr& tree, NodePtr node) {
        if (!tree) {
            tree = node;
        }
        else if (node->prio > tree->prio) {
            split(tree, node->key, node->l, node->r);
            tree = node;
        }
        else {
            insert(node->key < tree->key ? tree->l : tree->r,
                  node);
        }
        recalc(tree);
    }

    void erase(NodePtr tree, ll key) {
        if (!tree) return;
        if (tree->key == key) {
            merge(tree, tree->l, tree->r);
        }
        else {
            erase(key < tree->key ? tree->l : tree->r, key);
        }
        recalc(tree);
    }

    void print(NodePtr t, bool newline = true) {
        if (!t) return;
        print(t->l, false);
            cout << t->key << " ";
            print(t->r, false);
            if (newline) cout << endl;
        }
}
```

5.4 Segment Tree With Lazy Propagation

```
// Add to segment, get maximum of segment
struct LazySegTree {
    int n;
    vector<ll> t, lazy;
    LazySegTree(int _n) {
        n = _n; t = vector<ll>(4*n, 0); lazy = vector<ll>(4*n,
              0);
    }
    LazySegTree(vector<ll>& arr) {
        n = _n; t = vector<ll>(4*n, 0); lazy = vector<ll>(4*n,
              0);
        build(arr, 1, 0, n-1); // same as in simple SegmentTree
    }
    void push(int v) {
        t[v*2] += lazy[v];
        lazy[v*2] += lazy[v];
        t[v*2+1] += lazy[v];
        lazy[v*2+1] += lazy[v];
        lazy[v] = 0;
    }
    void update(int v, int tl, int tr, int l, int r, ll addend) {
        if (l > r)
            return;
        if (l == tl && tr == r) {
            t[v] += addend;
            lazy[v] += addend;
        } else {
            push(v);
            int tm = (tl + tr) / 2;
            update(v*2, tl, tm, l, min(r, tm), addend);
            update(v*2+1, tm+1, tr, max(l, tm+1), r, addend);
            t[v] = max(t[v*2], t[v*2+1]);
        }
    }

    int query(int v, int tl, int tr, int l, int r) {
        if (l > r || r < tl || l > tr) return -OO;
        if (l <= tl && tr <= r) return t[v];
        push(v);
        int tm = (tl + tr) / 2;
        return max(query(v*2, tl, tm, l, r),
                query(v*2+1, tm+1, tr, l, r));
    }
};

// Multiply every element on seg. by `addend`, query product of
      numbers in seg.
struct ProdTree {
    int n;
    vector<ll> t, lazy;
    ProdTree(int _n) {
        n = _n; t = vector<ll>(4*n, 1); lazy = vector<ll>(4*n,
              1);
    }
    void push(int v, int l, int r) {
        int mid = (l+r)/2;
        t[v*2] = (t[v*2]*pwr(lazy[v], mid-l+1, MOD))%MOD;
        lazy[v*2] = (lazy[v*2]*lazy[v])%MOD;
        t[v*2+1] = (t[v*2+1]*pwr(lazy[v], r-(mid+1)+1, MOD))
              %MOD;
        lazy[v*2+1] = (lazy[v*2+1]*lazy[v])%MOD;
        lazy[v] = 1;
    }
    void update(int v, int tl, int tr, int l, int r, ll addend) {
        if (l > r)
            return;
        if (l == tl && tr == r) {
            t[v] = (t[v]*pwr(addend, tr-tl+1, MOD))%MOD;
            lazy[v] = (lazy[v]*addend)%MOD;
        } else {
            push(v, tl, tr);
            int tm = (tl + tr) / 2;
            update(v*2, tl, tm, l, min(r, tm), addend);
            update(v*2+1, tm+1, tr, max(l, tm+1), r, addend);
            t[v] = (t[v*2] * t[v*2+1]) % MOD;
        }
```

```
    }

    ll query(int v, int tl, int tr, int l, int r) {
        if (l > r || r < tl || l > tr) return 1;
        if (l <= tl && tr <= r) {
            return t[v];
        }
        push(v, tl, tr);
        int tm = (tl + tr) / 2;
        return (query(v*2, tl, tm, l, min(r, tm)) * query(v*2+1,
               tm+1, tr, max(l, tm+1), r))%MOD;
    }
};
```

5.5 Fenwick Tree Range Update And Point Query

```
struct Fenwick {
    vector<ll> tree;
    vector<ll> arr;
    int n;
    Fenwick(vector<ll> _arr) {
        n = _arr.size();
        arr = _arr;
        tree = vector<ll>(n+2, 0);
    }
    void add(int i, ll val) { // arr[i] += val
        for(; i <= n; i += i&(-i)) tree[i] += val;
    }
    void add(int l, int r, ll val) {// arr[l..r] += val
        add(l, val);
        add(r+1, -val);
    }
    ll get(int i) { // arr[i]
        ll sum = arr[i-1]; // zero based
        for(; i > 0; i -= i&(-i)) sum += tree[i];
        return sum; // zero based
    }
};
```

5.6 Segment Tree

```
struct SegmentTree {
    int n;
    vector<ll> t;
    const ll IDENTITY = 0; // OO for min, -OO for max, ...
    ll f(ll a, ll b) {
        return a+b;
    }
    SegmentTree(int _n) {
        n = _n; t = vector<ll>(4*n, IDENTITY);
    }
    SegmentTree(vector<ll>& arr) {
        n = arr.size(); t = vector<ll>(4*n, IDENTITY);
        build(arr, 1, 0, n-1);
    }
    void build(vector<ll>& arr, int v, int tl, int tr) {
        if(tl == tr) { t[v] = arr[tl]; }
        else {
            int tm = (tl+tr)/2;
            build(arr, 2*v, tl, tm);
            build(arr, 2*v+1, tm+1, tr);
            t[v] = f(t[2*v], t[2*v+1]);
        }
    }
    // sum(1, 0, n-1, l, r)
    ll sum(int v, int tl, int tr, int l, int r) {
        if(l > r) return IDENTITY;
        if (l == tl && r == tr) return t[v];
        int tm = (tl+tr)/2;
        return f(sum(2*v, tl, tm, l, min(r, tm)), sum(2*v+1, tm
               +1, tr, max(l, tm+1), r));
    }
    // update(1, 0, n-1, i, v)
    void update(int v, int tl, int tr, int pos, ll newVal) {
        if(tl == tr) { t[v] = newVal; }
        else {
            int tm = (tl+tr)/2;
            if(pos <= tm) update(2*v, tl, tm, pos, newVal);
            else update(2*v+1, tm+1, tr, pos, newVal);
            t[v] = f(t[2*v],t[2*v+1]);
        }
    }
};
```

5.7 Fenwick Tree Point Update And Range Query

```
struct Fenwick {
    vector<ll> tree;
    int n;
    Fenwick(){}
    Fenwick(int _n) {
        n = _n;
        tree = vector<ll>(n+1, 0);
    }
    void add(int i, ll val) { // arr[i] += val
        for(; i <= n; i += i&(-i)) tree[i] += val;
    }
    ll get(int i) { // arr[i]
        return sum(i, i);
    }
    ll sum(int i) { // arr[1]+...+arr[i]
        ll ans = 0;
        for(; i > 0; i -= i&(-i)) ans += tree[i];
        return ans;
    }
    ll sum(int l, int r) {// arr[l]+...+arr[r]
        return sum(r) - sum(l-1);
    }
};
```

5.8 Disjoin Set Union

```
struct DSU {
    vector<int> par;
    vector<int> sz;

    DSU(int n) {
        FOR(i, 0, n) {
            par.pb(i);
            sz.pb(1);
        }
    }

    int find(int a) {
        return par[a] = par[a] == a ? a : find(par[a]);
    }

    bool same(int a, int b) {
        return find(a) == find(b);
    }

    void unite(int a, int b) {
        a = find(a);
        b = find(b);
        if(sz[a] > sz[b]) swap(a, b);
        sz[b] += sz[a];
        par[a] = b;
    }
};
```

5.9 Implicit Treap

```
template <typename T>
struct Node {
    Node* l, *r;
    ll prio, size, sum;
    T val;
    bool rev;
    Node() {}
    Node(T _val) : l(nullptr), r(nullptr), val(_val), size(1), sum(
          _val), rev(false) {
        prio = rand() ^ (rand() << 15);
    }
};
template <typename T>
struct ImplicitTreap {
    typedef Node<T>* NodePtr;
    int sz(NodePtr n) {
        return n ? n->size : 0;
```

```
}
ll getSum(NodePtr n) {
    return n ? n->sum : 0;
}

void push(NodePtr n) {
    if (n && n->rev) {
        n->rev = false;
        swap(n->l, n->r);
        if (n->l) n->l->rev ^= 1;
        if (n->r) n->r->rev ^= 1;
    }
}

void recalc(NodePtr n) {
    if (!n) return;
    n->size = sz(n->l) + 1 + sz(n->r);
    n->sum = getSum(n->l) + n->val + getSum(n->r);
}

void split(NodePtr tree, ll key, NodePtr& l, NodePtr& r) {
    push(tree);
    if (!tree) {
        l = r = nullptr;
    }
    else if (key <= sz(tree->l)) {
        split(tree->l, key, l, tree->l);
        r = tree;
    }
    else {
        split(tree->r, key-sz(tree->l)-1, tree->r, r);
        l = tree;
    }
    recalc(tree);
}

void merge(NodePtr& tree, NodePtr l, NodePtr r) {
    push(l); push(r);
    if (!l || !r) {
        tree = l ? l : r;
    }
    else if (l->prio > r->prio) {
        merge(l->r, l->r, r);
        tree = l;
    }
    else {
        merge(r->l, l, r->l);
        tree = r;
    }
    recalc(tree);
}

void insert(NodePtr& tree, T val, int pos) {
    if (!tree) {
        tree = new Node<T>(val);
        return;
    }
    NodePtr L, R;
    split(tree, pos, L, R);
    merge(L, L, new Node<T>(val));
    merge(tree, L, R);
    recalc(tree);
}

void reverse(NodePtr tree, int l, int r) {
    NodePtr t1, t2, t3;
    split(tree, l, t1, t2);
    split(t2, r - l + 1, t2, t3);
    if(t2) t2->rev = true;
    merge(t2, t1, t2);
    merge(tree, t2, t3);
}

void print(NodePtr t, bool newline = true) {
    push(t);
    if (!t) return;
    print(t->l, false);
    cout << t->val << " ";
    print(t->r, false);
    if (newline) cout << endl;
}

NodePtr fromArray(vector<T> v) {
    NodePtr t = nullptr;
    FOR(i, 0, (int)v.size()) {
        insert(t, v[i], i);
    }
    return t;
}

ll calcSum(NodePtr t, int l, int r) {
    NodePtr L, R;
    split(t, l, L, R);
    NodePtr good;
    split(R, r - l + 1, good, L);
    return getSum(good);
}
};
/* Usage: ImplicitTreap<int> t;
Node<int> tree = t.fromArray(someVector); t.reverse(tree, l, r);
    ...
*/
```

5.10 Fenwick Tree Range Update And Range Query

```
struct RangedFenwick {
    Fenwick F1, F2; // support range query and point update
    RangedFenwick(int _n) {
        F1 = Fenwick(_n+1);
        F2 = Fenwick(_n+1);
    }
    void add(int l, int r, ll v) { // arr[l..r] += v
        F1.add(l, v);
        F1.add(r+1, -v);
        F2.add(l, v*(l-1));
        F2.add(r+1, -v*r);
    }
    ll sum(int i) { // arr[1..i]
        return F1.sum(i)*i-F2.sum(i);
    }
    ll sum(int l, int r) { // arr[l..r]
        return sum(r)-sum(l-1);
    }
};
```

6 Strings

6.1 KMP

```
// Knuth-Morris-Pratt algorithm
vector<int> findOccurences(const string& s, const string& t) {
    int n = s.length();
    int m = t.length();
    string S = s + "#" + t;
    auto pi = prefixFunction(S);
    vector<int> ans;
    FOR(i, n+1, n+m+1) {
        if(pi[i] == n) {
            ans.pb(i-2*n);
        }
    }
    return ans;
}
```

6.2 Hashing

```
struct HashedString {
    const ll A1 = 999999929, B1 = 1000000009, A2 =
        1000000087, B2 = 1000000097;
    vector<ll> A1pwrs, A2pwrs;
    vector<pll> prefixHash;
    HashedString(const string& _s) {
        init(_s);
        calcHashes(_s);
    }
    void init(const string& s) {
        ll a1 = 1;
        ll a2 = 1;
        FOR(i, 0, (int)s.length()+1) {
            A1pwrs.pb(a1);
            A2pwrs.pb(a2);
            a1 = (a1*A1)%B1;
            a2 = (a2*A2)%B2;
        }
```

```
    }
    void calcHashes(const string& s) {
        pll h = {0, 0};
        prefixHash.pb(h);
        for(char c : s) {
            ll h1 = (prefixHash.back().first*A1 + c)%B1;
            ll h2 = (prefixHash.back().second*A2 + c)%B2;
            prefixHash.pb({h1, h2});
        }
    }
    pll getHash(int l, int r) {
        ll h1 = (prefixHash[r+1].first - prefixHash[l].first*A1pwrs
              [r+1-l]) % B1;
        ll h2 = (prefixHash[r+1].second - prefixHash[l].second*
              A2pwrs[r+1-l]) % B2;
        if(h1 < 0) h1 += B1;
        if(h2 < 0) h2 += B2;
        return {h1, h2};
    }
};
```

6.3 Prefix Function Automaton

```
// aut[oldPi][c] = newPi
vector<vector<int>> computeAutomaton(string s) {
    const char BASE = 'a';
    s += "#";
    int n = s.size();
    vector<int> pi = prefixFunction(s);
    vector<vector<int>> aut(n, vector<int>(26));
    for (int i = 0; i < n; i++) {
        for (int c = 0; c < 26; c++) {
            if (i > 0 && BASE + c != s[i])
                aut[i][c] = aut[pi[i-1]][c];
            else
                aut[i][c] = i + (BASE + c == s[i]);
        }
    }
    return aut;
}
vector<int> findOccurs(const string& s, const string& t) {
    auto aut = computeAutomaton(s);
    int curr = 0;
    vector<int> occurs;
    FOR(i, 0, (int)t.length()) {
        int c = t[i]-'a';
        curr = aut[curr][c];
        if(curr == (int)s.length()) {
            occurs.pb(i-s.length()+1);
        }
    }
    return occurs;
}
```

6.4 Prefix Function

```
// pi[i] is the length of the longest proper prefix of the substring
      s[0..i] which is also a suffix
// of this substring
vector<int> prefixFunction(const string& s) {
    int n = (int)s.length();
    vector<int> pi(n);
    for (int i = 1; i < n; i++) {
        int j = pi[i-1];
        while (j > 0 && s[i] != s[j])
            j = pi[j-1];
        if (s[i] == s[j])
            j++;
        pi[i] = j;
    }
    return pi;
}
```

6.5 Aho Corasick Automaton

```
// alphabet size
const int K = 70;

// the indices of each letter of the alphabet
int intVal[256];
void init() {
    int curr = 2;
    intVal[1] = 1;
    for(char c = '0'; c <= '9'; c++, curr++) intVal[(int)c] =
          curr;
    for(char c = 'A'; c <= 'Z'; c++, curr++) intVal[(int)c] =
          curr;
    for(char c = 'a'; c <= 'z'; c++, curr++) intVal[(int)c] =
          curr;
}

struct Vertex {
    int next[K];
    vector<int> marks;
    // ^ this can be changed to int mark = -1, if there will be
          no duplicates
    int p = -1;
    char pch;
    int link = -1;
    int exitLink = -1;
    // ^ exitLink points to the next node on the path of suffix
          links which is marked
    int go[K];

    // ch has to be some small char
    Vertex(int __p=-1, char ch=(char)1) : p(__p), pch(ch) {
        fill(begin(next), end(next), -1);
        fill(begin(go), end(go), -1);
    }
};

vector<Vertex> t(1);

void addString(string const& s, int id) {
    int v = 0;
    for (char ch : s) {
        int c = intVal[(int)ch];
        if (t[v].next[c] == -1) {
            t[v].next[c] = t.size();
            t.emplace_back(v, ch);
        }
        v = t[v].next[c];
    }
    t[v].marks.pb(id);
}

int go(int v, char ch);

int getLink(int v) {
    if (t[v].link == -1) {
        if (v == 0 || t[v].p == 0)
            t[v].link = 0;
        else
            t[v].link = go(getLink(t[v].p), t[v].pch);
    }
    return t[v].link;
}

int getExitLink(int v) {
    if(t[v].exitLink != -1) return t[v].exitLink;
    int l = getLink(v);
    if(l == 0) return t[v].exitLink = 0;
    if(!t[l].marks.empty()) return t[v].exitLink = l;
    return t[v].exitLink = getExitLink(l);
}

int go(int v, char ch) {
    int c = intVal[(int)ch];
    if (t[v].go[c] == -1) {
        if (t[v].next[c] != -1)
            t[v].go[c] = t[v].next[c];
        else
            t[v].go[c] = v == 0 ? 0 : go(getLink(v), ch);
    }
    return t[v].go[c];
}

void walkUp(int v, vector<int>& matches) {
    if(v == 0) return;
    if(!t[v].marks.empty()) {
        for(auto m : t[v].marks) matches.pb(m);
    }
    walkUp(getExitLink(v), matches);
}

// returns the IDs of matched strings.
// Will contain duplicates if multiple matches of the same string
      are found.
```

```
vector<int> walk(const string& s) {
    vector<int> matches;
    int curr = 0;
    for(char c : s) {
        curr = go(curr, c);
        if(!t[curr].marks.empty()) {
            for(auto m : t[curr].marks) matches.pb(m);
        }
        walkUp(getExitLink(curr), matches);
    }
    return matches;
}
/* Usage:
 * addString(strs[i], i);
 * auto matches = walk(text);
 * .. do what you need with the matches - count, check if some
        id exists, etc ..
 * Some applications:
 * - Find all matches: just use the walk function
 * - Find lexicographically smallest string of a given length that
        doesn't match any of the given strings:
 * For each node, check if it produces any matches (it either
        contains some marks or walkUp(v) returns some marks).
 * Remove all nodes which produce at least one match. Do DFS
        in the remaining graph, since none of the remaining
        nodes
 * will ever produce a match and so they're safe.
 * - Find shortest string containing all given strings:
 * For each vertex store a mask that denotes the strings which
        match at this state. Start at (v = root, mask = 0),
 * we need to reach a state (v, mask=2^n-1), where n is the
        number of strings in the set. Use BFS to transition
        between states
 * and update the mask.
 */
```

6.6 Suffix Array

```
vector<int> sortCyclicShifts(string const& s) {
    int n = s.size();
    const int alphabet = 256; // we assume to use the whole
        ASCII range
    vector<int> p(n), c(n), cnt(max(alphabet, n), 0);
    for (int i = 0; i < n; i++)
        cnt[s[i]]++;
    for (int i = 1; i < alphabet; i++)
        cnt[i] += cnt[i-1];
    for (int i = 0; i < n; i++)
        p[--cnt[s[i]]] = i;
    c[p[0]] = 0;
    int classes = 1;
    for (int i = 1; i < n; i++) {
        if (s[p[i]] != s[p[i-1]])
            classes++;
        c[p[i]] = classes - 1;
    }
    vector<int> pn(n), cn(n);
    for (int h = 0; (1 << h) < n; ++h) {
        for (int i = 0; i < n; i++) {
            pn[i] = p[i] - (1 << h);
            if (pn[i] < 0)
                pn[i] += n;
        }
        fill(cnt.begin(), cnt.begin() + classes, 0);
        for (int i = 0; i < n; i++)
            cnt[c[pn[i]]]++;
        for (int i = 1; i < classes; i++)
            cnt[i] += cnt[i-1];
        for (int i = n-1; i >= 0; i--)
            p[--cnt[c[pn[i]]]] = pn[i];
        cn[p[0]] = 0;
        classes = 1;
        for (int i = 1; i < n; i++) {
            pair<int, int> cur = {c[p[i]], c[(p[i] + (1 << h)) % n
                ]};
            pair<int, int> prev = {c[p[i-1]], c[(p[i-1] + (1 << h))
                % n]};
            if (cur != prev)
                ++classes;
            cn[p[i]] = classes - 1;
        }
        c.swap(cn);
    }
    return p;
}
vector<int> constructSuffixArray(string s) {
    s += "$"; // <- this must be smaller than any character in
        s
    vector<int> sorted_shifts = sortCyclicShifts(s);
    sorted_shifts.erase(sorted_shifts.begin());
    return sorted_shifts;
}
```

7 Math

7.1 Chinese Remainder Theorem

Let's say we have some numbers m_i, which are all mutually coprime. Also, let $M = \prod_i m_i$. Then the system of congruences

$$\begin{cases} x \equiv a_1 \pmod{m_1} \\ x \equiv a_2 \pmod{m_2} \\ \dots \\ x \equiv a_k \pmod{m_k} \end{cases}$$

is equivalent to $x \equiv A \pmod{M}$ and there exists a unique number A satisfying $0 \le A \le M$.

Solution for two: $x \equiv a_1 \pmod{m_1}, x \equiv a_2 \pmod{m_2}$. Let $x = a_1 + km_1$. Substituting into the second congruence: $km_1 \equiv a_2 - a_1 \pmod{m_2}$. Then, $k = (m_1)^{-1}_{m_2}(a_2 - a_1) \pmod{m_2}$. and we can easily find x. This can be extended to multiple equations by solving them one-by-one.

If the moduli are not coprime, solve the system $y \equiv 0 \pmod{\frac{m_1}{g}}, y \equiv \frac{a_2 - a_1}{g} \pmod{\frac{m_2}{g}}$ for y. Then let $x \equiv gy + a_1 \pmod{\frac{m_1 m_2}{g}}$.

7.2 Modular Inverse

```
bool invWithEuclid(ll a, ll m, ll& aInv) {
    ll x, y, g;
    if(!solveEqNonNegX(a, m, 1, x, y, g)) return false;
    aInv = x;
    return true;
}
// Works only if m is prime
ll invFermat(ll a, ll m) {
    return pwr(a, m-2, m);
}
// Works only if gcd(a, m) = 1
ll invEuler(ll a, ll m) {
    return pwr(a, phi(m)-1, m);
}
```

7.3 FFT With Modulo

```
bool isGenerator(ll g) {
    if (pwr(g, M - 1) != 1) return false;
    for (ll i = 2; i*i <= M - 1; i++) {
        if ((M - 1) % i == 0) {
            ll q = i;
            if (isPrime(q)) {
                ll p = (M - 1) / q;
                ll pp = pwr(g, p);
                if (pp == 1) return false;
            }
            q = (M - 1) / i;
            if (isPrime(q)) {
                ll p = (M - 1) / q;
                ll pp = pwr(g, p);
                if (pp == 1) return false;
            }
        }
    }
    return true;
}
```

```
namespace FFT {
    ll n;
    vector<ll> r;
    vector<ll> omega;
    ll logN, pwrN;

    void initLogN() {
        logN = 0;
        pwrN = 1;
        while (pwrN < n) {
            pwrN *= 2;
            logN++;
        }
        n = pwrN;
    }

    void initOmega() {
        ll g = 2;
        while (!isGenerator(g)) g++;
        ll G = 1;
        g = pwr(g, (M - 1) / pwrN);
        FOR(i, 0, pwrN) {
            omega[i] = G;
            G *= g;
            G %= M;
        }
    }

    void initR() {
        r[0] = 0;
        FOR(i, 1, pwrN) {
            r[i] = r[i / 2] / 2 + ((i & 1) << (logN - 1));
        }
    }

    void initArrays() {
        r.clear();
        r.resize(pwrN);
        omega.clear();
        omega.resize(pwrN);
    }

    void init(ll n) {
        FFT::n = n;
        initLogN();
        initArrays();
        initOmega();
        initR();
    }

    void fft(ll a[], ll f[]) {
        for (ll i = 0; i < pwrN; i++) {
            f[i] = a[r[i]];
        }
        for (ll k = 1; k < pwrN; k *= 2) {
            for (ll i = 0; i < pwrN; i += 2 * k) {
                for (ll j = 0; j < k; j++) {
                    auto z = omega[j*n / (2 * k)] * f[i + j + k] %
                        M;
                    f[i + j + k] = f[i + j] - z;
                    f[i + j] += z;
                    f[i + j + k] %= M;
                    if (f[i + j + k] < 0) f[i + j + k] += M;
                    f[i + j] %= M;
                }
            }
        }
    }
}
```

7.4 Extended Euclidean Algorithm

```
// ax+by=gcd(a,b)
void solveEq(ll a, ll b, ll& x, ll& y, ll& g) {
    if(b==0) {
        x = 1;
        y = 0;
        g = a;
        return;
    }
    ll xx, yy;
    solveEq(b, a%b, xx, yy, g);
    x = yy;
    y = xx-yy*(a/b);
}
// ax+by=c
bool solveEq(ll a, ll b, ll c, ll& x, ll& y, ll& g) {
    solveEq(a, b, x, y, g);
    if(c%g != 0) return false;
    x *= c/g; y *= c/g;
    return true;
}
// Finds a solution (x, y) so that x >= 0 and x is minimal
bool solveEqNonNegX(ll a, ll b, ll c, ll& x, ll &y, ll& g) {
    if(!solveEq(a, b, c, x, y, g)) return false;
    ll k = x*g/b;
    x = x - k*b/g;
    y = y + k*a/g;
    if(x < 0) {
        x += b/g;
        y -= a/g;
    }
    return true;
}
```

7.5 Formulas

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$; $\sum_{i=1}^{n} i^2 = \frac{n(2n+1)(n+1)}{6}$; $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$; $\sum_{i=1}^{n} i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$; $\sum_{i=a}^{b} c^i = \frac{c^{b+1}-c^a}{c-1}, c \neq 1$; $\sum_{i=1}^{n} a_1 + (i-1)d = \frac{n(a_1+a_n)}{2}$; $\sum_{i=1}^{n} a_1 r^{i-1} = \frac{a_1(1-r^n)}{1-r}, r \neq 1$; $\sum_{i=1}^{\infty} ar^{i-1} = \frac{a_1}{1-r}, |r| \leq 1$.

7.6 Factorization With Sieve

```
// Use linear sieve to calculate minDiv
vector<pll> factorize(ll x) {
    vector<pll> res;
    ll prev = -1;
    ll cnt = 0;
    while(x != 1) {
        ll d = minDiv[x];
        if(d == prev) {
            cnt++;
        } else {
            if(prev != -1) res.pb({prev, cnt});
            prev = d;
            cnt = 1;
        }
        x /= d;
    }
    res.pb({prev, cnt});
    return res;
}
```

7.7 FFT

```
namespace FFT {
    int n;
    vector<int> r;
    vector<complex<ld>> omega;
    int logN, pwrN;

    void initLogN() {
        logN = 0;
        pwrN = 1;
        while (pwrN < n) {
            pwrN *= 2;
            logN++;
        }
        n = pwrN;
    }

    void initOmega() {
        FOR(i, 0, pwrN) {
            omega[i] = { cos(2 * i*PI / n), sin(2 * i*PI / n) };
        }
    }

    void initR() {
        r[0] = 0;
        FOR(i, 1, pwrN) {
            r[i] = r[i / 2] / 2 + ((i & 1) << (logN - 1));
        }
    }
```

```
    void initArrays() {
        r.clear();
        r.resize(pwrN);
        omega.clear();
        omega.resize(pwrN);
    }

    void init(int n) {
        FFT::n = n;
        initLogN();
        initArrays();
        initOmega();
        initR();
    }

    void fft(complex<ld> a[], complex<ld> f[]) {
        FOR(i, 0, pwrN) {
            f[i] = a[r[i]];
        }
        for (ll k = 1; k < pwrN; k *= 2) {
            for (ll i = 0; i < pwrN; i += 2 * k) {
                for (ll j = 0; j < k; j++) {
                    auto z = omega[j*n / (2 * k)] * f[i + j + k];
                    f[i + j + k] = f[i + j] - z;
                    f[i + j] += z;
                }
            }
        }
    }
}
```

All other solutions can be found like this:

$$x' = x - k\frac{b}{g}, y' = y + k\frac{a}{g}, k \in \mathbb{Z}$$

7.8 Sprague Grundy Theorem

We have a game which fulfills the following requirements:

- There are two players who move alternately.
- The game consists of states, and the possible moves in a state do not depend on whose turn it is.
- The game ends when a player cannot make a move.
- The game surely ends sooner or later.
- The players have complete information about the states and allowed moves, and there is no randomness in the game.

Grundy Numbers. The idea is to calculate Grundy numbers for each game state. It is calculated like so: $mex(\{g_1, g_2, ..., g_n\})$, where $g_1, g_2, ..., g_n$ are the Grundy numbers of the states which are reachable from the current state. mex gives the smallest nonnegative number that is not in the set ($mex(\{0, 1, 3\}) = 2, mex(\emptyset) = 0$). If the Grundy number of a state is 0, then this state is a losing state. Otherwise it's a winning state.

Grundy's Game. Sometimes a move in a game divides the game into subgames that are independent of each other. In this case, the Grundy number of a game state is $mex(\{g_1, g_2, ..., g_n\}), g_k = a_{k,1} \oplus a_{k,2} \oplus ... \oplus a_{k,m}$ meaning that move k divides the game into m subgames whose Grundy numbers are $a_{i,j}$.

Example. We have a heap with n sticks. On each turn, the player chooses a heap and divides it into two nonempty heaps such that the heaps are of different size. The player who makes the last move wins the game. Let $g(n)$ denote the Grundy number of a heap of size n. The Grundy number can be calculated by going though all possible ways to divide the heap into two parts. E.g. $g(8) = mex(\{g(1) \oplus g(7), g(2) \oplus g(6), g(3) \oplus g(5)\})$. Base case: $g(1) = g(2) = 0$, because these are losing states.

7.9 Euler Totient Function

```
// Number of numbers x < n so that gcd(x, n) = 1
ll phi(ll n) {
    if(n == 1) return 1;
    auto f = factorize(n);
    ll res = n;
    for(auto p : f) {
        res = res - res/p.first;
    }
    return res;
}
```

7.10 Simpson Integration

```
const int N = 1000 * 1000; // number of steps (already
        multiplied by 2)

double simpsonIntegration(double a, double b){
    double h = (b - a) / N;
    double s = f(a) + f(b); // a = x_0 and b = x_2n
    for (int i = 1; i <= N - 1; ++i) {
        double x = a + h * i;
        s += f(x) * ((i & 1) ? 4 : 2);
    }
    s *= h / 3;
    return s;
}
```

7.11 Burnside's Lemma

Let G be a finite group that acts on a set X. For each g in G let X^g denote the set of elements in X that are fixed by g. Burnside's lemma asserts the following formula for the number of orbits:

$$|X/G| = \frac{1}{|G|}\sum_{g\in G}|X^g|.$$

Example. Coloring a cube with three colors.

Let X be the set of 3^6 possible face color combinations. Let's count the sizes of the fixed sets for each of the 24 rotations:

- one 0-degree rotation which leaves all 3^6 elements of X unchanged
- six 90-degree face rotations, each of which leaves 3^3 elements of X unchanged
- three 180-degree face rotation, each of which leaves 3^4 elements of X unchanged
- eight 120-degree vertex rotations, each of which leaves 3^2 elements of X unchanged
- six 180-degree edge rotations, each of which leaves 3^3 elements of X unchanged

The average is then $\frac{1}{24}(3^6 + 6\cdot 3^3 + 3\cdot 3^4 + 8\cdot 3^2 + 6\cdot 3^3) = 57$. For n colors: $\frac{1}{24}(n^6 + 3n^4 + 12n^3 + 8n^2)$.

Example. Coloring a circular stripe of n cells with two colors.

X is the set of all colored striped (it has 2^n elements), G is the group of rotations (n elements - by 0 cells, by 1 cell, ..., by $(n-1)$ cells). Let's fix some K and find the number of stripes that are fixed by the rotation by K cells. If a stripe becomes itself after rotation by K cells, then its 1st cell must have the same color as its $(1 + K \mod n)$-th cell, which is in turn the same as its $(1 + 2K \mod n)$-th cell, etc., until $mK \mod n = 0$. This will happen when $m = n/gcd(K, n)$. Therefore, we have $n/gcd(K, n)$ cells that must all be of the same color. The same will happen when starting from the second cell and so on. Therefore, all cells are separated into $gcd(K, n)$ groups, with each group being of one color, and that yields $2^{gcd(K,n)}$ choices. That's why the answer to the original problem is $\frac{1}{n}\sum_{k=0}^{n-1} 2^{gcd(k,n)}$.

7.12 Linear Sieve

```
ll minDiv[MAXN+1];
vector<ll> primes;

void sieve(ll n){
    FOR(k, 2, n+1){
        minDiv[k] = k;
    }
    FOR(k, 2, n+1) {
        if(minDiv[k] == k) {
            primes.pb(k);
        }
        for(auto p : primes) {
            if(p > minDiv[k]) break;
            if(p*k > n) break;
            minDiv[p*k] = p;
        }
    }
}
```

7.13 Big Integer Multiplication With FFT

```
complex<ld> a[MAX_N], b[MAX_N];
complex<ld> fa[MAX_N], fb[MAX_N], fc[MAX_N];
complex<ld> cc[MAX_N];

string mul(string as, string bs) {
    int sgn1 = 1;
    int sgn2 = 1;
    if (as[0] == '-') {
        sgn1 = -1;
        as = as.substr(1);
    }
    if (bs[0] == '-') {
        sgn2 = -1;
        bs = bs.substr(1);
    }
    int n = as.length() + bs.length() + 1;
    FFT::init(n);
    FOR(i, 0, FFT::pwrN) {
        a[i] = b[i] = fa[i] = fb[i] = fc[i] = cc[i] = 0;
    }
    FOR(i, 0, as.size()) {
        a[i] = as[as.size() - 1 - i] - '0';
    }
    FOR(i, 0, bs.size()) {
        b[i] = bs[bs.size() - 1 - i] - '0';
    }
    FFT::fft(a, fa);
    FFT::fft(b, fb);
    FOR(i, 0, FFT::pwrN) {
        fc[i] = fa[i] * fb[i];
    }
    // turn [0,1,2,...,n-1] into [0, n-1, n-2, ..., 1]
    FOR(i, 1, FFT::pwrN) {
        if (i < FFT::pwrN - i) {
            swap(fc[i], fc[FFT::pwrN - i]);
        }
    }
    FFT::fft(fc, cc);
    ll carry = 0;
    vector<int> v;
    FOR(i, 0, FFT::pwrN) {
        int num = round(cc[i].real() / FFT::pwrN) + carry;
        v.pb(num % 10);
        carry = num / 10;
    }
    while (carry > 0) {
        v.pb(carry % 10);
        carry /= 10;
    }
    reverse(v.begin(), v.end());
    bool start = false;
    ostringstream ss;
    bool allZero = true;
    for (auto x : v) {
        if (x != 0) {
            allZero = false;
            break;
        }
    }
    if (sgn1*sgn2 < 0 && !allZero) ss << "-";
    for (auto x : v) {
        if (x == 0 && !start) continue;
        start = true;
        ss << abs(x);
    }
    if (!start) ss << 0;
    return ss.str();
}
```

7.14 Gaussian Elimination

```
// The last column of a is the right-hand side of the system.
// Returns 0, 1 or oo - the number of solutions.
// If at least one solution is found, it will be in ans
int gauss (vector < vector<ld> > a, vector<ld> & ans) {
    int n = (int) a.size();
    int m = (int) a[0].size() - 1;

    vector<int> where (m, -1);
    for (int col=0, row=0; col<m && row<n; ++col) {
        int sel = row;
        for (int i=row; i<n; ++i)
            if (abs (a[i][col]) > abs (a[sel][col]))
                sel = i;
        if (abs (a[sel][col]) < eps)
            continue;
        for (int i=col; i<=m; ++i)
            swap (a[sel][i], a[row][i]);
        where[col] = row;

        for (int i=0; i<n; ++i)
            if (i != row) {
                ld c = a[i][col] / a[row][col];
                for (int j=col; j<=m; ++j)
                    a[i][j] -= a[row][j] * c;
            }
        ++row;
    }

    ans.assign (m, 0);
    for (int i=0; i<m; ++i)
        if (where[i] != -1)
            ans[i] = a[where[i]][m] / a[where[i]][i];
    for (int i=0; i<n; ++i) {
        ld sum = 0;
        for (int j=0; j<m; ++j)
            sum += ans[j] * a[i][j];
        if (abs (sum - a[i][m]) > eps)
            return 0;
    }

    for (int i=0; i<m; ++i)
        if (where[i] == -1)
            return oo;
    return 1;
}
```

8 Misc

8.1 Builtin GCC Stuff

- __builtin_clz(x): the number of zeros at the beginning of the bit representation.
- __builtin_ctz(x): the number of zeros at the end of the bit representation.
- __builtin_popcount(x): the number of ones in the bit representation.
- __builtin_parity(x): the parity of the number of ones in the bit representation.
- __gcd(x, y): the greatest common divisor of two numbers.
- __int128_t: the 128-bit integer type. Does not support input/output.

8.2 Binary Exponentiation

```
ll pwr(ll a, ll b, ll m) {
    if(a == 1) return 1;
    if(b == 0) return 1;
    a %= m;
    ll res = 1;
    while (b > 0) {
        if (b & 1)
            res = res * a % m;
        a = a * a % m;
        b >>= 1;
    }
    return res;
}
```

8.3 Big Integer

```
const int base = 1000000000;
const int base_digits = 9;
struct bigint {
    vector<int> a;
    int sign;
    int size() {
        if (a.empty()) return 0;
        int ans = (a.size() - 1) * base_digits;
        int ca = a.back();
        while (ca) ans++, ca /= 10;
        return ans;
    }
    bigint operator^(const bigint &v) {
        bigint ans = 1, x = *this, y = v;
        while (!y.isZero()) {
            if (y % 2) ans *= x;
            x *= x, y /= 2;
        }
        return ans;
    }
    string to_string() {
        stringstream ss;
        ss << *this;
        string s;
        ss >> s;
        return s;
    }
    int sumof() {
        string s = to_string();
        int ans = 0;
        for (auto c : s) ans += c - '0';
        return ans;
    }
    bigint() : sign(1) {}
    bigint(long long v) {
        *this = v;
    }
    bigint(const string &s) {
        read(s);
    }
    void operator=(const bigint &v) {
        sign = v.sign;
        a = v.a;
    }
    void operator=(long long v) {
        sign = 1;
        a.clear();
        if (v < 0)
            sign = -1, v = -v;
        for (; v > 0; v = v / base)
            a.push_back(v % base);
    }
    bigint operator+(const bigint &v) const {
        if (sign == v.sign) {
            bigint res = v;
            for (int i = 0, carry = 0; i < (int)max(a.size(), v.a.
                size()) || carry; ++i) {
                if (i == (int)res.a.size()) res.a.push_back(0);
                res.a[i] += carry + (i < (int)a.size() ? a[i] : 0);
                carry = res.a[i] >= base;
                if (carry) res.a[i] -= base;
            }
            return res;
        }
        return *this - (-v);
    }
    bigint operator-(const bigint &v) const {
        if (sign == v.sign) {
            if (abs() >= v.abs()) {
                bigint res = *this;
                for (int i = 0, carry = 0; i < (int)v.a.size() ||
                    carry; ++i) {
                    res.a[i] -= carry + (i < (int)v.a.size() ? v.a[i] :
                        0);
                    carry = res.a[i] < 0;
                    if (carry) res.a[i] += base;
                }
                res.trim();
                return res;
            }
            return -(v - *this);
        }
        return *this + (-v);
    }
    void operator*=(int v) {
        if (v < 0) sign = -sign, v = -v;
        for (int i = 0, carry = 0; i < (int)a.size() || carry; ++i) {
            if (i == (int)a.size()) a.push_back(0);
            long long cur = a[i] * (long long)v + carry;
            carry = (int)(cur / base);
            a[i] = (int)(cur % base);
        }
        trim();
    }
    bigint operator*(int v) const {
        bigint res = *this;
        res *= v;
        return res;
    }
    void operator*=(long long v) {
        if (v < 0) sign = -sign, v = -v;
        for (int i = 0, carry = 0; i < (int)a.size() || carry; ++i) {
            if (i == (int)a.size()) a.push_back(0);
            long long cur = a[i] * (long long)v + carry;
            carry = (int)(cur / base);
            a[i] = (int)(cur % base);
        }
        trim();
    }
    bigint operator*(long long v) const {
        bigint res = *this;
        res *= v;
        return res;
    }
    friend pair<bigint, bigint> divmod(const bigint &a1, const
        bigint &b1) {
        int norm = base / (b1.a.back() + 1);
        bigint a = a1.abs() * norm;
        bigint b = b1.abs() * norm;
        bigint q, r;
        q.a.resize(a.a.size());
        for (int i = a.a.size() - 1; i >= 0; i--) {
            r *= base;
            r += a.a[i];
            int s1 = r.a.size() <= b.a.size() ? 0 : r.a[b.a.size()];
            int s2 = r.a.size() <= b.a.size() - 1 ? 0 : r.a[b.a.size()
                - 1];
            int d = ((long long)base * s1 + s2) / b.a.back();
            r -= b * d;
            while (r < 0) r += b, --d;
```

```cpp
        q.a[i] = d;
    }
    q.sign = a1.sign * b1.sign;
    r.sign = a1.sign;
    q.trim();
    r.trim();
    return make_pair(q, r / norm);
}
bigint operator/(const bigint &v) const {
    return divmod(*this, v).first;
}
bigint operator%(const bigint &v) const {
    return divmod(*this, v).second;
}
void operator/=(int v) {
    if (v < 0) sign = -sign, v = -v;
    for (int i = (int)a.size() - 1, rem = 0; i >= 0; --i) {
        long long cur = a[i] + rem * (long long)base;
        a[i] = (int)(cur / v);
        rem = (int)(cur % v);
    }
    trim();
}
bigint operator/(int v) const {
    bigint res = *this;
    res /= v;
    return res;
}
int operator%(int v) const {
    if (v < 0) v = -v;
    int m = 0;
    for (int i = a.size() - 1; i >= 0; --i)
        m = (a[i] + m * (long long)base) % v;
    return m * sign;
}
void operator+=(const bigint &v) {
    *this = *this + v;
}
void operator-=(const bigint &v) {
    *this = *this - v;
}
void operator*=(const bigint &v) {
    *this = *this * v;
}
void operator/=(const bigint &v) {
    *this = *this / v;
}
bool operator<(const bigint &v) const {
    if (sign != v.sign) return sign < v.sign;
    if (a.size() != v.a.size())
        return a.size() * sign < v.a.size() * v.sign;
    for (int i = a.size() - 1; i >= 0; i--)
        if (a[i] != v.a[i])
            return a[i] * sign < v.a[i] * sign;
    return false;
}
bool operator>(const bigint &v) const {
    return v < *this;
}
bool operator<=(const bigint &v) const {
    return !(v < *this);
}
bool operator>=(const bigint &v) const {
    return !(*this < v);
}
bool operator==(const bigint &v) const {
    return !(*this < v) && !(v < *this);
}
bool operator!=(const bigint &v) const {
    return *this < v || v < *this;
}
void trim() {
    while (!a.empty() && !a.back()) a.pop_back();
    if (a.empty()) sign = 1;
}
bool isZero() const {
    return a.empty() || (a.size() == 1 && !a[0]);
}
bigint operator-() const {
    bigint res = *this;
    res.sign = -sign;
    return res;
}
bigint abs() const {
    bigint res = *this;
    res.sign *= res.sign;
    return res;
}
long long longValue() const {
    long long res = 0;
    for (int i = a.size() - 1; i >= 0; i--) res = res * base + a[i
        ];
    return res * sign;
}
friend bigint gcd(const bigint &a, const bigint &b) {
    return b.isZero() ? a : gcd(b, a % b);
}
friend bigint lcm(const bigint &a, const bigint &b) {
    return a / gcd(a, b) * b;
}
void read(const string &s) {
    sign = 1;
    a.clear();
    int pos = 0;
    while (pos < (int)s.size() && (s[pos] == '-' || s[pos] ==
        '+')) {
        if (s[pos] == '-') sign = -sign;
        ++pos;
    }
    for (int i = s.size() - 1; i >= pos; i -= base_digits) {
        int x = 0;
        for (int j = max(pos, i - base_digits + 1); j <= i; j
            ++)
            x = x * 10 + s[j] - '0';
        a.push_back(x);
    }
    trim();
}
friend istream &operator>>(istream &stream, bigint &v) {
    string s;
    stream >> s;
    v.read(s);
    return stream;
}
friend ostream &operator<<(ostream &stream, const bigint
    &v) {
    if (v.sign == -1) stream << '-';
    stream << (v.a.empty() ? 0 : v.a.back());
    for (int i = (int)v.a.size() - 2; i >= 0; --i)
        stream << setw(base_digits) << setfill('0') << v.a[i
            ];
    return stream;
}
static vector<int> convert_base(const vector<int> &a, int
    old_digits, int new_digits) {
    vector<long long> p(max(old_digits, new_digits) + 1);
    p[0] = 1;
    for (int i = 1; i < (int)p.size(); i++)
        p[i] = p[i - 1] * 10;
    vector<int> res;
    long long cur = 0;
    int cur_digits = 0;
    for (int i = 0; i < (int)a.size(); i++) {
        cur += a[i] * p[cur_digits];
        cur_digits += old_digits;
        while (cur_digits >= new_digits) {
            res.push_back(int(cur % p[new_digits]));
            cur /= p[new_digits];
            cur_digits -= new_digits;
        }
    }
    res.push_back((int)cur);
    while (!res.empty() && !res.back()) res.pop_back();
    return res;
}
typedef vector<long long> vll;
static vll karatsubaMultiply(const vll &a, const vll &b) {
    int n = a.size();
    vll res(n + n);
    if (n <= 32) {
        for (int i = 0; i < n; i++)
            for (int j = 0; j < n; j++)
                res[i + j] += a[i] * b[j];
        return res;
    }
    int k = n >> 1;
    vll a1(a.begin(), a.begin() + k);
    vll a2(a.begin() + k, a.end());
    vll b1(b.begin(), b.begin() + k);
    vll b2(b.begin() + k, b.end());

    vll a1b1 = karatsubaMultiply(a1, b1);
    vll a2b2 = karatsubaMultiply(a2, b2);

    for (int i = 0; i < k; i++) a2[i] += a1[i];
    for (int i = 0; i < k; i++) b2[i] += b1[i];
```

```
        vll r = karatsubaMultiply(a2, b2);
        for (int i = 0; i < (int)a1b1.size(); i++) r[i] -= a1b1[i];
        for (int i = 0; i < (int)a2b2.size(); i++) r[i] -= a2b2[i];

        for (int i = 0; i < (int)r.size(); i++) res[i + k] += r[i];
        for (int i = 0; i < (int)a1b1.size(); i++) res[i] += a1b1[i
            ];
        for (int i = 0; i < (int)a2b2.size(); i++) res[i + n] +=
            a2b2[i];
        return res;
    }
    bigint operator*(const bigint &v) const {
        vector<int> a6 = convert_base(this->a, base_digits, 6);
        vector<int> b6 = convert_base(v.a, base_digits, 6);
        vll x(a6.begin(), a6.end());
        vll y(b6.begin(), b6.end());
        while (x.size() < y.size()) x.push_back(0);
        while (y.size() < x.size()) y.push_back(0);
        while (x.size() & (x.size() - 1)) x.push_back(0), y.
            push_back(0);
        vll c = karatsubaMultiply(x, y);
        bigint res;
        res.sign = sign * v.sign;
        for (int i = 0, carry = 0; i < (int)c.size(); i++) {
            long long cur = c[i] + carry;
            res.a.push_back((int)(cur % 1000000));
            carry = (int)(cur / 1000000);
        }
        res.a = convert_base(res.a, 6, base_digits);
        res.trim();
        return res;
    }
};
```

8.4 Mo's Algorithm

Mo's algorithm processes a set of range queries on a static array. Each query is to calculate something base on the array values in a range $[a, b]$. The queries have to be known in advance. Let's divide the array into blocks of size $k = O(\sqrt{n})$. A query $[a_1, b_1]$ is processed before query $[a_2, b_2]$ if $\lfloor \frac{a_1}{k} \rfloor < \lfloor \frac{a_2}{k} \rfloor$ or $\lfloor \frac{a_1}{k} \rfloor = \lfloor \frac{a_2}{k} \rfloor$ and $b_1 < b_2$.

Example problem: counting number of distinct values in a range. We can process the queries in the described order and keep an array count, which knows how many times a certain value has appeared. When moving the boundaries back and forth, we either increase count$[x_i]$ or decrease it. According to value of it, we will know how the number of distinct values has changed (e.g. if count$[x_i]$ has just become 1, then we add 1 to the answer, etc.).

8.5 Ternary Search

```
double ternary_search(double l, double r) {
    while (r - l > eps) {
        double m1 = l + (r - l) / 3;
        double m2 = r - (r - l) / 3;
        double f1 = f(m1);
        double f2 = f(m2);
        if (f1 < f2)
            l = m1;
        else
            r = m2;
    }
    return f(l); //return the maximum of f(x) in [l, r]
}
```