Foundations 2 **Heriot Watt University**
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Foundations 2 Assignment: Part 4 3. April 2013

1 Part 4

`part-4.c` is the main file for the third part of the assignment.

part-4.c expects there to be a file called input.json to run, prints an appropriate error if this is not the case, however. Output is printed to the file **output.txt**.

The new method `diagonalize` has been added to deal with diagonalization of functions. The function takes three arguments, a function that maps to a function: V1, an enumerator which takes the value if V1(x)(y) and transforms it and a nullReturn which is used as a marker when V1(x)(y) does not exist.

In the case of diagonalization, $x = y$ so that it is the diagonals of what can be visualised as a two dimensional array. The function `diagonalize` returns a function that represents a unique row for the set of V1.

1.1 Conditions

For $F \notin ran(V1)$ to hold $V2(V1(i)(i)) \neq V1(i)(i)$ where $i \in dom(V1)$ must be true.

Another issue with the current implementation is in the choice of null value placement. It is quite possible to select a value for this $E3$ such that a row exists where that is the case.

It would not be possible to use this implementation in the case of infinite sets, as there is no halting. In fact there is no actual checking if the implementation is in fact diagonalised. It simply returns F. If a check for F was added into the program it would be possible to handle infinite inputs. As subsets $ran(V1)$ could be used to create F and checked if they are contained. The program could then halt if F is in fact part of the value of that particular subset.

1.2 Relation to Cantor's Diagonalization Argument

Cantor's theorem states that the cardinality of the *powerset* of a countable set is in the cardinality of an uncountable infinite. This can be proved by creating a function whose value is an element of the powerset of an infinite set such that the argument must both be contained and not contained in the corresponding set.

Cantor's method uses an enumeration function, F on some set, $V1$ such that $F \in dom(V1) \to T$ where T is some set. It holds that there is some k such that for all elements $F(k)(i)$ it holds that $F(k)(i) \neq F(i)(i)$ where $i \in dom(V1)$.

The method implemented here is similar as it creates a function $G \in dom(V1) \to Q$. This method does differ however as it is applied on finite sets, so a placeholder is used if values are not contained within a set. For Cantor's method it is all ρS whose elements can be of infinite cardinality. Because of the use of a placeholder, the method does not always hold. The conditions for this to hold are that $V2(V1(i)(i)) \neq V1(i)(i)$ where $i \in dom(V1) \wedge i \in dom(ran(V1))$.