UNIVERSITY OF TORONTO

HYPERBOLIC SURFACES

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Contents

Goal
Prove the Nielson Realization Problem.

Kerckhoff (paper in annals)

Sources

- Katok - Fuchsian groups.
- Farb and Margalit - A primer on mapping class groups
- Hubbard - Teichmüller Theory Vol 1

Course webpage
math.toronto.edu/mbourque

Chapter 0 : Overview

Classification of close oriented surfaces

Surfaces have symmetries and we can try to classify them.

Any finite subgroup of $\mathrm{Isom}^+(S^2)$ isormorphic to $\mathbb{Z}_n$, D_n, Rot(tetrahedron), Rot(cube) (which is equal to Rot(octahedron) or Rot(dodecatedron).

Isom^+ is the set of orientation preserving isometries, S^2 is the 2-dimensional sphere.

What about the symmetry of the torus?

We can have $\mathbb{Z}_4$ symmetries or $\mathbb{Z}_6$ symmetries. That's it!

What about hyperbolic surfaces?

Q1: What is the largest number of orientation preserving isometries of a closed hyperbolic surface of genus g?

$[\mathrm{Isom}(S) : \mathrm{Isom}^+(S)] = 2$.

A1: by a theorem of Hurwitz, $|\mathrm{Isom}^+(S)| \leq 42 \underbrace{|\chi(S)|}_{|2-2g|}$

The bound is attained by infinitely many g but not all.

Q2: What is the largest possible order of such an isometry?

A2: $4g+2$ - this is realized by taking a regular 4g + 2 - g on $\mathbb{H}$ with opposite sides glued by isometries.