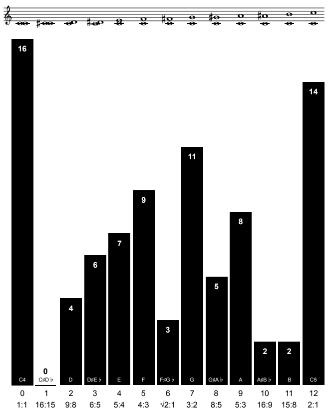
Prelude



Tonic Affinity

s	Name	Ratio	Example	Affinity
0	Tonic	1 : 1	$C4 : C4$	16
1	Minor 2nd	16 : 15	$C\sharp : C4$	0
2	Major 2nd	9 : 8	$D : C4$	4
3	Minor 3rd	6 : 5	$D\sharp : C4$	6
4	Major 3rd	5 : 4	$E : C4$	7
5	Perfect 4th	4 : 3	$F : C4$	9
6	Tritone	$\sqrt{2} : 1$	$F\sharp : C4$	3
7	Perfect 5th	3 : 2	$G : C4$	11
8	Minor 6th	8 : 5	$G\sharp : C4$	5
9	Major 6th	5 : 3	$A : C4$	8
10	Minor 7th	16 : 9	$A\sharp : C4$	2
11	Major 7th	15 : 8	$B : C4$	2
12	Octave	2 : 1	$C5 : C4$	14

Affinity Definition:

$\frac{\omega_i}{\omega_j} = \prod_{p \in \mathbb{P}} p^{\alpha_{sp}},\ s = |i-j|$ $\quad d_s = \sum_{p \in \mathbb{P}} |p\alpha_{sp}|$ $\quad a_s = d_{max} - d_s$

Minor 2nd Example: $i = 1,\ j = 0,\ s = 1$

$\alpha_{1p} = \{\alpha_{11} = 0, \alpha_{12} = 4, \alpha_{13} = -1, \alpha_{15} = -1, \alpha_{17} = 0\}$

$\frac{\omega_1}{\omega_0} = \frac{16}{15}$ $\quad d_1 = d_{max} = 16$ $\quad a_1 = 16 - 16 = 0$

3D Example of 5D Prime Space
Minor 2nd

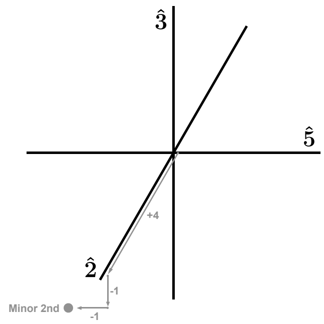
$$\frac{\omega_1}{\omega_0} = \frac{16}{15}$$

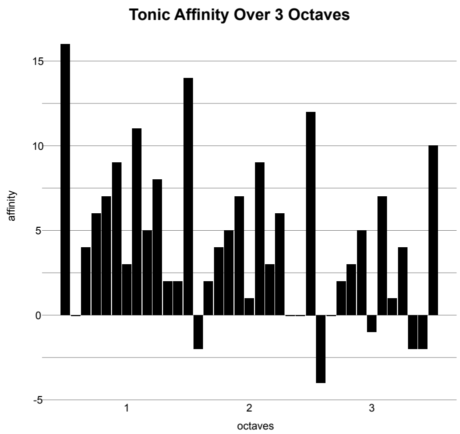
$$\alpha_{1p} = \{\alpha_{11} = 0, \alpha_{12} = 4, \alpha_{13} = -1, \alpha_{15} = -1, \alpha_{17} = 0\}$$

$$\underbrace{\left|(4 \times 2)\ \hat{\mathbf{2}} + (-1 \times 3)\ \hat{\mathbf{3}} + (-1 \times 5)\ \hat{\mathbf{5}}\right|}_{\text{L1 Norm}} = 16$$

$$d_1 = d_{max} = 16$$

$$a_1 = 16 - 16 = 0$$




Tonic Affinity Over 3 Octaves
affinity
15
10
5
0
-5
1
2
3
octaves

Prelude:
Where's the Music?

Octave Affinity

s	Name	Ratio	Example	Affinity
12	12 12	1 : 1	$C5 : C5$	16
11	11 12	15 : 16	$B : C5$	0
10	10 12	8 : 9	$B\flat : C5$	4
9	9 12	5 : 6	$A : C5$	6
8	8 12	4 : 5	$A\flat : C5$	7
7	7 12	3 : 4	$G : C5$	9
6	6 12	$1 : \sqrt{2}$	$G\flat : C5$	3
5	5 12	2 : 3	$F : C5$	11
4	4 12	5 : 8	$E : C5$	5
3	3 12	3 : 5	$E\flat : C5$	8
2	2 12	9 : 16	$D : C5$	2
1	1 12	8 : 15	$D\flat : C5$	2
0	0 12	1 : 2	$C4 : C5$	14

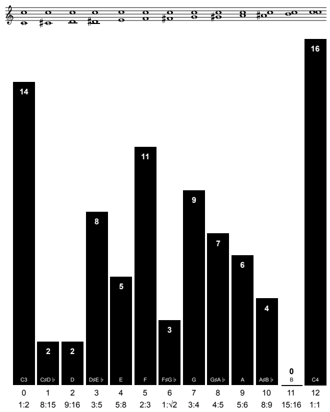
Affinity Definition:

$$\frac{\omega_i}{\omega_j} = \prod_{p \in \mathbb{P}} p^{\alpha_{sp}},\ s = |i - j| \qquad d_s = \sum_{p \in \mathbb{P}} |p\alpha_{sp}| \qquad a_s = d_{max} - d_s$$

'1 12' Example: $i = 1,\ j = 12,\ s = 11$

$\alpha_{11\ p} = \{\alpha_{11\ 1} = 0,\ \alpha_{11\ 2} = 3,\ \alpha_{11\ 3} = -1,\ \alpha_{11\ 5} = -1,\ \alpha_{11\ 7} = 0,\ \}$

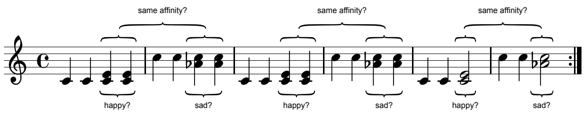
$$\frac{\omega_1}{\omega_0} = \frac{8}{15} \qquad d_{11} = 14 \qquad a_{11} = 16 - 14 = 2$$



does the absolute value
match experimental data?

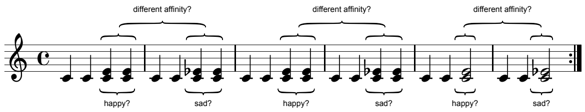
$$a_s = d_{max} - \sum_{p \in \mathbb{P}} |p\alpha_{sp}|$$

0:4 versus 8:12



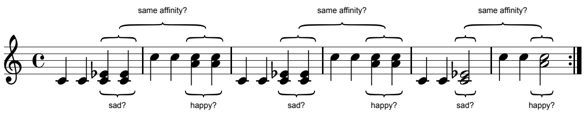
Interval	Ratio	Example	Affinity
0 4 (Major 3rd)	5 : 4	$E : C4$	7
8 12	4 : 5	$A\flat : C5$	7

0:4 versus 0:3

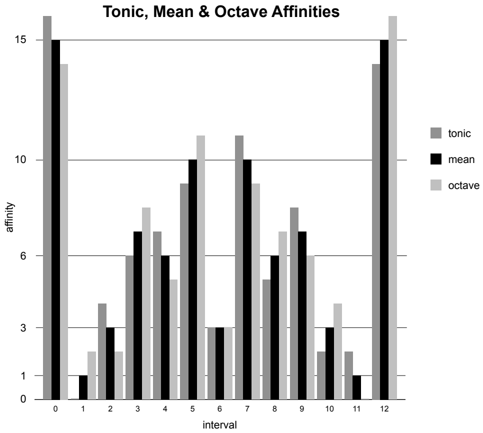


Interval	Ratio	Example	Affinity
0 4 (Major 3rd)	$5:4$	$E:C4$	7
0 3 (minor 3rd)	$6:5$	$E\flat:C4$	6

0:3 versus 9:12

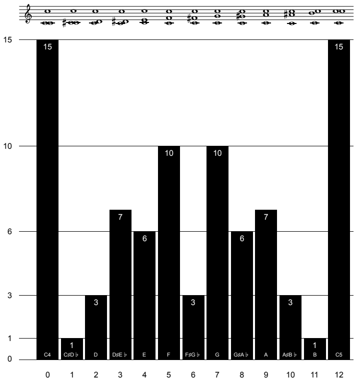


Interval	Ratio	Example	Affinity
0 3 (minor 3rd)	6:5	$C4 : E\flat$	6
9 12	5:6	$A : C5$	6

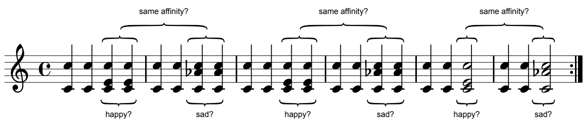


Tonic, Mean & Octave Affinities
affinity
15
10
6
3
1
0
0
1
2
3
4
5
6
7
8
9
10
11
12
interval
tonic
mean
octave

Tonic-Octave Affinity

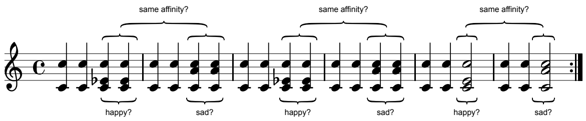


0:4:12 versus 0:8:12



Chord	Explicit Intervals	Implicit Intervals	Example	Mean Affinity
0 4 12	4,12	8	$C4 : E : C5$	9
0 8 12	8,12	4	$C4 : A\flat : C5$	9

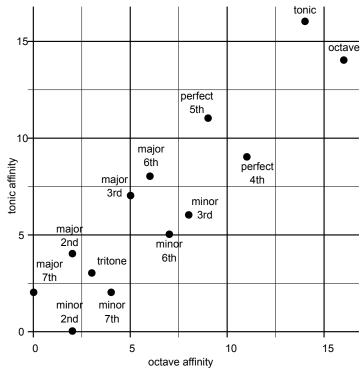
0:3:12 versus 0:9:12



Chord	Explicit Intervals	Implicit Intervals	Example	Mean Affinity
0 3 12	3,12	9	$C4 : E\flat : C5$	9.67
0 9 12	9,12	3	$C4 : A : C5$	9.67

Tonic Affinity v Octave Affinity

Name	Tonic Affinity	Octave Affinity
Tonic	16	14
Minor 2nd	0	2
Major 2nd	4	2
Minor 3rd	6	8
Major 3rd	7	5
Perfect 4th	9	11
Tritone	3	3
Perfect 5th	11	9
Minor 6th	5	7
Major 6th	8	6
Minor 7th	2	4
Major 7th	2	0
Octave	14	16

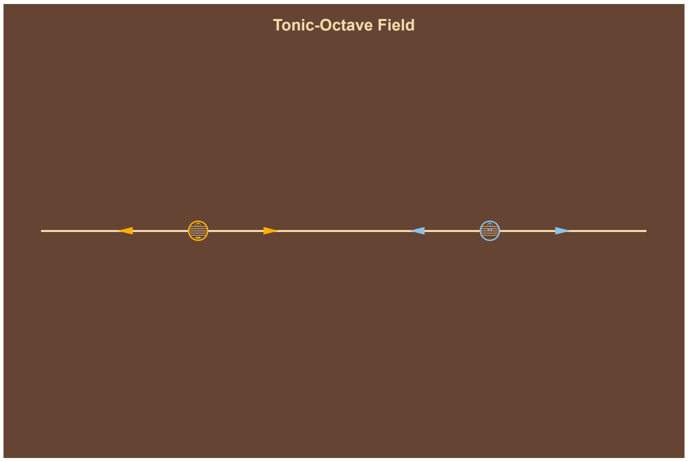


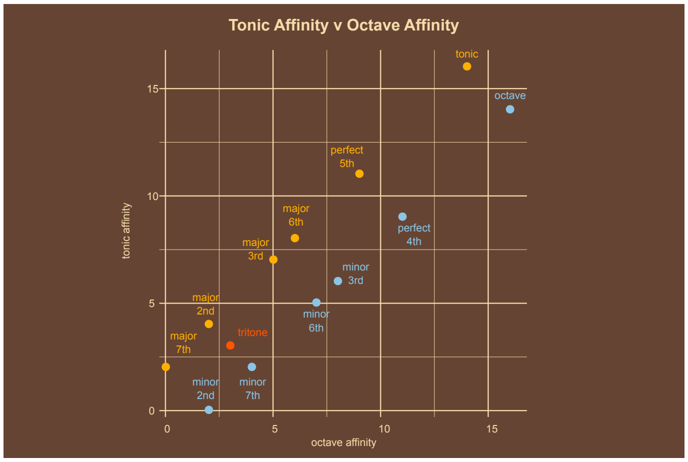
Field Theory

Helmholtz's Theorem

$$\boldsymbol{F} = -\boldsymbol{\nabla}\Phi + \boldsymbol{\nabla} \times \boldsymbol{A}$$

Change in Time?
Diverge?
Rotate?


Tonic-Octave Field



Tonic Affinity v Octave Affinity
tonic affinity
octave affinity
0
5
10
15
tonic
octave
perfect 5th
major 6th
perfect 4th
major 3rd
minor 3rd
major 2nd
minor 6th
tritone
major 7th
minor 2nd
minor 7th

Change in Time?	No.
Diverge?	No.
Rotate?	No.

Laplacian Field

$$\frac{\partial \boldsymbol{F}}{\partial t} = 0$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{F} = 0$$

$$\boldsymbol{\nabla} \times \boldsymbol{F} = 0$$

$$\boldsymbol{\nabla}^2 \Phi = 0$$

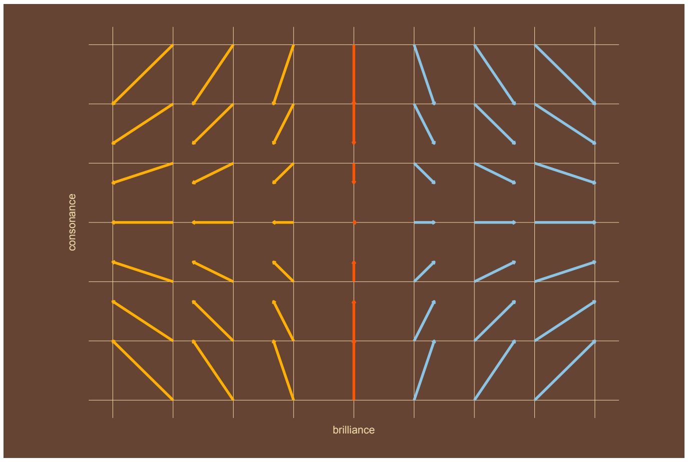
Generic Solution to Laplace's Equation

$$z = x + iy \Rightarrow z^2$$

$$\Phi(z) = \phi(x, y) + i\psi(x, y)$$

$$\psi = 2xy + \kappa$$

$$\boldsymbol{\nabla}\psi = 2x\hat{\boldsymbol{i}} - 2y\hat{\boldsymbol{j}}$$

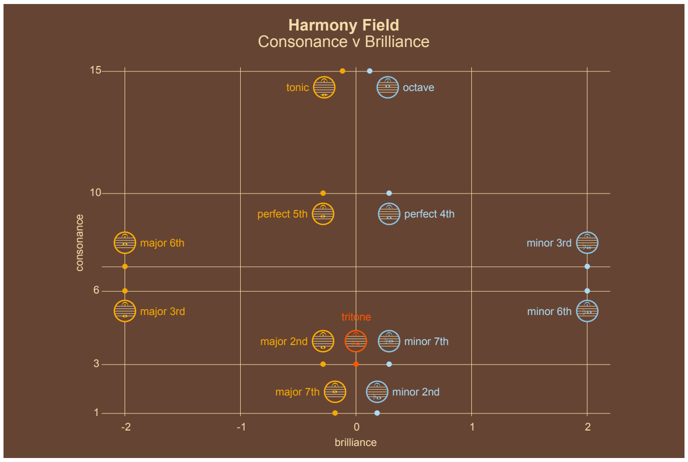

consonance
brilliance

Musical Harmony Field

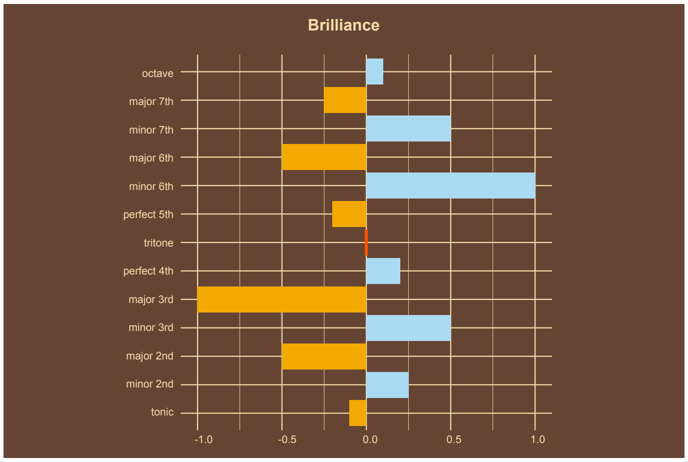
$$\boldsymbol{H} = \boldsymbol{R}(\theta)\boldsymbol{\nabla}\psi, \ \theta = -\frac{\pi}{4}$$

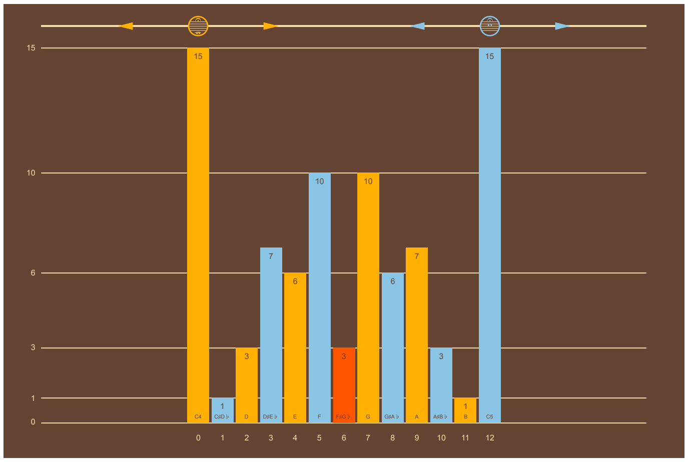
$$\boldsymbol{H} = B\hat{\boldsymbol{b}} - C\hat{\boldsymbol{c}}$$

$$B = \frac{\beta}{C-\mu}, \ \beta = \{\text{-}1, 0, 1\}, \ \mu = \frac{1}{S}\sum_{s\in\{3,4,8,9\}} C_s$$

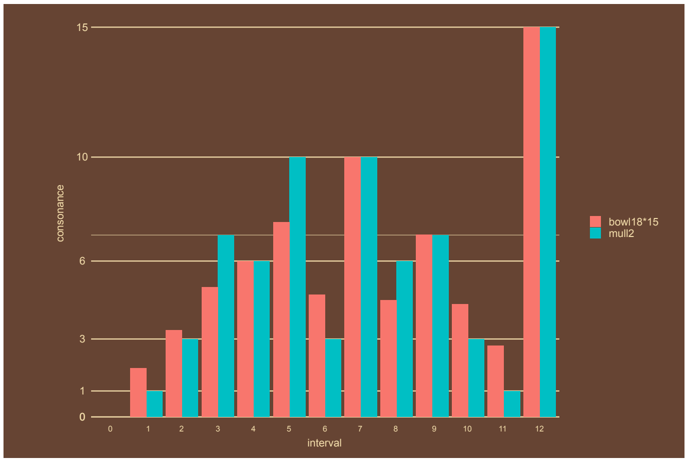


Harmony Field
Consonance v Brilliance
15
10
6
3
1
consonance
tonic
octave
perfect 5th
perfect 4th
major 6th
minor 3rd
major 3rd
minor 6th
tritone
major 2nd
minor 7th
major 7th
minor 2nd
-2
-1
0
1
2
brilliance


Brilliance
octave
major 7th
minor 7th
major 6th
minor 6th
perfect 5th
tritone
perfect 4th
major 3rd
minor 3rd
major 2nd
minor 2nd
tonic
-1.0
-0.5
0.0
0.5
1.0



15
10
6
3
1
0
15
1
3
7
6
10
3
10
6
7
3
1
15
C4
C♯D♭
D
D♯E♭
E
F
F♯G♭
G
G♯A♭
A
A♯B♭
B
C5
0
1
2
3
4
5
6
7
8
9
10
11
12

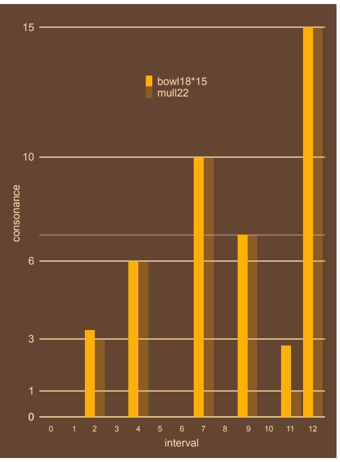


15
10
6
3
1
0
consonance
0
1
2
3
4
5
6
7
8
9
10
11
12
interval
bowl18*15
mull2

Name	Mulloy 2022	Bowling 2018 * 15.0	difference
Minor 2nd	1	1.875	0.875
Major 2nd	3	3.330	0.330
Minor 3rd	7	4.995	-2.005
Major 3rd	6	6.000	0.000
Perfect 4th	10	7.500	-2.500
Tritone	3	4.710	1.710
Perfect 5th	10	10.005	0.005
Minor 6th	6	4.500	-1.500
Major 6th	7	7.005	0.005
Minor 7th	3	4.335	1.335
Major 7th	1	2.745	1.745
Octave	15	15.000	0.000

mean absolute difference for 5 major intervals: **0.417**

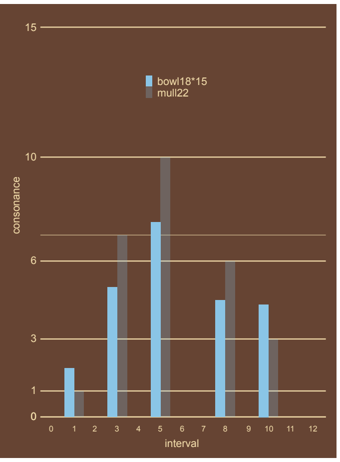
mean absolute difference for 5 minor intervals: **1.642**

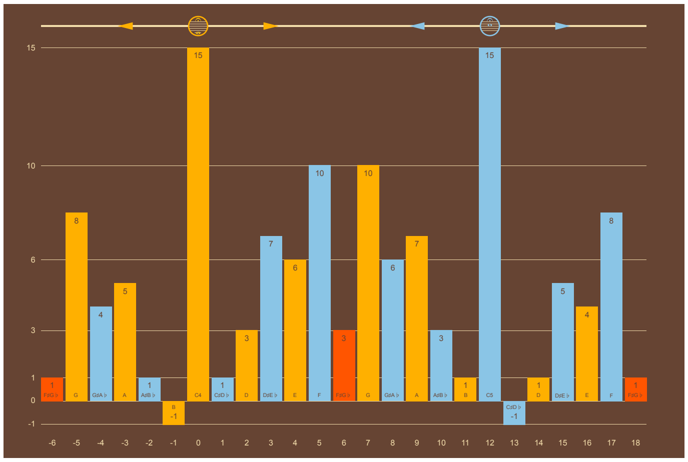


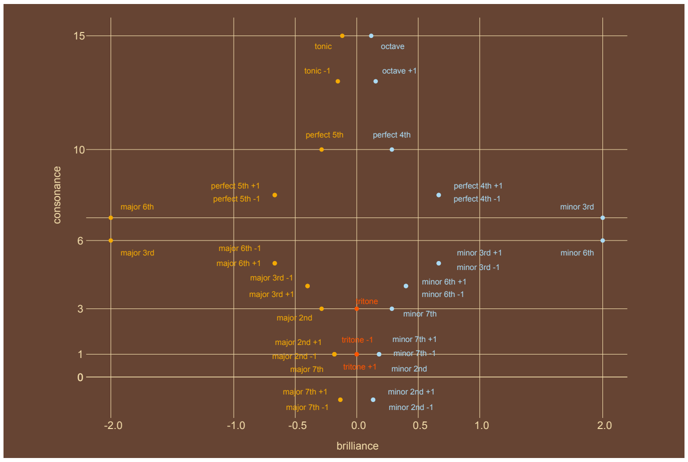
Name	**Mulloy 2022**	**Bowling 2018 * 15.0**	**difference**
Minor 2nd	1	1.875	0.875
Major 2nd	3	3.330	0.330
Minor 3rd	7	4.995	-2.005
Major 3rd	6	6.000	0.000
Perfect 4th	10	7.500	-2.500
Tritone	3	4.710	1.710
Perfect 5th	10	10.005	0.005
Minor 6th	6	4.500	-1.500
Major 6th	7	7.005	0.005
Minor 7th	3	4.335	1.335
Major 7th	1	2.745	1.745
Octave	15	15.000	0.000

mean absolute difference for 5 major intervals: **0.417**

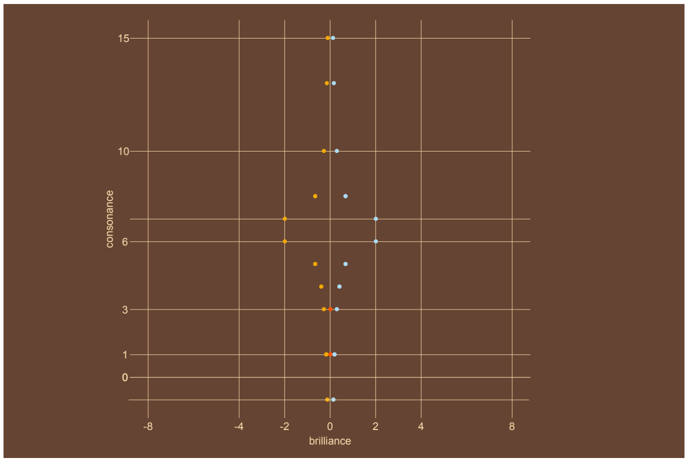
mean absolute difference for 5 minor intervals: **1.642**

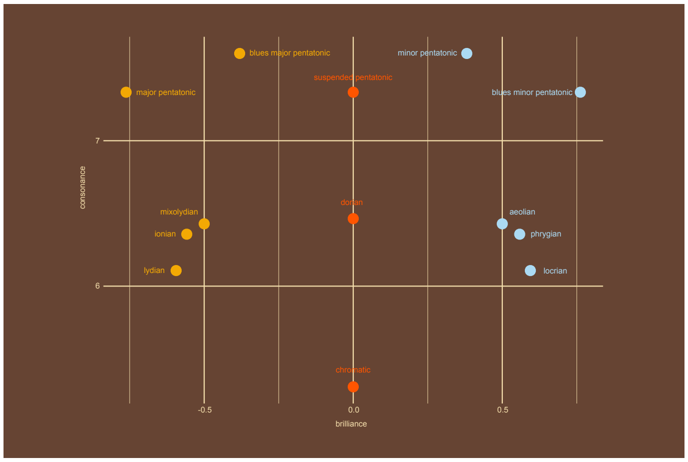



15
10
6
3
1
0
-1
F♯G♭ 1
G 8
G♯A♭ 4
A 5
A♯B♭ 1
B -1
C4 15
C♯D♭ 1
D 3
D♯E♭ 7
E 6
F 10
F♯G♭ 3
G 10
G♯A♭ 6
A 7
A♯B♭ 3
B 1
C5 15
C♯D♭ -1
D 1
D♯E♭ 5
E 4
F 8
F♯G♭ 1
-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

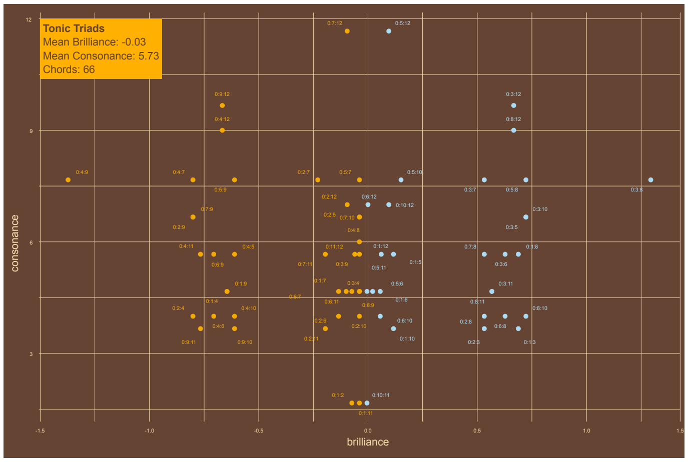


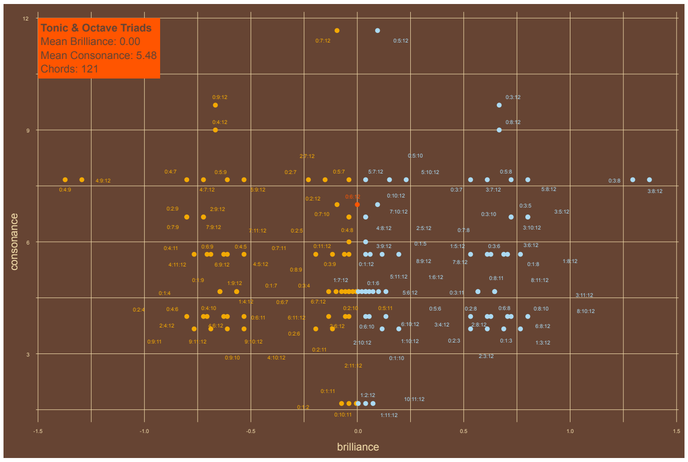
tonic
octave
tonic -1
octave +1
perfect 5th
perfect 4th
perfect 5th +1
perfect 5th -1
perfect 4th +1
perfect 4th -1
major 6th
minor 3rd
major 3rd
minor 6th
major 6th -1
major 6th +1
minor 3rd +1
minor 3rd -1
major 3rd -1
major 3rd +1
minor 6th +1
minor 6th -1
tritone
major 2nd
minor 7th
major 2nd +1
tritone -1
minor 7th +1
major 2nd -1
minor 7th -1
major 7th
tritone +1
minor 2nd
major 7th +1
minor 2nd +1
major 7th -1
minor 2nd -1
15
10
6
3
1
0
consonance
-2.0
-1.0
-0.5
0.0
0.5
1.0
2.0
brilliance


consonance
brilliance
15
10
6
3
1
0
-8
-4
-2
0
2
4
8

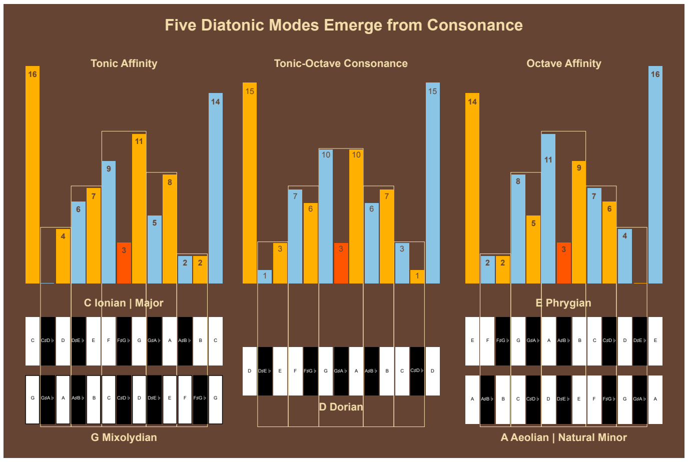


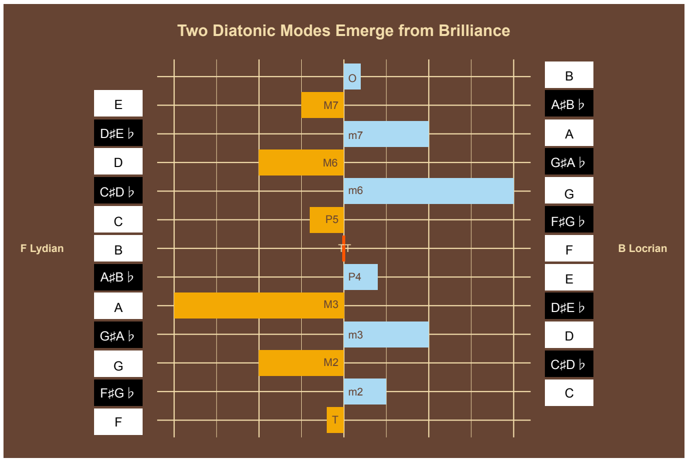
blues major pentatonic
minor pentatonic
suspended pentatonic
major pentatonic
blues minor pentatonic
7
consonance
dorian
mixolydian
aeolian
ionian
phrygian
lydian
locrian
6
chromatic
-0.5
0.0
0.5
brilliance


Tonic Triads
Mean Brilliance: -0.03
Mean Consonance: 5.73
Chords: 66
consonance
brilliance

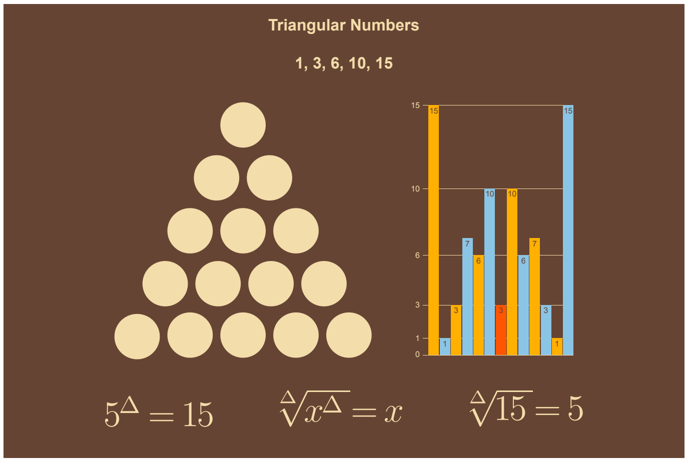

Tonic & Octave Triads
Mean Brilliance: 0.00
Mean Consonance: 5.48
Chords: 121
consonance
brilliance

Other Observations


Five Diatonic Modes Emerge from Consonance
Tonic Affinity
Tonic-Octave Consonance
Octave Affinity
C Ionian | Major
D Dorian
E Phrygian
G Mixolydian
A Aeolian | Natural Minor

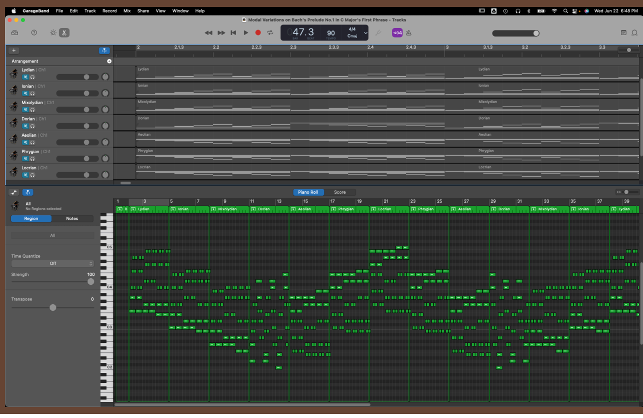


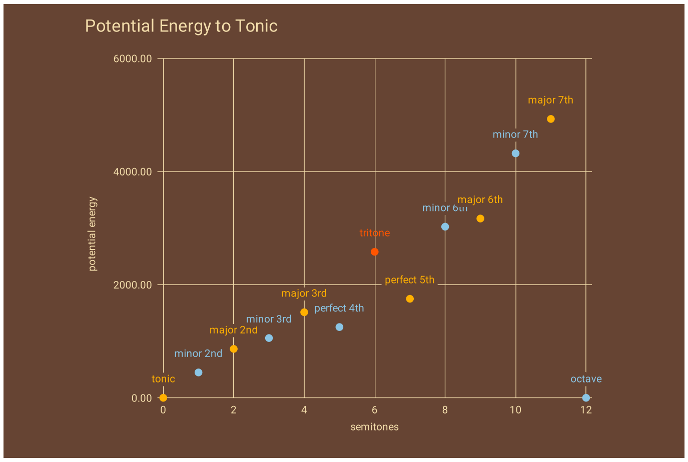
Two Diatonic Modes Emerge from Brilliance
F Lydian
B Locrian
E
D♯E♭
D
C♯D♭
C
B
A♯B♭
A
G♯A♭
G
F♯G♭
F
B
A♯B♭
A
G♯A♭
G
F♯G♭
F
E
D♯E♭
D
C♯D♭
C
O
M7
m7
M6
m6
P5
TT
P4
M3
m3
M2
m2
T

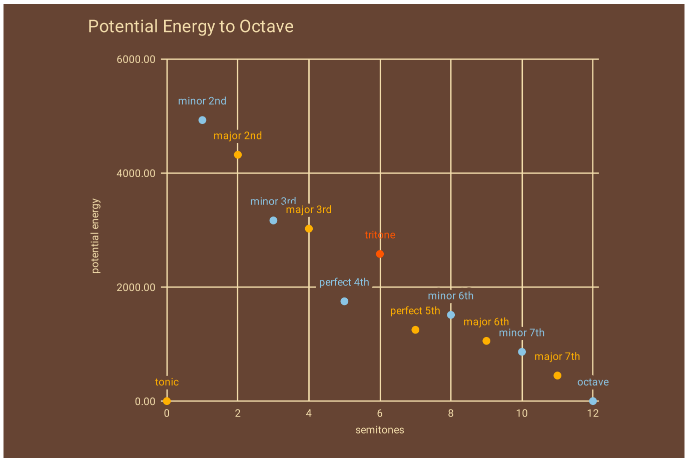


Triangular Numbers
1, 3, 6, 10, 15
$5^{\Delta} = 15$
$\sqrt[\Delta]{x^{\Delta}} = x$
$\sqrt[\Delta]{15} = 5$

Reflection Symmetry Among Diatonic Modes




Potential Energy to Tonic
6000.00
4000.00
2000.00
0.00
potential energy
0
2
4
6
8
10
12
semitones
tonic
minor 2nd
major 2nd
minor 3rd
major 3rd
perfect 4th
tritone
perfect 5th
major 6th
minor 7th
major 7th
octave


Potential Energy to Octave
potential energy
semitones
6000.00
4000.00
2000.00
0.00
0
2
4
6
8
10
12
minor 2nd
major 2nd
minor 3rd
major 3rd
tritone
perfect 4th
minor 6th
perfect 5th
major 6th
minor 7th
major 7th
tonic
octave

Next Steps