A Field Theory of Musical Harmony

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Abstract

The abstract serves both as a general introduction to the topic and as a brief, non-technical summary of the main results and their implications. Authors are advised to check the author instructions for the journal they are submitting to for word limits and if structural elements like subheadings, citations, or equations are permitted.

Keywords: Applied Physics, Music Theory

1 Introduction

The Introduction section, of referenced text [**?**] expands on the background of the work (some overlap with the Abstract is acceptable). The introduction should not include subheadings.

Springer Nature does not impose a strict layout as standard however authors are advised to check the individual requirements for the journal they are planning to submit to as there may be journal-level preferences. When preparing your text please also be aware that some stylistic choices are not supported in full text XML (publication version), including coloured font. These will not be replicated in the typeset article if it is accepted.

2 Words

1. The interaction of musical pitches can be described with the help of the concept of a field of force. Namely, instead of saying that one pitch acts on another, we may say that the pitch creates a field around itself; a certain force then acts on every other pitch located in this field.
2. Consonance and brilliance are not different phenomena but two aspects of the same thing.

3. Brilliance is the musical property of a pitch that causes it to experience a force when placed in a harmonic field. Brilliance can be major or minor. Like pitches attract each other and unlike pitches repel each other. An object with an absence of brilliance is referred to as neutral.
4. from Nature: Theoretical physics is the development of mathematical formalisms and computational protocols for describing all aspects of objects found in the world around us and their interaction. This can involve both providing models for understanding empirical results or constructing self-logical theories for explain phenomena beyond current experiments. https://www.nature.com/subjects/theoretical-physics
5. adjust this from bowl18 to point out that studies have focused only on the tonic: Previous evaluations have focused on the two-tone combinations ("dyads") that define the chromatic scale, a set of 12 tones over an octave used in much music worldwide (Table S1). Studies of dyadic consonance have been repeated many times over the last century and, despite some variation in consonance ranking, listeners broadly agree on the dyads heard as the most and least attractive (12, 32). Surprisingly, comparable perceptual data are not available for more complex tone combinations, such as triads (three-tone chords) and tetrads (four-tone chords).

3 Results

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4 This is an example for first level head—section head

4.1 This is an example for second level head—subsection head

4.1.1 This is an example for third level head—subsubsection head

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5 Equations

1. The interaction of musical pitches can be described with the help of the concept of a field of force. Namely, instead of saying that one pitch acts on another, we may say that the pitch creates a field around itself; a certain force then acts on every other pitch located in this field.

5.1 Harmonostatics

We start with Helmholtz's fundamental theorom of vector calculus.

$$\boldsymbol{F} = -\boldsymbol{\nabla}\Phi + \boldsymbol{\nabla} \times \boldsymbol{H} \tag{1}$$

We will explore the creation of a harmonic field that is steady with respect to time, irrotational and divergence free. We can think of this as harmonostatics.

$$\boldsymbol{H} = \boldsymbol{\nabla}\Phi \tag{2}$$

$$\boldsymbol{\nabla} \times \boldsymbol{H} = 0 \tag{3}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{H} = 0 \tag{4}$$

The intuition for setting the divergence of **H** to zero is that brilliance (major-minor) seems less detectable when consonance is high or low. Experimental data seem to show that the most brilliant chords are heard in the middle of the one-octave consonance range. So in a sense brilliance seems to stretch out as consonance is compressed and vice versa. In other words, harmony seems to be incompressible.

The principle of superposition allows us to use simple sums and averages to understand how multiple pitches in the field impact the harmony of other pitches in the field so that we can construct measures of harmony for chords and scales with any number of pitches.

5.2 Prime Factors Affinity

$$\frac{\omega_i}{\omega_j} = \prod_{p \in \mathbb{P}} p^{\alpha_{rp}},\ r = |i - j| \tag{5}$$

$$d_r = \sum_{p \in \mathbb{P}} |p\alpha_{rp}| \tag{6}$$

$$a_r = d_{max} - d_r \tag{7}$$

5.3 Tonic-Octave Field with Prime Factors Affinity

$$\boldsymbol{\nabla} \cdot \boldsymbol{H} = \eta(\hat{\boldsymbol{z}}, \theta)\boldsymbol{\nabla}^{\mathbf{2}}\psi = 0 \tag{8}$$

$$z = x + iy \Rightarrow z^2 \tag{9}$$

$$\Phi(z) = \phi(x, y) + i\psi(x, y) \tag{10}$$

$$\psi = 2xy + \Omega \tag{11}$$

For ψ, x is the affinity with respect to the octave pitch, y is affinity with respect to the tonic pitch and Ω is an range constant for how many octaves to include above and below the primary octave.

$$\boldsymbol{H} = \eta(\hat{\boldsymbol{z}}, \theta)\boldsymbol{\nabla}\psi \tag{12}$$

$$\boldsymbol{H} = (2x\hat{\boldsymbol{x}} - 2y\hat{\boldsymbol{y}})\cos(\theta) = 2B\hat{\boldsymbol{b}} - 2C\hat{\boldsymbol{c}} \tag{13}$$

$$B = \frac{\beta}{C - \kappa} \; where \; \beta = \{-1, 0, 1\} \; and \; \kappa = \sqrt[\Delta]{C_{max}} \tag{14}$$

$$C_I = \frac{1}{N}\sum_{i\in I}\sum_{j\in I} C_{|i-j|} \tag{15}$$

$$B_I = \frac{1}{N}\sum_{i\in I}\sum_{j\in I}\frac{B_i + B_j}{2} = \frac{1}{N}\sum_{i\in I} B_i \tag{16}$$

where I is the set of intervals of a chord or scale.

$$N = \frac{n!}{(n-r)!} \tag{17}$$

78 for chromatic 28 for 8 note scale, 21 for 7 note scale, 6 for 4-note chord 3 for 3-note chord

$$W = -\int_a^b \boldsymbol{H}\cdot d\boldsymbol{s} = \psi(b) - \psi(a) \tag{18}$$

$$W = (|\boldsymbol{H_i}| - |\boldsymbol{H_\tau}|)\, s \tag{19}$$

Equations in LaTeX can either be inline or on-a-line by itself ("display equations"). For inline equations use the `$...$` commands. E.g.: The equation $H\psi = E\psi$ is written via the command `$H \psi = E \psi$`.

$$\begin{aligned} D_\mu &= \partial_\mu - ig\frac{\lambda^a}{2}A^a_\mu \\ F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^a_\nu \end{aligned} \tag{20}$$

Notice the use of `\nonumber` in the align environment at the end of each line, except the last, so as not to produce equation numbers on lines where no equation numbers are required. The `\label{}` command should only be used at the last line of an align environment where `\nonumber` is not used.

$$Y_\infty = \left(\frac{m}{\mathrm{GeV}}\right)^{-3}\left[1 + \frac{3\ln(m/\mathrm{GeV})}{15} + \frac{\ln(c_2/5)}{15}\right] \tag{21}$$

The class file also supports the use of `\mathbb{}`, `\mathscr{}` and `\mathcal{}` commands. As such `\mathbb{R}`, `\mathscr{R}` and `\mathcal{R}` produces $\mathbb{R}$, $\mathscr{R}$ and $\mathcal{R}$ respectively (refer Subsubsection 4.1.1).

6 Tables

Tables can be inserted via the normal table and tabular environment. To put footnotes inside tables you should use `\footnotetext[]{...}` tag. The footnote appears just below the table itself (refer Tables 4 and 5). For the corresponding footnotemark use `\footnotemark[...]`

Table 1 Interval-Tonic Affinity

τ [1]	Name	Ratio [2]	Example	Affinity
0	Tonic	1 : 1	$C4 : C4$	16
1	Minor 2nd	16 : 15	$C\sharp : C4$	0
2	Major 2nd	9 : 8	$D : C4$	4
3	Minor 3rd	6 : 5	$D\sharp : C4$	6
4	Major 3rd	5 : 4	$E : C4$	7
5	Perfect 4th	4 : 3	$F : C4$	9
6	Tritone [3]	$\sqrt{2} : 1$	$F\sharp : C4$	3
7	Perfect 5th	3 : 2	$G : C4$	11
8	Minor 6th	8 : 5	$G\sharp : C4$	5
9	Major 6th	5 : 3	$A : C4$	8
10	Minor 7th	16 : 9	$A\sharp : C4$	2
11	Major 7th	15 : 8	$B : C4$	2
12	Octave	2 : 1	$C5 : C4$	14

[1]Semitones from the tonic, τ.

[2]Frequency ratio with respect to the tonic as *interval* : *tonic*.

[3]For tritone ratio and affinity, see the methods section.

In case of double column layout, tables which do not fit in single column width should be set to full text width. For this, you need to use `\begin{table*} ... \end{table*}` instead of `\begin{table} ... \end{table}` environment. Lengthy tables which do not fit in textwidth should be set as rotated table. For this, you need to use `\begin{sidewaystable} ... \end{sidewaystable}` instead of `\begin{table*} ... \end{table*}` environment. This environment puts tables rotated to single column width. For tables rotated to double column width, use `\begin{sidewaystable*} ... \end{sidewaystable*}`.

7 Figures

As per the LaTeX standards you need to use eps images for LaTeX compilation and `pdf/jpg/png` images for `PDFLaTeX` compilation. This is one of the major difference between LaTeX and `PDFLaTeX`. Each image should be from a single input .eps/vector image file. Avoid using subfigures. The command for inserting images for LaTeX and `PDFLaTeX` can be generalized. The package used to insert images in `LaTeX/PDFLaTeX` is the graphicx package. Figures can be inserted via the normal figure environment as shown in the below example:

Table 2 Interval-Octave Affinity

o [1]	Name	Ratio [2]	Example	Affinity
$\overline{0}$	Octave	$1:1$	$C5:C5$	16
$\overline{1}$	Major 7th	$15:16$	$B:C5$	0
$\overline{2}$	Minor 7th	$8:9$	$B\flat:C5$	4
$\overline{3}$	Major 6th	$5:6$	$A:C5$	6
$\overline{4}$	Minor 6th	$4:5$	$A\flat:C5$	7
$\overline{5}$	Perfect 5th	$3:4$	$G:C5$	9
$\overline{6}$	Tritone [3]	$1:\sqrt{2}$	$G\flat:C5$	3
$\overline{7}$	Perfect 4th	$2:3$	$F:C5$	11
$\overline{8}$	Major 3rd	$5:8$	$E:C5$	5
$\overline{9}$	Minor 3rd	$3:5$	$E\flat:C5$	8
$\overline{10}$	Major 2nd	$9:16$	$D:C5$	2
$\overline{11}$	Minor 2nd	$8:15$	$D\flat:C5$	2
$\overline{12}$	Tonic	$1:2$	$C4:C5$	14

[1]Semitones from the octave, o.

[2]Frequency ratio with respect to the octave as *interval* : *octave*.

[3]For tritone ratio and affinity, see the methods section.

Table 3 Tonic-Octave Affinity

Name	Tonic Affinity	Octave Affinity
Tonic	16	14
Minor 2nd	0	2
Major 2nd	4	2
Minor 3rd	6	8
Major 3rd	7	5
Perfect 4th	9	11
Tritone	3	3
Perfect 5th	11	9
Minor 6th	5	7
Major 6th	8	6
Minor 7th	2	4
Major 7th	2	0
Octave	14	16

```
\begin{figure}[<placement-specifier>]
\centering
\includegraphics{<eps-file>}
\caption{<figure-caption>}\label{<figure-label>}
\end{figure}
```

In case of double column layout, the above format puts figure captions/images to single column width. To get spanned images, we need to provide `\begin{figure*} ... \end{figure*}`.

Table 4 Frequency Ratio Prime Numbers

		Prime Factors					
r	Ratio	2	3	5	7	Disaffinity	Affinity
0	1 : 1	0	0	0	0	0	16
1	16 : 15	4	-1	-1	0	16	0
2	9 : 8	-3	2	0	0	12	4
3	6 : 5	1	1	-1	0	10	6
4	5 : 4	-2	0	1	0	9	7
5	4 : 3	2	-1	0	0	7	9
6	$\sqrt{2}$: 1	see methods				13	3
7	3 : 2	-1	1	0	0	5	11
8	8 : 5	3	0	-1	0	11	5
9	5 : 3	0	-1	1	0	8	8
10	16 : 9	4	-2	0	0	14	2
11	15 : 8	-3	1	1	0	14	2
12	2 : 1	1	0	0	0	2	14

Table 5 Example of a lengthy table which is set to full textwidth

	Element 1[1]			Element 2[2]		
Project	Energy	σ_{calc}	σ_{expt}	Energy	σ_{calc}	σ_{expt}
Element 3	990 A	1168	1547 ± 12	780 A	1166	1239 ± 100
Element 4	500 A	961	922 ± 10	900 A	1268	1092 ± 40

Note: This is an example of table footnote. This is an example of table footnote this is an example of table footnote this is an example of table footnote this is an example of table footnote this is an example of table footnote.

[1]Example for a first table footnote.

[2]Example for a second table footnote.

For sample purpose, we have included the width of images in the optional argument of `\includegraphics` tag. Please ignore this.

8 Algorithms, Program codes and Listings

Packages `algorithm`, `algorithmicx` and `algpseudocode` are used for setting algorithms in LaTeX using the format:

```
\begin{algorithm}
\caption{<alg-caption>}\label{<alg-label>}
\begin{algorithmic}[1]
. . .
\end{algorithmic}
\end{algorithm}
```

You may refer above listed package documentations for more details before setting `algorithm` environment. For program codes, the "program"

Table 6 Tables which are too long to fit, should be written using the "sidewaystable" environment as shown here

	Element 1[1]			Element[2]		
Projectile	Energy	σ_{calc}	σ_{expt}	Energy	σ_{calc}	σ_{expt}
Element 3	990 A	1168	1547 ± 12	780 A	1166	1239 ± 100
Element 4	500 A	961	922 ± 10	900 A	1268	1092 ± 40
Element 5	990 A	1168	1547 ± 12	780 A	1166	1239 ± 100
Element 6	500 A	961	922 ± 10	900 A	1268	1092 ± 40

Note: This is an example of table footnote this is an example of table footnote this is an example of table footnote this is an example of table footnote this is an example of table footnote.

[1]This is an example of table footnote.

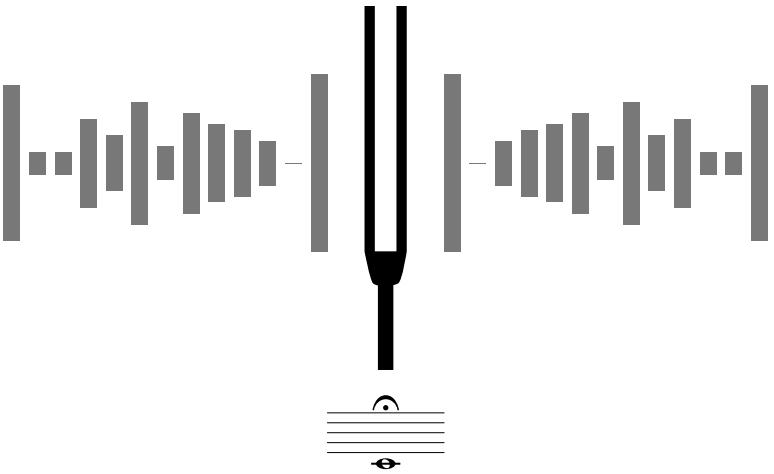

Fig. 1 This is a widefig. This is an example of long caption this is an example of long caption this is an example of long caption this is an example of long caption

package is required and the command to be used is `\begin{program} ... \end{program}`. A fast exponentiation procedure:

```
begin
  for i := 1 to 10 step 1 do
      expt(2, i);
      newline() od                    Comments will be set flush to the right margin
where
proc expt(x, n) ≡
  z := 1;
  do if n = 0 then exit fi;
      do if odd(n) then exit fi;
          comment: This is a comment statement;
          n := n/2; x := x ∗ x od;
      {n > 0};
      n := n − 1; z := z ∗ x od;
  print(z).
end
```

Similarly, for `listings`, use the `listings` package. `\begin{lstlisting} ... \end{lstlisting}` is used to set environments similar to `verbatim` environment. Refer to the `lstlisting` package documentation for more details.

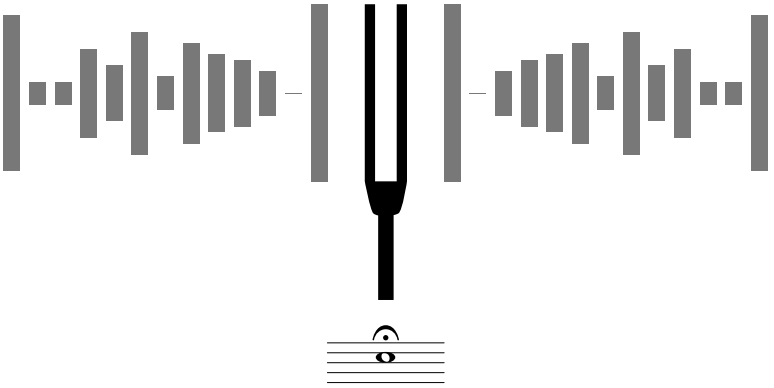

Fig. 2 This is a widefig. This is an example of long caption this is an example of long caption this is an example of long caption this is an example of long caption

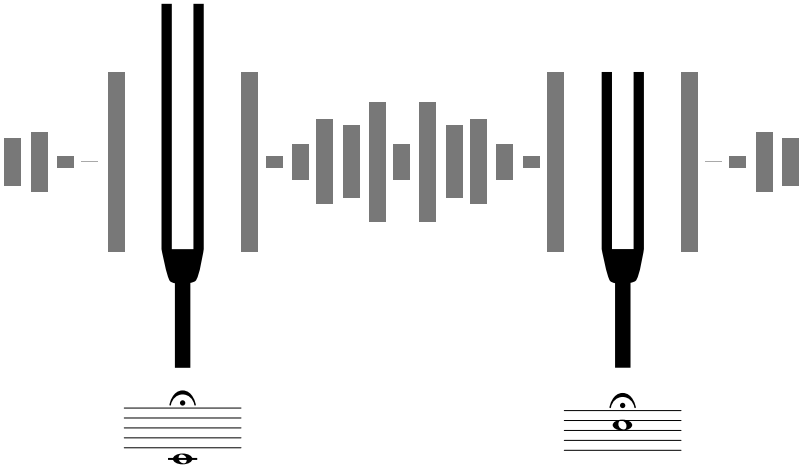

Fig. 3 A tonic-octave musical harmony field created by two pitches 12 semitones apart.

```
for i:=maxint to 0 do
begin
{ do nothing }
end;
Write('Case insensitive ');
Write('Pascal keywords.');
```

Algorithm 1 Calculate $y = x^n$

Require: $n \geq 0 \vee x \neq 0$
Ensure: $y = x^n$
1: $y \Leftarrow 1$
2: **if** $n < 0$ **then**
3: $X \Leftarrow 1/x$
4: $N \Leftarrow -n$
5: **else**
6: $X \Leftarrow x$
7: $N \Leftarrow n$
8: **end if**
9: **while** $N \neq 0$ **do**
10: **if** N is even **then**
11: $X \Leftarrow X \times X$
12: $N \Leftarrow N/2$
13: **else**[N is odd]
14: $y \Leftarrow y \times X$
15: $N \Leftarrow N - 1$
16: **end if**
17: **end while**

9 Cross referencing

Environments such as figure, table, equation and align can have a label declared via the `\label{#label}` command. For figures and table environments use the `\label{}` command inside or just below the `\caption{}` command. You can then use the `\ref{#label}` command to cross-reference them. As an example, consider the label declared for Figure **??** which is `\label{fig1}`. To cross-reference it, use the command `Figure \ref{fig1}`, for which it comes up as "Figure **??**".

To reference line numbers in an algorithm, consider the label declared for the line number 2 of Algorithm 1 is `\label{algln2}`. To cross-reference it, use the command `\ref{algln2}` for which it comes up as line 2 of Algorithm 1.

9.1 Details on reference citations

Standard LaTeX permits only numerical citations. To support both numerical and author-year citations this template uses `natbib` LaTeX package. For style guidance please refer to the template user manual.

Here is an example for `\cite{...}`: [**?**]. Another example for `\citep{...}`: [**?**]. For author-year citation mode, `\cite{...}` prints Jones et al. (1990) and `\citep{...}` prints (Jones et al., 1990).

All cited bib entries are printed at the end of this article: [**?**], [**?**], [**?**], [**?**], [**?**], [**?**], [**?**], [**?**], [**?**], [**?**] and [**?**].

10 Examples for theorem like environments

For theorem like environments, we require `amsthm` package. There are three types of predefined theorem styles exists—`thmstyleone`, `thmstyletwo` and `thmstylethree`

`thmstyleone`	Numbered, theorem head in boldont and theorem text in italic style
`thmstyletwo`	Numbered, theorem head in roman font and theorem text in italic style
`thmstylethree`	Numbered, theorem head in bold font and theorem text in roman style

For a quote environment, use `\begin{quote}...\end{quote}`

> Quoted text example. Aliquam porttitor quam a lacus. Praesent vel arcu ut tortor cursus volutpat. In vitae pede quis diam bibendum placerat. Fusce elementum convallis neque. Sed dolor orci, scelerisque ac, dapibus nec, ultricies ut, mi. Duis nec dui quis leo sagittis commodo.

Sample body text. Sample body text. Sample body text. Sample body text. Sample body text (refer Figure **??**). Sample body text. Sample body text. Sample body text (refer Table 6).

11 Methods

Topical subheadings are allowed. Authors must ensure that their Methods section includes adequate experimental and characterization data necessary for others in the field to reproduce their work. Authors are encouraged to include RIIDs where appropriate.

Ethical approval declarations (only required where applicable) Any article reporting experiment/s carried out on (i) live vertebrate (or higher invertebrates), (ii) humans or (iii) human samples must include an unambiguous statement within the methods section that meets the following requirements:

1. Approval: a statement which confirms that all experimental protocols were approved by a named institutional and/or licensing committee. Please identify the approving body in the methods section
2. Accordance: a statement explicitly saying that the methods were carried out in accordance with the relevant guidelines and regulations
3. Informed consent (for experiments involving humans or human tissue samples): include a statement confirming that informed consent was obtained from all participants and/or their legal guardian/s

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12 Western Tradition

1. nearest neighbor symmetry versus tonic-octave symmetry around the 2nds, 3rds, 6ths and 7ths (is there a curve on the plot?)

13 Emergence of Diatonic

1. tonic bias pleasantness gives ionian and mixolydian
2. octave bias pleasantness gives phrygian and aeolian
3. tonic-octave consonance gives dorian
4. tonic-octave brilliance gives lydian and locrian

14 Locrian the Sublime

A composer of the Western, tonic (and therefore major) biased tradition, when confronted with Locrian mode, will ask, "Where's the Perfect Fifth? Where's the Major 7th? Without the perfect 5th how will my chords will be consonant and my cadences strong?" If the composer were to frame those questions in a slightly different way the sense of doom would disappear. How?

Fig. 4 Locrian mode with no perfecet fifth and no major seventh.

We know from equation the consonance is the same when regardless of the ordering of the numerator and denominator of the frequency ratio. We also know from equation that the potential energy of cadences is a function of the abolsute value of the distance between the notes.

If the composer were to reframe the questions as, "Where's the 3:2 or 2:3 interval? Where's the +-8 interval?" then he would see that the Locrian mode is a fully capable and arguably the most sublime of all the modes for composition.

Include sheet music for Bach in Locrian.

Introduce the image of the piano with the mirror. With a mirror placed to the left of the home note then - with one exception, whatever the pianist plays, the mirror image pianist is playing the same piece perfectly well in

the reflected mode: Ionian-Phrygian, Mixolydian-Aeolian and Lydian-Locrian. The exception is Dorian. For Dorian mode, we must place the mirror directly in the center of the tritone.

Now show Bach's piece in all modes with sheet music first and then including piano roll.

15 Number Theory

1. consonance 1,3,6,10,15 are triangular numbers, other than 7 which is extremely brilliant
2. Pythagoras tetraktys of the decad (cite: James Tenney History of Consonance and Dissonance p. 12)
3. maybe point out that the reason major-mior emerges is that all the major-minor mirrors are exactly 2 apart?

16 General Solutions

1. Frequency-based ψ
2. add time t.
3. add amplitude A and intensity I to tie back into energy physics
4. and probably $\frac{\partial I}{\partial t}$
5. is harmony always divergence free? seems like yes.
6. however, it seems that there might be rotation when we start maaking music over time.
7. so does ψ become a vector field?
8. η variations ... what does this mean? related to *psi*? will the angles change with new measures of affinity.
9. $\rho > 0$ presence of other pitches
10. where's magnetism? does that come with harmonodynamics?
11. Higher Prime Limits / Larger Range of Just Intontation Microtunings in One Octave

17 Conclusion

Many powerful music-theoretic ideas emerge from a tonic-bias model of musical harmony: pitch classes, chord inversions, the circle of fifths.

Yet a concept that does not formally emerge (but is informally known) is a model for brilliance (brightness: major versus minor).

In our theory, any test pitch (with the exception of the tritone) that is introduced into the tonic-octave pitch field must take on a major-minor value.

Supplementary information. If your article has accompanying supplementary file/s please state so here.

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Acknowledgments. Acknowledgments are not compulsory. Where included they should be brief. Grant or contribution numbers may be acknowledged.

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Appendix A Section title of first appendix

An appendix contains supplementary information that is not an essential part of the text itself but which may be helpful in providing a more comprehensive understanding of the research problem or it is information that is too cumbersome to be included in the body of the paper.