AWBS kinetic modeling of electrons with nonlocal Ohms law in plasmas relevant to inertial confinement fusion

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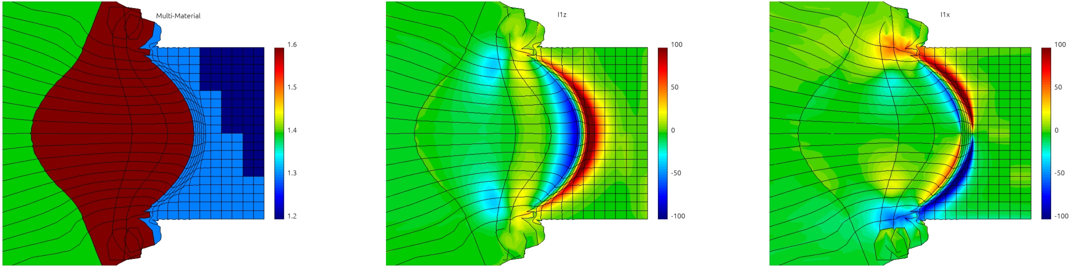

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Hydrodynamic model of plasma

Fokker-Planck/Boltzmann transport equation

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \nabla_x f + \frac{q_e}{m_e}\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c} \times \boldsymbol{B}\right) \cdot \nabla_{\boldsymbol{v}} f = \sigma \nabla_{\boldsymbol{v}} \cdot \int \frac{|\boldsymbol{v} - \boldsymbol{v}'|^2 \mathbf{I} - (\boldsymbol{v} - \boldsymbol{v}') \otimes \boldsymbol{v} - \boldsymbol{v}')}{|\boldsymbol{v} - \boldsymbol{v}'|^3} \left(\nabla_{\boldsymbol{v}} f(\boldsymbol{v}) f(\boldsymbol{v}') - f(\boldsymbol{v}) \nabla_{\boldsymbol{v}} f(\boldsymbol{v}')\right) \mathrm{d}\boldsymbol{v}'. \quad (1)$$

Fluid equations in Lagrangian frame (velocity space moments of (1))

$$\begin{aligned}
\frac{\mathrm{d}\rho}{\mathrm{d}t} &= -\rho \nabla \cdot \boldsymbol{U}, \\
\rho \frac{\mathrm{d}\boldsymbol{U}}{\mathrm{d}t} &= \nabla \cdot \sigma + \boldsymbol{j} \times \boldsymbol{B}, \\
\rho \frac{\mathrm{d}\varepsilon}{\mathrm{d}t} &= \sigma : \nabla \boldsymbol{U} - \nabla \cdot \boldsymbol{q}_e.
\end{aligned}$$

Microscopic closure

$$\begin{aligned}
\sigma &= -\rho \int (\boldsymbol{v} - \boldsymbol{U}) \otimes (\boldsymbol{v} - \boldsymbol{U})\, f \,\mathrm{d}\boldsymbol{v}^3 \approx -\mathbf{I}p + \tilde{\sigma}(\nabla \boldsymbol{U}), \\
\boldsymbol{q}_e &= \frac{m_e}{2} \int |\boldsymbol{v} - \boldsymbol{U}|^2 (\boldsymbol{v} - \boldsymbol{U})\, f \,\mathrm{d}\boldsymbol{v}^3 \approx \frac{m_e}{2} \int_{4\pi} \boldsymbol{n} \int_0^\infty |\boldsymbol{v}|^5 f \,\mathrm{d}|\boldsymbol{v}|\, \mathrm{d}\boldsymbol{n} \approx -\kappa(T^{2.5}) \nabla T, \\
\boldsymbol{j} &= q_e \int (\boldsymbol{v} - \boldsymbol{U})\, f \,\mathrm{d}\boldsymbol{v}^3.
\end{aligned}$$

Local diffusive transport regime

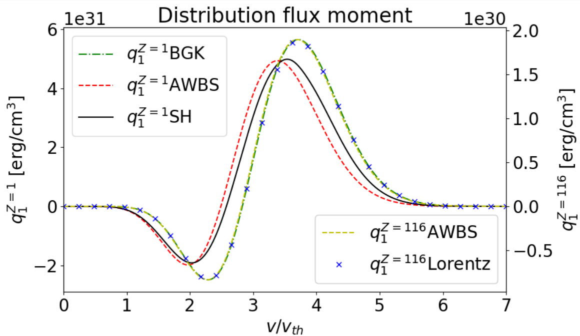
Collision operators

$$C_{FP} = \Gamma_{ee} \int \nabla_{\mathbf{v}} \nabla_{\mathbf{v}} (\mathbf{v} - \tilde{\mathbf{v}}) \cdot (f \nabla_{\tilde{\mathbf{v}}} f - f \nabla_{\mathbf{v}} f) \, d\tilde{\mathbf{v}} + \frac{\nu_{ei}}{2} \frac{\partial}{\partial \mu} \left((1 - \mu^2) \frac{\partial f}{\partial \mu} \right), \tag{2}$$

$$C_{BGK} = \nu_e (f_M - f) + \frac{Z + 4.2}{Z + 0.24} \frac{\nu_{ei}}{2} \frac{\partial}{\partial \mu} \left((1 - \mu^2) \frac{\partial f}{\partial \mu} \right), \tag{3}$$

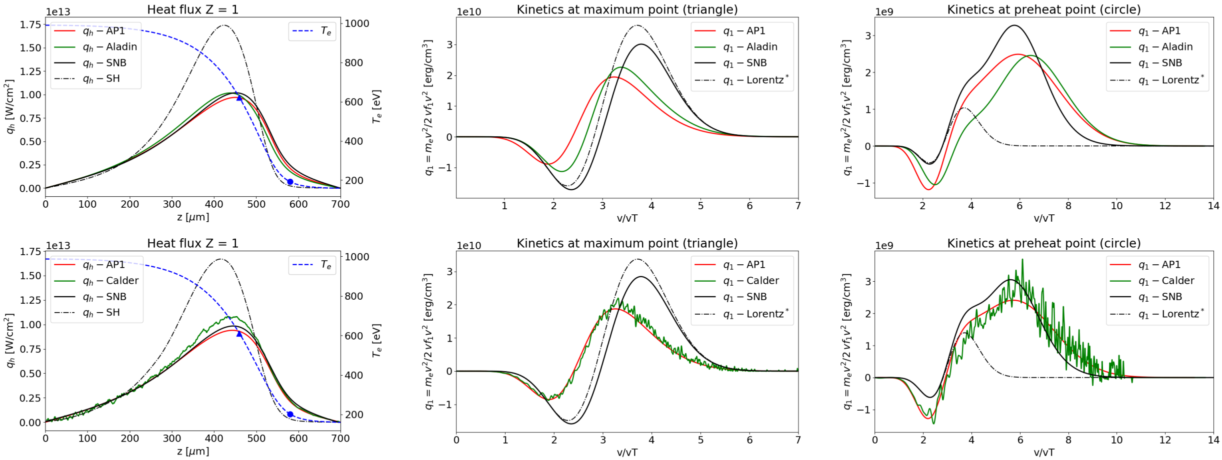
$$C_{FP0} = v \nu_e \frac{\partial}{\partial v} \left[C(f_0) f_0 + D(f_0) \frac{\partial f_0}{\partial v} \right] + \frac{Z + 4.2}{Z + 0.24} \frac{\nu_{ei}}{2} \frac{\partial}{\partial \mu} \left((1 - \mu^2) \frac{\partial f}{\partial \mu} \right), \tag{4}$$

$$C_{AWBS} = v \frac{\nu_e}{2} \frac{\partial}{\partial v} (f - f_M) + \frac{\nu_{ei} + \frac{\nu_e}{2}}{2} \frac{\partial}{\partial \mu} \left((1 - \mu^2) \frac{\partial f}{\partial \mu} \right). \tag{5}$$



	$Z = 1$	$Z = 2$	$Z = 4$	$Z = 16$	$Z = 116$
$\bar{\Delta}\mathbf{q}_{AWBS}$	0.057	0.004	0.037	0.021	0.004

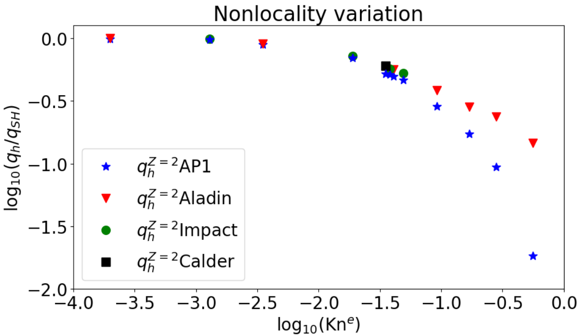
Nonlocal electron transport regime - PIC/VFP0/AWBS



Electron velocity limit - friction vs. $\boldsymbol{E}$ stopping

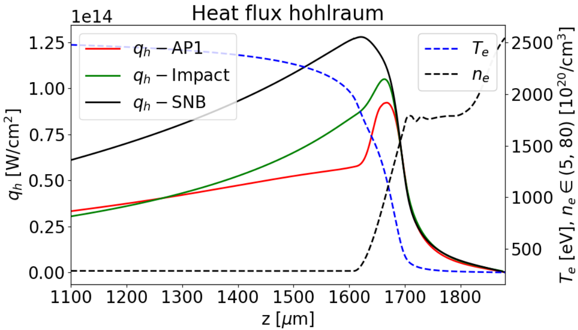
$$\frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\nabla f + \frac{q_e}{m_e}\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c}\times\boldsymbol{B}\right)\cdot\frac{\partial f}{\partial \Omega} = \left(v\frac{\nu_e}{2} - \frac{q_e}{m_e}\boldsymbol{v}\cdot\boldsymbol{E}\right)\frac{\partial f}{\partial v} - v\frac{\nu_e}{2}\frac{\partial f_M}{\partial v} + \frac{\nu_{ei}+\frac{\nu_e}{2}}{2}\frac{\partial^2 f}{\partial \Omega^2}.$$

Kn^e	10^{-4}	10^{-3}	10^{-2}	10^{-1}	1
v_{lim}/v_{th}	70.8	22.4	7.3	3.1	1.8



$\boldsymbol{E}$ stopping overtakes collisions for Kn$>$ 0.1.

NIF hohlraum - plasma profiles provided by HYDRA



SNB approach

$$\tilde{f} = f_M + \delta f_0 + \Omega \cdot (\boldsymbol{f}_{1M} + \delta \boldsymbol{f}_1)\,,$$

$$\boldsymbol{f}_{1M} = -\frac{Z + 0.24}{Z + 4.2}\lambda_{ei} f_M \left(\frac{\nabla n_e}{n_e} + \left(\frac{v^2}{2v_{th}^2} - \frac{3}{2}\right)\frac{\nabla T_e}{T_e} - \frac{q_e \boldsymbol{E}_L}{m_e v_{th}^2}\right), \qquad \boldsymbol{E}_L = \frac{m_e v_{th}^2}{q_e}\left(\frac{\nabla n_e}{n_e} + \frac{5}{2}\frac{\nabla T_e}{T_e}\right).$$

Nonlocal Ohm's law

We found that the quality of the electric field evaluation is essential for a correct plasma modeling

Nonlocal current in plasma

$$\boldsymbol{j}(f,\boldsymbol{E}) = e\int v\boldsymbol{f_1} v^2\,\mathrm{d}v = e\int v\frac{\nu_{ei}^2\boldsymbol{E}^* + \boldsymbol{\omega}_B\,\boldsymbol{\omega}_B\cdot\boldsymbol{E}^* + \nu_{ei}\,\boldsymbol{\omega}_B\times\boldsymbol{E}^*}{\nu_{ei}(\boldsymbol{\omega}_B^2 + \nu_{ei}^2)} v^2\,\mathrm{d}v, \tag{6}$$

where $\boldsymbol{E}^* = v\frac{\nu_e}{2}\frac{\partial\boldsymbol{f_1}}{\partial v} - v\nabla f_0 - \frac{q_e}{m_e}\boldsymbol{E}\frac{\partial f_0}{\partial v}$ is an effective electric field in plasma. A comparison to the *Generalized Ohm's law* shows a correct local behavior of (6), especially that $\nabla\times\nabla f_0 \sim \nabla\times\frac{\nabla p_e}{n_e} \sim \nabla n_e\times\nabla T_e$

Generalized Ohm's law vs. nonlocal Ohm's law

$$\begin{aligned}
\boldsymbol{E} &= (\boldsymbol{R}_T - \nabla p_e) &+& \quad\frac{\boldsymbol{j}}{\sigma} &+& \quad\boldsymbol{j}\times\boldsymbol{B},\\
\boldsymbol{E}v\frac{q_e}{m_e}\frac{\partial f_0}{\partial v} &= v^2\nabla f_0 &+& \quad v^2\frac{\nu_e}{2}\frac{\partial\boldsymbol{f_1}}{\partial v} - v\nu_{ei}\boldsymbol{f_1} &+& \quad v\boldsymbol{f_1}\times\frac{q_e}{m_e c}\boldsymbol{B}.
\end{aligned}$$

Nonlocal-MHD

HYDRODYNAMICS

$$
\begin{aligned}
\frac{\mathrm{d}\rho}{\mathrm{d}t} &= -\rho\nabla\cdot\boldsymbol{u},\\
\rho\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} &= -\nabla(p_i+p_e)+\boldsymbol{j}(f,\boldsymbol{E})\times\boldsymbol{B},\\
\rho\left(\frac{\partial\varepsilon_i}{\partial T_i}\frac{\mathrm{d}T_i}{\mathrm{d}t}+\frac{\partial\varepsilon_i}{\partial\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t}\right) &= -p_i\nabla\cdot\boldsymbol{u}-G(T_i-T_e),\\
\rho\left(\frac{\partial\varepsilon_e}{\partial T_e}\frac{\mathrm{d}T_e}{\mathrm{d}t}+\frac{\partial\varepsilon_e}{\partial\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t}\right) &= -p_e\nabla\cdot\boldsymbol{u}-\nabla\cdot\boldsymbol{q}_e(f)+G(T_i-T_e)+Q_{\mathrm{IB}}(\boldsymbol{E}_L),
\end{aligned}
$$

MAXWELL EQUATIONS

$$
\begin{aligned}
\nabla\times\boldsymbol{E} &= -\frac{1}{c}\frac{\mathrm{d}\boldsymbol{B}}{\mathrm{d}t},\\
\nabla\times\boldsymbol{B} &= \frac{4\pi}{c}\boldsymbol{j}(f,\boldsymbol{E}),
\end{aligned}
$$

AWBS KINETICS OF ELECTRONS

$$
\frac{\partial f}{\partial t}+\boldsymbol{v}\cdot\nabla f+(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})\cdot\nabla_{\boldsymbol{v}}f = v\frac{\nu_e}{2}\frac{\partial}{\partial v}\left(f-f_{MB}(T_e)\right)+\frac{\nu_{ei}+\frac{\nu_e}{2}}{2}\frac{\partial^2 f}{\partial\Omega^2}.
$$

Nonlocal-MHD $\Longleftrightarrow$ aiming HIGH

Height 250 m, overhang 40 m, Visera, Riglos, Spain.

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