An efficient kinetic modeling in plasmas by using the AWBS transport equation

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Abstract

The AWBS Boltzmann transport equation for electrons equipped with a simplified e-e collision operator [?] provides an efficient, yet physically relevant, kinetic extension compared the classical Spitzer-Harm heat flux (SH) based on local approximation and flux-limiting. This classical approach is widely used in plasma kinetics models coupled to hydrodynamics. Even though SH reflects the electron-electron collision effect, the essential physical properties of the electron transport cannot be modeled with an explicitly local model. A simple form of the AWBS model opens a way to couple kinetics to hydrodynamic codes while describing the important physics correctly. Since the electron-electron collision effect becomes especially important in the case of low ion potential, we focus on presenting results related to low Z plasmas.

Keywords: kinetics; hydrodynamics; nonlocal electron transport; laser-heated plasmas.

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1. Introduction

2. The AWBS nonlocal transport model

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3. BGK, AWBS, and Fokker-Planck models in diffusive regime

We can try to find an approximate solution while using the first term of expansion in λ_e and muas

$$\tilde{f}(z,v,\mu) = f^0(z,v) + f^1(z,v)\lambda_{ei}\mu. \tag{1}$$

3.1. The BGK diffusive electron transport

$$\boldsymbol{n}\cdot\nabla f + \frac{1}{v}\left[\tilde{\boldsymbol{E}}\cdot\boldsymbol{n}\frac{\partial f}{\partial v} + \frac{\tilde{\boldsymbol{E}}\cdot\boldsymbol{e}_\phi - v\tilde{\boldsymbol{B}}\cdot\boldsymbol{e}_\theta}{v}\frac{\partial f}{\partial \phi} + \frac{\tilde{\boldsymbol{E}}\cdot\boldsymbol{e}_\theta + v\tilde{\boldsymbol{B}}\cdot\boldsymbol{e}_\phi}{v\sin(\phi)}\frac{\partial f}{\partial \theta}\right] = \frac{(f_M - f)}{\lambda^e} + \frac{1}{2\lambda^{ei}}\left(\frac{\partial}{\partial \mu}\left((1-\mu^2)\frac{\partial f}{\partial \mu}\right) + \frac{1}{\sin^2(\phi)}\frac{\partial^2 f}{\partial \theta^2}\right), \tag{2}$$

where $\mu = \cos(\phi)$, λ_e is the electron-electron mean free path, and λ_{ei} is the electron-ion mean free path. We also approximate $\lambda_e = \bar{Z}\lambda_{ei}$.

Clearly, $\frac{\partial \tilde{f}}{\partial \theta} = 0$, and if $\tilde{\boldsymbol{B}} = \tilde{B}_z\boldsymbol{e}_z$, there is no effect of magnetic field. We also assume, that $\nabla f = \frac{\partial f}{\partial z}\boldsymbol{e}_z$ and appropriately $\tilde{\boldsymbol{E}} = \tilde{E}_z\boldsymbol{e}_z$. From the orientation of the Cartesian basis vectors and spherical basis vectors, one can find $\tilde{\boldsymbol{E}}\cdot\boldsymbol{n} = \tilde{E}_z\cos(\phi) = \mu$ and $\tilde{\boldsymbol{E}}\cdot\boldsymbol{e}_\phi = -\tilde{E}_z\sin(\phi)$. As a result, the analyzed BGK equation reads

$$\mu\frac{\partial}{\partial z}\left(f^0 + f^1\lambda_{ei}\mu\right) + \frac{1}{v}\left[\tilde{E}_z\mu\frac{\partial}{\partial v}\left(f^0 + f^1\lambda_{ei}\mu\right) - \frac{\tilde{E}_z\sin(\phi)}{v}\frac{\partial}{\partial \phi}\left(f^0 + f^1\lambda_{ei}\mu\right)\right] = \frac{(f_M - (f^0 + f^1\lambda_{ei}\mu))}{\lambda_e} + \frac{1}{2\lambda_{ei}}\frac{\partial}{\partial \mu}\left((1-\mu^2)\frac{\partial}{\partial \mu}\left(f^0 + f^1\lambda_{ei}\mu\right)\right), \tag{3}$$

$$\mu\frac{\partial f^0}{\partial z} + \mu^2\frac{\partial}{\partial z}\left(f^1\lambda_{ei}\right) + \frac{\tilde{E}_z}{v}\left[\mu\frac{\partial f^0}{\partial v} + \mu^2\frac{\partial}{\partial v}\left(f^1\lambda_{ei}\right) + \frac{1-\mu^2}{v}f^1\lambda_{ei}\right] = \frac{f_M - f^0}{\bar{Z}\lambda_{ei}} - \mu\frac{1}{\bar{Z}}f^1 - \mu f^1, \tag{4}$$

consequently, we have the following anisotropy expansion $\mu^0, \mu^1, \mu^2, ...$ equations

$$\begin{aligned}
\frac{f_M - f^0}{\bar{Z}\lambda_{ei}} &= \frac{1}{v} f^1 \lambda_{ei}, \\
\frac{\partial f^0}{\partial z} + \frac{\tilde{E}_z}{v}\frac{\partial f^0}{\partial v} &= -\frac{1}{\bar{Z}} f^1 - f^1, \\
\frac{\partial}{\partial z}\left(f^1\lambda_{ei}\right) + \frac{\tilde{E}_z}{v}\left[\frac{\partial}{\partial v}\left(f^1\lambda_{ei}\right) - \frac{1}{v} f^1\lambda_{ei}\right] &= 0,
\end{aligned}$$

which lead to the definitions

$$f^0 = f_M + \frac{1}{v} f^1 \bar{Z}\lambda_{ei}^2, \tag{5}$$

$$\begin{aligned}
f^1 &= -\frac{\bar{Z}}{\bar{Z}+1}\left[\frac{\partial f^0}{\partial z} + \frac{\tilde{E}_z}{v}\frac{\partial f^0}{\partial v}\right] \\
&= -\frac{\bar{Z}}{\bar{Z}+1}\left[\frac{1}{\rho}\frac{\partial \rho}{\partial z} + \left(\frac{v^2}{2v_{th}^2} - \frac{3}{2}\right)\frac{1}{T}\frac{\partial T}{\partial z} - \frac{\tilde{E}_z}{v_{th}^2}\right] f_M
\end{aligned} \tag{6}$$

In order to ensure the plasma to be quasi-neutral, the zero-current condition

$$\boldsymbol{j} = \int_0^\infty \int_{4\pi} q_e v \boldsymbol{n} f \,\mathrm{d}\boldsymbol{n}\; v^2 \,\mathrm{d}v = \boldsymbol{0}, \tag{7}$$

can be achieved by providing a consistent electric field in (14), i.e.

$$\tilde{\boldsymbol{E}} = \frac{v_{th}^2 \int_{4\pi} \boldsymbol{n}\otimes\boldsymbol{n}\cdot\int_0^\infty v f_M \frac{\lambda}{\alpha}\left(\frac{\nabla\rho}{\rho} + \left(\frac{v^2}{2v_{th}^2} - \frac{3}{2}\right)\frac{\nabla T}{T}\right) v^2 \,\mathrm{d}v\,\mathrm{d}\boldsymbol{n}}{\int_{4\pi} \boldsymbol{n}\otimes\boldsymbol{n}\cdot\int_0^\infty v f_M \frac{\lambda}{\alpha} v^2 \,\mathrm{d}v\,\mathrm{d}\boldsymbol{n}}, \tag{8}$$

which may be further simplified as

$$\tilde{\boldsymbol{E}} = \frac{\int_0^\infty f_M \frac{1}{2}\frac{\nabla T}{T} v^9 \,\mathrm{d}v}{\int_0^\infty f_M v^7 \,\mathrm{d}v} + v_{th}^2\left(\frac{\nabla\rho}{\rho} - \frac{3}{2}\frac{\nabla T}{T}\right) = v_{th}^2\left(\frac{\nabla\rho}{\rho} + \frac{5}{2}\frac{\nabla T}{T}\right), \tag{9}$$

where it is worth mentioning, that the part $f_M + \frac{v\lambda}{\alpha}\frac{\partial f_1}{\partial v}$ of the distribution does not contribute to the current since it is isotropic. One can write the quasi-neutral distribution function explicitly distinguishing between original part (blue color) and E field correction (red color) as

$$f \approx f_M\left(1 - \frac{\lambda}{\alpha}\boldsymbol{n}\cdot\left(\frac{v^2}{2v_{th}^2} - \frac{3}{2} - \frac{5}{2}\right)\frac{\nabla T}{T}\right) + \frac{v\lambda}{\alpha}\frac{\partial f_1}{\partial v}. \tag{10}$$

which leads to the resulting heat flux

$$\boldsymbol{q}_H = \int_{4\pi}\int_0^\infty \frac{m_e v^2}{2} v \boldsymbol{n} f v^2 \,\mathrm{d}v\,\mathrm{d}\boldsymbol{n} = \frac{4\pi}{3}\frac{m_e}{2}\frac{1}{\alpha\sigma\rho}\int_0^\infty \left(\frac{v^2}{2v_{th}^2} - \frac{3}{2} - \frac{5}{2}\right) v^9 f_M \,\mathrm{d}v \frac{\nabla T}{T}.$$

Based on the Gauss integral formula

$$\int v^{2s+1}\exp\left(-\frac{v^2}{2v_{th}^2}\right)\mathrm{d}v = \frac{s!\,(2v_{th}^2)^{s+1}}{2}$$

and Maxwell-Boltzmann distribution (**??**) the heat flux can be written as

$$\boldsymbol{q}_H = \frac{4\pi}{3}\frac{m_e}{2}\frac{1}{\alpha\sigma\rho}\frac{\rho}{v_{th}^3\,(2\pi)^{3/2}}\frac{4!\,2^4 v_{th}^{10}}{T}\left(5 - \frac{3}{2} - \frac{5}{2}\right)\nabla T = \frac{m_e}{\alpha\sigma}\frac{128}{\sqrt{2\pi}}\left(\frac{k_B}{m_e}\right)^{\frac{7}{2}} T^{\frac{5}{2}}\nabla T. \tag{11}$$

In conclusion, equation (11) provides nothing else than the well known Lorentz approximation heat flux and its nonlinearity 2.5 in temperature. What is worth mentioning is the effect of E field (quasi-neutrality), which reduces the flux of about 71.4% (also assuming constant density).

Finally, one can find the approximate solution

$$\tilde{f} = f_M - \lambda_{ei}\frac{\bar{Z}}{\bar{Z}+1}\left(\frac{v^2}{2v_{th}^2} - \frac{3}{2} - \alpha\right)\frac{\boldsymbol{n}\cdot\nabla T}{T} f_M. \tag{12}$$

3.2. The AWBS diffusive electron transport

The AWBS electron transport equation in 6D reads

$$\boldsymbol{n}\cdot\nabla f + \frac{1}{v}\left[\tilde{\boldsymbol{E}}\cdot\boldsymbol{n}\frac{\partial f}{\partial v} + \frac{\tilde{\boldsymbol{E}}\cdot\boldsymbol{e}_\phi - v\tilde{\boldsymbol{B}}\cdot\boldsymbol{e}_\theta}{v}\frac{\partial f}{\partial \phi} + \frac{\tilde{\boldsymbol{E}}\cdot\boldsymbol{e}_\theta + v\tilde{\boldsymbol{B}}\cdot\boldsymbol{e}_\phi}{v\sin(\phi)}\frac{\partial f}{\partial \theta}\right] =$$
$$\frac{v}{\lambda^e}\frac{\partial}{\partial v}(f - f_M) + \left(\frac{1}{\lambda_{ei}} + \frac{1}{\lambda_e}\right)\frac{1}{2}\left(\frac{\partial}{\partial \mu}\left((1-\mu^2)\frac{\partial f}{\partial \mu}\right) + \frac{1}{\sin^2(\phi)}\frac{\partial^2 f}{\partial \theta^2}\right), \tag{13}$$

where $\mu = \cos(\phi)$, λ_e is the electron-electron mean free path, and λ_{ei} is the electron-ion mean free path, and $\lambda_e = \bar{Z}\lambda_{ei}$.

We can try to find an approximate solution while using the first term of expansion in λ_e and μ as

$$\tilde{f}(z, v, \mu) = f^0(z, v) + f^1(z, v)\lambda_{ei}\mu. \tag{14}$$

Clearly, $\frac{\partial \tilde{f}}{\partial \theta} = 0$, and if $\tilde{\boldsymbol{B}} = \tilde{B}_z \boldsymbol{e}_z$, there is no effect of magnetic field. We also assume, that $\nabla f = \frac{\partial f}{\partial z}\boldsymbol{e}_z$ and appropriately $\tilde{\boldsymbol{E}} = \tilde{E}_z \boldsymbol{e}_z$. From the orientation of the Cartesian basis vectors and spherical basis vectors, one can find $\tilde{\boldsymbol{E}}\cdot\boldsymbol{n} = \tilde{E}_z \cos(\phi) = \mu$ and $\tilde{\boldsymbol{E}} \cdot \boldsymbol{e}_\phi = -\tilde{E}_z \sin(\phi)$. As a result, the analyzed AWBS equation reads

$$\mu\frac{\partial}{\partial z}\left(f^0 + f^1\lambda_{ei}\mu\right) + \frac{1}{v}\left[\tilde{E}_z\mu\frac{\partial}{\partial v}\left(f^0 + f^1\lambda_{ei}\mu\right) - \frac{\tilde{E}_z\sin(\phi)}{v}\frac{\partial}{\partial \phi}\left(f^0 + f^1\lambda_{ei}\mu\right)\right] =$$
$$\frac{v}{\lambda_e}\frac{\partial}{\partial v}\left(\left(f^0 + f^1\lambda_{ei}\mu\right) - f_M\right) + \frac{\bar{Z}+1}{2\lambda_{ei}\bar{Z}}\frac{\partial}{\partial \mu}\left((1-\mu^2)\frac{\partial}{\partial \mu}\left(f^0 + f^1\lambda_{ei}\mu\right)\right), \tag{15}$$

$$\mu\frac{\partial f^0}{\partial z} + \mu^2\frac{\partial}{\partial z}\left(f^1\lambda_{ei}\right) + \frac{\tilde{E}_z}{v}\left[\mu\frac{\partial f^0}{\partial v} + \mu^2\frac{\partial}{\partial v}\left(f^1\lambda_{ei}\right) + \frac{1-\mu^2}{v}f^1\lambda_{ei}\right] =$$
$$\frac{v}{\bar{Z}\lambda_{ei}}\frac{\partial}{\partial v}\left(f^0 - f_M\right) + \mu\frac{v}{\bar{Z}\lambda_{ei}}\frac{\partial(f^1\lambda_{ei})}{\partial v} - \mu\frac{\bar{Z}+1}{\bar{Z}}f^1, \tag{16}$$

consequently, we have the following anisotropy expansion $\mu^0, \mu^1, \mu^2, ...$ equations

$$\begin{aligned}
\frac{v}{\bar{Z}\lambda_{ei}}\frac{\partial}{\partial v}\left(f^0 - f_M\right) &= \frac{1}{v}f^1\lambda_{ei}, \\
\frac{\partial f^0}{\partial z} + \frac{\tilde{E}_z}{v}\frac{\partial f^0}{\partial v} &= \frac{v}{\bar{Z}\lambda_{ei}}\frac{\partial(f^1\lambda_{ei})}{\partial v} - \frac{\bar{Z}+1}{\bar{Z}}f^1, \\
\frac{\partial}{\partial z}\left(f^1\lambda_{ei}\right) + \frac{\tilde{E}_z}{v}\left[\frac{\partial}{\partial v}\left(f^1\lambda_{ei}\right) - \frac{1}{v}f^1\lambda_{ei}\right] &= 0,
\end{aligned}$$

which lead to the definitions

$$\frac{\partial}{\partial v}\left(f^0 - f_M\right) = \frac{1}{v^2}f^1\bar{Z}\lambda_{ei}^2, \tag{17}$$

$$\begin{aligned}
\frac{v}{\bar{Z}\lambda_{ei}}\frac{\partial(f^1\lambda_{ei})}{\partial v} - \frac{\bar{Z}+1}{\bar{Z}}f^1 &= \frac{\partial f^0}{\partial z} + \frac{\tilde{E}_z}{v}\frac{\partial f^0}{\partial v} \\
\frac{v}{\bar{Z}}\frac{\partial f^1}{\partial v} + \frac{4}{\bar{Z}}f^1 - \frac{\bar{Z}+1}{\bar{Z}}f^1 &= \frac{\partial f^0}{\partial z} + \frac{\tilde{E}_z}{v}\frac{\partial f^0}{\partial v} \\
\frac{\partial f^1}{\partial v} + \frac{1}{v}(3-\bar{Z})f^1 &= \frac{\bar{Z}}{v}\left(\frac{1}{\rho}\frac{\partial \rho}{\partial z} + \left(\frac{v^2}{2v_{th}^2} - \frac{3}{2}\right)\frac{1}{T}\frac{\partial T}{\partial z} - \frac{\tilde{E}_z}{v_{th}^2}\right)f_M
\end{aligned} \tag{18}$$

3.3. The Fokker-Planck diffusive electron transport

[2], [3], [4]

3.4. Summary of BGK, AWBS, and Fokker-Planck diffusion

4. Benchmarking the AWBS nonlocal transport model

4.1. Review of simulation codes

4.1.1. C7

4.1.2. ALADIN

4.1.3. IMPACT

4.1.4. CALDER

4.2. Simulation results

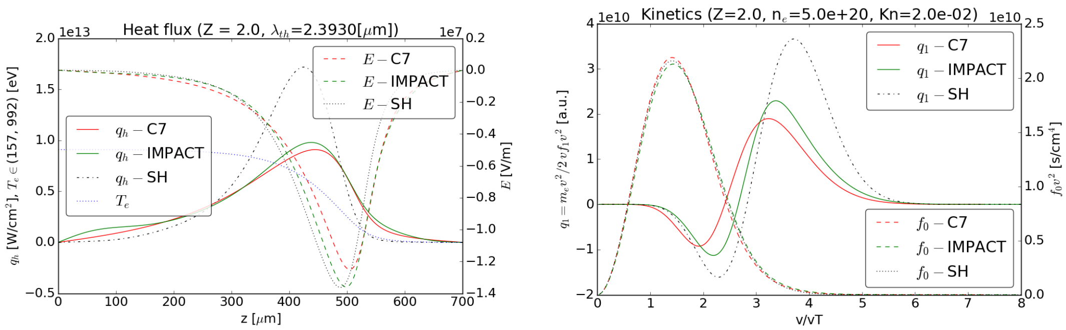


Figure 1: Left: correct steady solution. Right: correct comparison to kinetic profiles by IMPACT.

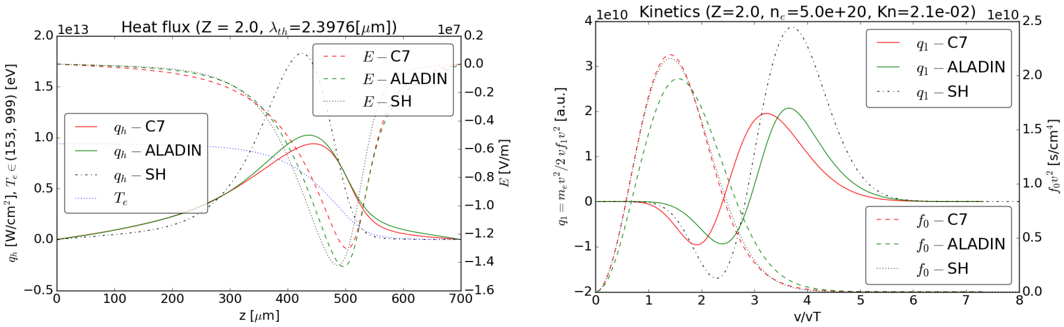


Figure 2: Left: correct steady solution. Right: time and point to be precised by ALADIN.

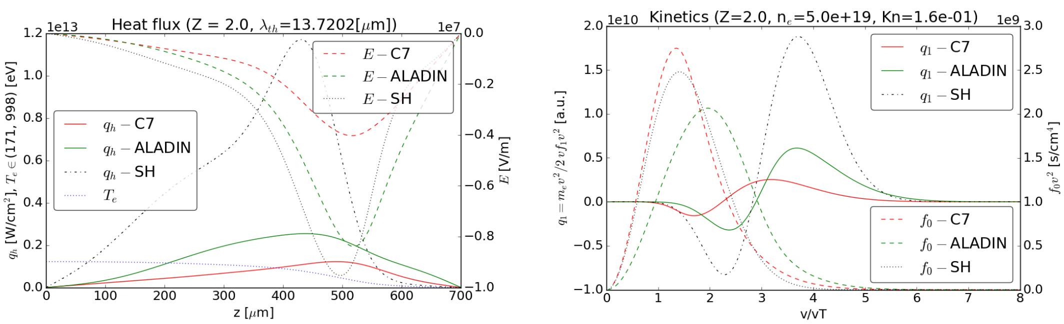


Figure 3: Left: Does not look as steady solution. Right: time and point to be precised by ALADIN.

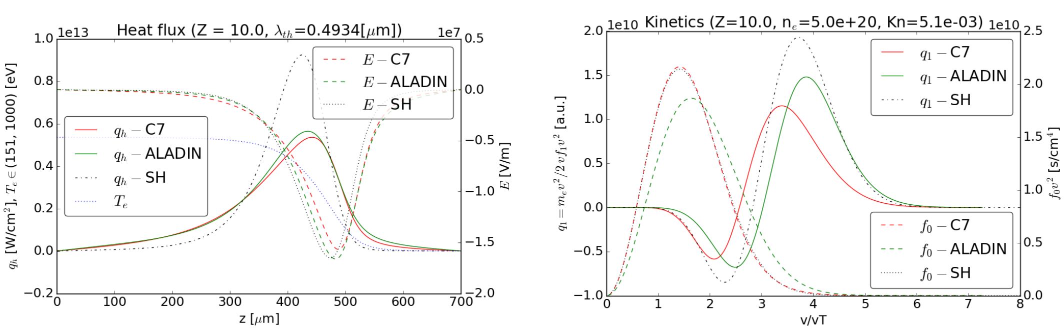
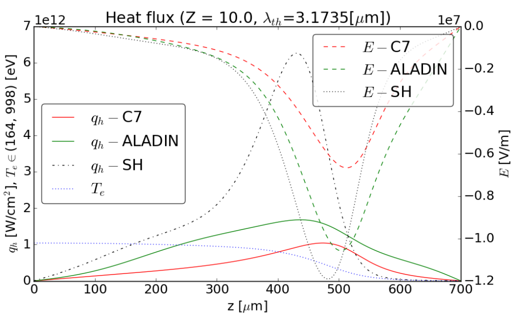


Figure 4: Left: correct steady solution. Right: time and point to be precised by ALADIN.



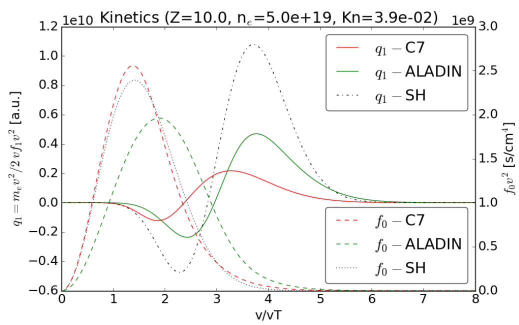


Figure 5: Left: Does not look as steady solution. Right: time and point to be precised by ALADIN.

5. Conclusions

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