Nonlocal Magnetic-Field Generation in Plasmas without Density Gradients

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Conventional theories of magnetic-field generation by laser pulses in collisional plasmas require the presence of density gradients or anisotropic pressure. Using the first two-dimensional Fokker-Planck code to self-consistently include magnetic fields, we find that magnetic fields can be spontaneously generated when a collisional plasma is nonuniformly heated even though $\nabla n = 0$ and the pressure is purely isotropic. These magnetic fields, which can become strong enough to significantly affect transport, are attributed to nonlocal effects that are missing in the standard, local theories.

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Strong magnetic fields can be generated when an intense laser pulse interacts with a plasma. Fields as large as several mega-Gauss have been directly measured in the blowoff plasma in front of solid targets [1,2] and attributed to a mechanism that occurs when the plasma density gradient ∇n and temperature gradient ∇T are not collinear, i.e., $\partial_t \mathbf{B} \propto -\nabla n \times \nabla T$. Strong magnetic fields can severely alter both the direction and size of heat flows in plasmas, according to linear transport theory [3]; hence they can play an important role in laser-plasma-based fusion schemes such as direct and indirect drive inertial confinement fusion (ICF). Anomalously large electron temperatures were recently observed in hohlraums and attributed to suppression of heat flow by magnetic fields [4]. Recently, numerical studies relevant to the fast ignitor variant of ICF have predicted self-generated magnetic fields ranging from 30–1000 MG inside solid density plasmas [5–7]. Production of energetic γ rays [8] and ions [5] has also been observed in experiments and simulations under similar conditions. A proper understanding of how magnetic fields are generated in laser-plasma interactions is crucial for the successful development of laser-fusion schemes and laser-plasma-based sources of energetic γ rays and ions.

The generation of magnetic fields in collisional plasmas has previously been considered. Linear and local theories of magnetic-field generation (reviewed in [9]), e.g., $\nabla n \times \nabla T$ magnetic-field generation [1], assume the plasma is very close to local thermodynamic equilibrium and therefore set $f_0 = f_M$, where f_0 is the isotropic part of the electron distribution and $f_M = n_e/(2\pi k_B T_e/m_e)^{3/2} \exp(-m_e v^2/2k_B T_e)$ is the Maxwellian distribution. They are based on linear transport theories, such as Braginskii's transport equations [3] and Spitzer-Härm thermal conduction [10], and thus are valid when n and T change negligibly over distances comparable to the mean-free path for collisions. Density gradients (in addition to temperature gradients) and/or anisotropic pressure are prerequisites for magnetic-field growth in this regime. Two-temperature (hot and cold) Maxwellian fluid models of magnetic-field generation, where $f_0 = (f_M)_c + (f_M)_h$, have also been considered [6,11]. Extra source terms such as $\nabla n_c \times \nabla n_h$ then appear. Davies *et al.* [12] developed a hybrid particle/fluid code to describe fast electron transport and magnetic-field generation in solid density plasmas. In his model a beam of energetic electrons, represented by particles, is injected into a background of thermal electrons, represented by a fluid. This model predicts formation of magnetic fields inside the uniform density target which focus the fast electrons thereby enabling them to traverse through many microns of dense plasma. Particle-in-cell (PIC) simulations [7,13] have also predicted self-generated magnetic fields inside dense, uniform plasmas, again in the presence of highly anisotropic heating (i.e., beams of fast electrons), but under conditions where collisional effects are less important.

In this Letter, we present a new magnetic-field generation mechanism based on nonlocal transport in collisional laser-produced plasmas. It is well known that nonlocal models are needed to properly describe heat flow down steep temperature gradients [14]. We show that nonlocal effects are also important for magnetic-field generation and produce new phenomena not found in local theories. We find that spontaneous magnetic-field generation can occur under conditions which have not been considered in hybrid or PIC simulations and which according to linear and local theories imply $\partial_t B = 0$: (i) there are *no density gradients,* (ii) the *pressure is purely isotropic,* and (iii) *isotropic heating* is applied to the plasma (i.e., no beams of energetic particles are injected into the plasma, only thermal energy). In other words, none of the usual conditions for magnetic-field self-generation in collisional plasmas ($\nabla n \neq 0$, anisotropic pressure, or anisotropic heating via beams) are in fact essential. The magnetic-field generation mechanism presented here is essentially the nonlocal analog of the standard $\nabla n \times \nabla T$ source and is driven by nonparallel gradients of high velocity moments of f_0. No assumptions about the form of f_0 are made here. The local theories miss this effect because setting $f_0 = f_M$ *forces* these gradients to be parallel.

We consider a planar, solid density plasma with a nonevolving, uniform, density profile which initially is unmagnetized and has a uniform temperature but is subsequently heated nonuniformly on one surface by a laser pulse. We model nonlocal electron energy transport from the absorption region into the plasma using our newly developed two-dimensional (2D) Fokker-Planck (FP) code, IMPACT (implicit magnetized plasma and collisional transport). IMPACT is the *first* 2D FP code, designed to model laser-plasma transport phenomena, that *self-consistently* includes magnetic fields. The only other FP code with self-consistent magnetic fields [15] is 1D and hence cannot describe magnetic-field generation. As other FP codes [15,16], IMPACT uses a reduced description of the electron velocity distribution; $f(\mathbf{v},\mathbf{r},t) \sim f_0(v,\mathbf{r},t) + \mathbf{f}_1(v,\mathbf{r},t)\cdot\hat{\mathbf{v}}$, which is appropriate when collisions are important. This splits the full FP equation, $[\partial_t + \mathbf{v}\cdot\nabla_r - |e|/m_e(\mathbf{E} + \mathbf{v}\times\mathbf{B})\cdot\nabla_v]f(\mathbf{v},\mathbf{r},t) = (\delta f/\delta t)_{\mathrm{coll}}$, into coupled equations for the evolution of f_0 and $\mathbf{f}_1$. It uses an implicit finite difference algorithm to update *both* the electric field components $E_x(x,y)$, $E_y(x,y)$ and the distribution functions $f_0(x,y,v)$ and $f_x(x,y,v)$, $f_y(x,y,v)$ (f_x and f_y are the components of $\mathbf{f}_1$) while simultaneously ensuring that the new $\mathbf{f}_1$ yields a current that satisfies $\mu_0\mathbf{j} = \nabla\times\mathbf{B}$, at every time step. Faraday's law is used to update the magnetic field. IMPACT currently uses the Lorentz approximation (valid for high Z), whereby electron-electron (e-e) collisions are neglected in the equation for $\mathbf{f}_1$—electron-ion (e-i) scattering is retained of course. The code does include the electron inertia term $\partial\mathbf{f}_1/\partial t$, however. Hydrodynamic motion is neglected. Full details of IMPACT will be published elsewhere.

The electron density, ionization state, and initial temperature of the plasma are taken to be $n_e = 3\times10^{23}\ \mathrm{cm}^{-3}$, $Z = 10$, and $T_{e0} = 11$ keV, respectively. Under these conditions the mean-free-path and collision time for angular scattering of an electron (moving at the initial thermal speed $v_{t0} = \sqrt{2k_bT_{e0}/m_e}$) by ions are $\lambda_{ei} = 1\ \mu$m and $\tau_{ei} = 16$ fs, respectively, the Coulomb logarithm is $\ln\Lambda_{ei} = 6.3$, and $\lambda_{ei}/\delta_c = 100$, where $\delta_c = c/\sqrt{n_e e^2/\epsilon_0 m_e}$ is the collisionless skin depth. The size of the computational domain is $L_x = 12\ \mu$m by $L_y = 2\ \mu$m. The x and y axes are taken to be aligned perpendicular and parallel to the surface, respectively. The heating spatial profile shown in Fig. 1 is applied for $0 < t < 640$ fs. It is described by $\partial_t T_e(x,y) = W_0[1 - \tanh\{[x - x_0(y)]/d\}]$ with $x_0(y) = (L_x/4) + (L_y/4)\cos(\pi y/L_y)$ and $W_0 = 85$ eV/fs, $d = L_x/12$. The heating mechanism is isotropic in velocity space and changes f_0 in such a way that if transport were turned off f_0 would maintain a Maxwellian profile while the temperature increased. Reflective boundary conditions and 150×25 grid points are used in the x and y directions, respectively. The v grid is defined by $\Delta v = v_{t0}/5$ and $v_{\max} = 28v_{t0}$ and the time step is $\Delta t = 6.4$ fs. The initial electron temperature of 11 keV is an appropriate point at which to start simulating magnetic-field generation in the vicinity of the laser absorption region. At and above this temperature resistive diffusion of the magnetic field is much slower than thermal diffusion, at the high plasma density considered here. In principle it would be possible to start at a lower value of T_{e0} so long as $\ln\Lambda_{ei}$ remains large enough for the FP equation to be valid.

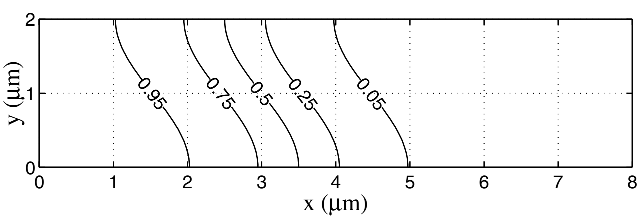

FIG. 1. Normalized profile of the isotropic heating source applied for $0 < \tau < 640$ fs.

The plasma is initially devoid of magnetic fields, but after just 80 fs of heating a substantial magnetic field, oriented in the $+$ve z-direction, has already formed as depicted in Fig. 2(a). The Hall parameter $\omega\tau \propto BT^{3/2}/n$ (where $\omega = |e|B/m_e$ is the electron cyclotron frequency and τ is the distribution averaged e-i collision time) almost reaches a value of 0.1 which indicates that this spontaneous magnetic field is becoming strong enough to affect transport. Comparison with the heating profile (Fig. 1) shows that the magnetic field grows most rapidly where the gradient and shear of that profile are largest, whereas no field grows along the upper and lower edges because the profile is unsheared there. The magnetic field's magnitude and profile continue to grow and evolve with further heating. Figure 2(b) shows that by 800 fs parts of the plasma are strongly magnetized and the field profile is considerably more complicated, with the direction of $\mathbf{B}$ now varying from region to region. Indeed, the Hall parameter reaches $\omega\tau \approx 9$ (equivalent to $B \approx 2.4$ MG) and the large magnetic field severely deflects the direction of the heat flow, as can be seen in Fig. 3, so that it does not correspond at all to ∇T. This heat flow deflection in turn results in localized heating deeper into the plasma as witnessed by the curved temperature contours between $5\ \mu\mathrm{m} < x < 6.5\ \mu$m in Fig. 4.

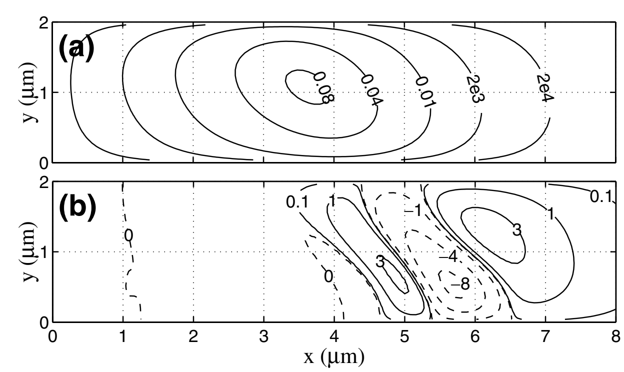

FIG. 2. Magnetic field profiles after (a) 80 fs and (b) 800 fs of nonuniform heating (as shown in Fig. 1). The Hall parameter corresponding to the magnetic field strength, $\omega\tau \propto BT^{3/2}/n$, is shown.

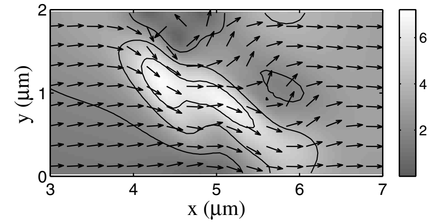

FIG. 3. The strongly deflected heat flow at 800 fs. Arrows represent the heat flow direction only, while the gray scale represents its magnitude in units of $q_{fs}/1000$ [where $q_{fs} = (2/3)n_e(k_B T_e)^{3/2}/m_e^{1/2}$ is the free streaming heat flow]. The contours show where $q/q_{fs} = \{2, 4, 6\} \times 10^{-3}$.

At first sight the appearance and growth of the magnetic field, as described above, seem surprising since $\nabla n_e = 0$ at all times (this is guaranteed as a consequence of neglecting the displacement current) and the model deals only with scalar pressure. Anisotropic pressure, which is necessary for several magnetic-field generating instabilities [9] (e.g., collisional Weibel), is not included. It turns out that the mechanism responsible for the magnetic-field in Fig. 2 is a nonlocal effect that arises when the temperature scale length L_T starts becoming comparable to the transport mean-free path, as will now be shown. The "standard" prerequisites for spontaneous magnetic-field generation in a collisional plasma stem from the generalized Ohms law which describes momentum balance. In our case (stationary, cold ions) and neglecting electron inertia it is $|e|n_e\mathbf{E} = -\nabla p_e - \nabla \cdot \mathbf{\Pi}_e + \mathbf{j}_e \times \mathbf{B} + |e|n_e\alpha \cdot \mathbf{j}_e - n_e\beta \cdot \nabla T_e$, where $\mathbf{\Pi}_e$, α, and β are the traceless stress, electrical resistivity, and thermoelectric tensors, respectively. Effects due to anisotropic pressure appear through $\mathbf{\Pi}_e$. Inserting this electric field into Faraday's law shows that only the $-\nabla p_e/|e|n_e$ and $-(\nabla \cdot \mathbf{\Pi}_e)/|e|n_e$ terms are genuine sources of magnetic field. The term involving the thermoelectric tensor β has been shown not to be a source term [17]. Ohm's law is itself derived by taking the $(4\pi/3)\int_0^\infty v^3\,dv$ velocity moment of the $\mathbf{f}_1$ equation, $-\mathbf{f}_1\nu'/v^3 + C_{ee1} = v\nabla f_0 + \frac{2}{5}v\nabla \cdot \mathbf{f}_2 - \frac{|e|}{m_e}[\mathbf{E}\partial_v f_0 + (2/5v^3)\mathbf{E} \cdot \partial_v(v^3\mathbf{f}_2) + \mathbf{B} \times \mathbf{f}_1]$ and making the local approximation, $f_0 = f_M$. The effect of electron-electron collisions on $\mathbf{f}_1$ is represented by C_{ee1}, while $\nu' = 4\pi n_i(Ze^2/4\pi\epsilon_0 m_e)^2\ln\Lambda$ is the velocity independent part of the electron-ion angular scattering frequency $\nu_{ei} = \nu'/v^3$.

The nonlocal origin of the magnetic field in Fig. 2 can be understood by obtaining a more general expression for Ohm's law that dispenses with the local approximation. In the absence of $\mathbf{f}_2$ (i.e., pressure is isotropic), neglecting e-e collisions, and before electric currents and magnetic fields are present, the $(4\pi/3)\int_0^\infty v^6\,dv$ moment of the $\mathbf{f}_1$ equation yields [11] $\mathbf{E} = -(m_e/|e|)\nabla(n_e\langle v^5\rangle)/6n_e\langle v^3\rangle$, where $\langle v^m\rangle = (4\pi/n_e)\int_0^\infty f_0 v^{m+2}\,dv$ are moments of f_0 and thus $\partial_t\mathbf{B} = -(m_e/6|e|n_e^2\langle v^3\rangle^2)\nabla(n_e\langle v^3\rangle) \times \nabla(n_e\langle v^5\rangle)$, valid for arbitrary f_0. This shows that a magnetic field can be generated in a uniform density plasma if the gradients in $\langle v^3\rangle$ and $\langle v^5\rangle$ are nonparallel which is possible only if f_0 is not Maxwellian. It is easy to see that when $f_0 = f_M$ the preceding expressions for $\mathbf{E}$ and $\partial_t\mathbf{B}$ reduce to $\mathbf{E} = -\{(\nabla p_e)/n_e + \beta_0\nabla T_e\}/|e|$ [where $\beta_0 = (3/2)k_B$] and $\partial_t\mathbf{B} = -(k_B/|e|n_e)\nabla n_e \times \nabla T_e$ because the moments become $\langle v^m\rangle = (2/\sqrt{\pi})\Gamma[(m+3)/2](2k_B T_e/m_e)^{m/2}$. Therefore magnetic-field generation when $\nabla n_e \to 0$ is overlooked. Nonlocal transport is responsible for generating the *angle* between the velocity moment gradients. This can be understood by considering diffusion of different velocity groups in the electron distribution through the plasma. Because $\lambda_{mfp} \propto v^4$ (and consequently the diffusion coefficient $D \propto v^5$), high velocity electrons move through the plasma more readily than low velocity ones. When the temperature scale length L_T becomes comparable to or less than $(10$–$100)\overline{\lambda}_{mfp}$ (the distribution averaged mean-free path), the hot and cold portions of f_0 can develop different spatial profiles: fast electrons are able to spread themselves out through the plasma while slower electrons remain "bottled up" by collisions with ions. Also, electron-electron collisions are unable to return the distribution to a Maxwellian. Ripples in the profile of $\langle v^5\rangle$ will therefore smooth out faster than ripples in $\langle v^3\rangle$ due to nonlocal transport which in turn generates the angle between the velocity moment gradients. The actual angle between $\nabla\langle v^3\rangle$ and $\nabla\langle v^5\rangle$ obtained from the nonuniform heating simulation at 400 fs is shown in Fig. 5. The region of reversed angle at this time explains the appearance of areas of reversed magnetic field later on [e.g., Fig. 2(b)].

We can derive a linearized equation for the nonlocal magnetic field that is seeded by a given initial temperature perturbation,

$$\ddot{\mathbf{B}}_{\rm NL} \approx 185(m_e\lambda^4/\tau^3T^2|e|)[\nabla\delta T \times \nabla(\nabla^2\delta T)], \quad (1)$$

where $T(\mathbf{r}) = T_0 + \delta T(\mathbf{r})$ and $\delta T(\mathbf{r}) < T_0$. Here λ and τ are the e-i mean-free path and collision

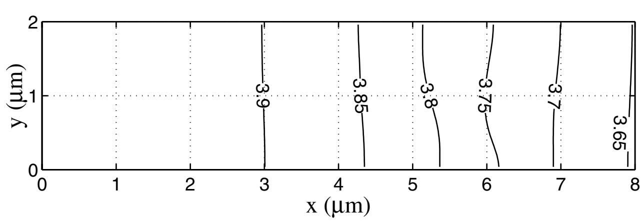

FIG. 4. Temperature profile at 800 fs. $(T - T_0)/T_0$ is plotted.

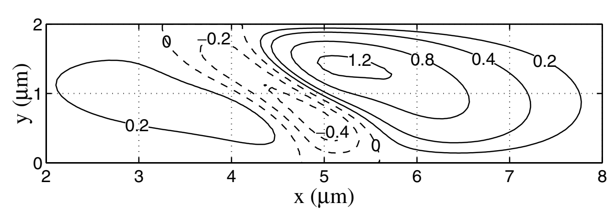

FIG. 5. Angle between $\nabla\langle v^5\rangle$ and $\nabla\langle v^3\rangle$ at 400 fs in degrees. Positive angles are defined for $\nabla\langle v^3\rangle$ lying counterclockwise of $\nabla\langle v^5\rangle$ (in the range 0 to 180 deg).

time at the local T, Z, and n. For simplicity, electron-electron collisions (in the f_0 equation) and electron inertia are neglected. We assume that f_0 starts off as a single Maxwellian; hence initially $\nabla\langle v^3\rangle$ and $\nabla\langle v^5\rangle$ are parallel and thus $\dot{\mathbf{B}} = 0$ at $t = 0$. On the other hand, $\ddot{\mathbf{B}} \neq 0$ at $t = 0$ is perfectly possible because $\ddot{\mathbf{B}} \propto \nabla(\partial_t\langle v^5\rangle) \times \nabla\langle v^3\rangle - \nabla(\partial_t\langle v^3\rangle) \times \nabla\langle v^5\rangle$ and the 3rd and 5th velocity moments of f_0 can evolve differently due to nonlocal transport. Derivation of Eq. (1) is too involved to describe fully here. It suffices to say that several moments of both the f_0 and $\mathbf{f}_1$ equations are taken; then these moment equations and $\mathbf{E} \propto -\nabla\langle v^5\rangle/6\langle v^3\rangle$ are combined together. The linearized equation is valid so long as $\omega\tau \ll 1$, the deviation of f_0 from a Maxwellian is small, and diffusion of the temperature profile is negligible. The magnetic-field profile that grows from an initial temperature perturbation during an IMPACT simulation (with electron inertia and *e-e* collisions turned off) matches the profile predicted by Eq. (1) at early times when nonlocal magnetic-field growth is still in the linear regime. Only the self-consistent solution of the full equation set, as performed by IMPACT, will yield the long term nonlinear and nonlocal magnetic-field growth to values large enough to affect transport. Nevertheless Eq. (1) is useful since it yields the scaling of the mechanism with temperature perturbation amplitude and scale length plus suggests what perturbation geometry is required. The scaling is given by $(\omega\tau)_{\mathrm{NL}} \propto G(t/\tau)^2(\delta T/T)^2(L_T/\lambda)^{-4}$, where G is a factor that depends on the temperature profile's geometry. The $(L_T/\lambda)^{-4}$ dependence (and the need for no density gradient) means the mechanism is well suited to magnetic-field generation inside solid targets irradiated by short, intense laser pulses where L_T is small. The form of Eq. (1) implies that any profile with a *varying curvature,* e.g., elliptically and ellipsoidally symmetric, and rippled planar, will generate a magnetic field. Only profiles possessing perfect symmetry, e.g., planar, cylindrical, and spherical, will not generate B. Finally, the presence of the high order derivative $\nabla(\nabla^2\delta T)$ in Eq. (1) is indicative of the mechanism's nonlocality. It is worth mentioning that local two-temperature models [6,11] cannot describe magnetic-field self-generation from a *single temperature* Maxwellian perturbation, as Eq. (1) does, since they neglect nonlocal effects. Though hybrid codes [12] include nonlocal effects via the particles, they are not well suited to describing the nonlocal magnetic-field mechanisms considered here, especially when $T_c = T_h$ and $\delta T \ll T_0$. Their artificial separation of the electron distribution into hot and cold components becomes problematic when the energies of electrons in the hot and cold parts become comparable [18]. FP codes model the full range of electron speeds in one distribution, so overcome these problems.

In summary, we have found a new mechanism for self-generation of magnetic fields by thermal sources in collisional plasmas, which is based on nonlocal transport. It demonstrates the importance of nonlocal effects to magnetic-field generation. The mechanism works in the absence of density gradients without the need for anisotropic pressure or nonthermal heating mechanisms (e.g., beams). Simply starting with a nonuniform temperature perturbation or alternatively nonuniformly applying a thermal heating source [i.e., heats $f(v)$ isotropically in v] is sufficient. These are conditions that imply $\partial_t B = 0$ according to linear and local theories (e.g., the $\nabla n \times \nabla T$ source) and have not been addressed before in nonlinear kinetic simulations. Physically, nonlocal magnetic-field generation arises when collisions cannot maintain a Maxwellian distribution as heat flows down steep temperature gradients which allows $\nabla \times \mathbf{E}$ to develop. Two-dimensional Fokker-Planck simulations, which are the first to self-consistently include magnetic fields, show that the nonlocal mechanism can spontaneously grow a magnetic field that is large enough to significantly affect the transport of heat from the surface into the bulk, resulting in localized heating several microns inside a thick target.

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