

An efficient kinetic modeling in plasmas by using the AWBS transport equation

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Abstract

The AWBS Boltzmann transport equation for electrons equipped with a simplified e-e collision operator [?] provides an efficient, yet physically relevant, kinetic extension compared the classical Spitzer-Harm heat flux (SH) based on local approximation and flux-limiting. This classical approach is widely used in plasma kinetics models coupled to hydrodynamics. Even though SH reflects the electron-electron collision effect, the essential physical properties of the electron transport cannot be modeled with an explicitly local model. A simple form of the AWBS model opens a way to couple kinetics to hydrodynamic codes while describing the important physics correctly. Since the electron-electron collision effect becomes especially important in the case of low ion potential, we focus on presenting results related to low Z plasmas.

Keywords: kinetics; hydrodynamics; nonlocal electron transport; laser-heated plasmas.

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17 1. Introduction

18 2. The AWBS nonlocal transport model

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f + \tilde{\mathbf{E}} \cdot \nabla_{\mathbf{v}} f = v \frac{\nu_e}{2} \frac{\partial}{\partial v} (f - f_M) + \frac{\nu_{ei} + \frac{\nu_e}{2}}{2} \frac{\partial^2 f}{\partial \mathbf{n}^2}, \quad (1)$$

19 [1]

20 3. BGK, AWBS, and Fokker-Planck models in diffusive regime

21 We can try to find an approximate solution while using the first term of
22 expansion in λ_e and $muas$

$$\tilde{f}(z, v, \mu) = f^0(z, v) + f^1(z, v) \lambda_{ei} \mu. \quad (2)$$

23 3.1. The BGK diffusive electron transport

$$\begin{aligned} \mathbf{n} \cdot \nabla f + \frac{1}{v} \left[\tilde{\mathbf{E}} \cdot \mathbf{n} \frac{\partial f}{\partial v} + \frac{\tilde{\mathbf{E}} \cdot \mathbf{e}_\phi - v \tilde{\mathbf{B}} \cdot \mathbf{e}_\theta}{v} \frac{\partial f}{\partial \phi} + \frac{\tilde{\mathbf{E}} \cdot \mathbf{e}_\theta + v \tilde{\mathbf{B}} \cdot \mathbf{e}_\phi}{v \sin(\phi)} \frac{\partial f}{\partial \theta} \right] = \\ \frac{(f_M - f)}{\lambda_e} + \frac{1}{2\lambda_{ei}} \left(\frac{\partial}{\partial \mu} \left((1 - \mu^2) \frac{\partial f}{\partial \mu} \right) + \frac{1}{\sin^2(\phi)} \frac{\partial^2 f}{\partial \theta^2} \right), \quad (3) \end{aligned}$$

24 where $\mu = \cos(\phi)$, λ_e is the electron-electron mean free path, and λ_{ei} is
25 the electron-ion mean free path. We also approximate $\lambda_e = \bar{Z} \lambda_{ei}$.

Clearly, $\frac{\partial \tilde{f}}{\partial \theta} = 0$, and if $\tilde{\mathbf{B}} = \tilde{B}_z \mathbf{e}_z$, there is no effect of magnetic field. We also assume, that $\nabla f = \frac{\partial f}{\partial z} \mathbf{e}_z$ and appropriately $\tilde{\mathbf{E}} = \tilde{E}_z \mathbf{e}_z$. From the orientation of the Cartesian basis vectors and spherical basis vectors, one can find $\tilde{\mathbf{E}} \cdot \mathbf{n} = \tilde{E}_z \cos(\phi) = \mu$ and $\tilde{\mathbf{E}} \cdot \mathbf{e}_\phi = -\tilde{E}_z \sin(\phi)$. As a result, the analyzed BGK equation reads

$$\begin{aligned} \mu \frac{\partial}{\partial z} (f^0 + f^1 \lambda_{ei} \mu) + \frac{1}{v} \left[\tilde{E}_z \mu \frac{\partial}{\partial v} (f^0 + f^1 \lambda_{ei} \mu) - \frac{\tilde{E}_z \sin(\phi)}{v} \frac{\partial}{\partial \phi} (f^0 + f^1 \lambda_{ei} \mu) \right] = \\ \frac{(f_M - (f^0 + f^1 \lambda_{ei} \mu))}{\lambda_e} + \frac{1}{2\lambda_{ei}} \frac{\partial}{\partial \mu} \left((1 - \mu^2) \frac{\partial}{\partial \mu} (f^0 + f^1 \lambda_{ei} \mu) \right), \quad (4) \end{aligned}$$

$$\mu \frac{\partial f^0}{\partial z} + \mu^2 \frac{\partial}{\partial z} (f^1 \lambda_{ei}) + \frac{\tilde{E}_z}{v} \left[\mu \frac{\partial f^0}{\partial v} + \mu^2 \frac{\partial}{\partial v} (f^1 \lambda_{ei}) + \frac{1 - \mu^2}{v} f^1 \lambda_{ei} \right] = \frac{f_M - f^0}{\bar{Z} \lambda_{ei}} - \mu \frac{1}{\bar{Z}} f^1 - \mu f^1, \quad (5)$$

consequently, we have the following anisotropy expansion $\mu^0, \mu^1, \mu^2, \dots$ equations

$$\begin{aligned} \frac{f_M - f^0}{\bar{Z} \lambda_{ei}} &= \frac{\tilde{E}_z}{v^2} f^1 \lambda_{ei}, \\ \frac{\partial f^0}{\partial z} + \frac{\tilde{E}_z}{v} \frac{\partial f^0}{\partial v} &= -\frac{1}{\bar{Z}} f^1 - f^1, \\ \frac{\partial}{\partial z} (f^1 \lambda_{ei}) + \frac{\tilde{E}_z}{v} \left[\frac{\partial}{\partial v} (f^1 \lambda_{ei}) - \frac{1}{v} f^1 \lambda_{ei} \right] &= 0, \end{aligned}$$

which lead to the definitions

$$f^0 = f_M + \frac{1}{v} f^1 \bar{Z} \lambda_{ei}^2, \quad (6)$$

$$\begin{aligned} f^1 &= -\frac{\bar{Z}}{\bar{Z} + 1} \left[\frac{\partial f^0}{\partial z} + \frac{\tilde{E}_z}{v} \frac{\partial f^0}{\partial v} \right] \\ &= -\frac{\bar{Z}}{\bar{Z} + 1} \left[\frac{1}{\rho} \frac{\partial \rho}{\partial z} + \left(\frac{v^2}{2v_{th}^2} - \frac{3}{2} \right) \frac{1}{T} \frac{\partial T}{\partial z} - \frac{\tilde{E}_z}{v_{th}^2} \right] f_M \end{aligned} \quad (7)$$

In order to ensure the plasma to be quasi-neutral, the zero-current condition

$$\mathbf{j} = \int_0^\infty \int_{4\pi} q_e v \mathbf{n} f \, d\mathbf{n} \, v^2 \, dv = \mathbf{0}, \quad (8)$$

can be achieved by providing a consistent electric field in (15), i.e.

$$\tilde{\mathbf{E}} = \frac{v_{th}^2 \int_{4\pi} \mathbf{n} \otimes \mathbf{n} \cdot \int_0^\infty v f_M \frac{\lambda}{\alpha} \left(\frac{\nabla \rho}{\rho} + \left(\frac{v^2}{2v_{th}^2} - \frac{3}{2} \right) \frac{\nabla T}{T} \right) v^2 \, dv \, d\mathbf{n}}{\int_{4\pi} \mathbf{n} \otimes \mathbf{n} \cdot \int_0^\infty v f_M \frac{\lambda}{\alpha} v^2 \, dv \, d\mathbf{n}}, \quad (9)$$

which may be further simplified as

$$\tilde{\mathbf{E}} = \frac{\int_0^\infty f_M \frac{1}{2} \frac{\nabla T}{T} v^9 \, dv}{\int_0^\infty f_M v^7 \, dv} + v_{th}^2 \left(\frac{\nabla \rho}{\rho} - \frac{3}{2} \frac{\nabla T}{T} \right) = v_{th}^2 \left(\frac{\nabla \rho}{\rho} + \frac{5}{2} \frac{\nabla T}{T} \right), \quad (10)$$

where it is worth mentioning, that the part $f_M + \frac{v\lambda}{\alpha} \frac{\partial f_1}{\partial v}$ of the distribution does not contribute to the current since it is isotropic. One can write the quasi-neutral distribution function explicitly distinguishing between original part (blue color) and E field correction (red color) as

$$f \approx f_M \left(1 - \frac{\lambda}{\alpha} \mathbf{n} \cdot \left(\frac{v^2}{2v_{th}^2} - \frac{3}{2} - \frac{5}{2} \right) \frac{\nabla T}{T} \right) + \frac{v\lambda}{\alpha} \frac{\partial f_1}{\partial v}. \quad (11)$$

which leads to the resulting heat flux

$$\mathbf{q}_H = \int_{4\pi} \int_0^\infty \frac{m_e v^2}{2} v \mathbf{n} f v^2 dv d\mathbf{n} = \frac{4\pi}{3} \frac{m_e}{2} \frac{1}{\alpha \sigma \rho} \int_0^\infty \left(\frac{v^2}{2v_{th}^2} - \frac{3}{2} - \frac{5}{2} \right) v^9 f_M dv \frac{\nabla T}{T}.$$

Based on the Gauss integral formula

$$\int v^{2s+1} \exp \left(-\frac{v^2}{2v_{th}^2} \right) dv = \frac{s! (2v_{th}^2)^{s+1}}{2}$$

and Maxwell-Boltzmann distribution (??) the heat flux can be written as

$$\mathbf{q}_H = \frac{4\pi}{3} \frac{m_e}{2} \frac{1}{\alpha \sigma \rho} \frac{\rho}{v_{th}^3} \frac{1}{(2\pi)^{3/2}} \frac{4!}{T} \frac{2^4 v_{th}^{10}}{T} \left(\frac{5}{2} - \frac{3}{2} - \frac{5}{2} \right) \nabla T = \frac{m_e}{\alpha \sigma} \frac{128}{\sqrt{2\pi}} \left(\frac{k_B}{m_e} \right)^{\frac{7}{2}} T^{\frac{5}{2}} \nabla T. \quad (12)$$

In conclusion, equation (12) provides nothing else than the well known Lorentz approximation heat flux and its nonlinearity 2.5 in temperature. What is worth mentioning is the effect of E field (quasi-neutrality), which reduces the flux of about 71.4% (also assuming constant density).

Finally, one can find the approximate solution

$$\tilde{f} = f_M - \lambda_{ei} \frac{\bar{Z}}{\bar{Z} + 1} \left(\frac{v^2}{2v_{th}^2} - \frac{3}{2} - \alpha \right) \frac{\mathbf{n} \cdot \nabla T}{T} f_M. \quad (13)$$

3.2. The AWBS diffusive electron transport

The AWBS electron transport equation in 6D reads

$$\mathbf{n} \cdot \nabla f + \frac{1}{v} \left[\tilde{\mathbf{E}} \cdot \mathbf{n} \frac{\partial f}{\partial v} + \frac{\tilde{\mathbf{E}} \cdot \mathbf{e}_\phi - v \tilde{\mathbf{B}} \cdot \mathbf{e}_\theta}{v} \frac{\partial f}{\partial \phi} + \frac{\tilde{\mathbf{E}} \cdot \mathbf{e}_\theta + v \tilde{\mathbf{B}} \cdot \mathbf{e}_\phi}{v \sin(\phi)} \frac{\partial f}{\partial \theta} \right] = \frac{v}{\lambda^e} \frac{\partial}{\partial v} (f - f_M) + \left(\frac{1}{\lambda_{ei}} + \frac{1}{\lambda_e} \right) \frac{1}{2} \left(\frac{\partial}{\partial \mu} \left((1 - \mu^2) \frac{\partial f}{\partial \mu} \right) + \frac{1}{\sin^2(\phi)} \frac{\partial^2 f}{\partial \theta^2} \right), \quad (14)$$

where $\mu = \cos(\phi)$, λ_e is the electron-electron mean free path, and λ_{ei} is the electron-ion mean free path, and $\lambda_e = \bar{Z}\lambda_{ei}$.

We can try to find an approximate solution while using the first term of expansion in λ_e and μ as

$$\tilde{f}(z, v, \mu) = f^0(z, v) + f^1(z, v)\lambda_{ei}\mu. \quad (15)$$

Clearly, $\frac{\partial \tilde{f}}{\partial \theta} = 0$, and if $\tilde{\mathbf{B}} = \tilde{B}_z \mathbf{e}_z$, there is no effect of magnetic field. We also assume, that $\nabla f = \frac{\partial f}{\partial z} \mathbf{e}_z$ and appropriately $\tilde{\mathbf{E}} = \tilde{E}_z \mathbf{e}_z$. From the orientation of the Cartesian basis vectors and spherical basis vectors, one can find $\tilde{\mathbf{E}} \cdot \mathbf{n} = \tilde{E}_z \cos(\phi) = \mu$ and $\tilde{\mathbf{E}} \cdot \mathbf{e}_\phi = -\tilde{E}_z \sin(\phi)$. As a result, the analyzed AWBS equation reads

$$\begin{aligned} \mu \frac{\partial}{\partial z} (f^0 + f^1 \lambda_{ei} \mu) + \frac{1}{v} \left[\tilde{E}_z \mu \frac{\partial}{\partial v} (f^0 + f^1 \lambda_{ei} \mu) - \frac{\tilde{E}_z \sin(\phi)}{v} \frac{\partial}{\partial \phi} (f^0 + f^1 \lambda_{ei} \mu) \right] = \\ \frac{v}{\lambda_e} \frac{\partial}{\partial v} ((f^0 + f^1 \lambda_{ei} \mu) - f_M) + \frac{\bar{Z} + 1}{2\lambda_{ei}\bar{Z}} \frac{\partial}{\partial \mu} \left((1 - \mu^2) \frac{\partial}{\partial \mu} (f^0 + f^1 \lambda_{ei} \mu) \right), \end{aligned} \quad (16)$$

$$\begin{aligned} \mu \frac{\partial f^0}{\partial z} + \mu^2 \frac{\partial}{\partial z} (f^1 \lambda_{ei}) + \frac{\tilde{E}_z}{v} \left[\mu \frac{\partial f^0}{\partial v} + \mu^2 \frac{\partial}{\partial v} (f^1 \lambda_{ei}) + \frac{1 - \mu^2}{v} f^1 \lambda_{ei} \right] = \\ \frac{v}{\bar{Z}\lambda_{ei}} \frac{\partial}{\partial v} (f^0 - f_M) + \mu \frac{v}{\bar{Z}\lambda_{ei}} \frac{\partial (f^1 \lambda_{ei})}{\partial v} - \mu \frac{\bar{Z} + 1}{\bar{Z}} f^1, \end{aligned} \quad (17)$$

consequently, we have the following anisotropy expansion $\mu^0, \mu^1, \mu^2, \dots$ equations

$$\begin{aligned} \frac{v}{\bar{Z}\lambda_{ei}} \frac{\partial}{\partial v} (f^0 - f_M) &= \frac{\tilde{E}_z}{v^2} f^1 \lambda_{ei}, \\ \frac{\partial f^0}{\partial z} + \frac{\tilde{E}_z}{v} \frac{\partial f^0}{\partial v} &= \frac{v}{\bar{Z}\lambda_{ei}} \frac{\partial (f^1 \lambda_{ei})}{\partial v} - \frac{\bar{Z} + 1}{\bar{Z}} f^1, \\ \frac{\partial}{\partial z} (f^1 \lambda_{ei}) + \frac{\tilde{E}_z}{v} \left[\frac{\partial}{\partial v} (f^1 \lambda_{ei}) - \frac{1}{v} f^1 \lambda_{ei} \right] &= 0, \end{aligned}$$

52 which lead to the definitions

$$\begin{aligned}
\frac{\partial}{\partial v} (f^0 - f_M) &= \frac{1}{v^2} f^1 \bar{Z} \lambda_{ei}^2, \\
\frac{v}{\bar{Z} \lambda_{ei}} \frac{\partial(f^1 \lambda_{ei})}{\partial v} - \frac{\bar{Z} + 1}{\bar{Z}} f^1 &= \frac{\partial f^0}{\partial z} + \frac{\tilde{E}_z}{v} \frac{\partial f^0}{\partial v} \\
\frac{v}{\bar{Z}} \frac{\partial f^1}{\partial v} + \frac{4}{\bar{Z}} f^1 - \frac{\bar{Z} + 1}{\bar{Z}} f^1 &= \frac{\partial f^0}{\partial z} + \frac{\tilde{E}_z}{v} \frac{\partial f^0}{\partial v} \\
\frac{\partial f^1}{\partial v} + \frac{1}{v} (3 - \bar{Z}) f^1 &= \frac{\bar{Z}}{v} \left(\frac{1}{\rho} \frac{\partial \rho}{\partial z} + \left(\frac{v^2}{2v_{th}^2} - \frac{3}{2} \right) \frac{1}{T} \frac{\partial T}{\partial z} - \frac{\tilde{E}_z}{v_{th}^2} \right) f^1
\end{aligned} \tag{18}$$

53 3.3. The Fokker-Planck diffusive electron transport

$$v_{2th} = \sqrt{\frac{2k_B T}{m_e}} = 1/j,$$

54

$$\begin{aligned}
A &= -\frac{m_e^2 v_{2th}^2 \tilde{\mathbf{E}}}{2\pi e^4 n_e \ln \Lambda} = -\frac{mE}{2\pi j^2 e^3 n_e \ln \Lambda}, \\
B &= \frac{m_e^2 v_{2th}^4 |\nabla T|}{2\pi e^4 n_e \ln \Lambda T} = \frac{2k_B^2 T |\nabla T|}{\pi e^4 n_e \ln \Lambda},
\end{aligned}$$

55

$$\frac{A}{B} = -\frac{|\tilde{\mathbf{E}}|T}{v_{2th}^2 |\nabla T|},$$

56

$$\tilde{\mathbf{E}} = -\frac{3}{2} \frac{v_{2th}^2}{2} \frac{\gamma_T}{\gamma_E} \frac{\nabla T}{T},$$

57 From Eq. (24) CSR, we can write the form of f_1 including both ∇T and $\tilde{\mathbf{E}}$
58 effects as

$$f_1(v, \theta) = \cos(\theta) \frac{B}{\bar{Z}} \left(d_T(v/v_{2th}) + \frac{A}{B} d_E(v/v_{2th}) \right) f_M(v),$$

59 where in the case of vanishing current one gets

$$\frac{A}{B} = \frac{3}{2} \frac{\gamma_T}{2\gamma_E},$$

60 i.e.

$$f_1(v, \theta) = \cos(\theta) \frac{m_e^2}{4\pi e^4 \ln \Lambda} \frac{v_{2th}^4}{\bar{Z}} \left(2d_T(v/v_{2th}) + \frac{3}{2} \frac{\gamma_T}{\gamma_E} d_E(v/v_{2th}) \right) \frac{f_M(v)}{n_e} \frac{\nabla T}{T}, \quad (20)$$

61 where $d_T(x) = \bar{Z}D_T(x)/B$ and $d_E(x) = \bar{Z}D_E(x)/A$ are represented by nu-
 62 merical values in TABLE I and TABLE II in [5], respectively. In the case of
 63 high \bar{Z} limit, $\gamma_T \rightarrow 1$, $\gamma_E \rightarrow 1$, $d_E(x) = x^4$, and $d_T(x) = x^4(2.5 - x^2)/2$ [5],
 64 which leads to the standard Lorentz gas model

$$f_1(v, \theta) = \cos(\theta) \frac{m_e^2}{4\pi e^4 \ln \Lambda} \frac{v^4}{\bar{Z}} \left(4 - \frac{v^2}{v_{2th}^2} \right) \frac{f_M(v)}{n_e} \frac{\nabla T}{T}, \quad (21)$$

65 [2], [3], [4]

66 3.4. Summary of BGK, AWBS, and Fokker-Planck diffusion

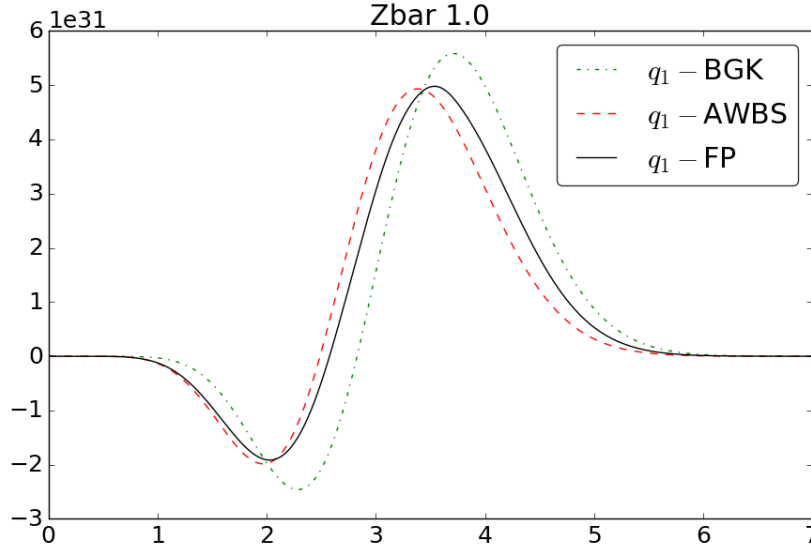


Figure 1: The flux velocity moment of the anisotropic part of the electron distribution function in low Z plasmas in diffusive regime.

	$\bar{Z} = 1$	$\bar{Z} = 2$	$\bar{Z} = 4$	$\bar{Z} = 16$	$\bar{Z} \rightarrow \infty$
$error(\mathbf{q}_{AWBS})$	0.057	0.004	0.038	0.049	0.000

Table 1: Relative $error(\mathbf{q}_{AWBS}) = |\mathbf{q}_{AWBS} - \mathbf{q}_{SH}|/\mathbf{q}_{SH}$ of the AWBS kinetic model equation (1) showing the discrepancy (maximum around 5%) with respect to the original solution of the heat flux given by Spitzer and Harm [5].

67 4. Benchmarking the AWBS nonlocal transport model

68 4.1. Review of simulation codes

69 4.1.1. C7

70 Since in the laser heated plasmas the Knudsen number $\text{Kn} = \frac{v_{th}}{\nu_t(v_{th})L} \in$
71 $(0, 1)$, i.e. the collisionality in the kinetics of electrons plays always an im-
72 portant effect for thermal-like particles, the electron distribution function
73 can be treated as out-of-equilibrium approximation

$$f = f_M + \delta f, \quad (22)$$

where the consequent AWBS model reads

$$\begin{aligned} v\mathbf{n} \cdot \nabla(f_M + \delta f) + \tilde{\mathbf{E}} \cdot \mathbf{n} \left(\frac{\partial f_M}{\partial v} + \frac{\partial \delta f}{\partial v} \right) + \frac{\tilde{\mathbf{E}} \cdot \mathbf{e}_\theta}{v} \frac{\partial \delta f}{\partial \theta} = \\ v \frac{\nu_e}{2} \frac{\partial \delta f}{\partial v} + \left(\nu_{ei} + \frac{\nu_e}{2} \right) (f_0 - f_M - \delta f), \end{aligned} \quad (23)$$

or its 1D version

$$\begin{aligned} v\mu \frac{\partial}{\partial z}(f_M + \delta f) + \tilde{E}\mu \left(\frac{\partial f_M}{\partial v} + \frac{\partial \delta f}{\partial v} \right) + \frac{\tilde{E}_z(1 - \mu^2)}{v} \frac{\partial \delta f}{\partial \mu} = \\ v \frac{\nu_e}{2} \frac{\partial \delta f}{\partial v} + \left(\nu_{ei} + \frac{\nu_e}{2} \right) (f_0 - f_M - \delta f), \end{aligned} \quad (24)$$

74 where $\tilde{\mathbf{E}} \cdot \mathbf{e}_\theta = \tilde{E}_z \sin(\theta)$ and $\frac{\partial}{\partial \theta} = \sin(\theta) \frac{\partial}{\partial \mu}$, $\mu = \cos(\theta)$.

- 75 4.1.2. ALADIN
76 4.1.3. IMPACT
77 4.1.4. CALDER
78 4.2. Simulation results

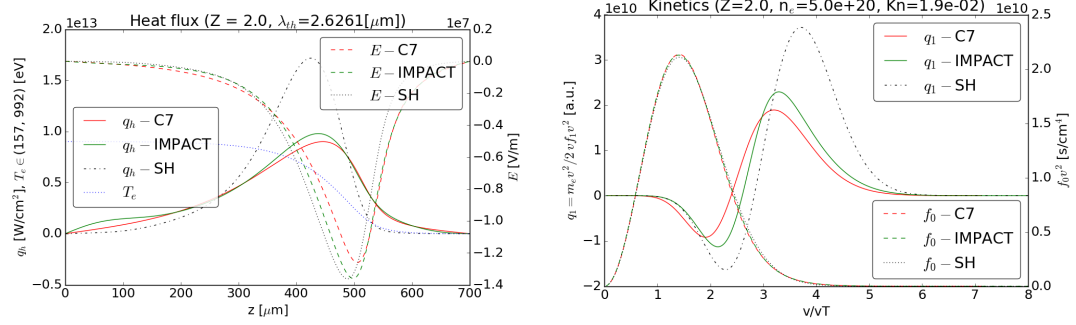


Figure 2: Snapshot 12 ps. Left: correct steady solution. Right: correct comparison to kinetic profiles at point 437 μm by IMPACT.

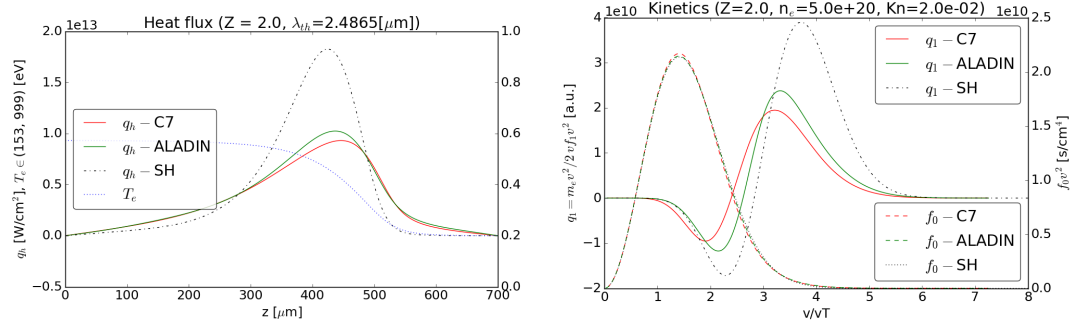


Figure 3: Snapshot 12 ps. Left: correct steady solution. Right: correct comparison to kinetic profiles at point 442 μm by ALADIN.

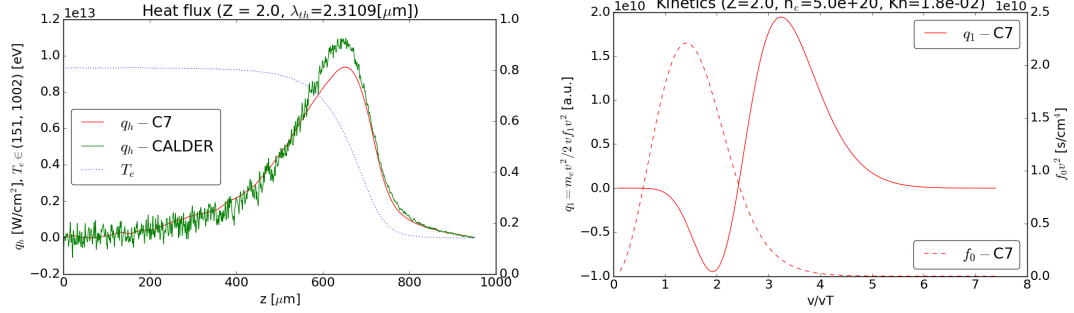


Figure 4: Snapshot 11 ps. Left: correct steady solution. Right: Kinetic profiles at point of maximum flux by C7. Kinetics profiles by CALDER to be added.

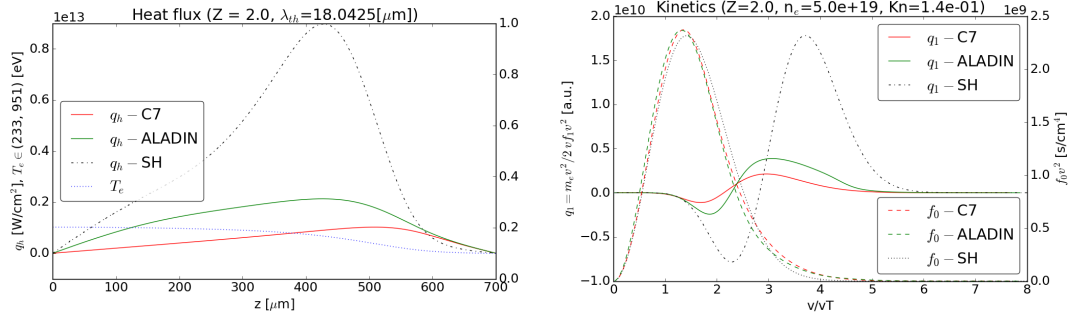


Figure 5: Snapshot 12 ps. Left: correct steady solution. Right: correct comparison to kinetic profiles at point 480 μm by ALADIN.

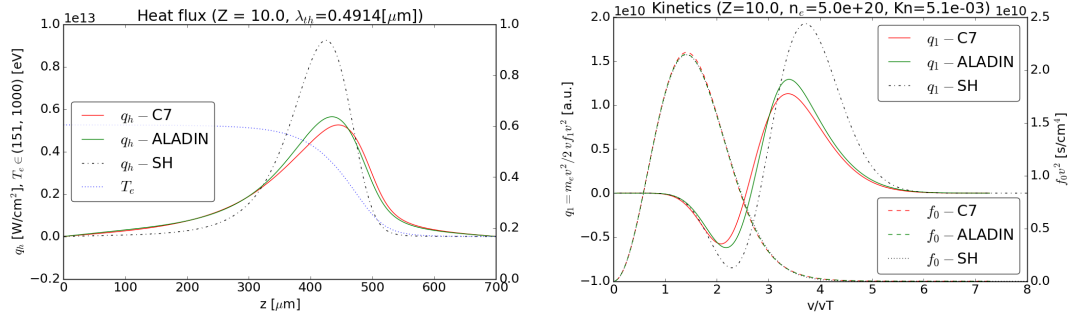


Figure 6: Snapshot 12 ps. Left: correct steady solution. Right: correct comparison to kinetic profiles at point 442 μm by ALADIN.

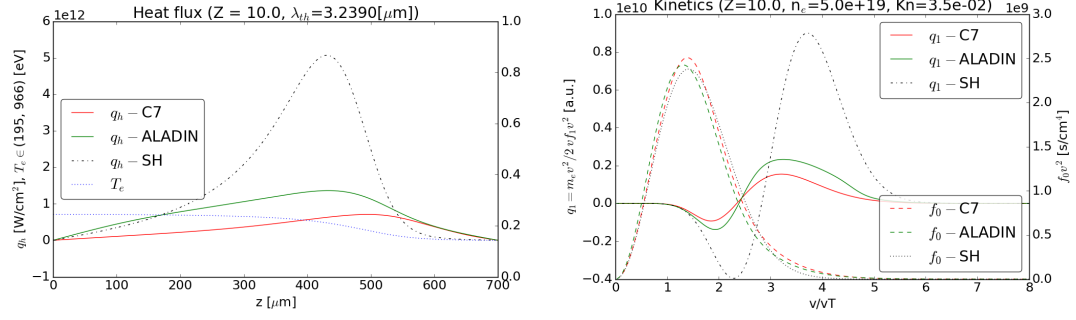


Figure 7: Snapshot 12 ps. Left: correct steady solution. Right: correct comparison to kinetic profiles at point $480 \mu\text{m}$ by ALADIN.

79 **5. Conclusions**

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