# Nonlocal Magneto-Hydrodynamic Model and Its Applications to Laser Plasmas
*Serious Numerics for Serious Physics*

Milan Holec

LLNL - ClubMED,
Livermore, CA,
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1. Introduction to Nonlocal Magneto-Hydrodynamic model (Nonlocal-MHD)
   - Radiation-Hydrodyanmics
   - Nonlocal electron transport

# Radiation-Hydrodynamic model of plasma

## Boltzmann transport equation

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \nabla_x f + \frac{q_e}{m_e}\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c} \times \boldsymbol{B}\right) \cdot \nabla_{\boldsymbol{v}} f = \sigma \nabla_{\boldsymbol{v}} \cdot \int \frac{|\boldsymbol{v} - \boldsymbol{v}'|^2 \mathbf{I} - (\boldsymbol{v} - \boldsymbol{v}') \otimes \boldsymbol{v} - \boldsymbol{v}')}{|\boldsymbol{v} - \boldsymbol{v}'|^3} (\nabla_{\boldsymbol{v}} f(\boldsymbol{v}) f(\boldsymbol{v}') - f(\boldsymbol{v}) \nabla_{\boldsymbol{v}} f(\boldsymbol{v}')) \mathrm{d}\boldsymbol{v}'$$

## Fluid equations in Lagrangian frame

$$\begin{aligned} \frac{\mathrm{d}\rho}{\mathrm{d}t} &= -\rho \nabla \cdot \boldsymbol{U} \\ \rho \frac{\mathrm{d}\boldsymbol{U}}{\mathrm{d}t} &= \nabla \cdot \sigma \\ \rho \frac{\mathrm{d}\varepsilon}{\mathrm{d}t} &= \sigma : \nabla \boldsymbol{U} - \nabla \cdot (\boldsymbol{q}_e + \boldsymbol{q}_R) \end{aligned}$$

## Microscopic closure

$$\begin{aligned} \sigma &= -\rho \int (\boldsymbol{v} - \boldsymbol{U}) \otimes (\boldsymbol{v} - \boldsymbol{U}) \, f \, \mathrm{d}\boldsymbol{v}^3 \approx -\mathbf{I} p + \tilde{\sigma}(\nabla \boldsymbol{U}) \\ \boldsymbol{q}_e &= \frac{\rho}{2} \int |\boldsymbol{v} - \boldsymbol{U}|^2 (\boldsymbol{v} - \boldsymbol{U}) \, f \, \mathrm{d}\boldsymbol{v}^3 \approx \frac{\rho}{2} \int_{4\pi} \boldsymbol{n} \int_0^\infty |\boldsymbol{v}|^5 f \, \mathrm{d}|\boldsymbol{v}| \, \mathrm{d}\boldsymbol{n} \approx -\kappa(T^{2.5}) \nabla T \\ \boldsymbol{q}_R &= \int_{4\pi} \boldsymbol{n} \int_\nu I_\nu^p \, \mathrm{d}\nu \, \mathrm{d}\boldsymbol{n} \approx -\int_\nu \frac{c}{\chi_\nu} \nabla (f_\nu E_\nu) \, \mathrm{d}\nu \end{aligned}$$

## BGK collision operator

$$
\begin{aligned}
C_{ei}(f^e, f^i) &\overset{m_e/m_i}{\approx} \sigma \nabla_v \cdot \left( \frac{1}{v} \left( \mathbf{I} - \frac{\boldsymbol{v} \otimes \boldsymbol{v}}{v^2} \right) \cdot \nabla_v f^e \right) \\
&\overset{\text{spher}}{\approx} \frac{\sigma}{v^3} \frac{1}{\sin(\phi)} \left[ \frac{\partial}{\partial \phi} \left( \sin(\phi) \frac{\partial f^e}{\partial \phi} \right) + \frac{1}{\sin(\phi)} \frac{\partial^2 f^e}{\partial \theta^2} \right] \\
&\overset{f^0 + f^1 \cos(\phi)}{\approx} -\frac{2\sigma}{v^3} f_1^e \cos(\phi) = 2\nu_{ei} \left( f_0^e - f^e \right)
\end{aligned}
$$

Then, the electron distribution function can be governed by the BGK Boltzmann transport equation

$$
\boldsymbol{n} \cdot \nabla_x f^e + \frac{q_e}{m_e |\boldsymbol{v}|} \boldsymbol{E} \cdot \nabla_{\boldsymbol{v}} f^e = \frac{2(f_{MB}(|\boldsymbol{v}|, T_e) - f^e)}{\lambda_{ei}},
$$

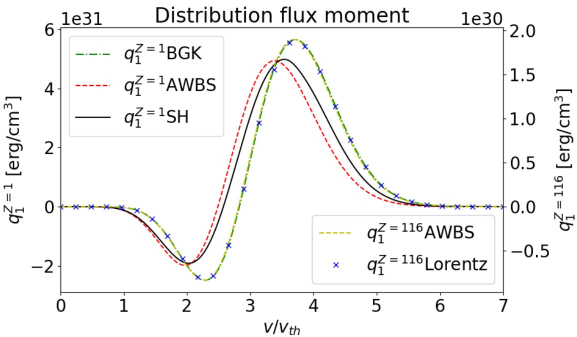
where $f_{MB}$ is the Maxwell-Boltzmann equilibrium distribution and $\lambda_{ei}$ the electron mean free path when scattered on ions.

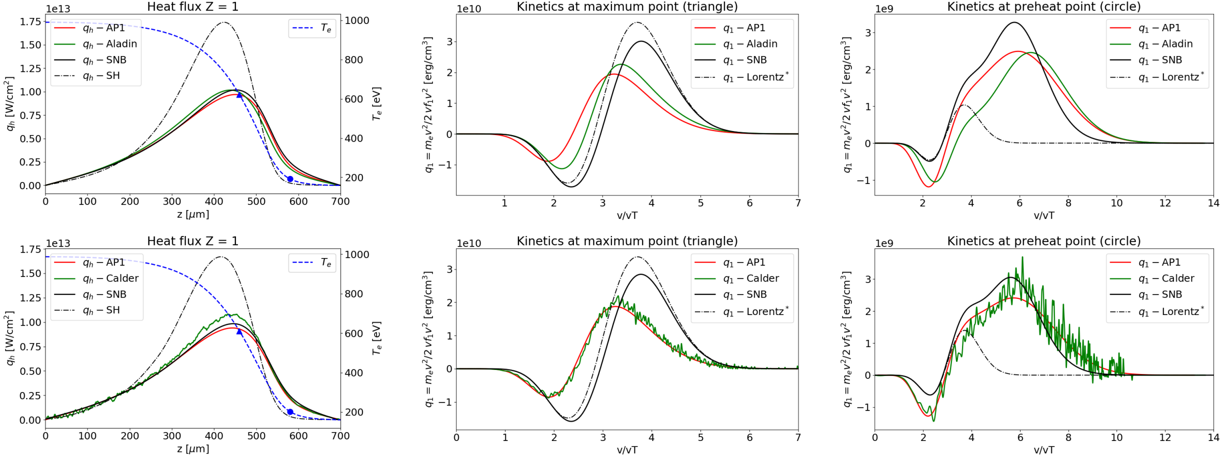
## Chapman-Enskog approximation in small parameter $\lambda_{ei}$

$$
f^e = f_0 + \lambda_{ei} f_1 + O(\lambda_{ei}{}^2) \approx f_{MB}(|\boldsymbol{v}|, T_e) - f_{MB}(|\boldsymbol{v}|, T_e) g(\bar{Z}) \left( \frac{|\boldsymbol{v}|^2}{2 v_{T_e}^2} - 4 \right) \boldsymbol{n} \cdot \frac{\lambda_{ei} \nabla T_e}{T_e}
$$

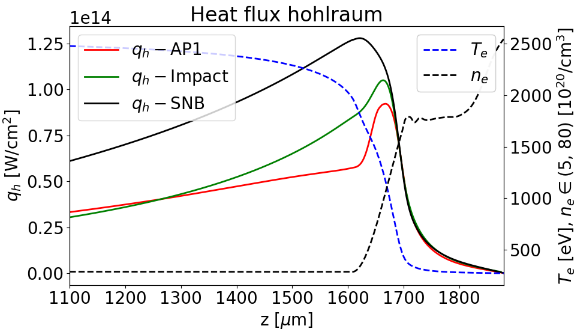
$$
\rightarrow \boldsymbol{q}_{SH} = \kappa_{SH} T_e^{\frac{5}{2}} \nabla T_e
$$

$$
\frac{\lambda_{ei} f_1}{f_0} = 0.25 \left( \frac{|\boldsymbol{v}|^2}{2 v_{T_e}^2} - 4 \right) \boldsymbol{n} \cdot \frac{\lambda_{ei} (|\boldsymbol{v}|^4) \nabla T_e}{T_e} < 0.1 \quad \longrightarrow \quad \mathrm{Kn}^e = \frac{\lambda_{ei} \nabla T_e}{T_e} < 7.5 \times 10^{-4}
$$


Distribution flux moment
1e31
1e30
6
4
2
0
−2
2.0
1.5
1.0
0.5
0.0
−0.5
0 1 2 3 4 5 6 7
$q_1^{Z=1}$ [erg/cm$^3$]
$q_1^{Z=116}$ [erg/cm$^3$]
$v/v_{th}$
$q_1^{Z=1}$BGK
$q_1^{Z=1}$AWBS
$q_1^{Z=1}$SH
$q_1^{Z=116}$AWBS
$q_1^{Z=116}$Lorentz



Heat flux Z = 1
1e13
$q_h$ – AP1
$q_h$ – Aladin
$q_h$ – SNB
$q_h$ – SH
$T_e$
$q_h$ [W/cm$^2$]
$T_e$ [eV]
z [$\mu$m]
Kinetics at maximum point (triangle)
1e10
$q_1$ – AP1
$q_1$ – Aladin
$q_1$ – SNB
$q_1$ – Lorentz*
$q_1 = m_e v^2/2\, v f_1 v^2$ [erg/cm$^3$]
v/vT
Kinetics at preheat point (circle)
1e9
Heat flux Z = 1
$q_h$ – AP1
$q_h$ – Calder
$q_h$ – SNB
$q_h$ – SH
Kinetics at maximum point (triangle)
$q_1$ – AP1
$q_1$ – Calder
$q_1$ – SNB
$q_1$ – Lorentz*
Kinetics at preheat point (circle)



Heat flux hohlraum
1e14
1.25
1.00
0.75
0.50
0.25
0.00
$q_h$ [W/cm$^2$]
$q_h$ − AP1
$q_h$ − Impact
$q_h$ − SNB
$T_e$
$n_e$
2500
2000
1500
1000
500
$T_e$ [eV], $n_e \in (5, 80)$ [$10^{20}$/cm$^3$]
1100
1200
1300
1400
1500
1600
1700
1800
z [$\mu$m]

# Nonlocal electron transport & Maxwell equations

## AWBS electron transport equation

$$\boldsymbol{v}\cdot\nabla_{\boldsymbol{x}} f + (\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})\cdot\nabla_{\boldsymbol{v}} f = v\frac{\nu_e}{2}\frac{\partial}{\partial v}\left(f - f_{MB}(T_e)\right) + \left(\nu_{ei} + \frac{\nu_e}{2}\right)(f_0 - f)$$

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\nabla_{\boldsymbol{x}} f + (\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})\cdot\nabla_{\boldsymbol{v}} f = v\frac{\nu_e}{2}\frac{\partial}{\partial v}\left(F(f_0)f_0 + D(f_0)\frac{\partial f_0}{\partial v}\right) + \nu_{ei}(f_0 - f)$$

## Generalized Ohm's law vs. nonlocal Ohm's law

$$\begin{aligned}
\boldsymbol{E} &= (\boldsymbol{R}_T - \nabla p_e) + \frac{\boldsymbol{j}}{\sigma} + \boldsymbol{j}\times\boldsymbol{B} \\
\boldsymbol{E}\int v\frac{\partial f_0}{\partial v}v^2\,\mathrm{d}v &= \int v^2\nabla f_0 v^2\,\mathrm{d}v + \int v\nu_{ei}\boldsymbol{f_1}v^2\,\mathrm{d}v + \int v\boldsymbol{f_1}\times\boldsymbol{B}v^2\,\mathrm{d}v \qquad (1)
\end{aligned}$$

$$\begin{aligned}
\nabla\times\boldsymbol{E} &= -\frac{1}{c}\frac{\partial\boldsymbol{B}}{\partial t} && \text{(life of magnetic field } \boldsymbol{B} \text{ in fluid frame)} \\
\nabla\times\boldsymbol{B} &= \frac{4\pi}{c}\boldsymbol{j} && \text{(quasi-neutrality} \nabla\cdot\boldsymbol{j} = 0\text{)} \\
\boldsymbol{j} &= C(\boldsymbol{E}, f) && \text{(from nonlocal electron transport model (1))}
\end{aligned}$$

# *Nonlocal-MHD* $\Longleftrightarrow$ aiming HIGH

Height 250 m, overhang 40 m, Visera, Riglos, Spain.

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