

An efficient kinetic modeling in hydrodynamics using the AWBS transport equation

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Abstract

The AWBS Boltzmann transport equation for electrons equipped with a simplified e-e collision operator [1] provides an efficient, yet physically relevant, kinetic extension compared the classical Spitzer-Harm heat flux (SH) based on local approximation and flux-limiting. This classical approach is widely used in plasma kinetics models coupled to hydrodynamics. Even though SH reflects the electron-electron collision effect, the essential physical properties of the electron transport cannot be modeled with an explicitly local model. A simple form of the AWBS model opens a way to couple kinetics to hydrodynamic codes while describing the important physics correctly. Since the electron-electron collision effect becomes especially important in the case of low ion potential, we focus on presenting results related to low Z plasmas.

Keywords: kinetics; hydrodynamics; nonlocal electron transport; laser-heated plasmas.

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12 **1. The Fokker-Planck equation**

13 Use equation Hu (1, 2, 3) in text, (10), which puts together (4, 5, 7), and
14 write definitions (11, 12) using (5, 7, 13) and after define (6, 8) and to close
15 write (31) from Tzoufras.

16 *1.1. Spherical harmonics expansion*

17 Write Hu (14, 15) and full (1) in spherical coordinates. Then explicitly
18 write (17, 18, 23, 24, 25). Further define spherical harmonics expansion and
19 from Tzoufras (34, 35, 36, 37).

20 *1.2. The linearized Fokker-Planck equation for low anisotropy*

21 Define the anisotropic perturbation as in Tzoufras and then use the equa-
22 tions (32, 33) and the harmonic expansions (38, 39, 40) and the most impor-
23 tantly (41) for one-kind particles. Finally, write explicitly (41) for the case
24 f_1^0 and write set of integrals I, J and constants C_1, \dots, C_6 , which will be used
25 to calculate FP equation solution for diffusive conditions.

26 **2. AWBS-P1 modeling of laser heated plasmas**

27 *2.1. Model equations*

The AWBS electron transport equation reads

$$v\mathbf{n} \cdot \nabla f + \tilde{\mathbf{E}} \cdot \mathbf{n} \frac{\partial f}{\partial v} + \frac{\tilde{\mathbf{E}} \cdot \mathbf{e}_\phi - v \tilde{\mathbf{B}} \cdot \mathbf{e}_\theta}{v} \frac{\partial f}{\partial \phi} = v\nu_e \frac{\partial}{\partial v} (f - f_M) + (\nu_{ei} + \nu_e)(f_0 - f),$$

28 where ν_e is the electron-electron collision frequency, ν_{ei} is the electron-ion
29 collision frequency, and $\nu_{ei} = \bar{Z}\nu_e$.

30 In order to eliminate the dimensions of the above transport problem
31 the first-two-moment model based on approximation

$$f = \frac{f_0}{4\pi} + \frac{3}{4\pi} \mathbf{n} \cdot \mathbf{f}_1,$$

32 can be adopted and reads

$$\begin{aligned}\nu_e v \frac{\partial}{\partial v} (f_0 - \tilde{f}_M) &= v \nabla \cdot \mathbf{f}_1 + \tilde{\mathbf{E}} \cdot \frac{\partial \mathbf{f}_1}{\partial v} + \frac{2}{v} \tilde{\mathbf{E}} \cdot \mathbf{f}_1, \\ \nu_e v \frac{\partial}{\partial v} \mathbf{f}_1 - \nu_t \mathbf{f}_1 &= v \nabla \cdot (\mathbf{A} f_0) + \tilde{\mathbf{E}} \cdot \frac{\partial (\mathbf{A} f_0)}{\partial v} + \tilde{\mathbf{B}} \times \mathbf{f}_1,\end{aligned}$$

33 where $\tilde{f}_M = 4\pi f_M$ and the closure matrix takes the form

$$\mathbf{A} = \frac{1}{3} \mathbf{I}.$$

34 Since in the laser heated plasmas the Knudsen number $\text{Kn} = \frac{v_{th}}{\nu_t(v_{th})L} \in$
 35 $(0, 1)$, i.e. the collisionality in the kinetics of electrons plays always an im-
 36 portant effect for thermal-like particles, the electron distribution function
 37 can be treated as out-of-equilibrium approximation

$$f = f_M + \delta f, \quad (1)$$

where the consequent AWBS model reads

$$\begin{aligned}v \mathbf{n} \cdot \nabla (f_M + \delta f) + \tilde{\mathbf{E}} \cdot \mathbf{n} \frac{\partial f_M}{\partial v} + \tilde{\mathbf{E}} \cdot \mathbf{n} \frac{\partial \delta f}{\partial v} + \frac{\tilde{\mathbf{E}} \cdot \mathbf{e}_\phi - v \tilde{\mathbf{B}} \cdot \mathbf{e}_\theta}{v} \frac{\partial \delta f}{\partial \phi} = \\ v \nu_e \frac{\partial \delta f}{\partial v} + (\nu_{ei} + \nu_e)(f_0 - f_M - \delta f),\end{aligned} \quad (2)$$

38 or its P1 approximation equivalent

$$f = \frac{4\pi f_M + \delta f_0}{4\pi} + \frac{3}{4\pi} \mathbf{n} \cdot \mathbf{f}_1. \quad (3)$$

39 where the moment model reads

$$\nu_e v \frac{\partial \delta f_0}{\partial v} = v \nabla \cdot \mathbf{f}_1 + \tilde{\mathbf{E}} \cdot \frac{\partial \mathbf{f}_1}{\partial v} + \frac{2}{v} \tilde{\mathbf{E}} \cdot \mathbf{f}_1, \quad (4)$$

$$\begin{aligned}\nu_e v \frac{\partial \mathbf{f}_1}{\partial v} - \nu_t \mathbf{f}_1 &= \frac{v}{3} \nabla \delta f_0 + \frac{\tilde{\mathbf{E}}}{3} \frac{\partial \delta f_0}{\partial v} + \tilde{\mathbf{B}} \times \mathbf{f}_1 \\ &+ \frac{v}{3} \nabla \tilde{f}_M + \frac{\tilde{\mathbf{E}}}{3} \frac{\partial \tilde{f}_M}{\partial v}.\end{aligned} \quad (5)$$

40 *2.2. A consistent treatment of $\tilde{\mathbf{E}}$ field*

41 The plasma conditions providing an appropriate electric field are the best
42 expressed via the definition of current

$$\mathbf{q}_c(\mathbf{x}) = \int_v v \mathbf{f}_1(\mathbf{x}) v^2 dv,$$

43 which can be directly expressed from (5) as

$$\mathbf{q}_c = \int_v \left(\frac{\nu_e v^2}{\nu_t} \frac{\partial \mathbf{f}_1}{\partial v} - \frac{v^2}{3\nu_t} \nabla (\tilde{f}_M + \delta f_0) - \frac{v}{3\nu_t} \frac{\partial (\tilde{f}_M + \delta f_0)}{\partial v} \tilde{\mathbf{E}} \right) v^2 dv, \quad (6)$$

44 where the \mathbf{B} field and \mathbf{E} field scattering effect (angular) have been omitted.

45 Then, the current can be easily evaluated based on

$$\begin{aligned} a_0(\mathbf{x}) &= \int_v \frac{v}{3\nu_t} \frac{\partial (\tilde{f}_M + \delta f_0)}{\partial v}(\mathbf{x}) v^2 dv, \\ \mathbf{b}_0(\mathbf{x}) &= \int_v \left(\frac{v^2}{3\nu_t} \nabla (\tilde{f}_M(\mathbf{x}) + \delta f_0(\mathbf{x})) - \frac{\nu_e v^2}{\nu_t} \frac{\partial \mathbf{f}_1(\mathbf{x})}{\partial v} \right) v^2 dv, \end{aligned}$$

46 as the following generalization of the Ohm's law

$$\mathbf{q}_c(\mathbf{x}) = -\mathbf{b}_0(\mathbf{x}) - a_0(\mathbf{x}) \tilde{\mathbf{E}}(\mathbf{x}),$$

47 where one needs the actual distribution function f values.

48 It is straightforward to find the *zero current* formula for the electric field

49

$$\tilde{\mathbf{E}}(\mathbf{x}) = -\frac{\mathbf{b}_0(\mathbf{x})}{a_0(\mathbf{x})}. \quad (7)$$

50 *2.3. AWBS model analysis*

The AWBS transport equation can be written as the following

$$\left(v\nu_e - \tilde{\mathbf{E}} \cdot \mathbf{n} \right) \frac{\partial \delta f}{\partial v} = v\mathbf{n} \cdot \nabla (f_M + \delta f) + \tilde{\mathbf{E}} \cdot \mathbf{n} \frac{\partial f_M}{\partial v} + (\nu_{ei} + \nu_e)(f_M + \delta f - f_0), \quad (8)$$

51 in order to stress the effect of force applied to electrons, i.e. the effect of
52 friction described by ν_e and the Lorentz force effect via $\tilde{\mathbf{E}}$, and their compe-
53 titution.

54 The same reformulation can be written for the moment AWBS model

$$\begin{aligned}\frac{\partial \delta f_0}{\partial v} &= \frac{1}{\nu_e v} \left(v \nabla \cdot \mathbf{f}_1 + \tilde{\mathbf{E}} \cdot \frac{\partial \mathbf{f}_1}{\partial v} \right), \\ \nu_e v \frac{\partial \mathbf{f}_1}{\partial v} - \nu_t \mathbf{f}_1 &= \frac{v}{3} \nabla \delta f_0 + \frac{\tilde{\mathbf{E}}}{3} \frac{\partial \delta f_0}{\partial v} + \frac{v}{3} \nabla \tilde{f}_M + \frac{\tilde{\mathbf{E}}}{3} \frac{\partial \tilde{f}_M}{\partial v},\end{aligned}$$

55 and it takes the following form

$$\left(\nu_e v \mathbf{I} - \frac{\tilde{\mathbf{E}} \tilde{\mathbf{E}}}{3 \nu_e v} \right) \cdot \frac{\partial \mathbf{f}_1}{\partial v} = \frac{v}{3} \nabla \delta f_0 + \frac{\tilde{\mathbf{E}}}{3 \nu_e} \nabla \cdot \mathbf{f}_1 + \frac{v}{3} \nabla \tilde{f}_M + \frac{\tilde{\mathbf{E}}}{3} \frac{\partial \tilde{f}_M}{\partial v} + \nu_t \mathbf{f}_1,$$

56 which is especially instructive in 1D

$$\left(\nu_e v - \frac{\tilde{E}_z^2}{3 \nu_e v} \right) \frac{\partial f_{1z}}{\partial v} = \frac{v}{3} \frac{\partial \delta f_0}{\partial z} + \frac{\tilde{E}_z}{3 \nu_e} \frac{\partial f_{1z}}{\partial z} + \frac{v}{3} \frac{\partial \tilde{f}_M}{\partial z} + \frac{\tilde{E}_z}{3} \frac{\partial \tilde{f}_M}{\partial v} + \nu_t f_{1z}, \quad (9)$$

57 because it gives a "reverse-time-like evolution" condition

$$\sqrt{3} \nu_e > \frac{|\tilde{E}_z|}{v}. \quad (10)$$

$$\frac{\partial f_{1z}}{\partial v} = \frac{3 \nu_e v}{3(\nu_e v)^2 - \tilde{E}_z^2} \left(\frac{v}{3} \frac{\partial \delta f_0}{\partial z} + \frac{\tilde{E}_z}{3 \nu_e} \frac{\partial f_{1z}}{\partial z} + \frac{v}{3} \frac{\partial \tilde{f}_M}{\partial z} + \frac{\tilde{E}_z}{3} \frac{\partial \tilde{f}_M}{\partial v} + \nu_t f_{1z} \right), \quad (11)$$

58 because it gives a "reverse-time-like evolution" condition

$$3(\nu_e v)^2 - \tilde{E}_z^2 \neq 0,$$

59 or a numerical stability formulation because it gives a "reverse-time-like evolution" condition

$$|3(\nu_e v)^2 - \tilde{E}_z^2| > \epsilon. \quad (12)$$

61 which can be obtained from

$$\left(3(\nu_e v)^2 - \tilde{E}_z^2 \right)^2 - \epsilon^2 = 0. \quad (13)$$

62 2.4. "Reverse-time-like evolution" model by splitting

63 Full separation of advection and E field

$$\begin{aligned}\nu_e v \frac{\partial \mathbf{f}_1^{\nu_e}}{\partial v} &= \frac{v}{3} \nabla \delta f_0^{\nu_e} + \frac{v}{3} \nabla \tilde{f}_M + \nu_t \mathbf{f}_1^{\nu_e}, \\ \frac{\tilde{\mathbf{E}} \tilde{\mathbf{E}}}{3\nu_e v} \cdot \frac{\partial \mathbf{f}_1^{\tilde{\mathbf{E}}}}{\partial v} &= -\frac{\tilde{\mathbf{E}}}{3\nu_e} \nabla \cdot \mathbf{f}_1^{\tilde{\mathbf{E}}} - \frac{\tilde{\mathbf{E}}}{3} \frac{\partial \tilde{f}_M}{\partial v},\end{aligned}$$

64 or separation of stable "bulk" E field effect and implicit E field effect

$$\begin{aligned}\nu_e v \frac{\partial \mathbf{f}_1^{\nu_e}}{\partial v} &= \frac{v}{3} \nabla \delta f_0^{\nu_e} + \frac{v}{3} \nabla \tilde{f}_M + \nu_t \mathbf{f}_1^{\nu_e} + \frac{\tilde{\mathbf{E}}}{3} \frac{\partial \tilde{f}_M}{\partial v}, \\ \frac{\tilde{\mathbf{E}} \tilde{\mathbf{E}}}{3\nu_e v} \cdot \frac{\partial \mathbf{f}_1^{\tilde{\mathbf{E}}}}{\partial v} &= -\frac{\tilde{\mathbf{E}}}{3\nu_e} \nabla \cdot \mathbf{f}_1^{\tilde{\mathbf{E}}},\end{aligned}$$

65 and the complete effect of diffusion in velocity space reads

$$\frac{\partial \mathbf{f}_1}{\partial v} = \frac{\partial \mathbf{f}_1^{\nu_e}}{\partial v} + \frac{\partial \mathbf{f}_1^{\tilde{\mathbf{E}}}}{\partial v},$$

66 2.5. "Friction" model

67 In order to obey (13), an additional friction $\nu_{\tilde{\mathbf{E}}}$ can be introduced as

$$\begin{aligned}|\tilde{\mathbf{E}}| &= |\tilde{\mathbf{E}}^*| + \nu_{\tilde{\mathbf{E}}} v, \\ \nu_e + \nu_{\tilde{\mathbf{E}}} &= \frac{|\tilde{\mathbf{E}}^*|}{v},\end{aligned}$$

68 which is then applied to perturbation δf_0 as

$$\begin{aligned}(\nu_e + \nu_{\tilde{\mathbf{E}}}) v \frac{\partial \delta f_0}{\partial v} &= v \nabla \cdot \mathbf{f}_1 + \tilde{\mathbf{E}}^* \cdot \frac{\partial \mathbf{f}_1}{\partial v}, \\ \left(\nu_e + \frac{\nu_{\tilde{\mathbf{E}}}}{3}\right) v \frac{\partial \mathbf{f}_1}{\partial v} - \nu_t \mathbf{f}_1 &= \frac{v}{3} \nabla \delta f_0 + \frac{\tilde{\mathbf{E}}^*}{3} \frac{\partial \delta f_0}{\partial v} + \frac{v}{3} \nabla \tilde{f}_M + \frac{\tilde{\mathbf{E}}}{3} \frac{\partial \tilde{f}_M}{\partial v}.\end{aligned}$$

69 3. Simulation results

70 Three cases:

- 71 • constant $n_e = 5 \times 10^{20}$ [1/cm³], constant $\bar{Z} = 4$, T_e temperature profile
- 72 taken from IMPACT simulation at 12 ps, see Figure 1

- 73 • n_e, T_e, \bar{Z} profiles taken from HYDRA simulation of Gadolinium hohlraum
74 at 10 ps, see Figure 2 and Figure 3
- 75 • Detail of distribution function under diffusive conditions of hydrogen
76 raising the question of potentially outstanding properties of AWBS,
77 since the uncorrected AWBS result is very close to KIPP full collision
78 operator, see Figure 4 (consult Eq. (41) in Tzoufras OSHUN JCP 2011)

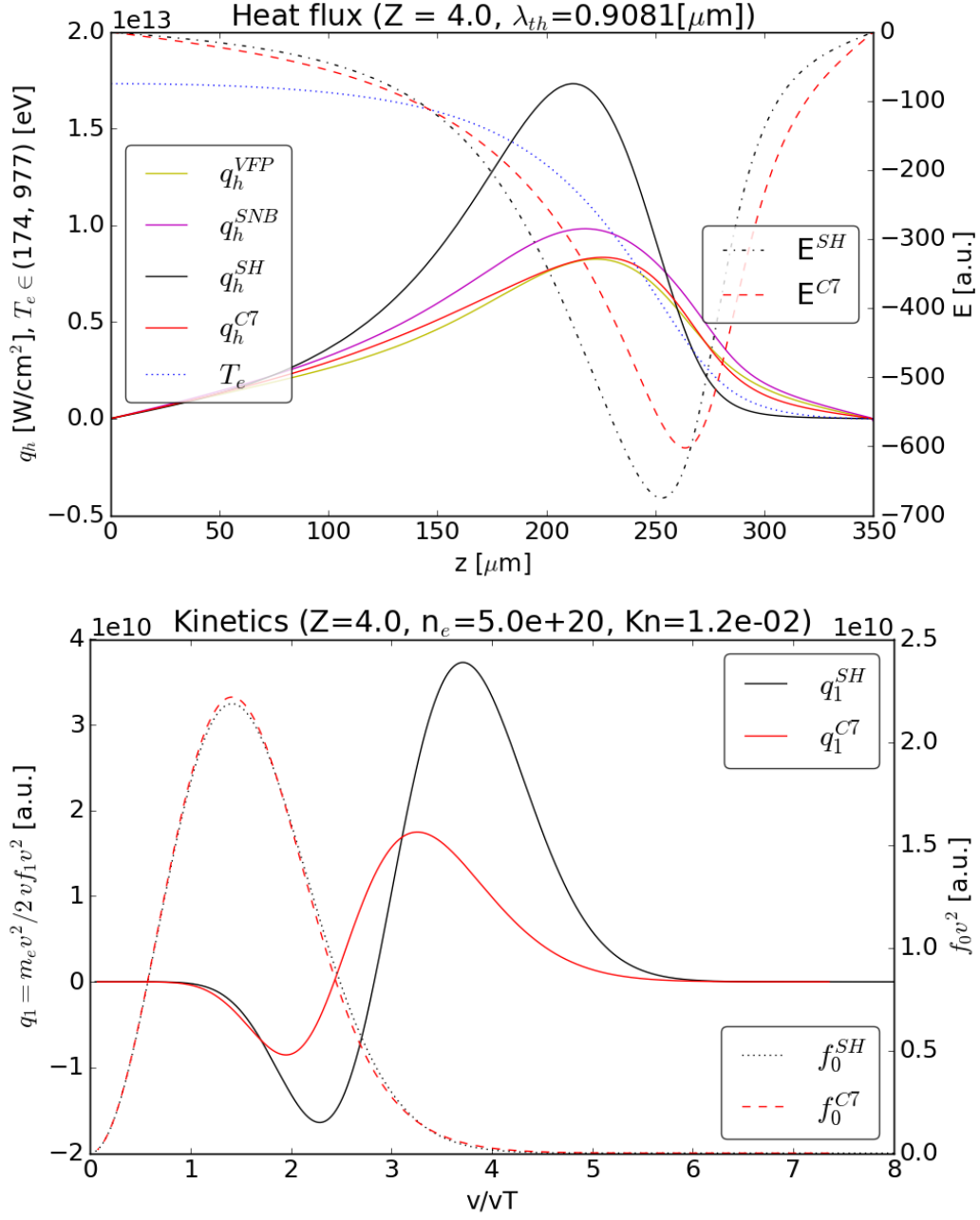


Figure 1: Philippe's preferred test $\bar{Z} = 4$ at 12 ps.

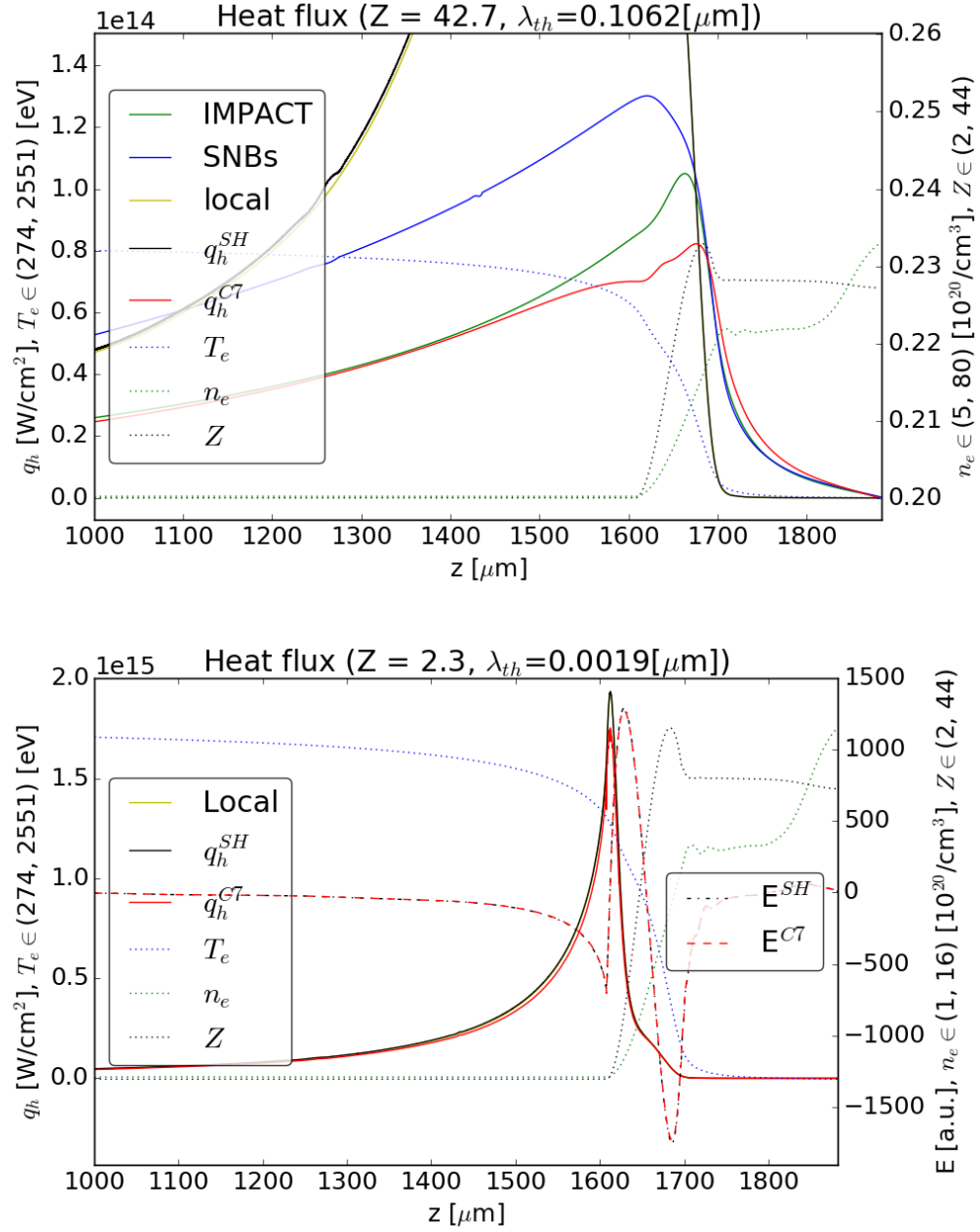


Figure 2:

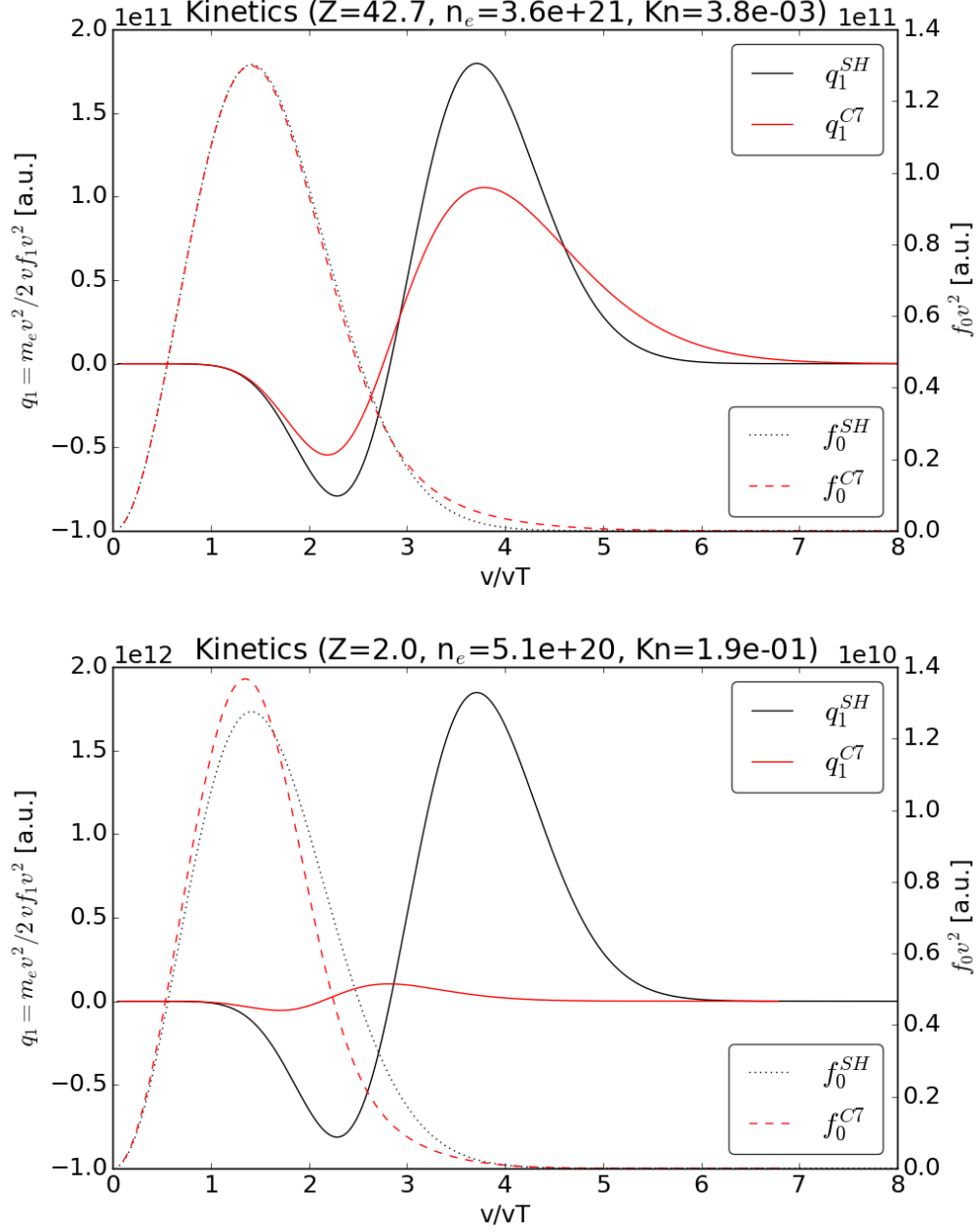


Figure 3: Kinetics profiles for max(flux) point and 1605 microns point for the case of 10ps VFP temperature profile, n_e and Z Hydra profiles.

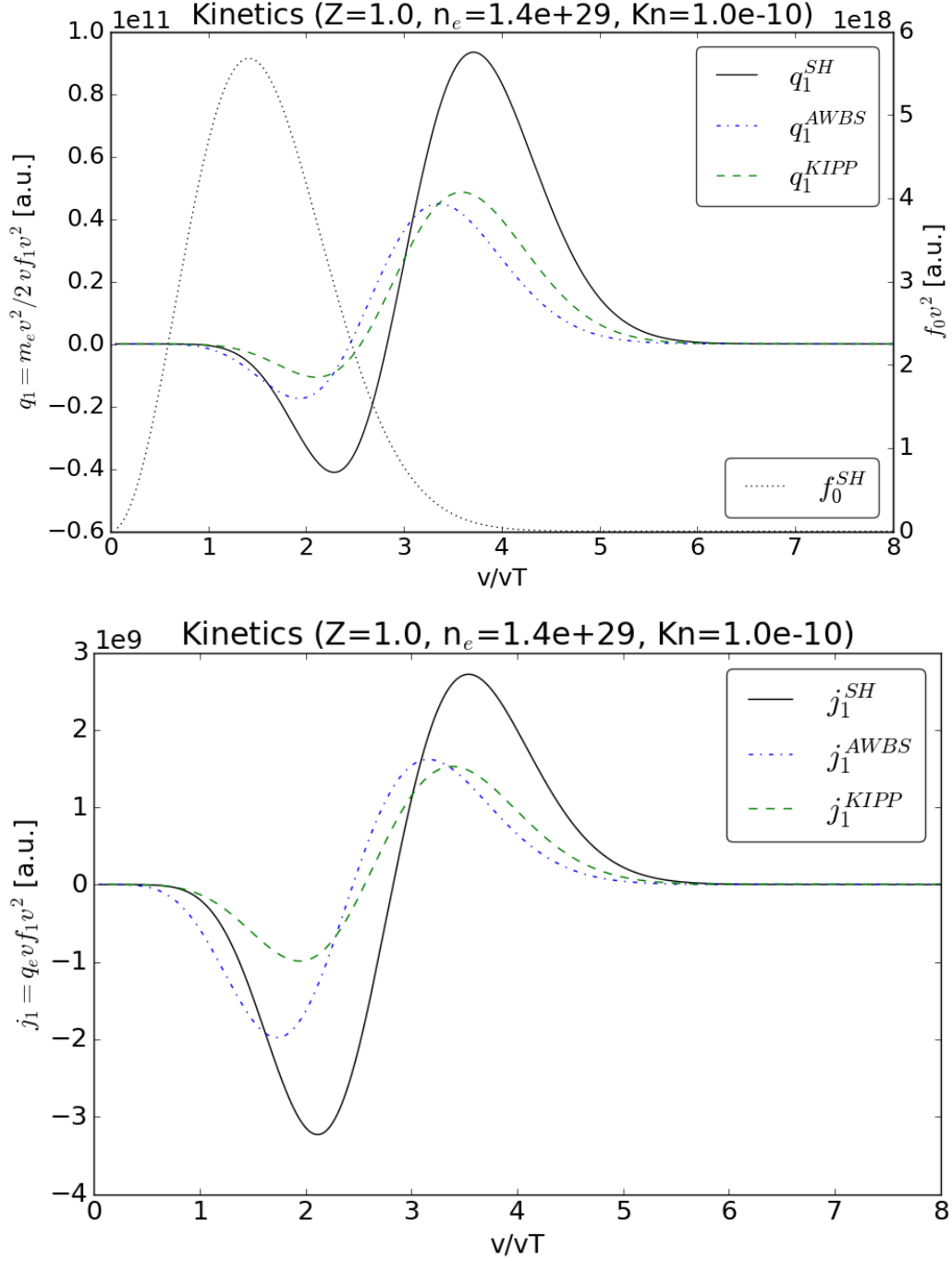


Figure 4: KIPP (by Johnathan) vs AWBS using $\lambda_{ei}^* = \frac{\bar{Z}+0.24}{\bar{Z}+4.2} \lambda_{ei}$, $\bar{Z} = 1$, $v_{th} = \sqrt{\frac{k_B T_e}{m_e}}$
 $f_1^{SH} = -\lambda_{ei}^*(v) \left(\frac{v^2}{2v_{th}^2} - 4 \right) \frac{\mathbf{n} \cdot \nabla T_e}{T_e} f_M$, $f_1^{KIPP} = -\lambda_{ei}^*(v) \left(\frac{3}{16} \frac{v^2}{v_{th}^2} - 1 - \frac{3}{2} \frac{v_{th}^2}{v^2} \right) \frac{\mathbf{n} \cdot \nabla T_e}{T_e} f_M$.

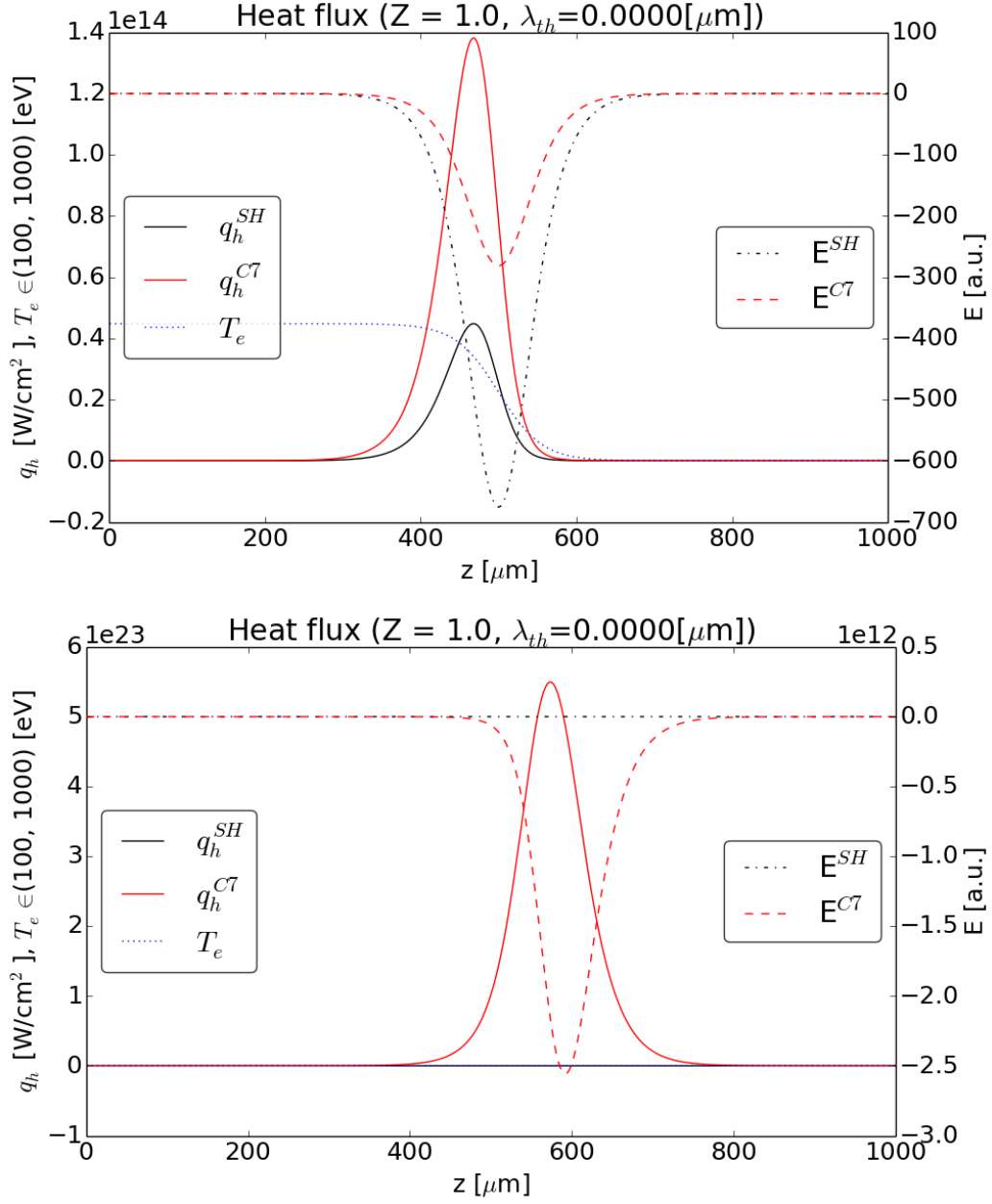


Figure 5: Decelerating (top) vs. accelerating (bottom) computations. Zeroth E field iteration, i.e. no E field effect, of the diffusion regime conditions.

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80 transport by not quite maxwell-boltzmann distributions, Phys. Rev. Lett.
81 57 (1986) 1887–1890.