An efficient kinetic modeling in hydrodynamics using the AWBS transport equation

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Abstract

The AWBS Boltzmann transport equation for electrons equipped with a simplified e-e collision operator [1] provides an efficient, yet physically relevant, kinetic extension compared the classical Spitzer-Harm heat flux (SH) based on local approximation and flux-limiting. This classical approach is widely used in plasma kinetics models coupled to hydrodynamics. Even though SH reflects the electron-electron collision effect, the essential physical properties of the electron transport cannot be modeled with an explicitly local model. A simple form of the AWBS model opens a way to couple kinetics to hydrodynamic codes while describing the important physics correctly. Since the electron-electron collision effect becomes especially important in the case of low ion potential, we focus on presenting results related to low Z plasmas.

Keywords: kinetics; hydrodynamics; nonlocal electron transport; laser-heated plasmas.

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1. The Fokker-Planck equation

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\nabla_{\boldsymbol{x}} f + (\tilde{\boldsymbol{E}} + \boldsymbol{v}\times\tilde{\boldsymbol{B}})\cdot\nabla_{\boldsymbol{v}} f = -\nabla_{\boldsymbol{v}}\cdot\sum_b \boldsymbol{S}_c^{t/b},$$

where the collision flux of test particles (labeled f) colliding on field particles (labeled $\tilde{f}$) takes the Landau-Fokker-Planck (LFP) form

$$\boldsymbol{S}_c^{t/b} = \Gamma^{t/b}\int \nabla_{\boldsymbol{v}}\nabla_{\boldsymbol{v}}\boldsymbol{s}\cdot\left[f(\boldsymbol{v})\frac{m_t}{m_b}\nabla_{\tilde{\boldsymbol{v}}}\tilde{f}(\tilde{\boldsymbol{v}}) - \tilde{f}(\tilde{\boldsymbol{v}})\nabla_{\boldsymbol{v}} f(\boldsymbol{v})\right]\,\mathrm{d}\tilde{\boldsymbol{v}},$$

where $\Gamma^{t/b} = \frac{4\pi\bar{Z}_t^2\bar{Z}_b^2 q^4 \ln\Lambda}{m_t^2}$, $\boldsymbol{s} = \boldsymbol{v} - \tilde{\boldsymbol{v}}$, and $\frac{\mathbf{I}}{s} - \frac{\boldsymbol{s}\boldsymbol{s}}{s^3} = \nabla_{\boldsymbol{v}}\nabla_{\boldsymbol{v}}\boldsymbol{s}$ was used. The LFP integral collision model can written in the form introduced by Rosenbluth 1957

$$\left(\frac{\partial f}{\partial t}\right)_b = -\nabla_{\boldsymbol{v}}\cdot\boldsymbol{S}_c^{t/b} = -\Gamma^{t/b}\left[\nabla_{\boldsymbol{v}}\cdot(f\nabla_{\boldsymbol{v}} h_b) - \frac{\nabla_{\boldsymbol{v}}\nabla_{\boldsymbol{v}} : (f\nabla_{\boldsymbol{v}}\nabla_{\boldsymbol{v}} g_b)}{2}\right], \quad (1)$$

where $\left(\frac{\partial f}{\partial t}\right)_b$ expresses the rate of change in the distribution function of test particles f due to collisions with background field particles (distribution function $\tilde{f}$) and where the complicated nature of collisions is modeled by the Rosenbluth potentials

$$h_b(\boldsymbol{v}) = \frac{m_t + m_b}{m_b}\int \frac{\tilde{f}(\tilde{\boldsymbol{v}})}{|\boldsymbol{v} - \tilde{\boldsymbol{v}}|}\,\mathrm{d}\tilde{\boldsymbol{v}}, \quad g_b(\boldsymbol{v}) = \int \tilde{f}(\tilde{\boldsymbol{v}})|\boldsymbol{v} - \tilde{\boldsymbol{v}}|\,\mathrm{d}\tilde{\boldsymbol{v}},$$

which have the following properties

$$\nabla_{\boldsymbol{v}}\cdot\nabla_{\boldsymbol{v}} h_b = -4\pi\frac{m_t + m_b}{m_b}\Gamma^{t/b}\tilde{f}, \quad \nabla_{\boldsymbol{v}}\cdot\nabla_{\boldsymbol{v}} g_b = 2\frac{m_b}{m_t + m_b} h_b.$$

The Rosenbluth equation (1) can be further rewritten according to [Longmire, Conrad L. : Elementary Plasma Physics. Intersci. Pub., 1963] as

$$\left(\frac{\partial f}{\partial t}\right)_c = \sum_b \Gamma^{t/b}\left[4\pi\frac{m_t}{m_b}\tilde{f}f + \frac{m_b - m_t}{m_t + m_b}\nabla_{\boldsymbol{v}} h_b\cdot\nabla_{\boldsymbol{v}} f + \frac{\nabla_{\boldsymbol{v}}\nabla_{\boldsymbol{v}} g_b : \nabla_{\boldsymbol{v}}\nabla_{\boldsymbol{v}} f}{2}\right], \quad (2)$$

which was also published in Shkarofsky 1966 and used in Tzoufras 2011.

1.1. *The linearized Fokker-Planck equation for low anisotropy*

Define the anisotropic perturbation as in Tzoufras and then use the equations (32, 33) and the harmonic expansions (38, 39, 40) and the most importantly (41) for one-kind particles. Finally, write explicitly (41) for the case f_1^0 and write set of integrals I, J and constants $C_1, .., C_6$, which will be used to calculate FP equation solution for diffusive conditions.

If we write the distribution function as its isotropic and anisotropic parts, i.e. $f = f_0 + \delta f$ and $\tilde{f} = \tilde{f}_0 + \delta\tilde{f}$, then the linearized LFP operator for low anisotropy of order $O(\delta f^2, \delta\tilde{f}^2)$ reads

$$
\begin{aligned}
\frac{1}{\Gamma^{t/b}}\left(\frac{\partial f_0}{\partial t}\right)_b &= 4\pi\frac{m_t}{m_b}\tilde{f}_0 f_0 + \frac{m_b - m_t}{m_t + m_b}\nabla_{\boldsymbol{v}} h(\tilde{f}_0)\cdot\nabla_{\boldsymbol{v}} f \\
&+ \frac{\nabla_{\boldsymbol{v}}\nabla_{\boldsymbol{v}} g(\tilde{f}_0) : \nabla_{\boldsymbol{v}}\nabla_{\boldsymbol{v}} f_0}{2}, \qquad (3)\\
\frac{1}{\Gamma^{t/b}}\left(\frac{\partial \delta f}{\partial t}\right)_b &= 4\pi\frac{m_t}{m_b}\left(\tilde{f}_0\delta f + f_0\delta\tilde{f}\right) \\
&+ \frac{m_b - m_t}{m_t + m_b}\left(\nabla_{\boldsymbol{v}} h(\tilde{f}_0)\cdot\nabla_{\boldsymbol{v}}\delta f + \nabla_{\boldsymbol{v}} f_0\cdot\nabla_{\boldsymbol{v}} h(\delta\tilde{f})\right) \\
&+ \frac{\nabla_{\boldsymbol{v}}\nabla_{\boldsymbol{v}} g(\tilde{f}_0) : \nabla_{\boldsymbol{v}}\nabla_{\boldsymbol{v}}\delta f}{2} + \nabla_{\boldsymbol{v}}\nabla_{\boldsymbol{v}} f_0 : \frac{\nabla_{\boldsymbol{v}}\nabla_{\boldsymbol{v}} g(\delta\tilde{f})}{2}. \qquad (4)
\end{aligned}
$$

$$
\begin{aligned}
f &= f_0 + \sum_{l=1}^{\infty}\sum_{m=-l}^{l} f_l^m(v) P_l^{|m|}(\cos\theta)\exp^{im\phi}, \\
\tilde{f} &= \tilde{f}_0 + \sum_{l=1}^{\infty}\sum_{m=-l}^{l} \tilde{f}_l^m(v) P_l^{|m|}(\cos\theta)\exp^{im\phi},
\end{aligned}
$$

$$
\begin{aligned}
I_j(\tilde{f}_l^m) &= \frac{4\pi}{v^j}\int_0^v \tilde{f}_l^m(u) u^{j+2}\,\mathrm{d}u, \\
J_j(\tilde{f}_l^m) &= \frac{4\pi}{v^j}\int_v^\infty \tilde{f}_l^m(u) u^{j+2}\,\mathrm{d}u,
\end{aligned}
$$

$$
\frac{1}{\Gamma^{t/b}}\left(\frac{\partial f_0}{\partial t}\right)_b = \frac{1}{3v^2}\frac{\partial}{\partial v}\left[\frac{3m_t}{m_b} f_0 I_0(\tilde{f}_0) + v\left(I_2(\tilde{f}_0) + J_{-1}(\tilde{f}_0)\right)\frac{\partial f_0}{\partial v}\right],
$$

$$\frac{1}{\Gamma^{t/b}}\left(\frac{\partial f_l^m}{\partial t}\right)_b = 4\pi\frac{m_t}{m_b}\left[\tilde{f}_0 f_l^m + f_0\tilde{f}_l^m\right]$$
$$+\frac{m_t - m_b}{m_b v^2}\left[\frac{\partial f_0}{\partial v}\left(\frac{l+1}{2l+1}I_l(\tilde{f}_l^m) - \frac{l}{2l+1}J_{-1-l}(\tilde{f}_l^m)\right) + I_0(\tilde{f}_0)\frac{\partial f_l^m}{\partial v}\right]$$
$$+\frac{I_2(\tilde{f}_0)+J_{-1}(\tilde{f}_0)}{3v}\frac{\partial^2 f_l^m}{\partial v^2} + \frac{-I_2(\tilde{f}_0)+2J_{-1}(\tilde{f}_0)+3I_0(\tilde{f}_0)}{3v^2}\frac{\partial f_l^m}{\partial v}$$
$$-\frac{l(l+1)}{2}\frac{-I_2(\tilde{f}_0)+2J_{-1}(\tilde{f}_0)+3I_0(\tilde{f}_0)}{3v^3}f_l^m$$
$$\frac{1}{2v}\frac{\partial^2 f_0}{\partial v^2}\left[C_1 I_{l+2}(\tilde{f}_l^m) + C_1 J_{-l-1}(\tilde{f}_l^m) + C_2 I_l(\tilde{f}_l^m) + C_2 J_{1-l}(\tilde{f}_l^m)\right]$$
$$\frac{1}{v^2}\frac{\partial f_0}{\partial v}\left[C_3 I_{l+2}(\tilde{f}_l^m) + C_4 J_{1-l}(\tilde{f}_l^m) + C_5 J_{-l-1}(\tilde{f}_l^m) + C_6 I_l(\tilde{f}_l^m)\right] \quad (5)$$

$$C_1 = \frac{(l+1)(l+2)}{(2l+1)(2l+3)}, C_2 = -\frac{(l-1)l}{(2l+1)(2l-1)}, C_3 = -\frac{l(l+1)/2+(l+1)}{(2l+1)(2l+3)},$$
$$C_4 = \frac{l(l+1)/2 - l}{(2l+1)(2l-1)}, C_5 = \frac{(l+2)-l(l+1)/2}{(2l+1)(2l+3)}, C_6 = \frac{l(l+1)/2+(l-1)}{(2l+1)(2l-1)},$$

In the case of massive background particles $m_t/m_b << 1$ in equilibrium and comparable temperatures $T_b \approx T_b$, i.e. slow-non-moving background, the isotropic distribution function can be approximated by $\tilde{f}_0^{slow} = n_{slow}\delta(v)/(4\pi v^2)$, and since all integrals $I_j(\tilde{f}_0^{slow}), J_j(\tilde{f}_0^{slow})$ vanish except $I_0(\tilde{f}_0^{slow}) = n_{slow}$, equation (5) reduces to

$$\frac{1}{\Gamma^{t/slow}}\left(\frac{\partial f_l^m}{\partial t}\right)_{slow} = -\frac{l(l+1)}{2}\frac{n_{slow}}{v^3}f_l^m \quad (6)$$

where n_{slow} is the density of slow massive particles. Consequently, the effect of collisions on slow massive particles leads to scattering but no change in velocity, i.e. energy, of test particles.

1.2. Plasma Fokker-Planck equation in diffusive regime

$$\left(\frac{\partial f_0}{\partial t}\right)_e = \frac{\Gamma^{e/e}}{3v^2}\frac{\partial}{\partial v}\left[3f_0 I_0(f_0) + v\left(I_2(f_0)+J_{-1}(f_0)\right)\frac{\partial f_0}{\partial v}\right],$$

$$\left(\frac{\partial f_l^m}{\partial t}\right)_e = \Gamma^{e/e}\Bigg[8\pi f_0 f_l^m - \frac{l(l+1)}{2}\frac{3I_0(f_0)-I_2(f_0)+2J_{-1}(f_0)}{3v^3}f_l^m + \frac{I_2(f_0)+J_{-1}(f_0)}{3v}\frac{\partial^2 f_l^m}{\partial v^2} + \frac{3I_0(f_0)-I_2(f_0)+2J_{-1}(f_0)}{3v^2}\frac{\partial f_l^m}{\partial v} + \frac{1}{2v}\frac{\partial^2 f_0}{\partial v^2}\left[C_1 I_{l+2}(f_l^m) + C_1 J_{-l-1}(f_l^m) + C_2 I_l(f_l^m) + C_2 J_{1-l}(f_l^m)\right] + \frac{1}{v^2}\frac{\partial f_0}{\partial v}\left[C_3 I_{l+2}(f_l^m) + C_4 J_{1-l}(f_l^m) + C_5 J_{-l-1}(f_l^m) + C_6 I_l(f_l^m)\right]\Bigg] \quad (7)$$

$$C_1 = \frac{2}{5}, C_2 = 0, C_3 = -\frac{1}{5}, C_4 = 0, C_5 = \frac{2}{15}, C_6 = \frac{1}{3},$$

$$\left(\frac{\partial f_1}{\partial t}\right)_{e+i} = \Gamma^{e/e}\Bigg[8\pi f_0 f_1 - \left[\frac{3I_0(f_0)-I_2(f_0)+2J_{-1}(f_0)}{3v^3} + \frac{\bar{Z}n_e}{v^3}\right]f_1 + \frac{3I_0(f_0)-I_2(f_0)+2J_{-1}(f_0)}{3v^2}\frac{\partial f_1}{\partial v} + \frac{I_2(f_0)+J_{-1}(f_0)}{3v}\frac{\partial^2 f_1}{\partial v^2} + \frac{1}{5v}\frac{\partial^2 f_0}{\partial v^2}\left[I_3(f_1)+J_{-2}(f_1)\right] + \frac{1}{15v^2}\frac{\partial f_0}{\partial v}\left[5I_1(f_1) - 3I_3(f_1) + 2J_{-2}(f_1)\right]\Bigg] \quad (8)$$

$$f \approx f_M + \cos(\theta)\lambda_{ei}(v)\left[\frac{v^2}{2v_{th}^2} - 4 + D\left(v\right)\right]f_M,$$

$$v\frac{\partial f_M}{\partial z} - \frac{v\tilde{E}_z}{v_{th}^2}f_M = \Gamma^{e/e}\Bigg[8\pi f_M f_1 - \frac{3I_0(f_M)-I_2(f_M)+2J_{-1}(f_M)+3\bar{Z}n_e}{3v^3}f_1 + \frac{3I_0(f_M)-I_2(f_M)+2J_{-1}(f_M)}{3v^2}\frac{\partial f_1}{\partial v} + \frac{I_2(f_M)+J_{-1}(f_M)}{3v}\frac{\partial^2 f_1}{\partial v^2} + \frac{f_M}{15vv_{th}^2}\left[\frac{3v^2}{v_{th}^2}\left[I_3(f_1)+J_{-2}(f_1)\right] - 5\left[I_1(f_1)+J_{-2}(f_1)\right]\right]\Bigg]$$

$$
\left[\frac{I_2(f_M)+J_{-1}(f_M)}{3v}\right]\frac{\partial^2 f_1}{\partial v^2}+\left[\frac{3I_0(f_M)-I_2(f_M)+2J_{-1}(f_M)}{3v^2}\right]\frac{\partial f_1}{\partial v}
+\left[8\pi f_M-\frac{3I_0(f_M)-I_2(f_M)+2J_{-1}(f_M)+3\bar{Z}n_e}{3v^3}\right]f_1=
\frac{1}{\Gamma^{e/e}}\left[v\frac{\partial f_M}{\partial z}-\frac{v\tilde{E}_z}{v_{th}^2}f_M\right]-\frac{f_M}{15vv_{th}^2}\left[\frac{3v^2}{v_{th}^2}\left[I_3(f_1)+J_{-2}(f_1)\right]-5\left[I_1(f_1)+J_{-2}(f_1)\right]\right]
\tag{9}
$$

Integration of (9) from $\infty\to 0$

$$
\begin{aligned}
a^{n-0.5}\frac{df^n-df^{n-1}}{-\Delta v} &= b^{n-0.5}df^{n-1}+c^{n-0.5}f_1^{n-1}+d^{n-0.5},\\
\frac{f_1^n-f_1^{n-1}}{-\Delta v} &= df^{n-1},
\end{aligned}
$$

$$
\begin{aligned}
\left(\frac{a^{n-0.5}}{\Delta v}-b^{n-0.5}\right)df^{n-1}-c^{n-0.5}f_1^{n-1} &= d^{n-0.5}+\frac{a^{n-0.5}}{\Delta v}df^n,\\
-df^{n-1}+\frac{1}{\Delta v}f_1^{n-1} &= \frac{1}{\Delta v}f_1^n,
\end{aligned}
$$

$$
\begin{bmatrix}-c^{n-0.5} & \frac{a^{n-0.5}}{\Delta v}-b^{n-0.5}\\ \frac{c^{n-0.5}}{\Delta v} & -c^{n-0.5}\end{bmatrix}\begin{bmatrix}f_1^{n-1}\\ df^{n-1}\end{bmatrix}=\begin{bmatrix}d^{n-0.5}+\frac{a^{n-0.5}}{\Delta v}df^n\\ \frac{c^{n-0.5}}{\Delta v}f_1^n\end{bmatrix},
$$

$$
\begin{bmatrix}-c^{n-0.5} & \frac{a^{n-0.5}}{\Delta v}-b^{n-0.5}\\ 0 & \frac{1}{\Delta v}\left(\frac{a^{n-0.5}}{\Delta v}-b^{n-0.5}\right)-c^{n-0.5}\end{bmatrix}\begin{bmatrix}f_1^{n-1}\\ df^{n-1}\end{bmatrix}=
\begin{bmatrix}d^{n-0.5}+\frac{a^{n-0.5}}{\Delta v}df^n\\ \frac{1}{\Delta v}\left(c^{n-0.5}f_1^n+d^{n-0.5}+\frac{a^{n-0.5}}{\Delta v}df^n\right)\end{bmatrix},
$$

$$
\begin{aligned}
a^n &= \frac{I_2^n(f_M) + J_{-1}^n(f_M)}{3v^n}, \\
b^n &= \frac{3I_0^n(f_M) - I_2^n(f_M) + 2J_{-1}^n(f_M)}{3v^{n2}}, \\
c^n &= 8\pi f_M^n - \frac{3I_0^n(f_M) - I_2^n(f_M) + 2J_{-1}^n(f_M) + 3\bar{Z}n_e}{3v^{n3}}, \\
d^n &= \frac{1}{\Gamma^{e/e}}\left[v^n\frac{\partial f_M^n}{\partial z} - \frac{v^n\tilde{E}_z}{v_{th}^2}f_M^n\right] \\
&\quad - \frac{f_M^n}{15v^n v_{th}^2}\left[\frac{3v^{n2}}{v_{th}^2}\left[I_3^n(f_1) + J_{-2}^n(f_1)\right] - 5\left[I_1^n(f_1) + J_{-2}^n(f_1)\right]\right]
\end{aligned}
$$

$$
\begin{aligned}
I_0^n(g) = 4\pi\int_0^{v^n} g(u)u^2\,\mathrm{d}u, &\quad I_1^n(g) = \frac{4\pi}{v^n}\int_0^{v^n} g(u)u^3\,\mathrm{d}u, \\
I_2^n(g) = \frac{4\pi}{v^{n2}}\int_0^{v^n} g(u)u^4\,\mathrm{d}u, &\quad I_3^n(g) = \frac{4\pi}{v^{n3}}\int_0^{v^n} g(u)u^5\,\mathrm{d}u, \\
J_{-1}^n(g) = 4\pi v^n\int_{v^n}^{\infty} g(u)u\,\mathrm{d}u, &\quad J_{-2}^n(g) = 4\pi v^{n2}\int_{v^n}^{\infty} g(u)\,\mathrm{d}u,
\end{aligned}
$$

2. AWBS-P1 modeling of laser heated plasmas

2.1. Model equations

The AWBS electron transport equation reads

$$
v\boldsymbol{n}\cdot\nabla f + \tilde{\boldsymbol{E}}\cdot\boldsymbol{n}\frac{\partial f}{\partial v} + \frac{\tilde{\boldsymbol{E}}\cdot\boldsymbol{e}_\phi - v\tilde{\boldsymbol{B}}\cdot\boldsymbol{e}_\theta}{v}\frac{\partial f}{\partial \phi} = v\nu_e\frac{\partial}{\partial v}\left(f - f_M\right) + (\nu_{ei} + \nu_e)(f_0 - f),
$$

where ν_e is the electron-electron collision frequency, ν_{ei} is the electron-ion collision frequency, and $\nu_{ei} = \bar{Z}\nu_e$.

In order to eliminate the dimensions of the above transport problem the first-two-moment model based on approximation

$$
f = \frac{f_0}{4\pi} + \frac{3}{4\pi}\boldsymbol{n}\cdot\boldsymbol{f_1},
$$

can be adopted and reads

$$\begin{aligned}
\nu_e v \frac{\partial}{\partial v}\left(f_0 - \tilde{f}_M\right) &= v\nabla\cdot\boldsymbol{f_1} + \tilde{\boldsymbol{E}}\cdot\frac{\partial \boldsymbol{f_1}}{\partial v} + \frac{2}{v}\tilde{\boldsymbol{E}}\cdot\boldsymbol{f_1},\\
\nu_e v \frac{\partial}{\partial v}\boldsymbol{f_1} - \nu_t \boldsymbol{f_1} &= v\nabla\cdot(\mathbf{A} f_0) + \tilde{\boldsymbol{E}}\cdot\frac{\partial(\mathbf{A} f_0)}{\partial v} + \tilde{\boldsymbol{B}}\times\boldsymbol{f_1},
\end{aligned}$$

where $\tilde{f}_M = 4\pi f_M$ and the closure matrix takes the form

$$\mathbf{A} = \frac{1}{3}\mathbf{I}.$$

Since in the laser heated plasmas the Knudsen number Kn$= \frac{v_{th}}{\nu_t(v_{th})L} \in (0,1)$, i.e. the collisionality in the kinetics of electrons plays always an important effect for thermal-like particles, the electron distribution function can be treated as out-of-equilibrium approximation

$$f = f_M + \delta f, \tag{10}$$

where the consequent AWBS model reads

$$\begin{aligned}
v\boldsymbol{n}\cdot\nabla(f_M + \delta f) + \tilde{\boldsymbol{E}}\cdot\boldsymbol{n}\frac{\partial f_M}{\partial v} + \tilde{\boldsymbol{E}}\cdot\boldsymbol{n}\frac{\partial \delta f}{\partial v} + \frac{\tilde{\boldsymbol{E}}\cdot\boldsymbol{e}_\phi - v\tilde{\boldsymbol{B}}\cdot\boldsymbol{e}_\theta}{v}\frac{\partial \delta f}{\partial \phi} &= \\
v\nu_e\frac{\partial \delta f}{\partial v} + (\nu_{ei} + \nu_e)(f_0 - f_M - \delta f), &
\end{aligned} \tag{11}$$

or its P1 approximation equivalent

$$f = \frac{4\pi f_M + \delta f_0}{4\pi} + \frac{3}{4\pi}\boldsymbol{n}\cdot\boldsymbol{f_1}. \tag{12}$$

where the moment model reads

$$\nu_e v\frac{\partial \delta f_0}{\partial v} = v\nabla\cdot\boldsymbol{f_1} + \tilde{\boldsymbol{E}}\cdot\frac{\partial \boldsymbol{f_1}}{\partial v} + \frac{2}{v}\tilde{\boldsymbol{E}}\cdot\boldsymbol{f_1}, \tag{13}$$

$$\begin{aligned}
\nu_e v\frac{\partial \boldsymbol{f_1}}{\partial v} - \nu_t\boldsymbol{f_1} &= \frac{v}{3}\nabla\delta f_0 + \frac{\tilde{\boldsymbol{E}}}{3}\frac{\partial \delta f_0}{\partial v} + \tilde{\boldsymbol{B}}\times\boldsymbol{f_1}\\
&\quad + \frac{v}{3}\nabla\tilde{f}_M + \frac{\tilde{\boldsymbol{E}}}{3}\frac{\partial \tilde{f}_M}{\partial v}.
\end{aligned} \tag{14}$$

2.2. A consistent treatment of $\tilde{\boldsymbol{E}}$ field

The plasma conditions providing an appropriate electric field are the best expressed via the definition of current

$$\boldsymbol{q}_c(\boldsymbol{x}) = \int_v v\boldsymbol{f_1}(\boldsymbol{x})v^2\,\mathrm{d}v,$$

which can be directly expressed from (14) as

$$\boldsymbol{q}_c = \int_v \left(\frac{\nu_e v^2}{\nu_t}\frac{\partial \boldsymbol{f_1}}{\partial v} - \frac{v^2}{3\nu_t}\nabla\left(\tilde{f}_M + \delta f_0\right) - \frac{v}{3\nu_t}\frac{\partial\left(\tilde{f}_M + \delta f_0\right)}{\partial v}\tilde{\boldsymbol{E}}\right) v^2\,\mathrm{d}v, \quad (15)$$

where the $\boldsymbol{B}$ field and $\boldsymbol{E}$ field scattering effect (angular) have been omitted.

Then, the current can be easily evaluated based on

$$\begin{aligned} a_0(\boldsymbol{x}) &= \int_v \frac{v}{3\nu_t}\frac{\partial\left(\tilde{f}_M + \delta f_0\right)}{\partial v}(\boldsymbol{x})v^2\,\mathrm{d}v, \\ \boldsymbol{b}_0(\boldsymbol{x}) &= \int_v \left(\frac{v^2}{3\nu_t}\nabla\left(\tilde{f}_M(\boldsymbol{x}) + \delta f_0(\boldsymbol{x})\right) - \frac{\nu_e v^2}{\nu_t}\frac{\partial \boldsymbol{f_1}}{\partial v}(\boldsymbol{x})\right) v^2\,\mathrm{d}v, \end{aligned}$$

as the following generalization of the Ohm's law

$$\boldsymbol{q}_c(\boldsymbol{x}) = -\boldsymbol{b}_0(\boldsymbol{x}) - a_0(\boldsymbol{x})\tilde{\boldsymbol{E}}(\boldsymbol{x}),$$

where one needs the actual distribution function f values.

It is straightforward to find the *zero current* formula for the electric field

$$\tilde{\boldsymbol{E}}(\boldsymbol{x}) = -\frac{\boldsymbol{b}_0(\boldsymbol{x})}{a_0(\boldsymbol{x})}. \quad (16)$$

2.3. AWBS model analysis

The AWBS transport equation can be written as the following

$$\left(v\nu_e - \tilde{\boldsymbol{E}}\cdot\boldsymbol{n}\right)\frac{\partial \delta f}{\partial v} = v\boldsymbol{n}\cdot\nabla(f_M+\delta f) + \tilde{\boldsymbol{E}}\cdot\boldsymbol{n}\frac{\partial f_M}{\partial v} + (\nu_{ei}+\nu_e)(f_M+\delta f - f_0), \quad (17)$$

in order to stress the effect of force applied to electrons, i.e. the effect of friction described by ν_e and the Lorentz force effect via $\tilde{\boldsymbol{E}}$, and their competition.

The same reformulation can be written for the moment AWBS model

$$
\begin{aligned}
\frac{\partial \delta f_0}{\partial v} &= \frac{1}{\nu_e v}\left(v\nabla\cdot\boldsymbol{f_1} + \tilde{\boldsymbol{E}}\cdot\frac{\partial \boldsymbol{f_1}}{\partial v}\right),\\
\nu_e v\frac{\partial \boldsymbol{f_1}}{\partial v} - \nu_t\boldsymbol{f_1} &= \frac{v}{3}\nabla\delta f_0 + \frac{\tilde{\boldsymbol{E}}}{3}\frac{\partial \delta f_0}{\partial v} + \frac{v}{3}\nabla\tilde{f}_M + \frac{\tilde{\boldsymbol{E}}}{3}\frac{\partial \tilde{f}_M}{\partial v},
\end{aligned}
$$

and it takes the following form

$$
\left(\nu_e v\mathbf{I} - \frac{\tilde{\boldsymbol{E}}\tilde{\boldsymbol{E}}}{3\nu_e v}\right)\cdot\frac{\partial \boldsymbol{f_1}}{\partial v} = \frac{v}{3}\nabla\delta f_0 + \frac{\tilde{\boldsymbol{E}}}{3\nu_e}\nabla\cdot\boldsymbol{f_1} + \frac{v}{3}\nabla\tilde{f}_M + \frac{\tilde{\boldsymbol{E}}}{3}\frac{\partial \tilde{f}_M}{\partial v} + \nu_t\boldsymbol{f_1},
$$

which is especially instructive in 1D

$$
\left(\nu_e v - \frac{\tilde{E}_z^2}{3\nu_e v}\right)\frac{\partial f_{1_z}}{\partial v} = \frac{v}{3}\frac{\partial \delta f_0}{\partial z} + \frac{\tilde{E}_z}{3\nu_e}\frac{\partial f_{1_z}}{\partial z} + \frac{v}{3}\frac{\partial \tilde{f}_M}{\partial z} + \frac{\tilde{E}_z}{3}\frac{\partial \tilde{f}_M}{\partial v} + \nu_t f_{1_z}, \quad (18)
$$

because it gives a *”reverse-time-like evolution”* condition

$$
\sqrt{3}\nu_e > \frac{|\tilde{E}_z|}{v}. \quad (19)
$$

$$
\frac{\partial f_{1_z}}{\partial v} = \frac{3\nu_e v}{3(\nu_e v)^2 - \tilde{E}_z^2}\left(\frac{v}{3}\frac{\partial \delta f_0}{\partial z} + \frac{\tilde{E}_z}{3\nu_e}\frac{\partial f_{1_z}}{\partial z} + \frac{v}{3}\frac{\partial \tilde{f}_M}{\partial z} + \frac{\tilde{E}_z}{3}\frac{\partial \tilde{f}_M}{\partial v} + \nu_t f_{1_z}\right), \quad (20)
$$

because it gives a *”reverse-time-like evolution”* condition

$$
3(\nu_e v)^2 - \tilde{E}_z^2 \neq 0,
$$

or a numerical stability formulation because it gives a *”reverse-time-like evolution”* condition

$$
|3(\nu_e v)^2 - \tilde{E}_z^2| > \epsilon. \quad (21)
$$

which can be obtained from

$$
\left(3(\nu_e v)^2 - \tilde{E}_z^2\right)^2 - \epsilon^2 = 0. \quad (22)
$$

2.4. "Reverse-time-like evolution" model by splitting

Full separation of advection and E field

$$\begin{aligned}
\nu_e v \frac{\partial \boldsymbol{f_1}^{\nu_e}}{\partial v} &= \frac{v}{3}\nabla \delta f_0^{\nu_e} + \frac{v}{3}\nabla \tilde{f}_M + \nu_t \boldsymbol{f_1}^{\nu_e}, \\
\frac{\tilde{\boldsymbol{E}}\tilde{\boldsymbol{E}}}{3\nu_e v} \cdot \frac{\partial \boldsymbol{f_1}^{\tilde{\boldsymbol{E}}}}{\partial v} &= -\frac{\tilde{\boldsymbol{E}}}{3\nu_e}\nabla \cdot \boldsymbol{f_1}^{\tilde{\boldsymbol{E}}} - \frac{\tilde{\boldsymbol{E}}}{3}\frac{\partial \tilde{f}_M}{\partial v},
\end{aligned}$$

or separation of stable "bulk" E field effect and implicit E field effect

$$\begin{aligned}
\nu_e v \frac{\partial \boldsymbol{f_1}^{\nu_e}}{\partial v} &= \frac{v}{3}\nabla \delta f_0^{\nu_e} + \frac{v}{3}\nabla \tilde{f}_M + \nu_t \boldsymbol{f_1}^{\nu_e} + \frac{\tilde{\boldsymbol{E}}}{3}\frac{\partial \tilde{f}_M}{\partial v}, \\
\frac{\tilde{\boldsymbol{E}}\tilde{\boldsymbol{E}}}{3\nu_e v} \cdot \frac{\partial \boldsymbol{f_1}^{\tilde{\boldsymbol{E}}}}{\partial v} &= -\frac{\tilde{\boldsymbol{E}}}{3\nu_e}\nabla \cdot \boldsymbol{f_1}^{\tilde{\boldsymbol{E}}},
\end{aligned}$$

and the complete effect of diffusion in velocity space reads

$$\frac{\partial \boldsymbol{f_1}}{\partial v} = \frac{\partial \boldsymbol{f_1}^{\nu_e}}{\partial v} + \frac{\partial \boldsymbol{f_1}^{\tilde{\boldsymbol{E}}}}{\partial v},$$

2.5. "Friction" model

In order to obey (22), an additional friction $\nu_{\tilde{\boldsymbol{E}}}$ can be introduced as

$$\begin{aligned}
|\tilde{\boldsymbol{E}}| &= |\tilde{\boldsymbol{E}}^*| + \nu_{\tilde{\boldsymbol{E}}} v, \\
\nu_e + \nu_{\tilde{\boldsymbol{E}}} &= \frac{|\tilde{\boldsymbol{E}}^*|}{v},
\end{aligned}$$

which is then applied to perturbation δf_0 as

$$\begin{aligned}
(\nu_e + \nu_{\tilde{\boldsymbol{E}}})\, v \frac{\partial \delta f_0}{\partial v} &= v\nabla \cdot \boldsymbol{f_1} + \tilde{\boldsymbol{E}}^* \cdot \frac{\partial \boldsymbol{f_1}}{\partial v}, \\
\left(\nu_e + \frac{\nu_{\tilde{\boldsymbol{E}}}}{3}\right) v \frac{\partial \boldsymbol{f_1}}{\partial v} - \nu_t \boldsymbol{f_1} &= \frac{v}{3}\nabla \delta f_0 + \frac{\tilde{\boldsymbol{E}}^*}{3}\frac{\partial \delta f_0}{\partial v} + \frac{v}{3}\nabla \tilde{f}_M + \frac{\tilde{\boldsymbol{E}}}{3}\frac{\partial \tilde{f}_M}{\partial v}.
\end{aligned}$$

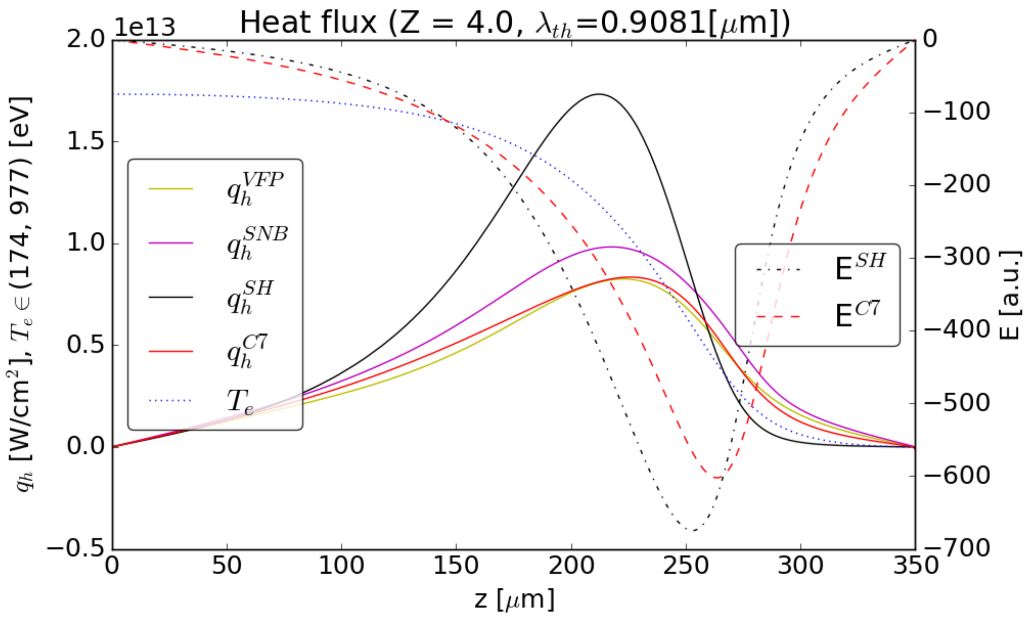
3. Simulation results

Three cases:

- constant $n_e = 5 \times 10^{20}$ [1/cm^3], constant $\bar{Z} = 4$, T_e temperature profile taken from IMPACT simulation at 12 ps, see Figure 1

- $n_e, T_e, \bar{Z}$ profiles taken from HYDRA simulation of Gadolinium hohlraum at 10 ps, see Figure 2 and Figure 3

- Detail of distribution function under diffusive conditions of hydrogen raising the question of potentially outstanding properties of AWBS, since the uncorrected AWBS result is very close to KIPP full collision operator, see Figure 4 (consult Eq. (41) in Tzoufras OSHUN JCP 2011)



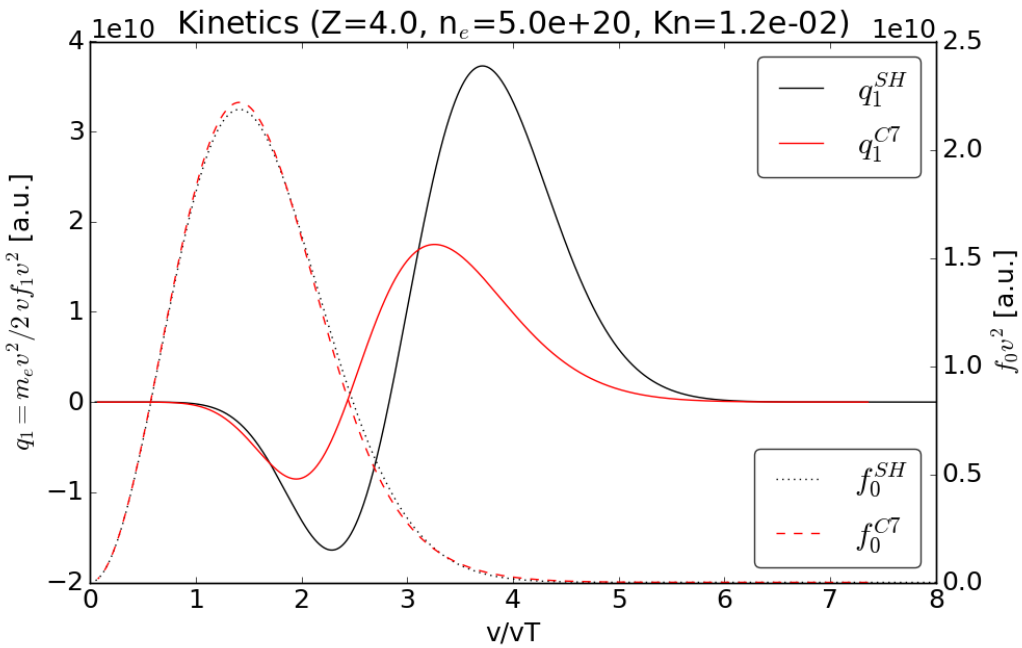
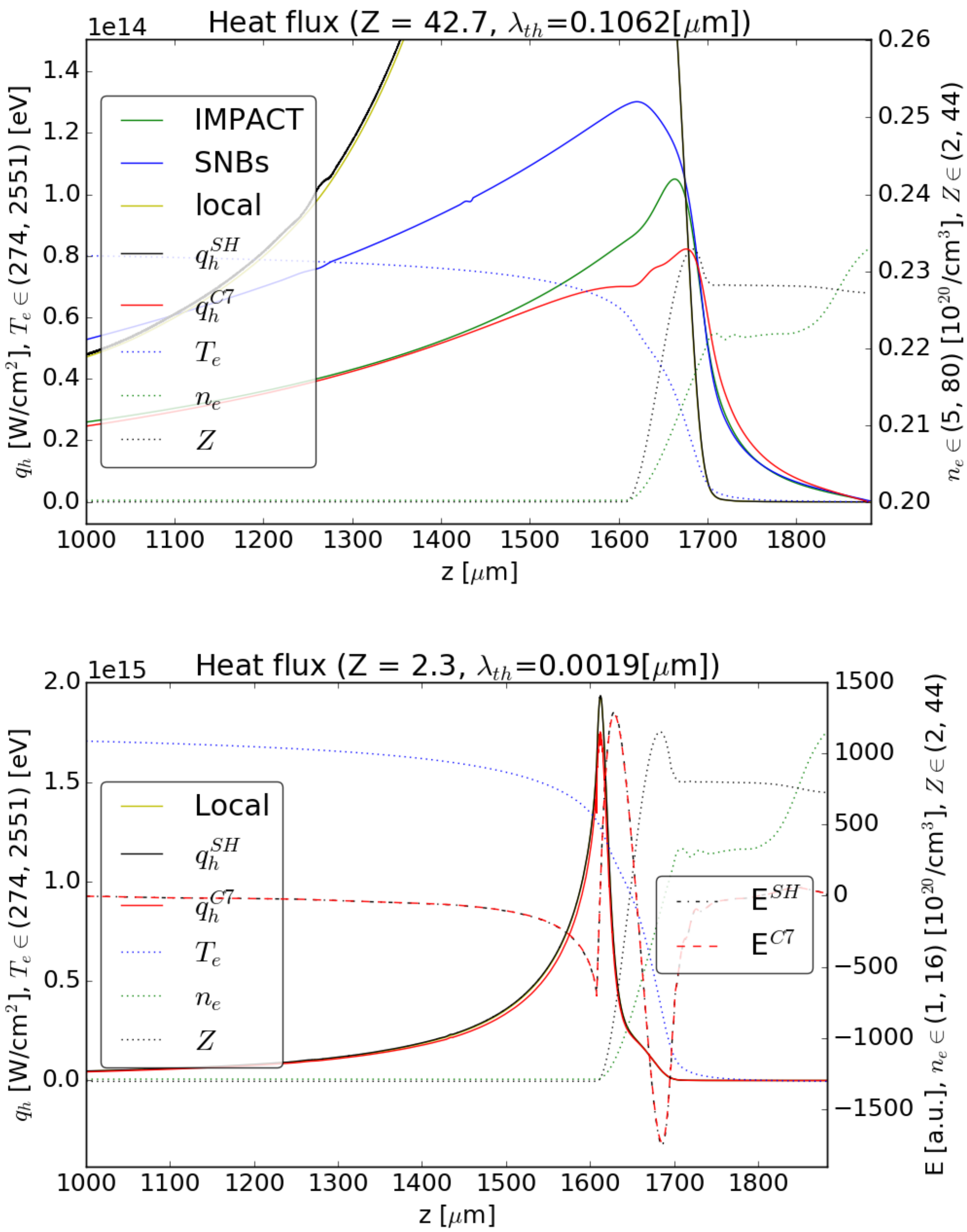
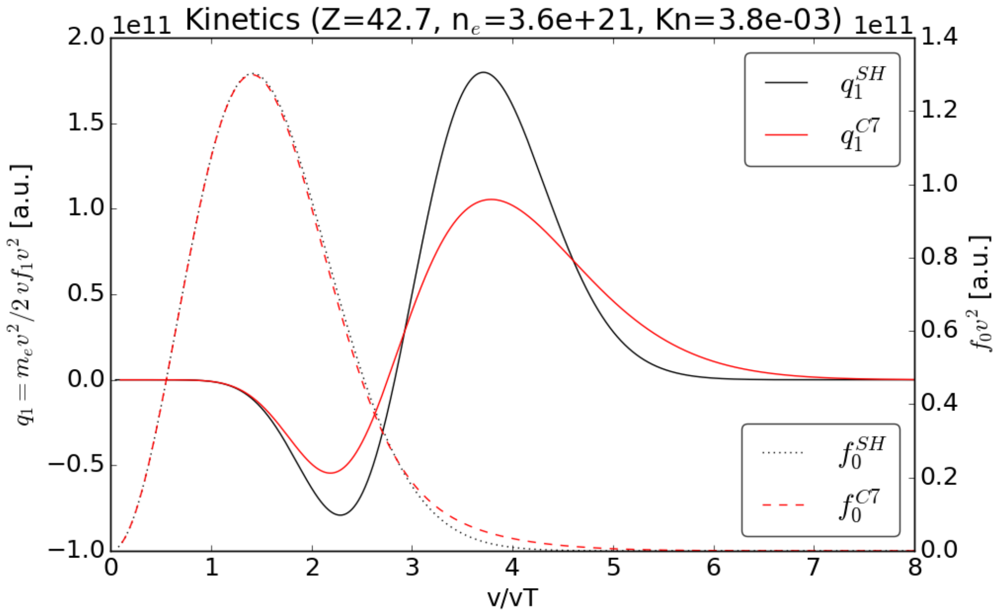


Figure 1: Philippe's preferred test $\bar{Z} = 4$ at 12 ps.



Heat flux (Z = 42.7, λ_{th}=0.1062[μm])
1e14
IMPACT
SNBs
local
q_h^{SH}
q_h^{C7}
T_e
n_e
Z
q_h [W/cm^2], $T_e \in$ (274, 2551) [eV]
$n_e \in$ (5, 80) [10^{20}/cm^3], $Z \in$ (2, 44)
z [μm]
Heat flux (Z = 2.3, λ_{th}=0.0019[μm])
1e15
Local
q_h^{SH}
q_h^{C7}
T_e
n_e
Z
E^{SH}
E^{C7}
q_h [W/cm^2], $T_e \in$ (274, 2551) [eV]
E [a.u.], $n_e \in$ (1, 16) [10^{20}/cm^3], $Z \in$ (2, 44)
z [μm]

Figure 2:



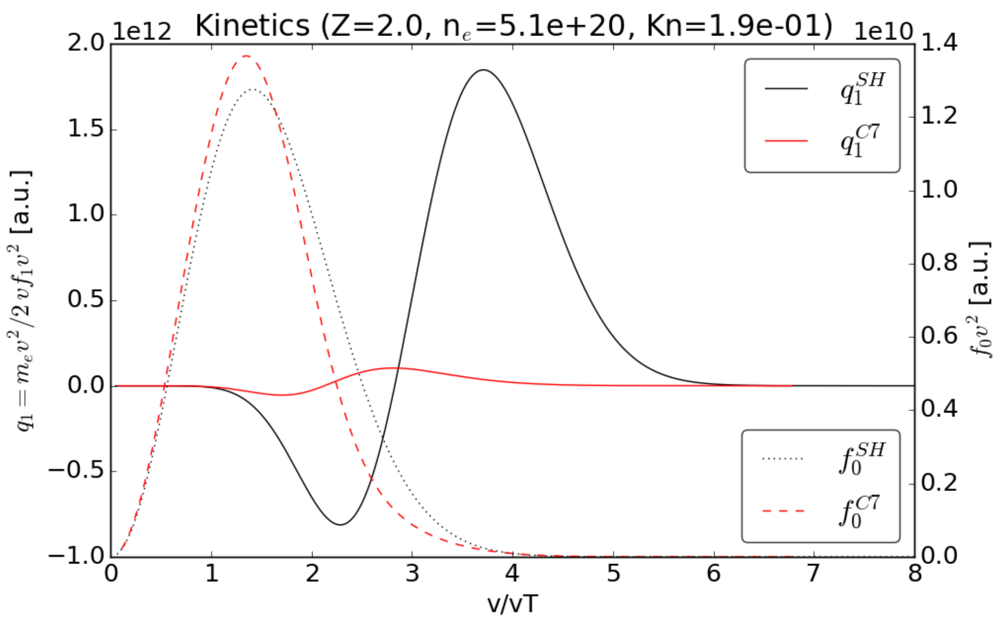


Figure 3: Kinetics profiles for max(flux) point and 1605 microns point for the case of 10ps VFP temperature profile, ne and Z Hydra profiles.

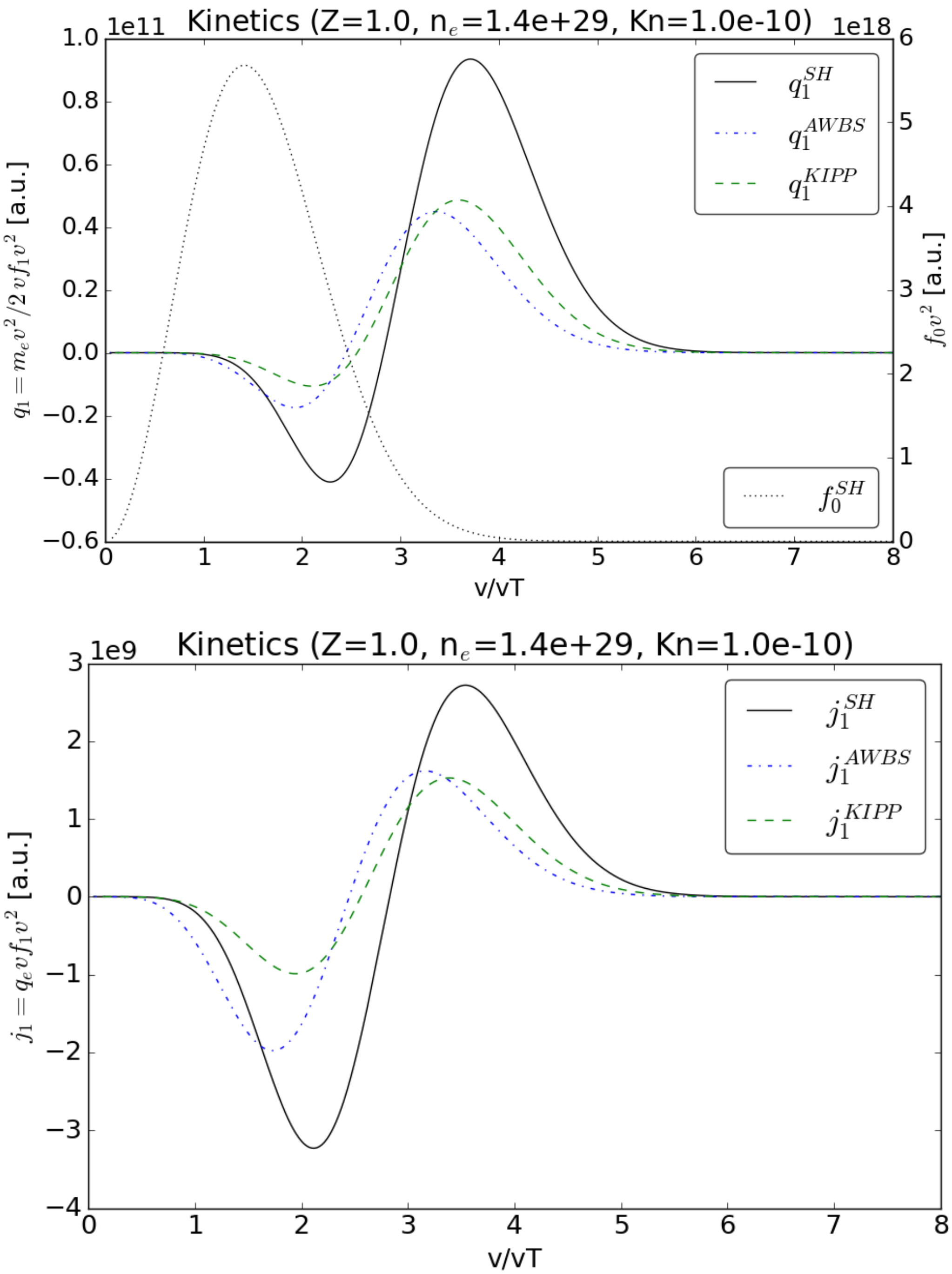


Figure 4: KIPP (by Johnathan) vs AWBS using $\lambda_{ei}^* = \frac{\bar{Z}+0.24}{\bar{Z}+4.2}\lambda_{ei}, \bar{Z} = 1, v_{th} = \sqrt{\frac{k_B T_e}{m_e}}$ $f_1^{SH} = -\lambda_{ei}^*(v)\left(\frac{v^2}{2v_{th}^2} - 4\right)\frac{\boldsymbol{n}\cdot\nabla T_e}{T_e} f_M, \quad f_1^{KIPP} = -\lambda_{ei}^*(v)\left(\frac{3}{16}\frac{v^2}{v_{th}^2} - 1 - \frac{3}{2}\frac{v_{th}^2}{v^2}\right)\frac{\boldsymbol{n}\cdot\nabla T_e}{T_e} f_M.$

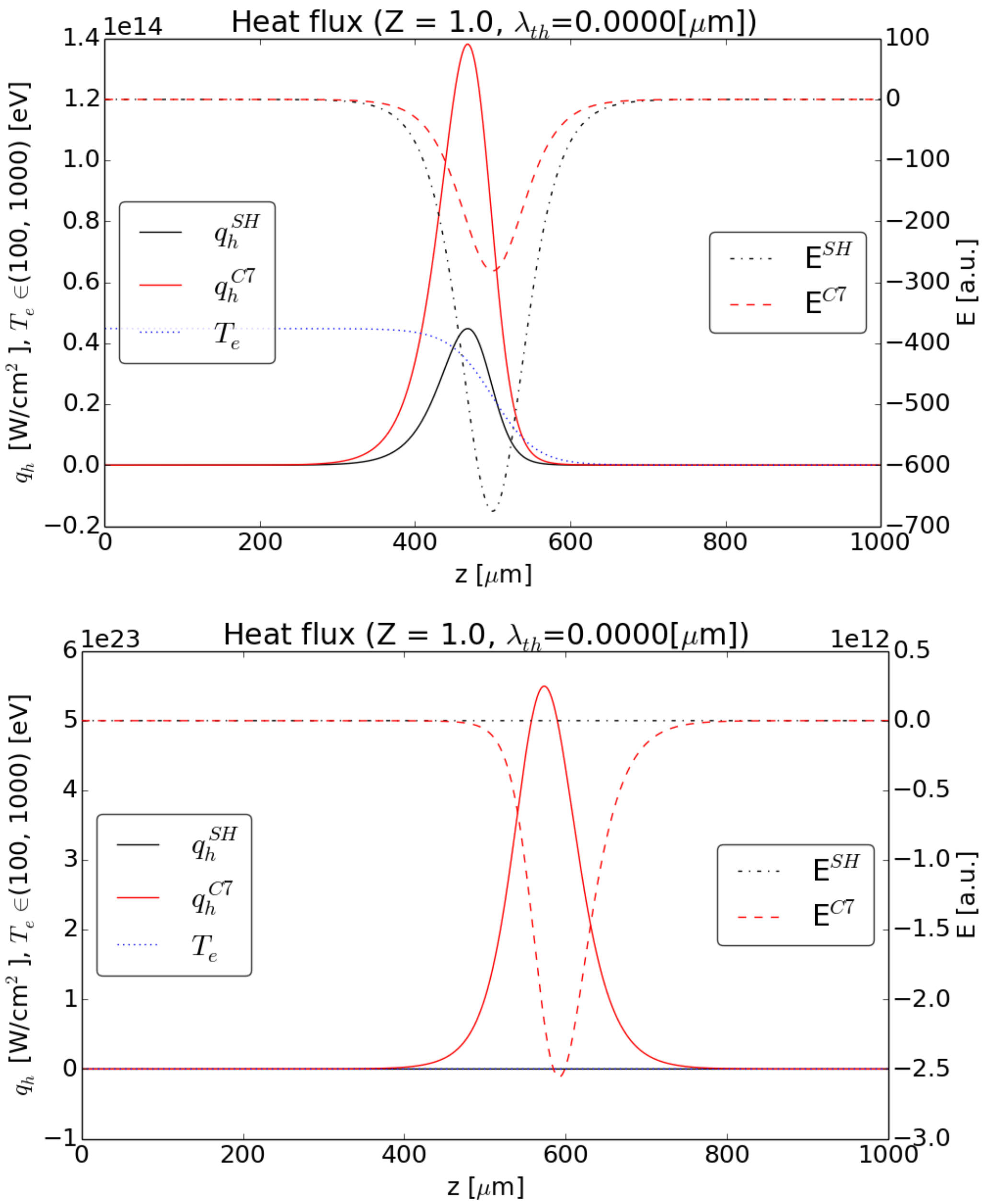


Figure 5: Decelerating (top) vs. accelerating (bottom) computations. Zeroth E field iteration, i.e. no E field effect, of the diffusion regime conditions.

[1] J. R. Albritton, E. A. Williams, I. B. Bernstein, Nonlocal electron heat transport by not quite maxwell-boltzmann distributions, Phys. Rev. Lett. 57 (1986) 1887–1890.