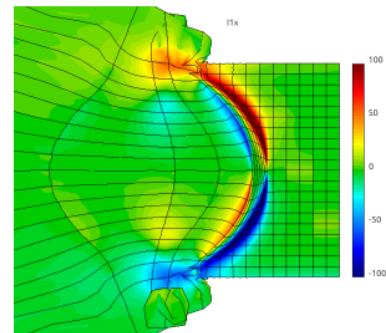
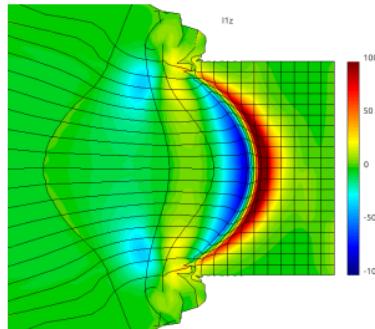
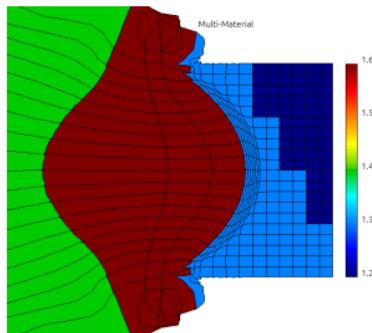


AWBS kinetic modeling of electrons with nonlocal Ohms law in plasmas relevant to inertial confinement fusion

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Hydrodynamic model of plasma

Fokker-Planck/Boltzmann transport equation

$$\frac{\partial \mathbf{f}}{\partial t} + \mathbf{v} \cdot \nabla_x \mathbf{f} + \frac{q_e}{m_e} \left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right) \cdot \nabla_{\mathbf{v}} \mathbf{f} = \sigma \nabla_{\mathbf{v}} \cdot \int \frac{|\mathbf{v} - \mathbf{v}'|^2 \mathbf{l} - (\mathbf{v} - \mathbf{v}') \otimes (\mathbf{v} - \mathbf{v}')}{|\mathbf{v} - \mathbf{v}'|^3} (\nabla_{\mathbf{v}} \mathbf{f}(\mathbf{v}) f(\mathbf{v}') - \mathbf{f}(\mathbf{v}) \nabla_{\mathbf{v}} f(\mathbf{v}')) d\mathbf{v}' \quad (1)$$

Fluid equations in Lagrangian frame (velocity space moments of (1))

$$\begin{aligned}\frac{d\rho}{dt} &= -\rho \nabla \cdot \mathbf{U} \\ \rho \frac{d\mathbf{U}}{dt} &= \nabla \cdot \boldsymbol{\sigma} + \mathbf{j} \times \mathbf{B} \\ \rho \frac{d\varepsilon}{dt} &= \boldsymbol{\sigma} : \nabla \mathbf{U} - \nabla \cdot \mathbf{q}_e\end{aligned}$$

Microscopic closure

$$\begin{aligned}\boldsymbol{\sigma} &= -\rho \int (\mathbf{v} - \mathbf{U}) \otimes (\mathbf{v} - \mathbf{U}) \mathbf{f} d\mathbf{v}^3 \approx -\mathbf{l} p + \tilde{\sigma}(\nabla \mathbf{U}) \\ \mathbf{q}_e &= \frac{m_e}{2} \int |\mathbf{v} - \mathbf{U}|^2 (\mathbf{v} - \mathbf{U}) \mathbf{f} d\mathbf{v}^3 \approx \frac{m_e}{2} \int_{4\pi} \mathbf{n} \int_0^\infty |\mathbf{v}|^5 \mathbf{f} d|\mathbf{v}| d\mathbf{n} \approx -\kappa(T^{2.5}) \nabla T \\ \mathbf{j} &= q_e \int (\mathbf{v} - \mathbf{U}) \mathbf{f} d\mathbf{v}^3\end{aligned}$$

Local diffusive transport regime

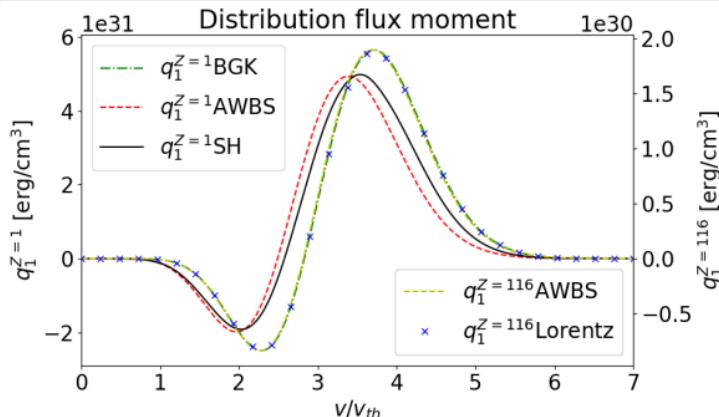
Collision operators

$$C_{FP} = \Gamma_{ee} \int \nabla_{\mathbf{v}} \nabla_{\mathbf{v}} (\mathbf{v} - \tilde{\mathbf{v}}) \cdot (f \nabla_{\tilde{\mathbf{v}}} f - f \nabla_{\mathbf{v}} f) d\tilde{\mathbf{v}} + \frac{\nu_{ei}}{2} \frac{\partial}{\partial \mu} \left((1 - \mu^2) \frac{\partial f}{\partial \mu} \right) \quad (2)$$

$$C_{BGK} = \nu_e (f_M - f) + \frac{Z + 4.2}{Z + 0.24} \frac{\nu_{ei}}{2} \frac{\partial}{\partial \mu} \left((1 - \mu^2) \frac{\partial f}{\partial \mu} \right) \quad (3)$$

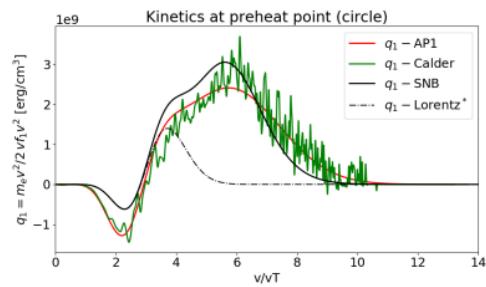
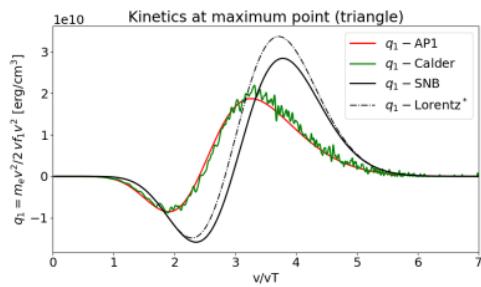
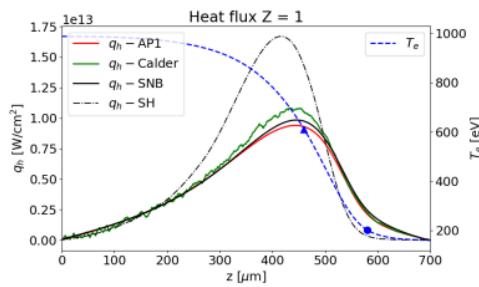
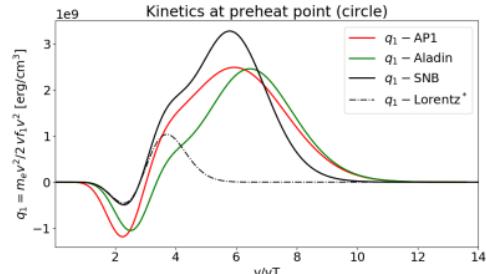
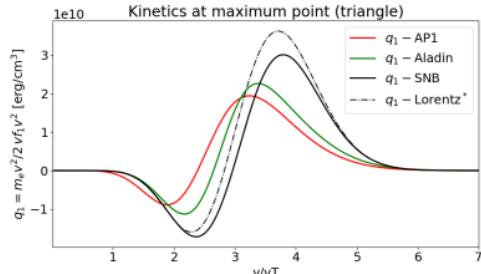
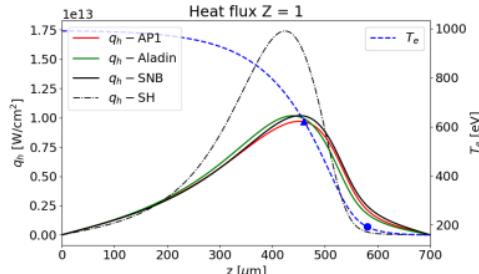
$$C_{FP0} = \nu \nu_e \frac{\partial}{\partial v} \left[C(f_0) f_0 + D(f_0) \frac{\partial f_0}{\partial v} \right] + \frac{Z + 4.2}{Z + 0.24} \frac{\nu_{ei}}{2} \frac{\partial}{\partial \mu} \left((1 - \mu^2) \frac{\partial f}{\partial \mu} \right) \quad (4)$$

$$C_{AWBS} = \nu \frac{\nu_e}{2} \frac{\partial}{\partial v} (f - f_M) + \frac{\nu_{ei} + \frac{\nu_e}{2}}{2} \frac{\partial}{\partial \mu} \left((1 - \mu^2) \frac{\partial f}{\partial \mu} \right) \quad (5)$$



	$Z = 1$	$Z = 2$	$Z = 4$	$Z = 16$	$Z = 116$
Δq_{AWBS}	0.057	0.004	0.037	0.021	0.004

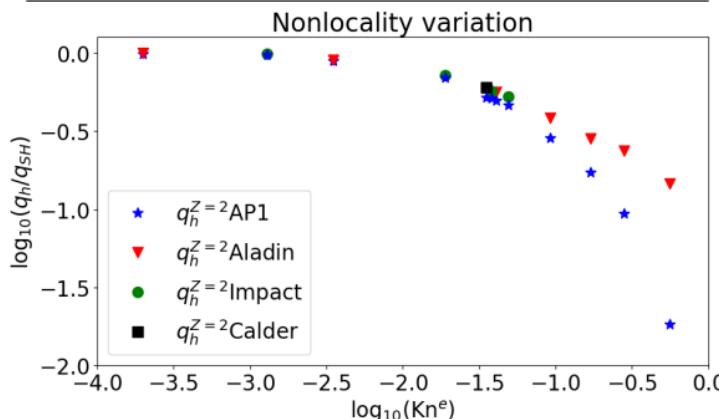
Nonlocal electron transport regime - PIC/VFP0/AWBS



Electron velocity limit - friction vs. \mathbf{E} stopping

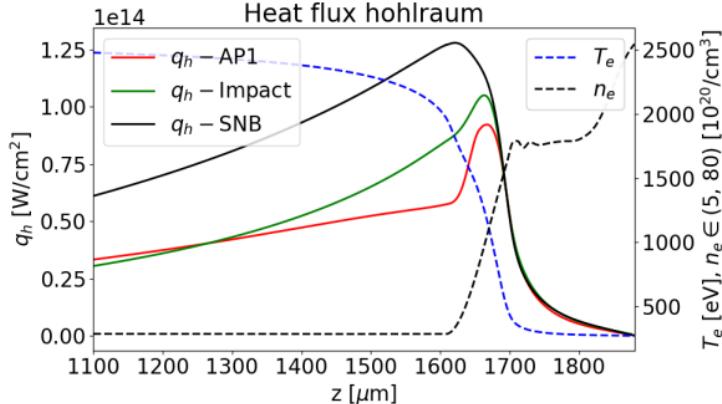
$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{q_e}{m_e} \left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right) \cdot \frac{\partial f}{\partial \Omega} = \left(v \frac{\nu_e}{2} - \frac{q_e}{m_e} \mathbf{v} \cdot \mathbf{E} \right) \frac{\partial f}{\partial v} - v \frac{\nu_e}{2} \frac{\partial f_M}{\partial v} + \frac{\nu_{ei} + \frac{\nu_e}{2}}{2} \frac{\partial^2 f}{\partial \Omega^2},$$

Kn^e	10^{-4}	10^{-3}	10^{-2}	10^{-1}	1
	70.8	22.4	7.3	3.1	1.8



\mathbf{E} stopping overtakes collisions for $\text{Kn} > 0.1$.

NIF hohlraum - plasma profiles provided by HYDRA



SNB approach

$$\tilde{f} = f_M + \delta f_0 + \Omega \cdot (\mathbf{f}_{1M} + \delta \mathbf{f}_1).$$

$$\mathbf{f}_{1M} = -\frac{Z+0.24}{Z+4.2} \lambda_{ei} f_M \left(\frac{\nabla n_e}{n_e} + \left(\frac{v^2}{2v_{th}^2} - \frac{3}{2} \right) \frac{\nabla T_e}{T_e} - \frac{q_e \mathbf{E}_L}{m_e v_{th}^2} \right), \quad \mathbf{E}_L = \frac{m_e v_{th}^2}{q_e} \left(\frac{\nabla n_e}{n_e} + \frac{5}{2} \frac{\nabla T_e}{T_e} \right)$$

Nonlocal Ohm's law

We found that the quality of the electric field evaluation is essential for a correct plasma modeling

Nonlocal current in plasma

$$j(f, \mathbf{E}) = e \int v \mathbf{f}_1 v^2 dv = e \int v \frac{\nu_{ei}^2 \mathbf{E}^* + \omega_B \omega_B \cdot \mathbf{E}^* + \nu_{ei} \omega_B \times \mathbf{E}^*}{\nu_{ei}(\omega_B^2 + \nu_{ei}^2)} v^2 dv, \quad (6)$$

where $\mathbf{E}^* = v \frac{\nu_e}{2} \frac{\partial \mathbf{f}_1}{\partial v} - v \nabla f_0 - \frac{q_e}{m_e} \mathbf{E} \frac{\partial f_0}{\partial v}$ is an effective electric field in plasma. A comparison to the *Generalized Ohm's law* shows a correct local behavior of (6), especially that $\nabla \times \nabla f_0 \sim \nabla \times \frac{\nabla p_e}{n_e} \sim \nabla n_e \times \nabla T_e$

Generalized Ohm's law vs. nonlocal Ohm's law

$$\begin{aligned} \mathbf{E} &= (\mathbf{R}_T - \nabla p_e) + \frac{\mathbf{j}}{\sigma} + \mathbf{j} \times \mathbf{B}, \\ \mathbf{E} v \frac{q_e}{m_e} \frac{\partial f_0}{\partial v} &= v^2 \nabla f_0 + v^2 \frac{\nu_e}{2} \frac{\partial \mathbf{f}_1}{\partial v} - v \nu_{ei} \mathbf{f}_1 + v \mathbf{f}_1 \times \frac{q_e}{m_e c} \mathbf{B}. \end{aligned}$$

Nonlocal-MHD

HYDRODYNAMICS

$$\begin{aligned}\frac{d\rho}{dt} &= -\rho \nabla \cdot \mathbf{u}, \\ \rho \frac{d\mathbf{u}}{dt} &= -\nabla(p_i + p_e) + \mathbf{j}(f, \mathbf{E}) \times \mathbf{B}, \\ \rho \left(\frac{\partial \varepsilon_i}{\partial T_i} \frac{dT_i}{dt} + \frac{\partial \varepsilon_i}{\partial \rho} \frac{d\rho}{dt} \right) &= -p_i \nabla \cdot \mathbf{u} - G(T_i - T_e), \\ \rho \left(\frac{\partial \varepsilon_e}{\partial T_e} \frac{dT_e}{dt} + \frac{\partial \varepsilon_e}{\partial \rho} \frac{d\rho}{dt} \right) &= -p_e \nabla \cdot \mathbf{u} - \nabla \cdot \mathbf{q}_e(f) + G(T_i - T_e) + Q_{IB}(\mathbf{E}_L),\end{aligned}$$

MAXWELL EQUATIONS

$$\begin{aligned}\nabla \times \mathbf{E} &= -\frac{1}{c} \frac{d\mathbf{B}}{dt}, \\ \nabla \times \mathbf{B} &= \frac{4\pi}{c} \mathbf{j}(f, \mathbf{E}),\end{aligned}$$

AWBS KINETICS OF ELECTRONS

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{v}} f = \mathbf{v} \frac{\nu_e}{2} \frac{\partial}{\partial \mathbf{v}} (f - f_{MB}(T_e)) + \frac{\nu_{ei} + \frac{\nu_e}{2}}{2} \frac{\partial^2 f}{\partial \Omega^2}.$$

Nonlocal-MHD \iff aiming HIGH



Height 250 m, overhang 40 m, Visera, Riglos, Spain.

Nonlocal-MHD \iff aiming HIGH



Height 250 m, overhang 40 m, Visera, Riglos, Spain.

Nonlocal-MHD \iff aiming HIGH



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