An efficient kinetic modeling in hydrodynamics using the AWBS transport equation

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Abstract

The AWBS Boltzmann transport equation for electrons equipped with a simplified e-e collision operator [1] provides an efficient, yet physically relevant, kinetic extension compared the classical Spitzer-Harm heat flux (SH) based on local approximation and flux-limiting. This classical approach is widely used in plasma kinetics models coupled to hydrodynamics. Even though SH reflects the electron-electron collision effect, the essential physical properties of the electron transport cannot be modeled with an explicitly local model. A simple form of the AWBS model opens a way to couple kinetics to hydrodynamic codes while describing the important physics correctly. Since the electron-electron collision effect becomes especially important in the case of low ion potential, we take a special in the case of low Z plasmas.

Keywords: kinetics; hydrodynamics; nonlocal electron transport; laser-heated plasmas.

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1. AWBS-P1 modeling of laser heated plasmas

1.1. Model equations

The AWBS electron transport equation reads

$$v\boldsymbol{n}\cdot\nabla f+\tilde{\boldsymbol{E}}\cdot\boldsymbol{n}\frac{\partial f}{\partial v}+\frac{\tilde{\boldsymbol{E}}\cdot\boldsymbol{e}_{\phi}-v\tilde{\boldsymbol{B}}\cdot\boldsymbol{e}_{\theta}}{v}\frac{\partial f}{\partial \phi}=v\nu_e\frac{\partial}{\partial v}\left(f-f_M\right)+(\nu_{ei}+\nu_e)(f_0-f),$$

where ν_e is the electron-electron collision frequency, ν_{ei} is the electron-ion collision frequency, and $\nu_{ei}=\bar{Z}\nu_e$.

In order to eliminate the dimensions of the above transport problem the first-two-moment model based on approximation

$$f=\frac{f_0}{4\pi}+\frac{3}{4\pi}\boldsymbol{n}\cdot\boldsymbol{f_1},$$

can be adopted and reads

$$\begin{aligned}
\nu_e v\frac{\partial}{\partial v}\left(f_0-\tilde{f}_M\right) &= v\nabla\cdot\boldsymbol{f_1}+\tilde{\boldsymbol{E}}\cdot\frac{\partial\boldsymbol{f_1}}{\partial v}+\frac{2}{v}\tilde{\boldsymbol{E}}\cdot\boldsymbol{f_1},\\
\nu_e v\frac{\partial}{\partial v}\boldsymbol{f_1}-\nu_t\boldsymbol{f_1} &= v\nabla\cdot(\mathbf{A}f_0)+\tilde{\boldsymbol{E}}\cdot\frac{\partial(\mathbf{A}f_0)}{\partial v}+\tilde{\boldsymbol{B}}\times\boldsymbol{f_1},
\end{aligned}$$

where $\tilde{f}_M=4\pi f_M$ and the closure matrix takes the form

$$\mathbf{A}=\frac{1}{3}\mathbf{I}.$$

Since in the laser heated plasmas the Knudsen number Kn$=\frac{v_{th}}{\nu_t(v_{th})L}\in(0,1)$, i.e. the collisionality in the kinetics of electrons plays always an important effect for thermal-like particles, the electron distribution function can be treated as out-of-equilibrium approximation

$$f=f_M+\delta f, \tag{1}$$

where the consequent AWBS model reads

$$v\boldsymbol{n}\cdot\nabla(f_M+\delta f)+\tilde{\boldsymbol{E}}\cdot\boldsymbol{n}\frac{\partial f_M}{\partial v}+\tilde{\boldsymbol{E}}\cdot\boldsymbol{n}\frac{\partial\delta f}{\partial v}+\frac{\tilde{\boldsymbol{E}}\cdot\boldsymbol{e}_{\phi}-v\tilde{\boldsymbol{B}}\cdot\boldsymbol{e}_{\theta}}{v}\frac{\partial\delta f}{\partial\phi}= v\nu_e\frac{\partial\delta f}{\partial v}+(\nu_{ei}+\nu_e)(f_0-f_M-\delta f), \tag{2}$$

or its P1 approximation equivalent

$$f = \frac{4\pi f_M + \delta f_0}{4\pi} + \frac{3}{4\pi}\boldsymbol{n}\cdot\boldsymbol{f_1}. \tag{3}$$

where the moment model reads

$$\begin{aligned}
\nu_e v \frac{\partial \delta f_0}{\partial v} &= v\nabla\cdot\boldsymbol{f_1} + \tilde{\boldsymbol{E}}\cdot\frac{\partial \boldsymbol{f_1}}{\partial v} + \frac{2}{v}\tilde{\boldsymbol{E}}\cdot\boldsymbol{f_1}, \qquad (4)\\
\nu_e v \frac{\partial \boldsymbol{f_1}}{\partial v} - \nu_t \boldsymbol{f_1} &= \frac{v}{3}\nabla\delta f_0 + \frac{\tilde{\boldsymbol{E}}}{3}\frac{\partial \delta f_0}{\partial v} + \tilde{\boldsymbol{B}}\times\boldsymbol{f_1}\\
&\quad + \frac{v}{3}\nabla\tilde{f}_M + \frac{\tilde{\boldsymbol{E}}}{3}\frac{\partial \tilde{f}_M}{\partial v}. \qquad (5)
\end{aligned}$$

1.2. A consistent treatment of $\tilde{\boldsymbol{E}}$ field

$$\boldsymbol{q}_c(\boldsymbol{x}) = \int_v v\boldsymbol{f_1}(\boldsymbol{x})v^2\,\mathrm{d}v,$$

from (5)

$$\boldsymbol{q}_c = \int_v \left(\frac{\nu_e v^2}{\nu_t}\frac{\partial \boldsymbol{f_1}}{\partial v} - \frac{v^2}{3\nu_t}\nabla\left(\tilde{f}_M + \delta f_0\right) - \frac{v}{3\nu_t}\frac{\partial\left(\tilde{f}_M + \delta f_0\right)}{\partial v}\tilde{\boldsymbol{E}}\right) v^2\,\mathrm{d}v, \tag{6}$$

$$\begin{aligned}
a_0(\boldsymbol{x}) &= \int_v \frac{v}{3\nu_t}\frac{\partial\left(\tilde{f}_M + \delta f_0\right)}{\partial v}(\boldsymbol{x})v^2\,\mathrm{d}v,\\
\boldsymbol{b}_0(\boldsymbol{x}) &= \int_v \left(\frac{v^2}{3\nu_t}\nabla\left(\tilde{f}_M(\boldsymbol{x}) + \delta f_0(\boldsymbol{x})\right) - \frac{\nu_e v^2}{\nu_t}\frac{\partial \boldsymbol{f_1}}{\partial v}(\boldsymbol{x})\right) v^2\,\mathrm{d}v,
\end{aligned}$$

$$\boldsymbol{q}_c(\boldsymbol{x}) = -\boldsymbol{b}_0(\boldsymbol{x}) - a_0(\boldsymbol{x})\tilde{\boldsymbol{E}}(\boldsymbol{x}),$$

$$\tilde{\boldsymbol{E}}(\boldsymbol{x}) = -\frac{\boldsymbol{b}_0(\boldsymbol{x})}{a_0(\boldsymbol{x})}. \tag{7}$$

1.3. AWBS model analysis

$$\left(v\nu_e - \tilde{\boldsymbol{E}}\cdot\boldsymbol{n}\right)\frac{\partial\delta f}{\partial v} = v\boldsymbol{n}\cdot\nabla(f_M+\delta f)+\tilde{\boldsymbol{E}}\cdot\boldsymbol{n}\frac{\partial f_M}{\partial v}+(\nu_{ei}+\nu_e)(f_M+\delta f-f_0), \tag{8}$$

$$\begin{aligned}\frac{\partial\delta f_0}{\partial v} &= \frac{1}{\nu_e v}\left(v\nabla\cdot\boldsymbol{f_1}+\tilde{\boldsymbol{E}}\cdot\frac{\partial\boldsymbol{f_1}}{\partial v}\right),\\ \nu_e v\frac{\partial\boldsymbol{f_1}}{\partial v}-\nu_t\boldsymbol{f_1} &= \frac{v}{3}\nabla\delta f_0+\frac{\tilde{\boldsymbol{E}}}{3}\frac{\partial\delta f_0}{\partial v}+\frac{v}{3}\nabla\tilde{f}_M+\frac{\tilde{\boldsymbol{E}}}{3}\frac{\partial\tilde{f}_M}{\partial v},\end{aligned}$$

$$\left(\nu_e v\mathbf{I}-\frac{\tilde{\boldsymbol{E}}\tilde{\boldsymbol{E}}}{3\nu_e v}\right)\cdot\frac{\partial\boldsymbol{f_1}}{\partial v} = \frac{v}{3}\nabla\delta f_0+\frac{\tilde{\boldsymbol{E}}}{3\nu_e}\nabla\cdot\boldsymbol{f_1}+\frac{v}{3}\nabla\tilde{f}_M+\frac{\tilde{\boldsymbol{E}}}{3}\frac{\partial\tilde{f}_M}{\partial v}+\nu_t\boldsymbol{f_1},$$

$$\left(\nu_e v-\frac{\tilde{E}_z^2}{3\nu_e v}\right)\frac{\partial f_{1_z}}{\partial v} = \frac{v}{3}\frac{\partial\delta f_0}{\partial z}+\frac{\tilde{E}_z}{3\nu_e}\frac{\partial f_{1_z}}{\partial z}+\frac{v}{3}\frac{\partial\tilde{f}_M}{\partial z}+\frac{\tilde{E}_z}{3}\frac{\partial\tilde{f}_M}{\partial v}+\nu_t f_{1_z}. \tag{9}$$

2. Simulation results

Two cases:

- constant $n_e = 5\times10^{20}$ [1/cm^3], constant $\bar{Z}=4$, T_e temperature profile taken from IMPACT simulation at 12 ps, see Figure 1
- $n_e, T_e, \bar{Z}$ profiles taken from HYDRA simulation of Gadolinium hohlraum at 10 ps, see Figure 2 and Figure 3

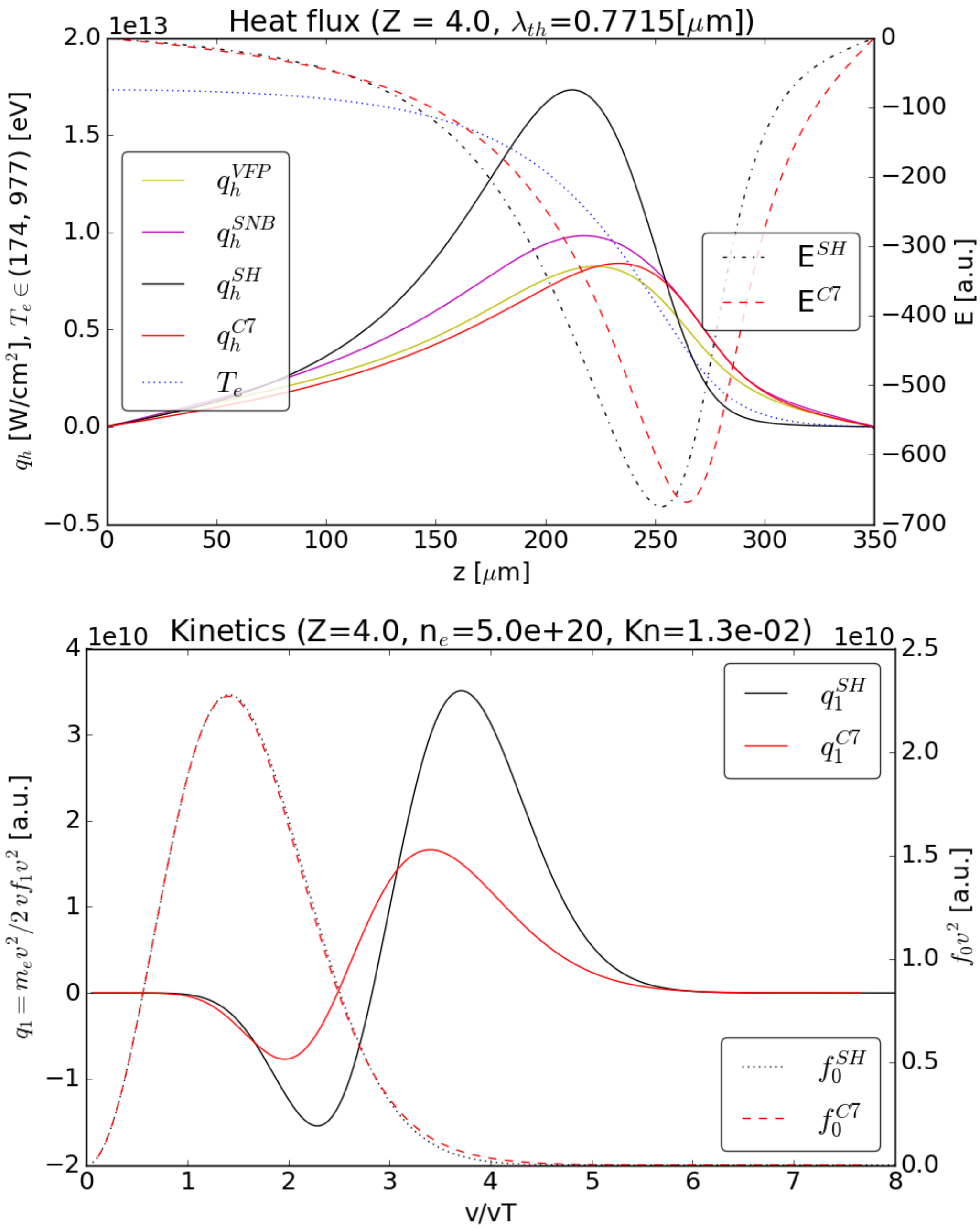
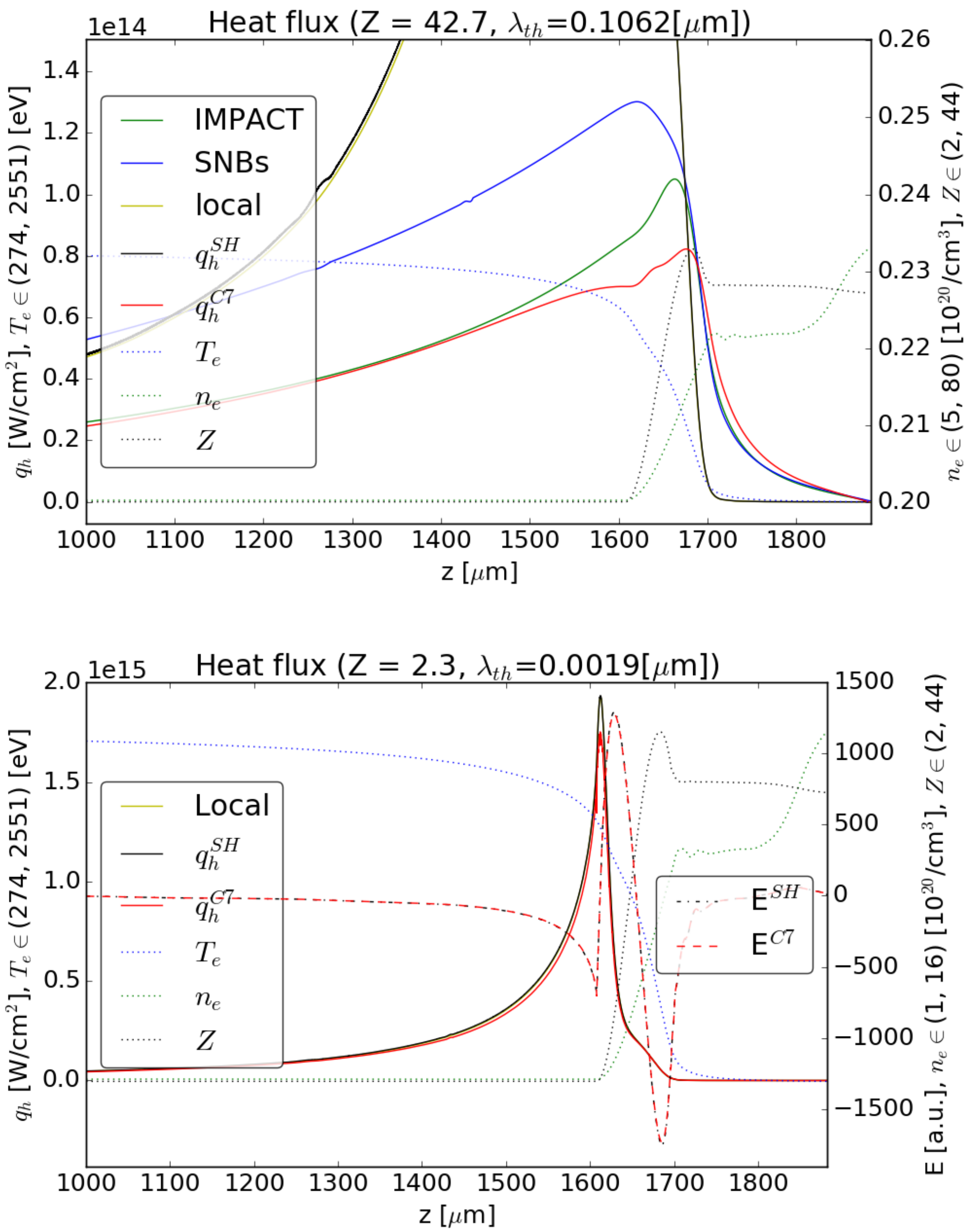


Figure 1: Philippe's preferred test $\bar{Z} = 4$ at 12 ps.



Heat flux (Z = 42.7, λ_{th}=0.1062[μm])
1e14
IMPACT
SNBs
local
q_h^{SH}
q_h^{C7}
T_e
n_e
Z
q_h [W/cm^2], $T_e \in$ (274, 2551) [eV]
$n_e \in$ (5, 80) [10^{20}/cm^3], $Z \in$ (2, 44)
z [μm]
Heat flux (Z = 2.3, λ_{th}=0.0019[μm])
1e15
Local
q_h^{SH}
q_h^{C7}
T_e
n_e
Z
E^{SH}
E^{C7}
q_h [W/cm^2], $T_e \in$ (274, 2551) [eV]
E [a.u.], $n_e \in$ (1, 16) [10^{20}/cm^3], $Z \in$ (2, 44)
z [μm]

Figure 2:

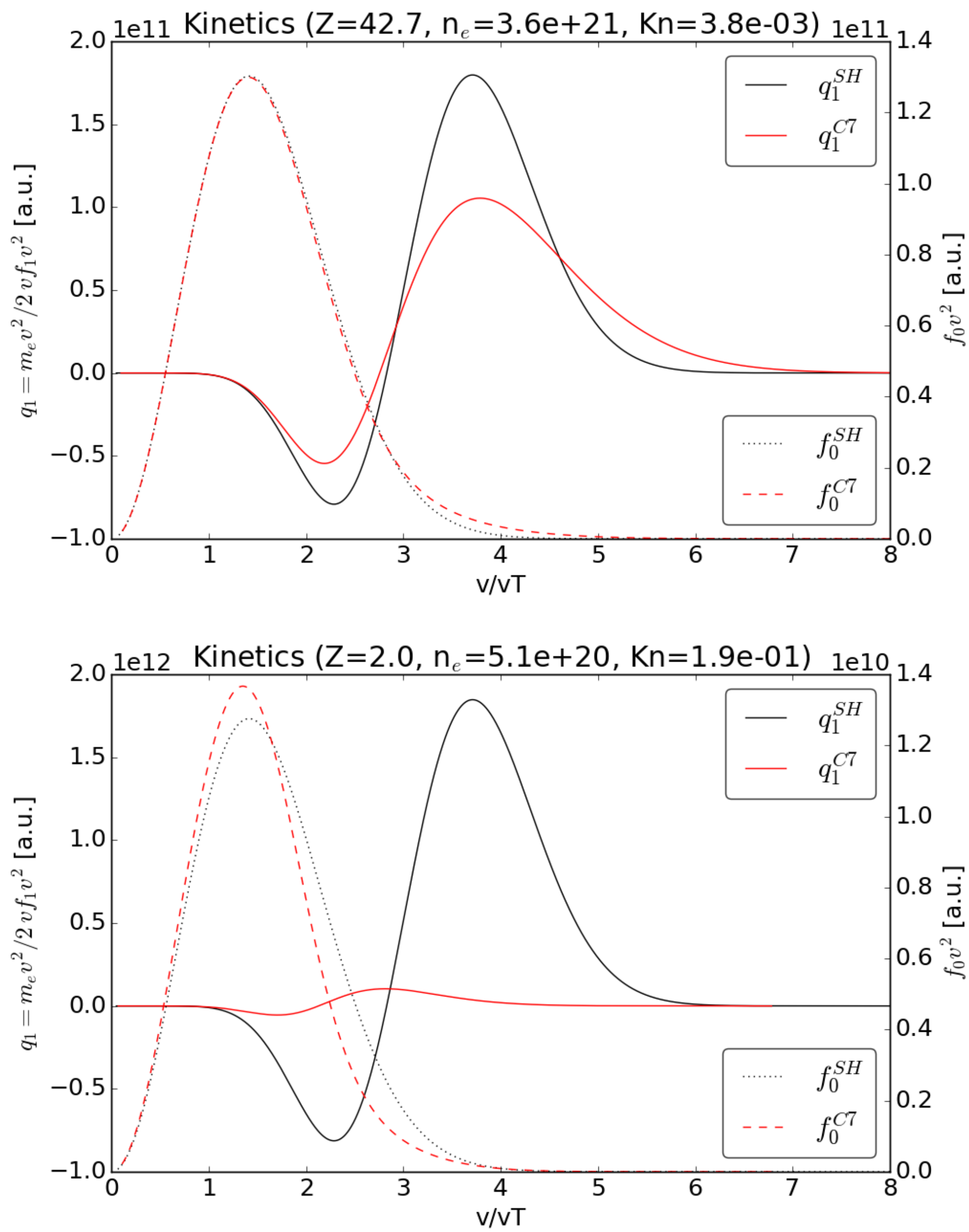


Figure 3: Kinetics profiles for max(flux) point and 1605 microns point for the case of 10ps VFP temperature profile, ne and Z Hydra profiles.

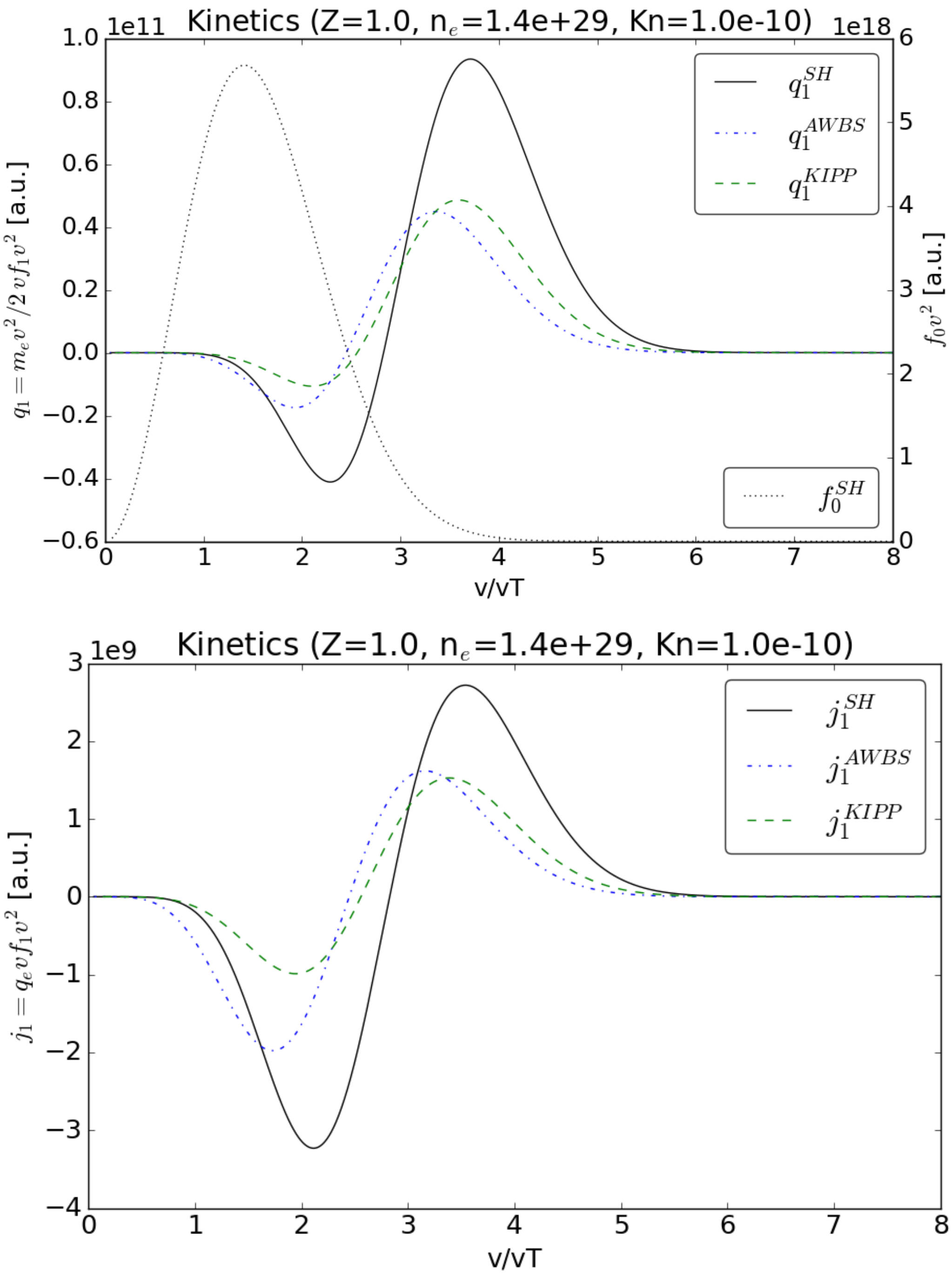
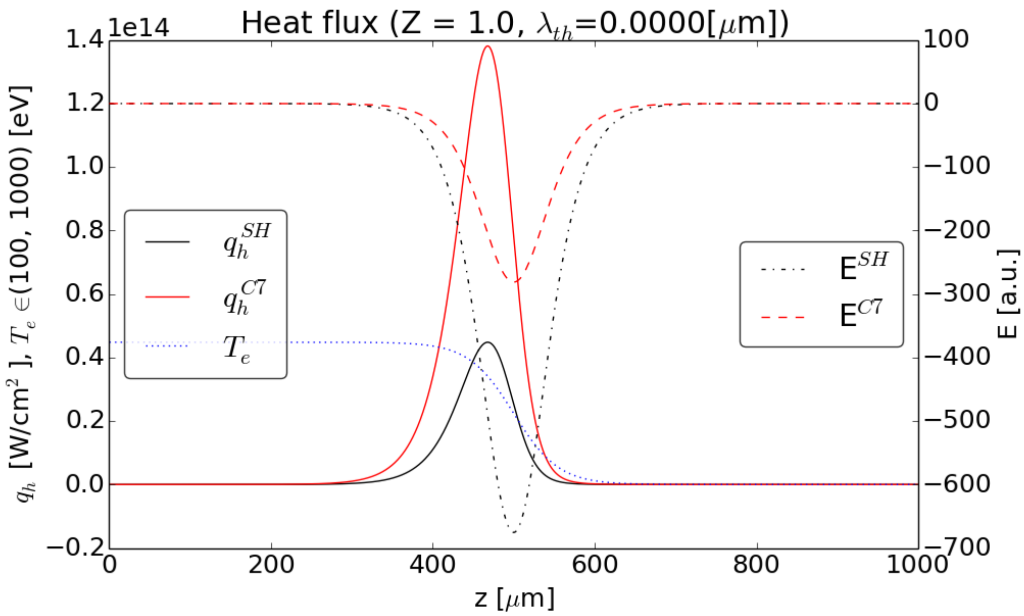


Figure 4: KIPP (by Johnathan) vs AWBS using $\lambda_{ei}^{*} = \frac{\bar{Z}+0.24}{\bar{Z}+4.2}\lambda_{ei}, \bar{Z} = 1, v_{th} = \sqrt{\frac{k_B T_e}{m_e}}$ $f_1^{SH} = -\lambda_{ei}^{*}(v)\left(\frac{v^2}{2v_{th}^2} - 4\right)\frac{\boldsymbol{n}\cdot\nabla T_e}{T_e} f_M, \quad f_1^{KIPP} = -\lambda_{ei}^{*}(v)\left(\frac{3}{16}\frac{v^2}{v_{th}^2} - 1 - \frac{3}{2}\frac{v_{th}^2}{v^2}\right)\frac{\boldsymbol{n}\cdot\nabla T_e}{T_e} f_M.$



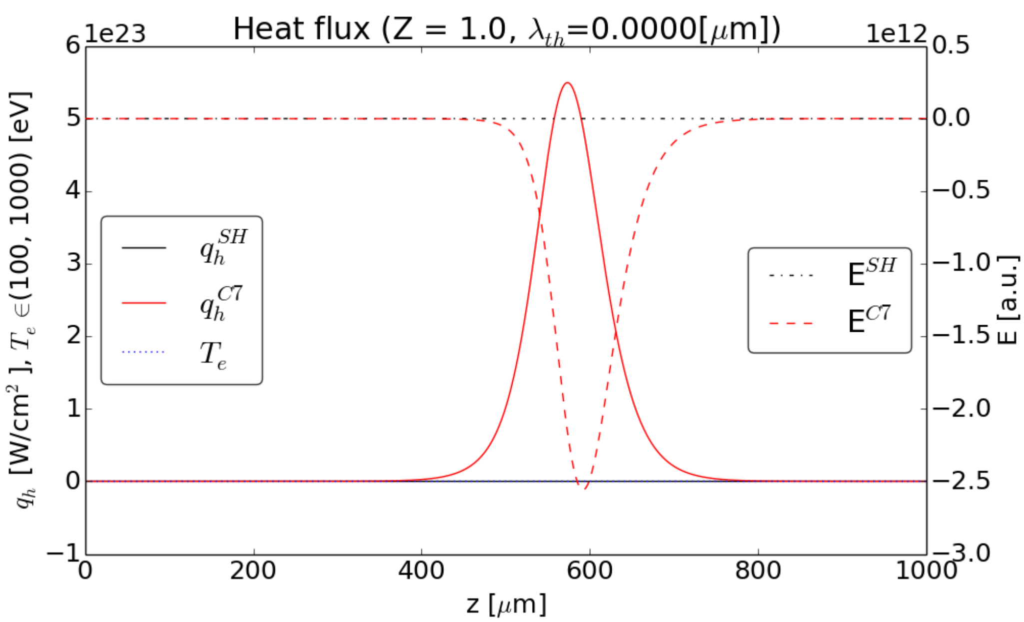


Figure 5: Decelerating (top) vs. accelerating (bottom) computations. Zeroth E field iteration, i.e. no E field effect, of the diffusion regime conditions.

[1] J. R. Albritton, E. A. Williams, I. B. Bernstein, Nonlocal electron heat transport by not quite maxwell-boltzmann distributions, Phys. Rev. Lett. 57 (1986) 1887–1890.