

Multi-dimensional High-Order FEM Methods for Electron Kinetics in ALE Hydrodynamics

SIAM Conference on Computational Science and Engineering (CSE19) – Spokane, Washington, USA



Milan Holec, Ben S. Southworth

February 25 - March 1, 2019



LLNL-PRES-768124

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under contract DE-AC52-07NA27344. Lawrence Livermore National Security, LLC

Lawrence Livermore National Laboratory

- 1 Motivation - Nonlocal Magneto-Hydrodynamic model (Nonlocal-MHD)**
- 2 S_N vs. P_N high-order finite element approach to kinetics on curvilinear meshes**
- 3 What we learned from physics/math/simulation to make the kinetics to be efficient?**
- 4 Conclusions**

Classical MHD

HYDRODYNAMICS *local* $\rightarrow \mathbf{q}_e = -\kappa_{SH} T_e^{2.5} \nabla T_e$

$$\begin{aligned}\frac{d\rho}{dt} &= -\rho \nabla \cdot \mathbf{u}, \\ \rho \frac{d\mathbf{u}}{dt} &= -\nabla p + \frac{c}{4\pi} \nabla \times \mathbf{B} \times \mathbf{B}, \\ \rho \frac{\partial \varepsilon}{\partial T} \frac{dT}{dt} &= -\rho \nabla \cdot \mathbf{u} + \nabla \cdot (\kappa_{SH} T_e^{2.5} \nabla T_e) + \sigma \mathbf{E} \cdot \mathbf{E},\end{aligned}$$

MAXWELL EQUATIONS *resistive* $\rightarrow \mathbf{j} = \sigma(\mathbf{E} + \mathbf{u}_e \times \mathbf{B})$

$$\begin{aligned}\nabla \times \mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \\ \nabla \times \mathbf{B} &= \frac{4\pi}{c} \sigma(\mathbf{E} + \mathbf{u}_e \times \mathbf{B}),\end{aligned}$$

KINETICS OF ELECTRONS *Landau – Fokker – Planck*

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{v}} f = \Gamma_e \int \nabla_{\mathbf{v}} \nabla_{\mathbf{v}} (\mathbf{v} - \tilde{\mathbf{v}}) \cdot (f \nabla_{\tilde{\mathbf{v}}} f - f \nabla_{\mathbf{v}} f) d\tilde{\mathbf{v}} + \frac{\nu_{ei}}{2} \frac{\partial^2 f}{\partial \Omega^2}.$$

Nonlocal-MHD

HYDRODYNAMICS 4D

$$\begin{aligned}\frac{d\rho}{dt} &= -\rho \nabla \cdot \mathbf{u}, \\ \rho \frac{d\mathbf{u}}{dt} &= -\nabla p + \mathbf{j}(f) \times \mathbf{B}, \\ \rho \frac{\partial \varepsilon}{\partial T} \frac{dT}{dt} &= -\rho \nabla \cdot \mathbf{u} - \nabla \cdot \mathbf{q}_e(f) + \mathbf{j}(f) \cdot \mathbf{E},\end{aligned}$$

MAXWELL EQUATIONS 4D

$$\begin{aligned}\nabla \times \mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \\ \nabla \times \mathbf{B} &= \frac{4\pi}{c} \mathbf{j}(f),\end{aligned}$$

KINETICS OF ELECTRONS 6D

$$\mathbf{v} \cdot \nabla f + (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{v}} f = v \tilde{\nu}_e \frac{\partial}{\partial \mathbf{v}} (f - f_{MB}(T_e)) + \nu_{scat} (f_0 - f).$$

M. Holec et al, *Phys. Plas.*, submitted (2019) / arXiv:1901.11378 .

3D velocity space discretizations

AWBS electron kinetic model 7D

$$\begin{aligned} C_V \frac{d T_e}{dt} &= -\nabla \cdot \mathbf{q}_e(f) + \mathbf{j}(f) \cdot \mathbf{E} + S_H, \\ &= \int_{4\pi} \int_v v \tilde{\nu}_e \frac{\partial f}{\partial v} v^4 dv d\mathbf{n} - \sigma T_e^{-0.5} + S_H, \\ \mathbf{v} \cdot \nabla f + (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{v}} f &= v \tilde{\nu}_e \frac{\partial}{\partial v} (f - f_{MB}(T_e)) + \nu_{scat} (f_0 - f). \end{aligned}$$

- C7 (S_N - discontinuous Galerkin FEM) 3D velocity space in spherical coordinates (v, ϕ, θ)

$$\mathbf{n} \cdot \nabla f + \frac{\mathbf{E} \cdot \mathbf{n}}{v} \frac{\partial f}{\partial v} + \frac{E_\phi - v B_\theta}{v^2} \frac{\partial f}{\partial \phi} + \frac{E_\theta + v B_\phi}{v^2 \sin(\phi)} \frac{\partial f}{\partial \theta} = \tilde{\nu}_e \frac{\partial}{\partial v} (f - f_M) + \frac{\nu_{scat}}{v} (f_0 - f).$$

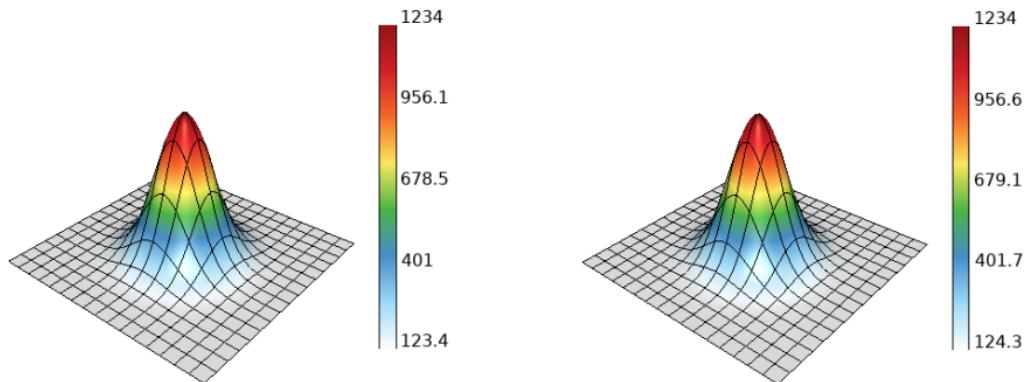
- AP1 (P_N (VEF $\xi = 1/3$) - continuous Galerkin mixed FEM) *Physicists like it!*

$$\begin{aligned} \xi \nabla \cdot \mathbf{f}_1 + \xi \frac{q_e}{m_e v} \mathbf{E} \cdot \left(\frac{\partial \mathbf{f}_1}{\partial v} + \frac{2}{v} \mathbf{f}_1 \right) &= \tilde{\nu}_e \frac{\partial}{\partial v} (f_0 - f_M), \\ \nabla f_0 + \frac{q_e}{m_e v} \mathbf{E} \frac{\partial f_0}{\partial v} + \frac{q_e \mathbf{B}}{m_e c v} \times \mathbf{f}_1 &= \tilde{\nu}_e \frac{\partial \mathbf{f}_1}{\partial v} - \frac{\nu_{scat}}{v} \mathbf{f}_1, \end{aligned}$$

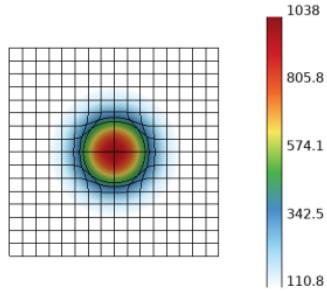
S_N upwind High-Order DG, AMG Approximate-Ideal-Relaxation (AIR) solver

$$\mathbf{M}_{(\nu_e - \frac{\mathbf{E} \cdot \mathbf{n}_d}{v})} \cdot \frac{\Delta \mathbf{f}_d}{\Delta v} - (\mathbf{n}_d \cdot \mathbf{G} + \mathbf{F}_d) \cdot (\tilde{\mathbf{f}}_d + \Delta \mathbf{f}_d) = \mathbf{M}_{(\frac{\nu_{scat}}{v})} \cdot (\tilde{\mathbf{f}}_d + \Delta \mathbf{f}_d) + \mathcal{S}_{(\tilde{\mathbf{f}}, \nu_{scat}, \mathbf{E}, \mathbf{B}, \nu_e \frac{\partial f_M}{\partial v})}$$

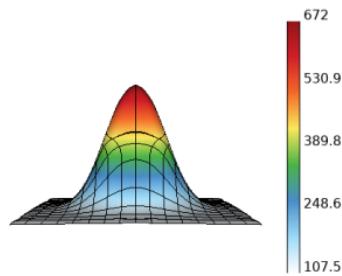
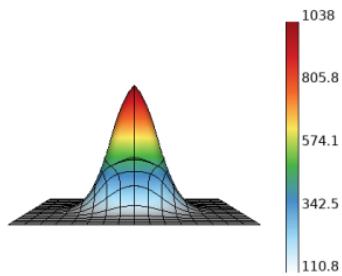
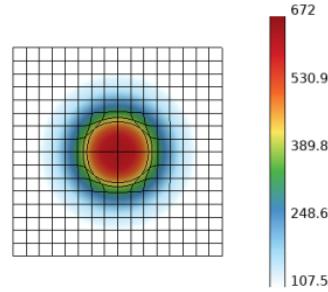
Velocity groups	32	64	128	256	order	
Backward Euler	1.595e-1	8.618e-2	4.456e-2	2.264e-2	0.98	
SDIRK2	3.217e-2	8.888e-3	2.322e-3	5.924e-3	1.97	
SDIRK3	2.455e-2	3.639e-3	4.887e-4	6.317e-05	2.96	
Hydro temperature		Kinetic temperature (50 groups)				



Local hydro temperature

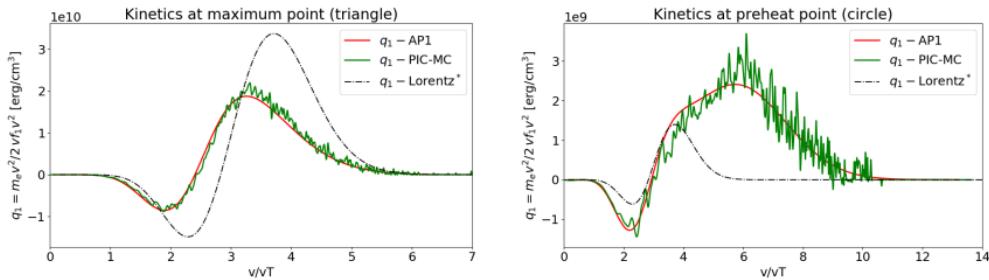
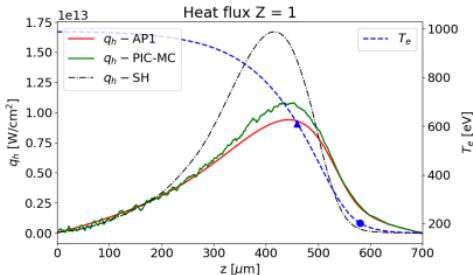


Nonlocal kinetic temperature



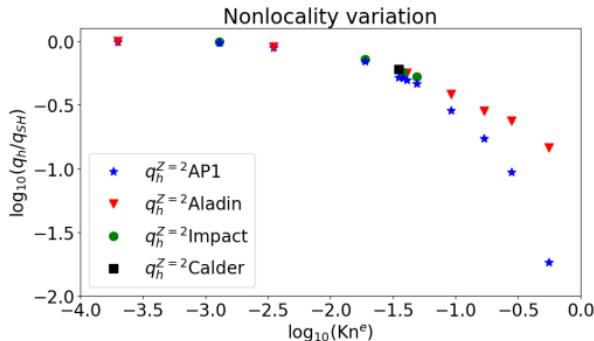
AP1 High-Order Mixed FEM formulation, fixed P1 angular discretization PCG(AMG)

$$\begin{aligned} \mathbf{M}_{(\nu_e)}^{L_2} \cdot \frac{d\mathbf{f}_0}{dv} - \mathbf{V}_{(\frac{\xi q_e E}{m_e v})}^{L_2} \cdot \frac{d\mathbf{f}_1}{dv} &= \mathbf{D}_{(\xi)}^{L_2} \cdot \mathbf{f}_1 + \mathbf{M}_{(\frac{\xi^2 q_e E}{m_e v^2})}^{L_2} \cdot \mathbf{f}_1 + \mathbf{S}_{(\nu_e \frac{\partial f_M}{\partial v})}^{L_2}, \\ \mathbf{M}_{(\nu_e)}^{H_1} \cdot \frac{d\mathbf{f}_1}{dv} - \mathbf{V}_{(\frac{q_e E}{m_e v})}^{H_1} \cdot \frac{d\mathbf{f}_0}{dv} &= \mathbf{G}^{H_1} \cdot \mathbf{f}_0 + \mathbf{M}_{(\frac{\nu_{scat}}{v})}^{H_1} \cdot \mathbf{f}_1 + \mathbf{C}_{(\frac{q_e B}{m_e c v})}^{H_1} \cdot \mathbf{f}_1, \end{aligned}$$



Electron velocity limit - friction vs. E stopping

$$\left(\tilde{\nu}_e - \frac{\mathbf{E} \cdot \mathbf{n}}{v}\right) \frac{\partial f}{\partial v} = \mathbf{n} \cdot \nabla f + \frac{\nu_{scat}}{v} (f - f_0) + \frac{E_\phi - v B_\theta}{v^2} \frac{\partial f}{\partial \phi} + \frac{E_\theta + v B_\phi}{v^2 \sin(\phi)} \frac{\partial f}{\partial \theta} + \tilde{\nu}_e \frac{\partial f_M}{\partial v}.$$



E stopping overtakes collisions for $Kn > 0.1$.

$Kn = \frac{\lambda}{L}$	10^{-4}	10^{-3}	10^{-2}	10^{-1}	1
v_{lim}/v_{th}	70.8	22.4	7.3	3.1	1.8

Adaptive DSA preconditioner for Hydro-Kinetics coupling

AWBS electron kinetic model 7D

$$\begin{aligned} C_V \frac{dT_e}{dt} &= \int_{\mathbf{v}} \sigma K(f) d\mathbf{v} - \sigma T_e^\alpha, \\ \mathbf{v} \cdot \nabla f + (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{v}} f &= v \tilde{\nu}_e \frac{\partial}{\partial \mathbf{v}} (f - f_{MB}(T_e)) + \nu_{scat} (f_0 - f). \end{aligned}$$

Continuum analysis of local ($\text{Kn} \ll 1$) transport regime \rightarrow DIFFUSION

$$C_V \frac{dT_e}{dt} = \nabla \cdot \lambda(T_e^{2.5}) \nabla T_e + O(\text{Kn}^2)$$

Precodnitioned fixed-point iteration $\mathbf{E}(\Delta \mathbf{T}) \equiv (\mathbf{I} - \mathbf{C}) \cdot \mathbf{M}_\sigma(\tilde{\mathbf{T}} + \Delta \mathbf{T})^\alpha - \mathbf{C} \cdot \mathbf{D}_\lambda(\tilde{\mathbf{T}} + \Delta \mathbf{T})^\alpha$

$$\mathbf{M}_{C_V} \cdot \frac{\Delta \mathbf{T}^{k+1}}{\Delta t} + \mathbf{E}(\Delta \mathbf{T}^{k+1}) - \mathbf{E}(\Delta \mathbf{T}^k) = \mathbf{K}_\sigma(f^k) - \mathbf{M}_\sigma(\tilde{\mathbf{T}} + \Delta \mathbf{T}^k)^\alpha.$$

Finally, we get an unconditionally stable backward Euler (SDIRK) fast iterating scheme

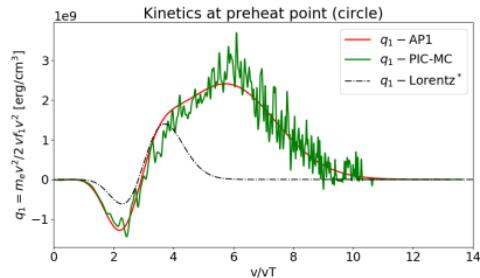
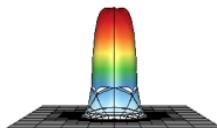
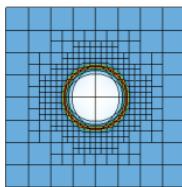
$$\mathbf{M}_{C_V} \cdot \frac{\Delta \mathbf{T}^{k+1}}{\Delta t} + (\mathbf{I} - \mathbf{C}) \cdot \mathbf{M}_\sigma(\tilde{\mathbf{T}} + \Delta \mathbf{T}^{k+1})^\alpha - \mathbf{C} \cdot \mathbf{D}_\lambda(\tilde{\mathbf{T}} + \Delta \mathbf{T}^{k+1})^\alpha = \mathbf{K}(f^k) - \mathbf{C} \cdot \mathbf{M}_\sigma(\tilde{\mathbf{T}} + \Delta \mathbf{T}^k)^\alpha - \mathbf{C} \cdot \mathbf{D}_\lambda(\tilde{\mathbf{T}} + \Delta \mathbf{T}^k)^\alpha,$$

where adaptive coefficient diffusion $\mathbf{C} \xrightarrow{\text{Kn} \ll 1} 1$ and nonlocal transport $\mathbf{C} \xrightarrow{\text{Kn} > 1} 0$ and $\mathbf{C} \in (1, 0)$ in between.

T. Haut et al, SIAM, submitted (2018) / arXiv:1810.11082 .

Conclusions

- 7D microscopic world of electrons in hydro simulations.
- S_N high-order DG finite element approach.
- P_1 high-order mixed finite element approach.
- E field dominated stopping (P_1 fails).
- Adaptive DSA preconditioner for Hydro-Kinetics coupling (ML on **C**).
- Algebraic Multigrid solver pAIR scales $\log(P)^{1.22}$.



MS85 - Developments in Algebraic Multigrid for Nonsymmetric and Hyperbolic Problems - Ben S. Southworth
<https://github.com/CEED/Laghos/tree/master/amr>
<https://mfem.org>
holec1@llnl.gov

Thank you for your attention. Any questions?



CASC
Center for Applied
Scientific Computing

Disclaimer: This document was prepared as an account of work sponsored by an agency of the United States government. Neither the United States government nor Lawrence Livermore National Security, LLC, nor any of their employees make any warranty, expressed or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represent that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States government or Lawrence Livermore National Security, LLC. The views and options of authors expressed herein do not necessarily state or reflect those of the United States government or Lawrence Livermore National Security, LLC, and shall not be used for advertising or product endorsement purposes.