High-order finite element modeling of M1-AWBS nonlocal electron transport

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Abstract

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1. Moments method

2. M1 model

Simplified Boltzmann transport equation of electrons relying in the use of AWBS collision-thermalization operator [1] reads

$$v\boldsymbol{n} \cdot \nabla f + \frac{q_e}{m_e} \left(\boldsymbol{E} + \frac{v}{c} \boldsymbol{n} \times \boldsymbol{B} \right) \cdot \nabla_{\boldsymbol{v}} f = \nu_e v \frac{\partial}{\partial v} \left(f - f_M \right) . \qquad (1)$$

In order to eliminate the dimensions of the transport problem (1) the two moment model referred to as *M1-AWBS*

$$\nu_e v \frac{\partial}{\partial v} \left(f_0 - f_M \right) = v \nabla \cdot \boldsymbol{f}_1 + \frac{q_e}{m_e v^2} \boldsymbol{E} \cdot \frac{\partial}{\partial v} \left(v^2 \boldsymbol{f}_1 \right) , \qquad (2)$$

$$\nu_e v \frac{\partial}{\partial v} \boldsymbol{f}_1 - \nu_t \boldsymbol{f}_1 = v \nabla \cdot \left(\mathbf{A} f_0 \right) + \frac{q_e}{m_e v^2} \boldsymbol{E} \cdot \frac{\partial}{\partial v} \left(v^2 \mathbf{A} f_0 \right)$$
$$+ \frac{q_e}{m_e v} \boldsymbol{E} \cdot \left(\mathbf{A} - \mathbf{I} \right) f_0 + \frac{q_e}{m_e c} \boldsymbol{B} \times \boldsymbol{f}_1 , \qquad (3)$$

where the anisotropy-closure matrix takes the form

$$\mathbf{A} = \frac{1}{3} \mathbf{I} + \frac{|\boldsymbol{f}_1|^2}{2 f_0^2} \left(1 + \frac{|\boldsymbol{f}_1|^2}{f_0^2} \right) \left(\frac{\boldsymbol{f}_1 \otimes \boldsymbol{f}_1}{|\boldsymbol{f}_1|^2} - \frac{1}{3} \mathbf{I} \right) . \qquad (4)$$

2.1. High-order finite element scheme

$$\rho \frac{\partial f_0}{\partial v} = \frac{\rho}{\nu_e} \mathbf{I} : \nabla \boldsymbol{f}_1 + \frac{q_e \rho}{m_e \nu_e v} \boldsymbol{E} \cdot \frac{\partial \boldsymbol{f}_1}{\partial v} + \frac{2 q_e \rho}{m_e \nu_e v^2} \boldsymbol{E} \cdot \boldsymbol{f}_1 + \rho \frac{\partial f_M}{\partial v} , \qquad (5)$$

$$\rho \frac{\partial \boldsymbol{f}_1}{\partial v} = \nabla \cdot \left(\frac{\rho \mathbf{A}}{\nu_e} f_0 \right) + \left(\frac{q_e \rho}{m_e \nu_e v^2} \boldsymbol{E} \cdot \left(3 \mathbf{A} - \mathbf{I} \right) - \nabla \frac{\rho}{\nu_e} \cdot \mathbf{A} \right) f_0$$
$$+ \frac{q_e \rho}{m_e \nu_e v} \boldsymbol{E} \cdot \frac{\partial}{\partial v} \left(\mathbf{A} f_0 \right) + \frac{q_e \rho}{m_e c \nu_e v} \boldsymbol{B} \times \boldsymbol{f}_1 + \frac{\rho \nu_t}{\nu_e v} \boldsymbol{f}_1 , \qquad (6)$$

$$\int_\Omega \boldsymbol{\phi} \otimes \boldsymbol{\phi}^T \rho \, \mathrm{d}\Omega \cdot \frac{\partial \boldsymbol{f_0}}{\partial v} = \int_\Omega \boldsymbol{\phi} \otimes \left(\frac{\rho}{\nu_e} \mathbf{I} : \nabla \boldsymbol{w}^T + \frac{2 q_e \rho}{m_e \nu_e v^2} \boldsymbol{E} \cdot \boldsymbol{w}^T \right) \mathrm{d}\Omega \cdot \boldsymbol{f}_1$$
$$+ \int_\Omega \boldsymbol{\phi} \otimes \frac{q_e \rho}{m_e \nu_e v} \boldsymbol{E} \cdot \boldsymbol{w}^T \, \mathrm{d}\Omega \cdot \frac{\partial \boldsymbol{f}_1}{\partial v} + \int_\Omega \boldsymbol{\phi} \otimes \boldsymbol{\phi}^T \rho \, \mathrm{d}\Omega \cdot \frac{\partial \boldsymbol{f_M}}{\partial v} , \qquad (7)$$

$$\int_\Omega \boldsymbol{w} \otimes \boldsymbol{w}^T \rho \, \mathrm{d}\Omega \cdot \frac{\partial \boldsymbol{f_1}}{\partial v} = - \int_\Omega \frac{\rho}{\nu_e} \mathbf{A} : \nabla \boldsymbol{w} \otimes \boldsymbol{\phi}^T \, \mathrm{d}\Omega \cdot \boldsymbol{f_0}$$

$$+ \int_\Omega \boldsymbol{w} \otimes \left(\frac{q_e \rho}{m_e \nu_e v^2} \boldsymbol{E} \cdot (3\mathbf{A} - \mathbf{I}) - \nabla \left(\frac{\rho}{\nu_e} \right) \cdot \mathbf{A} \right) \boldsymbol{\phi}^T \, \mathrm{d}\Omega \cdot \boldsymbol{f_0}$$

$$+ \int_\Omega \boldsymbol{w} \otimes \frac{q_e \rho}{m_e \nu_e v} \boldsymbol{E} \cdot \mathbf{A} \, \boldsymbol{\phi}^T \, \mathrm{d}\Omega \cdot \frac{\partial \boldsymbol{f_0}}{\partial v}$$

$$+ \int_\Omega \boldsymbol{w} \otimes \left(\frac{q_e \rho}{m_e c \nu_e v} \boldsymbol{B} \times \boldsymbol{w}^T + \frac{\rho \nu_t}{\nu_e v} \boldsymbol{w}^T \right) \, \mathrm{d}\Omega \cdot \boldsymbol{f_1} \,, \quad (8)$$

[2]

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