High-order finite element modeling of M1-AWBS nonlocal electron transport

Milan, Vladimir, Philippe[a]

[a] *Université de Bordeaux - CNRS - CEA, CELIA, UMR 5107, F-33405 Talence, France*

Abstract

Keywords: Nonlocal transport, Kinetics, MHD, High-order methods, FEM

Contents

1. Moments method

2. M1 model

Simplified Boltzmann transport equation of electrons relying in the use of AWBS collision-thermalization operator [1] reads

$$v\boldsymbol{n}\cdot\nabla f + \frac{q_e}{m_e}\left(\boldsymbol{E} + \frac{v}{c}\boldsymbol{n}\times\boldsymbol{B}\right)\cdot\nabla_{\boldsymbol{v}} f = \nu_e v \frac{\partial}{\partial v}\left(f - f_M\right)\,. \tag{1}$$

In order to eliminate the dimensions of the transport problem (1) the two moment model referred to as *M1-AWBS*

$$\begin{aligned}
\nu_e v \frac{\partial}{\partial v}\left(f_0 - f_M\right) &= v\nabla\cdot\boldsymbol{f}_1 + \frac{q_e}{m_e v^2}\boldsymbol{E}\cdot\frac{\partial}{\partial v}\left(v^2\boldsymbol{f}_1\right)\,, && (2)\\
\nu_e v \frac{\partial}{\partial v}\boldsymbol{f}_1 - \nu_t\boldsymbol{f}_1 &= v\nabla\cdot\left(\mathbf{A}f_0\right) + \frac{q_e}{m_e v^2}\boldsymbol{E}\cdot\frac{\partial}{\partial v}\left(v^2\mathbf{A}f_0\right) &&\\
&\quad + \frac{q_e}{m_e v}\boldsymbol{E}\cdot\left(\mathbf{A}-\mathbf{I}\right)f_0 + \frac{q_e}{m_e c}\boldsymbol{B}\times\boldsymbol{f}_1\,, && (3)
\end{aligned}$$

where the anisotropy-closure matrix takes the form

$$\mathbf{A} = \frac{1}{3}\mathbf{I} + \frac{|\boldsymbol{f}_1|^2}{2f_0^2}\left(1 + \frac{|\boldsymbol{f}_1|^2}{f_0^2}\right)\left(\frac{\boldsymbol{f}_1\otimes\boldsymbol{f}_1}{|\boldsymbol{f}_1|^2} - \frac{1}{3}\mathbf{I}\right)\,. \tag{4}$$

2.1. High-order finite element scheme

$$\begin{aligned}
\rho\frac{\partial f_0}{\partial v} &= \frac{\rho}{\nu_e}\mathbf{I}:\nabla\boldsymbol{f}_1 + \frac{q_e\rho}{m_e\nu_e v}\boldsymbol{E}\cdot\frac{\partial\boldsymbol{f}_1}{\partial v} + \frac{2q_e\rho}{m_e\nu_e v^2}\boldsymbol{E}\cdot\boldsymbol{f}_1 + \rho\frac{\partial f_M}{\partial v}\,, && (5)\\
\rho\frac{\partial\boldsymbol{f}_1}{\partial v} &= \nabla\cdot\left(\frac{\rho\mathbf{A}}{\nu_e}f_0\right) + \left(\frac{q_e\rho}{m_e\nu_e v^2}\boldsymbol{E}\cdot\left(3\mathbf{A}-\mathbf{I}\right) - \nabla\left(\frac{\rho}{\nu_e}\right)\cdot\mathbf{A}\right)f_0 &&\\
&\quad + \frac{q_e\rho}{m_e\nu_e v}\boldsymbol{E}\cdot\frac{\partial}{\partial v}\left(\mathbf{A}f_0\right) + \frac{q_e\rho}{m_e c\nu_e v}\boldsymbol{B}\times\boldsymbol{f}_1 + \frac{\rho\nu_t}{\nu_e v}\boldsymbol{f}_1\,, && (6)
\end{aligned}$$

$$\begin{aligned}
\int_\Omega \boldsymbol{\phi}\otimes\boldsymbol{\phi}^T\rho\,\mathrm{d}\Omega\cdot\frac{\partial\boldsymbol{f_0}}{\partial v} &= \int_\Omega\boldsymbol{\phi}\otimes\left(\frac{\rho}{\nu_e}\mathbf{I}:\nabla\boldsymbol{w}^T + \frac{2q_e\rho}{m_e\nu_e v^2}\boldsymbol{E}\cdot\boldsymbol{w}^T\right)\mathrm{d}\Omega\cdot\boldsymbol{f}_1\\
&+ \int_\Omega\boldsymbol{\phi}\otimes\frac{q_e\rho}{m_e\nu_e v}\boldsymbol{E}\cdot\boldsymbol{w}^T\,\mathrm{d}\Omega\cdot\frac{\partial\boldsymbol{f}_1}{\partial v} + \int_\Omega\boldsymbol{\phi}\otimes\boldsymbol{\phi}^T\rho\,\mathrm{d}\Omega\cdot\frac{\partial\boldsymbol{f_M}}{\partial v}\,, \quad (7)
\end{aligned}$$

$$
\begin{aligned}
\int_\Omega \boldsymbol{w} \otimes \boldsymbol{w}^T \rho \,\mathrm{d}\Omega \cdot \frac{\partial \boldsymbol{f}_1}{\partial v} = &- \int_\Omega \frac{\rho}{\nu_e} \mathbf{A} : \nabla \boldsymbol{w} \otimes \boldsymbol{\phi}^T \,\mathrm{d}\Omega \cdot \boldsymbol{f_0} \\
&+ \int_\Omega \boldsymbol{w} \otimes \left(\frac{q_e \rho}{m_e \nu_e v^2} \boldsymbol{E} \cdot (3\mathbf{A} - \mathbf{I}) - \nabla \left(\frac{\rho}{\nu_e} \right) \cdot \mathbf{A} \right) \boldsymbol{\phi}^T \,\mathrm{d}\Omega \cdot \boldsymbol{f_0} \\
&+ \int_\Omega \boldsymbol{w} \otimes \frac{q_e \rho}{m_e \nu_e v} \boldsymbol{E} \cdot \mathbf{A}\, \boldsymbol{\phi}^T \,\mathrm{d}\Omega \cdot \frac{\partial \boldsymbol{f_0}}{\partial v} \\
&+ \int_\Omega \boldsymbol{w} \otimes \left(\frac{q_e \rho}{m_e c \nu_e v} \boldsymbol{B} \times \boldsymbol{w}^T + \frac{\rho \nu_t}{\nu_e v} \boldsymbol{w}^T \right) \mathrm{d}\Omega \cdot \boldsymbol{f}_1 \,, \qquad (8)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{M}_0 \cdot \frac{\partial \boldsymbol{f_0}}{\partial v} &= \left(\mathbf{D}_0^T + \frac{2 q_e}{m_e v^2} \mathbf{E}_0 \right) \cdot \boldsymbol{f}_1 + \frac{q_e}{m_e v} \mathbf{E}_0 \cdot \frac{\partial \boldsymbol{f}_1}{\partial v} + \mathbf{M}_0 \cdot \frac{\partial \boldsymbol{f_M}}{\partial v} \,, \quad (9) \\
\mathbf{M}_1 \cdot \frac{\partial \boldsymbol{f}_1}{\partial v} &= -\mathbf{D}_1 \cdot \boldsymbol{f_0} + \mathbf{EI}_1 \cdot \boldsymbol{f_0} + \mathbf{E}_1 \cdot \frac{\partial \boldsymbol{f_0}}{\partial v} + \mathbf{B}_1 \cdot \boldsymbol{f}_1 + \mathbf{N} \cdot \boldsymbol{f}_1 \,, \quad (10)
\end{aligned}
$$

[2]

References

[1] J. R. Albritton, E. A. Williams, I. B. Bernstein, Nonlocal electron heat transport by not quite maxwell-boltzmann distributions, Phys. Rev. Lett. 57 (1986) 1887–1890.

[2] V. Dobrev, T. Kolev, R. Rieben, High-order curvilinear finite element methods for Lagrangian hydrodynamics, SIAM J. Sci. Comput. 34 (2012) B606–B641.