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Diffusion Synthetic Acceleration for Heterogeneous Domains, Compatible with Voids

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Abstract — *A standard approach to solving the S_N transport equations is to use source iteration with diffusion synthetic acceleration (DSA). Although this approach is widely used and effective on many problems, there remain some practical issues with DSA preconditioning, particularly on highly heterogeneous domains. For large-scale parallel simulation, it is critical that both (a) preconditioned source iteration converges rapidly and (b) the action of the DSA preconditioner can be applied using fast, scalable solvers, such as algebraic multigrid (AMG). For heterogeneous domains, these two interests can be at odds. In particular, there exist DSA diffusion discretizations that can be solved rapidly using AMG, but they do not always yield robust/fast convergence of the larger source iteration. Conversely, there exist robust DSA discretizations where source iteration converges rapidly on difficult heterogeneous problems, but fast parallel solvers like AMG tend to struggle applying the action of such operators. Moreover, very few current methods for the solution of deterministic transport are compatible with voids. This paper develops a new heterogeneous DSA preconditioner based on only preconditioning the optically thick subdomains. The resulting method proves robust on a variety of heterogeneous transport problems, including a linearized hohlraum mesh related to inertial confinement fusion. Moreover, the action of the preconditioner is easily computed using $\mathcal{O}(1)$ AMG iterations, convergence of the transport iteration typically requires 2 to 5× fewer iterations than current state-of-the-art "full" DSA, and the proposed method is trivially compatible with voids. On the hohlraum problem, rapid convergence is obtained by preconditioning less than 3% of the mesh elements with five to ten AMG iterations.*

Keywords — *Transport, discrete ordinates, high order, sweep, unstructured.*

Note — *Some figures may be in color only in the electronic version.*

I. INTRODUCTION

Solving particle transport equations arises in numerous fields of research such as nuclear reactor design, inertial confinement fusion (ICF), and medical imaging. Despite decades of research, however, this remains a computationally expensive and challenging problem. The simplified steady-state, monoenergetic neutral particle transport equation for the spatially and angularly dependent angular flux ψ is given by

$$\begin{aligned}\boldsymbol{\Omega}\cdot\ \ &\psi(\mathbf{x},\boldsymbol{\Omega})+\sigma_t(\mathbf{x})\psi(\mathbf{x},\boldsymbol{\Omega})\\ &=\frac{\sigma_s(\mathbf{x})}{4\pi}\int_{\mathbb{S}^2}\psi(\mathbf{x},\boldsymbol{\Omega}')d\boldsymbol{\Omega}'+q(\mathbf{x},\boldsymbol{\Omega})\ . \qquad (1)\end{aligned}$$

Here, $0\le\sigma_s\le\sigma_t$ are the spatially dependent scattering and total cross sections, respectively, and we assume isotropic scattering for simplicity. This equation is fundamental to

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