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A quadratic programming flux correction method for high-order DG discretizations of S_N transport

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ABSTRACT

We present a new flux-fixup approach for arbitrarily high-order discontinuous Galerkin (DG) discretizations of the S_N transport equation, and we demonstrate the compatibility of this approach with the Variable Eddington Factor (VEF) method [1,2]. The new fixup approach is sweep-compatible: during a transport sweep (block Gauss-Seidel iteration in which the scattering source is lagged), a local quadratic programming (QP) problem is solved in each spatial element to ensure that the solution satisfies certain physical constraints, including local particle balance. In this paper, we describe two choices of physical constraints, resulting in two variants of the method: QP Zero (QPZ) and QP Maximum Principle (QPMP). In QPZ, the finite element coefficients of the solution are constrained to be nonnegative. In QPMP, they are constrained to adhere to an approximate discrete maximum principle.

There are two primary takeaways in this paper. First, when the positive Bernstein basis is used for DG discretization, the QPMP method eliminates negativities, preserves high-order accuracy for smooth problems, and significantly dampens unphysical oscillations in the solution. The latter feature – the dampening of unphysical oscillations – is an improvement upon standard, simpler fixup approaches such as the approach described in [3] (denoted as the "zero and rescale" (ZR) method in this paper). This improvement comes at a moderate computational cost, but it is not prohibitive. Our results show that, even in an unrealistic worst-case scenario where 83% of the spatial elements require a fixup, the computational cost of performing a transport sweep with fixup is only ∼31% greater than performing one without fixup.

The second takeaway is that the VEF method can be used to accelerate the convergence of transport sweeps even when a fixup is applied. When optically thick regions are present, transport sweeps converge slowly, regardless of whether a fixup is applied, and acceleration is needed. However, attempting to apply standard diffusion synthetic acceleration (DSA) to fixed-up transport sweeps results in divergence for optically thick problems. Our results show that the same is not true for VEF. When VEF is combined with fixed-up transport sweeps, the result is a scheme that produces a nonnegative solution, converges

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