# DESIGN AND ANALYSIS OF ALGORITHMS

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# HOMEWORK 1

## Question 1

**Thread**:

For each of the following six functions, state its rate of growth using Θ notation; if possible, use one of the Basic Asymptotic Efficiency Classes from Levitin Table 2.2. Explain your reasoning in one line. Then sort the functions from lowest to highest order of growth.

1. 34n
2. n!
3. 2n+1
4. √(144 n)
5. (n – 4)!
6. 2log2n5
7. n4/200 + 100n3 + 500000 – n

**Answer:**

* ***State rate of growth using Θ notation:***

1. 34n ∈ Θ(n)

c1n0 ≤ 34n ≤ c2n0 with c1 = 33, c2 = 35, n0 = n ⬄ 33n ≤ 34n ≤ 35n for all n0 ≥ 0.

1. n! ∈ Θ(n)

c1n0! ≤ n! ≤ c2n0! with c1 = 1, c2 = 2, n0 = n ⬄ 1n! ≤ n! ≤ 2n! for all n0 ≥ 0.

1. 2n + 1 ∈ Θ(n)

c1n0 ≤ 2n + 1 ≤ c2n0 with c1 = 1, c2 = 3, n0 = n ⬄ n ≤ 2n + 1 ≤ 3n for all n0 ≥ 0.

1. √(144 n) ∈ Θ(√n)

√(c1n0) ≤ √(144 n) ≤ √(c2n0) with c1 = 143, c2 = 145, n0 = n ⬄ √(143n) ≤ √(144n) ≤ √(145n) for all n0 ≥ 0.

1. (n – 4)! ∈ Θ(n)

c1(n0 – 4)! ≤ (n - 4)! ≤ c2(n0 – 4)! with c1 = 1, c2 = 2, n0 = n ⬄ n! ≤ n! ≤ 2n! for all n ≥ 4.

1. 2log2n5 ∈ Θ(log2n5)

c1log2n05 ≤ 2log2n5 ≤ c2log2n05 with c1 = 1, c2 = 3, n0 = n ⬄ log2n5 ≤ 2log2n5 ≤ 3log2n5for all n0 ≥ 0.

1. n4/200 + 100n3 + 500000 – n ∈ Θ(n4)

c1n04+100n3+500000–n ≤ n4/200+100n3+500000–n ≤ c2n04+100n3+500000–n with c1 = 1, c2 = 2, n0 = n

⬄ n4+100n3+500000–n ≤ n4/200+100n3+500000–n ≤ 2n4+100n3+500000–n for all n.

* ***Sort the function from lowest to highest order of growth:***

Depends on the Basic Asymptotic Efficiency Classes from Levitin Table 2.2

We have

2log2n5

√(144 n)

2n+1

34n

n4/200 + 100n3 + 500000 – n

(n – 4)!

n!

## Question 2

**Thread:**

Prove (by using the deﬁnitions of the notations involved) that if

g(n) ∈ Ω(t(n)), then t(n) ∈ O(g(n)).

**Answer:**

With g(n) ∈ Ω(t(n)), there exists positive constant c and non-negative integer n0 such that:

g(n) ≥ c t(n) for all n ≥ n0

For all n ≥ N, we have

g(n) ≥ t(n)

For all n ≥ N, letting k = **,** we have

t(n) ≤ k g(n)

For all n ≥ N, as c > 0, k > 0, we can show that:

t(n) ∈ O(g(n)).

## Question 3

**Thread:**

p(n) = aknk + ak-1nk-1 + ak-2nk-2 + ak-3nk-3 + ... + a0 , where the ai are constants, is a polynomial of degree k. Prove that every polynomial of degree k belongs to Θ(nk).

**Answer:**

* p(n) ∈ O(nk):

p(n) = aknk + ak−1nk−1 + ... + a0

= ak . nk . (1 + + ... + )

≤ ak . nk . (1 + + ... + )

≤ ak . nk . (1 + + ... + )

for all n ≥ 1. Letting

c = ak . (1 + + ... + )

We have this p(n) ≤ cnk for all n ∈ N. Thus p(n) ∈ O(nk). (1)

* p(n) ∈ Ω(nk):

p(n) = aknk + ak−1nk−1 + ... + a0

= aknk . (1 + + ... + )

The expression within parentheses has limit 1 for n → ∞.

Thus, there is n0 ∈ N, such that this expression is ≥ for all n ≥ n0.

Then, for c = ak and n ≥ n0

p(n) ≥ aknk = cnk

Thus p(n) ∈ Ω(nk) (2)

Because of (1) and (2),we have p(n) ∈ Θ(nk)