ECONOMETRICS

M.A. Program in Economics
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Further Issues in Regression Analysis with Time Series Data

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Stationary Stochastic Process

- A stochastic process is stationary if for every collection of time indices $1 \le t_1 < \ldots < t_m$ the joint distribution of $(x_{t1}, \ldots, x_{tm})$ is the same as that of $(x_{t1+h}, \ldots x_{tm+h})$ for $h \ge 1$
- Thus, stationarity implies that the x_t's are identically distributed and that the nature of any correlation between adjacent terms is the same across all periods.

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Covariance Stationary Process

- A stochastic process is covariance stationary if
  1. $E(x_t)$ is constant,
  2. $Var(x_t)$ is constant
  3. For any t, $h \ge 1$, $Cov(x_t, x_{t+h})$ depends only on h and not on t
- Thus, this weaker form of stationarity requires only that the mean and variance are constant across time, and the covariance just depends on the distance across time.

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Weakly Dependent Time Series

- A stationary time series is weakly dependent if x_t and x_{t+h} are "almost independent" as h increases
- If for a covariance stationary process

$$Corr(x_t, x_{t+h}) \to 0 \text{ as } h \to \infty,$$

  we'll say this covariance stationary process is weakly dependent
- Want to still use law of large numbers (LLN) and Central Limit Theorem (CLT).

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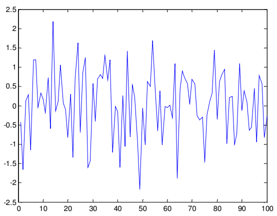
Weakly-dependent Time Series: Example

- **i.i.d.** (***independently and identically distributed***) process, or white noise process.
  - $E(x_t)=0$ for all t
  - $Var(x_t)$=constant, for all t
  - $Cov(x_t, x_{t+h})=0$, for all h
- For example, randomly drawn numbers from standard normal distribution
- This time series plot displays 100 random numbers drawn from N(0,1).



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An MA(1) Process

- A moving average process of order one [MA(1)] can be characterized as one where

$$x_t = e_t + \alpha_1 e_{t-1},$$

  $t = 1, 2, \ldots$ with e_t being an iid sequence with mean 0 and variance σ^2_e
- This is a stationary, weakly dependent sequence as variables 1 period apart are correlated, but 2 periods apart they are not.

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MA(1) Process

- Cov (x_t, x_{t+1}) = α_1. Var(e_t) = α_1. σ^2_e
- Var(x_t)= (1+ α^2_1). σ^2_e
- Cor (x_t, x_{t+1}) = α_1 / (1+ α^2_1).
- For example if α1=0.5, then Cor (x_t, x_{t+1}) =0.40
- When α_1 =1 first order autocorrelation will reach its highest value, 0.5.
- Cov (x_t, x_{t+2}) = 0 hence Cor (x_t, x_{t+2}) = 0
- In fact, second and higher autocorrelations will all be zero for MA(1) process.
- Short memory property

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MA(q) Process

$$x_t = e_t + \alpha_1 e_{t-1} + \alpha_2 e_{t-2} + \ldots + \alpha_q e_{t-q}$$

- Where e_t *is a white noise process*
- (q+1)th and higher autocorrelations will all be zero for an MA(q) process.
- An MA(q) process is always stationary
- An MA(q) process is weakly-dependent

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AR(1) Process

- An autoregressive process of order one [AR(1)] can be characterized as one where
  $$y_t = \rho y_{t-1} + e_t,$$
  t = 1, 2,... with e_t being an iid sequence with mean 0 and variance σ_e^2
- For this process to be stationary and weakly dependent, it must be the case that $|\rho| < 1$
- Autocorrelations:
  $$\text{Corr}(y_t, y_{t+h}) = \text{Cov}(y_t, y_{t+h})/(\sigma_y \sigma_y) = \rho_1^h$$

which becomes small as h increases

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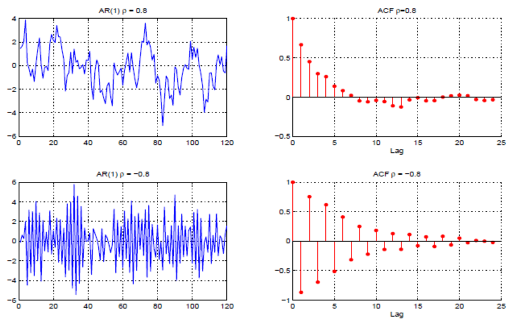
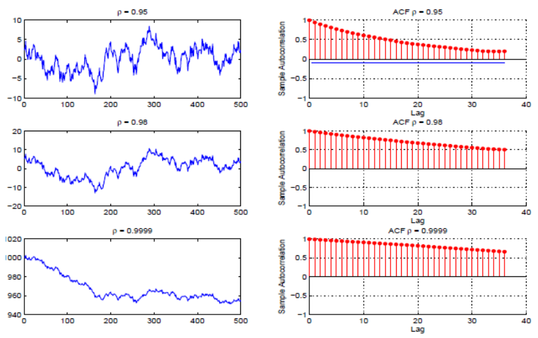
Two Simulated AR(1) Process and Correlograms



Highly Persistent AR(1) Processes



Trends Revisited

- A trending series cannot be stationary, since the mean is changing over time
- A trending series can be weakly dependent
- If a series is weakly dependent and is stationary about its trend, we will call it a **trend-stationary process:**

$$y_t = \beta_0 + \beta_1 t + e_t$$

- As long as a trend is included, all is well
- A trend-stationary process should not be confused with difference-stationary process
- Will need special tests to distinguish them.

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Assumptions for Consistency

- Linearity and Weak Dependence
- A weaker zero conditional mean assumption:

$$E(u_t|\mathbf{x}_t) = 0,$$

for each *t, which implies*

$$E(u_t) = 0, \mathrm{Cov}(x_{tj},u_t) = 0, j = 1,\ldots,k.$$

- No Perfect Collinearity
- Thus, for asymptotic unbiasedness (consistency), we can weaken the exogeneity assumptions somewhat relative to those for unbiasedness

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Example

- Let z(t1) be monthly growth rate of money supply and y(t) be inflation rate in the following model

$$y_t = \beta_0 + \beta_1 z_{t1} + \beta_2 z_{t2} + u_t.$$

$$E(u_t|z_{t1},z_{t2}) = 0.$$

- Also let us assume that last month's inflation rate affects this month's money growth rate:

$$z_{t1} = \delta_0 + \delta_1 y_{t-1} + v_t.$$

- We can still use z(t1) as an explanatory variable.

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Example: AR(1) Model

Consider the AR(1) model,

$$y_t = \beta_0 + \beta_1 y_{t-1} + u_t,$$

where the error u_t has a zero expected value, given all past values of y:

$$E(u_t|y_{t-1},y_{t-2},\ldots) = 0.$$

Combined, these two equations imply that

$$E(y_t|y_{t-1},y_{t-2},\ldots) = E(y_t|y_{t-1}) = \beta_0 + \beta_1 y_{t-1}.$$

Strict Exogeneity is not satisfied.
OLS will be biased but consistent. Bias will increase as the AR parameter approaches 1.

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Large-Sample Inference

- Weaker assumption of homoskedasticity:

$$\mathrm{Var}(u_t|\mathbf{x}_t) = \sigma^2, \text{ for each } t$$

- Weaker assumption of no serial correlation:

$$E(u_t u_s|\ \mathbf{x}_t, \mathbf{x}_s) = 0 \text{ for } t \neq s$$

- With these assumptions, we have asymptotic normality and the usual standard errors, *t* statistics, *F* statistics and *LM* statistics are valid

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Example: Efficient Market Hypothesis

New York Stock Exchange composite index. A strict form of the efficient markets hypothesis states that information observable to the market prior to week *t* should not help to predict the return during week *t*. If we use only past information on *y*, the EMH is stated as

$$E(y_t|y_{t-1},y_{t-2},\ldots) = E(y_t). \quad \textbf{(11.15)}$$

If (11.15) is false, then we could use information on past weekly returns to predict the current return. The EMH presumes that such investment opportunities will be noticed and will disappear almost instantaneously.

$$\widehat{return}_t = \underset{(.081)}{.180} + \underset{(.038)}{.059}\, return_{t-1}$$

$$n = 689, R^2 = .0035, \bar{R}^2 = .0020.$$

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Efficient Market Hypothesis : AR(2) Model

$$y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2} + u_t$$
$$E(u_t|y_{t-1},y_{t-2},\ldots) = 0.$$

$$H_0: \beta_1 = \beta_2 = 0.$$

If we add the homoskedasticity assumption $\mathrm{Var}(u_t|y_{t-1},y_{t-2}) = \sigma^2$, we can use a standard F statistic to test (11.18). If we estimate an AR(2) model for *return$_t$*, we obtain

$$\widehat{return}_t = \underset{(.081)}{.186} + \underset{(.038)}{.060}\, return_{t-1} - \underset{(.038)}{.038}\, return_{t-2}$$

$$n = 688, R^2 = .0048, \bar{R}^2 = .0019$$

(where we lose one more observation because of the additional lag in the equation). The two lags are individually insignificant at the 10% level. They are also jointly insignificant: using $R^2 = .0048$, the F statistic is approximately $F = 1.65$; the p-value for this F statistic (with 2 and 685 degrees of freedom) is about .193. Thus, we do no reject (11.18) at even the 15% significance level.

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Example: Expectations-augmented Phillips Curve

A linear version of the *expectations augmented Phillips curve* can be written as

$$inf_t - inf_t^e = \beta_1(unem_t - \mu_0) + e_t,$$

where μ_0 is the *natural rate of unemployment* and inf_t^e is the *expected* rate of inflation

Assume adaptive expectations:

$$inf_t^e = inf_{t-1}.$$

Now the model becomes

$$inf_t - inf_{t-1} = \beta_0 + \beta_1 unem_t + e_t$$

$$\Delta inf_t = \beta_0 + \beta_1 unem_t + e_t,$$

$\Delta inf_t = inf_t - inf_{t-1}$

$\beta_0 = -\beta_1\mu_0.$

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Example: Expectations-augmented Phillips Curve

$$\Delta \widehat{inf}_t = \underset{(1.38)}{3.03} - \underset{(.230)}{.543}\, unem_t$$

$$n = 48,\ R^2 = .108,\ \bar{R}^2 = .088.$$

A one-point increase in *unem* lowers unanticipated inflation by over one-half of a point. The effect is statistically significant. Natural rate of unemployment is:

$$\mu_0 = \beta_0/(-\beta_1),$$

$$\hat{\mu}_0 = \hat{\beta}_0/(-\hat{\beta}_1) = 3.03/.543 \approx 5.58.$$

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