ECONOMETRICS

M.A. Program in Economics
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Serial Correlation and Heteroskedasticity in Time Series Regression Analysis

Testing for AR(1) Serial Correlation

- Want to be able to test for whether the errors are serially correlated or not
- Want to test the null that $\rho = 0$ in

$$u_t = \rho u_{t-1} + e_t,\ t = 2,\ldots, n,$$

where u_t is the model error term and e_t is iid
- With strictly exogenous regressors, the test is very straightforward – simply regress the residuals on lagged residuals and use a t-test

Testing for AR(1) Serial Correlation: Strictly Exogenous Regressors

(i) Run the OLS regression of y_t on $x_{t1}, \ldots, x_{tk}$ and obtain the OLS residuals, $\hat{u}_t$, for all $t = 1,2,\ldots,n$.

(ii) Run the regression of

$$\hat{u}_t \text{ on } \hat{u}_{t-1}, \text{ for all } t = 2,\ldots,n, \quad \textbf{(12.14)}$$

obtaining the coefficient $\hat{\rho}$ on $\hat{u}_{t-1}$ and its t statistic, $t_{\hat{\rho}}$. (This regression may or may not contain an intercept; the t statistic for $\hat{\rho}$ will be slightly affected, but it is asymptotically valid either way.)

(iii) Use $t_{\hat{\rho}}$ to test H_0: $\rho = 0$ against H_1: $\rho \neq 0$ in the usual way. (Actually, since $\rho > 0$ is often expected a priori, the alternative can be H_0: $\rho > 0$.) Typically, we conclude that serial correlation is a problem to be dealt with only if H_0 is rejected at the 5% level. As always, it is best to report the p-value for the test.

Example: Phillips Curve

For the static Phillips curve, the regression in (12.14) yields $\hat{\rho} = .573$, $t = 4.93$, and p-value $= .000$ (with 48 observations). This is very strong evidence of positive, first order serial correlation. One consequence of this is that the standard errors and t statistics from Chapter 10 are not valid. By contrast, the test for AR(1) serial correlation in the expectations augmented curve gives $\hat{\rho} = -.036$, $t = -.297$, and p-value $= .775$ (with 47 observations): there is no evidence of AR(1) serial correlation in the expectations augmented Phillips curve.

Testing for AR(1) Serial Correlation (continued)

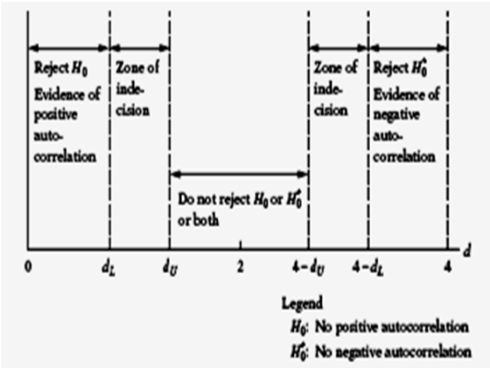
- An alternative is the Durbin-Watson (DW) statistic, which is calculated by many packages
- If the DW statistic is around 2, then we can reject serial correlation, while if it is significantly < 2 we cannot reject
- Critical values are difficult to calculate, making the t test easier to work with

Durbin-Watson Test

$$DW = \frac{\sum_{t=2}^{n}(\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{t=1}^{n}\hat{u}_t^2}. \qquad DW \approx 2(1 - \hat{\rho}).$$

All CLM assumptions, including normality, have to be satisfied.
Regressors must be strictly exogenous. We cannot include lagged dependent variable as a regressor.
Critical values consist of upper (dU) and lower limits (dL) that depend on sample size, n, and number of regressors, k.

Durbin-Watson Test



Testing for AR(1) Serial Correlation (continued)

- If the regressors are not strictly exogenous, then neither the t or DW test will work
- Regress the residual (or *y*) on the lagged residual and all of the *x*'s
- The inclusion of the *x*'s allows each x_{tj} to be correlated with u_{t-1}, so don't need assumption of strict exogeneity

Testing for AR(1) Serial Correlation without strictly exogenous regressors

(i) Run the OLS regression of y_t on $x_{t1}, \ldots, x_{tk}$ and obtain the OLS residuals, $\hat{u}_t$, for all $t = 1, 2, \ldots, n$.

(ii) Run the regression of

$$u_t \text{ on } x_{t1}, x_{t2}, \ldots, x_{tk}, \hat{u}_{t-1}, \text{ for all } t = 2, \ldots, n. \quad \textbf{(12.18)}$$

to obtain the coefficient $\hat{\rho}$ on $\hat{u}_{t-1}$ and its t statistic, $t_{\hat{\rho}}$.

(iii) Use $t_{\hat{\rho}}$ to test H_0: $\rho = 0$ against H_1: $\rho \neq 0$ in the usual way (or use a one-sided alternative).

Testing for Higher Order S.C.

- Can test for AR(*q*) serial correlation in the same basic manner as AR(1)
- Just include *q* lags of the residuals in the regression and test for joint significance
- Can use F test or LM test, where the LM version is called the **Breusch-Godfrey test** and is $(n\text{-}q)R^2$ using R^2 from residual regression
- Can also test for seasonal forms

Testing for Higher Order S.C.

(i) Run the OLS regression of y_t on $x_{t1}, \ldots, x_{tk}$ and obtain the OLS residuals, $\hat{u}_t$, for all $t = 1, 2, \ldots, n$.

(ii) Run the regression of

$$\hat{u}_t \text{ on } x_{t1}, x_{t2}, \ldots, x_{tk}, \hat{u}_{t-1}, \hat{u}_{t-2}, \ldots, \hat{u}_{t-q}, \text{ for all } t = (q+1), \ldots, n. \quad \textbf{(12.22)}$$

(iii) Compute the F test for joint significance of $\hat{u}_{t-1}, \hat{u}_{t-2}, \ldots, \hat{u}_{t-q}$ in (12.22). [The F statistic with y_t as the dependent variable in (12.22) can also be used, as it gives an identical answer.]

LM version is called Breusch-Godfrey test:

$$LM = (n - q)R^2_{\hat{u}},$$

Correcting for Serial Correlation

- Start with case of strictly exogenous regressors, and maintain all G-M assumptions except no serial correlation
- Assume errors follow AR(1) so

  $u_t = \rho u_{t-1} + e_t,\ t = 2, \ldots, n$
- $\mathrm{Var}(u_t) = \sigma^2_e/(1-\rho^2)$
- We need to try and transform the equation so we have no serial correlation in the errors

Correcting for S.C. (continued)

- Consider that since

$$y_t = \beta_0 + \beta_1 x_t + u_t ,$$

then

$$y_{t-1} = \beta_0 + \beta_1 x_{t-1} + u_{t-1}$$

- If you multiply the second equation by ρ, and subtract if from the first you get

$$y_t - \rho\, y_{t-1} = (1 - \rho)\beta_0 + \beta_1(x_t - \rho\, x_{t-1}) + e_t ,$$

since $e_t = u_t - \rho\, u_{t-1}$

- This quasi-differencing results in a model without serial correlation
- Need to know ρ.

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Feasible GLS Estimation

- Problem with this method is that we don't know ρ, so we need to get an estimate first
- Can just use the estimate obtained from regressing residuals on lagged residuals
- Depending on how we deal with the first observation, this is either called Cochrane-Orcutt or Prais-Winsten estimation
- CO procedure omits the first observation.
- PW transforms the first observation.

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Feasible GLS (continued)

- Often both Cochrane-Orcutt and Prais-Winsten are implemented iteratively
- This basic method can be extended to allow for higher order serial correlation, AR(q)
- Most statistical packages will automatically allow for estimation of AR models without having to do the quasi-differencing by hand

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Feasible GLS: Example

Dependent Variable: *inf*

Coefficient	OLS	Cochrane-Orcutt
unem	.468 (.289)	−.665 (.320)
intercept	1.424 (1.719)	7.580 (2.379)
$\hat{\rho}$	———	.774 (.091)
Observations *R*-Squared	49 .053	48 .086

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Serial Correlation-Robust Standard Errors

- What happens if we don't think the regressors are all strictly exogenous?
- It's possible to calculate serial correlation-robust standard errors, along the same lines as heteroskedasticity robust standard errors
- Idea is that want to scale the OLS standard errors to take into account serial correlation

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Serial Correlation-Robust Standard Errors (continued)

- Estimate normal OLS to get residuals, root MSE
- Run the auxiliary regression of x_{t1} on $x_{t2}, \ldots , x_{tk}$
- Form $\hat{a}_t$ by multiplying these residuals with $\hat{u}_t$
- Choose g – say 1 to 3 for annual data, then

$$\hat{v} = \sum_{t=1}^{n} \hat{a}_t^2 + 2\sum_{h=1}^{g} [1 - h/(g+1)] \left(\sum_{t=h+1}^{n} \hat{a}_t \hat{a}_{t-h} \right)$$

and $se(\hat{\beta}_1) = [SE/\hat{\sigma}]^2 \sqrt{\hat{v}}$, where SE is the usual OLS standard error of $\hat{\beta}_j$

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Heteroskedasticity in Time Series

- We saw that heteroskedasticity is generally a cross-section data problem.
- In most cases heteroskedasticity is ignored in time series contexts.
- But heteroskedasticity can be a problem in time series analysis too.
- What happens if we ignore this?
- If all other assumptions are satisfied OLS is still consistent but not efficient.
- We can still use heteroskedasticity and serial-correlation-robust standard errors and test statistics. (these are calculated automatically by most statistical packages)

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Heteroskedasticity in Time Series Analysis

- We can still use heteroskedasticity tests we saw before.
- For example, can use Breusch-Pagan test.
- In the first step of BP test we estimate the model using OLS and obtain residuals.
- In the second step, we regress squared residuals on all regressors in the model.
- Then we test the joint significance of the auxiliary regression in the second step using either F or LM version.
- If we find a signficant F or LM statistic then we reject the null hypothesis of "No heteroskedasticity (variance is constant)", ie there is heteroskedasticity.

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Heteroskedasticity in Time Series Analysis

- We can also use White test, if we believe that squared residuals may be nonlinearly related to x's.
- We saw the steps of this test before, so no need to reiterate.
- If we find that the model is heteroskedastic we can use WLS or FGLS instead of OLS.
- If residuals are serially correlated these tests will not be valid.
- If the residual variance is dynamically changing with time we need to use appropriate time series models to account for this.
- For example, many financial time series, especially return series, may have autoregressive changing variance structure.

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Heteroskedasticity in Time Series Analysis
Example: Efficient Market Hypothesis (EMH)

$$return_t = \beta_0 + \beta_1 return_{t-1} + u_t.$$

$\beta_1 = 0$ $\quad t_{\beta_1} = 1.55$ **EMH: valid???**

Auxiliary Regression for BP test

$$\hat{u}_t^2 = \underset{(0.43)}{4.66} - \underset{(0.201)}{1.104}\, return_{t-1} + residual_t$$

$$n = 689,\ R^2 = .042.$$

t-stat=-5.5

Srong evidence for heteroskedasticity

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Heteroskedasticity in Time Series Analysis
Example: Efficient Market Hypothesis (EMH)

$$\hat{u}_t^2 = \underset{(0.43)}{4.66} - \underset{(0.201)}{1.104}\, return_{t-1} + residual_t$$

$$n = 689,\ R^2 = .042.$$

- According to these results, when the return in the previous period is high, volatility in the current period tends to be lower.
- Although the expected value of returns does not depend on the previous period's return (as suggested by EMH test), we see that the variance is changing with return.
- This is a widely observed behavior in financial time series.

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Autoregressive Conditional Heteroskedasticity (ARCH)

- Consider the simple regression model:

$$y_t = \beta_0 + \beta_1 z_t + u_t,$$

- Suppose that error term follows the process:

$$\mathrm{E}(u_t^2 | u_{t-1}, u_{t-2}, \ldots) = \mathrm{E}(u_t^2 | u_{t-1}) = \alpha_0 + \alpha_1 u_{t-1}^2,$$

$$u_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + v_t, \quad \text{ARCH(1) Model}$$

$\alpha_0 > 0 \quad \alpha_1 \geq 0 \quad \alpha_1 < 1$

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Autoregressive Conditional Heteroskedasticity (ARCH)

- If all other Gauss-Markov assumptions are satisfied then OLS will be unbiased and consistent under ARCH errors.
- Why bother with this family of models?
- First, we can obtain more efficient estimaton procedures than OLS, such as WLS.
- Second, ARCH and its generalizations provide a useful framework to analyze dynamic behavior of variance (or volatility) especially for financial return series.

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ARCH Example

- ARCH(1) estimation results for stock market returns

$$\hat{u}_t^2 = \underset{(0.44)}{2.95} + \underset{(.036)}{.337}\,\hat{u}_{t-1}^2 + residual_t$$
$$n = 688,\ R^2 = .114.$$

High t-statistic indicates the presence of ARCH effects.

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