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Task 1

Given the function:

$$f(t) = e^{t^{11-11}} = e \quad \forall t \tag{1}$$

Here the solution starts.

Step 1. Determine if the function is piecewise continious(*Please, refer to Textbook: Elementary Differential Equations by William F. Trench. Brooks/Cole Thomson Learning 2001, p. 400*):

$\lim_{t \to 0^+} f(t) = \lim_{t \to 0^+} e =$ e

$\lim_{t \to \infty^-} f(t) = \lim_{t \to \infty^-} e =$ e

Since both limits are finite and the function is piecewise continuous on $[0; T]$ for every $T > 0$(Since the function is just constant), f is piecewise continious on the infinite interval $[0; +\infty[$.

Step 2. Determine if the function has exponential order(*Please, refer to Textbook: Elementary Differential Equations by William F. Trench. Brooks/Cole Thomson Learning 2001, p. 401*):

$\lim_{t \to \infty} \frac{f(t)}{e^{ct}} = \lim_{t \to \infty} \frac{e}{e^{ct}} = 0$ (only for $c > 0$)

Thus, the function f has positive exponential order.

Step 3. Determine the interval s for which we can define the Laplace transform $L(e)$(*Please, refer to Textbook: Elementary Differential Equations by William F. Trench. Brooks/Cole Thomson Learning 2001, p. 394*):

The function f is defined for $t \geq 0$ and let s be a real number.

$F(s) = \int_0^\infty e^{-st} f(t) dt = \int_0^\infty e^{-st} e dt = e \int_0^\infty e^{-st} dt = -e \lim_{b \to \infty} \frac{e^{-sb} - e^0}{s} = \frac{e}{s}$ for $s > 0$.

So, the improper integral above converges for $s > 0$.

Thus, the Laplace transform is defined for s in $[0; +\infty[$.

Answer:

1) f is piecewise continious on the infinite interval $[0; +\infty[$.

2) f has positive exponential order.

3) The Laplace transform is defined for s in $[0; +\infty[$.

Task 2

We need to solve the following system using elimination:

$$\begin{cases} y_1' = \quad y_1 - y_2 \\ y_2' = 11y_1 + 11y_2 \end{cases}$$

Before solving, we analyze the system above:a homogeneous system of 2 first-order linear homogeneous equations.

Here the solution starts.

Step 1. Using elimination method(*Please, refer to lecture 14, slides 8-9.*) and assuming that chain rule can be applied differentiate the first equation (for y_1') and instantiate y_2' to it:

$$y_1'' = y_1' - y_2' = y_1 - 11y_1 - 11y_2 \tag{2}$$

Step 2. Express y_2 from the first equation:

$$y_2 = y_1 - y_1' \tag{3}$$

Then substitute y_2 into (2):

$$y_1'' = y_1 - 11y_1 - 11(y_1 - y_1') \tag{4}$$

$$y_1'' - 12y_1' + 22y_1 = 0 \tag{5}$$

Now we obtained a homogeneous linear second-order ordinary differential equation with constant coefficients.

Step 3. Let us the method of characteristic equation(*Please, refer to lecture 8&9, slide 26*) to solve (5):

$$a^2 - 12a - 22 = 0 \tag{6}$$

Find roots of the characteristic(quadratic) equation:

$a_1 = \frac{12+\sqrt{56}}{2} = 6 + \sqrt{14}$

$a_2 = \frac{12-\sqrt{56}}{2} = 6 - \sqrt{14}$

So, we get 2 real roots, and $y_1 = c_1 e^{(6+\sqrt{14})x} + c_2 e^{(6-\sqrt{14})x}$ where $c_1 \in R$, $c_2 \in R$, $x \in R$ is the most general solution(*Please, refer to lecture 8&9, slide 27*) of the equation (5).

Then find y_1':

$y_1' = (6+\sqrt{14})c_1 e^{(6+\sqrt{14})x} + (6-\sqrt{14})c_2 e^{(6-\sqrt{14})x}$

Step 4. Find y_2 substituting y_1' into (3):

$y_2 = c_1 e^{(6+\sqrt{14})x} + c_2 e^{(6-\sqrt{14})x} - (6+\sqrt{14}) c_1 e^{(6+\sqrt{14})x} - (6-\sqrt{14}) c_2 e^{(6-\sqrt{14})x}$
$, \ where \ c_1 \in R, \ c_2 \in R, \ x \in R$

$y_2 = c_1 e^{(6+\sqrt{14})x}(-5-\sqrt{14}) + c_2 e^{(6-\sqrt{14})x}(-5+\sqrt{14})$

So, we obtained the most general solution for the homogeneous system of 2 equations with constant coefficients(*Please, refer to lecture 14, slide 10*) in the following form:

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = c_1 e^{(6+\sqrt{14})x} \begin{pmatrix} 1 \\ -5-\sqrt{14} \end{pmatrix} + c_2 e^{(6-\sqrt{14})x} \begin{pmatrix} 1 \\ -5+\sqrt{14} \end{pmatrix} \ where \ c_1 \in R, c_2 \in R, x \in R$$

Answer: $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = c_1 e^{(6+\sqrt{14})x} \begin{pmatrix} 1 \\ -5-\sqrt{14} \end{pmatrix} + c_2 e^{(6-\sqrt{14})x} \begin{pmatrix} 1 \\ -5+\sqrt{14} \end{pmatrix}$ where $c_1 \in R$, $c_2 \in R$, $x \in R$ is the most general solution.