Differential Equations

Lab 1

Task 1. Verify that the function y(x) is a solution of the differential equation on some interval, for any choice of the arbitrary constants appearing in the function.

1) $y = \frac{x^2}{3} + \frac{c}{x}; \quad xy' + y = x^2$

2) $y = \frac{1}{2} + ce^{-x^2}; \quad y' + 2xy = x$

3) $y = \frac{1+ce^{-\frac{x^2}{2}}}{1-ce^{-\frac{x^2}{2}}}; \qquad 2y' + x(y^2 - 1) = 0$

4) $y = \tan\left(\frac{x^3}{3} + c\right); \quad y' = x^2(1 + y^2)$

5) $y = x^{-1/2}(c_1 \sin x + c_2 \cos x) + 4x + 8;$
$x^2 y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 4x^3 + 8x^2 + 3x - 2$

Task 1. Explanation

1) $y = \frac{x^2}{3} + \frac{c}{x}; \quad xy' + y = x^2$

step1. calculate y’ If $y = \frac{x^2}{3} + \frac{c}{x}$, then $y' = \frac{2x}{3} - \frac{c}{x^2}$

step2. substitute y’ $xy' + y = x^2$

$$xy' + y = \frac{2x^2}{3} - \frac{c}{x} + \frac{x^2}{3} + \frac{c}{x} = x^2$$

Task 2. Solve the initial value problem.

1) $y' = -xe^x, \quad y(0) = 1$

2) $y' = x \sin x^2, \quad y\left(\sqrt{\frac{\pi}{2}}\right) = 1$

3) $y'' = x^4, \quad y(2) = -1, \quad y'(2) = -1$

4) $y''' = 2 + \sin 2x, \quad y(0) = 1, \quad y'(0) = -6, \quad y''(0) = 3$

Task 2. Explanation

1) $y' = -xe^x, \quad y(0) = 1$

step1.
general solution

$$y = -\int xe^x dx = -(xe^x - \int e^x dx) = -xe^x + e^x + C$$

step2.
initial value problem

$$y(0) = -0 + 1 + C = 1 \rightarrow C = 0$$

Answer:

$$y = e^x(1 - x)$$