№	$f(t)$	$F(p)$	№	$f(t)$	$F(p)$
1	$1(t)$	$\frac{1}{p}$	17	$\frac{1}{a^2}(1-\cos at)$	$\frac{1}{p(p^2+a^2)}$
2	C	$\frac{C}{p}$	18	$\frac{1}{a^2}(e^{at}-1-at)$	$\frac{1}{p^2(p-a)}$
3	t	$\frac{1}{p^2}$	19	$\operatorname{sh} at$	$\frac{a}{p^2-a^2}$
4	t^n	$\frac{n!}{p^{n+1}}$	20	$\operatorname{ch} at$	$\frac{p}{p^2-a^2}$
5	$\delta(t)$	1	21	$(t+\frac{1}{2}at^2)\,e^{at}$	$\frac{p}{(p-a)^3}$
6	e^{at}	$\frac{1}{p-a}$	22	$(1+2at+\frac{1}{2}a^2t^2)\,e^{at}$	$\frac{p^2}{(p-a)^3}$
7	$t^n e^{at}$	$\frac{n!}{(p-a)^{n+1}}$	23	$(1+at)\,e^{at}$	$\frac{p}{(p-a)^2}$
8	$\sin at$	$\frac{a}{p^2+a^2}$	24	$\cos^2 at$	$\frac{p^2+2a^2}{p(p^2+4a^2)}$
9	$\cos at$	$\frac{p}{p^2+a^2}$	25	$\sin^2 at$	$\frac{2a^2}{p(p^2+4a^2)}$
10	$t\sin at$	$\frac{2pa}{(p^2+a^2)^2}$	26	$\sin\frac{a}{\sqrt{2}}t\ \operatorname{sh}\frac{a}{\sqrt{2}}t$	$\frac{a^2p}{p^4+a^4}$
11	$t\cos at$	$\frac{p^2-a^2}{(p^2+a^2)^2}$	27	$\cos\frac{a}{\sqrt{2}}t\ \operatorname{ch}\frac{a}{\sqrt{2}}t$	$\frac{p^3}{p^4+a^4}$
12	$e^{at}\sin bt$	$\frac{b}{(p-a)^2+b^2}$	28	$\frac{1}{2}(\operatorname{sh} at-\sin at)$	$\frac{a^3}{p^4-a^4}$
13	$e^{at}\cos bt$	$\frac{p-a}{(p-a)^2+b^2}$	29	$\frac{1}{2}(\operatorname{ch} at-\cos at)$	$\frac{a^2p}{p^4-a^4}$
14	$\frac{1}{a}e^{-\frac{t}{a}}$	$\frac{1}{1+ap}$	30	$\frac{1}{2}(\operatorname{sh} at+\sin at)$	$\frac{ap^2}{p^4-a^4}$
15	$\frac{1}{a}(e^{at}-1)$	$\frac{1}{p(p-a)}$	31	$\frac{1}{2}(\operatorname{ch} at+\cos at)$	$\frac{p^3}{p^4-a^4}$
16	$\frac{e^{at}-e^{bt}}{a-b}$	$\frac{1}{(p-a)(p-b)}$	32	$\frac{a\,e^{at}-b\,e^{bt}}{a-b}$	$\frac{p}{(p-a)(p-b)}$

1.1. $\mathcal{L}^{-1}\left(\frac{3}{(s-7)^4}\right)$ - ?

$$\mathcal{L}(t^n e^{at}) = \frac{n!}{(s-a)^{n+1}}$$

$a=7,\ n=3:$ $\quad \frac{6}{(s-7)^4} = \frac{1}{2}\mathcal{L}(t^3 e^{7t})$

$$\mathcal{L}^{-1}\left(\frac{3}{(s-7)^4}\right) = \frac{1}{2}\mathcal{L}^{-1}\left(\frac{6}{(s-7)^4}\right) = \frac{1}{2}\cdot t^3 e^{7t}$$

Answer: $\mathcal{L}^{-1}\left(\frac{3}{(s-7)^4}\right) = \frac{1}{2}t^3 e^{7t}$

1.2 $\mathcal{L}^{-1}\left(\frac{s^2-4s+3}{(s^2-4s+5)^2}\right)$ - ?

Notice that $\frac{s^2-4s+3}{s^4-8s^3+26s^2-40s+25} =$

$$= \frac{1}{2}\left[\frac{1}{(s+i-2)^2} + \frac{1}{(s-i-2)^2}\right]$$

Then $\mathcal{L}^{-1}\left(\frac{s^2-4s+3}{(s^2-4s+5)^2}\right) = \frac{1}{2}\mathcal{L}^{-1}\left(\frac{1}{(s+i-2)^2}\right) +$

$+ \frac{1}{2}\mathcal{L}^{-1}\left(\frac{1}{(s-i-2)^2}\right) = \frac{1}{2}te^{t(2-i)} + \frac{1}{2}te^{t(2+i)}$

Answer: $\mathcal{L}^{-1}\left(\frac{s^2-4s+3}{(s^2-4s+5)^2}\right) = \frac{1}{2}t\left(e^{t(2-i)} + e^{t(2+i)}\right)$

11.3. $\mathcal{L}^{-1}\left(\frac{1}{s} - \frac{s}{s^2+1}\right) - ?$

$$\mathcal{L}^{-1}\left(\frac{1}{s} - \frac{s}{s^2+1}\right) = \mathcal{L}^{-1}\left(\frac{1}{s}\right) - \mathcal{L}^{-1}\left(\frac{s}{s^2+1}\right) =$$

$= 1 - \cos t$

Answer: $\mathcal{L}^{-1}\left(\frac{1}{s} - \frac{s}{s^2+1}\right) =$
$= 1 - \cos t$

11.4 $\mathcal{L}^{-1}\left(\frac{s+5}{s^2+6s+18}\right) - ?$

$$\frac{s+5}{s^2+6s+18} = \frac{s+5}{(s+3)^2+3^2}$$

$$\mathcal{L}^{-1}\left(\frac{s+5}{(s+3)^2+3^2}\right) = \frac{2}{3}\mathcal{L}^{-1}\left(\frac{3}{(s+3)^2+3^2}\right) + \mathcal{L}^{-1}\left(\frac{s+3}{(s+3)^2+3^2}\right) =$$

$$= \frac{2}{3}\left[e^{-3t}\sin 3t\right] + \left[e^{-3t}\cos 3t\right] = e^{-3t}\left(\frac{2}{3}\sin 3t + \cos 3t\right)$$

Answer: $\mathcal{L}^{-1}\left(\frac{s+5}{s^2+6s+18}\right) = e^{-3t}\left(\frac{2}{3}\sin 3t + \cos 3t\right)$

11.5 $\mathcal{L}^{-1}\left(\frac{3-(s+1)(s-2)}{(s+1)(s+2)(s-2)}\right) - ?$

$$\frac{-s^2+s+5}{(s+1)(s+2)(s-2)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s-2}$$

$$A(s+2)(s-2)+B(s+1)(s-2)+C(s+1)(s+2)=-s^2+s+5$$

$$As^2-4A+Bs^2-Bs-2B+Cs^2+2C+3Cs=-s^2+s+5$$

$$\begin{cases} A+B+C=-1 \\ -B+3C=1 \\ -4A-2B+2C=5 \end{cases} \qquad \begin{cases} A=-1 \\ B=-\frac{1}{4} \\ C=\frac{1}{4} \end{cases}$$

$$\mathcal{L}^{-1}\left(\frac{3-(s+1)(s-2)}{(s+1)(s+2)(s-2)}\right)=-1\cdot\mathcal{L}^{-1}\left(\frac{1}{s+1}\right)-\frac{1}{4}\cdot$$

$$\cdot\mathcal{L}^{-1}\left(\frac{1}{s+2}\right)+\frac{1}{4}\cdot\mathcal{L}^{-1}\left(\frac{1}{s-2}\right)=$$

$$=-1\cdot e^{-t}-\frac{1}{4}\cdot e^{-2t}+\frac{1}{4}e^{2t}$$

Answer: $\mathcal{L}^{-1}\left(\frac{3-(s+1)(s-2)}{(s+1)(s+2)(s-2)}\right)=-e^{-t}-\frac{1}{4}e^{-2t}+\frac{1}{4}e^{2t}$

11.6. $\mathcal{L}^{-1}\left(\frac{7+(s+4)(18-3s)}{(s-3)(s-1)(s+4)}\right)-?$

$$\frac{-3s^2+6s+65}{(s-3)(s-1)(s+4)}=\frac{A}{s-3}+\frac{B}{s-1}+\frac{C}{s+4}$$

$$A(s-1)(s+4)+B(s-3)(s+4)+C(s-3)(s-1)=$$

$$=s^2(A+B+C)+s(3A+B-4C)+(-4A-12B+3C)=-3s^2+6s+65$$

$$\begin{cases} A+B+C=-3 \\ 3A+B-4C=6 \\ -4A-12B+3C=65 \end{cases} \qquad \begin{cases} A=4 \\ B=-\frac{34}{5} \\ C=-\frac{1}{5} \end{cases}$$

$$\mathcal{L}^{-1}\left(\frac{7+(s+4)(18-3s)}{(s-3)(s-1)(s+4)}\right) = 4\mathcal{L}^{-1}\left(\frac{1}{s-3}\right) - \frac{34}{5}\mathcal{L}^{-1}\left(\frac{1}{s-1}\right)$$

$$-\frac{1}{5}\mathcal{L}^{-1}\left(\frac{1}{s+4}\right) = 4e^{3t} - \frac{34}{5}e^{t} - \frac{1}{5}e^{-4t}.$$

Answer: $\mathcal{L}^{-1}\left(\frac{7+(s+4)(18-3s)}{(s-3)(s-1)(s+4)}\right) = 4e^{3t} - \frac{34}{5}e^{t} - \frac{1}{5}e^{-4t}.$

11.7. $f(t) \leftrightarrow F(s) \quad a>0. \quad \mathcal{L}^{-1}(F(as-b)) - ?$

$$\int_0^{+\infty} [f(t)e^{-\lambda t}]e^{-st}\,dt = \int_0^{+\infty} [f(t)]e^{(-\lambda-s)t}\,dt =$$

$$= F(s+\lambda) \quad \Rightarrow \quad F(s+\lambda) \leftrightarrow e^{-\lambda t} f(t)$$

$$\mathcal{L}^{-1}(F(as-b)) = e^{bt}\mathcal{L}^{-1}(F(as)) =$$

$$= e^{bt}\mathcal{L}^{-1}\left(\int_0^{+\infty} e^{-ast} f(t)\,dt\right) = \{k = at\} =$$

$$= e^{bt}\mathcal{L}^{-1}\left(\frac{1}{a}\int_0^{+\infty} e^{-sk} f\left(\frac{k}{a}\right)dk\right) =$$

$$= \frac{e^{bt}}{a}\mathcal{L}^{-1}\left(\mathcal{L}\left(f\left(\frac{k}{a}\right)\right)\right) = \frac{e^{bt}}{a} f\left(\frac{t}{a}\right)$$

Answer: $\mathcal{L}^{-1}(F(as-b)) = \frac{e^{bt}}{a} f\left(\frac{t}{a}\right).$