№ 1) $y = \frac{x^2}{3} + \frac{C}{x}$; $xy' + y = x^2$

$y' = \frac{2}{3}x - \frac{C}{x^2}$; $x\left(\frac{2}{3}x - \frac{C}{x^2}\right) + \frac{x^2}{3} + \frac{C}{x} =$

$= x^2$ – solution

2) $y = \frac{1}{2} + Ce^{-x^2}$; $y' + 2xy = x$

$y' = -2xCe^{-x^2}$; $-2xCe^{-x^2} + 2x\left(\frac{1}{2} + Ce^{-x^2}\right) =$

$= x$ – solution

3) $y = \frac{1 + Ce^{-\frac{x^2}{2}}}{1 - Ce^{-\frac{x^2}{2}}}$; $2y' + x(y^2 - 1) = 0$

$y' = \frac{-xCe^{-\frac{x^2}{2}}\left(1 - Ce^{-\frac{x^2}{2}}\right) + xCe^{-\frac{x^2}{2}}\left(1 + Ce^{-\frac{x^2}{2}}\right)}{\left(1 - Ce^{-\frac{x^2}{2}}\right)^2} =$

$= \frac{-2xCe^{-\frac{x^2}{2}}}{\left(1 - Ce^{-\frac{x^2}{2}}\right)^2}$

$y^2 - 1 = \frac{C^2e^{-x^2} + 2Ce^{-\frac{x^2}{2}} + 1 - 1 - C^2e^{-x^2} + 2Ce^{-\frac{x^2}{2}}}{\left(1 - Ce^{-\frac{x^2}{2}}\right)^2} =$

$= -\frac{2y'}{x}$ – solution

4) $y = \tan\left(\frac{x^3}{3} + C\right)$, $y' = x^2(1+y^2)$

$y' = x^2 \cdot \frac{1}{\cos^2\left(\frac{x^3}{3}+C\right)}$

$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha}$ $\qquad 1+y^2 = \frac{1}{\cos^2\left(\frac{x^3}{3}+C\right)}$

$(1+y^2)x^2 = \frac{x^2}{\cos^2\left(\frac{x^3}{3}+C\right)} = y'$ - solution

5) $y = x^{-\frac{1}{2}}(C_1 \sin x + C_2 \cos x) + 4x + 8$

$x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 4x^3 + 8x^2 + 3x - 2$

$y' = -\frac{1}{2}x^{-\frac{3}{2}}(C_1 \sin x + C_2 \cos x) + x^{-\frac{1}{2}}(C_1 \cos x - C_2 \sin x) + 4$

$y'' = \frac{3}{4}x^{-\frac{5}{2}}(C_1 \sin x + C_2 \cos x) - \frac{1}{2}x^{-\frac{3}{2}}(C_1 \cos x - C_2 \sin x) - \frac{1}{2}x^{-\frac{3}{2}}(C_1 \cos x - C_2 \sin x) + x^{-\frac{1}{2}}(-C_1 \sin x - C_2 \cos x)$

$x^2y'' + xy' = \frac{3}{4}x^{-\frac{1}{2}}(C_1 \sin x + C_2 \cos x) - \frac{1}{2}x^{\frac{1}{2}}(C_1 \cos x - C_2 \sin x) - x^{\frac{3}{2}}(C_1 \sin x + C_2 \cos x) - \frac{1}{2}x^{-\frac{1}{2}}(C_1 \sin x + C_2 \cos x) + x^{\frac{1}{2}}(C_1 \cos x - C_2 \sin x) + 4x =$

$= \frac{1}{4} x^{-\frac{1}{2}} (C_1 \sin x + C_2 \cos x) - x^{\frac{1}{2}} (C_1 \cos x - C_2 \sin x) -$

$- x^{\frac{5}{2}} (C_1 \sin x + C_2 \cos x) + 4x,$

$x^2 y = x^{\frac{3}{2}} (C_1 \sin x + C_2 \cos x) + 4x^3 + 8x^2$

$(x^2 - \frac{1}{4}) y = x^{\frac{3}{2}} (C_1 \sin x + C_2 \cos x) + 4x^3 + 8x^2 -$

$- x^{-\frac{1}{2}} (C_1 \sin x - C_2 \cos x)$

$x^2 y'' + x y' + (x^2 - \frac{1}{4}) y = 4x^3 + 8x^2 + 3x - 2.$

- solution.

12. 1) $y' = -xe^x \qquad y(0) = 1$

$y = \int -x e^x dx = -xe^x + \int e^x dx =$

$v = -x \quad dv = -dx$

$du = e^x \quad du = e^x dx$

$= -xe^x + e^x + C$

$y' = -e^x - xe^x + e^x = -xe^x,$

$y(0) = 1: \quad 0 \cdot e^0 + e^0 + C = 1 \quad C = 0$

$y(x) = -xe^x + e^x$

2) $y' = x \sin x^2$ $\quad y(\sqrt{\frac{\pi}{2}}) = 1$

$y = -\frac{1}{2}\cos x^2 + C$

$y(\sqrt{\frac{\pi}{2}}) = -\frac{1}{2}\cos\frac{\pi}{2} + C = 1 \quad C = 1$

$y = -\frac{1}{2}\cos x^2 + 1$

3) $y'' = x^4$ $\quad y(2) = -1 \quad y'(2) = -1$

$y' = \frac{1}{5}x^5 + C_1$

$y = \frac{1}{30}x^6 + C_1 x + C_2$

$y(2) = \frac{1}{30}\cdot 64 + 2C_1 + C_2 = \frac{32}{15} + 2C_1 + C_2 = -1$

$y'(2) = \frac{32}{5} + C_1 = -1$

$\begin{cases} \frac{32}{15} + 2C_1 + C_2 = -1 \\ \frac{32}{5} + C_1 = -1 \end{cases}$

$C_1 = -1 - \frac{32}{5} = -\frac{37}{5}$

$C_2 = -1 + \frac{74}{5} - \frac{32}{15} = -\frac{15}{15} + \frac{222}{15} - \frac{32}{15} =$

$= \frac{175}{15} = \frac{35}{3}$

$y(x) = \frac{1}{30}x^6 - \frac{37}{5}x + \frac{35}{3}$

4) $y'' = 2 + 5\sin 2x$ $\quad y(0) = 1, \; y'(0) = -6$

$y''(0) = 3$

$y'' = 2x - \frac{1}{2}\cos 2x + C_1$

$y' = x^2 - \frac{1}{4}\sin 2x + C_1 x + C_2$

$$y = \frac{x^3}{3} + \frac{1}{8}\cos 2x + \frac{C_1}{2}x^2 + C_2 x + C_3$$

$$y''(0) = -\frac{1}{2} + C_1 = 1 \qquad C_1 = \frac{1}{2}$$

$$y'(0) = C_1 x + C_2 = \frac{x}{2} + C_2 = C_2 = -6; \quad C_2 = -6$$

$$y(0) = \frac{1}{8} + C_3 = 3 \quad C_3 = 3 - \frac{1}{8} = \frac{23}{24}$$

$$y(x) = \frac{x^3}{3} + \frac{1}{8}\cos 2x + \frac{1}{4}x^2 - 6x + \frac{23}{24}$$