Differential Equations

Lab 2

Linear Non homogeneous First Order Equations

$$y' + p(x)y = f(x).$$

Variation of parameters approach:

Step1. Solve complementary equation $y_1{}' + p(x)y_1 = 0$

Step2. find solution in the form $y = uy_1$

y_1 is a nontrivial solution of the complementary equation

u is function to be determined

$$y = uy_1, \quad \text{then} \quad y' = u'y_1 + uy_1'.$$

$$u'y_1 + u(y_1' + p(x)y_1) = f(x),$$

$$u'y_1 = f(x),$$

$$u' = f(x)/y_1(x).$$

Task 1. Find the general solution

1) $$y' + \frac{1}{x}y = \frac{7}{x^2} + 3$$

2) $$y' = 2x(x^2 + y)$$

3) $$y' + \frac{2x}{1+x^2}y = \frac{e^{-x}}{1+x^2}$$

4) $$(x-1)y' + 3y = \frac{1}{(x-1)^3} + \frac{sinx}{(x-1)^2}; y(0) = 1$$ solve i.v.p.

Separable Equations

A first order differential equation is separable if it can be written as

$$h(y)y' = g(x),$$

where the left side is a product of y' and a function of y and the right side is a function of x.

Example:

$$y' = -\frac{x}{y}$$

$$yy' = -x$$

$$\frac{y^2}{2} = -\frac{x^2}{2} + c$$

Task 2. Find the solution

1) $$y' + x(y^2 + y) = 0, y(2) = 1$$

2) $$y' + \frac{(y+1)(y-1)(y-2)}{x+1} = 0; y(1) = 0$$

3) $$xy' - 2y = \frac{x^6}{y + x^2}$$