

$$9.1. \quad y'' + y' - 2y = 0$$

This is linear homogenous second order ODE

$$\lambda^2 + \lambda - 2 = 0 \quad \lambda_1 = -2 \quad \lambda_2 = 1$$

$$y = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x} = C_1 e^{-2x} + C_2 e^x$$

$$\text{Answer: } y = C_1 e^{-2x} + C_2 e^x$$

$$9.2. \quad y'' - 4y = 0 \quad y(0) = 1 \quad y'(0) = 2$$

$$\lambda^2 - 4 = 0 \quad \lambda_1 = -2 \quad \lambda_2 = 2$$

$$y = C_1 e^{-2x} + C_2 e^{2x} \quad y' = -2C_1 e^{-2x} + 2C_2 e^{2x}$$

$$y(0) = 1 \quad 1 = C_1 e^{-2 \cdot 0} + C_2 e^{2 \cdot 0} = C_1 + C_2$$

$$y'(0) = 2 \quad 2 = -2C_1 e^{-2 \cdot 0} + 2C_2 e^{2 \cdot 0} = -2C_1 + 2C_2$$

Solve system of equations:

$$\begin{cases} C_1 + C_2 = 1 \\ -C_1 + C_2 = 1 \end{cases}$$

$$\begin{cases} C_1 = 0 \\ C_2 = 1 \end{cases}$$

$$y = e^{2x}$$

$$\text{Answer: } y = e^{2x}$$

$$9.3 \quad y'' + 4y = 0 \quad y(\pi) = -1 \quad y'\left(\frac{\pi}{2}\right) = -4$$

$$\lambda^2 + 4 = 0 \quad \lambda_1 = -2i \quad \lambda_2 = 2i$$

$$y = C_1 \sin(2x) + C_2 \cos(2x)$$

$$y' = 2C_1 \cos(2x) + 2C_2 (-\sin(2x))$$

$$y(\pi) = -1: \quad -1 = C_1 \sin(2\pi) + C_2 \cos(2\pi) = C_2$$

$$y'\left(\frac{\pi}{2}\right) = -4: \quad -4 = 2C_1 \cos(\pi) + 2C_2 (-\sin(\pi)) = -2C_1$$

Solve system of equations:

$$\begin{cases} -1 = C_2 \\ -4 = -2C_1 \end{cases} \quad \begin{cases} C_1 = +2 \\ C_2 = -1 \end{cases} \quad y = 2\sin(2x) - \cos(2x)$$

$$\text{Answer: } y = 2\sin(2x) - \cos(2x)$$

$$9.4 \quad y'' - 14y' + 49 = 0 \quad y(1) = 2 \quad y'(1) = 11$$

$$\lambda^2 - 14\lambda + 49 = 0 \quad \lambda_1 = \lambda_2 = 7$$

$$y = C_1 e^{7x} + C_2 x e^{7x} = C_1 e^{7x} + C_2 x e^{7x}$$

$$y' = 7C_1 e^{7x} + 7C_1 x e^{7x} + C_2 e^{7x}$$

$$y(1) = 2: \quad 2 = C_1 e^7 + C_2 e^7$$

$$y'(1) = 11: \quad 11 = 7C_1 e^7 + 7C_2 e^7 + C_2 e^7$$

Solve system of equations:

$$\begin{cases} C_1 = 5e^{-7} \\ C_2 = -3e^{-7} \end{cases} \quad y = 5e^{7(x-1)} - 3xe^{7(x-1)}$$

$$\text{Answer: } y = (5-3x)e^{7(x-1)}$$