The final examination on Ordinary Differential Equations

(Innopolis University, Fall semester 2020, BS-II)

It is distance asynchronous individual written test to check that students understand and can apply main definitions, concepts and techniques covered in the lectures on weeks 10-11 and 13-14. According to the Syllabus, (https://moodle.innopolis.university/mod/resource/view.php?id=29351), the final examination costs 20 points. Examination has one (parameterized) variant for all enrolled students with 2 obligatory (to attempt) tasks and 1 bonus task (for volunteers to enjoy a piece of theory – to be awarded by bonus points at the discretion of the lecturer). The timeline of the examination follows:

- Examination specification and problem set on Moodle - by Sunday, November 29, 2020.
- Consultation on technical issues of the examination – on Friday December 4, 2020, at regular lecture class time 9:00-10:30 using hybrid mode (Zoom).
- Due day and time for uploading examination papers on Moodle - the midnight from Friday December 4 to Saturday November 5, 2020.
- Publication of the grades for students on Moodle - by Friday December 11, 2020.
- Office hours for students to argue grades - Saturday December 12, 2020, from 10:00 to 14:00 in room 507 or online (Skype/Zoom).

The main grading criterion for written test will be “proof of individual work” while computational errors will be treated as tiny mistakes (at most one-point deduction for each individual task).

Rules

1. Students that already have earned amount of points that is sufficient for the overall grade that they like may/can skip (neither attempt nor submit) the final examination. – Recall that according to the Syllabus (https://moodle.innopolis.university/mod/resource/view.php?id=29351) the grade scale is as follows: A – at least 117, B – at least 88, C – at least 73, and D – less than 73 points.
2. Because the examination is final, the Late Submission Policy from the Syllabus (https://moodle.innopolis.university/mod/resource/view.php?id=29351), but late submission will be treated as fail and will becomes the case for the retake (to be managed by the Department of Education).
3. “Proof of individual work” rule means that students must upload individual solutions in two files: the source file (in one of 3 formats Word 2007 document .docx, Word document .doc, or application/x-tex .tex with document class article) and the result of PDF-conversion of the source file (i.e. pdf file).
4. Solutions must be well-structured and formatted, concise (each task – at most 2 pages with font 12pt) but detailed at least at level of lecture notes for the weeks (for example – see slides 18-19, 26 and 28 in lecture notes for weeks 10&11 at https://moodle.innopolis.university/pluginfile.php/90440/mod_resource/content/6/ODEw10_11fall20.pdf, slides 47-48 in lecture notes for the week 13 at https://moodle.innopolis.university/pluginfile.php/99354/mod_resource/content/1/ODEw13fall20.

pdf, and slides 11-12 in lecture notes for the week 14 at https://moodle.innopolis.university/pluginfile.php/100777/mod_resource/content/1/ODEw14fall20.pdf).

5. Each submitted file should be named by student first name and surname (for example NikolayShilov.docx and NikolayShilov.pdf)
6. On the top of the front page of each submission should start with student first name and surname followed by birthdate (in the format $day.month.year$, for example $24.04.1961$).
7. Submissions with scanned or photo images of hand-written solutions will be discarded without consideration!

Tasks

Task 1 (10 points)

Is function $e^{t^{(month-day)}}$ on $[0,+\infty[$ a piece-with continuous? Does it have exponential order? Where (i.e. for what real s in $[0,+\infty[$) you can guarantee that the Laplace transform $L\left(e^{t^{(month-day)}}\right)$ is defined? – Your answer must be proven and explained!

Task 2 (10 points)

Solve (using elimination) the following system: $y_1' = y_1 - y_2$ and $y_2' = (day)y_1 + (month)y_2$. – Do not forget to prove the your solution is the most general!

Bonus Task (a piece of theory for volunteers)

Find a power series solution for the equation $y' = xy + y^2$ (i.e. find the general term of the series and study convergence of the series).