№	$f(t)$	$F(p)$	№	$f(t)$	$F(p)$
1	$I(t)$	$\frac{1}{p}$	17	$\frac{1}{a^2}(1-\cos at)$	$\frac{1}{p(p^2+a^2)}$
2	C	$\frac{C}{p}$	18	$\frac{1}{a^2}(e^{at}-1-at)$	$\frac{1}{p^2(p-a)}$
3	t	$\frac{1}{p^2}$	19	$\operatorname{sh} at$	$\frac{a}{p^2-a^2}$
4	t^n	$\frac{n!}{p^{n+1}}$	20	$\operatorname{ch} at$	$\frac{p}{p^2-a^2}$
5	$\delta(t)$	1	21	$(t+\frac{1}{2}at^2)\,e^{at}$	$\frac{p}{(p-a)^3}$
6	e^{at}	$\frac{1}{p-a}$	22	$(1+2at+\frac{1}{2}a^2t^2)\,e^{at}$	$\frac{p^2}{(p-a)^3}$
7	$t^n e^{at}$	$\frac{n!}{(p-a)^{n+1}}$	23	$(1+at)\,e^{at}$	$\frac{p}{(p-a)^2}$
8	$\sin at$	$\frac{a}{p^2+a^2}$	24	$\cos^2 at$	$\frac{p^2+2a^2}{p(p^2+4a^2)}$
9	$\cos at$	$\frac{p}{p^2+a^2}$	25	$\sin^2 at$	$\frac{2a^2}{p(p^2+4a^2)}$
10	$t\sin at$	$\frac{2pa}{(p^2+a^2)^2}$	26	$\sin\frac{a}{\sqrt{2}}t\ \operatorname{sh}\frac{a}{\sqrt{2}}t$	$\frac{a^2p}{p^4+a^4}$
11	$t\cos at$	$\frac{p^2-a^2}{(p^2+a^2)^2}$	27	$\cos\frac{a}{\sqrt{2}}t\ \operatorname{ch}\frac{a}{\sqrt{2}}t$	$\frac{p^3}{p^4+a^4}$
12	$e^{at}\sin bt$	$\frac{b}{(p-a)^2+b^2}$	28	$\frac{1}{2}(\operatorname{sh} at-\sin at)$	$\frac{a^3}{p^4-a^4}$
13	$e^{at}\cos bt$	$\frac{p-a}{(p-a)^2+b^2}$	29	$\frac{1}{2}(\operatorname{ch} at-\cos at)$	$\frac{a^2p}{p^4-a^4}$
14	$\frac{1}{a}e^{-\frac{t}{a}}$	$\frac{1}{1+ap}$	30	$\frac{1}{2}(\operatorname{sh} at+\sin at)$	$\frac{ap^2}{p^4-a^4}$
15	$\frac{1}{a}(e^{at}-1)$	$\frac{1}{p(p-a)}$	31	$\frac{1}{2}(\operatorname{ch} at+\cos at)$	$\frac{p^3}{p^4-a^4}$
16	$\frac{e^{at}-e^{bt}}{a-b}$	$\frac{1}{(p-a)(p-b)}$	32	$\frac{a\,e^{at}-b\,e^{bt}}{a-b}$	$\frac{p}{(p-a)(p-b)}$

12 a.1. $y'' - 3y' - 4y = 4x - 5$ $y(0) = -1$ $y'(0) = 2$

$\mathcal{L}(y') = s\mathcal{L}(y) - y(0) = s\mathcal{L}(y) + 1$

$\mathcal{L}(y'') = s^2\mathcal{L}(y) - sy(0) - y'(0) = s^2\mathcal{L}(y) + s - 2$

$\mathcal{L}(y'' - 3y' - 4y) = \mathcal{L}(4x - 5)$

$\mathcal{L}(y'') - 3\mathcal{L}(y') - 4\mathcal{L}(y) = 4\mathcal{L}(x) - 5\mathcal{L}(1)$

$s^2\mathcal{L}(y) + s - 2 - 3s\mathcal{L}(y) - 3 - 4\mathcal{L}(y) = 4 \cdot \frac{1}{s^2} - 5\frac{1}{s}$

$\mathcal{L}(y)(s^2 - 3s - 4) = \frac{4}{s^2} - \frac{5}{s} - s + 5$

$\mathcal{L}(y) = \frac{-s^3 + 5s^2 - 5s + 4}{(s+1)(s-4)s^2} = \frac{2}{s} - \frac{1}{s^2} - \frac{3}{s+1}$

$y = \mathcal{L}^{-1}(\mathcal{L}(y)) = \mathcal{L}^{-1}\left(\frac{2}{s} - \frac{1}{s^2} - \frac{3}{s+1}\right) =$

$= 2\mathcal{L}^{-1}\left(\frac{1}{s}\right) - \mathcal{L}^{-1}\left(\frac{1}{s^2}\right) - 3\mathcal{L}^{-1}\left(\frac{1}{s+1}\right) = 2 \cdot 1 - x - 3 \cdot e^{-x} =$

$= 2 - x - 3e^{-x}$

Answer: $y = 2 - x - 3e^{-x}$

12 a.2. $y'' + 4y = (8x + 4)e^{2x}$ $y(0) = 4$ $y'(0) = -1$

$\mathcal{L}(y'' + 4y) = \mathcal{L}((8x + 4)e^{2x})$

$\mathcal{L}(y'') + 4\mathcal{L}(y) = 8\mathcal{L}(xe^{2x}) + 4\mathcal{L}(e^{2x})$

$s^2\mathcal{L}(y) - 4s + 1 + 4\mathcal{L}(y) = 8 \cdot \frac{1}{(s-2)^2} + 4\frac{1}{s-2}$

$\mathcal{L}(y)(s^2 + 4) = \frac{8}{(s-2)^2} + \frac{4}{s-2} + 4s - 1$

$\mathcal{L}(y) = \frac{4s^3 - 17s^2 + 24s - 4}{(s^2 + 4)(s-2)^2} = \frac{4s - 2}{s^2 + 4} + \frac{1}{(s-2)^2}$

$$y = \mathcal{L}^{-1}\left(\frac{4s-2}{s^2+4} + \frac{1}{(s-2)^2}\right) = 4\mathcal{L}^{-1}\left(\frac{s}{s^2+4}\right) - \mathcal{L}^{-1}\left(\frac{2}{s^2+4}\right)$$

$$+ \mathcal{L}^{-1}\left(\frac{1}{(s-2)^2}\right) = 4\cos 2x - \sin 2x + e^{2x}$$

Answer: $y = 4\cos 2x - \sin 2x + e^{2x}$

12 a.3. $y'' + y' - 2y = -5\cos x + 5\sin x \quad y(0)=6 \quad y'(0)=0$

$$s^2\mathcal{L}(y) - 6s + s\mathcal{L}(y) - 6 - 2\mathcal{L}(y) = -5\cdot\frac{s}{s^2+1} +$$

$$+ 5\cdot\frac{1}{s^2+1}$$

$$\mathcal{L}(y)(s^2+s-2) - 6s - 6 = \frac{-5s+5}{s^2+1}$$

$$\mathcal{L}(y) = \frac{6s^3+6s^2+s+11}{(s^2+1)(s-1)(s+2)} = \frac{s-2}{s^2+1} + \frac{4}{s-1} + \frac{1}{s+2}$$

$$y = \mathcal{L}^{-1}\left(\frac{s-2}{s^2+1} + \frac{4}{s-1} + \frac{1}{s+2}\right) = \mathcal{L}^{-1}\left(\frac{s}{s^2+1}\right) - 2\mathcal{L}^{-1}$$

$$\cdot\left(\frac{1}{s^2+1}\right) + 4\mathcal{L}^{-1}\left(\frac{1}{s-1}\right) + \mathcal{L}^{-1}\left(\frac{1}{s+2}\right) = \cos x - 2\sin x +$$

$$+ 4e^{x} + e^{-2x}$$

Answer: $y = \cos x - 2\sin x + 4e^{x} + e^{-2x}$

12 a.4 $y'' - 6y' + 15y = 2\sin 3x \quad y(0) = -1 \quad y'(0) = -4$

$$s^2 \mathcal{L}(y) + s + 4 - 6(s\mathcal{L}(y) + 1) + 15\mathcal{L}(y) = 2 \cdot \frac{3}{s^2+9}$$

$$\mathcal{L}(y)(s^2 - 6s + 15) + s + 4 - 6 = \frac{6}{s^2+9}$$

$$\mathcal{L}(y) = \frac{-s^3 + 2s^2 - 9s + 24}{(s^2+9)(s^2-6s+15)} = \frac{\frac{s}{2} - \frac{115}{10}}{s^2 - 6s + 15} +$$

$$+ \frac{\frac{s}{10} + \frac{1}{10}}{s^2+9} = -\frac{11}{10} \cdot \frac{s-3}{(s-3)^2 + 6} - \frac{4}{5\sqrt{6}} \cdot \frac{\sqrt{6}}{(s-3)^2+6} +$$

$$+ \frac{1}{10} \frac{s}{s^2+9} + \frac{1}{30} \cdot \frac{3}{s^2+9}$$

$$y = \mathcal{L}^{-1}\left(-\frac{11}{10} \frac{s-3}{(s-3)^2+6} - \frac{4}{5\sqrt{6}} \cdot \frac{\sqrt{6}}{(s-3)^2+6} + \frac{1}{10} \cdot \frac{s}{s^2+9} +\right.$$

$$\left. + \frac{1}{30} \cdot \frac{3}{s^2+9}\right) = -\frac{11}{10} e^{3x} \cos(\sqrt{6}x) - \frac{4}{5\sqrt{6}} e^{3x} \sin(\sqrt{6}x) +$$

$$+ \frac{1}{10} \cos(3x) + \frac{1}{30} \sin(3x)$$

Answer: $y = -\frac{11}{10} e^{3x} \cos(\sqrt{6}x) - \frac{4}{5\sqrt{6}} e^{3x} \sin(\sqrt{6}x) +$

$$+ \frac{1}{10} \cos(3x) + \frac{1}{30} \sin(3x)$$

12 a.5. $2y'' + 3y' - 2y = xe^{-2x} \quad y(0) = 0 \quad y'(0) = -2$

$$2(s^2 \mathcal{L}(y) + 2) + 3(s\mathcal{L}(y)) - 2\mathcal{L}(y) = \frac{1}{(s+2)^2}$$

$$\mathcal{L}(y)(2s^2 + 3s - 2) + 4 = \frac{1}{(s+2)^2}$$

$$\mathcal{L}(y) = \frac{-4s^2 - 16s - 15}{(s+2)^2 \cdot 2(s+2)(s-\frac{1}{2})} =$$

$$\mathcal{L}(y)=\frac{-\frac{96}{125}}{s-\frac{1}{2}}+\frac{\frac{96}{125}}{s+2}-\frac{\frac{2}{25}}{(s+2)^2}-\frac{\frac{1}{5}}{(s+2)^3}$$

$$y=\mathcal{L}^{-1}\left(\frac{-\frac{96}{125}}{s-\frac{1}{2}}+\frac{\frac{96}{125}}{s+2}-\frac{\frac{2}{25}}{(s+2)^2}-\frac{\frac{1}{5}}{(s+2)^3}\right)=$$

$$=-\frac{96}{125}e^{\frac{x}{2}}+\frac{96}{125}e^{-2x}-\frac{2}{25}xe^{-2x}-\frac{1}{10}x^2e^{-2x}$$

Answer: $y=-\frac{96}{125}e^{\frac{x}{2}}+\frac{96}{125}e^{-2x}-\frac{2}{25}xe^{-2x}-\frac{1}{10}x^2e^{-2x}$

12 b. 1. A. $\sum\limits_{n=0}^{\infty}\frac{1}{1+n^3}(x+6)^n$

$$\sum_{n=0}^{\infty}\frac{1}{1+n^3}(x+6)^n=\sum_{n=0}^{\infty}a_n(x-x_0)^n$$

$$L=\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\lim_{n\to\infty}\left|\frac{1+n^3}{1+(n+1)^3}\right|=1=$$

$$=\frac{1}{R}=1;\ R=1;\ x\in[x_0-R;\ x_0+R]=[-7;-5]$$

Answer: $R=1$ $x\in[-7;-5]$

B. $\sum\limits_{n=0}^{\infty}\frac{n}{n!}(x-3)^n$

$$L=\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\lim_{n\to\infty}\left(\frac{(n+1)!}{n+1}\cdot\frac{n}{n!}\right)=$$

$$=\lim_{n\to\infty}n=\infty=\frac{1}{R},\quad R=0.\quad x\in[x_0-R,x_0+R]=$$

$$=[3;3]$$

Answer: $R=0$, $x\in[3;3]$

C. $\sum_{n=0}^{\infty} \frac{(-1)^n n}{4^n}(x+3)^n$

$$L = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(-1)^{n+1}(n+1)}{4^{n+1}}\cdot\frac{4^n}{(-1)^n n}\right| =$$

$$= \lim_{n\to\infty}\left|\frac{-(n+1)}{4n}\right| = \frac{1}{4} = \frac{1}{R} \quad R=4$$

$$x \in [x_0 - R, x_0 + R] = [-7; 1]$$

Answer: $R=4$, $x \in [-7, 1]$

D. $\sum_{n=0}^{\infty} \frac{n!}{(n+1)^2+4n}(5x+3)^n$

$$\sum_{n=0}^{\infty}\frac{n!}{(n+1)^2+4n}(5x+3)^n = \sum_{n=0}^{\infty}\frac{n!\,5^n}{(n+1)^2+4n}\left(x+\frac{3}{5}\right)^n$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)!\cdot 5^{n+1}}{(n+2)^2+4(n+1)}\cdot\frac{(n+1)^2+4n}{n!\,5^n}\right| =$$

$$= \infty = \frac{1}{R}, \quad R=0 \quad x \in [x_0 - R, x_0 + R] = \left[-\frac{3}{5}; -\frac{3}{5}\right]$$

Answer: $R=0$, $x \in \left[-\frac{3}{5}; -\frac{3}{5}\right]$

12. b2. A $\quad y(x)=\sum_{n=0}^{\infty} a_n x^n$

$$(1+3x^2)\,y''+3x^2y'-2y$$

$$y'=\sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$y''=\sum_{n=2}^{\infty} n(n-1)\,a_n x^{n-2}$$

$$(1+3x^2)y''+3x^2y'-2y=\sum_{n=2}^{\infty} n(n-1)\,a_n x^{n-2}+$$

$$+3\sum_{n=2}^{\infty} n(n-1)\,a_n x^{n}+3\sum_{n=1}^{\infty} n a_n x^{n+1}-2\sum_{n=0}^{\infty} a_n x^n=$$

$$=\sum_{n=0}^{\infty}(n+2)(n+1)\,a_{n+2}x^n+3\sum_{n=0}^{\infty}(n+2)(n+1)\,a_{n+2}x^{n+2}+$$

$$+3\sum_{n=0}^{\infty}(n+1)\,a_{n+1}x^{n+2}-2\sum_{n=0}^{\infty} a_n x^n=$$

$$=\sum_{n=0}^{\infty}\big[(n+2)(n+1)\,a_{n+2}x^n+3(n+2)(n+1)\,a_{n+2}x^{n+2}+$$

$$+3(n+1)\,a_{n+1}x^{n+2}-2a_n x^n\big]$$

Answer: $(1+3x^2)y''+3x^2y'-2y=$

$$=\sum_{n=0}^{\infty}\big[(n+2)(n+1)\,a_{n+2}x^n+3(n+2)(n+1)\,a_{n+2}x^{n+2}+$$

$$+3(n+1)\,a_{n+1}x^{n+2}-2a_n x^n\big]$$

B $(1+2x^2)y''+(2-3x)y'+4y$

$$(1+2x^2)y''+(2-3x)y'+4y=\sum_{n=2}^{+\infty} n(n-1)a_n x^{n-2}+$$

$$+2\sum_{n=2}^{\infty} n(n-1)a_n x^n+2\sum_{n=1}^{\infty} n a_n x^{n-1}-3\sum_{n=1}^{\infty} n a_n x^n+$$

$$+4\sum_{n=0}^{\infty} a_n \cancel{x}^n=\sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^n+$$

$$+2\sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^{n+2}+2\sum_{n=0}^{\infty}(n+1)a_{n+1}x^n-$$

$$-3\sum_{n=0}^{\infty}(n+1)a_{n+1}x^{n+1}+4\sum_{n=0}^{\infty}a_n x^n=$$

$$=\sum_{n=0}^{\infty}[(n+2)(n+1)a_{n+2}x^n+2(n+2)(n+1)a_{n+2}$$

$$x^{n+2}+2(n+1)a_{n+1}x^n-3(n+1)a_{n+1}x^{n+1}+4a_n x^n]$$

Ansner: $(1+2x^2)y''+(2-3x)y'+4y=$

$$=\sum_{n=0}^{\infty}[(n+2)(n+1)a_{n+2}x^n+2(n+2)(n+1)a_{n+2}\cancel{x}^{n+2}+$$

$$+2(n+1)a_{n+1}x^n-3(n+1)a_{n+1}x^{n+1}+4a_n x^n]$$

C. $(1+x^2)y''+(2-x)y'+3y$

$$(1+x^2)y''+(2-x)y'+3y=\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2}+$$

$$+\sum_{n=2}^{\infty} n(n-1)a_n x^n+2\sum_{n=1}^{\infty} n a_n x^{n-1}-\sum_{n=1}^{\infty} n a_n x^n+$$

$$+3\sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^n + \sum_{n=0}^{\infty}(n+2)(n+1)$$

$$a_{n+2}x^{n+2} + 2\sum_{n=0}^{\infty}(n+1)a_{n+1}x^n - \sum_{n=0}^{\infty}(n+1)a_{n+1}x^{n+1} +$$

$$+3\sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty}\Big[(n+2)(n+1)a_{n+2}x^n + (n+2)(n+1)$$

$$a_{n+2}x^{n+2} + 2(n+1)a_{n+1}x^n - (n+1)a_{n+1}x^{n+1} +$$

$$+3a_n x^n\Big]$$

Answer: $(1+x^2)y'' + (2-x)y' + 3y =$

$$= \sum_{n=0}^{+\infty}\Big[(n+2)(n+1)a_{n+2}x^n + (n+2)(n+1)a_{n+2}x^{n+2} +$$

$$+2(n+1)a_{n+1}x^n - (n+1)a_{n+1}x^{n+1} + 3a_n x^n\Big]$$