First mid-term examination on Ordinary Differential Equations

(Innopolis University, Fall semester 2020, BS-III)

Description

It is distance asynchronous individual written test to check that students understand and can apply main definitions, concepts and techniques covered in the lectures on weeks 5-9. According to the Syllabus, (https://moodle.innopolis.university/mod/resource/view.php?id=29351), the each mid-term and the final examination cost is 20 points each. Examination has one (parameterized) variant for all enrolled students with 2 obligatory (to attempt) tasks and 1 bonus task (for volunteers to enjoy a piece of theory – to be awarded by bonus points at the discretion of the lecturer). The timeline of the examination follows:

- Examination problems to be uploaded to Moodle by Saturday October 31, 2020.
- Consultation on technical issues of the examination – on Friday November 6, 2020, at regular lecture class time 9:00-10:30 using hybrid mode.
- Individual solutions to be uploaded to Moodle for grading by Friday November 6, 2020.
- Submissions to be graded by Saturday November 14, 2020.

The main grading criterion for written test will be "proof of individual work" while computational errors will be treated as tiny mistakes (at most one-point deduction for each individual task).

Rules

1. "Proof of individual work" rule means that
   - students must upload individual solutions in two files: the source file (in one of 3 formats Word 2007 document .docx, Word document .doc, or application/x-tex .tex with any plain class like article) and the result of PDF-conversion of the source file (i.e. pdf file);
   - solutions must be well-structured and formatted, concise (each task – at most 2 pages with font 12pt) but detailed at least at level of lecture notes for the weeks (for example – see slides 6-7 and 10-11 in lecture notes for week at https://moodle.innopolis.university/mod/resource/view.php?id=43151).
2. Each submitted file should be named by student first name and surname (for example NikolayShilov.docx and NikolayShilov.pdf)
3. On the top of the front page of each submission should start with student first name and surname followed by birthdate (in the format $day.month.year$, for example $24.04.1961$).
4. Submissions with scanned or photo images of hand-written solutions will be discarded without consideration!

Tasks

Task 1 (10 points)

Characterize equation $y''y' = {}^1\!/_2$ in terms introduced in lectures for weeks 1-3 and 8-9. Since it matches two patterns for order reduction, solve the following initial value problem $y(1) = (day)$ and $y'(1) = (month)$ for this equation using any of these two methods that guarantees

uniqueness of the solution; use any of these method if you cannot guaranty uniqueness of the solution but declare it explicitly. The solution process must be explained!

Task 2 (10 points)

Characterize equation $y'' + (month)y' - (day)y = 0$ in terms introduced in lectures for weeks 1-3 and 8-9. Solve the equation analytically, i.e. find a solution as general as you can (i.e. if it is the most general then your solution must prove it explicitly). The solution process must be explained!

Bonus Task (a piece of theory for volunteers)

Let $a < b$ be real numbers, and $f: [a, b] \rightarrow \mathbf{R}$ be a continuous function. Solve analytically the equation $y'' = f(y)$, i.e. find a real-valued solution as general as you can (i.e. if it is the most general then your solution must proof it explicitly). The solution process must be explained!