3. 1) $y' + x(y^2 + y) = 0, \quad y(2) = 1$

$$\frac{dy}{dx} = -x(y^2+y) \qquad \frac{dy}{y^2+y} = -x\,dx$$

$$\int \frac{dy}{y^2+y} = \int \frac{dy}{y^2\left(\frac{1}{y}+1\right)} = \left\{ t = \frac{1}{y}+1,\ dy = -y^2\,dt \right\} =$$

$$= -\int \frac{dt}{t} = -\ln|t| + C$$

$$-\ln|t| = -\frac{x^2}{2} + C \qquad \ln\left|\frac{1}{y}+1\right| = \frac{x^2}{2} + C$$

$y(2) = 1: \quad \ln\left|\frac{1}{1}+1\right| = \frac{2^2}{2} + C; \ \ln 2 = 2 + C$

$C = \ln 2 - 2$

Answer: $\ln\left|\frac{1}{y}+1\right| = \frac{x^2}{2} + \ln 2 - 2$

2) $y' + \frac{(y+1)(y-1)(y-2)}{x+1} = 0, \quad y(1) = 0$

$$\frac{dy}{(y+1)(y-1)(y-2)} = -\frac{dx}{x+1}$$

$$\int \frac{dy}{(y+1)(y-1)(y-2)} = \frac{1}{6}\int \frac{dy}{y+1} - \frac{1}{2}\int \frac{dy}{y-1} + \frac{1}{3}\int \frac{dy}{y-2}$$

$$\ln|y+1| - 3\ln|y-1| + 2\ln|y-2| = -6\ln|x+1| + C$$

$y(1) = 0: \ \ln|1| - 3\ln|-1| + 2\ln|-2| = -6\ln|2| + C$

$C = 8\ln 2$

Answer: $\ln|y+1| - 3\ln|y-1| + 2\ln|y-2| = -6\ln|x+1| + 8\ln 2$

3) $y' = \dfrac{y}{x+2y-2}$

$y' = \dfrac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}$ $\quad \begin{Vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{Vmatrix} = \begin{Vmatrix} 0 & 1 \\ 1 & 2 \end{Vmatrix} = -1 \neq 0$

$\begin{cases} \alpha a_1 + \beta b_1 + c_1 = 0 \\ \alpha a_2 + \beta b_2 + c_2 = 0 \end{cases} \quad \begin{cases} \beta = 0 \\ \alpha + 2\beta - 2 = 0 \end{cases} \quad \begin{cases} \beta = 0 \\ \alpha = 2 \end{cases}$

$\begin{cases} x = \hat{x} + \alpha \\ y = \hat{y} + \beta \end{cases} \quad \begin{cases} x = \hat{x} + 2 \\ y = \hat{y} \end{cases}$

$\dfrac{d(\hat{y})}{d(\hat{x}+2)} = \dfrac{\hat{y}}{(\hat{x}+2)+2\hat{y}-2} = \dfrac{\hat{y}}{\hat{x}+2\hat{y}}; \quad \dfrac{d\hat{y}}{d\hat{x}} = \dfrac{\hat{y}}{\hat{x}+2\hat{y}}$

$\hat{y} = t\hat{x}, \quad \dfrac{d\hat{y}}{d\hat{x}} = t'\hat{x} + t, = \dfrac{t\hat{x}}{\hat{x}+2t\hat{x}} = \dfrac{t}{1+2t}$

$\hat{x}\dfrac{dt}{d\hat{x}} = \dfrac{t}{1+2t} - t = \dfrac{t-t-2t^2}{1+2t} = \dfrac{-2t^2}{1+2t}$

$dt \cdot \dfrac{1+2t}{-2t^2} = \dfrac{d\hat{x}}{\hat{x}}; \quad \dfrac{1}{2t} - \ln|t| = \ln|\hat{x}| + C$

$t = \dfrac{\hat{y}}{\hat{x}} = \dfrac{y}{x-2}; \quad \dfrac{x-2}{2y} - \ln\left|\dfrac{y}{x-2}\right| = \ln|x-2| + C$

$y=0$ solution $\rightarrow$

Answer: $\dfrac{x-2}{2y} - \ln\left|\dfrac{y}{x-2}\right| = \ln|x-2| + C$

& $y = 0$

4) $(x-2y-1)dx+(3(x-2y)+2)dy=0$

$(x-2y-1)dx=(-3x+6y-2)\,dy$

$\frac{dy}{dx}=\frac{x-2y-1}{-3x+6y-2}$ $\begin{cases}\alpha-2\beta-1=0\\-3\alpha+6\beta-2=0\end{cases}$

$\left\|\begin{matrix}1 & -2\\-3 & 6\end{matrix}\right\|=6-6=0$

$x-2y=z$; $-3x+6y=-3z$

$dz=d(x-2y)=dx-2dy$, $dy=\frac{dx-dz}{2}$

$\frac{1}{2}\,\frac{dx-dz}{dx}=\frac{z-1}{-3z-2}$ $1-\frac{dz}{dx}=\frac{2z-2}{-3z-2}$

$1+\frac{2z-2}{3z+2}=\frac{5z}{3z+2}=\frac{dz}{dx}$ $dx=dz\cdot\frac{3z+2}{5z}$

$z=0$
$y=\frac{x}{2}$
solution

$\int dx=\frac{3}{5}\int dz+\frac{3}{5}\int\frac{dz}{z}$; $x=\frac{3}{5}z+\frac{2}{5}\ln|z|+C$

$5x=3z+2\ln|z|+C$

$5x=3(x-2y)+2\ln|x-2y|+C$

Answer: $5x=3(x-2y)+2\ln|x-2y|+C$

& $y=\frac{x}{2}$