Repeat-Until-Success Circuit on *SliQSim*

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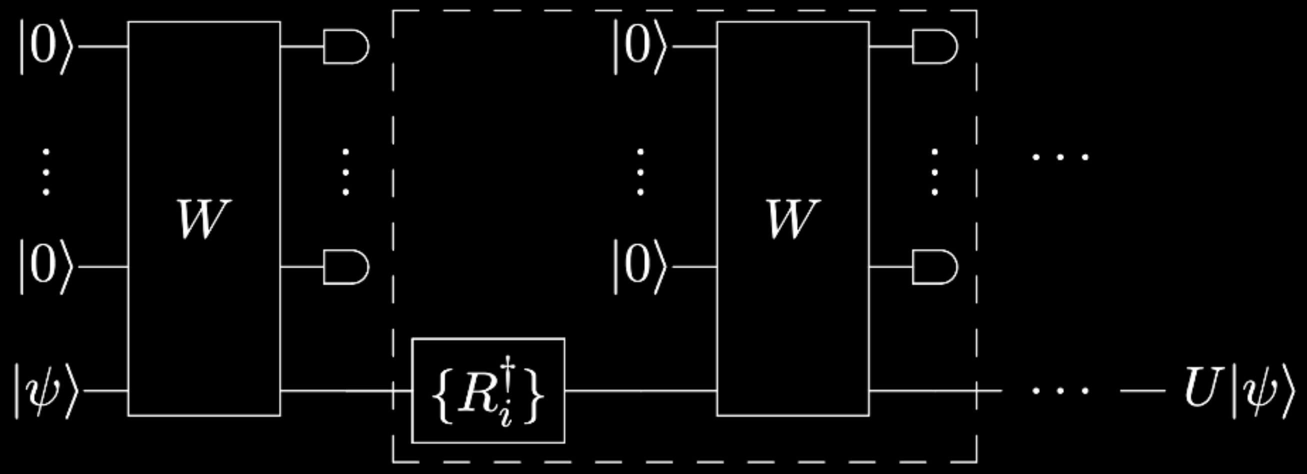
Repeat-Until-Success Circuit

- Non-deterministic decomposition approach
- Valid only when the measured ancilla in a specific state
- With less (expected) cost compared to many other methods

$R_i\,|\phi\rangle$ = state subsequent to measurements revealing failure



Method	T **count**
Phase kickback [KSV02]	$O(\log 1/\epsilon)$ (implementation dependent)
PAR [JWM+12]	$O(\log 1/\epsilon)$
Selinger [Sel12]	$4\log(1/\epsilon) + 11$
Ross-Selinger [RS14]	$3\log(1/\epsilon) + O(\log\log 1/\epsilon)$
KMM [KMM12b]	$3.21\log_2(1/\epsilon) - 6.93$
Floating-point [WK13]	$1.14\log_2(10^\gamma) + 8\log_2(10^{-\gamma}/\epsilon)$
RUS (axial)	$1.26\log_2(1/\epsilon) - 3.53$

RUS Ckt Simulating…

In our research:

- We can manually decide the state into which the ancillae bits collapse.
- Desired performance is determined by **time / space cost**, not T gate count.
- Circuits can include Clifford + T gates, and **beyond**.

However, when concerning real physical computations, restrictions come into play.

RUS Implementation on *SliQSim*

Flow

1. Encountering `RUS` Gate
2. Build `bigBDD`
3. Move the to-be-measured qubits to top
4. Measure along the desired branch
5. Collapse the states & Reset measured qubits

Encountering RUS gate

- A notation similar gates in `OPENQASM 2.0`

  `rus q1, q2, …, [s1], [s2], …`

- `s = (s1, s2, s3, …, sn)` is the desired Z-measure value of `q = (q1, q2, q3, …, qn)`.

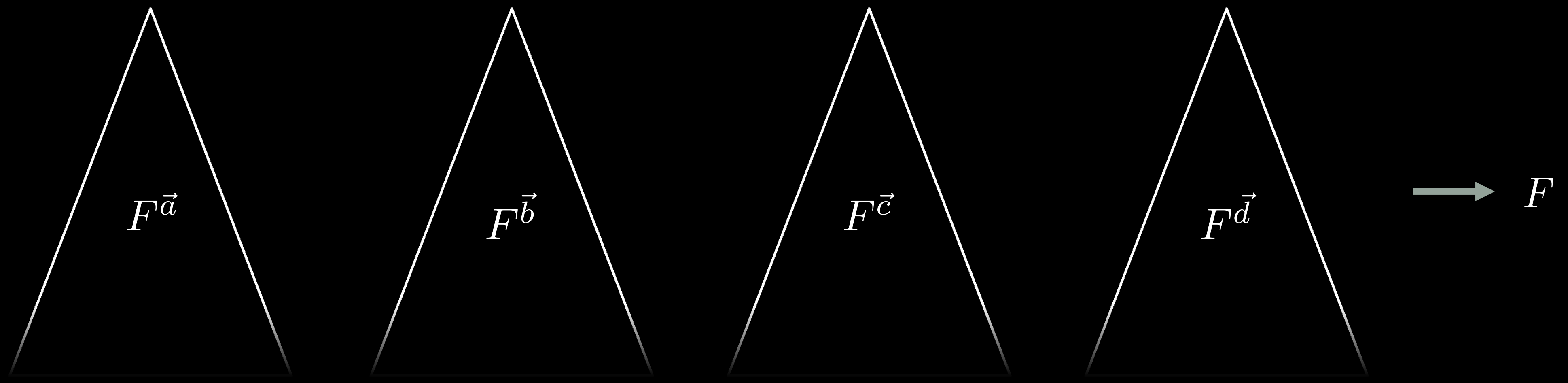
tests > bell_state_rus.qasm

```
OPENQASM 2.0;
include "qelib1.inc";
qreg q[2];
creg c[2];
h q[0];
cx q[0],q[1];
rus q[0],[0];

measure q[0] -> c[0];
measure q[1] -> c[1];
```

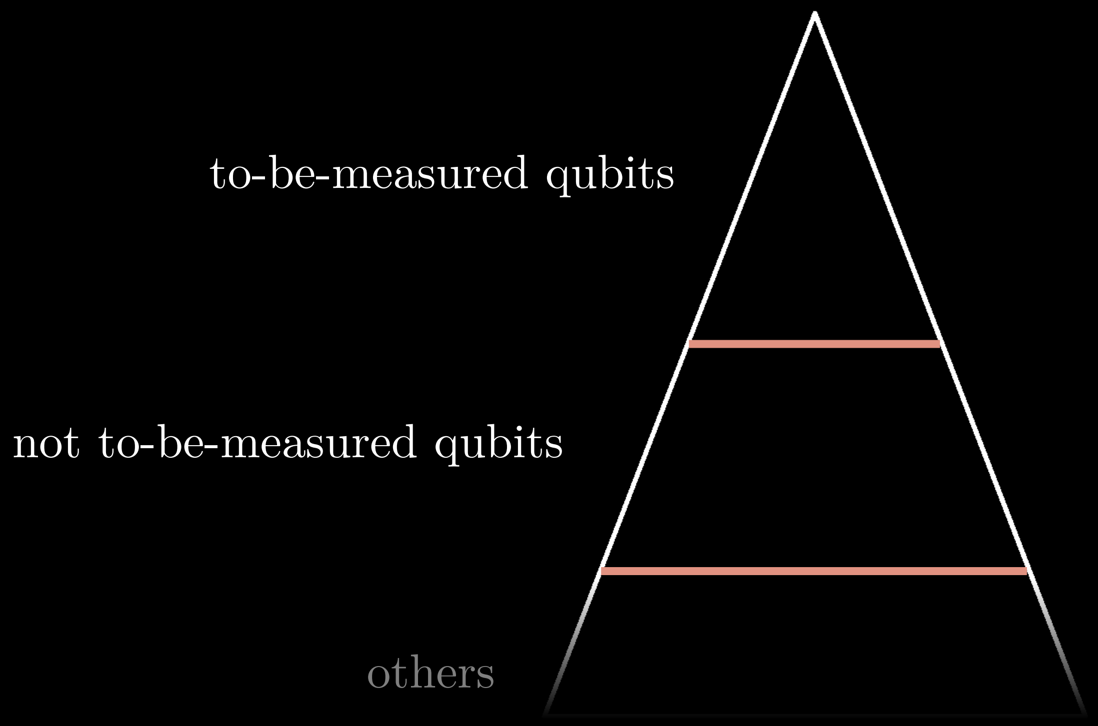
Build `bigBDD`

- Use `Cudd_bddAnd` and `Cudd_bddOr` to build the `bigBDD` F for the state vector.



Move the to-be-measured qubits to top

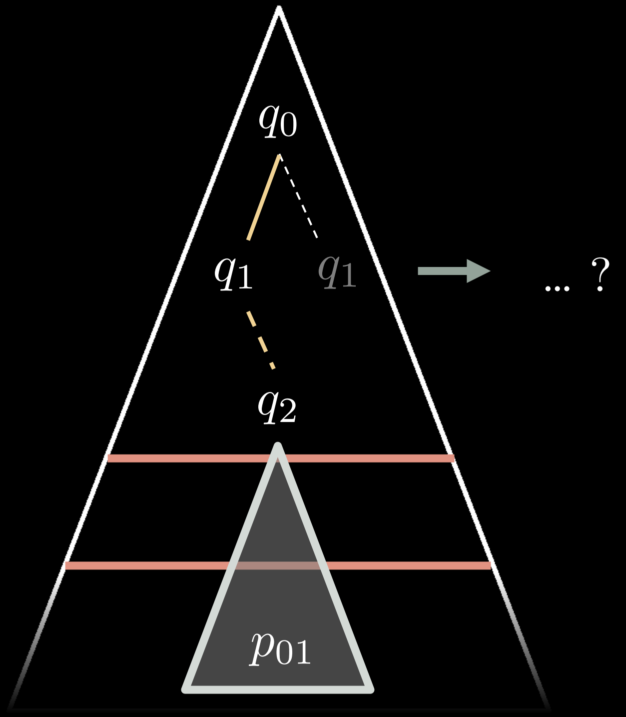
- Use `Cudd_ShuffleHeap` to move to-be-measured qubits to top of F.



Measure along the desired state branch

- Trace along the branch with $\mathbf{q} = \mathbf{s}$
- Record the final probability p
- `rus_normalize_factor` $/= \sqrt{p}$

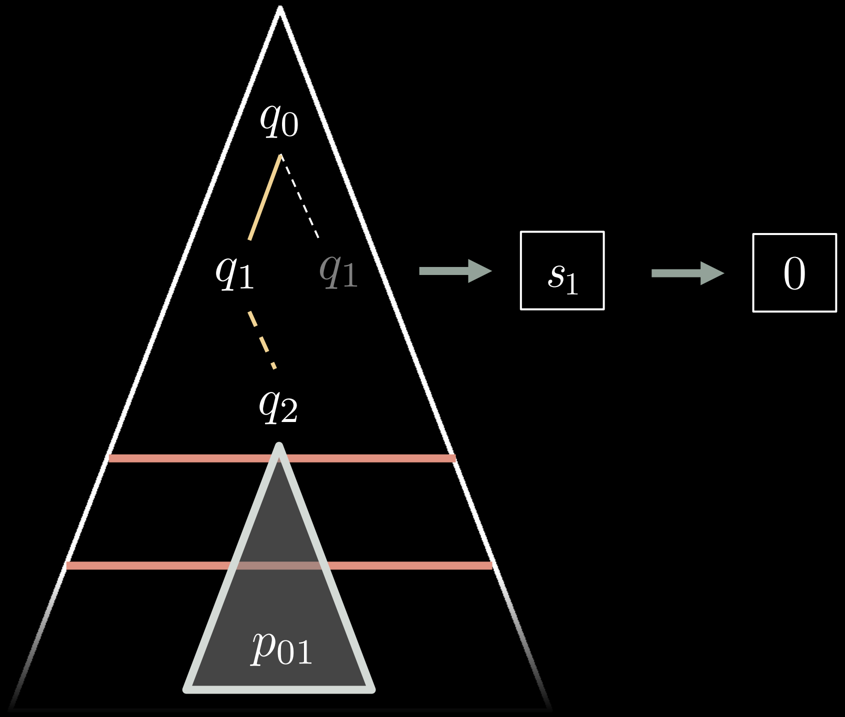
$$p_{ij} = \mathsf{Pr}[q_i = |j\rangle]$$



Collapse the state & Reset measured qubits

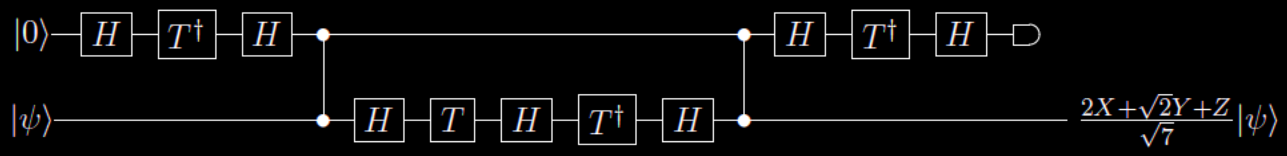
- For each qubit, collapse it into its corresponding state, by `Cudd_Cofactor`
- Set the measured qubits to 0

$$p_{ij} = \mathsf{Pr}[q_i = |j\rangle]$$



Experiments

Exp1: $\frac{2X + \sqrt{2}Y + Z}{\sqrt{7}}$



Compared with qiskit simulation result,

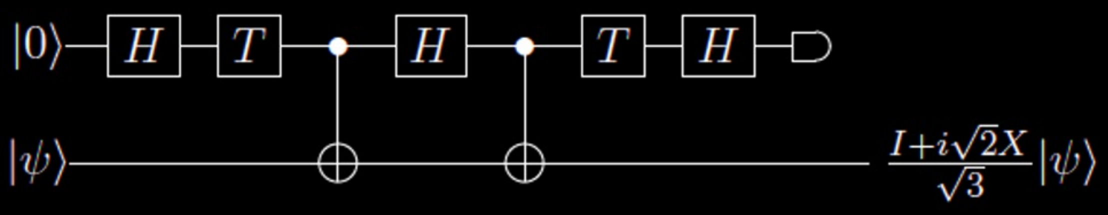
$|\psi\rangle = |0\rangle$

- qiskit: `[0.37796447+0.j  ,  0.75592895+0.53452248j]`
- ***SliQSim***: **`[0.267261-0.267261i, 0.912487-0.156558i]`** (With global phase $e^{0.785i}$)

$|\psi\rangle = |+\rangle$

- qiskit: `[0.80178373-0.37796447j, 0.26726124+0.37796447j]`
- ***SliQSim***: **`[0.299685-0.834208i, 0.456243+0.078279i]`** (With global phase $e^{0.785i}$)

Exp2: $\frac{I+\sqrt{2}X}{\sqrt{3}}$



- Execute 4096 times (to check the precision loss)

Compared with qiskit simulation result,

$|\psi\rangle = |0\rangle$

- qiskit: [`-0.35404678, -0.93522771i]`
- ***SliQSim***: **`[-0.354047, -0.935228i]`**

$|\psi\rangle = (0.0854 + 0.354i)\,|0\rangle + (0.146 - 0.354i)\,|1\rangle$

- qiskit: `[-0.63285076-0.26213537j, 0.27880398-0.67309235j]`
- ***SliQSim***: **`[-0.632851-0.262135j, 0.278804-0.673092j]`**

Future Work

RUS Circuit Synthesis

- Adopt existing RUS Circuit Synthesis Algorithms to support small-angle rotation as well as other non - Clifford + T gates.
- Design new RUS Synthesis Algorithm for further minimizing number of T counts if physical implementation is required.
- 3 D. V. Lindberg and H. K. H. Lee, “Optimization under constraints by applying an asymmetric entropy measure,” J. Comput. Graph. Statist., vol. 24, no. 2, pp. 379–393, Jun. 2015, doi: 10.1080/10618600.2014.901225. [Online]. Available: [3]

Reference

- 1 Y.-H. Tsai, J.-H. R. Jiang, and C.-S. Jhang, “Bit-slicing the Hilbert space: Scaling up accurate quantum circuit simulation,” in Proc. ACM/IEEE Design Automation Conference (DAC), 2021. [Online]. Available: [1]
- 2 A. Bocharov, M. Roetteler, and K. M. Svore, “Efficient synthesis of universal repeat-until-success quantum circuits,” Phys. Rev. Lett., vol. 114, no. 8, p. 080502, Feb. 2015. [Online]. Available: [2]
- 3 D. V. Lindberg and H. K. H. Lee, “Optimization under constraints by applying an asymmetric entropy measure,” J. Comput. Graph. Statist., vol. 24, no. 2, pp. 379–393, Jun. 2015, doi: 10.1080/10618600.2014.901225. [Online]. Available: [3]
- [4] A. Paetznick and K. M. Svore, “Repeat-Until-Success: Non-deterministic decomposition of single-qubit unitaries,” in Proc. IEEE International Conference on Quantum Computing and Engineering (QCE), 2014, pp. 146–152. [Online].