HierarchicalForecast: A Reference Framework for Hierarchical Forecasting

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Abstract

Large collections of time series data are commonly organized into structures with different levels of aggregation; examples include product and geographical groupings. It is often important to ensure that the forecasts are coherent so that the predicted values at disaggregate levels add up to the aggregate forecast. The growing interest of the Machine Learning community in hierarchical forecasting systems indicates that we are in a propitious moment to ensure that scientific endeavors are grounded on sound baselines. For this reason, we put forward the HierarchicalForecast library, which contains preprocessed publicly available datasets, evaluation metrics, and a compiled set of statistical baseline models. Our Python-based reference framework aims to bridge the gap between statistical and econometric modeling, and Machine Learning forecasting research.

Keywords: Hierarchical Forecasting, Econometrics, Datasets, Evaluation, Benchmarks

1. Introduction

Multivariate time series data can often be organized into hierarchical structures with different levels of aggregation. Independently forecasting all the series is unlikely to produce *coherent* forecasts, that is, forecasts which satisfy the aggregation constraints as the original data. In a nutshell, hierarchical time series forecasting is a multitask forecasting problem with a set of linear aggregation constraints to be satisfied.

While summing forecasts for the most disaggregated level (called *bottom-up*) will provide coherent forecasts, it can perform poorly on highly disaggregated series. Novel hierarchical forecasting methods first generate independent forecasts for each series (called *base* forecasts), then reconcile them to produce coherent forecasts (Wickramasuriya et al., 2019).

There is substantial interest on Hierarchical Forecasting from both industry and academia, as shown by the international forecasting competitions GEFCOM2012 (Hong et al., 2014) and M5 (Makridakis et al., 2021), and the Machine Learning (ML) community's growing interest in the topic (Rangapuram et al., 2021; Han et al., 2021; Paria et al., 2021; Olivares et al., 2021; Kamarthi et al., 2022; Panagiotelis et al., 2023).

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An enabling condition for the systematic development of useful forecasting methods is the ability to empirically evaluate and compare newly proposed methods with state-of-the-art and well-established baselines. However, ML research on hierarchical forecasting faces two challenges. First, while Python continues to grow in popularity among the ML community (Piatetsky, 2018), it lacks many relevant statistical and econometric modeling packages (often originally developed in the R language). As a result, researchers must build Python bridges to access the R baselines' implementations. Rapid, substantial development in statistical methods for hierarchical forecasting exacerbates this problem. Secondly, the Python global interpreter lock limits its programs to use a single thread, which prevents us from taking advantage of the available multi-core resources to speed up the software. When implemented naively, statistical baselines in Python take excessively long execution times, surpassing those of more complex methods, and discouraging their use.

We introduce the open-source benchmark library HierarchicalForecast to tackle these challenges¹. Our work builds upon Python's fastest open-source ETS/ARIMA² implementations and well-performing neural forecasting methods to improve the availability, utility, and adoption of hierarchical forecast reference baselines.

2. Library description (/features)

Compared to existing hierarchical forecasting software libraries, HierarchicalForecast has the following distinctive features:

Minimal dependencies. Our library is built with minimal dependencies using NumPy for linear algebra and array operations (Harris et al., 2020), Pandas for data manipulation (McKinney, 2010) and sklearn for predictive modeling (Pedregosa et al., 2011). We compute base forecasts using the statsforecast package (Garza et al., 2022), which provides the fastest implementations of AutoARIMA and AutoETS (Hyndman and Khandakar, 2008) based on NumBa (Lam et al., 2015). This just-in-time compiler optimizes Python's NumPy code to reach execution speed attainable with native C language code.

Comprehensive set of hierarchical forecasting methods. Some hierarchical forecasting Python implementations are available in the following packages: gluonts (Alexandrov et al., 2020), darts (Herzen et al., 2022), scikit-hts (Mazzaferro, 2019), sktime (Löning et al., 2019), and pyhts (Zhang et al., 2022). However, as seen in Table 1, each of these libraries only hosts a subset of the State-Of-The-Art (SOTA) methods. Our library provides unified access to a comprehensive set of these methods and enables robust performance validation of the implementations to ensure the Python community's access to efficient and reliable baselines. HierarchicalForecast's curated collection of reference algorithms includes BottomUp (Orcutt et al., 1968; Dunn et al., 1976), TopDown (Gross and Sohl, 1990; Fliedner, 1999), MiddleOut, MinTrace (Wickramasuriya et al., 2019), and ERM (Ben Taieb and Koo, 2019) for point forecasting, and it is the only Python library so far that includes SOTA probabilistic forecasting methods, including PERMBU (Ben Taieb et al., 2017), NORMALITY (Wickramasuriya, 2023), and BOOTSTRAP (Panagiotelis et al., 2023).

^{1.} License: CC-by 4.0, see https://creativecommons.org/licenses/by/4.0/. Code and documentation are available in https://github.com/Nixtla/hierarchicalforecast.

^{2.} Autoregressive Integrated Moving Average (ARIMA) and Exponential Smoothing (ETS) are two of the most important univariate forecasting baseline methods.

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	BottomUp	Point F. Methods ottomUp TopDown MiddleOut Comb MinTrace							Probabilistic F. Methods ERM BOOTSTRAP NORMALITY PERMBU						
Library	Боссомор	(f)	(a)	(p)	(f)	(ols)	(wls)	(ols)	(ws)	(wv)	(cf)	(lasso)		NOIGIALITI	Тышьо
g hierarchicalforecast	/	1	1	1	✓	1	1	1	1	/	/	1	✓	/	1
darts/hts	✓	1	X	X	X	1	1	1	1	1	X	X	X	X	X
scikit-hts	/	X	×	X	X	1	1	1	1	✓	X	X	X	X	X
A pyhts	✓	X	X	X	Х	✓	✓	✓	✓	✓	X	X	X	X	Х
fable	/	1	1	1	1	1	1	1	1	1	Х	Х	Х	Х	Х
⊈ hts	✓	1	1	1	✓	1	1	1	1	1	×	X	✓	✓	X
gluonts/hts	/	X	X	X	X	1	1	1	1	1	1	✓	X	X	✓

Table 1: Availability of hierarchical forecasting methods in considered software libraries.

Forecast evaluation and visualization. Our library facilitates a complete forecast evaluation across the levels of the hierarchical structure. It includes multiple standard accuracy measures for point forecasts. Furthermore, it also includes multiple scoring rules to evaluate probabilistic forecasts, such as the multivariate logarithmic and energy scores (Panagiotelis et al., 2023) and the univariate scaled continuous ranked probability score (sCRPS) (Gneiting, 2011; Olivares et al., 2021; Makridakis et al., 2022). In addition to the forecast accuracy evaluation tools, the package provides specialized visualization tools.

Hierarchical time series datasets. The library provides access to five Pandas datasets and the aggregation utils to create them. Australian Labour monthly reports (Australian Bureau of St. 2019), SF Bay Area daily Traffic measurements (Dua and Graff, 2017), Quarterly Australian Tourism-S visits (Tourism Australia, Canberra, 2005), Monthly Australian Tourism-L visits (Tourism Australia, Canberra, 2019), and daily Wiki2 article views (Anava et al., 2018). Each dataset is accompanied by metadata capturing its seasonality/frequency, the forecast horizon used in previous publications, its corresponding hierarchical aggregation constraints matrix, and the names of its levels.

3. Related Software

We refer to Januschowski et al. (2019), and Siebert et al. (2021) for complete open-source forecasting software surveys. We describe packages relevant to HierarchicalForecast. Classic statistical and econometric time series models, such as ARIMA, ETS, and GARCH have low-level NumPy implementations in multiple libraries, including statsmodels, tf_sts, kats, and pyflux. Currently, the statsforecast package provides the fastest NumBa implementations of these methods, and it has been adopted in multiple popular open-source Python frameworks, such as darts and sktime. Other libraries, including gluonts have Python API R-connections that enable comparisons with well-established methods at the cost of R dependency frictions. Finally, higher-level libraries, including darts, sktime, tslearn, pmdarima, and seglearn, provide API access to various time series forecasting models.

4. Usage Example and Benchmarks

The code example below highlights the usability and wide rate of available reconciliation methods in HierarchicalForecast. It predicts eight months of the 57 series of the Labour dataset using AutoARIMA base model and later reconciles the base predictions using the BottomUp, TopDown, and MinTrace methods. We generate prediction intervals with 80% and 90% coverage using the BOOTSTRAP technique.

Table 2: Mean sCRPS of ARIMA-based coherent forecasts, 95% confidence, 10 seeds.

DATASET	BottomUp	$^{\tt BOOTSTRAP}_{\tt TopDown}{}^{1,2}$	MinTrace	BottomUp	$\begin{array}{c} {\tt NORMALITY} \\ {\tt TopDown}^{1,2} \end{array}$	MinTrace	BottomUp	$\begin{array}{c} {\tt PERMBU}^1 \\ {\tt TopDown}^1 \end{array}$	MinTrace
Tourism-S Tourism-L	$\begin{array}{c} 0.006 \!\pm\! 0.001 \\ 0.067 \!\pm\! 0.002 \\ 0.089 \!\pm\! 0.001 \\ 0.142 \!\pm\! 0.001 \\ 0.417 \!\pm\! 0.002 \end{array}$	$0.070\pm0.001 \\ 0.120\pm0.001 \\ -$	$0.050\pm0.001 \\ 0.089\pm0.001 \\ 0.131\pm0.001$	0.076±0.000 0.083±0.000 0.169±0.000	0.071±0.000 0.117±0.000 -	$0.055\pm0.000 \\ 0.086\pm0.000 \\ 0.133\pm0.000$	0.076±0.001 0.084±0.001 -	0.063±0.001 0.103±0.001	0.052±0.001 0.084±0.001

¹ TopDown/PERMBU results are unavailable because, they cannot be applied to group hierarchical structures.

```
from statsforecast.core import StatsForecast
from statsforecast.models import AutoARIMA
from datasetsforecast.hierarchical import HierarchicalData
from hierarchicalforecast.core import HierarchicalReconciliation
from hierarchicalforecast.evaluation import HierarchicalEvaluation
from hierarchicalforecast.methods import BottomUp, TopDown, MinTrace
# Load Labour dataset
Y_df, S_df, tags = HierarchicalData.load('./data', 'Labour')
Y_df = Y_df.set_index('unique_id')
# Compute base AutoARIMA predictions and reconcile them
fcst = StatsForecast(df=Y_df, models=[AutoARIMA(season_length=12)],
                     freq='MS', n_jobs=-1)
Y_hat_df = fcst.forecast(h=8, fitted=True)
Y_fitted_df = fcst.forecast_fitted_values()
# Define reconcilers
reconcilers = [BottomUp(),
               TopDown(method='average_proportions'),
               MinTrace(method='ols')]
# Reconcile
hrec = HierarchicalReconciliation(reconcilers=reconcilers)
Y_rec_df = hrec.reconcile(Y_hat_df, S_df, tags, Y_df=Y_fitted_df,
                          intervals_method='bootstrap', level=[80,90])
```

Table 2 shows the overall average sCRPS for various HierarchicalForecast reconciliation methods, along with the measurement's 95% confidence intervals for the five datasets. These experimental results are aligned with previous studies' reports (Wickramasuriya et al., 2019; Ben Taieb and Koo, 2019; Rangapuram et al., 2021; Olivares et al., 2021).

5. Conclusion and Plans

We present HierarchicalForecast, a Python open-source library dedicated to hierarchical time series forecasting. The library integrates publicly available processed datasets, evaluation metrics, and a curated set of highly efficient statistical baselines. We provide examples and references to extensive experiments to show how to use the baselines and evaluate their empirical performance. This work will help the Machine Learning forecasting community by bridging the gap between statistical and econometric modeling and providing benchmark tools for developing novel hierarchical forecasting algorithms compared to the well-established methods. We intend to continue maintaining and improving the repository and promoting collaboration across the forecasting research community.

 $^{^2}$ The combinations NORMALITY-TopDown and BOOTSTRAP-TopDown are yet to be implemented, this has never been done before.

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