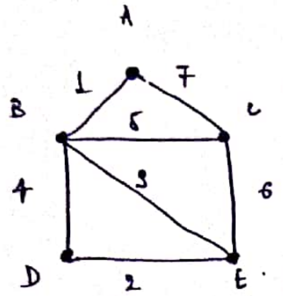
ORCS ASSIGNMENT on Unit 5

1.

a) Consider the network given as in the figure below. find the minimum spanning tree using Kruskal's Algorithm



→ Kruskal's algorithm is to find the minimum cost spanning tree using greedy approach

Step 1:

Given arcs and their weights

ARC	WEIGHT
A-B	1
A-C	7
B-C	5
B-D	4

ARC	WEIGHT
B-E	3
C-E	6
D-E	2

Step 2:

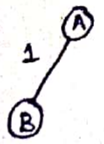
Sort the weights in ascending order

ARC	WEIGHT
A-B	1
D-E	2
B-E	3
B-D	4

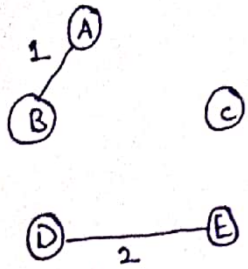
ARC	WEIGHT
B-C	5
C-E	6
A-C	7

Step-3: Form the edges such that no closed cycle is formed

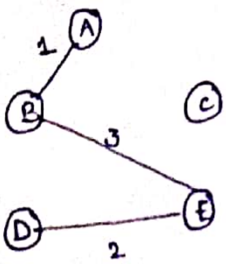
i) Add A-B



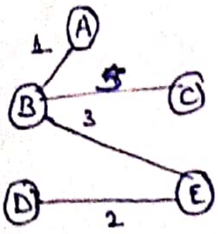
ii) Add D-E



iii) Add B-E



iv) Add B-C



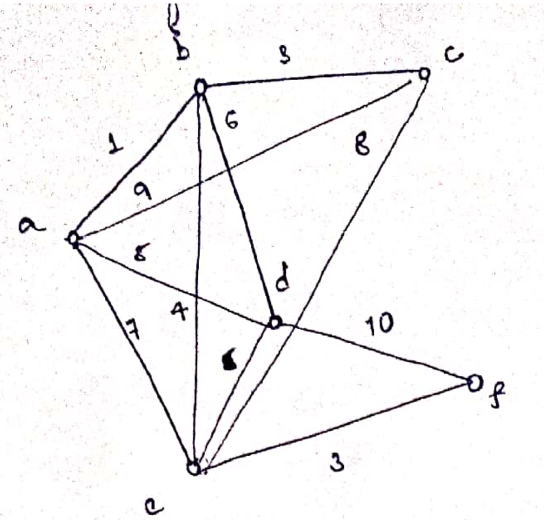
Adding B-D, C-E, A-C forms cycle.

Hence they are discarded.

Number of edges = $n-1 = 5-1 = 4$ ✓

where number of vertices = $n = 5$.

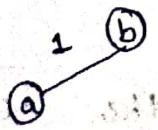
1 b] Consider the network given as in the figure below find the minimum spanning tree using prims algorithm



	a	b	c	d	e	f
a	0	1	9	5	7	∞
b	1	0	3	6	4	∞
c	9	3	0	∞	8	∞
d	5	6	∞	0	5	10
e	7	4	∞	5	0	3
f	∞	∞	∞	10	3	0

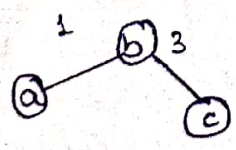
Step 1: consider Row 'a'. The smallest weight is '1'.

Eliminate a^{th} row and b^{th} column.

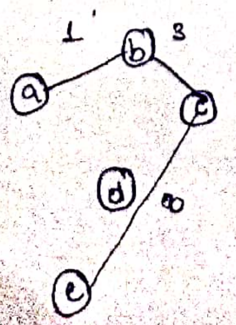


Step 2: consider Row 'b'. Least value is 3.

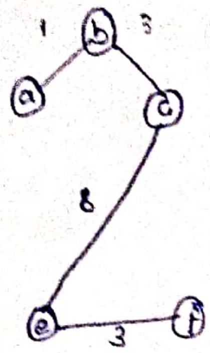
Eliminate and add to the graph.



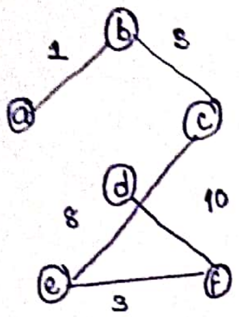
Step 3: in Row c, least value is 8.



Step-4: In Row c, least value is 8.



Step-5: In Row F, least value is 10.



Num of vertices = 6.

Number of edges = $n-1 = 6-1 = 5$.

∴ Minimum spanning tree is obtained.

Minimal cost $= 1+3+8+3+10$

$= 25$.

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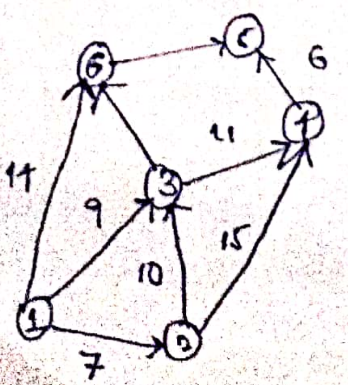
(2) Find the shortest path from node 1 to nodes of the distance network shown in figure below using Dijkstra Algorithm.



Let $G=(V,E)$, $V=\{1,2,3,4,5,6\}$

i) $P1=\{1\}$ $T1=\{2,3,4,5,6\}$

$l(3)=9$

$l(2)=7$

$l(6)=14$

$l(4)=\infty$

$l(5)=\infty$

Min weight is 7 which is 1-2.

ii) $P2=\{1,2\}$ $T2=\{3,4,5,6\}$

$l(3) = \min\{\text{old } l(3), l(2)+w(2,3)\}$

$= \min\{9, 7+10\} = \min\{9,17\} = 9.$

$l(4) = \min\{\text{old } l(4), l(2)+w(2,4)\}$

$= \min\{\infty, 7+15\} = 22.$

$l(5) = \min\{\text{old } l(5), l(2)+w(2,5)\}$

$= \min\{\infty, 7+\infty\} = \infty$

$l(6) = \min\{\text{old } l(6), l(2)+w(2,6)\}$

$= \min\{14, 7+\infty\} = 14$

iii) $P3=\{1,2,3\}$ $T3=\{4,5,6\}.$

$l(4) = \min\{\text{old } l(4), l(3)+w(3,4)\}$

$= \min\{22, 9+11\} = 20.$

$l(5) = \min\{\text{old } l(5)\ l(3)+w(3,5)\}$

$= \min\{\infty, 9+\infty\} = \infty$

$l(6) = \min\{\text{old } l(6), l(3)+w(3,6)\}$

$= \min\{14, 9+20\} = 14$

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iv)

$P_4 = \{1,2,3,6\}$ $\quad T_4 = \{4,5\}$

$l(4) = \min\{\text{old } l(4),\ l(6) + w(6,4)\}$

$= \min\{20,\ 11+\infty\} = 20.$

$l(5) = \min\{\text{old } l(5),\ l(6) + w(5,6)\}$

$= \min\{\infty,\ 11+9\} = 20.$

$\therefore\ P_5 = \{1,2,3,6,4,5\}.$

	1	2	3	4	5	6
1	0	∞	∞	∞	∞	∞
1,2	-	$\boxed{7}$	9	∞	∞	14
1,2,3		-	$\boxed{9}$	22	∞	14
1,2,3,6			-	20	∞	$\boxed{11}$
1,2,3,6,4				$\boxed{20}$	20	-
1,2,3,6,4,5					$\boxed{20}$	

$\therefore$ The path is 1-3-6-5 $\Rightarrow$ 20.