10/2/2020

UNIT-4

GAME THEORY

→ Finite & Infinite Game

→ pay-off

→ pure and Mix strategy

→ strategy

Two person zero sum game

The Game which involves only two players is called two person Game. If the Algebraic sum of gains and loss of two players is zero then the game is referred to as two person zero sum game.

In the game, gain of one player is equal to the loss of the other player we consider pay-off matrix

Player X

$$\begin{array}{cc} & \begin{array}{cc} I & II \end{array} \\ \text{Player } y \begin{array}{c} I \\ II \\ III \end{array} & \begin{bmatrix} -50 & -4 \\ 4 & -3 \\ 5 & 3 \end{bmatrix} \end{array}$$

It has the following points

→ There are only two players. x and y in the game.

→ The player 'y' has 3 strategies where as the Player 'x' has only two strategies

→ Both the players are opposite for the strategies I i.e The pay-off for y is -50 (loss of y 50 units). This implies that the game for the player 'x' will be 50 units the game for the player 'x'.

4 - gain for player 'y' for strategy II and loss of player 'x' for strategy I.

3 - gain of player 'y' for strategy III

It is a two person game so the corresponding pay-off matrix for player 'x'.

$$\begin{array}{c|ccc} & I & II & III \\ \hline I & 50 & -4 & -5 \\ II & 4 & 3 & -3 \end{array}$$

3 - gain of player II for strategy II

Maxi-Min and Mini-Max criteria of optimality

(or)

Maxi-Min and MiniMax principle

We can determine the best strategy for each player. These criteria are based on

1) Minimization of maximum loss

2) Maximization of minimum profit.

Gain with Saddle point (pure strategies)

A point in the pay-off matrix of the Game is called saddle point where the maxi-min and mini-max of the players are intersect i.e mini-max = maxi-min

Remarks 1) A saddle point, if exists is the value of Game.

2) If Maximin = mini-max = 0 then Game is fair

3) If Maxi-Min = Mini-Max ≠ 0 then it is said to be strictly ~~dom~~ determinable.

Find the saddle point for the game

$$\begin{array}{cc|ccc|}
 & & \mathrm{I} & \mathrm{II} & \mathrm{III} \\
 & \mathrm{I} & -4 & +12 & -4 \\
\text{Player A} & \mathrm{II} & -5 & -6 & -5 \\
 & \mathrm{III} & -6 & 17 & -8
\end{array}$$

○ - Minimum

□ - Maximum

Solution for Player A

Minimum value of row I

$\min\{-4, 12, -4\} = -4$

Minimum value of row II

$\min\{-5, -6, -5\} = -6$

Minimum value of row III

$\min\{-6, 17, -8\} = -8$

for Player B

max of I col $= \max\{-4, -5, -6\} = -4$

max of II col $= \max\{12, -6, 17\} = 17$

max of III col $= \max\{-4, -5, -8\} = -4$

max min $= -4$ (choose max value from min value i.e from Player A)

mini max $= -4$ (choose min value from max values i.e from Player B)

maximin = mini-max

	I	II	III	
I	-4 ✓	12	-4 ✓	$\boxed{-4}$
II	-5	-6	-5	-6
III	-6	17	-8	-8
	$\boxed{-4}$	$\boxed{17}$	$\boxed{-4}$	

Here we got two saddle points

mini-max = max-min = -4

It means saddle point exists at the cells (1, 1) and (1, 3) then the value of Game is -4.

→ The best optimal strategies for the Player A will be I as first strategy have -4 (i.e saddle points) for player B best strategy will be I, III

$$\begin{array}{cc} & \begin{array}{ccc} I & II & III \end{array} \\ \begin{array}{c} I \\ \text{Player A } II \\ III \end{array} & \begin{bmatrix} -3 & -2 & -3 \\ 2 & 0 & 2 \\ 5 & -2 & -4 \end{bmatrix} \end{array}$$

consider two person zero sum game matrix which represents pay-off matrix to the player A. Find the optimal strategy

for player A

$\min\{-3, -2, -3\} = -3$

$\min\{2, 0, 2\} = 0$

$\min\{5, -2, -4\} = -4$

for player B

$\max\{-3, 2, 5\} = 5$

$\max\{-2, 0, -2\} = 0$

$\max\{-3, 2, -4\} = 2$

$$\begin{array}{ccc} & \begin{array}{ccc} I & II & III \end{array} & \\ \begin{array}{c} I \\ II \\ III \end{array} & \begin{bmatrix} -3 & -2 & -3 \\ 2 & 0 & 2 \\ 5 & -2 & -4 \end{bmatrix} & \begin{array}{c} -3 \\ \boxed{0} \\ -4 \end{array} \text{ maximin} \\ & \begin{array}{ccc} \boxed{5} & \boxed{0} & \boxed{2} \end{array} & \end{array}$$

minimax

minimax = maximin = 0

The game is fair. The saddle point is (2,2) and the value of game is '0'.

For Player A II is the best strategy

for Player B II is the best strategy

Note

$$\max_{i} \min_{j} V_{ij} = \min_{j} \max_{i} V_{ij}$$

$$\underline{V} \qquad \overline{V}$$

→ Consider the following game. Find the Range of the values p and q. Determine ~~which will~~ the entry (2,2) as saddle point for the game.

$$
\begin{array}{cccc|c}
 & & B & & \\
 & I & II & III & \\
A \quad I & 2 & 4 & 5 & 2 \\
II & 10 & 7 & q & \boxed{7} \\
III & 4 & p & 6 & \\
\hline
 & 10 & \boxed{7} & &
\end{array}
$$

saddle point = (2,2)

$V = 7$

The value of p must be $p \le 7$ because it is maximum among the remaining.

The value of q must be $q \ge 7$ because it is the ~~m~~ minimum in that row.

→

$$\begin{array}{cc} & B \\ A & \begin{array}{c|ccccc|c} & & & & & & \text{max min} \\ \text{I} & 9 & 3 & 1 & 8 & 0 & 0 \\ \text{II} & 9 & 5 & 4 & 6 & 7 & \boxed{4} \\ \text{III} & 2 & 6 & 3 & 3 & 8 & 2 \\ \text{IV} & 5 & 4 & 2 & 2 & 1 & 1 \\ \hline & \boxed{9} & \boxed{6} & \boxed{4} & \boxed{8} & \boxed{8} & \end{array} \end{array}$$

saddle point = (2, 3)

· max min = 4

min max = 4

max min = min max = 4

value of game = 4

for Player A, the optimal strategy is II

for Player B, the optimal strategy is III

→ Given that the pay off matrix of a certain game G for two players X and Y

$$\begin{array}{cc} & Y \\ X & \begin{array}{c|cc} & \text{I} & \text{II} \\ \hline \text{I} & 10 & 15 \\ \text{II} & -10 & \alpha \end{array} \end{array}$$

Find the value of game

let ignore the α

$$\begin{array}{cc|c} \left[\begin{array}{cc} 10 & 15 \\ -10 & \alpha \end{array}\right] & & \begin{array}{c} \boxed{10} \\ -10 \end{array} \\ \boxed{10} \quad \boxed{15} & & \end{array}$$

Saddle Point = (1, 1)

minimax = maximin = 10 ≠ 0

so the game is strictly determinable whatever the value of α. The game is not fair

for Player A, optimal strategy is I

for Player B, optimal strategy is I

→ Find IBFS using least cost cell method and optimize the solution

	D1	D2	D3	D4	D5	
O1	73	40	9	79	20	8
O2	62	93	96	8	13	~~7~~ 3
O3	96	65	80	50	65	9
O4	57	58	29	12	87	3
O5	56	23	87	18	12	5
	6	8	10	~~4~~ 0	4	

73	40	9	20	~~8~~ 0
62	93	96	13	3
96	65	80	65	9
57	58	29	87	3
56	23	87	12	5
6	8	~~10~~ 2	4	

62	93	96	13	8
96	65	80	65	9
57	58	29	87	3
56	23	87 $\boxed{4}$	12	81
6	8	2	40	

62	93	96	3
96	65	80	9
57	58	29	3
~~56~~ $\boxed{1}$	~~23~~	~~87~~	~~10~~
6	8	2	
	7		

62	93	96	3
96	65	80	9
57	58 $\boxed{2}$	29	31
61	7	20	

62	93	3
96	65	9
$\boxed{1}$ ~~57~~	~~58~~	~~10~~
65	7	

$\boxed{3}$ ~~62~~	~~93~~	~~30~~
96	65	9
52	7	

96 $\boxed{7}$	65	92
2	7	

$\boxed{2}$ 96	20
2	

73	40	$\boxed{8}$ 9	79	20	8
$\boxed{3}$ 62	93	96	$\boxed{4}$ 8	13	7
$\boxed{2}$ 96	$\boxed{7}$ 65	80	50	65	9
$\boxed{1}$ 57	58	$\boxed{2}$ 29	12	82	3
56	$\boxed{1}$ 23	87	18	$\boxed{4}$ 12	5
6	8	10	4	4	

No. of allocations = 9

$m+n-1 = 5+5-1 = 10-1 = 9$

No. of allocations $= m+n-1$

$\therefore$ The Problem is not degenaracy.

$T.C = 9(8) + 62(3) + 8(4) + 96(2) + 65(7) + 57(1) + 29(2) + 23(1) + 12(4)$

$= 1123$

U-V Method

Matrix of occupied cells

-	-	9	-	-	u_1
62	-	-	8	-	u_2
96	65	-	-	-	u_3
57	-	29	-	-	u_4
-	23	-	-	12	u_5
v_1	v_2	v_3	v_4	v_5	

$u_1 + v_3 = 9$

$u_2 + v_1 = 62$

$u_2 + v_4 = 8$

$u_3 + v_1 = 96$

$u_3 + v_2 = 65$

$u_4 + v_1 = 57$

$u_4 + v_3 = 29$

$u_5 + v_2 = 23$

$u_5 + v_5 = 12$

$v_1 = 0$

$u_2 = 62$

$u_3 = 96$

$u_4 = 57$

$62 + v_4 = 8$

$v_4 = -54$

$\cancel{u_5}$ $u_5 - 31 = 23$

$u_5 = 54$

$96 + v_2 = 65$

$v_2 = -31$

$57 + v_3 = 29$

$v_3 = -28$

$u_1 - 28 = 9$

$u_1 = 37$

$54 + v_5 = 12$

$v_5 = -42$

c_{ij}

73	40	-	79	20
-	93	96	-	13
-	-	80	50	65
-	58	-	12	87
56	-	87	18	-

$u_{ij} + v_{ij}$

37	6	-	-17	-5	37
-	31	34	-	20	62
-	-	68	42	54	96
-	26	-	3	15	57
54	-	26	0	-	54
0	-31	-28	-54	-42	

$d_{ij} = c_{ij} - (u_{ij} + v_{ij})$

36	34	-	96	25
-	62	62	-	-7
-	-	12	8	11
-	32	-	9	72
2	-	61	18	-

all $d_{ij} \ngeq 0$

		$\boxed{8}$		
$\boxed{3}$ −			$\boxed{4}$	$\boxed{}$ +
$\boxed{2}$ +	$\boxed{7}$ −			
$\boxed{1}$		$\boxed{2}$		
	$\boxed{1}$ +			$\boxed{4}$ −

		$\boxed{8}$ 9		
$\boxed{0}$ 62			$\boxed{4}$ 8	$\boxed{3}$ 13
$\boxed{5}$ 96	$\boxed{4}$ 65			
$\boxed{1}$ 57		$\boxed{2}$ 29		
	$\boxed{4}$ 23			$\boxed{1}$ 12

Total cost $= 9(8) + 62(0) + 8(4) + 13(3) + 96(5) + 65(4)$
$+ 57(1) + 29(2) + 23(4) + 12(1) = 1102$

−	−	9	−	−	u_1
−	−	−	8	13	u_2
96	65	−	−	−	u_3
57	−	29	−	−	u_4
−	23	−	−	12	u_5
v_1	v_2	v_3	v_4	v_5	

$u_1 + v_3 = 9$

$u_2 + v_4 = 8$

$u_2 + v_5 = 13$

$u_3 + v_1 = 96$

$u_3 + v_2 = 65$

$u_4 + v_1 = 57$

$u_4 + v_3 = 29$

$u_5 + v_2 = 23$

$u_5 + v_5 = 12$

$u_1 = 0$

$v_3 = 9$

$u_4 + 9 = 29$

$u_4 = 20$

$20 + v_1 = 57$

$v_1 = 37$

$u_3 + 37 = 96$

$u_3 = 59$

$18 + v_4 = 8$

$v_4 = -10$

$\cancel{u_3 + v}$

$59 + v_2 = 65$

$v_2 = 6$

$u_5 + 6 = 23$

$u_5 = 17$

$17 + v_5 = 12$

$v_5 = -5$

$u_2 - 5 = 13$

$u_2 = 18$

c_{ij}

73	40	–	79	20
62	93	96	–	–
–	–	80	50	65
–	58	–	12	87
56	–	87	18	–

$u_{ij} + v_{ij}$

37	6	–	-10	-5	0
55	24	27	–	–	18
–	–	68	49	54	59
–	26	–	10	15	20
54	–	26	7	–	17
37	6	9	-10	-5	

$d_{ij} = c_{ij} - (u_{ij} + v_{ij})$

36	34	–	89	25
7	69	69	–	–
–	–	12	1	11
–	32	–	2	72
2	–	61	11	–

all $d_{ij} > 0$

so the op. cost 1102 is optimal.

							p
7	10	7	4	7	8	5	3
5	1	5	5	3	$\boxed{2}$ 3	64	2
4	3	7	9	1	9	2	2
4	6	9	0	0	8	9	4
4	4	6	2	4	20		
1	2	2	4	1	5 ↑		

p (last row)

						p
7	10	7	4	7	5	3
5	1	5	5	3	4	2
4	3	7	9	1	2	2
4	6	9	$\boxed{2}$ 0	0	97	4
4	4	6	02	4		
1	2	2	4 ↑	1		

p (last row)

					p
7	10	7	7	5	3
5	1	5	3	4	2
4	3	7	1	2	2
4	6	9	$\boxed{4}$ 0	73	4 ←
4	4	6	40		
1	2	2	1		

p (last row)

				p
7	10	7	5	3
5 $\boxed{4}$	1	5	40	4 ←
4	3	7	2	1
4	6	9	3	2
4	04	6		
1	2	2		

p (last row)

			P
7	7	5	0
4	7	2	3
$\boxed{3}$ 4	9	~~3~~0	5
1 ~~4~~	6		
3	2		

P

			P
7	7	5	0
$\boxed{1}$ 4	7	~~2~~1	3
~~1~~0	6		
3	0		

P

		P
7	5	2
$\boxed{1}$ 7	10	2
~~6~~5		
0		

P

		P
$\boxed{5}$ 7	~~8~~0	7
~~5~~0		
7		

P

7	10	$\boxed{5}$ 7	4	7	8	5
5	$\boxed{4}$ 1	5	5	3	$\boxed{2}$ 3	6
$\boxed{1}$ 4	3	$\boxed{1}$ 7	9	1	9	2
$\boxed{3}$ 4	6	9	$\boxed{2}$ 0	$\boxed{4}$ 0	8	9
4	4	6	2	4	2	

No. of allocations = 8

$m+n-1 = 4+6-1 = 10-1 = 9$

No. of allocations $< m+n-1$

The problem is degenaracy.

7	10	$\boxed{5}$ 7	4	7	8
5	$\boxed{4}$ 1	5	5	3	$\boxed{2}$ 3
$\boxed{1}$ 4	3	$\boxed{1}$ 7	9	$\boxed{8}$ 1	9
$\boxed{3}$ 4	6	9	$\boxed{2}$ 0	$\boxed{4}$ 0	8

No. of allocations = 9

$m + n - 1 = 4 + 6 - 1 = 9$

No. of allocations = $m + n - 1$

$\therefore$ The problem is not degenaracy

$T.C = 4(1) + 4(3) + 1(4) + 7(5) + 7(1) + 0(2) + 0(4) + 3(2)$

$= 4 + 12 + 4 + 35 + 7 + 0 + 0 + 6$

$= 68$

Matrix of occupied cells

-	-	7	-	-	-	u_1
-	1	-	-	-	3	u_2
4	-	7	-	1	-	u_3
4	-	-	0	0	-	u_4
V_1	V_2	V_3	V_4	V_5	V_6	

$u_1 + V_3 = 7$

$u_2 + V_2 = 1$

$u_2 + V_6 = 3$

$u_4 + V_1 = 4$

$u_4 + V_4 = 0$

$u_4 + V_5 = 0$

$u_3 + V_1 = 4$

$u_3 + V_3 = 7$

$u_3 + V_5 = 1$

~~$u_1 = 0$~~

~~$V_3 = 7$~~

~~$u_3 = 0$~~

~~$V_1 = 4$~~

~~$V_5 = 1$~~

$u_4 = 0$

$V_1 = 4$

$V_4 = 0$

$V_5 = 0$

$u_3 = \cancel{0}$ 0/1

$u_2 = 0$

$V_2 = 1$

$V_6 = 3$

$u_1 = \cancel{0}$

$u_3 = 0$

$V_1 = 4$

$V_3 = 7$

$V_5 = 1$

$u_4 = 0$

$V_1 = 4$

$V_4 = 0$

$V_5 = 0$

10	8	12	9	3	15
4	4	6	6	7	12
15	7	11	13	8	16
8	8	4	7	6 33	43

The problem is unbalanced

10	8	12	9	3	0	15
4	4	6	6	7	0	12
15	7	11	13	8	0	16
8	8	4	7	6	10	43

The problem is balanced.

								P
	10	8	12	9	3	0	15	3
	4	4	6	6	7	0	12	4
	15	7	11	13	8	10 0	~~16~~ 6	7
	8	8	4	7	6	~~10~~ 0		
P	6	3	5	3	4	0		

							P
	10	8	12	9	3	15	5
	8 4	4	6	6	7	~~12~~ 4	3
	15	7	11	13	8	6	1
	~~8~~ 0	8	4	7	6		
P	6	3	5	3	4		

						P
	8	12	9	$\boxed{6}$ 3	~~15~~ 9	5
	4	6	6	7	4	3
	7	11	13	8	6	1
	8	4	7	0 ~~6~~		
P	3	5	3	4		

					P
	8	12	9	9	1
	4	$\boxed{4}$ 6	6	~~4~~	2
	7	11	13	6	4
	8	0 ~~4~~	7		
P	3	5	3		

				P
	8	9	9	1
	$\boxed{6}$ 7	13	~~8~~ 6	6
	2 ~~8~~	7		
P	1	4		

				P
	8 $\boxed{7}$	9	9 2	1
	2	~~7~~ 0		
P	8	9		

			P
	$\boxed{2}$ 8	~~2~~ 0	8
	~~2~~ 0		
P	8		

10	$\boxed{2}$ 8	12	$\boxed{7}$ 9	$\boxed{6}$ 3	0	15
$\boxed{8}$ 4	4	$\boxed{4}$ 6	6	7	0	12
15	$\boxed{6}$ 7	11	13	8	$\boxed{10}$ 0	16
8	8	4	7	6	10	

$T.C = 4(8)+8(2)+7(6)+6(4)+9(7)+3(6)+0(10)$

$= 32+16+42+24+63+18+0$

$= 195$

No. of allocations $= 7$

$m+n-1 = 3+6-1 = 9-1 = 8$

No. of allocations $< m+n-1$

The problem is degenaracy

10	$\boxed{2}$ 8	12	$\boxed{7}$ 9	$\boxed{6}$ 3	$\boxed{\varepsilon}$ 0	15
$\boxed{8}$ 4	4	$\boxed{4}$ 6	6	7	0	12
15	$\boxed{6}$ 7	11	13	8	$\boxed{10}$ 0	16
8	8	4	7	6	10	

No. of allocations $= 8$

$m+n-1 = 3+6-1 = 8$

No. of allocations $= m+n-1$

$\therefore$ The problem is ~~degenarac~~ not degenaracy.

-	8	-	9	3	0	u_1
4	-	6	-	-	-	u_2
-	7	-	-	-	0	u_3
v_1	v_2	v_3	v_4	v_5	v_6	

$u_1 + v_2 = 8$ $\qquad u_2 + v_1 = 4$

$u_1 + v_4 = 9$ $\qquad u_2 + v_3 = 6$

$u_1 + v_5 = 3$ $\qquad u_3 + v_2 = 7$

$u_1 + v_6 = 0$ $\qquad u_3 + v_6 = 0$

$u_1 = 0$ $\qquad u_3 = 0$ $\qquad u_2 = 0$

$v_2 = 8$ $\qquad v_2 = 7$ $\qquad v_1 = 4$

$v_4 = 9$ $\qquad \cancel{u_2} \qquad v_3 = 6$

$v_5 = 3$ $\qquad u_1 = 1$

$v_6 = 0$ $\qquad v_4 = 8$

$v_5 = 2$

$v_6 = -1$

$u_3 = 1$

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→ Finite and infinite Games - definition

A Game is called finite if it has a finite number of Points in a Game at which one of the Player select an alternative from the set of actions (strategies) otherwise It is known as infinite

Pay off

The result of the playing game is called the payoff. It is measured in terms of objective firm

* Pay off Matrix

It is the table that represent the result or payoff of different strategies of game. In two person game the name at the top of the table represents the name of the player that made the pay~~ment~~ m.

We consider a game in which only two players M and N wheras M has m course of actions and player N has n course of actions. Thus the payoff matrix is represented as

M's pay-off matrix

$$
\begin{array}{c|c}
 & N \\
 & \begin{array}{cccccc} 1 & 2 & 3 & \cdots & \cdots & n \end{array} \\
M \begin{array}{c} 1 \\ 2 \\ 3 \\ \vdots \\ m \end{array} & \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{31} & a_{32} & \cdots & a_{3n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}
\end{array}
$$

N's pay-off matrix

$$
N \begin{bmatrix} -a_{11} & -a_{21} & \cdots & -a_{m1} \\ -a_{12} & -a_{22} & \cdots & -a_{m2} \\ \vdots & \vdots & & \vdots \\ -a_{1n} & -a_{2n} & \cdots & -a_{mn} \end{bmatrix}
$$

Remarks
① Row indicates the course of actions of M

② column indicates the course of actions of N

③ The cell entries a_{ij} represent in the payoff matrix ~~capit~~ M if M select the course of action 'i' and N selects 'j'.

④

→ For two persons game the cell entries N pay off matrix will be negative of the corresponding pay-off matrix M.

→ The cell entries a_{ij} will be positive, negative or zero value.

→ In two person zero sum game, the gain of the one player is less than the other player and his payoff matrix is called rectangular and it is "rectangular game".

→ only one pay-off matrix is used by both the players.

Strategy

The strategy of the player is predefined rule (decision), from which the player select the course of action from his own list during the game.

Pure strategy

It is the particular course of action that can be selected by the player in advance and use it in every play.

Mixed strategy

If a player can select more than one course of actions from own list of actions with fixed probability distribution is called Mixed strategy.

Mathematically Mixed strategy is a set of non negative real numbers their sum will be unity.

→ set is a collection of well defined distinct elements.

→ Let $\{x_1, x_2, \dots x_m\}$ be mixed strategy

$\Rightarrow x_1 + x_2 + \dots x_m = 1, \quad x_i \geq 0, i = 1 \dots m$

$$\Downarrow$$

$$\sum_{i=1}^{m} x_i = 1$$

if $x_r = 1, \; x_i = 0$ then $r \neq i$

$r \Rightarrow$ pure strategy.

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Optimal Strategy

It is that course of actions which is played by the player at the most important turning point during the game: such strategy is called optimal strategy.

It is directly proportional to the pay of the player.

Pure strategy is the special case of mixed strategy.

Value of game

It is the expected gain to a player if ~~the~~ tie and his opponent during the game follow, the optimal strategy.

(Value of game)

mini max = maxi min

$\bar{V} = \underline{V} = V$, ~~the~~

The game is fair if ~~the~~ $V = 0$

→ Find the value of game

$$\begin{array}{c|cccc c|c} & & & & & & \text{Maximin } (\underline{v}) \\ & \left[\begin{array}{ccccc} -2 & 0 & 0 & \boxed{5} & 3 \\ 3 & 2 & \boxed{1} & 2 & 2 \\ -4 & -3 & 0 & -2 & \boxed{6} \\ \boxed{5} & \boxed{3} & -4 & -2 & -6 \end{array}\right] & & & & & \begin{array}{c} -2 \\ \boxed{1} \\ -4 \\ -6 \end{array} \\ \text{Mini-max } (\bar{v}) & \begin{array}{ccccc} 5 & 3 & 1 & 5 & 6 \end{array} & & & & & \end{array}$$

saddle point = (2, 3)

optimal strategy for player A = 2

optimal strategy for player B = 3

Value of game,

$\bar{v} = \underline{v} = v$

$v = 1$

$v \neq 0$, The game is unfair

→

$$\begin{array}{c|c|c} & & \text{Maximin } (\underline{v}) \\ & \left[\begin{array}{ccc} \boxed{-2} & 15 & \boxed{-2} \\ -5 & -6 & -4 \\ -5 & \boxed{20} & -8 \end{array}\right] & \begin{array}{c} \boxed{-2} \\ -6 \\ -8 \end{array} \\ \text{Minimax } (\bar{v}) & \begin{array}{ccc} \boxed{-2} & 20 & \boxed{-2} \end{array} & \end{array}$$

Two saddle points

(1,1) (1,3), value = -2

Player A has '1' optimal strategy

Player B has '2' optimal strategies

→

$$
\begin{array}{c|c}
\begin{bmatrix} 15 & 2 & 3 \\ 6 & 5 & 7 \\ -7 & 4 & 0 \end{bmatrix} & \begin{array}{c} \text{maximin} \\ 2 \\ 5 \\ -7 \end{array} \\
\text{minimax} \quad 15 \quad 5 \quad 7 &
\end{array}
$$

saddle point = (2,2), value = 5

for player A ~~has~~ optimal strategy = 2

for player B " " = 2

→

$$
\begin{array}{c|c}
\begin{bmatrix} 1 & 7 & 3 & 4 \\ 5 & 6 & 4 & 5 \\ 7 & 2 & 0 & 3 \end{bmatrix} & \begin{array}{c} \text{maximin} \\ 1 \\ 4 \\ 0 \end{array} \\
\text{Minimax} \quad 7 \quad 7 \quad 4 \quad 5 &
\end{array}
$$

saddle point = (2,3), value = 4

$$
\begin{array}{c|ccc|c}
 & & & & \text{Maximin} \\
 & \begin{bmatrix} -3 & -1 & \boxed{6} \\ 2 & \boxed{0} & 2 \\ \boxed{5} & -2 & -4 \end{bmatrix} & & & \begin{matrix} -3 \\ \boxed{0} \\ -4 \end{matrix} \\
\text{Minimax} & 5 \quad \boxed{0} \quad 6 & & &
\end{array}
$$

Saddle point $= (0,0)$

Value $= 0$

$$
\begin{array}{c|c|c}
 & & \text{Maximin} \\
 & \begin{bmatrix} \boxed{1} & 2 & \boxed{1} \\ 0 & -4 & -1 \\ \boxed{1} & \boxed{3} & -2 \end{bmatrix} & \begin{matrix} \boxed{1} \\ -4 \\ -2 \end{matrix} \\
\text{Minimax} & \boxed{1} \quad 3 \quad \boxed{1} &
\end{array}
$$

Saddle point $= (1,1)\ (1,3)$

Value $= 1$

$$
\begin{array}{c|c|c}
 & & \text{Maximin} \\
 & \begin{bmatrix} \boxed{6} & 8 & \boxed{6} \\ 4 & \boxed{12} & 2 \end{bmatrix} & \begin{matrix} 6 \\ 2 \end{matrix} \\
\text{Minimax} & 6 \quad 12 \quad 6 &
\end{array}
$$

Saddle point $= (1,1)\ (1,3)$

→

$$\begin{bmatrix} 3 & -4 & 8 \\ -8 & \boxed{5} & -6 \\ \boxed{6} & -7 & 6 \end{bmatrix} \begin{array}{c} \text{maximin} \\ \boxed{-4} \\ -8 \\ -7 \end{array}$$

Minimax 6 5 8

No saddle point

→

$$\begin{bmatrix} -3 & \boxed{1} \\ \boxed{3} & -1 \end{bmatrix} \begin{array}{c} -3 \\ -1 \end{array}$$

3 1

No saddle point

imp → Player A can choose his strategy from {A1, A2, A3} and player B can choose from {B1, B2}. The rule of the game is state that the payment should be made in accordance with selection of the strategy.

Pair	Payment to be made
(A1, B1)	Player A plays Rs 1 to player B
(A1, B2)	Player B plays Rs 6 to Player A
(A2, B1)	,, B ,, Rs 2 to ,, A
(A2, B2)	,, B ,, ,, 4 to ,, A
(A3, B1)	,, A ,, ,, 2 to ,, B

(A_3, B_2) Player A has to pay Rs 6 to B

find payoff matrix and value of the game.

$$\begin{array}{c c} & \begin{array}{cc} B_1 & B_2 \end{array} & \text{Maximin} \\ A \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \begin{bmatrix} -1 & 6 \\ \boxed{2} & 4 \\ -2 & -6 \end{bmatrix} & \begin{array}{c} -1 \\ \boxed{2} \\ -6 \end{array} \\ & \begin{array}{cc} 2 & 6 \end{array} & \end{array}$$

saddle point = (2,1)

Value = 2

→ solve and optimize the assignment problem

	1	2	3	4
		subject		
1	25	44	33	35
2	33	40	40	43
3	40	35	33	30
4	44	45	28	35
5	45	35	38	40
6	40	49	40	46

No. of rows ≠ No. of Columns

The problem is unbalanced

so add the columns with zeroes

	1	2	3	4	5	6
1	25	44	33	35	0	0
2	33	40	40	43	0	0
3	40	35	33	30	0	0
4	44	45	28	35	0	0
5	45	35	38	40	0	0
6	40	49	40	46	0	0

row subtraction:-

25	44	33	35	0	0
33	40	40	43	0	0
40	35	33	30	0	0
44	45	28	35	0	0
45	35	38	40	0	0
40	49	40	46	0	0

column subtraction & assignment

$\boxed{0}$	9	5	5	$\cancel{0}$	$\cancel{0}$
8	5	12	13	$\boxed{0}$	$\cancel{0}$
15	$\cancel{0}$	5	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$
19	10	$\boxed{0}$	5	$\cancel{0}$	$\cancel{0}$
20	$\boxed{0}$	10	10	$\cancel{0}$	$\cancel{0}$
15	14	12	16	$\cancel{0}$	$\boxed{0}$

No. of assigned zeroes = No. of rows/columns

The optimal solution is

Faculty	Subject	
1	1	25
2	5	0
3	4	30
4	3	28
5	2	35
6	6	0
		118

→

Sales region

Sales man	1	2	3	4
1	5	11	8	9
2	5	7	9	7
3	7	8	9	9
4	6	8	11	12

No. of rows = No. of columns

∴ The problem is balanced.

The problem is Maximization

so to convert it in to minimization

subtract each element ~~overall maximum~~ element

from overall maximum ~~all~~ elements.

overall maximum = 12

7	1	4	3
7	5	3	5
5	4	3	3
6	4	1	0

row subtraction

6	0	3	2
4	2	0	2
2	1	0	0
6	4	1	0

column subtraction and assignment

4	$\boxed{0}$	3	2
2	2	$\boxed{0}$	2
$\boxed{0}$	1	~~0~~	~~0~~
4	4	1	$\boxed{0}$

No. of assignment zeroes = No. of rows/ Columns.

The optimal assignment is

Salesman   Salesregion:

1	2	11
2	3	9
3	1	7
4	4	12
		39

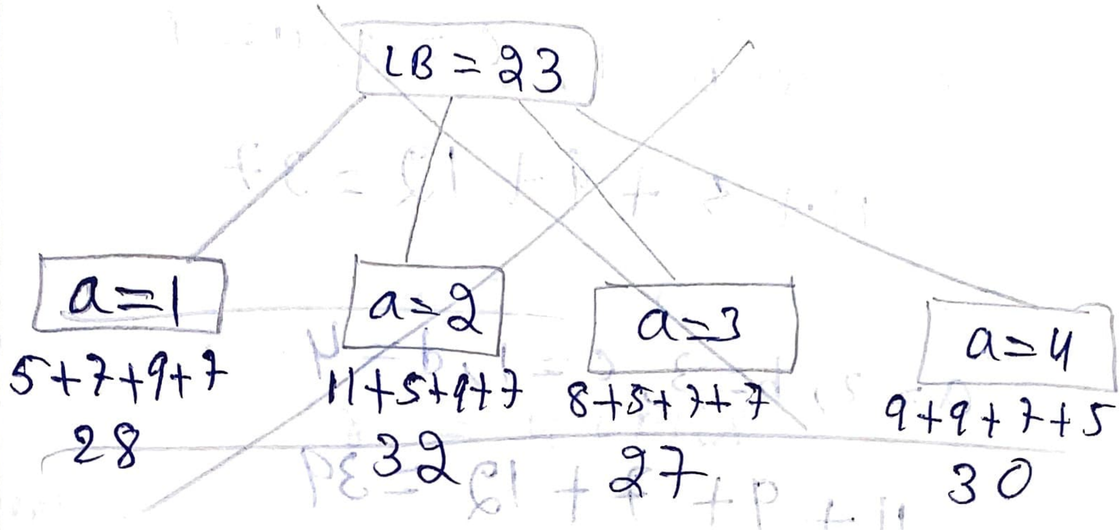
Using ~~Branch and Bound~~

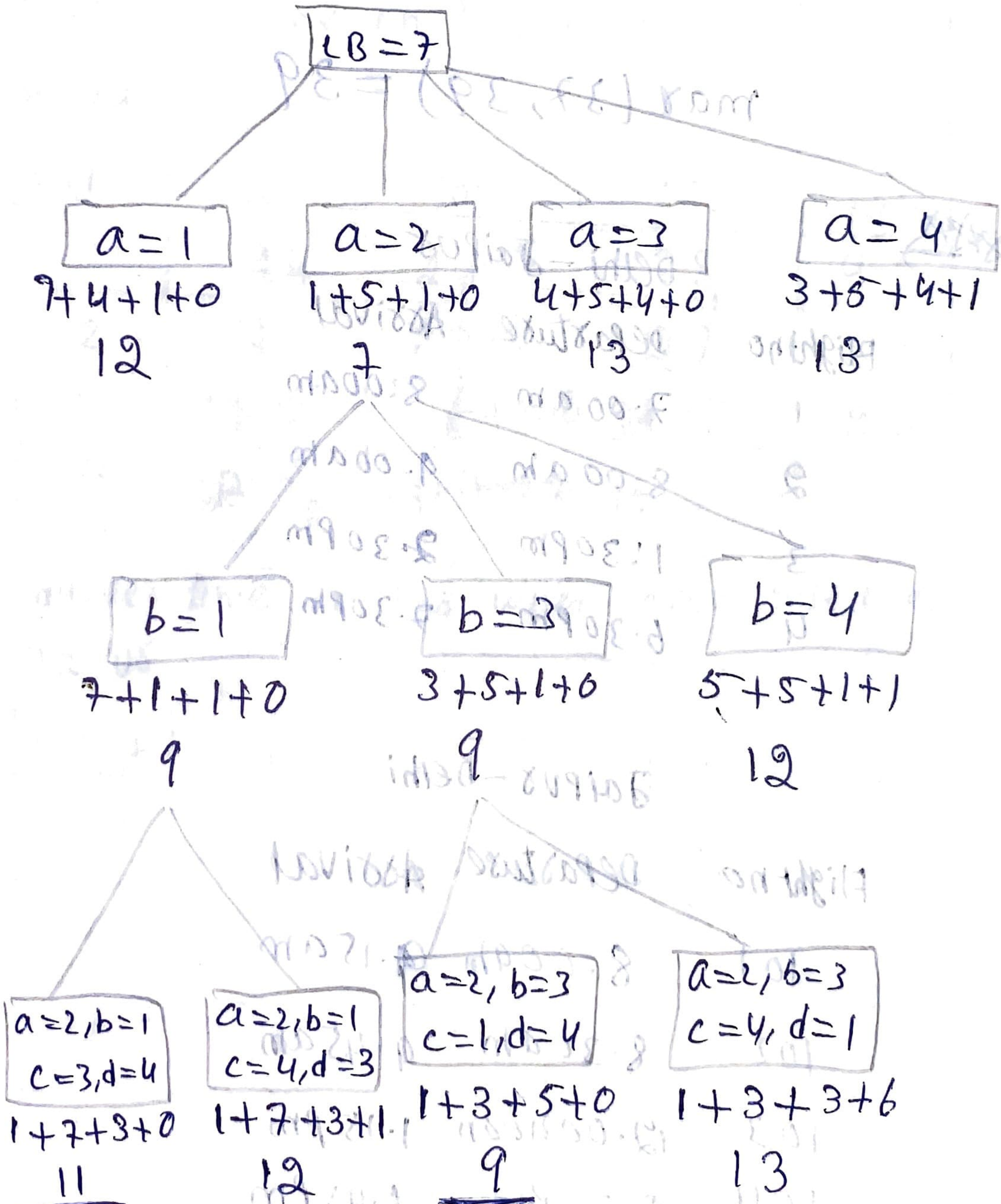
~~$LB = 5+5+7+6$~~

~~$= 23$~~

	1	2	3	4
a	7	1	4	3
b	7	5	3	5
c	5	4	3	3
d	6	4	1	0

$LB = 1+3+3+0 = 7$


LB = 23
a=1
5+7+9+7
28
a=2
11+5+9+7
32
a=3
8+5+7+7
27
a=4
9+9+7+5
30


LB = 7
a=1
7+4+1+0
12
a=2
1+5+1+0
7
a=3
4+5+4+0
13
a=4
3+5+4+1
13
b=1
7+1+1+0
9
b=3
3+5+1+0
9
b=4
5+5+1+1
12
a=2, b=1
c=3, d=4
1+7+3+0
11
a=2, b=1
c=4, d=3
1+7+3+1
12
a=2, b=3
c=1, d=4
1+3+5+0
9
a=2, b=3
c=4, d=1
1+3+3+6
13

$a = 2, b = 1, c = 3, d = 4$

$11 + 5 + 9 + 12 = 37$

$a = 2, b = 3, c = 1, d = 4$

$11 + 9 + 7 + 12 = 39$

$\max(37, 39) = 39$

Delhi – Jaipur		
Flight no	Departure	Arrival
1	7.00 am	8:00 am
2	8.00 am	9.00 am
3	1:30 pm	2.30 pm
4	6.30 pm	7.30 pm

Jaipur – Delhi		
Flight no	Departure	Arrival
101	8.00 am	9.15 am
102	8.30 am	9.45 am
103	12.00 noon	1.15 pm
104	5.30 pm	6.45 pm

$\begin{array}{r} 16 \\ 12 \\ \hline 28 \\ +9 \\ \hline 37 \end{array}$

$\begin{array}{r} 20 \\ 19 \\ \hline 39 \end{array}$

Minimum layover time between flight is 5 hours.

15 minutes = 1unit

1 hr = 4 units

sol

Layover time in hrs when crew based at Delhi.

Flight	101	102	103	104
1	24	24.5	28	9.5
2	23	23.5	27	8.5
3	17.5	6+12=18	21.5	27
4	~~23.5~~ 12.5	13 ~~2+12=24~~	16.5	22

Layover time in hrs when crew based at Jaipur

Flight	101	102	103	104
101	21.75	22.75	29.75	
102				
103				
104				

Maxi-Min principle

This principle maximize the minimum gains of a player A. The minimum gains with respect to the alternatives of A's irrespective of B alternatives are obtained first. The maximum of these minimum gains is known as the maximin value and the corresponding ~~the~~ alternatives is called maximum strategy.

Minimax principle

This principle minimize the maximum loss of player B. The maximum losses with respect to the alternatives of B's irrespective of A's alternatives are obtained first.

The minimum of these maximum loss is known as the minimax value and the corresponding alternative is called minimum strategy.

Saddle point

In a game, if the maximin value is equal to the minimax value then the game is said to have a saddle point.

If the game has a saddle point, then each player has the pure strategy.

Two person zero sum game

In a game with two players, if the gain of one player is equal to the loss of another player then it is called two person zero sum game.

Application of Assignment

salesman to areas

workers to machines

military operations

→ Applications of transportation, game theory

↓

Flipkart

→ Find the saddle point and strategies

→ what is the objective function of assignme-nt programming.

→ Transportation, assignment similarities and difference

→ when UV method of transportation problem reaches to optimality

→ $m+n-1$ = no. of allocations

→ Generalized format / mathematical formulation of assignment problem.

Row scanning no. of lines drawn horizontally to cover zeroes.

1. starting from the first row ask the following question. Is there exactly one zero in that row?
If yes mark a square around that zero entry and draw a vertical line passing through that zero otherwise skip that zero.
2. After scanning the last row. Check whether all the zeroes are covered with lines if yes

Column scanning replace above row with column

Branch and Bound

~~At~~ label K the row marked as K of the assignment problem will be assigned with the best column of the assignment problem

If there is a tie on the lower bound then the terminal node at the lower most label is to be considered for the further branching.

Stopping rule:

If the minimum lower bound happening to be at any one of the terminal nodes at n-1th label the optimality is reached.

→ degeneracy and non-degeneracy

→ what is the reason to optimize the feasible solution?

→

5	11	8	9
5	7	9	7
7	8	9	9
8	8	11	12

Maximization problem

UB = Row maximum = 11 + 9 + 9 + 12

UB = 41

UB = 41

a = 1

5 + 8 + 11 + 12

36

a = 2

11 + 7 + 11 + 12

41

a = 3

8 + 7 + 8 + 12

~~36~~ 35

a = 4

9 + 7 + 8 + 11

~~36~~ 35

b = 1

5 + 11 + 11 + 12

39

b = 3

9 + 8 + 11 + 12

40

b = 4

7 + 8 + 11 + 11

37

a = 2, b = 3
c = 1, d = 4

11 + 9 + 7 + 12

39

a = 2, b = 3
c = 4, d = 1

11 + 9 + 9 + 8

37

18
23
41

13
23
36

20
22
42

4/3/2020

Game with no saddle point (Mixed strategy)

If the game does not contain saddle point then maximin and minimax criteria are not applicable. Therefore, The player can not use pure strategy.

Rectangular Game without saddle point

expectation of the payoff matrix $E(x,y)$ in a Game whose Payoff matrix is $\{v_{ij}\}$

$$E(x,y) = \sum_{i=1}^{m} \sum_{j=1}^{n} (x_i v_{ij}) y_j$$

x and y are mixed strategy of Player A and Player B.

strategic saddle point

If $\min_{y} \max_{x} E(x,y) = E(x_0, y_0) = \max_{x} \min_{y} E(x,y)$

x_0, y_0 called strategic saddle point of the game where x_0, y_0 defines optimal strategy.

The value of . game

$$V = E(x_0, y_0)$$

→ In a game of matching coins with two Players suppose one Player wins Rs.2 when there are two heads and wins nothing when there are two tails and loss is Rs 1 when there are one head and one tail. Determine the pay off matric, the Best strategy for each Player and value of the game.

		H	T	$\underline{V}$
A	H	2	-1	-1
	T	-1	0	-1
	$\overline{V}$	2	0	

A - coln
B - row

$0 \neq -1$

∴ The game has no saddle point

Let the player A plays H with probability 'x' and T with probability $1-x$. so that $x + 1 - x = 1$. Then if the player B plays H all the time then

A's expected ~~game~~ gain is

$$E(A, H) = 2(x) + (-1)(1-x)$$

$$= 2x - 1 + x$$

$$= 3x - 1$$

similarly if the player B plays T all the time the A's expected gain is

$$E(A, T) = (-1)x + 0(1-x)$$

$$= -x$$

It can be shown mathematically if the player A choose x such that

$$E(A, H) = E(A, T) = E(A)$$

$$3x - 1 = -x$$

$$4x = 1$$

$$x = \frac{1}{4}$$

$$1 - x = \frac{3}{4}$$

Best strategy for player A is $\left(\frac{1}{4}, \frac{3}{4}\right)$

So the expected gain for the player A (i.e value of the game)

col I $E(A) = \frac{1}{4} \times 2 + (-1)\left(\frac{3}{4}\right)$

$= \frac{2}{4} - \frac{3}{4}$

$= \frac{-1}{4}$

col II $E(A) = \frac{1}{4}(-1) + 0\left(\frac{3}{4}\right)$

$= -\frac{1}{4}$

Let the player B plays H with probability 'y' and T with probability $1-y$, so that $y+1-y=1$

If the player A plays H all the time

i.e then B's expected gain is

$E(B,H) = 2(y) + (-1)(1-y)$

$= 2y - 1 + y = 3y - 1$

Similarly if the player A plays T all the time then B's expected gain is

$E(B,T) = (-1)y + 0(1-y)$

$= -y$

It can be shown mathematically
is the player B chooses y such that

$$E(B,H) = E(B,T) = E(B)$$

$$3y - 1 = -y$$

$$4y = 1$$

$$y = \frac{1}{4}$$

$$1 - y = \frac{3}{4}$$

Best strategy for player B $\left(\frac{1}{4}, \frac{3}{4}\right)$

so that the expected gain for the player B is

Row I $\quad E(B) = \frac{1}{4}(2) + \frac{3}{4}(-1)$

$$= \frac{2}{4} - \frac{3}{4} = \frac{-1}{4}$$

Row II $\quad E(B) = \frac{1}{4}(-1) + \frac{3}{4}(0)$

$$= \frac{-1}{4}$$

$\Rightarrow$

		B: y I	$1-y$ II	
A	x I	1	$\boxed{3}$	1
	$1-x$ II	$\boxed{4}$	2	$\boxed{2}$
		4	3	

$3 \neq 2$

$\therefore$ no saddle point

(c)

$$E(A, \mathrm{I}) = E(A, \mathrm{II})$$

$$1(x) + 4(1-x) = 3(x) + 2(1-x)$$

$$x + 4 - 4x = 3x + 2 - 2x$$

$$-3x + 4 = x + 2$$

$$2 = 4x$$

$$x = \frac{1}{2}$$

$$1 - x = \frac{1}{2}$$

$$\left(\frac{1}{2}, \frac{1}{2}\right)$$

$$E(B, \mathrm{I}) = E(B, \mathrm{II})$$

$$1(y) + 3(1-y) = 4(y) + 2(1-y)$$

$$y + 3 - 3y = 4y + 2 - 2y$$

$$-2y + 3 = 2y + 2$$

$$4y = 1$$

$$y = \frac{1}{4}$$

$$1 - y = \frac{3}{4}$$

$$\left(\frac{1}{4}, \frac{3}{4}\right)$$

$$E(A) = \frac{1}{2}(1) + \frac{1}{2}(4)$$

$$= \frac{1}{2} + \frac{4}{2} = \frac{5}{2}$$

or

$$E(A) = \frac{1}{2}(3) + \frac{1}{2}(2) = \frac{3}{2} + \frac{2}{2} = \frac{5}{2}$$

$$E(B) = \frac{1}{4}(1) + \frac{3}{4}(3) = \frac{1}{4} + \frac{9}{4} = \frac{10}{4} = \frac{5}{2}$$

$$E(B) = \frac{1}{4}(4) + \frac{3}{4}(2) = \frac{4}{4} + \frac{6}{4} = \frac{10}{4} = \frac{5}{2}$$

$$\begin{array}{ccc|c|c|c}
 & & & \overset{y}{I} & \overset{1-y}{II} & \\
 & & & B & & \\
\hline
A & x & I & -4 & 6 & -4 \\
\hline
 & 1-x & II & 2 & -3 & \boxed{-3} \\
\hline
 & & & \boxed{2} & 6 &
\end{array}$$

$$2 \neq -3$$

$$E(A, I) = E(A, II)$$

$$-4(x) + 2(1-x) = 6(\cancel{y}\,x) - 3(1-\cancel{y}\,x)$$

$$-4x + 2 - 2x = 6x - 3 + 3x$$

$$-6x + 2 = 9x - 3$$

$$5 = 15x \qquad \left(\frac{1}{3}, \frac{2}{3}\right)$$

$$x = \frac{1}{3}$$

$$1 - x = \frac{2}{3}$$

$$E(B, I) = E(B, II)$$

$$-4(y) + 6(1-y) = 2(y) + (-3)(1-y)$$

$$-4y + 6 - 6y = 2y - 3 + 3y$$

$$-10y + 6 = 5y - 3$$

$$9 = 15y$$

$$y = \frac{3}{5}$$

$$1 - y = \frac{2}{5}$$

$$\left(\frac{3}{5}, \frac{2}{5}\right)$$

$$E(A) = \frac{1}{3}(-4) + 2\left(\frac{2}{3}\right)$$

$$= -\frac{4}{3} + \frac{4}{3} = 0$$

$$E(A) = \frac{1}{3}(6) + \frac{2}{3}(-3) = \frac{6}{3} - \frac{6}{3} = 0$$

$$E(B) = \frac{3}{5}(-4) + \frac{2}{5}(6) = -\frac{12}{5} + \frac{12}{5} = 0$$

$$E(B) = \frac{3}{5}(2) + \frac{2}{5}(-3) = \frac{6}{5} - \frac{6}{5} = 0$$

$$\begin{array}{cc|c|c|c}
 & & y & 1-y & \\
 & & \text{I} & \text{II} & \\
\hline
x & \text{I} & 6 & -3 & \boxed{-3} \\
\hline
1-x & \text{II} & -3 & 0 & \boxed{-3} \\
\hline
 & & 6 & \boxed{0} &
\end{array}$$

$$0 \neq -3$$

$$E(A, \text{I}) = E(A, \text{II})$$

$$6(x) - 3(1-x) = -3(x) + 0(1-x)$$

$$6x - 3 + 3x = -3x$$

$$9x - 3 = -3x$$

$$12x = 3$$

$$x = \frac{1}{4}$$

$$1 - x = \frac{3}{4} \qquad \left(\frac{1}{4}, \frac{3}{4}\right)$$

$$E(B, \text{I}) = E(B, \text{II})$$

$$6(y) - 3(1-y) = -3(y) + 0(1-y)$$

$$6y - 3 + 3y = -3y$$

$$12y = 3$$

$$y = \frac{1}{4}$$

$$1 - y = \frac{3}{4}$$

$$E(A) = \frac{1}{4}(6) + \frac{3}{4}(-3) = \frac{6}{4} - \frac{9}{4} = \frac{-3}{4}$$

$E(A) = \frac{1}{4}(-3) + 0\left(\frac{3}{4}\right) = \frac{-3}{4}$

$E(B) = \frac{1}{4}(6) + \frac{3}{4}(-3) = \frac{6}{4} - \frac{9}{4} = \frac{-3}{4}$

$E(B) = \frac{1}{4}(-3) + \frac{3}{4}(0) = -\frac{3}{4}$

Arithmetic method

let the pay off matrix of two person, zero sum game

$$\begin{array}{cc} & B \\ & \begin{array}{cc} \mathrm{I} & \mathrm{II} \end{array} \\ A \begin{array}{c} \mathrm{I} \\ \mathrm{II} \end{array} & \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \end{array}$$

let the Player A select first row as strategy with probability x and for second row the probability is $1-x$

$$x + 1 - x = 1$$

similarly for B 1st column probability is y and 2nd column probability is $1-y$

$$y + 1 - y = 1$$

$V = a_{11}x + a_{21}(1-x)$

$$x = \frac{a_{22} - a_{21}}{(a_{11} - a_{12}) + (a_{22} - a_{21})}$$

$$1 - x = \frac{a_{11} - a_{12}}{(a_{11} - a_{12}) + (a_{22} - a_{21})}$$

$\rightarrow$

$$\begin{bmatrix} 10 & 7 \\ 8 & 9 \end{bmatrix} \begin{matrix} 7 \\ \boxed{8} \end{matrix}$$

$$\begin{matrix} 10 & 9 \end{matrix}$$

$9 \neq 8$

no saddle point

$$\begin{array}{c|c|c|c} & \text{Row diff} & \text{Ratio} & \text{Interchange ratio} \\ \begin{bmatrix} 10 & 7 \\ 8 & 9 \end{bmatrix} & \begin{matrix} 3 \\ 1 \end{matrix} & \begin{matrix} 3/4 \\ 1/4 \end{matrix} & \begin{matrix} 1/4 \\ 3/4 \end{matrix} \end{array}$$

$\left(1/4, \frac{3}{4}\right)$

Col. diff $\quad 2 \quad 2$

Ratio $\quad 2/4 = 1/2 \quad 2/4 = 1/2$

Inter change ratio $\quad 1/2 \quad 1/2 \quad \left(\frac{1}{2}, \frac{1}{2}\right)$

$$\begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{matrix}$$

$\left.\begin{matrix} a_{11} - a_{12} \\ a_{22} - a_{21} \end{matrix}\right\}$ Row diff

$a_{11} - a_{21}$; $a_{22} - a_{21} \Rightarrow$ Col - diff

$$V = \frac{1}{4}(10) + \frac{3}{4}(8) = \frac{10}{4} + \frac{24}{4} = \frac{34}{4} = \frac{17}{2}$$

$$= \frac{1}{4}(7) + \frac{3}{4}(9) = \frac{7}{4} + \frac{27}{4} = \frac{34}{4} = \frac{17}{2}$$

$$= \frac{1}{2}(10) + \frac{1}{2}(7) = \frac{10}{2} + \frac{7}{2} = \frac{17}{2}$$

$$= \frac{1}{2}(8) + \frac{1}{2}(9) = \frac{8}{2} + \frac{9}{2} = \frac{17}{2}$$

A game has the following payoff Matrix

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix} \end{array}$$

Prove that $E(x,y) = 1-2x\left(y-\frac{1}{2}\right)$ and deduce that in solution of the game the player A follows pure strategies where player B has infinite number of mixed strategies.

(Given that x is the probability that player A select his first strategy and y is the probability that player B select his first strategy)

7/3/2020

$\rightarrow$ 1) $\begin{bmatrix} 5 & 4 \\ 6 & 8 \end{bmatrix}$ 2) $\begin{bmatrix} 4 & 12 \\ 8 & 4 \end{bmatrix}$ 3) $\begin{bmatrix} 5 & 12 \\ 6 & 4 \end{bmatrix}$

1) $\begin{bmatrix} 5 & 4 \\ 6 & 8 \end{bmatrix} \begin{matrix} 4 \\ \boxed{6} \end{matrix}$

$\begin{matrix} 6 & 8 \end{matrix}$

Maximin = Minimax $(6 = 6)$

$\therefore$ saddle point $= (2, 1)$

Value of game $= 6$

2)

$$\begin{bmatrix} 4 & 12 \\ 8 & 4 \end{bmatrix} \begin{matrix} \boxed{4} \\ \boxed{4} \end{matrix}$$

$$\begin{matrix} 8 & 12 \end{matrix}$$

$8 \neq 4$

$\therefore$ There is no saddle point

$$\begin{array}{c|c|c|c} & & \text{Row diff} & \text{ratio} & \text{interchange Ratio} \\ \begin{bmatrix} 4 & 12 \\ 8 & 4 \end{bmatrix} & & \begin{matrix} -8 \\ \boxed{-4} \end{matrix} & \begin{matrix} -8/12 = +2/3 \\ -4/12 = +1/3 \end{matrix} & \begin{matrix} 1/3 \\ 2/3 \end{matrix} \end{array}$$

$\left(\frac{1}{3}, \frac{+2}{3}\right)$

Col diff $\quad -4 \quad -8$

Ratio $\quad -4/12 = -1/3 \quad -8/12 = 2/3$

$\left(+\frac{2}{3}, +\frac{1}{3}\right)$

$$V = +\frac{1}{3} \times 4 + \frac{2}{3} \times 8 = +\frac{4}{3} + \frac{16}{3} = +\frac{20}{3} = 6.33$$

$$V = \frac{1}{3} \times 12 + \frac{2}{3} \times 4 = \frac{12}{3} + \frac{8}{3} = \frac{20}{3} = 6.33$$

$$V = \frac{2}{3} \times 4 + \frac{1}{3} \times 12 = \frac{8}{3} + \frac{12}{3} = \frac{20}{3} = 6.33$$

$$V = \frac{2}{3} \times 8 + \frac{1}{3} \times 4 = \frac{16}{3} + \frac{4}{3} = \frac{20}{3} = 6.33$$

subgame method

$$\begin{bmatrix} 5 & 4 & 12 \\ 6 & 8 & 4 \end{bmatrix}$$

3)

$$\begin{array}{c|c|c|c}
 & \begin{bmatrix} 5 & 12 \\ 6 & 4 \end{bmatrix} & \text{Row diff} & \text{Ratio} & \text{Interchange} \\
 & & -7 & -7/-9 = 7/9 & 2/9 \\
 & & -2 & -2/-9 = 2/9 & 7/9 \\
\text{Col diff} & -1 \quad -8 & & & \\
\text{Ratio} & -1/-9 = 1/9 \quad -8/-9 = 8/9 & & & \\
\text{Interchange} & 8/9 \quad 1/9 & & &
\end{array}$$

$\left(\frac{2}{9}, \frac{7}{9}\right)$

Interchange $\left(\frac{8}{9}, \frac{1}{9}\right)$

$$V = \frac{2}{9}\times 5 + \frac{7}{9}\times 6 = \frac{10}{9} + \frac{42}{9} = \frac{52}{9} = 5.7$$

$$V = \frac{2}{9}\times 12 + \frac{7}{9}\times 4 = \frac{24}{9} + \frac{28}{9} = \frac{52}{9} = 5.7$$

$$V = \frac{8}{9}\times 5 + \frac{1}{9}\times 12 = \frac{40}{9} + \frac{12}{9} = \frac{52}{9} = 5.7$$

$$V = \frac{8}{9}\times 6 + \frac{1}{9}\times 4 = \frac{48}{9} + \frac{4}{9} = \frac{52}{9} = 5.7$$

subgame

$$\begin{bmatrix} 5 & 4 & 12 \\ 6 & 8 & 4 \end{bmatrix}$$

Divide the matrix into possible submatrix of size 2x2

$$\begin{bmatrix} 5 & 4 \\ 6 & 8 \end{bmatrix} \begin{bmatrix} 4 & 12 \\ 8 & 4 \end{bmatrix} \begin{bmatrix} 5 & 12 \\ 6 & 4 \end{bmatrix}$$

Calculate the value of the game for each submatrix.

The submatrix which has the highest value ~~valume~~ of value of game it is the value of game of the original payoff matrix

Here the value of game of original matrix is 6.33

9/3/2020

Principle of dominance

If the game contains saddle point of size 2x2 (or 2xn or mx2) then the value of game exists. on the other hand if the saddle point doesn't exist then by using this principle we reduce the given matrix 2x2.

Rules
i) if all the elements of a row ≤ corresponding element of other row then ~~rated~~ Row is dominated by first row.

ii) if all the elements of column ≥ corresponding element of other column then column is dominated by the first column

iii) By the average of two or more strategies pure strategy will be dominated

iv) By deleting dominants row or column we reduce the size of payoff matrix

→

	I	II	III	IV	V	VI	
1	4	2	0	2	1	1	0
2	4	3	1	3	2	2	$\boxed{1}$
3	4	3	7	-5	1	2	-5
4	4	3	4	-1	2	2	-1
5	4	3	3	-2	2	2	-2
	4	3	7	3	2	2	

$1 \neq 2$

Maximin ≠ minimax

∴ no saddle point

size of matrix = (5×6)

step I: Game has no saddle point Thus we use the principle of dominance to reduce the matrix.

In Matrix 1st and 5th Row will be dominated by 2nd and 4th Row then reduce payoff matrix.

2	4	3	1	3	2	2	$\boxed{1}$
3	4	3	7	-5	1	2	-5
4	4	3	4	-1	2	2	-1
	4	3	7	3	2	2	

$1 \neq -2$

no saddle point

Also col I, II, ~~III~~, VI are dominated by column V

1	3	2	$\boxed{1}$
7	-5	1	-5
4	-1	2	-1
$\boxed{7}$	$\boxed{3}$	$\boxed{2}$	

$1 \neq 2$

no saddle point