Floyd's Algo

- Used to find shorest path in a weighted graph
- Travel maps containing driving distance from one point to another
  - Represented by tables
  - shortest distance from point A to point B by intersection of row and column
  - Route may pass through other cities represented in the table.

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· Dijkstra is single - source, shortest - path algo.
This means they only Compute the shortest path from a single source.
Floyd - warshall, computes the shortest distances between every pair of vertices in the input graph.

F-W Algo is extremely useful in networking
It is more effective at managing multiple stops on the route because it can calculate the shortest paths between all relevant nodes.

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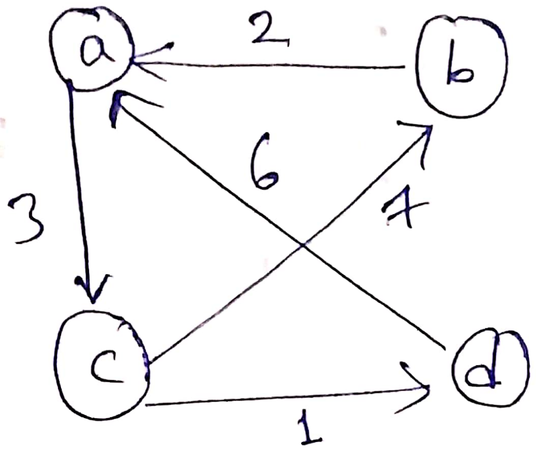
Floyd's Algo (all pair shortest path)



step 1 Construct D^0

if $i == j$ w_{ij} = "—"

else if $i \xrightarrow{v} j = \{v\}$

else ∞

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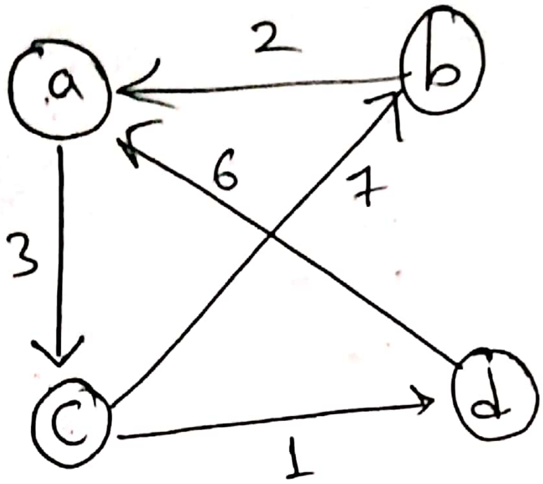
Floyd's Algo (all pair shortest path)



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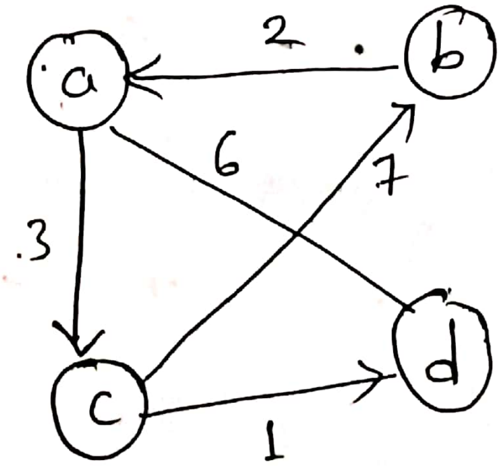

$$D^0 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & — & \infty & 3 & \infty \\ b & 2 & — & \infty & \infty \\ c & \infty & 7 & — & 1 \\ d & 6 & \infty & \infty & — \end{array}$$

step-2 $w_{ij}^{k} = \min\left(w_{ij}^{k-1},\ \underset{(\text{Row})}{w_{ik}^{k-1}} + \underset{(\text{Col})}{w_{kj}^{k-1}}\right)$

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$$D_0 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & - & \infty & 3 & \infty \\ b & 2 & - & \infty & \infty \\ c & \infty & 7 & - & 1 \\ d & 6 & \infty & \infty & - \end{array}, \quad D^{(1)} = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & - & \infty & 3 & \infty \\ b & 2 & - & & \\ c & \infty & & - & \\ d & 6 & & & - \end{array}$$

$$\begin{array}{cccc} a & b & c & d \end{array}$$

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$$
D_0 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & - & \infty & 3 & \infty \\ b & 2 & - & \infty & \infty \\ c & \infty & 7 & - & 1 \\ d & 6 & \infty & \infty & - \end{array}
\quad , \quad
D^{(1)} = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & - & \infty & 3 & \infty \\ b & 2 & - & & \infty \\ c & \infty & 7 & - & 1 \\ d & 6 & \infty & & - \end{array}
$$

$(k-1)$

$$D_0 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & - & \infty & 3 & \infty \\ b & 2 & - & \infty & \infty \\ c & \infty & 7 & - & 1 \\ d & 6 & \infty & \infty & - \end{array} \quad , \quad D^{(1)} = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & - & \infty & 3 & \infty \\ b & 2 & - & 5 & \infty \\ c & \infty & 7 & - & 1 \\ d & 6 & \infty & & - \end{array}$$

$\boxed{R_1, C_1}$ $\quad w_{ij}^{(k)} = \min\left(w_{ij}^{(k-1)}, w_{ik}^{(k-1)} + w_{kj}^{(k-1)}\right)$

Row $= k = 1$ $\quad w_{23} = \min(\infty, 2+3) = 5$

Col $= k = 1$ $\quad \left\{ \therefore w_{23} = M(w_{23}, w_{21} + w_{13}) \right\}$

$$w_{43} = \min(w_{43}, w_{41} + w_{13})$$

$$= \min(\infty, 6 + 3)$$

$$D_0 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & - & \infty & 3 & \infty \\ b & 2 & - & \infty & \infty \\ c & \infty & 7 & - & 1 \\ d & 6 & \infty & \infty & - \end{array}$$

$b = 9$

$$D^{(1)} = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & - & \infty & 3 & \infty \\ b & 2 & - & 5 & \infty \\ c & \infty & 7 & - & 1 \\ d & 6 & \infty & 9 & - \end{array}$$

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$$D^{(1)} = \begin{array}{c|cccc|} & a & b & c & d \\ \hline a & - & \infty & 3 & \infty \\ b & 2 & - & 5 & \infty \\ c & \infty & 7 & - & 1 \\ d & 6 & \infty & 9 & - \end{array} \quad , \quad D^{(2)} = \begin{array}{c|cccc|} & a & b & c & d \\ \hline a & - & \infty & & \\ b & 2 & - & 5 & \infty \\ c & & 7 & - & \\ d & & \infty & & - \end{array}$$

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$$D^{(1)} = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & - & \infty & 3 & \infty \\ b & 2 & - & 5 & \infty \\ c & \infty & 7 & - & 1 \\ d & 6 & \infty & 9 & - \end{array} , \quad D^{(2)} = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & - & \infty & 3 & \infty \\ b & 2 & - & 5 & \infty \\ c & 9 & 7 & - & 1 \\ d & 6 & \infty & 9 & - \end{array}$$

$k=2$ $\quad w_{31}^{(2)} = \min\left(w_{31}^{(1)}, w_{32}^{(1)} + w_{21}^{(1)}\right) = \min(\infty, 7+2) = 9$

$\left\{ w_{ij}^{k} = \min\left(w_{ij}^{k-1}, w_{ik}^{k-1} + w_{kj}^{k-1}\right)\right.$

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$$D^{(2)} = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \left[\begin{array}{cccc} - & \infty & 3 & \infty \\ 2 & - & 5 & \infty \\ 9 & 7 & - & 1 \\ 6 & \infty & 9 & - \end{array} \right] \end{array}$$

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$$D^{(2)} = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & \left[\begin{matrix} - \\ 2 \\ 9 \\ 6 \end{matrix}\right. & \begin{matrix} \infty \\ - \\ 7 \\ \infty \end{matrix} & \begin{matrix} 3 \\ 5 \\ - \\ 9 \end{matrix} & \left.\begin{matrix} \infty \\ \infty \\ 1 \\ - \end{matrix}\right] \end{array}$$

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & - & \infty & 3 & \infty \\ b & 2 & - & 5 & \infty \\ c & 9 & 7 & - & 1 \\ d & 6 & \infty & 9 & - \end{array}$$

,

$$D^{(3)} = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & - & \bigcirc & 3 & \bigcirc \\ b & \bigcirc & - & 5 & \bigcirc \\ c & 9 & 7 & - & 1 \\ d & \bigcirc & \bigcirc & 9 & - \end{array}$$

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$$D^{(2)} = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & - & \infty & 3 & \infty \\ b & 2 & - & 5 & \infty \\ c & 9 & 7 & - & 1 \\ d & 6 & \infty & 9 & - \end{array} \quad , \quad D^{(3)} = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & - & 0 & 3 & 0 \\ b & 0 & - & 5 & 0 \\ c & 9 & 7 & - & 1 \\ d & 0 & 0 & 9 & - \end{array}$$

$$w_{ij}^{k} = \min\left(w_{ij}^{k-1},\ w_{ik}^{(k-1)} + w_{kj}^{(k-1)}\right)$$

$$w_{12}^{(3)} = \min\left(w_{12}^{(2)},\ w_{13}^{(2)} + w_{32}^{(2)}\right)$$

$$= \min\left(\infty,\ 3+7\right) = 10$$

$$w_{14}^{(3)} = \min\left(w_{14}^{(2)},\ w_{13}^{(2)} + w_{34}^{(2)}\right)$$

$$= \min\left(\infty,\ 3+1\right) = 4$$

$w_{21}^{(3)} = 2$, $w_{41}^{(3)} = 6$, $w_{42}^{(3)} = 16$, $w_{24}^{(3)} = 6$

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$w_{21} = 2, \quad w_{41} = 6, \quad w_{42} = 10$

$$D^{(2)} = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} - & \infty & 3 & \infty \\ 2 & - & 5 & \infty \\ 9 & 7 & - & 1 \\ 6 & \infty & 9 & - \end{bmatrix} \end{array}, \quad D^{3} = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} - & 10 & 3 & 4 \\ 2 & - & 5 & 6 \\ 9 & 7 & - & 1 \\ 6 & 16 & 9 & - \end{bmatrix} \end{array}$$

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$$D^{(3)} = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & - & 10 & 3 & 4 \\ b & 2 & - & 5 & 6 \\ c & 9 & 7 & - & 1 \\ d & 6 & 16 & 9 & - \end{array}, \quad D^{(4)} = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & - & 10 & & 4 \\ b & & - & & 6 \\ c & & & - & 1 \\ d & 6 & 16 & 9 & - \end{array}$$

$w_{12}^{(4)} = 10$, $w_{13}^{(4)} = 3$,

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$$
D^{(4)} \quad \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} - & 10 & 3 & 4 \\ 2 & - & 5 & 6 \\ 7 & 7 & - & 1 \\ 6 & 16 & 9 & - \end{bmatrix} \end{array}
$$

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