Part 2. Questions (20%)

1. (10%)

Tree A

$$\begin{array}{c} (400, 400) \\ \swarrow \quad \searrow \\ (300, \underline{100}) \qquad (\underline{100}, 300) \end{array}$$

Tree B.

$$\begin{array}{c} (400, 400) \\ \swarrow \quad \searrow \\ (\underline{200}, 400) \qquad (200, \underline{0}) \end{array}$$

(1) misclassification rate $= \dfrac{\text{total misclass}}{\text{total } C_1 + C_2}$

$$\text{mis\_rate}(A) = \frac{100+100}{800} = 0.25$$

$$\text{mis\_rate}(B) = \frac{200+0}{800} = 0.25 = \text{mis\_rate}(A) \ \#$$

(2) cross-entropy $E = -\sum_{k=1}^{K} P_k \log_2 P_k$

$$E_A = \frac{400}{800} I\left(\frac{300}{400}, \frac{100}{400}\right) + \frac{400}{800} I\left(\frac{100}{400}, \frac{300}{400}\right)$$

$$= \frac{1}{2}\left[-\left(\frac{3}{4}\right)\log_2\left(\frac{3}{4}\right) - \left(\frac{1}{4}\right)\log_2\left(\frac{1}{4}\right)\right] + \frac{1}{2}\left[-\left(\frac{1}{4}\right)\log_2\left(\frac{1}{4}\right) - \left(\frac{3}{4}\right)\log_2\left(\frac{3}{4}\right)\right]$$

$$= -\left(\frac{3}{4}\right)\log_2\left(\frac{3}{4}\right) - \left(\frac{1}{4}\right)\log_2\left(\frac{1}{4}\right) = 0.811$$

$$E_B = \frac{600}{800} I\left(\frac{400}{600}, \frac{200}{600}\right) + \frac{200}{800} I\left(\frac{200}{200}, 0\right)$$

$$= \frac{3}{4}\left[-\left(\frac{2}{3}\right)\log_2\left(\frac{2}{3}\right) - \left(\frac{1}{3}\right)\log_2\left(\frac{1}{3}\right)\right] + \frac{1}{4}\left[-\log_2(1)\right]$$

$$= 0.689 < E_A \ \#$$

(3) Gini index $G = 1 - \sum_{k=1}^{K} P_k^2$

$$G_A = \left\{1 - \left[\left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2\right]\right\} + \left\{1 - \left[\left(\frac{1}{4}\right)^2 + \left(\frac{3}{4}\right)^2\right]\right\} = \frac{3}{4} = 0.75$$

$$G_B = \left\{1 - \left[\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2\right]\right\} + \left\{1 - \left[1^2 + 0\right]\right\} = \frac{4}{9} = 0.44 < G_A \ \#$$

2, (10%)

minimize : $E_{x,t}\left[e^{-ty(x)}\right] = \sum_t \int e^{-ty(x)} p(t|x)\, P(x)\, dx$

$$\alpha_{t+1} = \arg\min\, (e^{\alpha} - e^{-\alpha})\, \epsilon_t + e^{-\alpha}$$

$$\frac{\partial}{\partial\alpha} (e^{\alpha} - e^{-\alpha})\, \epsilon_t + e^{-\alpha} = e^{\alpha}\epsilon_t + e^{-\alpha}\epsilon_t - e^{-\alpha}$$

$$= e^{2\alpha}\epsilon_t + \epsilon_t - 1$$

$$e^{2\alpha}\epsilon_t + \epsilon_t - 1 = 0$$

$$e^{2\alpha} = \frac{1-\epsilon_t}{\epsilon_t}$$

$$\alpha = \frac{1}{2}\ln\frac{1-\epsilon_t}{\epsilon_t} \quad \#$$

where $\epsilon_t = P(t=-1\,|\,x)$, $1-\epsilon_t = P(t=1\,|\,x)$