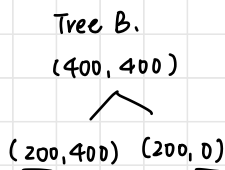
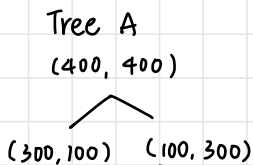


Part 2. Questions (20%)

1. (10%)



(1) misclassification rate = $\frac{\text{total misclass}}{\text{total } C_1 + C_2}$

$$\text{mis_rate}(A) = \frac{100 + 100}{800} = 0.25$$

$$\text{mis_rate}(B) = \frac{200 + 0}{800} = 0.25 = \text{mis_rate}(A) \#$$

(2) cross-entropy $E = -\sum_{k=1}^K P_k \log_2 P_k$

$$\begin{aligned} E_A &= \frac{400}{800} I\left(\frac{300}{400}, \frac{100}{400}\right) + \frac{400}{800} I\left(\frac{100}{400}, \frac{300}{400}\right) \\ &= \frac{1}{2} \left[-\left(\frac{3}{4}\right) \log_2 \left(\frac{3}{4}\right) - \left(\frac{1}{4}\right) \log_2 \left(\frac{1}{4}\right) \right] + \frac{1}{2} \left[-\left(\frac{1}{4}\right) \log_2 \left(\frac{1}{4}\right) - \left(\frac{3}{4}\right) \log_2 \left(\frac{3}{4}\right) \right] \\ &= -\left(\frac{3}{4}\right) \log_2 \left(\frac{3}{4}\right) - \left(\frac{1}{4}\right) \log_2 \left(\frac{1}{4}\right) = 0.811 \end{aligned}$$

$$\begin{aligned} E_B &= \frac{600}{800} I\left(\frac{400}{600}, \frac{200}{600}\right) + \frac{200}{800} I\left(\frac{200}{200}, 0\right) \\ &= \frac{3}{4} \left[-\left(\frac{2}{3}\right) \log_2 \left(\frac{2}{3}\right) - \left(\frac{1}{3}\right) \log_2 \left(\frac{1}{3}\right) \right] + \frac{1}{4} \left[-\log_2 (1) \right] \\ &= 0.689 < E_A \# \end{aligned}$$

(3) Gini index $G = 1 - \sum_{k=1}^K P_k^2$

$$G_A = \left\{ 1 - \left[\left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2 \right] \right\} + \left\{ 1 - \left[\left(\frac{1}{4}\right)^2 + \left(\frac{3}{4}\right)^2 \right] \right\} = \frac{3}{4} = 0.75$$

$$G_B = \left\{ 1 - \left[\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 \right] \right\} + \left\{ 1 - [1^2 + 0] \right\} = \frac{4}{9} = 0.44 < G_A \#$$

2. (10%)

$$\text{minimize: } E_{x,t} [e^{-ty(x)}] = \sum_t \int e^{-ty(x)} p(t|x) p(x) dx$$

$$\alpha_{t+1} = \arg \min (e^{\alpha} - e^{-\alpha}) \epsilon_t + e^{-\alpha}$$

$$\begin{aligned} \frac{\partial}{\partial \alpha} (e^{\alpha} - e^{-\alpha}) \epsilon_t + e^{-\alpha} &= e^{\alpha} \epsilon_t + e^{-\alpha} \epsilon_t - e^{-\alpha} \\ &= e^{2\alpha} \epsilon_t + \epsilon_t - 1 \end{aligned}$$

$$e^{2\alpha} \epsilon_t + \epsilon_t - 1 = 0$$

$$e^{2\alpha} = \frac{1 - \epsilon_t}{\epsilon_t}$$

$$\alpha = \frac{1}{2} \ln \frac{1 - \epsilon_t}{\epsilon_t} \quad \#$$

where $\epsilon_t = P(t=-1|x)$, $1 - \epsilon_t = P(t=1|x)$

