Online Supplement for

A simple but usually fast branch-and-bound algorithm for the capacitated facility location problem

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A. Comparison with CAPLOC

Ryu's and Guignard's (1992) CAPLOC algorithm is based on the same Lagrangean relaxation as our method BB-SG and applies basically the same simple subgradient procedure for computing the Lagrangean dual bound. CAPLOC is one of the last implementions of a subgradient-based branch-and-bound for the CFLP in the literature. We thus include a comparison between this method and BB-SG. CAPLOC uses 100 subgradient iterations at the root node and at most 25 iterations at each other node in the search tree. In contrast to BB-SG, CAPLOC applies a depth-first search strategy and, as as shown in the paper, a quite different heuristic branching rule. Moreover, at the top node after completion of the subgradient procedure, CAPLOC tries to fix as many y_j variables as possible by means of an extensive Lagrangean probing. Let (x^B, y^B) denote the best feasible solution CAPLOC found so far and let z_B denote its objective value. Variables y_j are temporarily fixed to $1 - y_j^B$ and a limited number of subgradient steps is applied. If the resulting lower bound is no smaller than z_B, the binary variable y_j is pegged to y_j^B. If the method branches on variable y_j, then the branch $y_j = 1$ is always investigated first.

We used a FORTRAN code of Ryu and Guignard, but replaced the out-of-kilter method used in this code for solving the transportation problems by calls to the network simplex algorithm of CPLEX's callable library. The code was then compiled and run on the same machine as BB-SG. Table 17 compares CAPLOC and BB-SG on the test problem instances from Klose and Görtz (2007) and Table 18 summarizes these results by again averaging over the capacity ratio r. As can be seen from Table 17 and 18, BB-SG is far superior to CAPLOC. (The number of enumerated nodes reported for CAPLOC in this table does not include the branches investigated in the Lagrangean probing.) On the test problems of Table 17, BB-SG was on average about 55 times faster than CAPLOC. Basically, both algorithms are based

Table 17: Comparison of CAPLOC and BB-SG

$\|I\| \times \|J\|$	CAPLOC N	CAPLOC T	BB-SG N	BB-SG D	BB-SG T
		$r = 3$			
100×100	11	0.36	9	3	0.43
200×100	293	5.36	58	7	1.29
200×200	586	12.12	39	6	2.18
500×100	8424	402.48	794	14	37.94
500×200	22470	978.93	678	16	54.80
max	54679	2773.29	1733	19	91.79
mean	6357	279.85	316	9	19.33
		$r = 5$			
100×100	67	0.84	15	4	0.44
200×100	810	13.61	133	10	3.07
200×200	365	10.33	49	6	2.63
500×100	13427	591.76	437	13	21.73
500×200	250202	29314.56	5061	17	434.98
max	850016	119911.52	12419	21	1319.37
mean	52974	5986.22	1139	10	92.57
		$r = 10$			
100×100	13	0.34	13	4	0.28
200×100	504	12.31	37	5	1.36
200×200	76	3.63	54	8	2.16
500×100	5193	298.16	83	7	6.45
500×200	65110	4973.16	1312	13	85.32
max	265228	21419.38	4495	18	230.96
mean	14179	1057.52	300	7	19.11

N = number of nodes enumerated, T = CPU time in seconds, D = maximal depth of the search tree reached.

on the same methodology; they apply the same Lagrangean relaxation and use subgradient optimization for lower bounding. BB-SG however greatly benefits from, firstly, the best lower bound search and, secondly and mostly, from better branching decisions based on a reasonable estimate of a primal solution.

Since also the test problem instances from the OR-library are of a size and difficulty that can be managed by CAPLOC, we also compared both methods on these test problems. Each of these problem instances is of size $|I| \times |J| = 1000 \times 100$. Table 19 shows the results, which again indicate the superiority of BB-SG. The ORLIB instances are easier to solve than the instances of Table 17, but still BB-SG is about 17 times faster than CAPLOC on these instances.

Table 18: Summarized comparison of CAPLOC and BB-SG

	CAPLOC		BB-SG		
$\lvert I\rvert \times \lvert J\rvert$	N	T	N	D	T
100×100	30	0.51	12	4	0.38
200×100	536	10.43	76	7	1.91
200×200	342	8.69	47	7	2.32
500×100	9015	430.80	438	11	22.04
500×200	112594	11755.55	2350	15	191.70
mean	24503	2441.20	585	9	43.67

Table 19: Comparison of CAPLOC and BB-SG on ORLIB instances

	CAPLOC		BB-SG		
problem	N	T	N	D	T
capa1	30	26.55	9	3	2.91
capa2	8	23.84	7	2	2.89
capa3	7	26.86	9	3	2.18
capa4	1	21.22	1	0	0.84
capb1	1	10.12	1	0	1.66
capb2	1510	361.92	27	5	11.06
capb3	280	243.44	29	6	11.49
capb4	4	25.12	17	7	4.32
capc1	95	61.25	9	3	3.40
capc2	569	164.91	59	8	12.63
capc3	22	52.19	11	4	6.01
capc4	18	51.87	5	2	2.40
mean	212	89.11	15	4	5.15

B. Using the volume algorithm instead of subgradient optimization

We also compared BB-SG with the same branch-and-bound method, where however instead of subgradient optimization the volume algorithm is used for computing lower bounds (BB-VA). In contrast to BB-SG, the volume algorithm estimates primal solutions by taking an exponentially smoothed average of the generated Lagrangean solutions. The guessed primal solution is in particular also used to determine a direction into which the method searches for improved dual solutions, whilst BB-SG simply makes a step in direction of a subgradient. In our experiments, we used a smoothing parameter $\mu = 0.1$. We also experimented with taking simple arithmetic averages, but this showed to be far less effective. Barahona and Anbil (2000) and Barahona and Chudak (2005) suggested to choose

Table 20: Comparison of BB-SG and BB-VA on the instances from Klose and Görtz (2007)

	BB-SG			BB-VA		
$\lvert I\rvert \times \lvert J\rvert$	N	D	T	N	D	T
			$r = 3$			
100×100	9	3	0.43	91	42	0.62
200×100	58	7	1.29	523	60	25.51
200×200	39	6	2.18	31	8	3.96
500×100	794	14	37.94	1321	58	259.42
500×200	678	16	54.80	1282	108	417.11
max	1733	19	91.79	3485	200	1035.95
mean	316	9	19.33	650	55	141.32
			$r = 5$			
100×100	15	4	0.44	136	32	1.78
200×100	133	10	3.07	227	35	10.69
200×200	49	6	2.63	232	39	10.40
500×100	437	13	21.73	497	44	95.84
500×200	5061	17	434.98	4490	92	1598.30
max	12419	21	1319.37	9459	128	4055.70
mean	1139	10	92.57	1116	48	343.40
			$r = 10$			
100×100	13	4	0.28	84	33	0.49
200×100	37	5	1.36	73	18	3.06
200×200	54	8	2.16	727	134	13.43
500×100	83	7	6.45	73	10	21.23
500×200	1312	13	85.32	1594	95	520.52
max	4495	18	230.96	5181	200	1675.48
mean	300	7	19.11	510	58	111.75

N = number of nodes enumerated, T = CPU time in seconds, D = maximal depth of the search tree reached.

$\mu = \max\{\mu_{\max}/10, \min\{\mu^*, \mu_{\max}\}\}$, where μ^* minimizes $\|\mu s^t + (1-\mu)g^t\|$ and s^t and g^t respectively denote the current subgradient and direction. The parameter $\mu_{\max}$ is initially set to 0.1 and halved if $Z_D(\lambda)$ is not increased by 1 % in 100 consecutive iterations. Table 20 compares the performance of the two methods on the test problem instances from Klose and Görtz (2007). Table 21 again summarizes these results by additionally taking averages over the different ratios r. BB-SG significantly outperformed the method based on the volume algorithm. On average, BB-SG was 4 to 5 times faster than BB-VA. The clear superiority of BB-SG suggests that the method will still do better than BB-VA even if the smoothing parameter μ is determined in a more sophisticated way. We therefore refrained from extending the comparison to the other types of test problems. In our computational experiments, we

Table 21: Summarized comparison of BB-SG and BB-VA

	BB-SG			BB-VA		
$\lvert I\rvert \times \lvert J\rvert$	N	D	T	N	D	T
100×100	12	4	0.38	104	36	0.96
200×100	76	7	1.91	274	38	13.09
200×200	47	7	2.32	330	60	9.26
500×100	438	11	22.04	630	37	125.50
500×200	2350	15	191.70	2455	98	845.31
mean	585	9	43.67	759	54	198.82

observed that the method based on the volume algorithm showed at times weak dual convergence behavior and difficulties to get close enough to the optimal Lagrangean dual solution. In such cases, the branch-and-bound method went deep down the enumeration tree and enumerated too many nodes. In contrast to BB-SG, the method based on the volume algorithm also needs to average the solutions x^t and thus usually required more computational effort per node than BB-SG.

C. Detailed results obtained with BB-SG and CPLEX on the new instances

Tables 22–25 show the results obtained with BB-SG on each single of the newly generated test instances. In these tables, the column headed UB denotes the final upper bound computed by the method. In the rare cases that optimality could not be proven and UB may thus differ from the optimal solution value, we additionally indicate the percentage gap

$$G = 100\frac{UB - LB}{LB - V_{\text{const}}}$$

between the upper bound (UB) and the global lower bound (LB) computed by the method. In addition to the number N of nodes enumerated, the maximum depth D of the search tree reached and the required CPU time T in seconds, the table also shows the upper and lower bound UB_0 and LB_0 as well as the corresponding percentage gap $G_0 = 100\frac{UB_0 - LB_0}{LB_0 - V_{\text{const}}}$ and CPU time T_0 after completion of the root node computations.

Table 22: Detailed results with BB-SG on instances with ratio $r = 5$

no.	LB_0	UB_0	G_0	T_0	UB	G	N	D	T
				$\lvert I\rvert \times \lvert J\rvert = 300 \times 300$					
1	50536.34	50675.42	0.29	1.65	50638.73		2357	17	91.24
2	49163.26	49261.65	0.21	1.81	49261.65		617	12	28.35
3	49804.67	49879.18	0.16	1.90	49879.18		441	12	18.75
4	50391.39	50449.52	0.12	1.98	50449.52		485	13	23.25
5	49602.42	49730.09	0.27	2.38	49721.6		10631	24	543.44
				$\lvert I\rvert \times \lvert J\rvert = 500 \times 500$					
1	78596.67	78662.68	0.09	4.20	78662.68		2211	24	257.05
2	81950.44	82087.28	0.17	4.76	82057.96		26691	22	3383.02
3	80654.81	80807.53	0.19	4.46	80763.30		15755	27	1440.32
4	80887.50	81060.48	0.22	4.99	80988.59		52475	27	5299.12
5	78339.19	78456.88	0.15	4.97	78429.26		2741	19	252.13
				$\lvert I\rvert \times \lvert J\rvert = 700 \times 700$					
1	111786.87	111936.5	0.14	10.38	111890.01		39537	34	9172.19
2	112488.62	112609.24	0.11	10.95	112581.89		133053	27	21525.98
3	112522.47	112674.77	0.14	9.19	112628.49		166285	33	34942.47
4	111235.92	111404.19	0.15	9.73	111333.92		141045	30	28952.73
5	112040.17	112176.53	0.12	11.95	112124.09		62559	22	13557.74
				$\lvert I\rvert \times \lvert J\rvert = 1000 \times 1000$					
1	155750.12	155928.55	0.12	25.95	155907.13	0.04	59252		76047.67[a]
2	160282.05	160421.46	0.09	48.15	160380.33	0.02	58684		52244.50[a]
3	159877.75	160006.89	0.08	30.11	159971.71		147221	24	56346.88
4	158117.04	158191.3	0.05	24.31	158189.69		49757	25	17453.25
5	159127.22	159246.22	0.08	36.71	159228.79	0.02	59160		47446.83[a]
				$\lvert I\rvert \times \lvert J\rvert = 1500 \times 300$					
1	65336.30	66015.08	1.20	17.90	65630.64		43375	35	11487.98
2	65639.00	66121.05	0.84	17.24	65831.00		15379	25	3798.34
3	67365.93	68173.71	1.37	22.68	67537.85		29891	25	8136.86
4	67527.16	67768.75	0.41	25.13	67670.7		5069	28	1662.28
5	67408.53	68081.74	1.15	19.87	67578.34		14107	24	4046.64
				$\lvert I\rvert \times \lvert J\rvert = 1500 \times 600$					
1	105153.27	105491.73	0.34	49.67	105381.44	0.13	45804		57589.38[a]
2	106524.67	106906.73	0.38	34.90	106736.07	0.05	45530		54212.58[a]
3	107159.05	107360.62	0.20	29.52	107296.34	0.03	44206		44071.32[a]
4	103927.00	104222.05	0.30	34.94	104051.34		87677	37	41896.71
5	104030.93	104233.81	0.21	30.82	104171.76		51135	39	23833.66

[a] Terminated due to insufficient memory.

Table 23: Detailed results with BB-SG on instances with ratio $r = 10$

no.	LB_0	UB_0	G_0	T_0	UB	G	N	D	T
				$\lvert I\rvert \times \lvert J\rvert = 300 \times 300$					
1	28379.10	28612.01	0.88	2.66	28474.02		293	13	16.18
2	26863.02	27452.85	2.35	3.13	27008.17		527	14	53.10
3	28720.44	28899.79	0.66	2.73	28826.24		1179	16	76.62
4	27550.64	27812.18	1.01	3.18	27662.75		641	19	35.41
5	28576.28	28775.06	0.74	3.87	28682.42		917	18	51.89
				$\lvert I\rvert \times \lvert J\rvert = 500 \times 500$					
1	44968.21	45120.95	0.36	7.53	45081.83		3921	22	455.12
2	43337.23	43420.27	0.20	5.66	43420.27		273	17	33.40
3	44750.39	45247.48	1.17	9.66	44899.66		23257	28	3458.19
4	43232.94	43715.91	1.18	7.32	43396.79		22153	23	2794.76
5	43395.43	43511.99	0.28	3.73	43511.99		1129	23	93.76
				$\lvert I\rvert \times \lvert J\rvert = 700 \times 700$					
1	59419.52	59641.45	0.39	13.61	59572.55		71615	30	14022.36
2	60669.36	60968.87	0.52	13.95	60832.31		44023	23	7468.60
3	61956.16	62273.71	0.54	23.17	62120.52	0.03	237975	30	100000.42[b]
4	59922.88	60279.51	0.62	22.06	60055.82		95551	31	20122.04
5	60759.07	61144.97	0.67	21.75	60920.86	0.04	167490		84648.77[a]
				$\lvert I\rvert \times \lvert J\rvert = 1000 \times 1000$					
1	85199.39	85410.13	0.26	29.45	85384.46	0.10	47515	22	100000.02[b]
2	82862.72	83230.43	0.46	28.52	83054.52	0.11	64427	23	100002.41[b]
3	82752.14	82948.62	0.25	29.42	82864.38		59797	27	25859.98
4	84167.45	84484.64	0.39	33.96	84331.57	0.06	80936		73527.65[a]
5	83281.81	83423.84	0.18	20.78	83394.66		56473	33	18786.89
				$\lvert I\rvert \times \lvert J\rvert = 1500 \times 300$					
1	49485.08	50183.41	1.70	16.80	49730.88		20261	30	4433.79
2	50043.44	53297.01	7.86	25.79	50255.27		7581	19	3458.74
3	49758.92	50472.54	1.73	19.32	49974.14		8605	22	1970.75
4	49637.00	49990.93	0.87	18.12	49869.27		5233	24	1466.44
5	50688.49	51617.69	2.24	22.27	50925.93		5877	23	1938.76
				$\lvert I\rvert \times \lvert J\rvert = 1500 \times 600$					
1	65622.47	66162.13	0.91	34.44	65859.94		152305	34	68850.16
2	63283.38	63770.96	0.85	29.41	63493.10		36909	27	13032.79
3	66908.67	67751.39	1.39	46.20	67117.28	0.07	61290		57054.54[a]
4	65050.71	66208.88	1.96	47.31	65277.42	0.14	60512		73016.79[a]
5	64905.24	65477.99	0.97	33.06	65151.72	0.07	61718		51813.71[a]

[a]Terminated due to insufficient memory.
[b]Time limit of 100,000 seconds reached.

Table 24: Detailed results with BB-SG on instances with ratio $r = 15$

no.	LB_0	UB_0	G_0	T_0	UB	G	N	D	T
				$\lvert I\rvert \times \lvert J\rvert = 300 \times 300$					
1	21221.35	21253.35	0.16	1.34	21253.35		13	5	1.63
2	21727.45	21889.87	0.82	2.33	21814.36		73	8	4.74
3	22359.79	22801.81	2.15	3.34	22529.39		1941	17	130.95
4	22109.56	22168.57	0.29	1.59	22168.57		91	9	3.91
5	22397.66	22693.60	1.44	3.18	22523.41		917	17	44.26
				$\lvert I\rvert \times \lvert J\rvert = 500 \times 500$					
1	33344.42	33564.90	0.71	6.79	33435.69		1019	21	123.56
2	33404.05	33636.30	0.75	6.78	33507.69		329	13	44.14
3	33809.71	34327.96	1.64	8.78	33959.00		2277	15	497.95
4	33572.46	34004.57	1.38	8.56	33709.19		15603	20	1576.54
5	33694.21	34023.50	1.05	9.72	33833.95		6947	23	823.59
				$\lvert I\rvert \times \lvert J\rvert = 700 \times 700$					
1	45005.45	45471.68	1.10	15.03	45104.89		5183	24	1147.89
2	44925.98	45361.63	1.03	15.63	45100.44		59121	30	8749.40
3	44881.10	45290.37	0.97	13.83	45032.72		34687	26	5503.07
4	44876.02	45233.16	0.84	17.72	44992.00		4219	16	852.30
5	46055.14	46651.17	1.38	18.55	46197.17		22581	21	6428.29
				$\lvert I\rvert \times \lvert J\rvert = 1000 \times 1000$					
1	62269.68	62810.95	0.92	34.33	62522.86	0.14	71933	30	100000.17[b]
2	62304.19	63112.36	1.37	37.66	62492.39	0.08	75877	23	100001.02[b]
3	61833.64	62409.74	0.98	36.37	62025.59		149679	31	56430.57
4	62254.01	62702.64	0.76	45.92	62404.53		105551	31	43359.93
5	62227.37	62696.64	0.80	40.24	62401.91	0.02	218013		100000.04[b]
				$\lvert I\rvert \times \lvert J\rvert = 1500 \times 300$					
1	45352.02	48194.56	7.79	20.90	46379.99		1007	18	391.25
2	45352.02	48194.56	7.79	21.12	45562.90		13541	22	2995.91
3	45702.20	46522.07	2.24	20.32	45872.96		1181	17	497.83
4	46365.16	47322.75	2.58	17.69	46456.43		441	14	199.65
5	46412.23	46843.68	1.15	21.11	46516.67		259	17	138.23
				$\lvert I\rvert \times \lvert J\rvert = 1500 \times 600$					
1	54309.54	54883.32	1.19	30.16	54285.13		82527	41	26848.46
2	54309.54	54883.32	1.19	30.37	54459.33		2995	16	1341.99
3	54073.30	54954.50	1.85	27.45	54273.96		14707	23	4902.58
4	54477.07	55707.77	2.54	35.37	54688.17		18575	23	6923.03
5	54462.86	55316.64	1.77	32.91	54626.06		13159	25	5939.51

[a] Terminated due to insufficient memory.
[b] Time limit of 100,000 seconds reached.
[c] CPLEX terminated with a segmentation fault.

Table 25: Detailed results with BB-SG on instances with ratio $r = 20$

no.	LB_0	UB_0	G_0	T_0	UB	G	N	D	T
				$\lvert I\rvert \times \lvert J\rvert = 300 \times 300$					
1	19353.68	20139.60	4.46	2.58	19519.92		547	14	27.52
2	18988.23	19157.20	0.98	2.50	19090.29		261	11	15.32
3	19568.58	20131.79	3.15	2.73	19618.22		49	8	8.50
4	19232.46	19408.21	1.00	2.09	19323.48		115	11	9.28
5	19223.25	19838.62	3.51	3.20	19287.12		111	9	17.28
				$\lvert I\rvert \times \lvert J\rvert = 500 \times 500$					
1	29119.11	29919.65	2.99	8.23	29294.42		12015	20	978.32
2	28290.21	28617.32	1.25	7.58	28485.57		5711	25	520.09
3	29013.00	29492.73	1.78	7.44	29183.51		3687	17	506.92
4	29127.23	30201.66	4.01	6.82	29245.49		815	17	119.12
5	28984.38	29446.83	1.73	8.31	29128.85		1787	15	223.61
				$\lvert I\rvert \times \lvert J\rvert = 700 \times 700$					
1	38303.52	38435.07	0.37	10.60	38426.66		1875	19	297.97
2	37747.32	37871.85	0.36	10.04	37863.00		1037	19	200.63
3	37646.41	38039.44	1.12	21.61	37794.64		16285	20	2763.86
4	37969.17	38233.73	0.75	13.15	38083.00		2485	21	353.92
5	37345.65	38337.34	2.86	15.85	37542.15		17903	22	2967.16
				$\lvert I\rvert \times \lvert J\rvert = 1000 \times 1000$					
1	51225.21	51875.93	1.36	32.44	51408.02		24523	26	11397.95
2	51527.68	52197.28	1.39	37.00	51721.28	0.06	120158		67046.95[a]
3	51063.10	51846.41	1.64	28.18	51363.95	0.17	92283	31	100000.65[b]
4	52128.18	52866.89	1.51	30.95	52309.10		19775	23	7938.05
5	50717.44	51762.52	2.19	34.89	50859.51		18899	25	16720.70
				$\lvert I\rvert \times \lvert J\rvert = 1500 \times 300$					
1	44484.99	45125.19	1.81	18.56	44723.91		3529	30	863.72
2	44305.96	46823.95	7.18	25.90	44441.89		12131	20	5689.69
3	44000.82	45987.56	5.64	25.03	44096.78		1197	16	573.94
4	44502.06	45303.35	2.28	21.80	44705.09		6429	23	1687.60
5	42778.26	45317.23	7.46	26.71	43000.61		18649	25	7589.52
				$\lvert I\rvert \times \lvert J\rvert = 1500 \times 600$					
1	49625.45	51875.65	5.16	36.31	49838.08		35243	24	30271.56
2	49503.72	51389.37	4.36	31.92	49648.02		7231	24	5684.19
3	49740.92	51378.59	3.76	29.45	49880.14		5223	23	4891.04
4	49906.19	50741.60	1.92	26.95	50094.26		7815	20	3060.95
5	49745.93	50951.77	2.78	34.12	49905.11		4231	21	3153.47

[a] Terminated due to insufficient memory.
[b] Time limit of 100,000 seconds reached.

Table 26: Detailed results with CPLEX 8.0 on instances with ratio $r = 5$

no.	LB_0	UB_0	G_0	T_0	UB	G	N	T
				$\|I\| \times \|J\| = 300 \times 300$				
1	50531.49	51523.46	2.04	2.68	50638.73		27373	931.93
2	49163.58	49266.48	0.22	3.18	49261.65		15668	559.36
3	49805.96	49889.44	0.17	3.49	49879.18		1597	73.63
4	50386.02	51343.63	1.97	3.64	50449.52		22438	1096.29
5	49594.27	49889.53	0.62	6.71	49721.60		21470	3457.00
				$\|I\| \times \|J\| = 500 \times 500$				
1	78595.83	79412.46	1.07	14.04	78662.68		88662	30704.46
2	81948.22	82451.43	0.63	12.21	82057.96	0.03	75000	35475.21[a]
3	80664.80	80864.01	0.25	12.04	80763.30		59042	6288.02
4	80886.32	81478.61	0.75	14.24	80988.59		126818	15146.69
5	78349.27	78726.54	0.50	12.82	78429.26		29921	3498.95
				$\|I\| \times \|J\| = 700 \times 700$				
1	111788.09	112163.90	0.34	29.71	111923.52	0.07	70900	50528.57[a]
2	112491.51	112767.23	0.25	24.59	112581.89		442184	82444.34
3	112522.67	112968.32	0.41	28.51	112665.00	0.07	144500	38714.36[a]
4	111236.77	111794.62	0.51	26.04	111357.60	0.06	155400	35325.08[a]
5	112038.90	112897.88	0.79	27.53	112399.49	0.29	116900	35226.71[a]
				$\|I\| \times \|J\| = 1000 \times 1000$				
1	155756.18	156394.34	0.42	68.94	155933.90	0.06	85500	41650.08[a]
2	160283.66	160512.65	0.15	69.90	160401.46	0.04	96500	44810.08[a]
3	159878.54	160153.12	0.18	55.64	159991.70	0.03	98800	45194.62[a]
4	158124.52	158391.28	0.17	64.80	158194.50	0.02	48000	39434.93[a]
5	159130.20	159386.13	0.16	62.87	159270.35	0.06	80500	37940.90[a]

[a]Terminated due to insufficient memory.

Tables 26–29 show the similar figures that result when the instances are solved by means of the procedure based on CPLEX (version 8.0). Recall that we did not apply CPLEX to the instances with 1500 customers.

Finally, Tables 30–33 show the detailed results obtained with CPLEX 12.1, which we applied only to the smaller of the newly generated instances. This time, the tables additionally show the lower bound, LB_0^*, that results at the root node of the branch-and-cut tree if the generation of all available types of cuts, even disjunctive cuts, was set to the most aggressive mode. As can be seen from table 30–33, this strategy in some cases improves the root lower bound slightly, whilst in the majority of the cases almost the same and sometimes even a weaker lower bound is reached. The effort for solving the LP relaxation is, however, in all cases significantly increased.

Table 27: Detailed results with CPLEX 8.0 on instances with ratio $r = 10$

no.	LB_0	UB_0	G_0	T_0	UB	G	N	T
				$\lvert I\rvert \times \lvert J\rvert = 300 \times 300$				
1	28375.73	29082.89	2.66	16.08	28474.02		16422	2129.03
2	26865.43	27559.13	2.76	67.57	27008.17		4302	1889.55
3	28720.95	29482.88	2.82	8.39	28826.24		23969	1465.70
4	27571.41	28587.88	3.93	11.52	27662.75		9628	679.58
5	28578.73	28804.46	0.84	23.49	28682.42		5395	874.07
				$\lvert I\rvert \times \lvert J\rvert = 500 \times 500$				
1	44995.54	45173.49	0.42	121.25	45081.83		45037	20855.99
2	43341.54	43524.15	0.44	58.72	43420.27		24829	12012.48
3	44751.96	45468.44	1.69	67.66	44968.09	0.38	73300	44505.23[a]
4	43252.26	43944.31	1.70	106.92	40990.60	0.25	52200	48614.37[a]
5	43420.41	43511.99	0.22	77.47	43511.99		11031	2328.70
				$\lvert I\rvert \times \lvert J\rvert = 700 \times 700$				
1	59433.25	59580.92	0.26	125.55	59572.55	0.08	95800	40789.26[a]
2	60678.55	60978.70	0.52	165.20	60846.78	0.12	60000	56448.26[a]
3	61979.15	62862.70	1.49	196.79	62249.18	0.35	83400	37008.56[a]
4	59931.70	60435.25	0.88	207.06	60101.53	0.18	64700	72927.47[a]
5	60766.03	61223.87	0.79	138.39	61139.75	0.55	62400	30543.04[a]
				$\lvert I\rvert \times \lvert J\rvert = 1000 \times 1000$				
1	85202.69	86088.03	1.08	228.30	85549.72	0.38	61000	64249.75[a]
2	82868.83	83560.11	0.87	343.09	83394.50	0.59	54600	41384.11[a]
3	82756.58	83041.65	0.36	223.86	82933.14	0.15	57500	47629.46[a]
4	84175.56	84506.92	0.41	283.01	84506.92	0.35	48600	32129.46[a]
5	83291.57	83575.77	0.35	358.52	83394.66	0.07	42800	66488.31[a]

[a]Terminated due to insufficient memory.

Table 28: Detailed results with CPLEX 8.0 on instances with ratio $r = 15$

no.	LB_0	UB_0	G_0	T_0	UB	G	N	T
				$\lvert I\rvert \times \lvert J\rvert = 300 \times 300$				
1	21231.84	21253.35	0.11	37.83	21253.35		53	79.32
2	21729.93	22584.77	4.31	22.03	21814.36		820	145.76
3	22416.48	22613.19	0.95	36.32	22529.39		16858	5382.56
4	22113.15	22295.96	0.90	35.57	22168.57		744	154.62
5	22413.43	22766.02	1.71	34.60	22523.41		13522	2890.04
				$\lvert I\rvert \times \lvert J\rvert = 500 \times 500$				
1	33353.14	33584.27	0.74	224.33	33435.69		18186	19391.52
2	33437.61	33655.78	0.70	244.40	33507.69		6890	3722.35
3	33834.56	34432.40	1.90	302.60	33959.00	0.15	20300	30996.04[c]
4	33590.76	33785.23	0.62	178.73	33785.23	0.39	68400	32433.31[a]
5	33712.73	34307.49	1.90	224.79	34025.29	0.75	69100	29448.15[a]
				$\lvert I\rvert \times \lvert J\rvert = 700 \times 700$				
1	45028.55	45166.88	0.33	566.68	45104.89		43385	61078.82
2	44946.30	45847.37	2.13	493.15	45101.74	0.16	56500	73651.92[a]
3	44913.26	45100.81	0.44	572.65	45100.81	0.26	51000	60907.68[a]
4	44888.40	45597.58	1.68	464.58	44992.00	0.04	75694	100425.30[b]
5	46081.03	46629.32	1.27	569.25	46451.16	0.70	44900	39507.25[a]
				$\lvert I\rvert \times \lvert J\rvert = 1000 \times 1000$				
1	62286.34	62653.08	0.62	1130.04	62605.11	0.41	33300	77854.68[a]
2	62333.38	63094.70	1.29	1061.87	62796.25	0.71	31700	44235.52[a]
3	61860.69	62323.55	0.79	918.52	62186.92	0.44	39700	61674.28[a]
4	62266.71	62486.15	0.37	765.91	62471.88	0.22	32900	72169.08[a]
5	62256.73	63276.92	1.73	1214.60	62768.69	0.77	28400	49560.19[a]

[a] Terminated due to insufficient memory.
[b] Time limit of 100,000 seconds reached.
[c] CPLEX terminated with a segmentation fault.

Table 29: Detailed results with CPLEX 8.0 on instances with ratio $r = 20$

no.	LB_0	UB_0	G_0	T_0	UB	G	N	T
				$\lvert I\rvert \times \lvert J\rvert = 300 \times 300$				
1	19370.28	19995.58	3.55	78.75	19519.92		17175	4700.24
2	18999.97	19511.32	2.96	50.95	19090.29		5411	1397.63
3	19581.92	19946.94	2.04	67.51	19618.22		1857	465.16
4	19240.50	19505.35	1.51	79.44	19323.48		2344	785.32
5	19237.90	19327.41	0.51	61.37	19287.12		1531	537.66
				$\lvert I\rvert \times \lvert J\rvert = 500 \times 500$				
1	29144.64	29421.57	1.03	329.42	29417.09	0.66	44900	39519.52[a]
2	28312.92	28684.46	1.42	488.81	28485.57		34884	30797.72
3	29036.75	29873.39	3.11	431.98	29400.98	1.10	50900	31358.79[a]
4	29162.06	29364.88	0.76	525.35	29245.49		8422	9121.40
5	29009.09	29173.72	0.62	483.65	29128.85		42948	36609.82
				$\lvert I\rvert \times \lvert J\rvert = 700 \times 700$				
1	38333.06	38744.69	1.15	1405.23	38426.66		24988	55198.34
2	37753.19	38483.69	2.09	961.38	37863.00	0.12	54100	93763.46[a]
3	37677.98	38510.79	2.37	1125.09	37805.71	0.19	36261	100924.82[b]
4	37985.26	38110.98	0.36	1007.30	38083.00		42737	60715.71
5	37389.42	37890.76	1.44	1249.78	37580.17	0.27	35072	100895.36[b]
				$\lvert I\rvert \times \lvert J\rvert = 1000 \times 1000$				
1	51265.08	51824.79	1.17	2843.47	51516.16	0.41	19098	102639.95[b]
2	51552.70	52473.13	1.91	2102.90	52080.13	0.97	19700	48874.67[a]
3	51100.31	51593.60	1.03	2578.06	51593.60	0.88	17681	102355.67[b]
4	52171.71	52691.62	1.06	2299.10	52319.24	0.16	25842	102373.61[b]
5	50738.03	51251.93	1.08	2041.42	51066.12	0.60	19700	74882.71[a]

[a] Terminated due to insufficient memory.

[b] Time limit of 100,000 seconds reached.

Table 30: Detailed results with CPLEX 12.1 on instances with ratio $r = 5$

no.	LB_0^*	LB_0	UB_0	G_0	T_0	UB	G	N	T
				$\|I\| \times \|J\| = 300 \times 300$					
1	50551.08	50543.63	51523.46	2.01	3.44	50638.73		14308	860.60
2	49175.43	49176.00	49266.48	0.19	3.78	49261.65		4306	253.43
3	49813.83	49814.80	49889.44	0.16	5.02	49879.18		1168	87.15
4	50390.09	50389.13	51343.63	1.96	4.25	50449.52		18386	1265.89
5	49598.76	49595.66	49889.53	0.62	6.80	49721.60		18413	1857.19
				$\|I\| \times \|J\| = 500 \times 500$					
1	78603.25	78595.89	79412.46	1.07	12.07	78662.68		77923	16915.94
2	81964.68	81958.16	82451.43	0.62	12.86	82099.16	0.1	87600	14947.87[a]
3	80688.38	80670.26	80864.01	0.25	10.74	80763.30		59753	11188.91
4	80903.97	80894.58	81478.61	0.74	15.55	80988.59		95666	19737.28
5	78359.48	78352.21	78726.54	0.49	10.95	78429.26		14223	2841.48

[a]Terminated due to insufficient memory.

Table 31: Detailed results with CPLEX 12.1 on instances with ratio $r = 10$

no.	LB_0^*	LB_0	UB_0	G_0	T_0	UB	G	N	T
				$\|I\| \times \|J\| = 300 \times 300$					
1	28377.94	28378.54	29082.89	2.65	32.84	28474.02		18637	1917.48
2	26864.37	26864.37	27559.13	2.76	106.52	27008.17		8991	2777.96
3	28729.81	28726.99	28888.93	0.60	23.92	28826.24		16164	1487.44
4	27557.84	27556.46	28587.88	3.99	10.24	27662.75		13898	1270.20
5	28570.52	28570.52	28804.46	0.87	27.77	28682.42		4772	928.96
				$\|I\| \times \|J\| = 500 \times 500$					
1	44976.77	44976.77	45173.49	0.46	86.15	45081.83		11833	9729.23
2	43339.63	43337.64	43524.15	0.45	50.83	43420.27		10086	7234.52
3	44750.42	44750.42	45468.44	1.69	49.85	44999.08	0.39	61000	18042.18[a]
4	43250.96	43249.39	43944.31	1.70	73.75	43430.61	0.22	90500	38216.79[a]
5	43421.90	43419.78	43665.85	0.60	192.1	43511.99		5644	2126.09

[a]Terminated due to insufficient memory.

Table 32: Detailed results with CPLEX 12.1 on instances with ratio $r = 15$

no.	LB_0^*	LB_0	UB_0	G_0	T_0	UB	G	N	T
				$\lvert I\rvert \times \lvert J\rvert = 300 \times 300$					
1	21233.21	21233.21	21253.35	0.10	46.58	21253.35		68	86.20
2	21734.00	21734.00	21889.87	0.79	27.23	21814.46		725	204.44
3	22373.01	22373.01	22613.19	1.17	38.74	22529.39		7549	2202.97
4	22116.81	22116.81	22168.57	0.26	44.97	22168.57		268	145.41
5	22413.71	22413.71	22766.02	1.71	35.61	22523.41		9454	2192.01
				$\lvert I\rvert \times \lvert J\rvert = 500 \times 500$					
1	33353.13	33353.13	33584.27	0.74	165.58	33435.69		12198	14241.64
2	33438.55	33438.14	33585.77	0.48	254.96	33507.69		5212	3792.65
3	33834.13	33834.13	34432.40	1.90	266.89	33959.00		48249	56160.96
4	33592.20	33592.20	33785.23	0.61	135.87	33784.95	0.37	40500	22220.37[a]
5	33714.04	33713.47	34307.49	1.89	195.15	33833.95		83352	48196.26

[a]Terminated due to insufficient memory.

Table 33: Detailed results with CPLEX 12.1 on instances with ratio $r = 20$

no.	LB_0^*	LB_0	UB_0	G_0	T_0	UB	G	N	T
				$\lvert I\rvert \times \lvert J\rvert = 300 \times 300$					
1	19370.93	19370.93	19995.58	3.54	118.06	19519.92		4282	1724.64
2	18999.97	18999.97	19511.32	2.96	51.57	19090.29		3902	1617.81
3	19583.04	19583.04	19946.94	2.03	74.54	19618.22		1355	833.13
4	19241.08	19241.08	19505.35	1.51	96.34	19323.48		1465	960.18
5	19237.90	19237.90	19327.41	0.51	65.06	19287.12		1132	625.74
				$\lvert I\rvert \times \lvert J\rvert = 500 \times 500$					
1	29145.07	29145.07	29421.57	1.03	237.65	29294.42	0.28	36300	45213.79[a]
2	28292.82	28292.82	28684.46	1.50	316.64	28485.57		17355	16811.31
3	29035.84	29035.84	29912.94	3.26	339.73	29285.95	0.64	36600	28473.13[a]
4	29162.50	29162.32	30209.47	3.90	398.56	29245.49		4879	6601.23
5	29007.96	29007.96	29173.72	0.62	357.91	29128.85		21544	23172.49

[a]Terminated due to insufficient memory.