ADD/DROP PROCEDURES FOR THE CAPACITATED PLANT LOCATION PROBLEM

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Abstract

The capacitated plant location problem with linear transportation costs is considered. An algorithm based on ADD/DROP procedures is proposed. Exact rules are presented allowing an a priori opening or closing of facilities. Facilities left are submitted to an extension of the exact rules. Procedures are implemented with the help of lower and upper bounds using Lagrangean relaxation. Computational results are presented and comparisons with other algorithms are made.

Key words: Capacitated plant location problem, ADD/DROP procedures, Lagrangean relaxation, algorithms, combinatorial optimization.

1 INTRODUCTION

The capacitated plant location problem (CPLP) may be formulated as:

$$\begin{array}{lll} \text{minimize} & \sum_{k\in I}\sum_{j\in J} c_{kj}x_{kj} + \sum_{k\in I} f_k y_k & \\ \text{subject to} & \sum_{j\in J} x_{kj} \le a_k y_k & \forall k\in I \\ & \sum_{k\in I} x_{kj} = b_j & \forall j \in J \\ & x_{kj} \ge 0 & \forall k\in I, \forall j\in J \\ & y_k \in \{0,1\} & \forall k \in I \end{array}$$

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where I is the set of possible plant locations each with a maximum capacity a_k and fixed cost f_k, J is the set of demand centers each with a demand b_j, and c_{kj} is the unit transportation cost between a facility k and a consumer j. The variable x_{kj} represents the amount sent from k to j, and y_k means locating (or not) plant k.

The CPLP is a well known combinatorial optimization problem belonging to the class of the NP-Hard problems. For large instances there may be the need of reduction tests, problem relaxation and heuristic methods. Beasley [3] and Barcelo et al. [2] have used problem reduction and Lagrangean relaxation to solve the CPLP. Beasley [5] has developed a framework for producing Lagrangean heuristics for the capacitated and the uncapacitated plant location problems. A good comparison of heuristic methods for the CPLP is presented by Cornuejols et al. [7].

Here an algorithm for the CPLP based on dominance criteria between fixed and variable costs is presented. These criteria may work in an exact way or just as a heuristic method for determining the status of facilities. Jacobsen [8] and Mateus and Bornstein [9] have used them within ADD or DROP heuristics. More recently Bornstein and Azlan [6] have used these ideas within the simulated annealing framework.

Reduction tests and heuristics based on dominance criteria are presented. Lower and upper bounds for the cost increments are used in order to get an efficient implementation. These bounds can be obtained by Lagrangean relaxation. Computational results are presented and a comparison with other known algorithms is made.

2 THE ALGORITHM

The dominance criteria between fixed and variable (transportation) costs are often used in reduction tests, cutting down the original size of the problem by an a priori opening or closing of plants. They are also useful to develop heuristics. In order to state the tests and heuristics, we consider the unified approach presented in Bornstein and Azlan [6].

Given a subset $K \subseteq I$ of the potential plant locations, let

$$w(K) = \min \left\{ \sum_{k \in I} \sum_{j \in J} c_{kj} x_{kj} \mid x \in X(K) \right\}$$

represent the solution value of the transportation problem $T(K)$ associated with the CPLP, where

$$X(K) = \{x = [x_{kj}] \mid \sum_{j \in J} x_{kj} \le a_k, \forall k \in I; \sum_{k \in K} x_{kj} = b_j, \forall j \in J;$$
$$x_{kj} \ge 0, \forall k \in K, \forall j \in J\}.$$

If $X(K) = \emptyset$, we consider $w(K) = +\infty$.

The function $w(K)$ has the important property of supermodularity (Prop. 10 in Wolsey [12]). A function $w(K)$ is said to be supermodular (or equivalently $-w(K)$ submodular) if $w(K) - w(K \cup i) \le w(K') - w(K' \cup i)$ $\forall K' \subseteq K$ and $\forall i \notin K$ (see Wolsey [12] and Nemhauser and Wolsey [10]). Thus, supermodularity is a kind of concavity.

For $i \in I - K$ let $\delta_i(K) = w(K) - w(K \cup i) \ge 0$ evaluate the increase/decrease in transportation costs if we close/open facility i. Furthermore let $\Delta_i(K) = f_i - \delta_i(K)$ evaluate the balance between fixed and variable costs with respect to facility i. The function $\Delta_i(K)$ gives rise to criteria for opening or closing plants. Let K_0 and K_1 be the sets of closed and opened plants, i.e., $K_0 = \{i \in I \mid y_i = 0\}$ and $K_1 = \{i \in I \mid y_i = 1\}$. Facilities whose status is yet undefined remain in $K_2 = I - K_0 - K_1$. We can state the following rules and heuristics (see Akinc and Khumawala [1], Mateus and Bornstein [9] and Jacobsen [8]):

O-test: If $\Delta_i(K_1 \cup K_2 - i) \le 0$ then $y_i = 1$.
C-test: If $\Delta_i(K_1) \ge 0$ then $y_i = 0$.
O-heuristic: If $\Delta_i(K_1) < 0$ then $y_i = 1$.
C-heuristic: If $\Delta_i(K_1 \cup K_2 - i) \le 0$ then $y_i = 0$.

The algorithm consists of two phases. It starts with $K_0 = K_1 = \emptyset$ and $K_2 = I$ and repeatedly applies the tests and heuristics defined above. Phase 1 defines the status of all plants, ending with $K_2 = \emptyset$. Plants are assigned to K_1, K_0 and $\bar{K}_0$, where $\bar{K}_0$ is the set of temporarily closed plants. Phase 2 refines the solution reexamining the plants placed in $\bar{K}_0$. The number of plants assigned to $\bar{K}_0$ depends on a parameter ϵ.

3 CONCLUSIONS AND COMPUTATIONAL RESULTS

A central dilemma of the CPLP is the conflict between the minimization of fixed and variable costs each driving the solution in a different direction. The minimization of transportation costs forces the solution towards putting a plant near each demand center while fixed costs are generally minimized by placing a central unit supplying all demand centers. So, the joint minimization of the costs means that there needs to be a compromise between the centralization and decentralization tendencies.

The algorithm presented in this paper moves around this dilemma, detecting dominances among the two types of costs and locating plants according to these dominances. It was implemented in FORTRAN 77 and tested with data obtained from Beasley's [4] OR-library. Comparison is made with the algorithms presented by Bornstein and Azlan [6], Van Roy [11] and Beasley [5]. The computational results show that it is able to tackle large scale problems finding almost always near optimal solutions at very low cost. For example, the average relative error was 0.18% for the 12 instances from Beasley [5] with 100 potential plant locations and 1000 demand centers.

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