量子信息学
lec 4

郝琰 516021910721
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1 Class 1

1.1 A^+ : some review

A^+: 算子函数，相当于把一个算子映射为另一个算子

1.1.1 定义、构造、唯一性

- A : H → H. A^* is the unique linear operator such that

$$(|\psi>, A|\psi>) = (A^+|\psi>, |\psi>)$$

- construct

$$< i|A^+|i> = (< i|A|i>)^*$$
$$A^+ = \sum_{ij}(< j|A|i>)^*|i>< j|$$

- unique

$$(|\psi>, A|\psi>) = (A^+|\psi>, |\psi>) = (B|\psi>, |\psi>)$$
$$(|\psi>, A^+|\psi>) = (|\psi>, B|\psi>)$$
$$(|\psi>, (A^+ - B)|\psi>) = 0$$
$$A^+ = B$$

A^+: 算子函数，相当于把一个算子映射为另一个算子

1.1.2 一些性质 (和转置性质相似)

- $$(A \bigotimes B)^+ = A^+ \bigotimes B^+$$

- $$(AB)^+ = B^+A^+$$

- tr()
- det()

1.2 补充

1.2.1 tr()

$$tr(A) = \sum_{i=1}^{d} < i|A|i >$$

- linear
$$tr(A + B) = tr(A) + tr(B)$$
- cyclic
$$tr(AB) = tr(BA)$$
- f: L(H) -> C ; linear and cyclic
$$f(H) = \lambda tr(H)$$

1.2.2 全体线性算子构成一个希尔伯特空间

$$L(H) = \{\text{A: A is linear operator over H}\}$$
$$< A, B >= tr(A^{+}B)$$

1.2.3 Schmidt procedure

对任意一组（n 个）线性独立的向量，存在同样个数的线性正交基使得其与前者张成同样的线性空间

1.3 idea 1: 柯西洗袜子

$$| < v|w > | \leq 1 \quad < v|v >= 1 \quad < w|w >= 1$$

首先用 w 扩充一组标准正交基，w 是这组基的第一个元素。

$$\begin{aligned} 1 =< v|v > &=< v| \sum_{i=1}^{d} |i >< i||v > \\ &=< v|w >< w|v > + < v| \sum_{i=2}^{d} |i >< i||v > \\ &\geq< v|w >< w|v > \end{aligned}$$

2 Class 2

2.1 idae 2: 定义、变换及测量

- H : state space; $|\psi >$ space vector.
$$< \psi|\psi >= 1 \quad |\psi >= \alpha|0 > +\beta|1 >$$
- $|\psi(t_) >$ and $|\psi(t_1) >$ 由一个酉变换联系

- measurement: M_m: 线性算子，H->H.

$$P(m) = ||M_m|\psi > ||^2$$
$$|\psi >= \frac{M_m|\psi >}{(P_m)^{\frac{1}{2}}}$$
$$\sum_m P(m) = 1$$
$$\sum_m < \psi|M_m^+ M_m|\psi >= 1$$
$$\sum_m M_m^+ M_m = 1$$
$$|m > ONS$$
$$M_m = ???$$

Actually the reference 1 gives the correct answer.

observable

A is observable.

$$A = A^+$$
$$A = \sum_{i=1}^{d} a_i |i >< i|$$

这样 A 就可以通过其特征值 a_i 作为 outcome 进行测量

测量的平均值

$$< A >_\psi = \sum_m \lambda_m p(m) = \sum_m \lambda_m < \psi|A|\psi >=< \psi| \sum_m \lambda_m P_m |\psi >=< \psi|A|\psi >= tr(A|\psi >< \psi|)$$

(方差)

2.2 idea 3:state discrimination

$$|\psi > \in \{|\psi_1 >, |\psi_2 >, ...\}$$

比如有两种可能，为了把真实系统恢复出来，需要做测量
用 $M_1, M_2, ...$ 做测量，把结果根据一定条件进行划分。怎么可以做到精确分辨?

正交

$$M_1 = |\psi_1 >< \psi_1|$$
$$M_2 = |\psi_2 >< \psi_2|$$
$$M_3 = I - |\psi_1 >< \psi_1| - |\psi_2 >< \psi_2|$$

either $M_1 = 1$ or $M_2 = 1$, M_3 does not exist.

不正交

$$\sum_i ||M_i|\psi_i||^2$$

对于 i 从 1 到 k，sum 为 1，对于 i 从 k+1 到 n，sum 为 1
则该问题可转化为

$$\exists E_1, E_2 \geq 0 \quad E_1 + E_2 = I \quad < \psi_1|E_1|\psi_1 >= 1 \quad < \psi_2|E_2|\psi_2 >= 1$$

以上为一个伪命题，

$$E_2|\psi_1 >= 0$$
$$E_2 = \lambda_2|\psi_1^\perp >< \psi_1^\perp|$$
$$E_1 = \lambda_1|\psi_2^\perp >< \psi_2^\perp|$$
$$\lambda_2| < \psi_2|\psi_1^\perp > | = 1$$
$$\lambda_2 \leq 1, \quad | < \psi_2|\psi_1^\perp > |! = 1$$

结论不成立

3 下节：超密集编码

reference 1

given by

$$p(m) = \langle\psi|M_m^\dagger M_m|\psi\rangle \; , \tag{2.92}$$

and the state of the system after the measurement is

$$\frac{M_m|\psi\rangle}{\sqrt{\langle\psi|M_m^\dagger M_m|\psi\rangle}} \; . \tag{2.93}$$

The measurement operators satisfy the *completeness equation*,

$$\sum_m M_m^\dagger M_m = I \; . \tag{2.94}$$

The completeness equation expresses the fact that probabilities sum to one:

$$1 = \sum_m p(m) = \sum_m \langle\psi|M_m^\dagger M_m|\psi\rangle \; . \tag{2.95}$$

This equation being satisfied for all $|\psi\rangle$ is equivalent to the completeness equation. However, the completeness equation is much easier to check directly, so that's why it appears in the statement of the postulate.

A simple but important example of a measurement is the *measurement of a qubit in the computational basis*. This is a measurement on a single qubit with two outcomes defined by the two measurement operators $M_0 = |0\rangle\langle 0|$, $M_1 = |1\rangle\langle 1|$. Observe that each measurement operator is Hermitian, and that $M_0^2 = M_0, M_1^2 = M_1$. Thus the completeness relation is obeyed, $I = M_0^\dagger M_0 + M_1^\dagger M_1 = M_0 + M_1$. Suppose the state being measured is $|\psi\rangle = a|0\rangle + b|1\rangle$. Then the probability of obtaining measurement outcome 0 is

$$p(0) = \langle\psi|M_0^\dagger M_0|\psi\rangle = \langle\psi|M_0|\psi\rangle = |a|^2. \tag{2.96}$$

Similarly, the probability of obtaining the measurement outcome 1 is $p(1) = |b|^2$. The state after measurement in the two cases is therefore

$$\frac{M_0|\psi\rangle}{|a|} = \frac{a}{|a|}|0\rangle \tag{2.97}$$

$$\frac{M_1|\psi\rangle}{|b|} = \frac{b}{|b|}|1\rangle. \tag{2.98}$$

图 1: approximation error graph