Introduction to Data Science Homework
hw 1

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Class 1

0.1 标准正交积 (ONS)

$$||\psi|| = (<\psi|\psi>)^{\frac{1}{2}}$$
$$H = span|e_1>, |e_2>, ..., |e_d>$$
$$H = span|1>, |2>, ..., |d>$$
$$|\psi> = \sum_{i=1}^{d} \alpha_i |i>$$
$$\alpha_i = <i|\psi>$$
$$\sum_{i=1}^{d} |i><i| = I_d$$

字空间上的单位矩阵：字空间上的投影算子

A : H -> H linear operator

The Heimition conjunction of A is defined as A daggger

$$(|\psi>, A|\psi>) =$$

A : H -> $H^{'}$

$$A = I_H A I_{H'}$$
$$\sum_{ij} <i|A|j> |i><j|$$
$$\sum_{ij} <e_i|A|e_j> |e_i><e_j|$$

$$A = \sum_{ij} < i|A|j > |i >< j|$$

A^{+} 共轭转置

$$A^{+} = \sum_{ij} < i|A^{+}|j > |i >< j|$$

$$A^{+} = \sum_{ij} (< i|A|j >)^{*} |i >< j|$$

$$A = |\psi >< \psi|,$$

- Hermitian operator A: H->H
- 123
- A is normal is $AA^{+} = A^{+}A$

$$A = \sum_{i=1}^{d} \alpha |i >< i|$$

$$A^{2} = \sum_{i=1}^{d} \alpha^{2} |i >< i|$$

$$A = \sum_{i=1}^{k} \lambda_i P_i$$

$$P_i P_j = \delta_{ij} P_i, P_i^{+} = P_i, \sum P_i = I$$

Unitary operator U

$$U^{+}U = I, U^{-1} = U^{+}$$

酉变换，相当于旋转，保持内积不变

$$(U|\psi >, U|\psi >) = (|\psi >, |\psi >)$$

$$U = \sum_{j=1}^{d} e^{i\theta j} |j >< j|$$

Class 2

Polar Decomposition

$$A = JU$$
$$A^+ = (JU)^+ = U^+ J^+$$
$$AA^+ = J^2, J = (AA^+)^{\frac{1}{2}}$$
$$A = UDU^+ =?$$

奇异值分解???

$$A = JU = V$$

把 A 写作普分解的好处

$$A = \sum_{i=1}^{k} \lambda_i P_i$$
$$A^n = \sum_{i=1}^{k} \lambda_i^n P_i$$

commutator

$$[A, B] = AB - BA$$
$$[A, B] = 0, AB = BA$$

iff

$$A = \sum_i \lambda_i |i >< i|$$
$$B = \sum_i \mu_i |i >< i|$$

特殊的情形，例如，B 测得 μ_i ,A 测得 λ_i。这就是对译子的特征.

反对译子:

$$\{A, B\} = AB + BA$$
$$\{A, B\} = 0, AB = -BA$$

some 应用

trace

$$tr(A) = \sum_{i=1}^{d} < i|A|i >$$
$$tr(AB) = tr(BA)$$
$$tr(A|\psi >< \psi|) =< \psi|A|\psi >$$

???

$$e^{i\theta \overrightarrow{n} \overrightarrow{\delta}}???$$

0.2 如何将两个空间张成一个空间

$$H = span\{|i >, i = 1, 2, 3, ...d\}$$
$$H^{'} = span\{|i >, i = 1, 2, 3, ...d^{'}\}$$
$$dim(H \oplus H^{'}) = dd^{'}$$

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Class 1

A^+ and some review

- A : H → H. A^* is the unique linear operator such that

$$(|\psi>, A|\psi>) = (A^+|\psi>, |\psi>)$$

- construct

$$< i|A^+|i> = (< i|A|i>)^*$$
$$A^+ = \sum_{ij} (< j|A|i>)^* |i>< j|$$

- unique

$$(|\psi>, A|\psi>) = (A^+|\psi>, |\psi>) = (B|\psi>, |\psi>)$$
$$(|\psi>, A^+|\psi>) = (|\psi>, B|\psi>)$$
$$(|\psi>, (A^+ - B)|\psi>) = 0$$
$$A^+ = B$$

  A^+: 算子函数，相当于把一个算子映射为另一个算子
  一些性质 (和转置性质相似)

- $$(A \bigotimes B)^+ = A^+ \bigotimes B^+$$

- $$(AB)^+ = B^+ A^+$$

- tr()
- det()

0.3 补充

0.3.1 tr()

$$tr(A) = \sum_{i=1}^{d} < i|A|i >$$

- linear
- cydic
- f: L(H) -> C ; linear and cydic

$$f = \lambda tr()$$

0.3.2 全体线性算子构成一个希尔伯特空间

$$L(H) = \{A : Aislinearoperatorover H\}$$
$$< A, B >= tr(A^+ B)$$

0.3.3 Schmidt procedure

对任意一组（n 个）线性独立的向量，存在同样个数的线性正交基使得其与前者张成同样的线性空间

0.4 idea 1: 柯西洗袜子

$$| < v|w > | \leq 1 \quad < v|v >= 1 \quad < w|w >= 1$$

首先用 w 扩充一组标准正交基，w 是这组基的第一个元素。

$$\begin{aligned} 1 =< v|v > &=< v| \sum_{i=1}^{d} |i >< i||v > \\ &=< v|w >< w|v > + < v| \sum_{i=2}^{d} |i >< i||v > \\ &\geq< v|w >< w|v > \end{aligned}$$