量子信息学
lec 5

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1 Class 1

1.1 Measurement Review

- 测得的结果
$$P(m) =< \psi|M_m^{+}M_m|\psi >$$
- 测量后的状态
$$|\psi_m >= \frac{M_m|\psi >}{\sqrt{P(m)}}$$
- $$\{|m >, m = 1, 2, 3, ...\} \quad M_m = |m >< m| \quad A = \sum_m \lambda_m |m >< m|$$
- $$\{P_m, m = 1, 2, ...\} \quad M_m = P_m \quad \sum_m P_m = I \quad A = \sum_m mP_m$$
- 若只关心测得的概率，不关心测量后的状态变成什么
POVM(positive operator-valued measure)
$$\{E_m : m = 1, 2, ..., k\} \quad E_m \geq 0 \quad \sum_m E_m = I$$
$$P(m) =< \psi|E_m|\psi >$$
$$E_m = M_m^{+}M_m$$
同时它也可以进行一些状态上的合并（E_m 为所需的状态集合）

1.2 idea 1: 复合系统 (第四条公理)

$$U + \text{projective measurement} \rightarrow \text{generalized measurement}$$
$$U|\psi > |0 >= \sum_m M_m|\psi > |m >$$
注: $|0 >$ 相当于 outcome, 是原本在环境之中的一个 space，现在把它放到 U 中。可以把整个的 global space 看作
$$\{U|\psi > |m >\}$$
$$U|\psi >^{'} |0 >= \sum_m M_m|\psi^{'} > |m >$$
$$(U|\psi >^{'} |0 >)^T U|\psi > |0 >=< \psi|\psi >$$
$$P(m) = \{I^A \bigotimes |m >\}$$

1.3 实际的测量

1.3.1 推导

- $$\{P_i, |\psi_i>\} \to \{P_i, U|\psi_i>\}$$

- $$\begin{aligned} P(m) = P(m|\psi_i)p_i &= \sum_i p_i < \psi_i|E_m^+ E_m|\psi_i > \\ &= \sum_i p_i tr(< \psi_i|E_m^+ E_m|\psi_i >) \\ &= tr(\sum_i p_i|\psi_i >< \psi_i|E_m^+ E_m) \\ &= tr(\rho E_m^+ E_m) \end{aligned}$$

$$\rho = \sum_i p_i|\psi_i >< \psi_i| \quad \text{density operator}$$

$$\rho \geq 0 \quad tr\rho = 1$$

系统状态可以用密度算子等价刻画

1.3.2 性质

- state ρ is positive operator with trace one
$$\rho = \sum_i p_i \rho_i$$

- $$\rho \to U\rho U^+$$

- $$p(m) = tr(\rho E_m^+ E_m)$$

- $$\rho_1 \bigotimes \rho_2 ... \bigotimes \rho_n$$

2 Class 2

2.1 idea 3: 应用 1

编码（量子操作），信道，解码（量子测量）
要求正交以精确传递
Alice and Bob 共享一个纠缠态

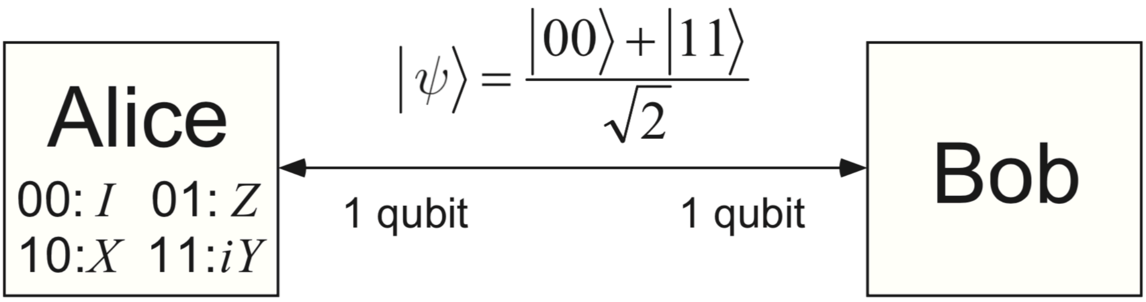

图 1: 1

$$
\begin{aligned}
00 &: |\psi\rangle \rightarrow \frac{|00\rangle + |11\rangle}{\sqrt{2}} \\
01 &: |\psi\rangle \rightarrow \frac{|00\rangle - |11\rangle}{\sqrt{2}} \\
10 &: |\psi\rangle \rightarrow \frac{|10\rangle + |01\rangle}{\sqrt{2}} \\
11 &: |\psi\rangle \rightarrow \frac{|01\rangle - |10\rangle}{\sqrt{2}}.
\end{aligned}
$$

图 2: 2

2.2 idea 4: 如何通过经典信道传量子信息

Send one qubit to Bob
要求传递的是状态，而先不管载体如何
Quantum teleportation(1993)
A->B(EPR)

$$A : |\psi >$$

- Alice performs a Bell measurement.

- Alice sends b0b1 to Bob
- Bob performs a unitary on this qubit based on b0b1

$$01:I^B \quad 01:X^B \quad 10:Z^B \quad 11:Y^B$$

get

$$\alpha|0> + \beta|1>$$

$$\begin{aligned}
&\frac{1}{\sqrt{2}}(\alpha|0>^A + \beta|1>^A)(|0^A 0^B> + |1^A 1^B>) \\
=&\frac{1}{\sqrt{2}}(\alpha|00>|0>^B + \alpha|01>|1>^B + \beta|10>|0>^B + \beta|11>|1>^B) \\
=&\frac{1}{2}(|\phi_{00}>(\lambda|0> + \beta|1>)^B + |\phi_{01}>(\lambda|1> + \beta|0>)^B + |\phi_{10}>(\lambda|0> - \beta|1>)^B + |\phi_{11}>(\lambda|1> - \beta|0>)^B)
\end{aligned}$$

2.3 idea 4

- 一个 judge 同时对 Alice 和 Bob 做测量，A:Q and R; B:S and T
- definite value for Q,R,S,T
- for 经典物理学

$$QS + RS + RT - QT \leq 2$$
$$E(QS) + E(RS) + E(RT) - E(QT) \leq 2$$

CHSH inequality
- for 量子力学

$$< Q \bigotimes S > + < R \bigotimes S > + < Q \bigotimes T > - < Q \bigotimes T > \leq 2\sqrt{2}$$
$$|\phi> = \frac{|01> - |10>}{\sqrt{2}}$$
$$Q = Z^A \quad R = X^A \quad S = \frac{-Z^B - X^B}{\sqrt{2}} \quad T = \frac{Z^B - X^B}{\sqrt{2}}$$

得到 $2\sqrt{2}$
贝尔不等式说明 reality or licadity 中至少有一个是不对的

tail

$$|\phi> = \frac{|00> + |11>}{\sqrt{2}}! = |a>|b>$$

Proof?

$$\rho^A = \frac{I}{2}$$

partial trace

$$tr_B(|a_1><a_2| \bigotimes |b_1><b_2|) = |a_1><a_2|tr(|b_1><b_2|) = |a_1><a_2|(<b_2|b_1>)$$
$$\rho^{AB} \quad \text{AB system}$$
$$\rho^A = tr_B \rho^{AB} = \sum_i < i^B|\rho^{AB}|r^B >$$