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T1 ________________	**73036**	F1 ________________
T2 ________________		F2 ________________
T3 ________________	Problem Chosen	F3 ________________
T4 ________________	**C**	F4 ________________

2019
MCM/ICM
Summary Sheet

E.P.A : Energy Profile Prediction And Analysis

Summary

The energy market of USA is facing complex circumstances. The contradiction between increasing energy consumption and energy shortage should be taken seriously. There are three stages we should go through and come to the final solution to achieve sustainability development of energy, especially renewable energy.

- Step 1:Find a concise model to characterize current situation.
  - Pick up 15 variables. There are 605 variables and 105744 data in total, which demands simplifying. So we pick up 14 variables. Their relationships can be described in three equations. The equations describe what does the total assumption of fossil fuels, the total assumption of renewable energy and the total energy assumption make up of.
  - Draw three pictures characterizing the three equations. The three pictures clearly showed the structure of energy consumption of the 50 years, pretty helpful for learning the whole situation.
- Step 2:Build two models dealing with the 14 variables.
  - Build Analytic Hierarchy Model to score the energy profile. We assume that there are 3 criteria affecting the judgment and list the importance of them. We calculate the scores for the four states using AHP. California ranks first among the four states. We use Consistency Index CI to judge its accuracy. And the final value of CI is pretty close to 0,which means the model is accurate.
  - Build Grey Forecasting Model. We build an array of certain variable's value of the 50 years. After 7 steps of execution, we can figure out the fitting function. We use the function to predict the future. Variance Ratio(VR) is used to describe the accuracy. For most points, the results are accurate.
- Step 3:Analyse the models and provide suggestions.
  - We found the similarities and difference of energy profile for the four states.Then we provided possible reasons for them.
  - We analysed the reasons why a few of our predictions are not accurate.
  - We provided reasonable goals and useful suggestions, for every single state as well as their compact.

Keywords: renewable energy; fossil fuels ; Analytic Hierarchy Process; Consistency Index; Grey Forecasting Model; similarities and difference; cooperative actions.

Memo

To: Governors of Arizona, California, New Mexico and Texas

From: Team 73036

Date: February 12,2018

Subject: Suggestions for Energy Management

- the State Profiles of 2009
  We judge an energy profile using three criteria: the renewable energy percentage (R), the energy consumption per person(P), the energy consumption of per unit increase of GDP(G).
  - Rank 1 :California
    R of California is nearly twice as much as Arizona and New Mexico, three times as Texas. It also has the lowest P and G
  - Rank 2:Arizona
    It has done better than New Mexico in all 3 criteria.
  - Rank 3:New Mexico.
    It is somewhere between Arizona and Texas.
  - Rank 4: Texas
    P of Texas is about twice as much as P of California and Arizona. And G of Texas is more than twice as much as G of California.
- Prediction without Policy Changes
  We analyze the prediction using four variables here: The total energy consumption(T), the consumption of fossil fuels(F), the consumption of nuclear energy(N), the consumption of renewable energy(P).The unit of all variables following is Billion Btu.

$$T = F + N + P$$

  - Arizona: From 2010 to 2050, the assumption of R is still small.And the proportion is stable. F is nearly 4 times as much as N and 40 times as much as R.
  - California: From 2010 to 2050, the percentage of N will become larger, while the percentage of F and P become smaller. In 2050, F is nearly 2 times as much as N and 4 times as much as R.
  - New Mexico: From 2010 to 2050, there is still no nuclear power in New Mexico, and the percentage of F and r is stable. in 2050, F is nearly 10 times as much as R.
  - Texas : From 2010 to 2050, the percentage of N and R is still pretty small. In 2050, F is nearly 9 times as much as N and 18 times as much as F.
- Recommend Goals
  - Arizona The proportion of renewable energy consumption in total energy consumption meets 4% in 2025 and 8% in 2050.
  - California In 2025, the renewable energy proportion remain at the same level of that in 2009 and in 2050, the proportion will increase to 15%.
  - New Mexico Renewable energy proportion meets 5% in 2025, and 10% in 2050.
  - Texas In 2025 wind energy will take up 4% of renewable energy and in 2050 wind energy will take up 8%, and renewable energy proportion will increase to 15%.

Contents

1 Introduction

1.1 Background

Efficient and renewable energy production and scientific cooperation strategy of energy usage are significant for American economy and environment. As **figure 1** [1] shows, the total energy consumption in American has been increasing fast. But the main contributors are petroleum, natural gas, coal and nuclear power, which are non-renewable and put great pressure on the environment. Consequently, efficient and renewable energy production is in demand to meet with the increasing energy demand and care for the environment. At the same time, scientific cooperation strategy is important between states to develop renewable energy together.

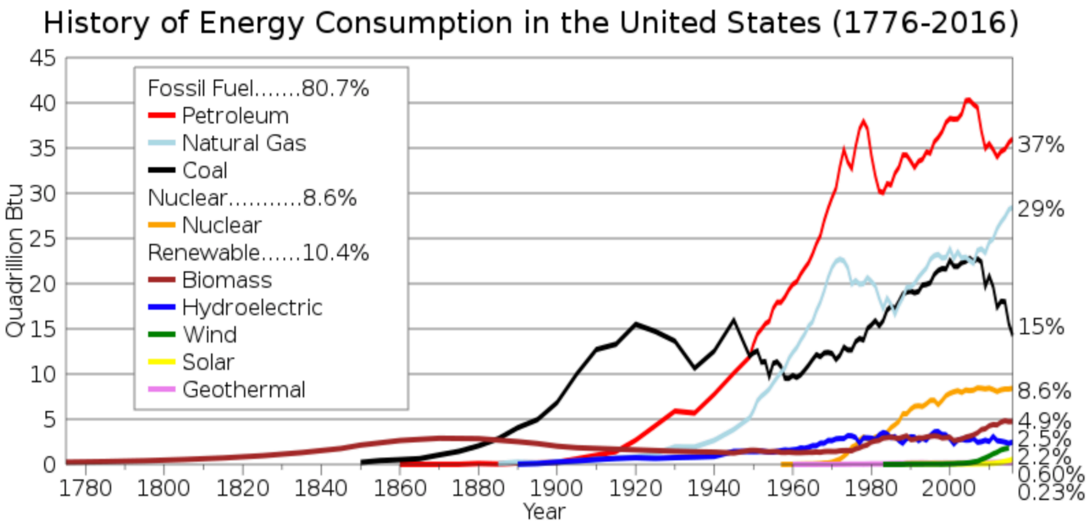

Figure 1: US energy consumption of 1776 - 2016.

Forming Interstate Compact among states is an efficient strategy under this circumstance. The varying geographies and industries of different states affect energy usage and production.If states can unit and complement one another, the extra energy produced by some states can solve the energy shortage of other states. And they can set scientific goals together. For instance, the Western Interstate Energy Compact (WIEC) formed in 1970 has done a lot ,including cooperation in deployment of solar thermal in the West and regionalization of electricity markets[2].

1.2 Our Work

Taking the background into consideration, our work concentrates on three dimensions to solve the problem.

- Analyze all the data provided, 605 variables and 105744 data in total.
- Build a Description Model. Select 15 variables from the 605 variables, and characterize the energy profile in pictures using 15 variables.
- Build a Analytic Hierarchy Model. Judge how each state has done in energy management using this model.
- Build a predicting Model. Calculate a fitting function using the historical data. Then predict how the Description Model in 2025 and 2050.

- Analyze the similarities and difference of energy profile for the four states. Bring up possible reasons.
- Provide respective goals and cooperation actions for the Interstate Energy Compact.

2 Assumptions

The models are built under these assumptions.

- We assume that nuclear electric power is not clean energy.

  Because the waste of nuclear fission is radioactive and harmful for creatures. But it can not be removed under current technology. The solution is commonly burying the waste deeply, which leaves potential hazards for the future.

- We assume that there will not be new type of energy invented before 2050.

  Because this period of time is not long enough for a new type of energy to be applied widespread and have significant influence.

- We only consider the assumption of energy while building models to simplify the analysis.

3 Nomenclature

Table 1: Nomenclature

Symbol	Definition
A	The positive reciprocal matrix used to calculate vector W
W_i	Weight of criteria i
CI	Consistency Index
y_i	The performance of state i
X	Original data sequence
$\overline{X}$	Result sequence of prediction
VR	Variance Ratio

4 Model Statement

4.1 Description Model of Energy Profile

We uses **15 variables** to describe the energy profile. Relationships of 15 variables are showed in figure 2.

We can describe the energy profile in three equations. The following are the equations and the pictures showing the data of the variables in each equation using the data of 50 years.

In the equations below, the name of energy type means the consumption of that type of energy.

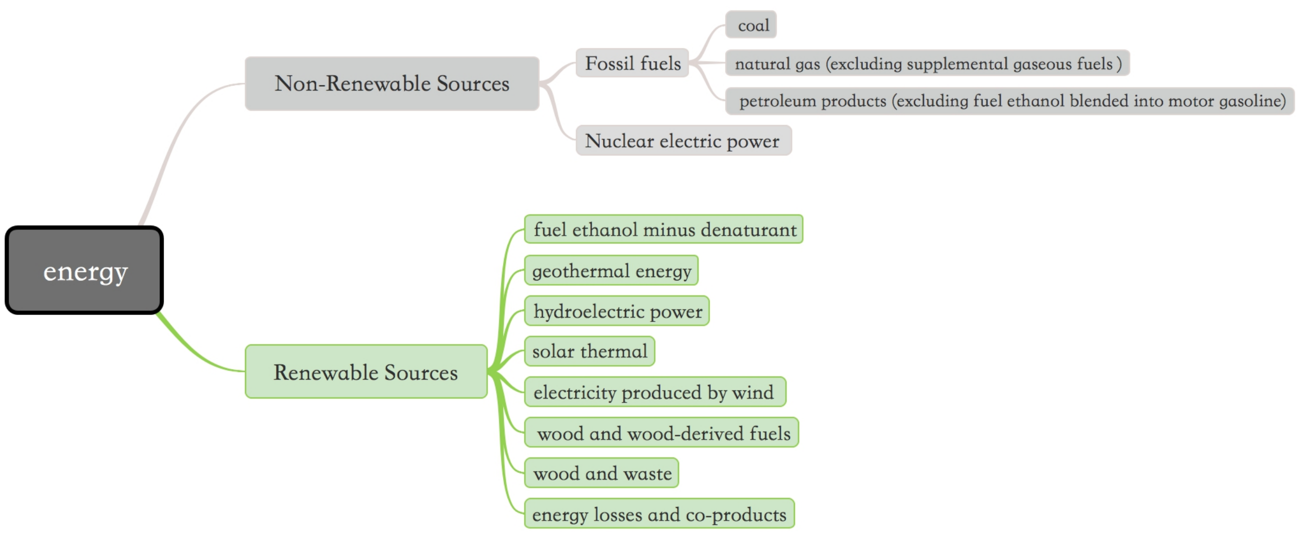

Figure 2: Energy classification tree.
There are 16 variables in total in this figure. The total consumption of "Non-Renewable Sources" is not included in the 15 variables we discuss.

- fossil fuels = coal + natural gas + petroleum products

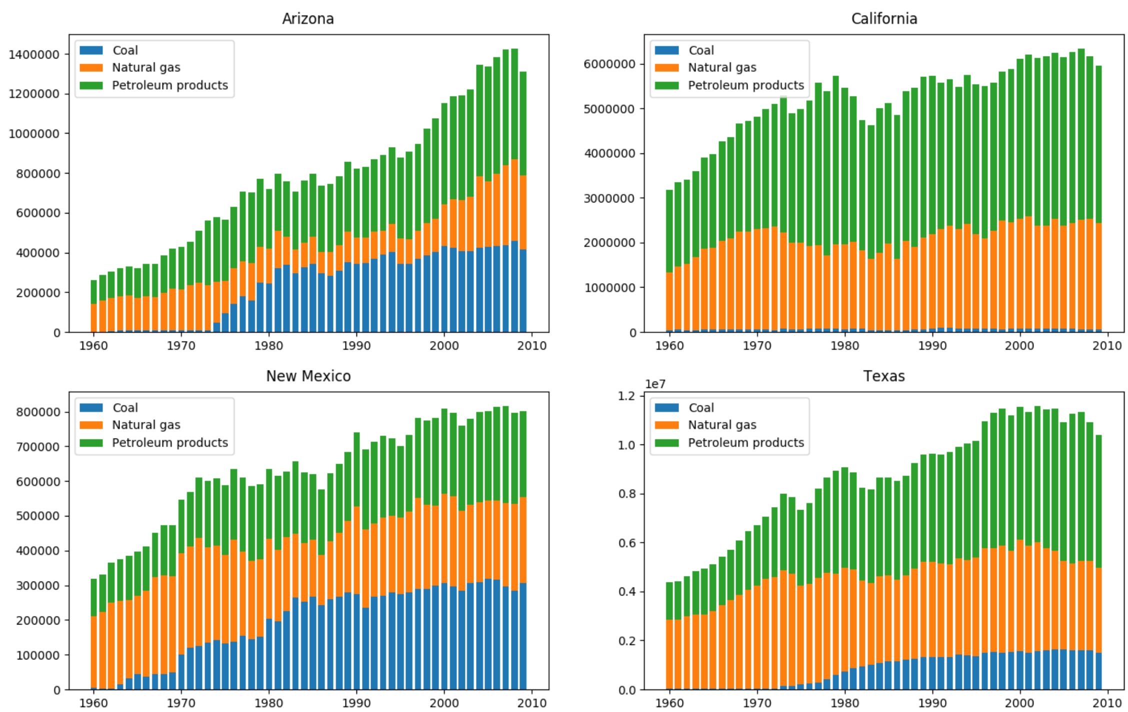

Figure 3: US fossil fuel energy consumption of 1960 - 2009.

- renewable energy = fuel ethanol + geothermal energy + hydroelectricity power + solar thermal energy + electricity produced by wind + wood and waste + energy losses and co-products

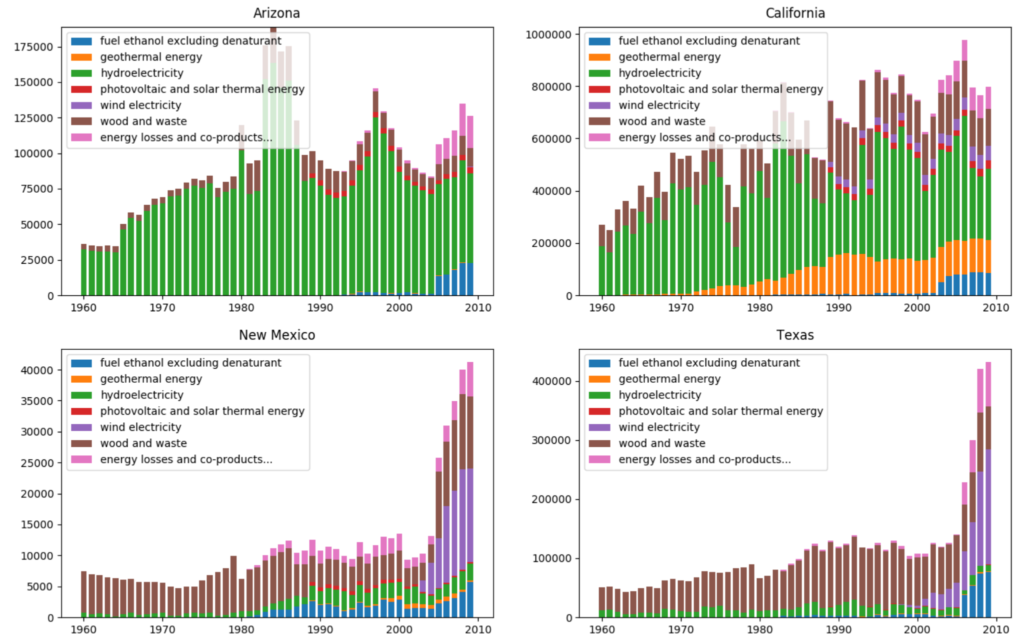

Figure 4: US renewable energy consumption of 1960 - 2009.

- total energy = fossil fuels + nuclear power + renewable energy

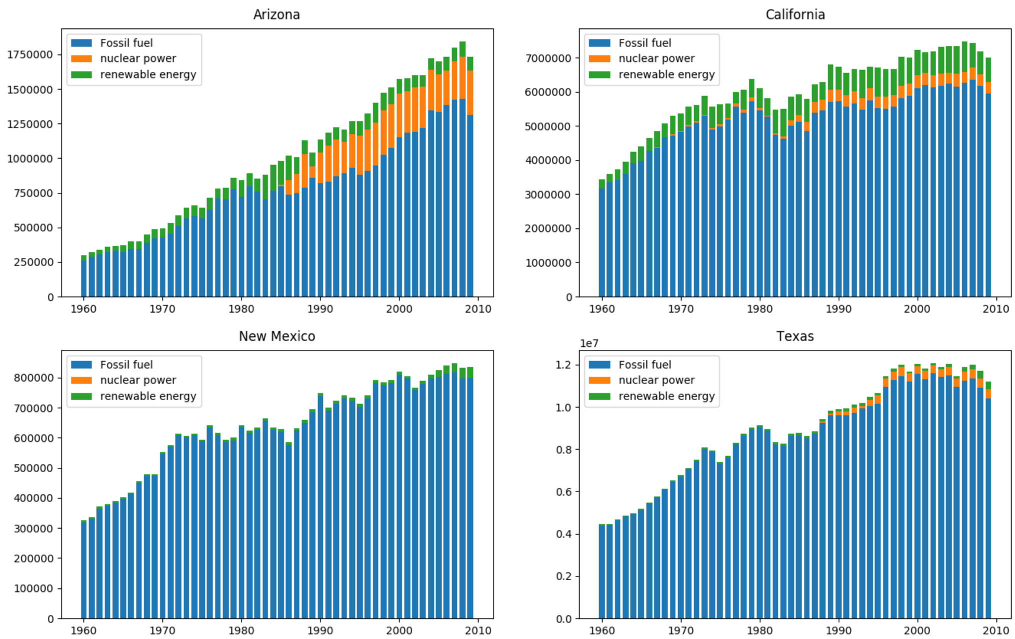

Figure 5: US total energy consumption of 1960 - 2009.

4.2 Analytic Hierarchy Model

Thanks to the work of Thomas L. Saaty in the 1970s[3]. We build our model on the basis of Analytic Hierarchy Process.

There are 3 factors which decide the excellence of energy profile. That is **the percentage of renewable energy consumption(R),the average amount of energy consumption per capita(P),the average amount of energy that a unit GDP increase takes(G)**.

- Step 1: Compute the vector of criteria weights.
  W_x means the importance of factor x.

$$A = \begin{array}{c c} & \begin{array}{ccc} R & P & G \end{array} \\ \begin{array}{c} R \\ P \\ G \end{array} & \begin{pmatrix} 1 & W_R/W_P & W_R/W_G \\ W_P/W_R & 1 & W_P/W_G \\ W_G/W_R & W_G/W_P & 1 \end{pmatrix} \end{array}$$

- Step 2: Calculate the largest eigenvalue λ_{max} and corresponding eigenvector Y of A.
  Assume corresponding eigenvector Y is

$$Y = [y_1, y_2, y_3]$$

y_1 : the weight of factor R y_2 : the weight of factor P y_3 : the weight of factor G

- Step 3: Consistency Analysis of the weight.
  Assume eigenvector satisfies that

$$\begin{pmatrix} 1 & W_R/W_P & W_R/W_G \\ W_P/W_R & 1 & W_P/W_G \\ W_G/W_R & W_G/W_P & 1 \end{pmatrix} \begin{pmatrix} W_R \\ W_P \\ W_G \end{pmatrix} = n \begin{pmatrix} W_R \\ W_P \\ W_G \end{pmatrix}$$

The Consistency Index (CI) is $CI = \dfrac{\alpha_{min} - 1}{n - 1}$

A perfectly consistent decision maker should always obtain CI=0, but small values of inconsistency may be tolerated. In particular, if

$$CI^{(1)} = \frac{CI}{RI} < 0.1$$

Then the the inconsistencies are tolerable. RI is the Random Index, i.e. the consistency index when the entries of A are completely random. According to the article[4], the values of RI for small problems (n 10) are shown in Table 1 .

m	2	3	4	5	6	7	8	9	10
RI	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.51

Table 2: Values of the Random Index (RI) for small problems

- Step 4: Assess the energy profile using each of the three criteria successively.
  Execute Step 1 to Step 3 for three times, corresponding with three factors.
  However, the vector of A and Y are different. C_i means the energy consumption(R) / the average amount of energy consumption per capita(P) / the average amount of energy that

a unit GDP increase takes(G) of state i.
Ar stands for Arizona, Ca stands for California, Ne stands for New Mexico, Te stands for Texas.

$$A' = \begin{array}{c} \\ Ar \\ Ca \\ Ne \\ Te \end{array} \begin{array}{c} \begin{array}{cccc} Ar & Ca & Ne & Te \end{array} \\ \begin{pmatrix} 1 & C_{Ar}/C_{Ca} & C_{Ar}/C_{Te} & C_{Ar}/C_{Te} \\ C_{Ca}/C_{Ar} & 1 & C_{Ca}/C_{Ne} & C_{Ca}/C_{Te} \\ C_{Ne}/C_{Ar} & C_{Ne}/C_{Ca} & 1 & C_{Ne}/C_{Te} \\ C_{TE}/CAr & C_{TE}/C_{Ca} & C_{Te}/C_{Ne} & 1 \end{pmatrix} \end{array}$$

At the same time.

$$Y' = [m_1, m_2, m_3, m_4]$$

y_1 : the performance of Arizona $\quad y_2$: the performance of California

y_3 : the performance of New Mexico $\quad y_4$: the performance of Texas

Notice: Different factors have different Y'.For instance, Y' for renewable energy consumption(R) can be described as

$$Y'^R = [m_1^R, m_2^R, m_3^R, m_4^R]$$

- Step 5: Synthesize the three factors using weighted average method for four states.
  For instance
$$ArizonaPerformance = y_1 * m_1^R + y_2 * m_1^P + y_3 * m_1^G$$

- Step 6: Consistency Analysis of the whole model.
$$CI_{total}^{(1)} = \frac{\sum_{j=1}^{3} CI(j) * y_j}{\sum_{j=1}^{3} RI(j) * y_j}$$

- Step 7: Compare the performance of Arizona, California, New Mexico and Texas. Then we can decide the excellence of energy profile for each state.

4.3 Grey Forecasting Model

The goal of this article is constructing a model to characterize how the energy profile of each of the four states has evolved from 1960 to 2009 and predict the energy profile of the four states in 2025 and 2050. Here we choose Grey Forecasting Model to do the analyses and prediction.

Thanks to the work of Shin-li Lu,Ching-I Li and Shih-hung(2013)[5]. We set up our model as the following steps.

- Step 1: Denote the original data sequence.
$$X^{(0)} = \left\{x_1^{(0)}, x_2^{(0)}, x_3^{(0)} ..., x_n^{(0)}\right\} \quad n \geq 4$$

- Step 2: Use accumulated generating operations (AGO) to form a new data series.
$$X^{(1)} = \left\{x_1^{(1)}, x_2^{(1)}, x_3^{(1)} ..., x_n^{(1)}\right\}$$

  where $x_1^{(1)} = x_1^{(0)}$
$$x_k^{(1)} = \sum_{i=1}^{k} x_i^{(0)} \quad k = 2, 3, 4..., n$$

- Step 3: Calculate background values Z.

$$Z = \{z_2, z_3, z_4, ..., z_n\}$$

$$z_k = (1-\alpha)x_k^{(0)} - \alpha x_{k-1}^{(0)} \quad k = 2, 3, 4, ..., n$$
$$0 < \alpha < 1, \quad \text{here we assume } \alpha = 0.5$$

- Step 4: Establish the grey differential equation.

$$\frac{dx_k^{(1)}}{dt} + ax_k^{(1)} = b. \quad (I)$$

(I) can also be written as

$$x_k^{(0)} + ax_k^{(1)} = b$$

We can caculate a and b in Step 5.

- Step 5: Define vector B, Y as

$$B = \begin{vmatrix} -z_2 & 1 \\ -z_3 & 1 \\ \vdots & \vdots \\ -z_n & 1 \end{vmatrix}$$

$$Y = [x_2^{(0)}, x_3^{(0)}, ..., x_n^{(0)}]^T$$

Caculate a and b.

$$[a, b]^T = (B^T B)^{-1} B^T Y$$

Solve (I) by using the least square method and the forecasting values can be obtained as

$$\begin{cases} \overline{x}_k^{(1)} = \left(x_1^{(0)} - \frac{b}{a}\right) e^{-a(k-1)} + \frac{b}{a} \\ \overline{x}_k^{(0)} = x_k^{(1)} - x_{k-1}^{(1)} \end{cases}$$

- Step 6: $\overline{X}$ are the results we predict.

$$\overline{X} = \{\overline{x}_1, \overline{x}_2, \overline{x}_3, ..., \overline{x}_n\}$$

- Step 7: Calculate the Variance Ratio for our prediction.
Define $\beta X^{(0)}$ as

$$\beta X^{(0)} = \left\{\beta x_2^{(0)}, \beta x_3^{(0)}, \beta x_4^{(0)}, ..., \beta x_n^{(0)}\right\}$$

$$\beta x_k^{(0)} = x_k^{(0)} - x_{k-1}^{(0)}$$

Compute S_1, S_2.

$$S_1 = \frac{\sum_{k=1}^{n}(x_k^{(0)} - \overline{x}_k^{(0)})^2}{n-1}$$

$$S_2 = \frac{\sum_{k=1}^{n}(\beta x_k^{(0)} - \beta \overline{x}_k^{(0)})^2}{n-1}$$

The **Variance Ratio (VR)** is

$$VR = \frac{S_2}{S_1}$$

Here is the standard judging the accuracy of the model using Variance Ratio.
Notice : The lower accurance level is, the more accurate the data is.

VR	(0,0.35)	[0,35,0.5)	[0.5, 0.65)	[0.65,1)
Accuracy Level	1	2	3	4

Table 3: Judge Standard of VR

5 Implementation

5.1 Analytic Hierarchy Model

- Step 1: Compute the vector of criteria weights.
  We assume that the importance of P (the average amount of energy consumption per capita) is the same as R (the percentage of renewable energy consumption) and four times as important as G (the average amount of energy that a unit of GDP increase takes). That is

$$\frac{W_R}{W_P} = 1 \qquad \frac{W_G}{W_P} = 0.25$$

$$A = \begin{array}{c} \\ R \\ P \\ G \end{array} \begin{array}{c} \begin{array}{ccc} R & P & G \end{array} \\ \begin{pmatrix} 1 & 1 & 4 \\ 1 & 1 & 4 \\ 0.25 & 0.25 & 1 \end{pmatrix} \end{array}$$

- Step 2: Caculate the largest eigenvalue λ_{max} and corresponding eigenvector Y of A.

$$Y = [0.4444, 0.4444, 0.1111]$$

- Step 3: Caculate $CI_0^{(1)} = 0$
  Assume that the Consistency Index for the vector of criteria weights is CI_0

$$\text{At this time,} \quad m = 3 \quad RI = 0.58$$

$$CI_0^{(1)} = \frac{CI_0}{RI} = 0$$

  So vector A is a consistent matrix.

- Step 4: Assess the energy profile using each of the three criteria successively.
  The data analysed are the amount of the three criteria in 2009.
  For a certain criterion, we can conclude the proportionality of the data amount between every state by judging the height of bars in figure 6. For a certain criterion, we can conclude the proportionality of the data amount between every state by judging the height of bars in figure 3. Build the following three matrixes and calculate its largest eigenvalue λ_{max} and corresponding eigenvector Y.
  The matrices built:
  For R (the percentage of new- able energy consumption)

$$A_R = \begin{array}{c} \\ Arizona \\ California \\ NewMexico \\ Texas \end{array} \begin{array}{c} \begin{array}{cccc} Arizona & California & NewMexico & Texas \end{array} \\ \begin{pmatrix} 1 & 0.25 & 4 & 4 \\ 4 & 1 & 8 & 8 \\ 0.25 & 0.125 & 1 & 1 \\ 0.25 & 0.125 & 1 & 1 \end{pmatrix} \end{array}$$

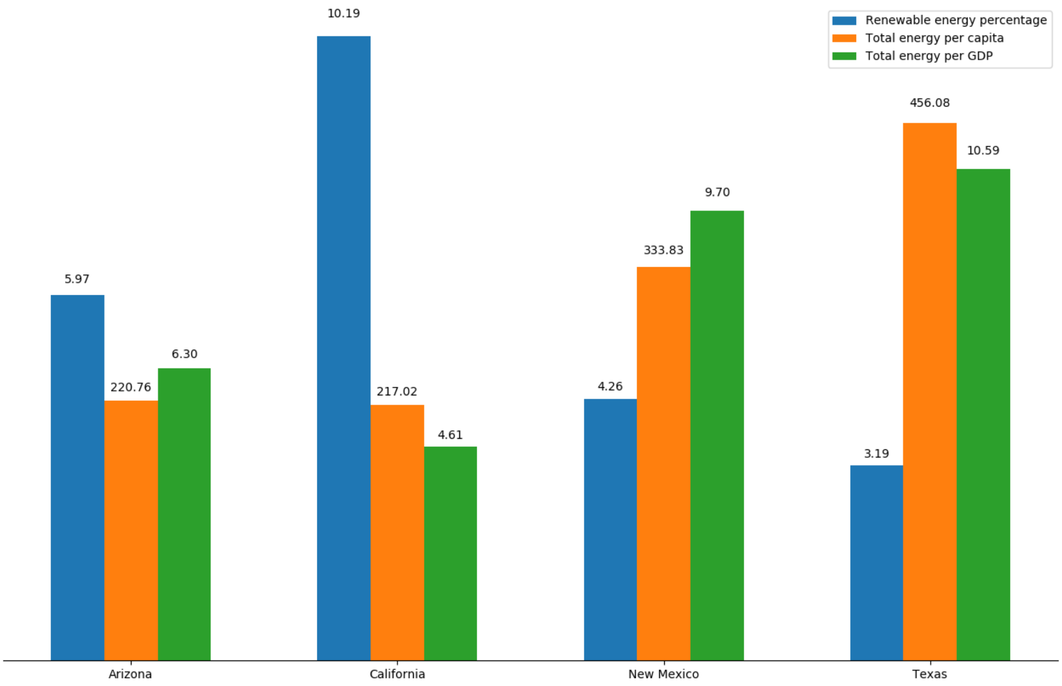

Figure 6: Normalization figure of the the four states' amount of R, P and G in 2009.
The unit of R is "%".
The unit of P is million Btu.
The unit of G is Thousand Btu per chained (2000) dollar.

For P (the average amount of energy consumption per capita)

$$A_R = \begin{array}{c c} & \begin{array}{cccc} Arizona & California & NewMexico & Texas \end{array} \\ \begin{array}{c} Arizona \\ California \\ NewMexico \\ Texas \end{array} & \begin{pmatrix} 1 & 1 & 0.5 & 0.25 \\ 1 & 1 & 0.5 & 0.25 \\ 2 & 2 & 1 & 0.5 \\ 4 & 4 & 2 & 1 \end{pmatrix} \end{array}$$

For G (the average amount of energy consumption per unit of GDP increase)

$$A_R = \begin{array}{c c} & \begin{array}{cccc} Arizona & California & NewMexico & Texas \end{array} \\ \begin{array}{c} Arizona \\ California \\ NewMexico \\ Texas \end{array} & \begin{pmatrix} 1 & 3 & 1/3 & 1/7 \\ 1/3 & 1/6 & 0.5 & 1/9 \\ 3 & 6 & 1 & 0.5 \\ 7 & 9 & 2 & 1 \end{pmatrix} \end{array}$$

After calculating the λ_{max} and corresponding eigenvector Y, compute $CI^{(1)} = CI/RI$ for these three models.The results:

$CI_R^{(1)}$	$CI_P^{(1)}$	$CI_G^{(1)}$
0.0225	0	0.0232

Table 4: The values of $CI_R^{(1)}$, $CI_P^{(1)}$ and $CI_G^{(1)}$

- Step 5: Synthesize the three factors using weighted average method for four states. The final scores each state get are showed in Table 5.

Arizona	California	New Mexico	Texas
0.1674	0.3460	0.1724	0.3142

Table 5: The final scores each state get.

- Step 6: Consistency Analysis of the whole model.

$$CI^{(1)}_{total} = 0.0126$$

  So the total Consistency Index is small enough.

- Step 7: As Step 5 shows, the energy profile of California is the best and that of Arizona is the worst.Because Step 6 has proved that the Consistency Index of the whole model is tolerable, we come to the conclusion that **the energy profile of California is an ideal one**.

5.2 Grey Forecasting Model

5.2.1 Prediction for Fossil Fuels

Predict the value of coal, natural gas and petroleum products.Using the method described in Model Statement, calculate the Variance Ratio.

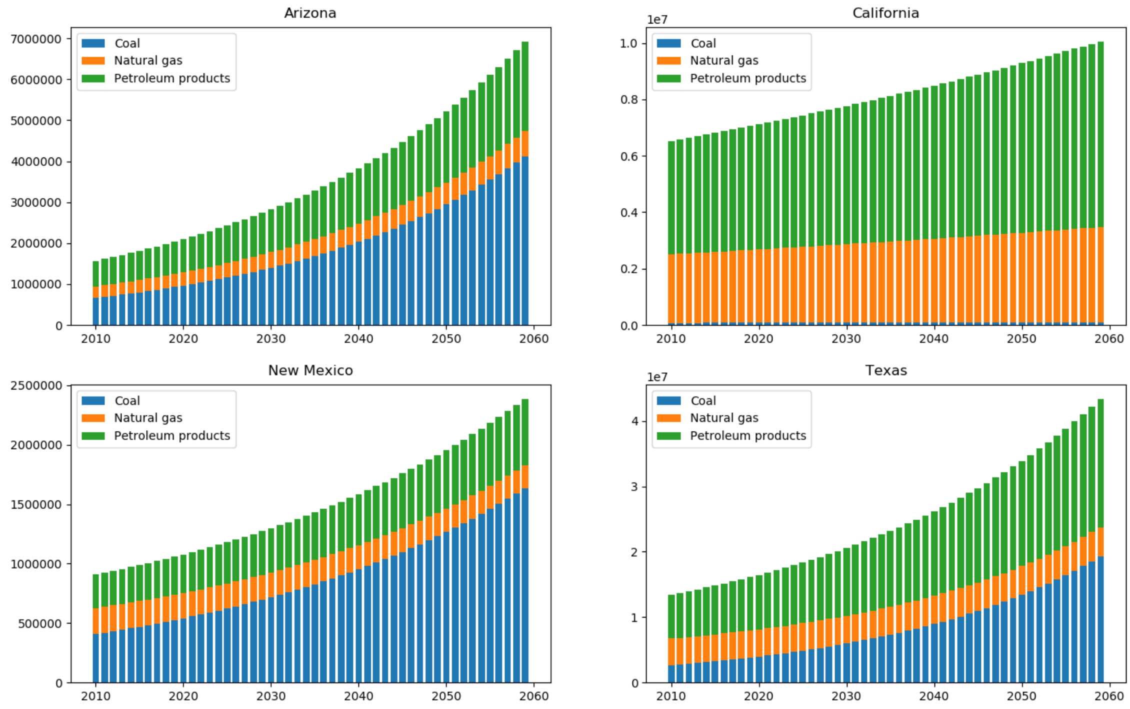

Figure 7: Prediction for Fossil Fuels.

	coal	natural gas	petroleum
Arizona	0.2623	2.2660	0.0864
California	3.4887	0.8730	0.4457
New Mexico	0.2872	12.4308	0.1845
Texas	0.2163	5.8416	0.0922

Table 6: VR of Fossil Fuels Predictions

5.2.2 Prediction for Renewable Energy

Predict the value of fuel ethanol, geothermal energy, hydroelectricity power, photovoltaic and solar thermal energy, wind electricity, wood and waste, energy losses and co-products.

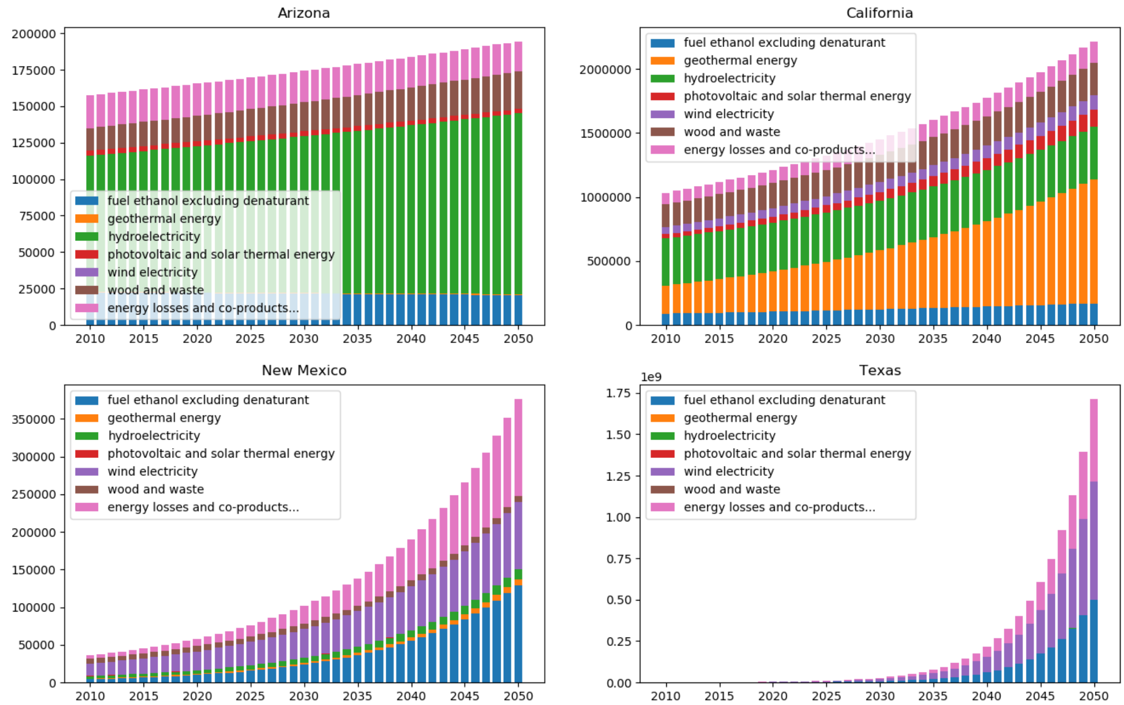

Figure 8: Prediction for Renewable Energy

Using the method described in Model Statement, caculate the Variance Ratio.

	Arizona	California	New Mexico	Texas
fuel ethanol	4.5743e-12	0.9376	1.1060	0.0370
geothermal energy	0.5058	0.3457	1.5215	0.0235
hydroelectricity power	7.3594	14.6927	0.4501	20.9098
photovoltaic and solar thermal energy	30.5735	1.0823	0.2216	0.1909
wind electricity	0	0.1753	0.3631	6.6149e-5
wood and waste	6.4654	2.1035	29.3199	0.9059
energy losses and co-products	4.5743e-12	0.9376	1.1060	0.0370

Table 7: VR of Renewable Energy Predictions

5.2.3 Prediction for Total Energy

Predict the value of fossil fuels, nuclear power and renewable energy.

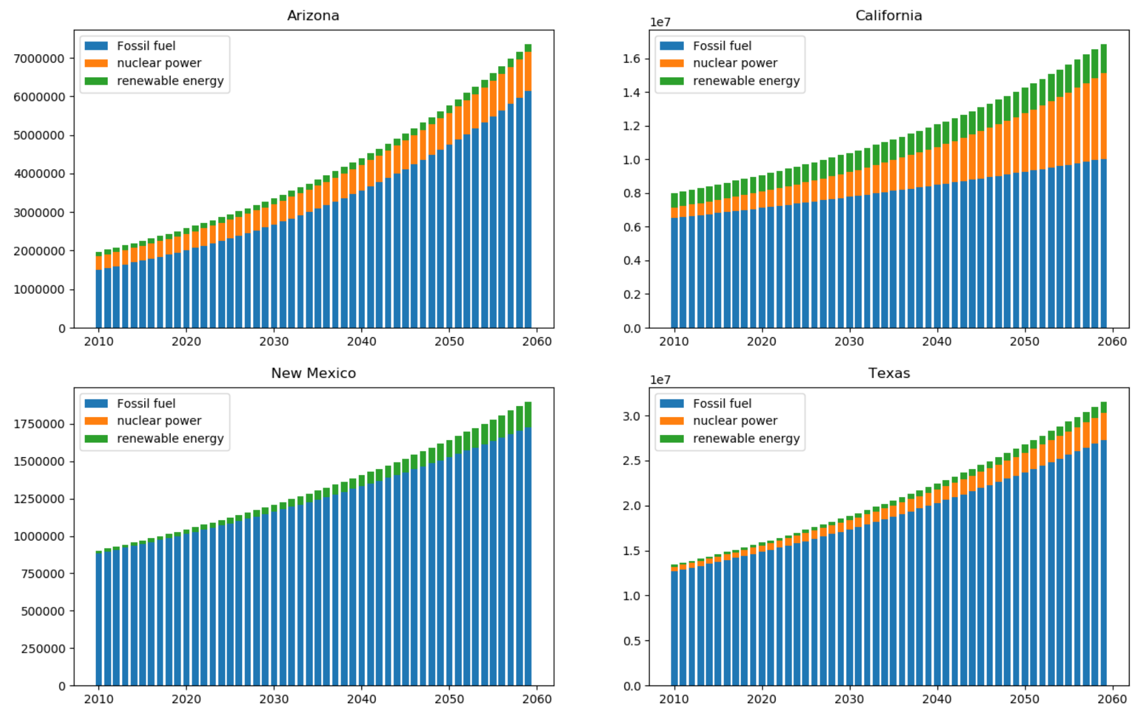

Figure 9: Prediction for Total Energy

Using the method described in Model Statement, calculate the Variance Ratio.

	fossil fuels	nuclear power	renewable energy
Arizona	0.0428	0.5866	3.3729
California	0.2498	0.2755	0.5194
New Mexico	0.1412	0	1.0488
Texas	0.1385	0.3960	0.5372

Table 8: VR of Fossil Fuels Predictions

Figure 10: Percentage of Renewable Energy Components

6 Similarities, Difference, Possible Reasons and Sensitivity Analysis

6.1 Usage of renewable energy

6.1.1 Arizona

- Renewable energy consumption of Arizona ranks third of the four states.
- Hydroelectric power dominated Arizona's renewable electricity generation for a quite long time.
- "Photovoltaic and solar thermal energy" and "energy generated from wood and waste" take only a small share.
- Recently, the consumption of fuel ethanol develops fast.

6.1.2 California

- California has the largest renewable energy consumption among the four states.
- California utilizes hydroelectric energy and geothermal thermal energy well.
- California is concentrating on developing fuel ethanol recently. The increasing speed of wind energy and solar thermal energy is high behind that of fuel ethanol.

6.1.3 New Mexico

- New Mexico has the least renewable energy consumption, which is only one twentieth of California's.
- Usage of Geothermal energy, photovoltaic and solar thermal energy both increased for a period of time, but decreased in the end.
- Hydroelectricity and fuel ethanol took a stable progression.
- Wind energy are developing fast recently.

6.1.4 Texas

- Texas's total renewable energy consumption ranks second, less than California.
- Texas has the largest total energy consumption.
- Early in the fifty years, energy generated from wood and waste along with a small amount of hydroelectricity were the majority of renewable energy generation.
- Recently, fuel ethanol and wind enrgy experienced a fast development.

6.2 Similarities and difference

6.2.1 Similarities

- Rising of renewable energy consumption

  Among the 50 years, the renewable energy consumption is mainly rising. The amount of renewable consumption of Arizona in 2009 nearly triples the amount of renewable consumption in 1969. The same thing happened in California. As for New Mexico, the number is 5. Texas witnessed the fastest increase among the four states, whose amount of renewable consumption in 2009 nearly septuples the amount of renewable consumption in 1969.

- Stable and low proportion of renewable energy in total energy consumption

  Proportion of renewable energy in total energy consumption of four states remain to be stable and low. The proportion of renewable energy in total energy consumption of Arizona takes about 5% - 15% , that of New Mexico and Texas takes about 3%, and that of California takes about 10%.

6.2.2 Difference

- Different level of renewable energy development

  The level of development of four states vary considerably. In the aspect of total consumption, California did the best, followed by Texas, Arizona and New Mexico. While in the aspect of share of renewable energy in total energy consumption, California still did best, followed by Arizona, Texas and New Mexico.

- Different constitution of renewable energy

  - Hydroelectricity

    Hydroelectricity is a conventional strength of Arizona and California, but is the weakness of Mew Mexico and Texas.

  - Wind energy

    Wind energy consumption of New Mexico and Texas was developing fast in recent years.But Arizona hardly developed wind energy. California was somewhere between them.

- Different potential of renewable energy

  - Arizona is potential in wind, geothermal energy, photovoltaic and solar thermal energy[6] .
  - California has potential in geothermal energy and wind energy[7].
  - New Mexico has potential develop geothermal and photovoltaic and solar thermal energy in the future[8].
  - Texas can continue to develop geothermal energy[9].

- Different developing speed

  Four states have differences in the developing speed of renewable energy. Arizona and Texas developed at a relatively stable pace. New Mexico and Texas did not develop fast in renewable energy until recently.

6.3 Possible influential factors

- Geography and resource distribution

  Texas has wide plains, which are convenient to build large quantities of wind farms to generate electricity utilizing wind energy[9]. Several Rivers which have large gaps flow through California and Arizona , so they are abundant in hydroelectricity[7][6]. California used little amount of coal because of California's small reserves of coal[7]. Its electricity generation mainly comes from natural gas and renewable energy[7].

- Population

  The populated California and Texas have large numbers of registered motor vehicles, which greatly booms the consumption of energy[7] [9]. On the contrary, with a relatively small population, New Mexico is low in energy demand[8].

- Industry

  Texas and California have many energy-intensive industries, such as petroleum refining and chemical manufacturing[7] [9]. Under the demand of massive cheap energy, the use of fossil fuels grows.

- Policy and standards

  "The California renewable portfolio standard (RPS), created in 2002, requires that 33% of retail sales of electricity in California come from eligible renewable resources by 2020 and 50% by 2050."[7]

6.4 Sensitivity Analysis

- "Fast Development" Trap
  If the amount of energy consumption developed too fast in the recent years, its prediction of exponential growth will result in an overoptimistic prediction or even unreasonable.

- Resources Limitations
  Although we dismiss the change of policy, the energy resources are limited. So the growth of energy consumption will be limited by the amount of resources. Our model cannot take that into consider.

- Technical Limitations
  Because of the instability of electricity generated from renewable energy, it is inconvenient to be integrated to power systems. As a result, it will possibly face difficulties and bottleneck in future development, which in return will restrain its growth of proportion in total energy consumption. Our model cannot predict that case.

7 Goals and Suggestions

7.1 Respective goals and actions

- Arizona

  Our goals for Arizona is that the proportion of renewable energy consumption in total energy consumption meets 4% in 2025 and 8% in 2050.

  In the end of twentieth century, nuclear power took more proportion. Because of the potential damage nuclear power may cause to the environment, we hope renewable energy will take more proportion in the future.

  By the analysis of end-use energy consumption, Arizona's transport sector consumed the most energy. So we assume that Arizona can develop fuel ethanol firstly and import electricity from New Mexico.

- California

  Our goals for California is that in 2025, the renewable energy proportion remains at the same level of that in 2009 and in 2050, the proportion increases to 15%.

  By prediction, we find renewable energy proportion slightly decreased. We assume it may be caused by the increase of total energy consumption.

  California can develop wind energy because of its inshore plains.

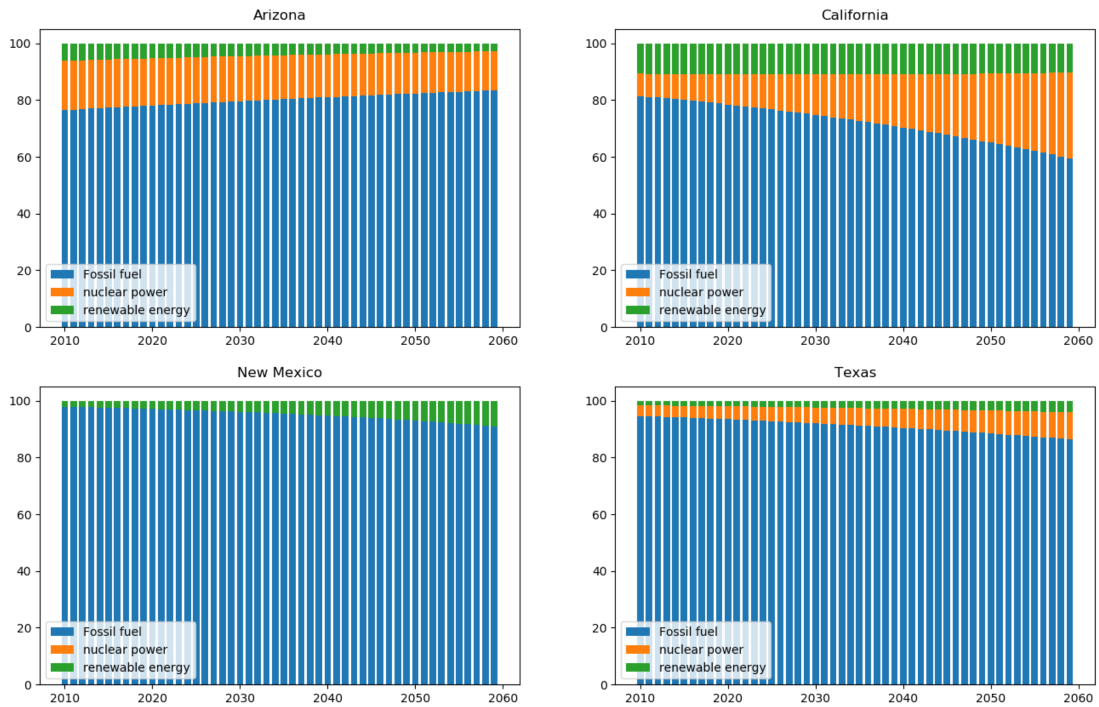

Figure 11: Prediction of Energy Proportion

- New Mwxico

  Our goal for New Mexico is that renewable energy proportion meets 5% in 2025, and 10% in 2050.

  By prediction, we find that energy generated from coal, wind and fuel ethanol will increase gradually. Because of the fast development of wind energy, we make an optimistic goal and suggest New Mexico developing wind energy.

- Texas

  Our goal for Texas is that in 2025 wind energy takes up 4% of renewable energy and in 2050 wind energy takes up 8%, and renewable energy proportion increases to 15%.

  By prediction, We find their wind energy and fuel ethanol will develop greatly. But its predicted total consumption is overoptimistic, so we set a reserved goal.

  However, prediction still imply that we can put emphasis on wind energy and fuel ethanol development.

7.2 Cooperative actions

- Interstate power transmission

  In need of reducing coal-fired power generation in Arizona and stimulating renewable energy demand in New Mexico, we suggest laying electricity transmission route between Arizona and New Mexico. It can not only supplement Arizona's energy gap and help Arizona adjust its energy structure, but also benefit the development of New Mexico.

- Promoting technical exchange

  New Mexico and Texas developed wind energy well recently. They can offer technical support to help California develop its underdeveloped wind energy to fulfill its great potential.

8 Reference

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Appendices

Appendix A Grey Prediction

```
# GM(1,1)
import numpy as np
from math import exp

alpha = 0.5
e = 1e-10

def pred_of_np(x_0, pre_num = 50):
    global alpha, e
    n = len(x_0)
    m = pre_num
    x_0 = x_0[-m:]
```

```
    h = 0
    for i in range(m):
        if (abs(x_0[i]) < e).all():
            h += 1
    x_0 = x_0[h:]
    m -= h
    if len(x_0) == 0:
        print((1e-16,0.0),0)
        return np.zeros(n)

    x_1 = []
    x_1.append(x_0[0])
    for i in range(1,m):
        x_1.append(x_1[-1]+x_0[i])
    x_1 = np.array(x_1)

    z_1 = np.array([0])
    z_1 = np.append(z_1, alpha * x_1[1:m] + (1 - alpha) * x_1[0:m-1])

    Y = x_0[1:][:,np.newaxis]

    B_1 = (-z_1[1:])[:,np.newaxis]
    B_2 = (np.zeros(m-1) + 1)[:,np.newaxis]

    B = np.mat(np.hstack([B_1, B_2]))
    B_t = B.transpose()
    if (z_1 == np.zeros(m)).all():
        u = [1e-16, 0]
    else:
        u = (B_t * B).I * B_t * Y
    u = np.array(u).flatten()
    a = u[0]
    b = u[1]

    x_0 = x_0.tolist()
    x_1 = x_1.tolist()
    x_0_pred = [x_0[0]]
    x_1_pred = [x_1[0]]
    for i in range(1, n + m):
        x_1_new = (x_1[0] - b / a) * exp(-a*i) + b / a
        x_0_pred.append(x_1_new - x_1_pred[-1])
        x_1_pred.append(x_1_new)

    for i in range(len(x_0_pred)):
        x_0_pred[i] = max(x_0_pred[i], 0)
    eps = np.array(x_0_pred[0:m]) - np.array(x_0)
    s1 = np.var(np.array(x_0_pred[0:m]))
    s2 = np.var(eps)
    if(abs(s1) < e):
        print((a,b),0)
    else:
        print(((a,b)), s2 / s1)
    return np.array(x_0_pred[m:])
```

Appendix B Analytic Hierarchy Process

```
fid = fopen('input.txt','r');
n1 = 3; n2 = 4;
a = [];
for i = 1:n1
    tmp = str2num(fgetl(fid));
    a=[a;tmp];
end
for i = 1:n1
    str1 = char(['b',int2str(i),'=[];']);
    str2 = char(['b',int2str(i),'=[b', int2str(i), ';tmp];']);
    eval(str1);
    for j = 1:n2
        tmp = str2num(fgetl(fid));
        eval(str2);
    end
end
ri = [0,0,0.58,0.90,1.12,1.24,1.32,1.41,1.45];
[x,y] = eig(a);
lambda = max(diag(y));
num = find(diag(y)==lambda);
w0 = x(:,num)/sum(x(:,num));
cr0 = (lambda-n1)/(n1-1)/ri(n1)
w1 = zeros(n2,n1);
```

```
for i = 1:n1
    [x,y] = eig(eval(char(['b',int2str(i)])));
    x = real(x);
    y = real(y);
    lambda = max(diag(y));
    num = find(diag(y)==lambda);
    w1(:,i) = x(:,num)./sum(x(:,num));
    cr1(i) = (lambda-n2)/(n2-1)/ri(n2);
end
cr1, ts = w1*w0 , cr = cr1*w0
```