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Assignment-1

Page No.
Date

Que 1:-

$t_{x=10} = 5\,sec$

$t_{x=50} = ?$

$kx^2 = t$

$k \times 100 = 5$

$k = \frac{1}{20}$

$t_{x=50} = \frac{1}{\cancel{20}} \times 5\cancel{0} \times \overset{25}{\cancel{50}}$

$= 125\,sec$

Que 2:-

$T_A = x^3 \quad , \quad T_B = 2x^2$

At Breaking point:-

$x^3 = 2x^2$

$\boxed{x=2}$

Que 3:-

$f(x) = x \cdot 2^x \quad . \quad g(x) = 4^n$

Applying limit rule:-

$\lim_{x\to\infty} \frac{f(x)}{g(x)} \Rightarrow \lim_{x\to\infty} \frac{x \cdot 2^x}{4^x} \Rightarrow \lim_{x\to\infty} \frac{x \cdot 2^x}{2^{2x}}$

Applying L' Hospital:-

$\lim_{x\to\infty} \frac{1}{2^x \log 2}$

Hence $f(x)$ is in $O(4^x)$.

Page No. Date

Que4 :-

Let any polynomial ~~funct~~, $P(x)$ and $\log(x)$ function

Using limit rule, $\lim_{x\to\infty} \frac{\log(x)}{P(x)} \left[\frac{\infty}{\infty}\right]$

Applying L'Hospital rule :-

$\lim_{x\to\infty} \frac{1}{x \cdot P'(x)}$

$= 0$

Thus, $\log x$ grows slower then all functions.

Let two log fun. $\frac{\log_a x}{\log_b(x+1)}$ and

Applying limit rule :-

$$\lim_{x\to\infty} \frac{\log_a x}{\log_b(x+1)} \Rightarrow \lim_{x\to\infty} \frac{\log_a x}{\log_b x + \log(1+1/x)}$$

$$\lim_{x\to\infty} \frac{\log_a x}{\log_b x}$$

$$\Rightarrow \lim_{x\to\infty} \frac{\ln b}{\ln a}$$

$\Rightarrow$ constant

Hence all log functions grow at same rate.

Date

Ans:-

$f(x) = x^4 + \log x + 17,\ g(x) = x^4$

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{x^4+\log x+17}{x^4}$$

$$\lim_{x\to\infty} \frac{4x^3+1/x}{4x^3}$$

$$\lim_{x\to\infty} \frac{4x^3}{4x^3} + \frac{1}{4x^4}$$

$\Rightarrow 1$ (constant)

Hence $f(x)$ is $O\log(x)$.

Que 7:-

(a) $k=1$

$k=2$

$k=3$

$\vdots$

$k=n-1$

$O(n-1) \Rightarrow O(n)$

(b)

for $i=1$	$j=2,3,---n$	$n-1$
$i=2$	$j=3,4,---n$	$n-2$
$\vdots$		
$i=n-1$	n	$(n-i)$

$(n-1)(n-2)+(n-3)+(n-4)+---+i+1$

$$\Rightarrow \frac{n(n-1)}{2}$$

$\Rightarrow O(n^2)$

Teacher Signature

Page No.
Date

Que 9:-

$T_A = 100^x$
$T_B = x^4$

Using limit rule :-

$$\lim_{x\to\infty} \frac{x^4}{100^x} \left(\frac{\infty}{\infty}\right)$$

$$\lim_{x\to\infty} \frac{4x^3}{100^x \log 100}$$

$$\lim_{x\to\infty} \frac{24}{100^x (\log(100))^4} = 0$$

Hence, T_A grows faster when $x \to \infty$

Que 10:-

$f(x) = x \log x, \; g(x) = \log(x!)$

$$\Rightarrow \log(1.2.3.4 \ldots (x-1)x)$$

$$\Rightarrow \log\left(\frac{x^{x/2}}{2}\right)$$

$$= \frac{x}{2} \log \frac{x}{2}$$

$$\Rightarrow \frac{x}{2}\log x - \frac{x}{2}\log 2$$

$$= \frac{x}{2}(\log x - \log 2) \simeq O(n \log n)$$

Hence, $f(n) \in \Theta g(n)$

Page No.
Date

Que 11.

(a) $2^{n-1} + 4^{n+1}$

for θ $\quad c_1 g(x) \leq f(x) \leq c_2 g(x)$

$$2^{n-1} + 2^{2n+2}$$

$$\frac{2^n}{2} + 4 \cdot 2^{2n}$$

$$\Rightarrow 2^{2n}\left(4 + \frac{1}{2^{n+1}}\right)$$

Highest order term 2^{2n}

$$\Rightarrow \theta(4^n)$$

(b)

$(x^2+6)^8$

Highest order term $= (x^2)^8$

$= x^{16}$

$$\theta(x^{16})$$

Que 12:-

$T_A = x^2$

$T_B = x+2$

Breaking point:-

$$x^2 = x+2$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2$$

Date

Que 5:-

a)

Avg. Case (θ)
Performs avg. no. of steps per input data of n elements. Averaged over all possible inputs.

Ex:- $\theta(n \log n)$

Worst case (O)
performs max. no. of steps on input data of n elements. Input is arbitrary and output is upper bound.

Ex:- $O(n^2)$

(b)

Worst case (O)
It is arbitrary and O/P is upper bound.

Asymptotically Bounded (θ)
Growth of a running time to within certain factors below.

Que 8:-

$$f(n) = n^2$$

$$1+2+3+\cdots+(n-1)+n = \frac{n(n+1)}{2}$$

whenever n doubles, the running time inc(↑) forefold.
Quad. algo. are for retativly smaller problems.