Diagonalization

Definition 1

We call matrix A similar to matrix B if there is an invertible matrix P such that $B = P^{-1}AP$

Diagonalization

If $B = P^{-1}AP$, then we see

$$B^n = \underbrace{P^{-1}APP^{-1}AP \cdots P^{-1}AP}_{n-\text{many}} = P^{-1}A^nP$$

Therefore, for any polynomial f, we have

$$f(B) = f(P^{-1}AP) = P^{-1}f(A)P$$

Diagonalization

Definition 2

A matrix A is called diagonalizable if it is similar to a diagonal matrix Λ

$$P^{-1}AP = \begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{pmatrix}$$