

Note: Preview of slides from (additionOfLinearTransformations.tex) by Qirui Li (<https://orcid.org/0000-0002-6042-1291>). For educational and non-commercial use only. Any unlawful use will be prosecuted.

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Transformation addition and matrix addition

Given linear transformation from V to W , we can construct new linear transformation by taking linear combinations of those linear transformation. This is corresponding to linear combination of matrices.

Definition 1

For two linear transformation $T : V \longrightarrow W$ and $S : V \longrightarrow W$, we can define their linear combination for any scalar $\lambda, \mu \in F$ by

$$\lambda T + \mu S : V \longrightarrow W, \vec{v} \mapsto \lambda T(\vec{v}) + \mu S(\vec{v})$$

Transformation addition and matrix addition

Proposition 1 Distribution law

Let V, W, U be vector spaces, then for the following linear transformation and arbitrary scalar $\lambda, \mu \in F$

$$V \xrightarrow[S]{T} W \xrightarrow{R} U$$

We have

$$R \circ (\lambda S + \mu T) = \lambda(R \circ S) + \mu(R \circ T);$$

For

$$U \xrightarrow{R} V \xrightarrow[S]{T} W$$

We have

$$(\lambda S + \mu T) \circ R = \lambda(S \circ R) + \mu(T \circ R);$$

Transformation addition and matrix addition

We leave the above proof as an Exercise since it is straightforward.

Transformation addition and matrix addition

Proposition 2

Let V, W be vector spaces,

\mathcal{E} basis for V \mathcal{F} basis for W

For a pair of linear transformation

$$V \begin{smallmatrix} T \\ S \end{smallmatrix} W$$

Suppose A is the matrix for T under bases \mathcal{E}, \mathcal{F} and B is the matrix for S under bases \mathcal{E}, \mathcal{F} . Then for any scalar $\mu, \lambda \in F$, the matrix for $\lambda T + \mu S$ under bases \mathcal{E}, \mathcal{F} is given by

$$\lambda A + \mu B.$$

Transformation addition and matrix addition

Proof: By what given, we have

$$T \left(\underbrace{\vec{e}_1 \quad \vec{e}_2 \quad \cdots \quad \vec{e}_n}_{\mathcal{E}} \right) = \left(\underbrace{\vec{w}_1 \quad \vec{w}_2 \quad \cdots \quad \vec{w}_m}_{\mathcal{F}} \right) A$$

$$S \left(\underbrace{\vec{e}_1 \quad \vec{e}_2 \quad \cdots \quad \vec{e}_n}_{\mathcal{E}} \right) = \left(\underbrace{\vec{w}_1 \quad \vec{w}_2 \quad \cdots \quad \vec{w}_m}_{\mathcal{F}} \right) B$$

So

$$\lambda T \left(\underbrace{\vec{e}_1 \quad \vec{e}_2 \quad \cdots \quad \vec{e}_n}_{\mathcal{E}} \right) = \left(\underbrace{\vec{w}_1 \quad \vec{w}_2 \quad \cdots \quad \vec{w}_m}_{\mathcal{F}} \right) \lambda A$$

$$\mu S \left(\underbrace{\vec{e}_1 \quad \vec{e}_2 \quad \cdots \quad \vec{e}_n}_{\mathcal{E}} \right) = \left(\underbrace{\vec{w}_1 \quad \vec{w}_2 \quad \cdots \quad \vec{w}_m}_{\mathcal{F}} \right) \mu B$$

Then

$$(\lambda T + \mu S) \left(\underbrace{\vec{e}_1 \quad \vec{e}_2 \quad \cdots \quad \vec{e}_n}_{\mathcal{E}} \right) = \left(\underbrace{\vec{w}_1 \quad \vec{w}_2 \quad \cdots \quad \vec{w}_m}_{\mathcal{F}} \right) (\lambda A + \mu B)$$