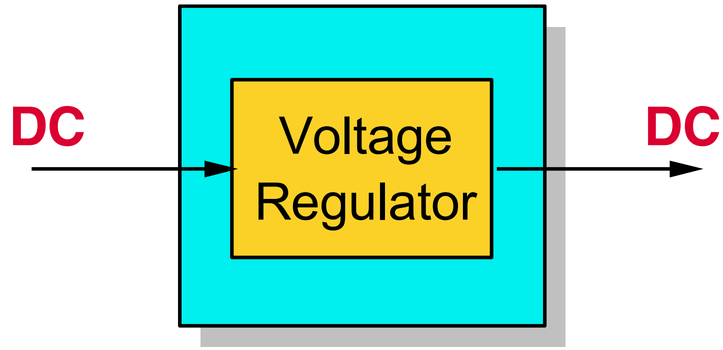
電源管理晶片設計與實作

Power Management IC Design and Implementation

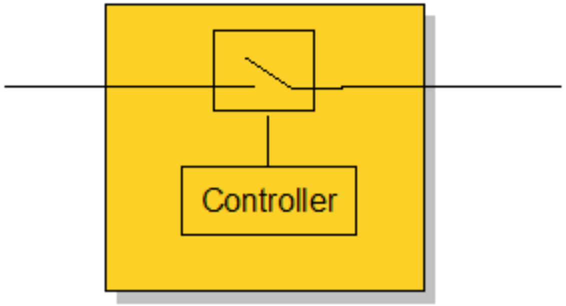
陳柏宏(Po-Hung Chen）副教授

Email : hakko@nctu.edu.tw
Phone : 035712121#54203

Voltage Regulators

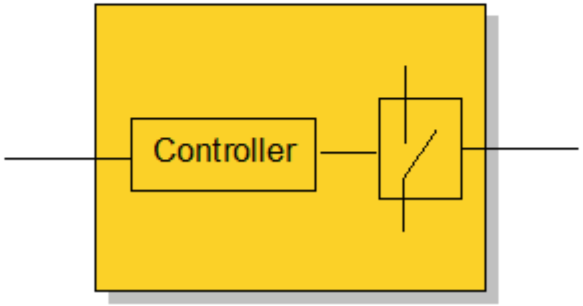


- Convert a DC Voltage to other Voltages
- Provide a "Clean" output Voltage



Linear Regulator

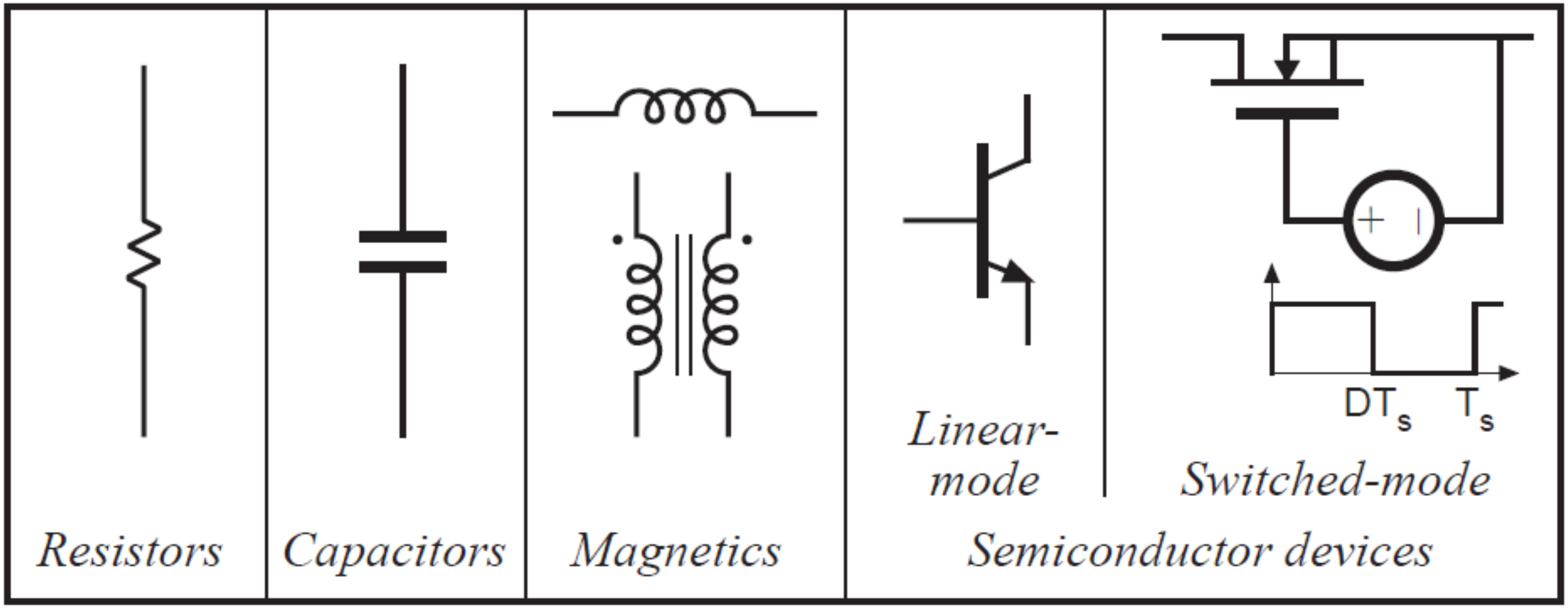
- Not very efficient at higher currents
- Low noise



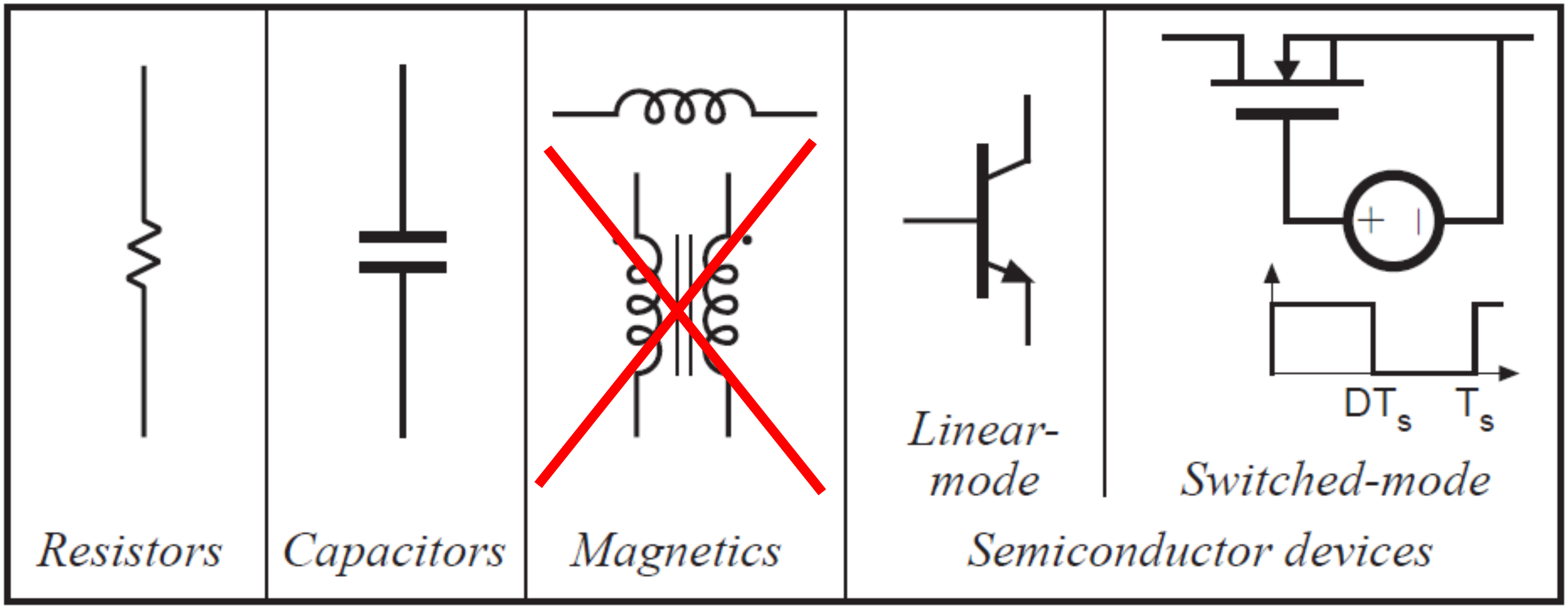
Switching Regulator

- Very efficient
- Switching noise

Devices Available to the Circuit Designer

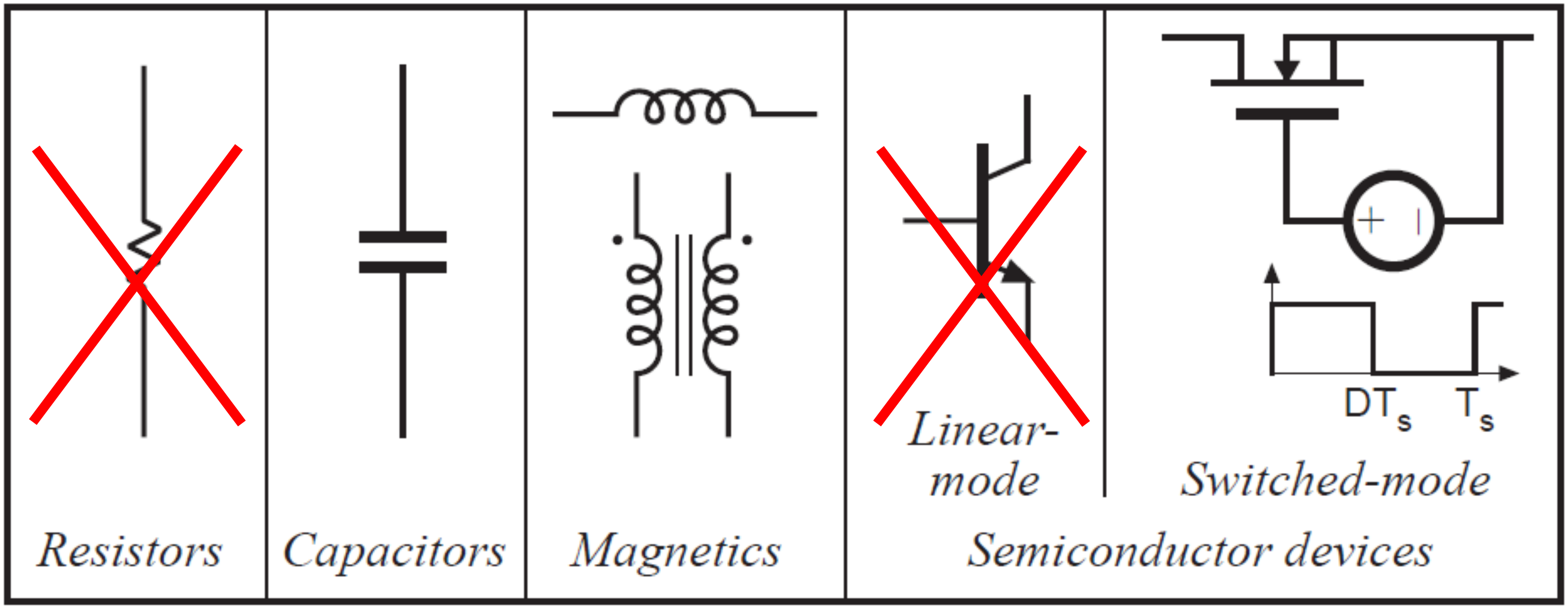


Devices Available to the Circuit Designer



Signal processing : avoid magnetics
→ Size is a problem

Devices Available to the Circuit Designer



Power processing : avoid lossy (resistive) elements
→ For high efficiency

Capacitors and inductors are important elements of switching converters because they ideally do not consume power.

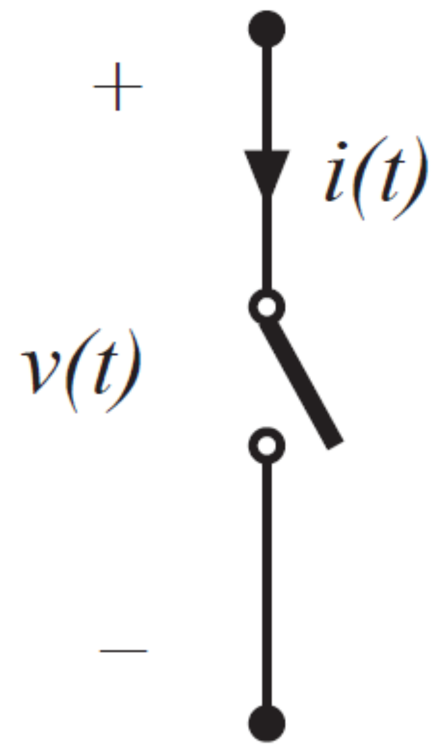
Power Loss in an Ideal Switch

Switch closed: $v(t) = 0$

Switch open: $i(t) = 0$

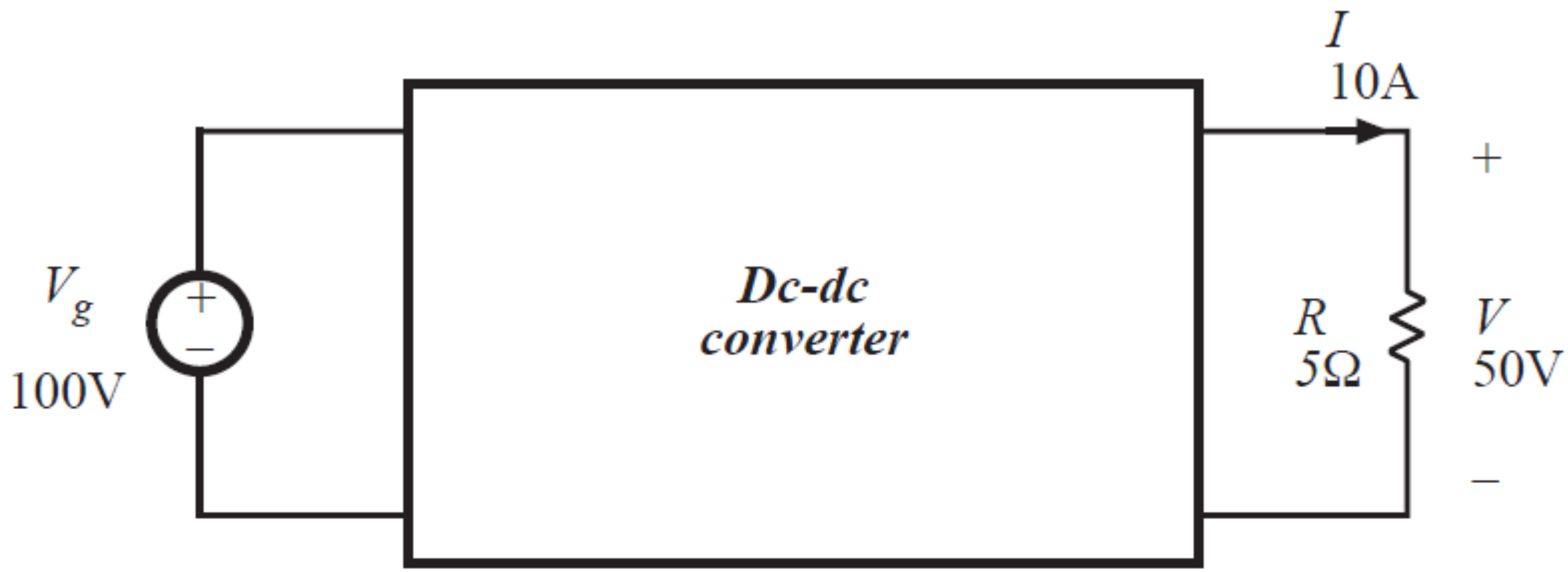
In either event: $p(t) = v(t)\, i(t) = 0$

Ideal switch consumes zero power



Capacitive and inductive elements, as well as switched-mode semiconductor devices are available for synthesis of high efficiency converters.

An Simple DC-DC Converter Example



Input source: 100V

Output load: 50V, 10A, 500W

How can this converter be realized?

General DC-DC Converter Topologies

(1)低電壓降電壓調整器

- ◆ Low-Dropout Regulator (LDO)

(2)切換電容式電源轉換電路

- ◆ Switched-Capacitor Power Converter

(3)切換式電源轉換電路

(電感)

- ◆ Switching-Mode Power Converter
- ◆ Buck Converter (Step Down)
- ◆ Boost Converter (Step UP)

Power Conversion Efficiency

- **Conversion Efficiency:**
  $\eta = P_{OUT}/P_{IN} = (V_{OUT}I_{OUT})/(V_{IN}I_{IN})$

- **Power Loss:**
  (1) Fixed loss
  (2) Conduction loss
  (3) Switching loss

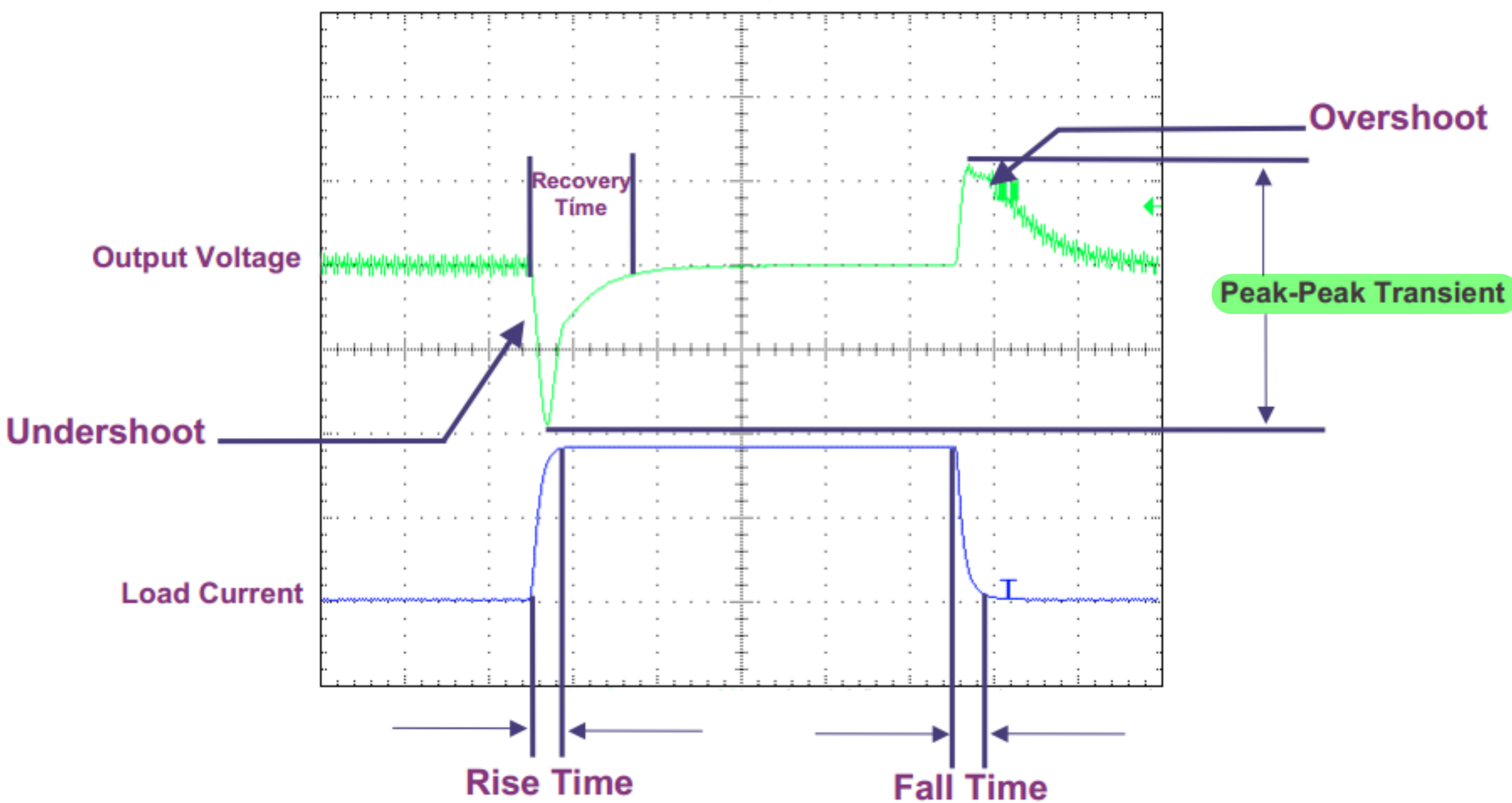
- **Fixed loss is mainly caused from the controller circuits which is independent to load current and switching frequency**

Specification in Power Converter

- Voltage and Current Rated
- Output Ripple
- Line Regulation input變化時的電壓穩壓狀況
  (Input)
- Load Regulation Load變化時的電壓穩壓狀況
- Transient Response
- Power Supply Rejection

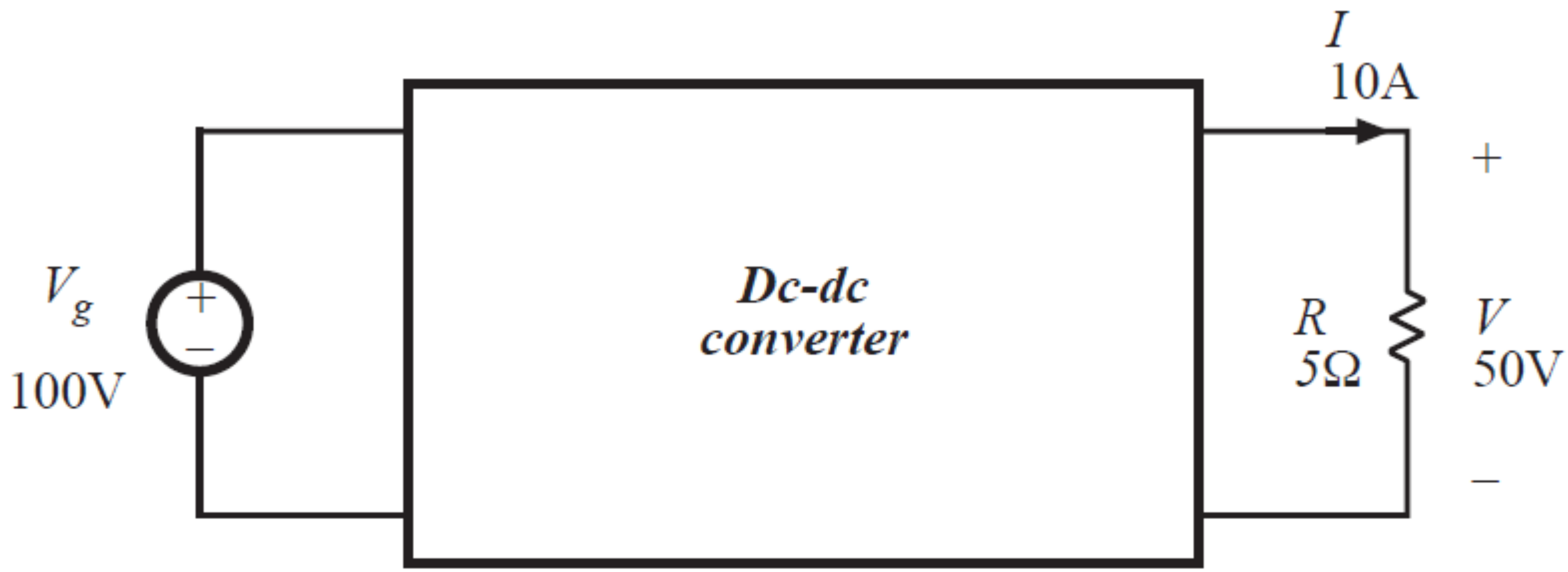
Transient Response

➤ **Example of transient response in DC-DC converter**



VICOR– Power Supply Design Consideration for High di/dt Loads

An Simple DC-DC Converter Example



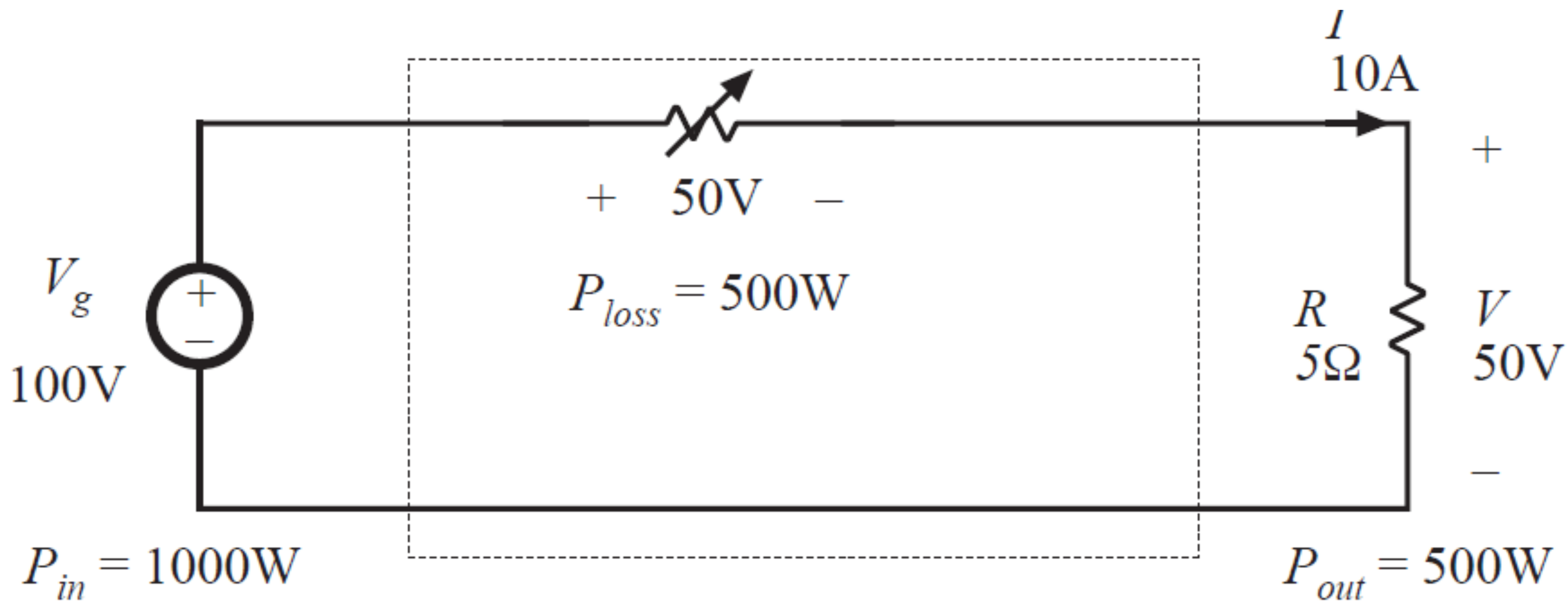
Input source: 100V

Output load: 50V, 10A, 500W

How can this converter be realized?

Dissipative Realization

- **Resistive voltage divider**

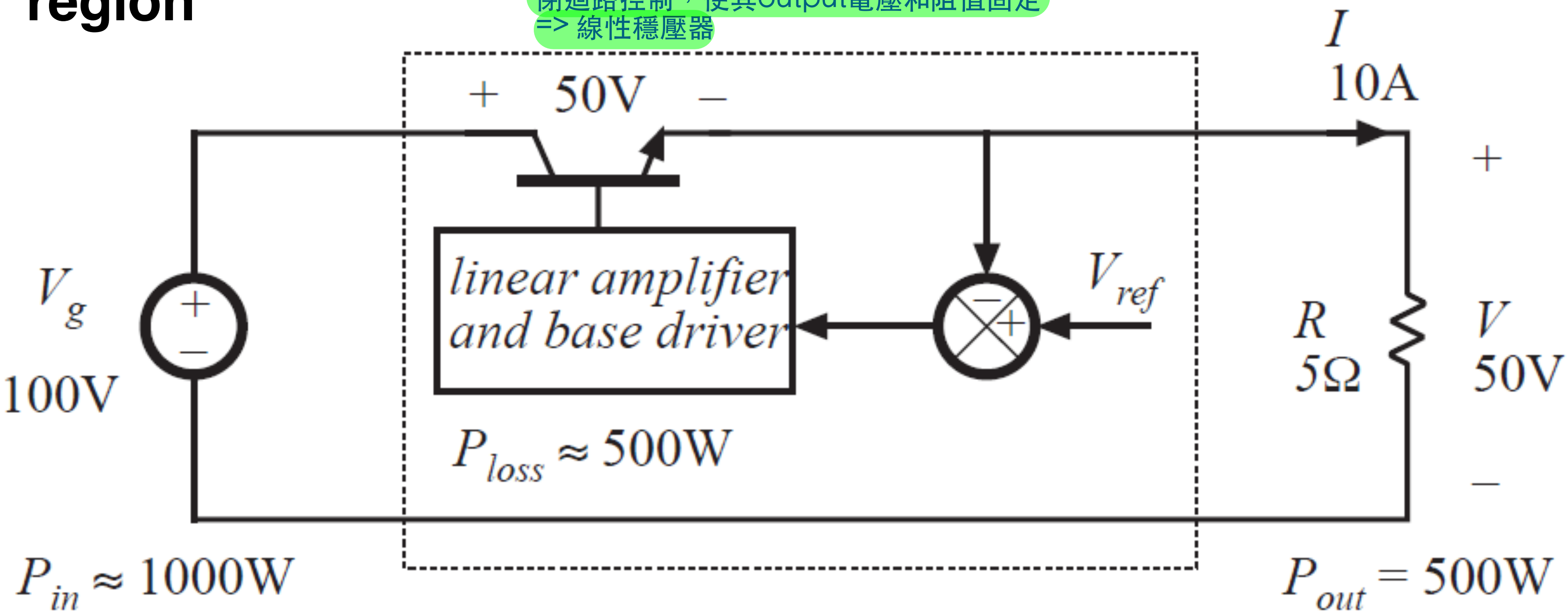


Efficiency : $$\eta = \frac{P_{OUT}}{P_{IN}} = \frac{V_{OUT} I_{OUT}}{V_{IN} I_{IN}} = \frac{V_{OUT}}{V_{IN}} = 50\%$$

Dissipative Realization

- Series pass regulator : transistor operates in active region

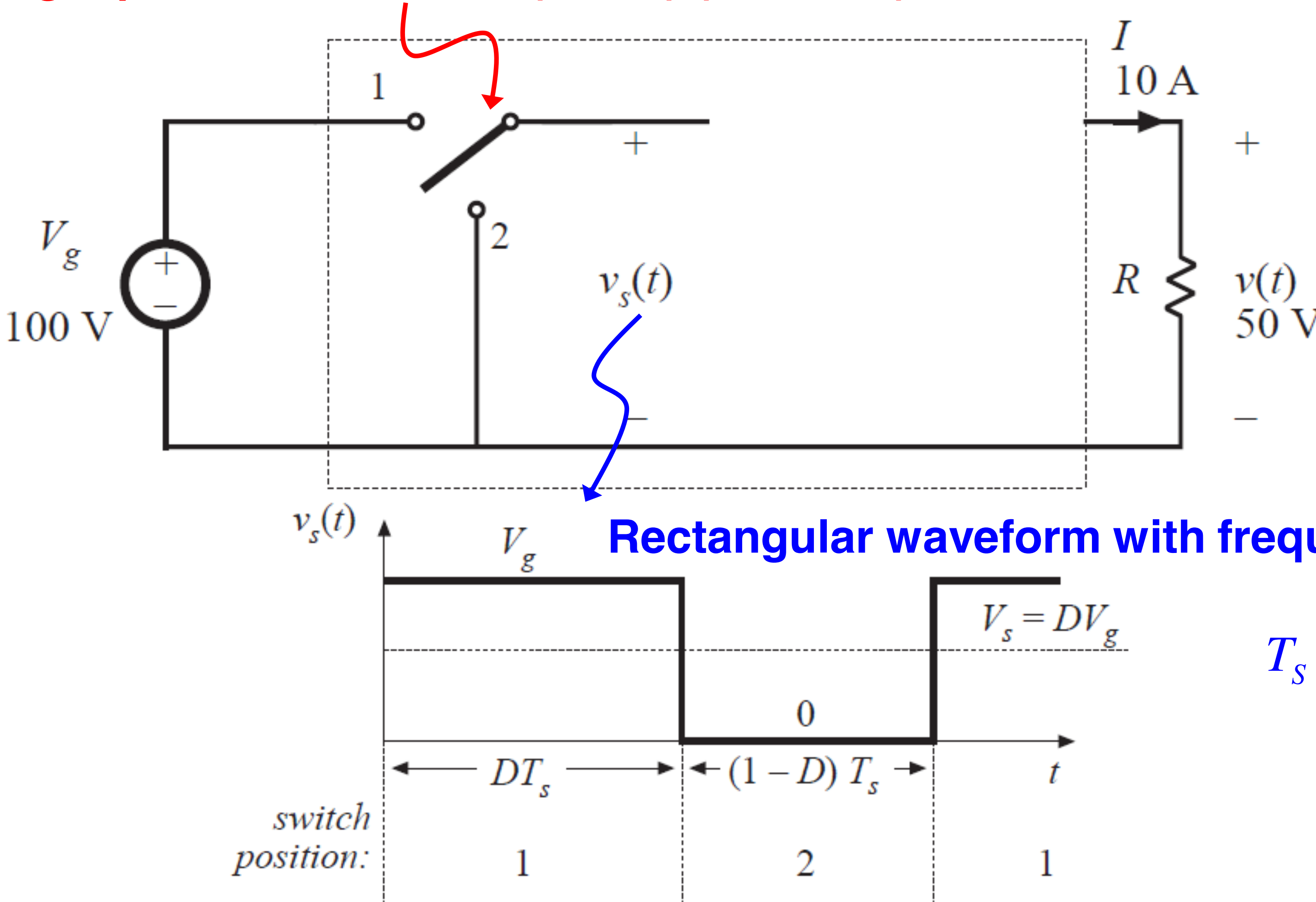
閉迴路控制，使其output電壓和阻值固定
=> 線性穩壓器



Series-pass linear regulators generally find modern application only at low power levels of few watts.

Use of a SPDT Switch

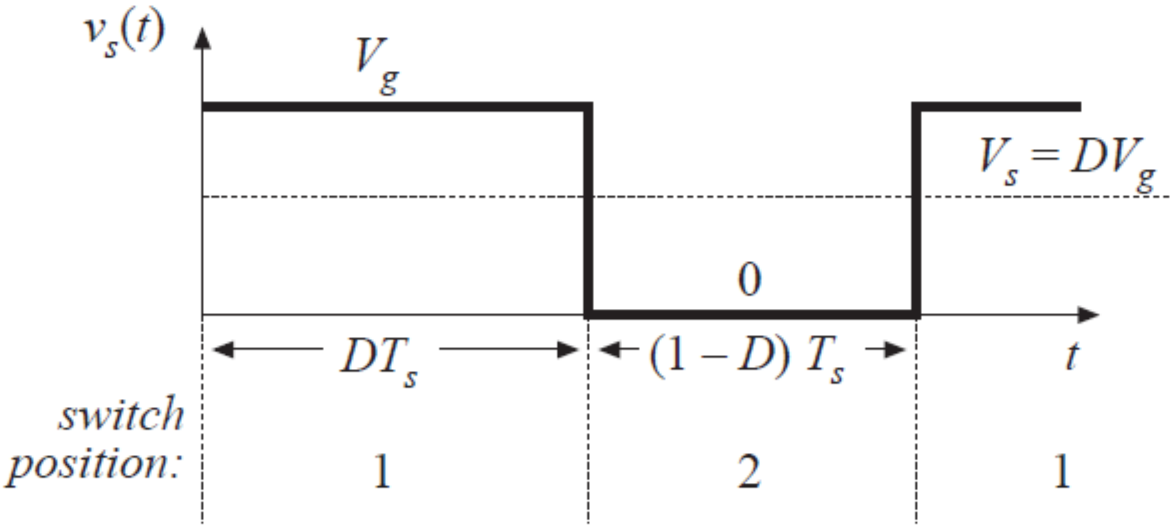
Single-pole double-throw (SPDT) (單極雙投).



Rectangular waveform with frequency f_S

$$T_S = \frac{1}{f_S}$$

The Switch Changes the DC Voltage Level



D = switch duty cycle
$0 \le D \le 1$

T_s = switching period

f_s = switching frequency
$= 1 / T_s$

From Fourier analysis :

DC component of $v_s(t)$ = average value:

$$V_s = \frac{1}{T_s} \int_0^{T_s} v_s(t)\, dt = DV_g$$

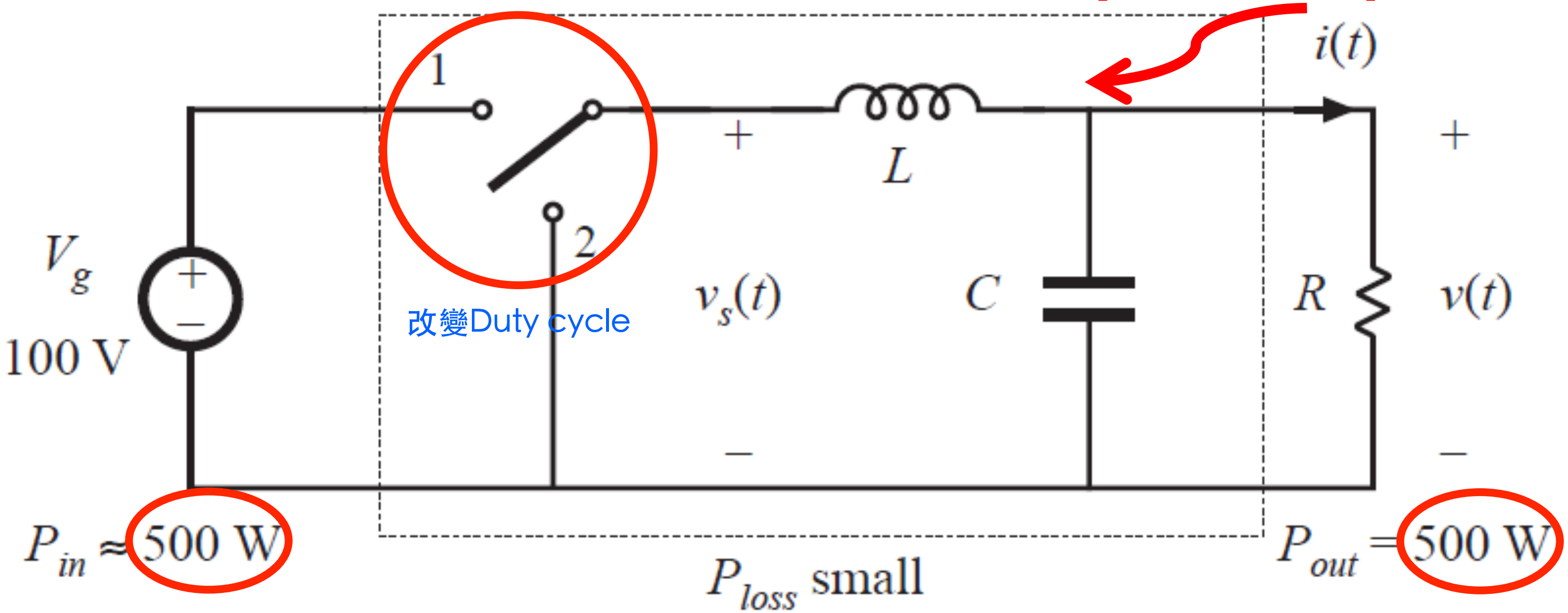
The switch changes the average voltage by a factor of D.

Addition of Low Pass Filter

取DC值

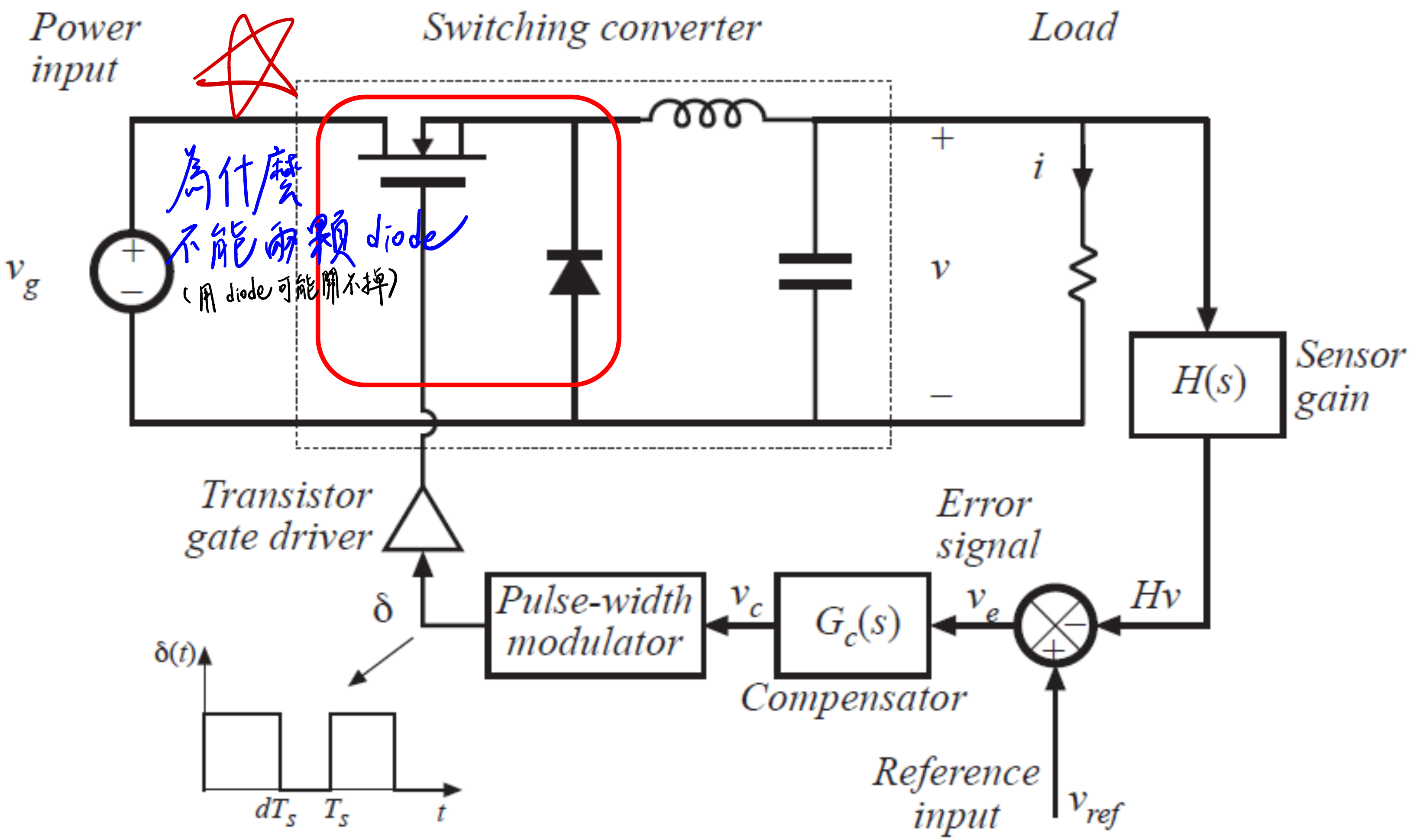
◆ **Addition of (ideally lossless) L-C low-pass filter, for removal of switching harmonics:**

2-pole low pass filter



- Choose filter cutoff frequency f_0 much smaller than switching frequency f_s → the filter only passes the dc component of v_S.
- This circuit is known as the "buck converter" **Ideally η=100%**

Control System for Output Voltage Regulation

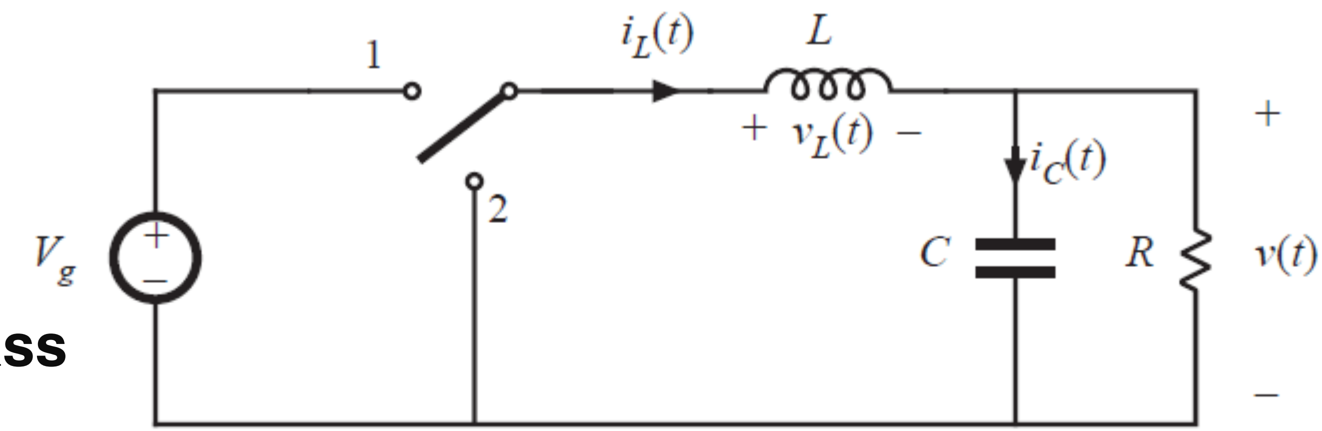


→ **Varies the duty cycle to cause the output voltage to follow a given reference (V_{ref}).**

Steady-State Converter Analysis

Output Waveform of Buck Converter

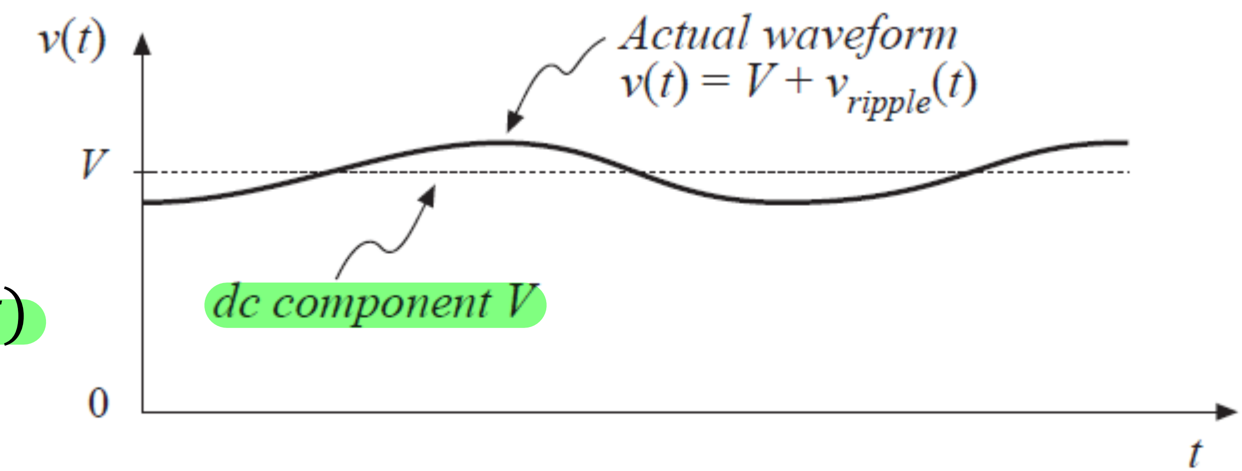
- Buck converter containing practical low-pass filter



- Actual output voltage waveform

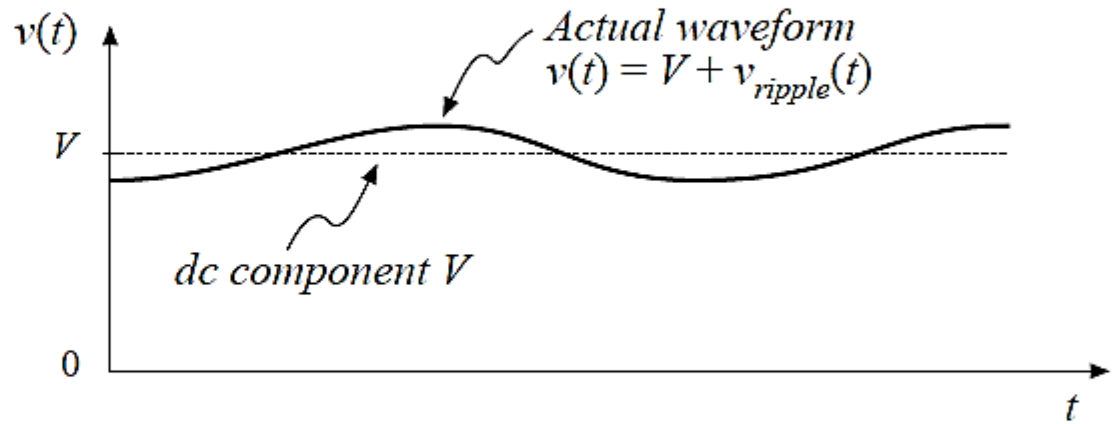
小擺幅

$$v(t) = V + v_{ripple}(t)$$



Small Ripple Approximation

$$v(t) = V + v_{ripple}(t)$$



- In a well-designed converter, the output voltage ripple is small. (<1%) Hence, the waveform can be easily determined by ignoring the ripple:

$$\left\| v_{ripple}(t) \right\| \ll V$$

$$v(t) \approx V$$

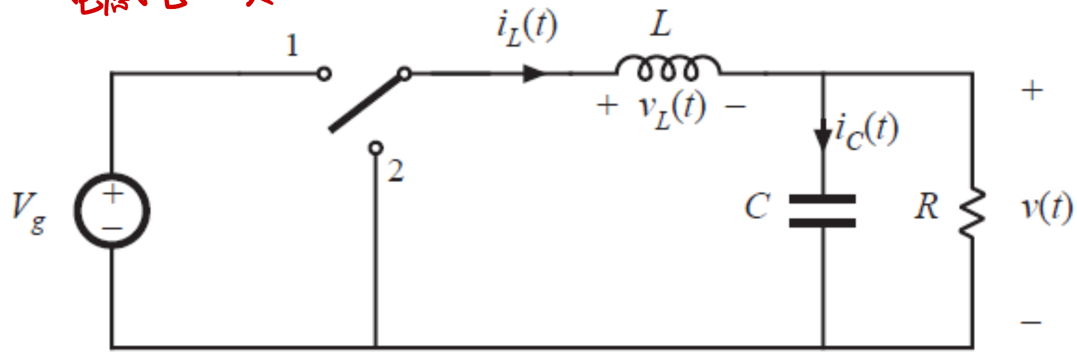
Small ripple approximation

Buck Converter Analysis

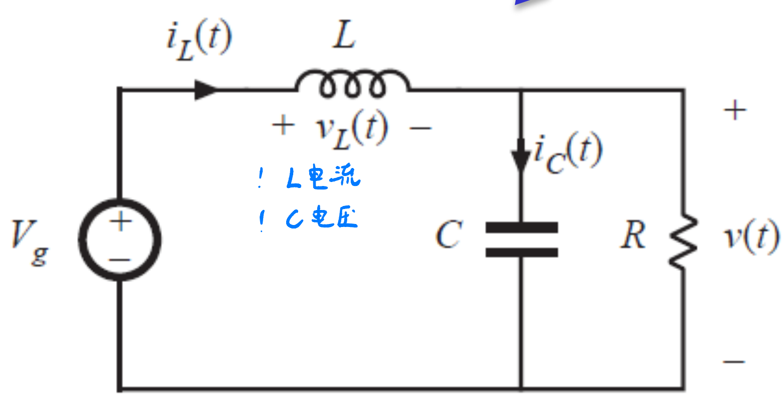
- **Analyze inductor current waveform by integrating inductor voltage**

電感電流變化

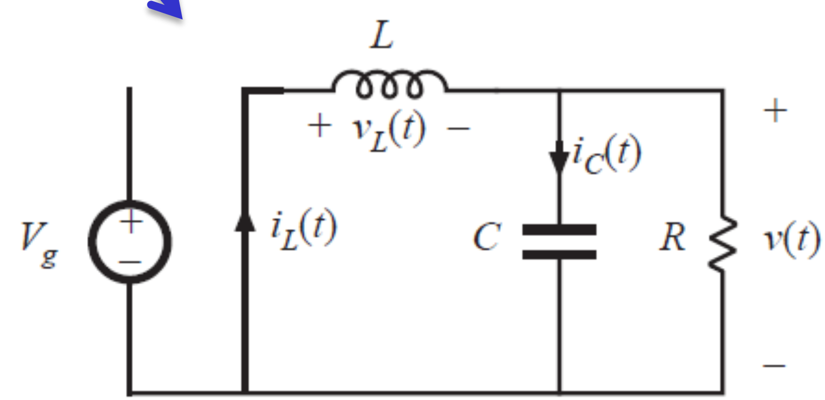
Original converter



Switch in position 1



Switch in position 2



Inductor Voltage and Current

Subinterval 1 : switch in position 1

$V_L(t) = V_g - V(t)$

$V_L(t) = L \cdot \frac{di}{dt}$, $\frac{di}{dt} = \frac{V_g - V}{L}$

- Inductor voltage

$$v_L = V_g - v(t)$$

- Small ripple approximation

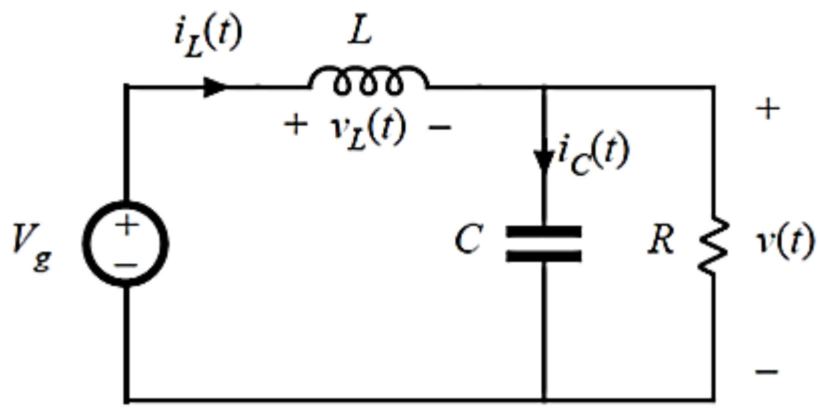
$$v_L \approx V_g - V$$

- Knowing the inductor voltage, we can now find the inductor current via

$$v_L(t) = L\frac{di_L(t)}{dt}$$

- Solve for the slope:

$$\frac{di_L(t)}{dt} = \frac{v_L(t)}{L} \approx \frac{V_g - V}{L}$$



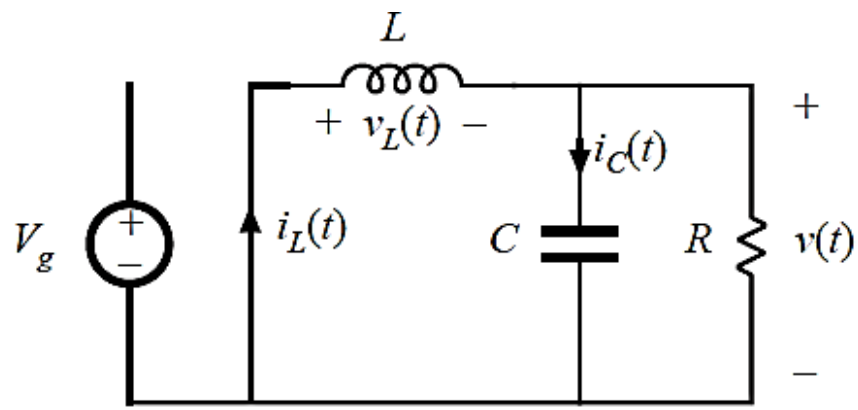
The inductor current changes with an essentially constant slope !

Inductor Voltage and Current

- **Subinterval 2 : switch in position 2**

$V_L(t) = -v(t) \approx -V$

$\frac{di(t)}{dt} = \frac{V_L(t)}{L} = \frac{-V}{L}$



- Inductor voltage

$$v_L(t) = -v(t)$$

- Small ripple approximation

$$v_L(t) \approx -V$$

- Knowing the inductor voltage, we can again find the inductor current via

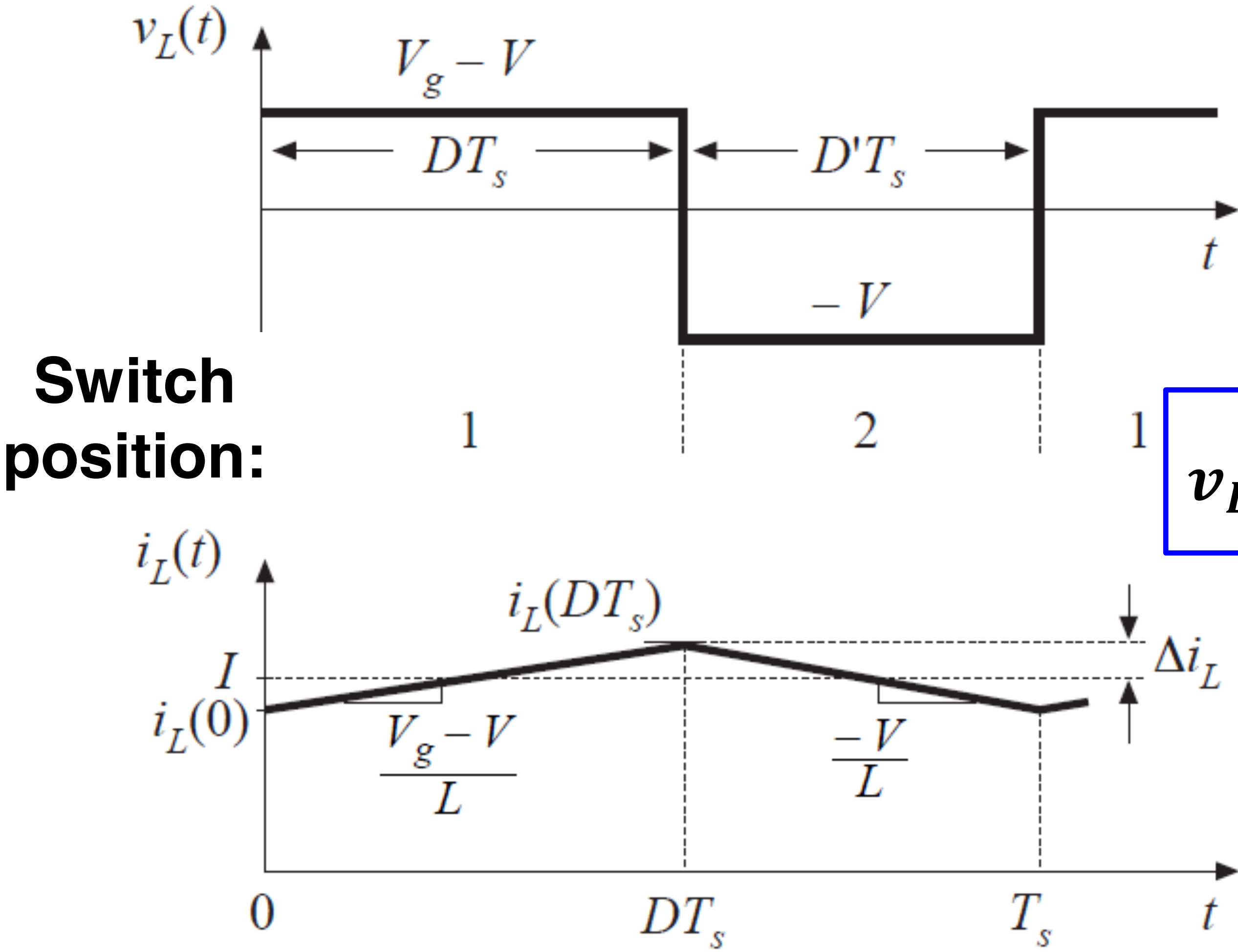
$$v_L(t) = L\frac{di_L(t)}{dt}$$

The inductor current changes with an essentially constant slope !

- Solve for the slope:

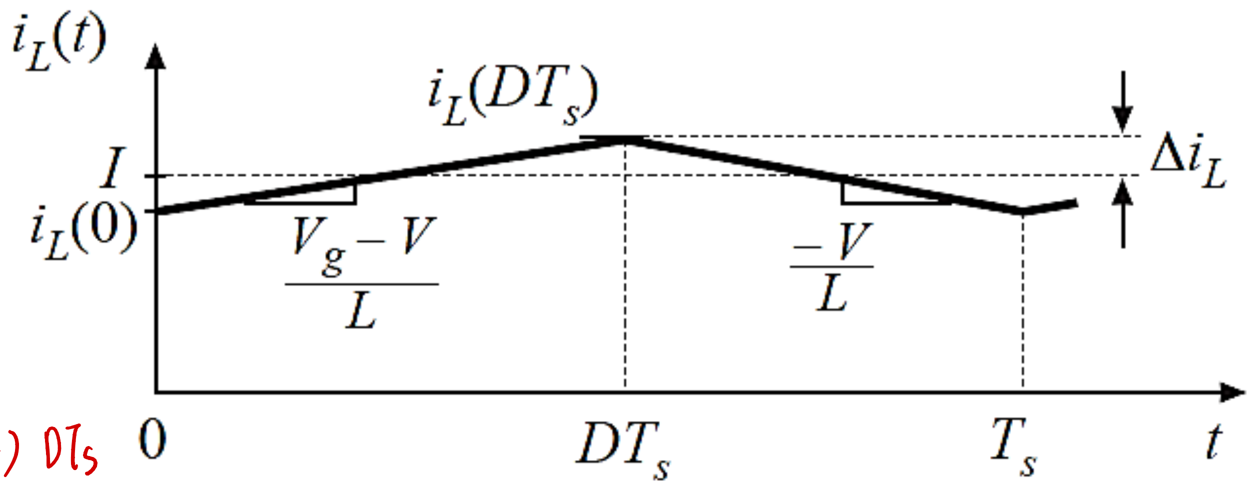
$$\frac{di_L(t)}{dt} \approx -\frac{V}{L}$$

Inductor Voltage and Current Waveforms



$$\boxed{v_L(t) = L\frac{di_L(t)}{dt}}$$

Ripple Magnitude of Inductor Current



$2 \cdot \Delta i_L = \left(\frac{V_g - V}{L}\right) DT_s$

$L = \frac{V_g - V}{2 \cdot \Delta i_L} DT_s$

$$(\text{change in } i_L) = (\text{slope})(\text{length of subinterval})$$

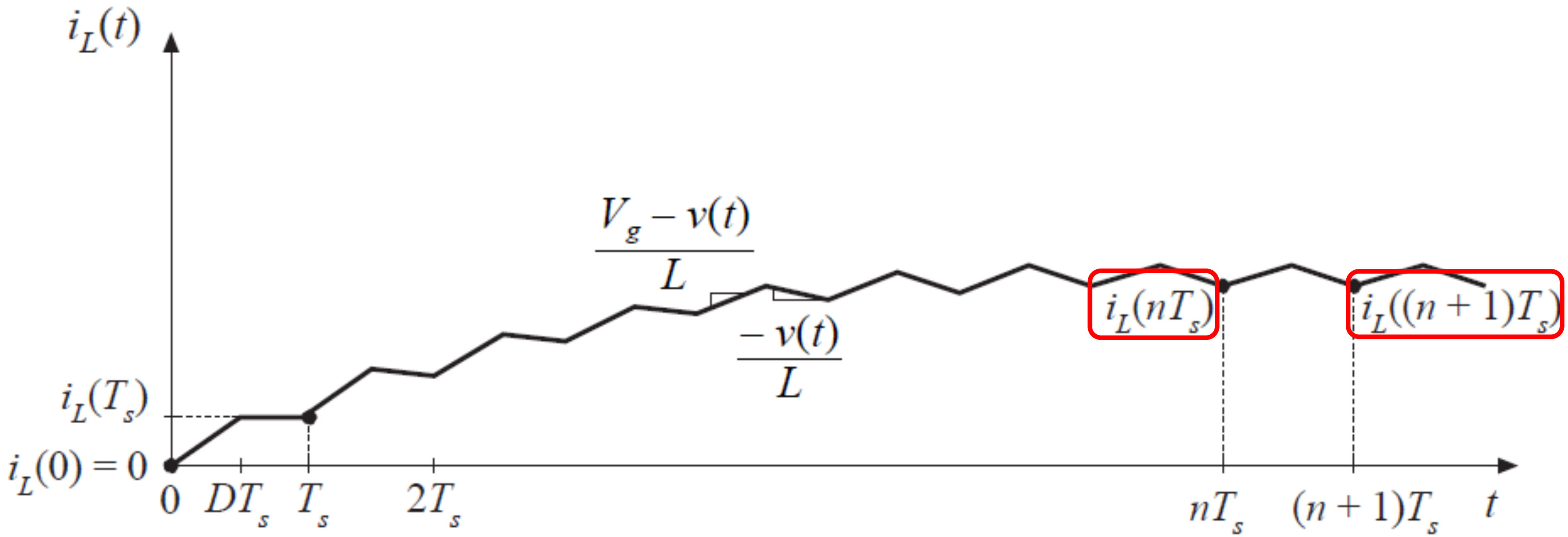
$$(2\Delta i_L) = \left(\frac{V_g - V}{L}\right)(DT_s)$$

$$\rightarrow (\Delta i_L) = \left(\frac{V_g - V}{2L}\right)(DT_s) \qquad \boxed{(L) = \left(\frac{V_g - V}{2\Delta i_L}\right)(DT_s)}$$

Select the value of inductance

Inductor Current Waveform

- During turn-on transient



- **When the converter operates in equilibrium**
  → $i_L((n+1)TS) = iL(nT_S)$

- **In steady state, net change in i_L over one switching period =0**

Inductor Voltage-Second Balance

Derivation

- Inductor defining relation:

$$v_L(t) = L\frac{di_L(t)}{dt}$$

- Integrate over one complete switching period:

$$i_L(T_s) - i_L(0) = \frac{1}{L}\int_0^{T_s} v_L(t)dt$$

- In periodic steady state, the net change in inductor current is zero:

$$0 = \int_0^{T_s} v_L(t)dt$$

- Hence, the total area (or volt-seconds) under the inductor voltage waveform is zero whenever the converter operates in steady state.

An equivalent form:

V_L在一个period内的average = 0

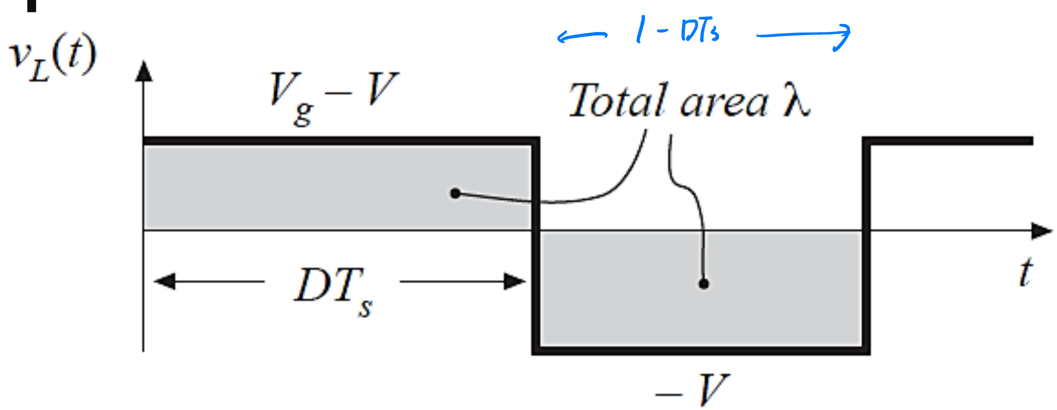
$$\boxed{0 = \frac{1}{T_s}\int_0^{T_s} v_L(t)dt = \langle v_L \rangle}$$

Inductor voltage second balance

→ **The average inductor voltage is zero in steady state.**

Inductor Voltage-Second Balance

- **Buck converter example**



- Inductor voltage waveform, previously derived:

- Integral of voltage waveform is area of rectangles:

$$\lambda = \int_0^{T_s} v_L(t)dt = \left(V_g - V\right)(DT_s) + (-V)(D'T_s)$$

$$(V_g - V)D\cancel{T_s} = V \cdot \cancel{T_s}(1-D)$$

$$V = V_g \cdot D$$

- Average voltage is:

$$\langle v_L \rangle = \frac{\lambda}{T_s} = D\left(V_g - V\right) + D'(-V)$$

- Equate to zero and solve for V:

$$0 = DV_g - (D + D')V = DV_g - V \quad \rightarrow \quad V = DV_g$$

Capacitor Charge Balance

Derivation

- **Capacitor defining relation:**

$$i_C(t) = C\frac{dv_C(t)}{dt}$$

i · t = Charge

- **Integrate over one complete switching period:**

$$v_C(T_s) - v_C(0) = \frac{1}{C}\int_0^{T_s} i_C(t)dt$$

Charge (Q)

- **In periodic steady state, the net change in capacitor voltage is zero:**

$$\boxed{0 = \frac{1}{T_s}\int_0^{T_s} i_C(t)dt = \langle i_C \rangle}$$

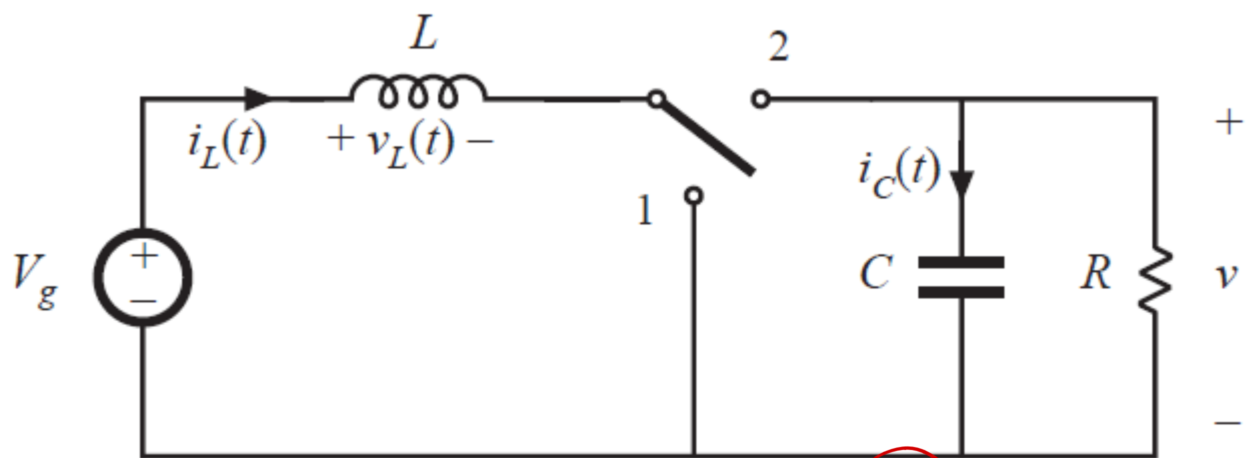
Amper second balance

Capacitor charge balance

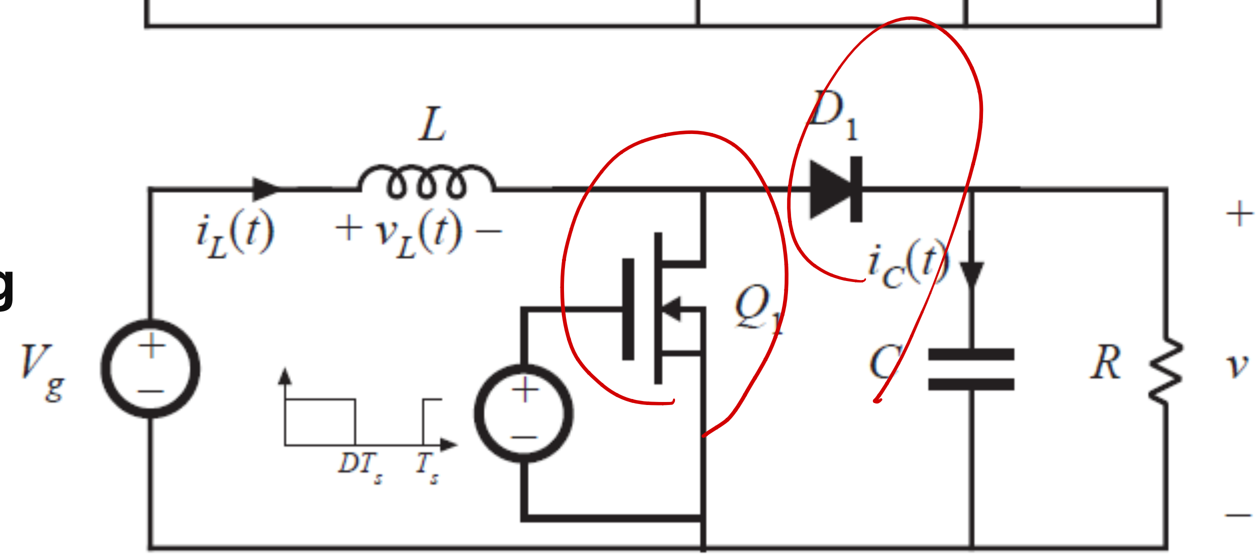
- **Hence, the total area (or charge) under the capacitor current waveform is zero whenever the converter operates in steady state. The average capacitor current is then zero.**

Boost Converter Example

- Boost converter with ideal switch

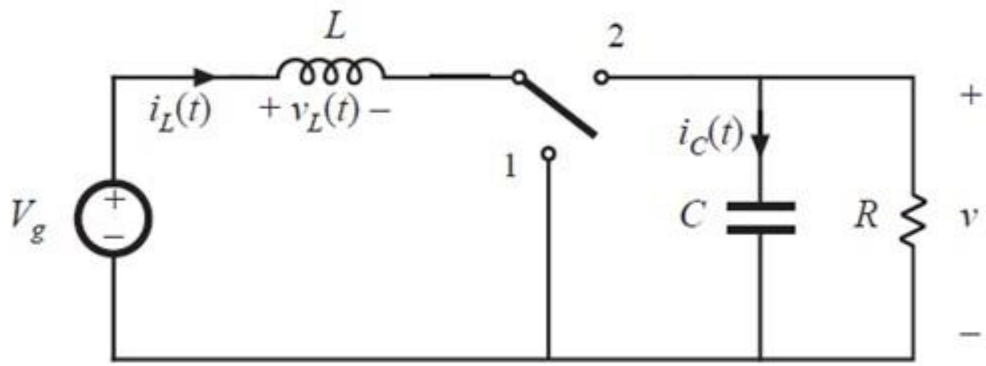


- Realization using power MOSFET and diode



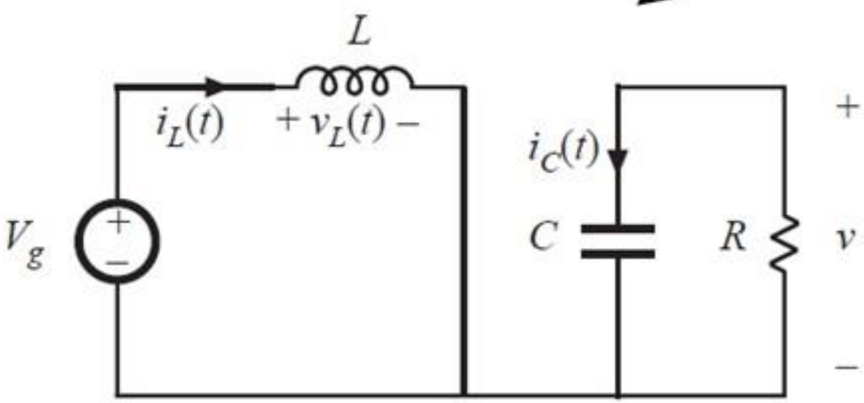
Boost Converter Analysis

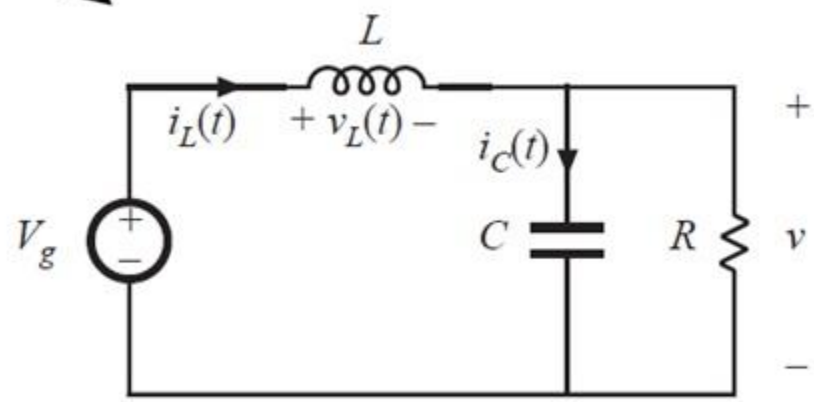
Original converter



Switch in position 1

Switch in position 2





Subinterval 1: Switch in Position 1

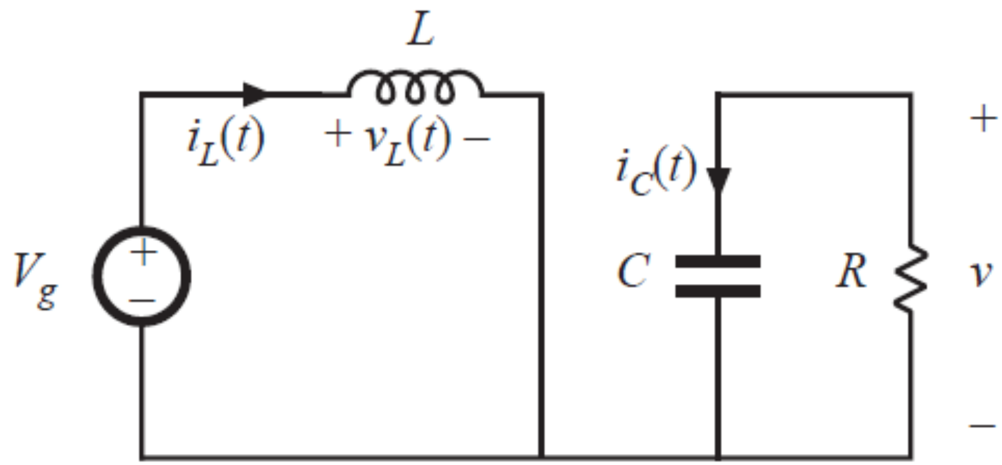
- Inductor voltage and capacitor current

$$v_L = V_g$$
$$i_C = -v/R$$

$V_L = V_g$

- Small ripple approximation

$$v_L = V_g$$
$$i_C = -V/R$$



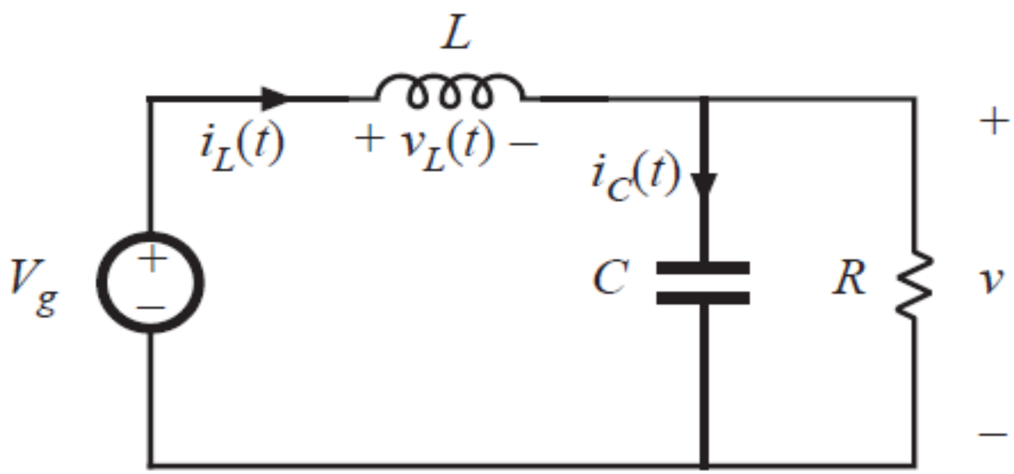
Subinterval 2: Switch in Position 2

- Inductor voltage and capacitor current

$$\boxed{\begin{aligned} v_L &= V_g - v \\ i_C &= i_L - v/R \end{aligned}}$$

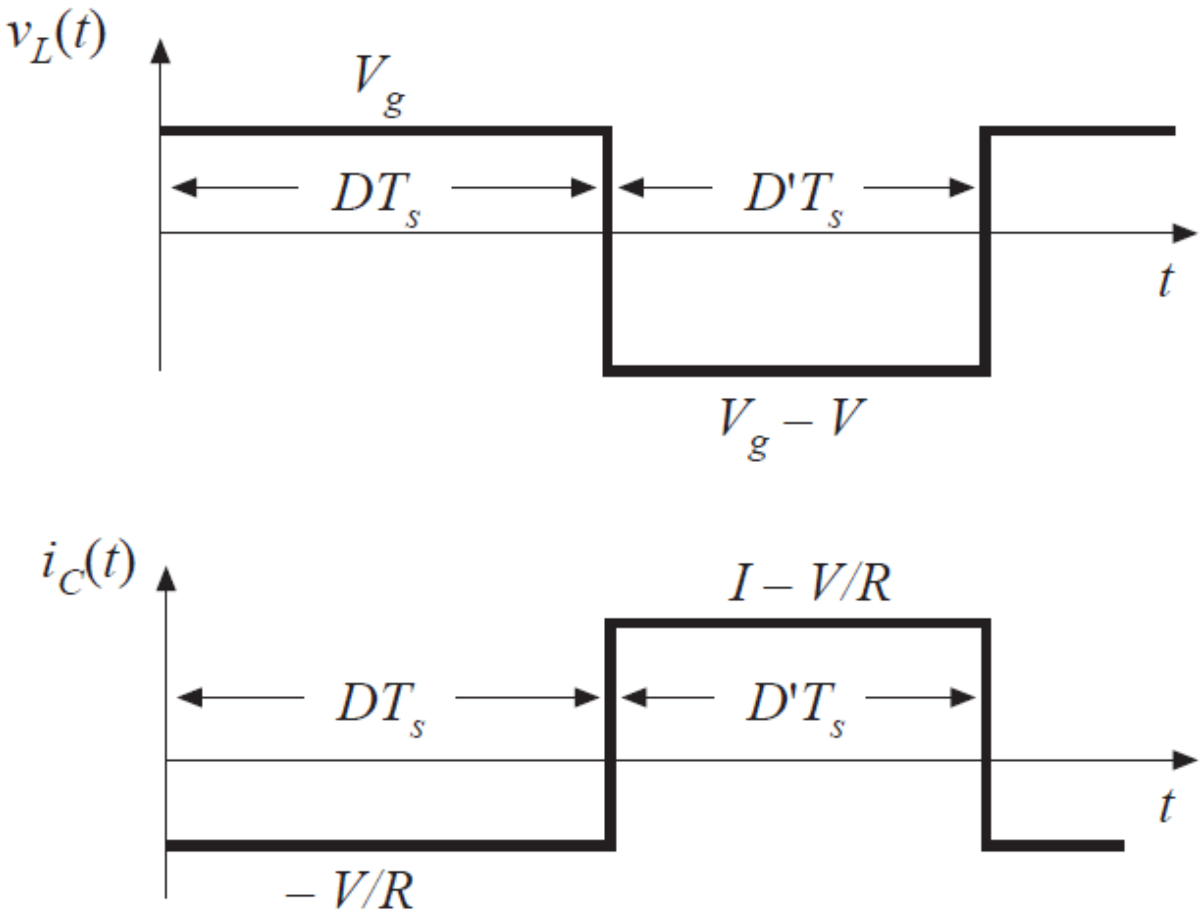
- Small ripple approximation

$$\boxed{\begin{aligned} v_L &= V_g - V \\ i_C &= i_L - V/R \end{aligned}}$$



Inductor Voltage and Capacitor Current

- **Waveforms**



Inductor Voltage-Second Balance

$$\int_0^{T_s} v_L(t) \cdot dt = 0$$

$$V_g \cdot D\cancel{T_s} + (V_g - V)(1-D)\cancel{T_s}$$

- Net volt-seconds applied to inductor over one switching period:

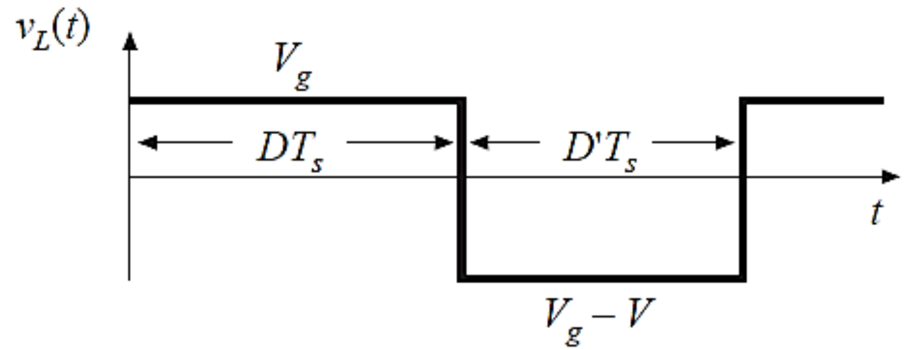
$$\int_0^{T_S} v_L(t)dt = (V_g)\,DT_S + \left(V_g - V\right)D'T_S$$

- Equate to zero and collect terms:

$$V_g(D + D') - VD' = 0$$

- Solve for V:

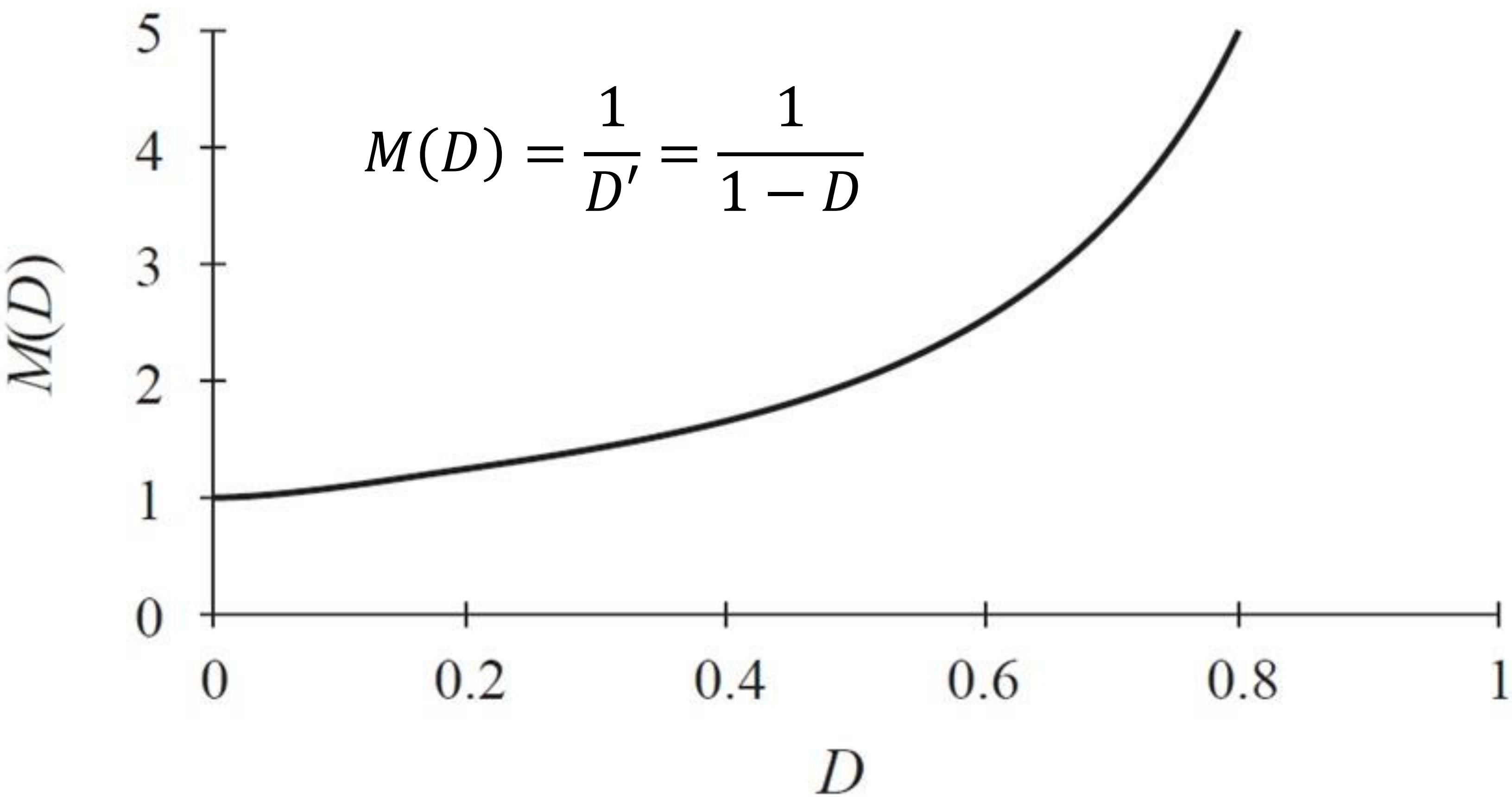
$$V = \frac{V_g}{D'}$$



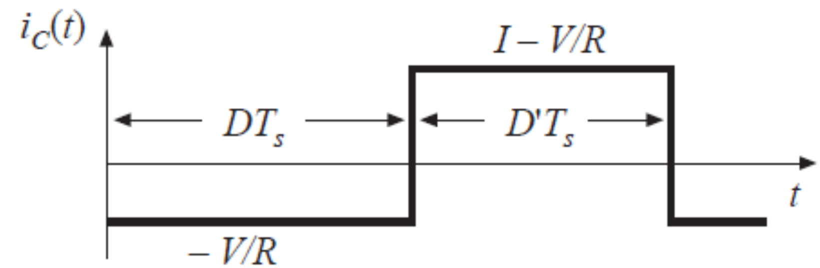
- The voltage conversion ratio is therefore

$$M(D) = \frac{V}{V_g} = \frac{1}{D'} = \frac{1}{1-D}$$

Conversion Ratio M(D) of Boost Converter



Inductor Current DC Component



- Capacitor charge balance:

$$\int_0^{T_S} i_C(t)dt = \left(-\frac{V}{R}\right) DT_S + \left(I - \frac{V}{R}\right) D'T_S$$

- Collect terms and equate to zero:

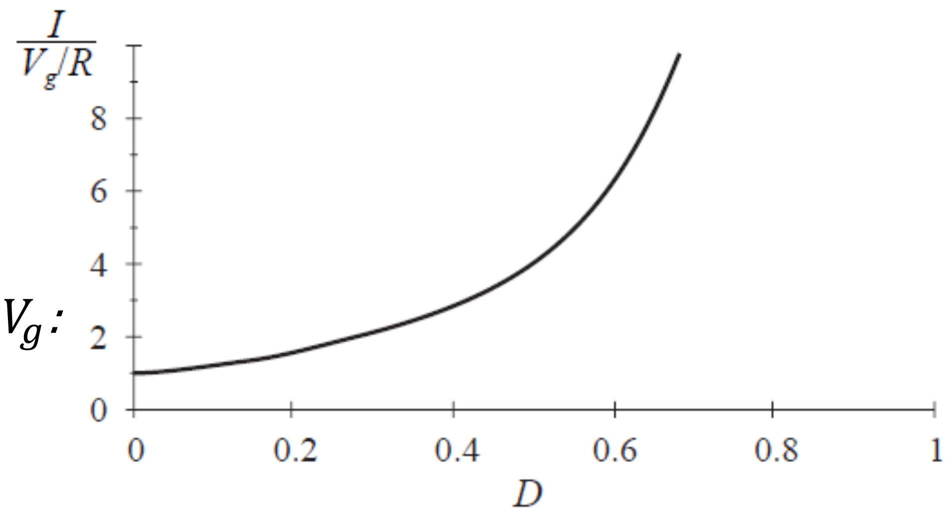
$$-\frac{V}{R}(D + D') + ID' = 0$$

- Solve for I:

$$I = \frac{V}{D'R}$$

- Eliminate V to express in terms of V_g:

$$I = \frac{V_g}{D'^2 R}$$



- **As D~1, *I* becomes very large and component non-idealities lead to large power loss.**

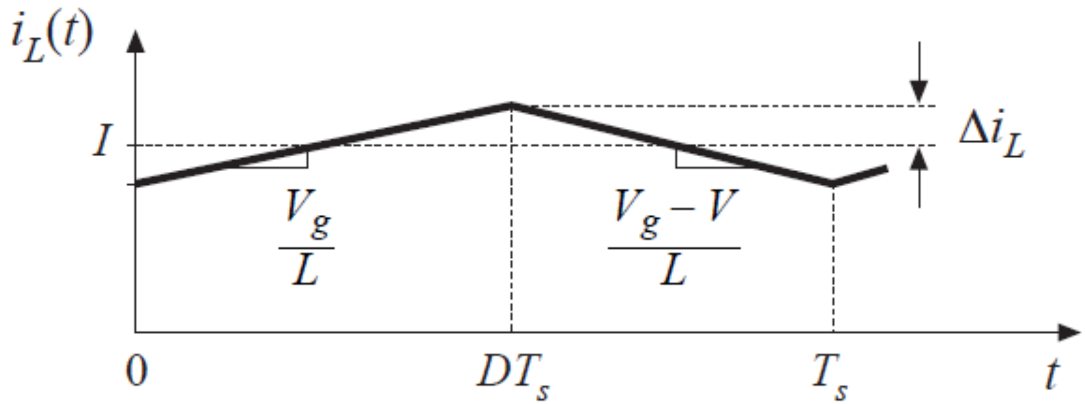
Inductor Current Ripple

- Inductor current slope during subinterval 1:

$$\frac{di_L(t)}{dt} = \frac{v_L(t)}{L} = \frac{V_g}{L}$$

- Inductor current slope during subinterval 2:

$$\frac{di_L(t)}{dt} = \frac{v_L(t)}{L} = \frac{V_g - V}{L}$$



- Change in inductor current during subinterval 1 is (*slope*)(*length of subinterval*):

$$2\Delta i_L = \frac{V_g}{L} DT_s$$

- Solve for peak ripple:

$$\boxed{\Delta i_L = \frac{V_g}{2L} DT_s}$$

- Choose *L* such that desired ripple magnitude is obtained

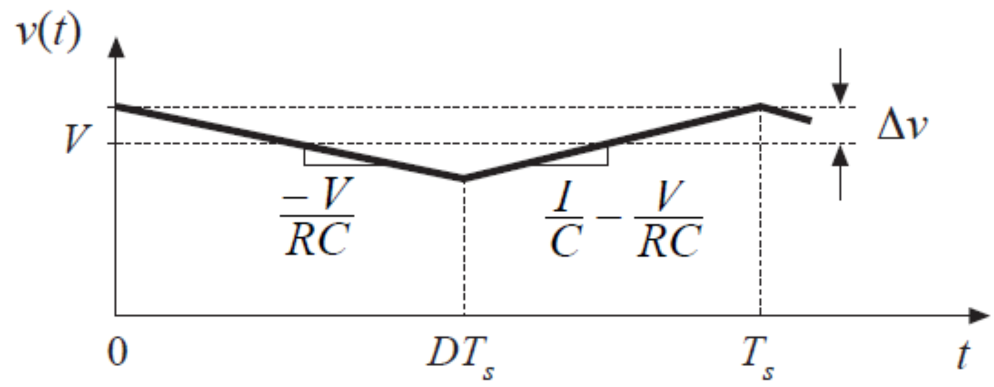
Capacitor Voltage Ripple

- Capacitor voltage slope during subinterval 1:

$$\frac{dv_C(t)}{dt} = \frac{i_C(t)}{C} = \frac{-V}{RC}$$

- Capacitor voltage slope during subinterval 2:

$$\frac{dv_C(t)}{dt} = \frac{i_C(t)}{C} = \frac{I}{C} - \frac{V}{RC}$$



- Change in capacitor voltage during subinterval 1 is (*slope*)(*length of subinterval*):

$$-2\Delta v = \frac{-V}{RC} DT_s$$

- Solve for peak ripple:

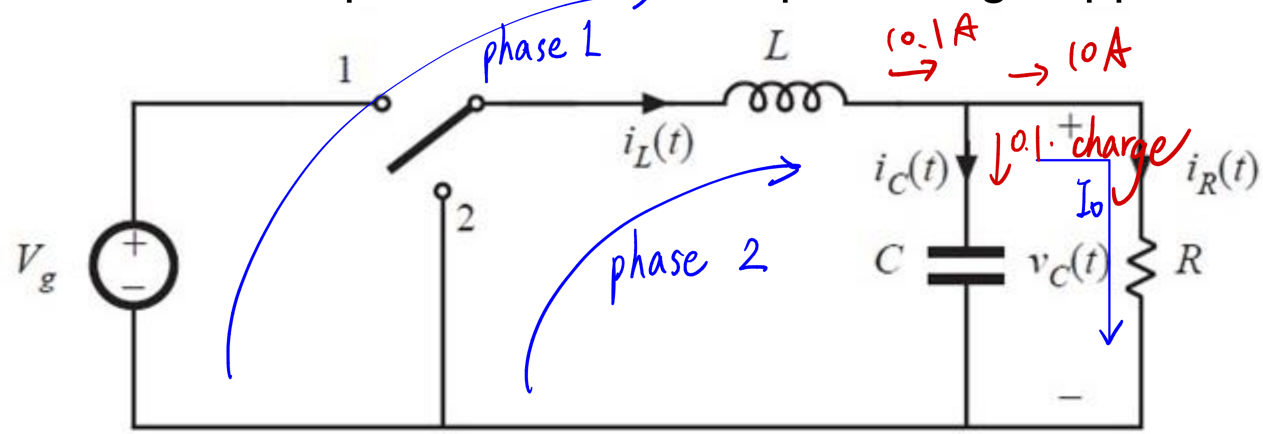
$$\boxed{\Delta v = \frac{V}{2RC} DT_s}$$

- Choose C such that desired voltage ripple magnitude is obtained
- In practice, capacitor *equivalent series resistance* (esr) leads to increased voltage ripple

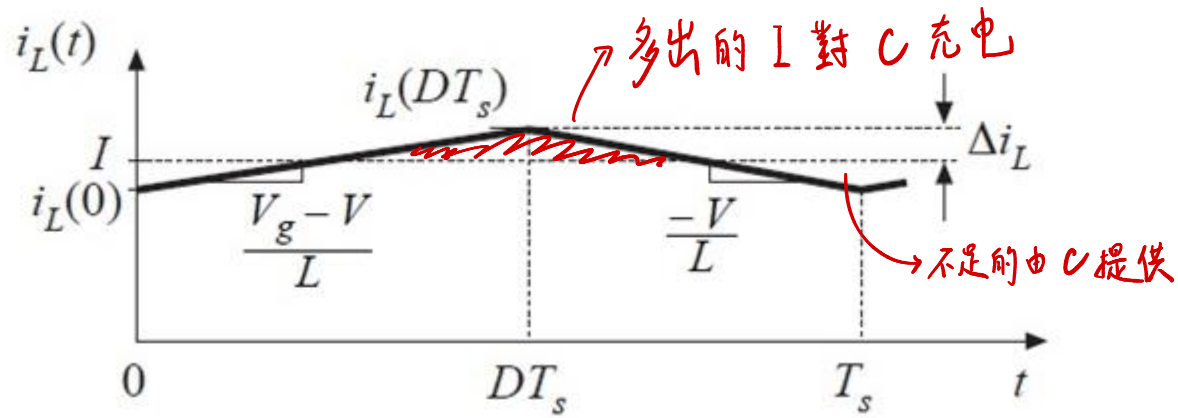
Estimating Ripple in Converters

- **Containing two-pole low-pass filters** steady state

  Buck converter example: Determine output voltage ripple



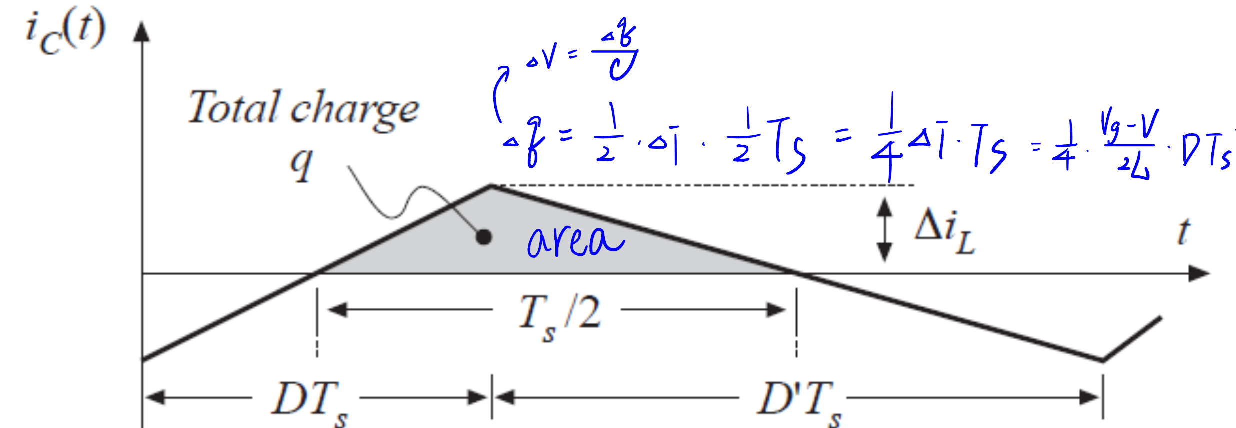
Inductor current waveform.
What is the capacitor current?



Capacitor Current and Voltage

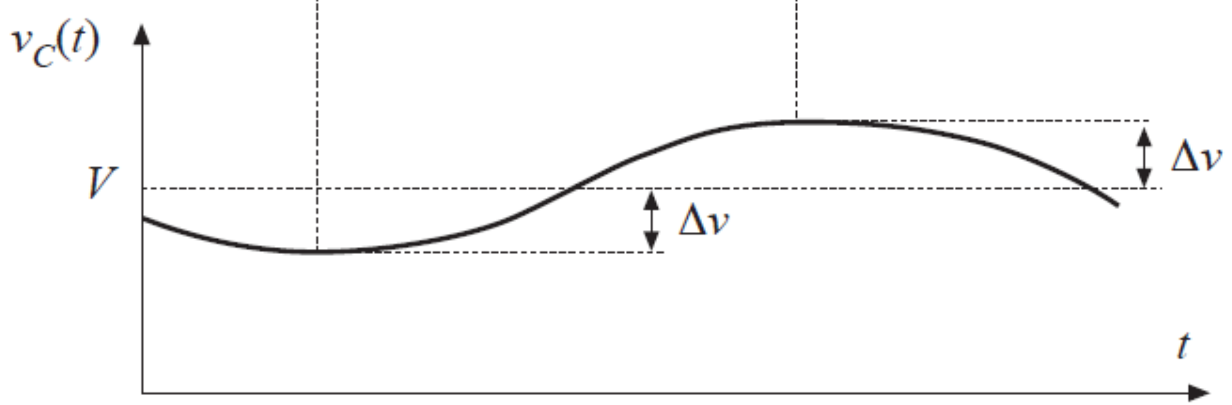
- **Buck example**

Must not neglect inductor current ripple!

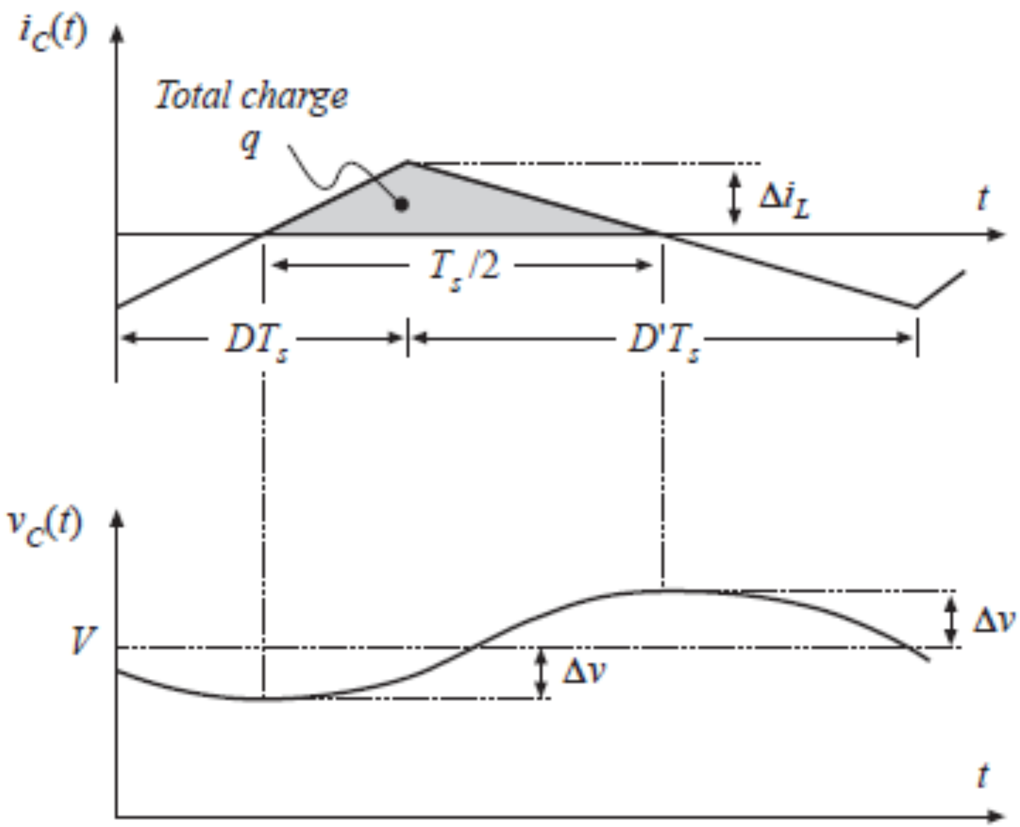


→ Output capacitor ripple arises from the inductor current ripple.

If the capacitor voltage ripple is small, then essentially all of the ac component of inductor current flows through the capacitor.



Estimating Capacitor Voltage Ripple Δv

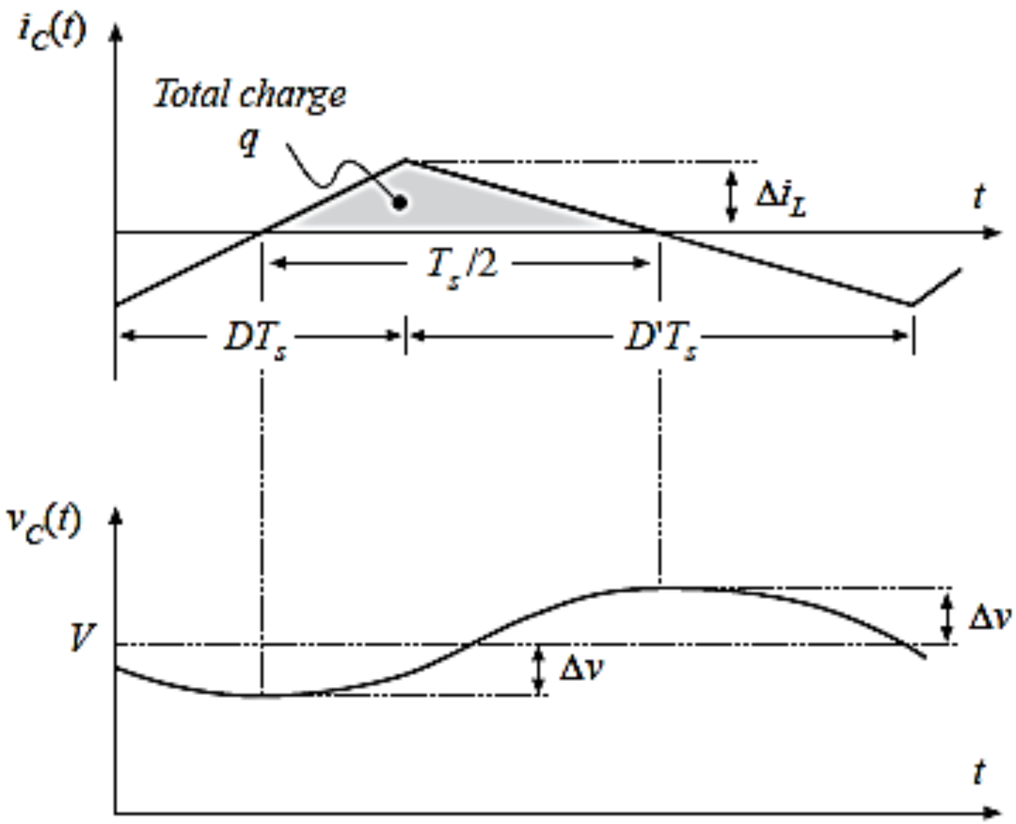


Current $\boldsymbol{i_C(t)}$ is positive for half of the switching period. This positive current causes the capacitor voltage $\boldsymbol{v_C(t)}$ to increase between its minimum and maximum extrema. During this time, the total charge $\boldsymbol{q}$ is deposited on the capacitor plates, where

$$q = C(2\Delta v)$$

$$(change\ in\ charge) = C(change\ in\ voltage)$$

Estimating Capacitor Voltage Ripple Δv



- The total charge q is the area of the triangle, as shown:

$$q = \frac{1}{2}\Delta i_L \frac{T_s}{2}$$

- Eliminate q and solve for Δv:

$$\boxed{\Delta v = \frac{\Delta i_L T_s}{8C}}$$

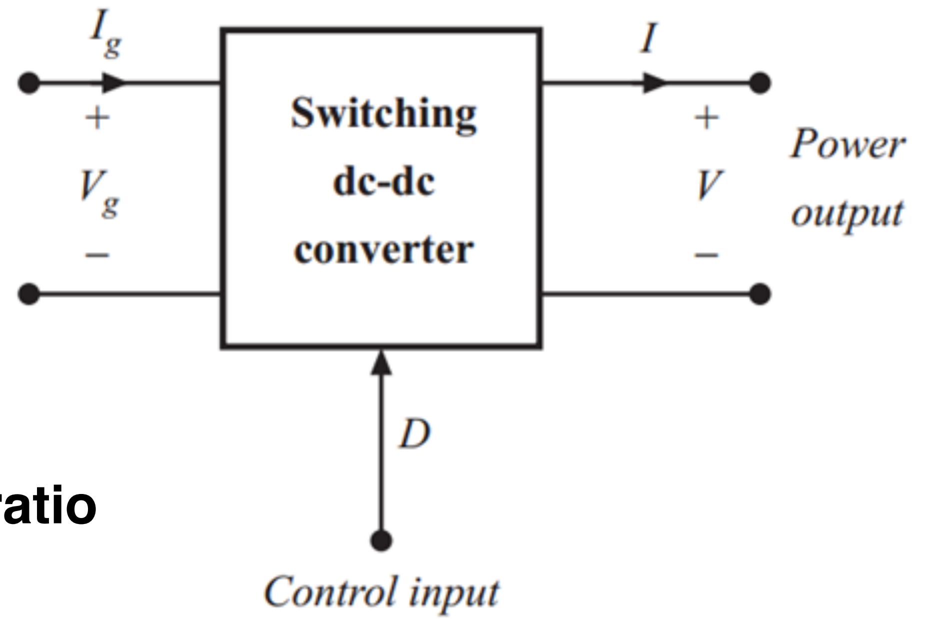
- Note:

In practice, capacitor equivalent series resistance (esr) further increases Δv.

Equivalent Circuit Modeling

The DC Transformer Model

- Basic equations of an ideal dc-dc converter:



$$P_{in} = P_{out} \quad (\eta = 100\%)$$
$$V_g I_g = VI$$

$$V = M(D)V_g$$
$$I_g = M(D)I$$

Ideal conversion ratio

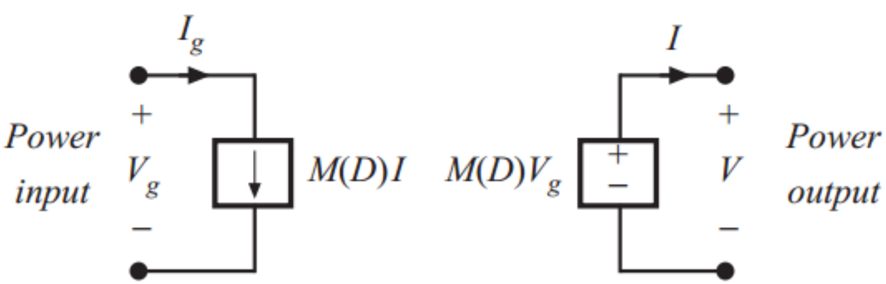
These equations are valid in steady-state.
During transients, energy storage within filter elements may cause $P_{in} \neq P_{out}$.

Equivalent Circuits in Ideal DC-DC Converter

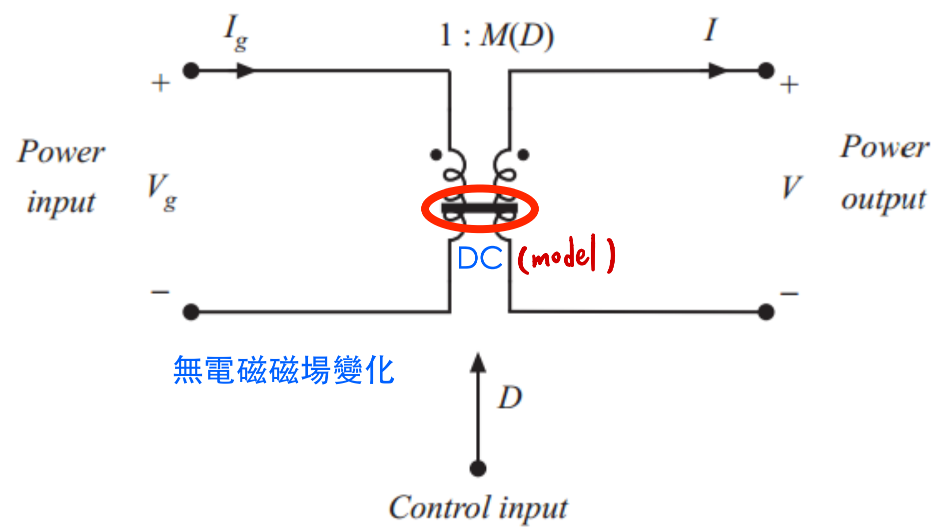
$\boxed{P_{in} = P_{out}}$ $\boxed{V_g I_g = VI}$ $\boxed{V = M(D)V_g}$ $\boxed{I_g = M(D)I}$

modeling

Dependent sources

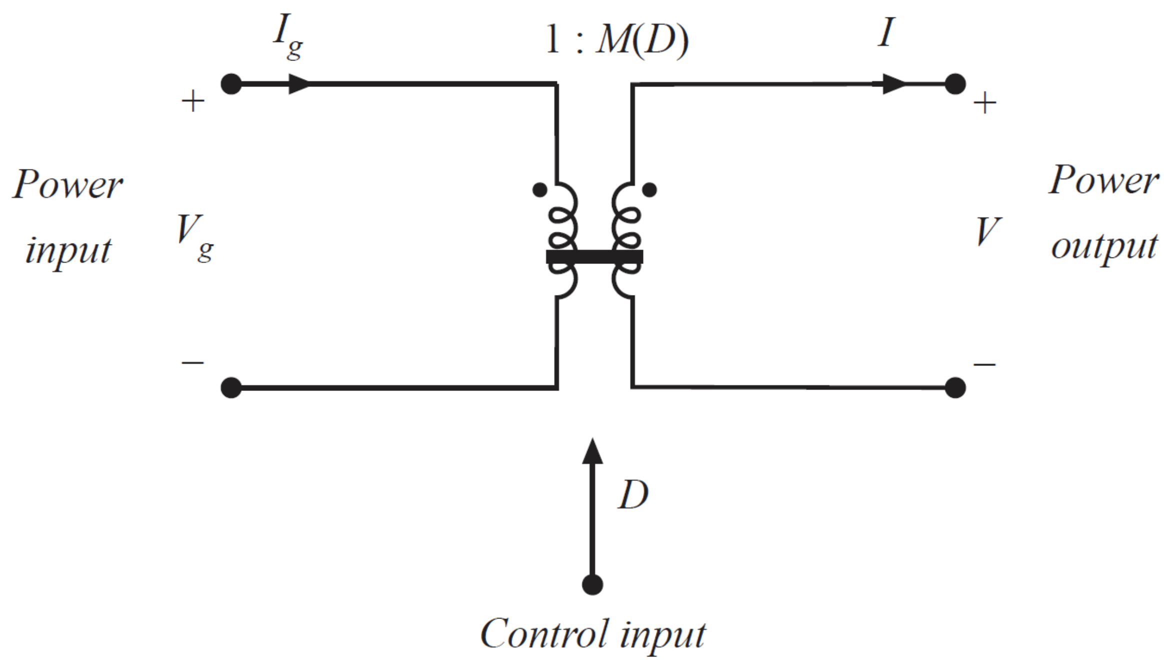


DC transformer



The DC Transformer Model

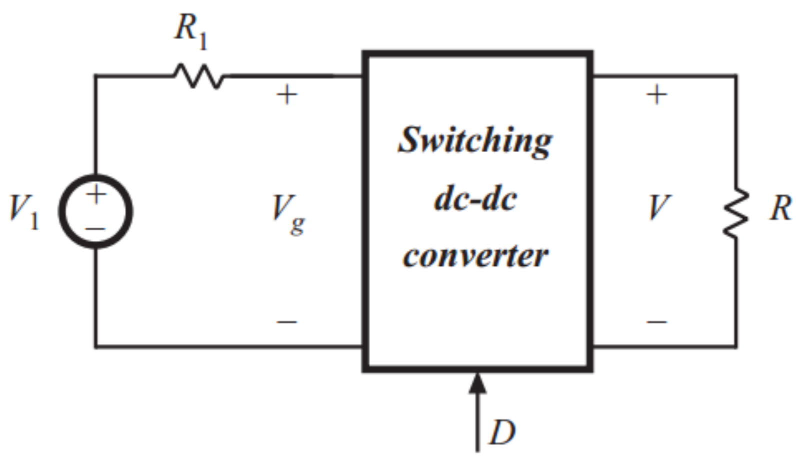
- Models basic properties of ideal dc-dc converter:
- Conversion of dc voltages and currents, ideally with 100% efficiency
- Conversion ratio M controllable via duty cycle



- Solid line denotes ideal transformer model, capable of passing dc voltages and currents
- Time-invariant model (no switching) which can be solved to find dc components of converter waveforms (Constant duty cycle)

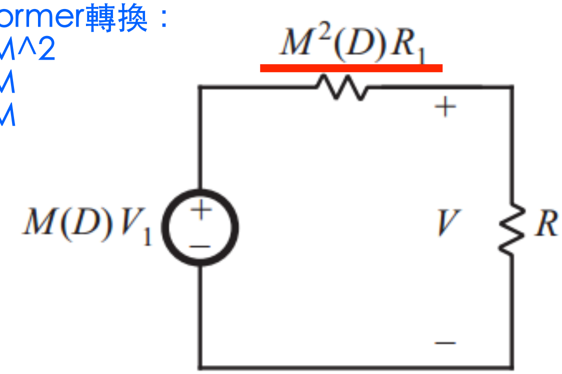
Example: DC Transformer Model

1. **Original system**

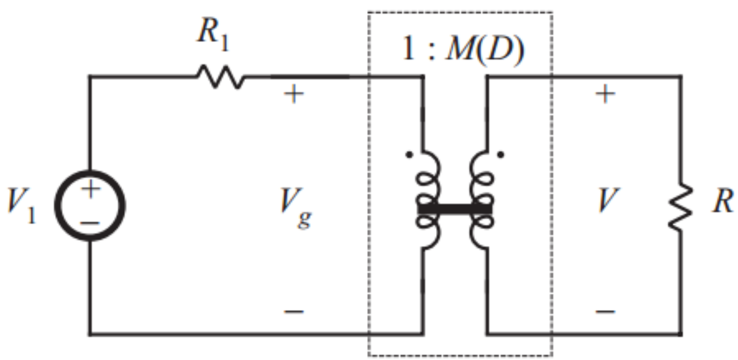


3. **Push source through transformer**

transformer轉換：
電阻*M^2
電壓*M
電流/M



2. **Insert dc transformer mode**



4. **Solve circuit**

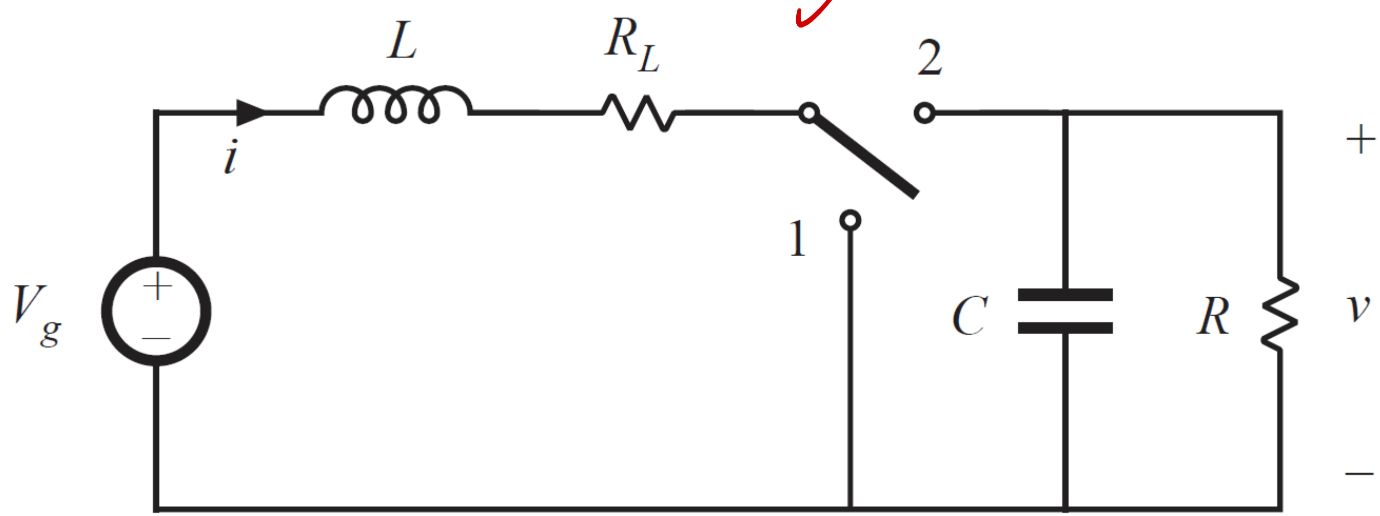
$$V = M(D)V_1 \frac{R}{R + M^2(D)R_1}$$

Inclusion of Inductor Copper Loss

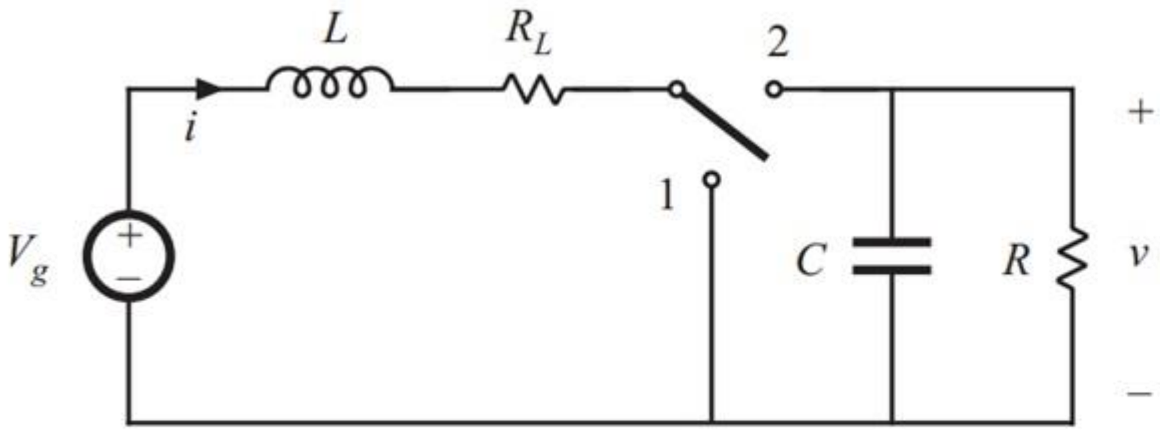
- DC transformer model can be extended, to include converter nonidealities.
- Example: inductor copper loss (resistance of winding):

ESR

- Insert this inductor model into boost converter circuit:

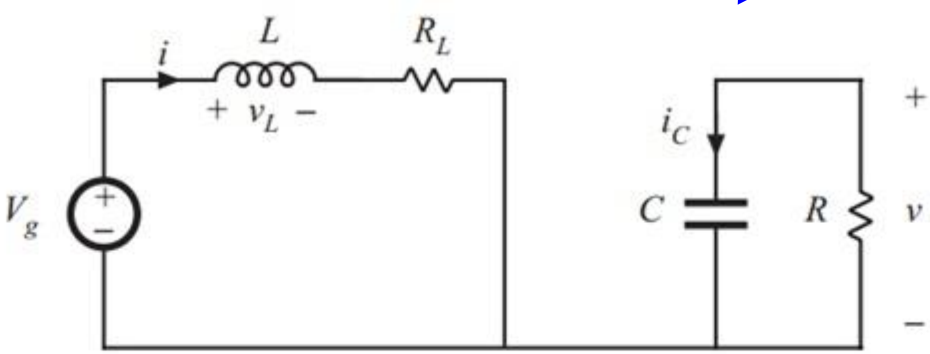


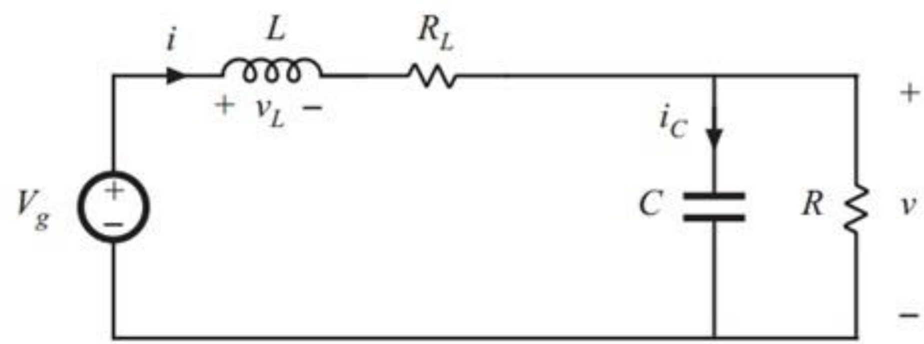
Analysis of Non-Ideal Boost Converter



Switch in position 1

Switch in position 2

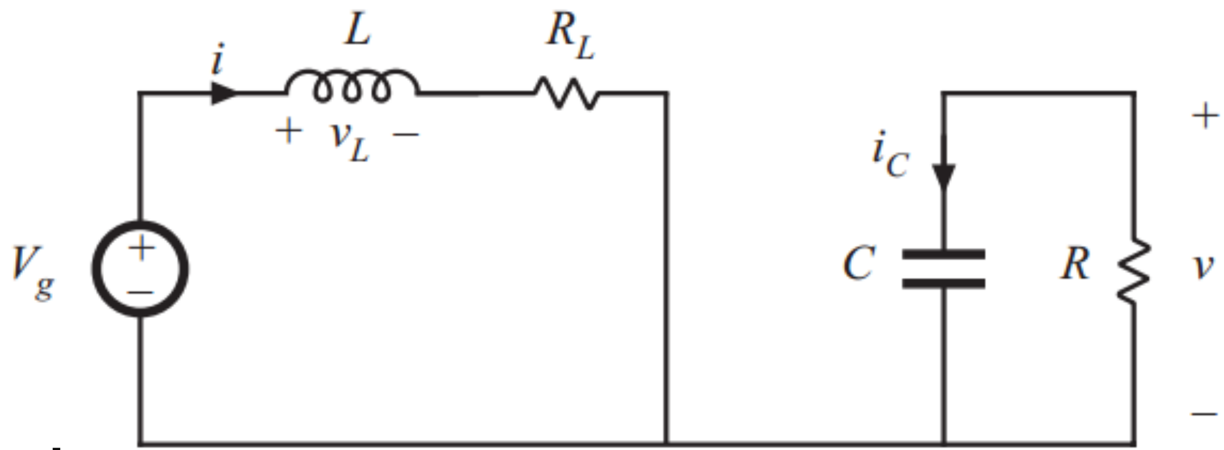




Circuit Equations, Switch in Position 1

- **Inductor current and capacitor voltage:**

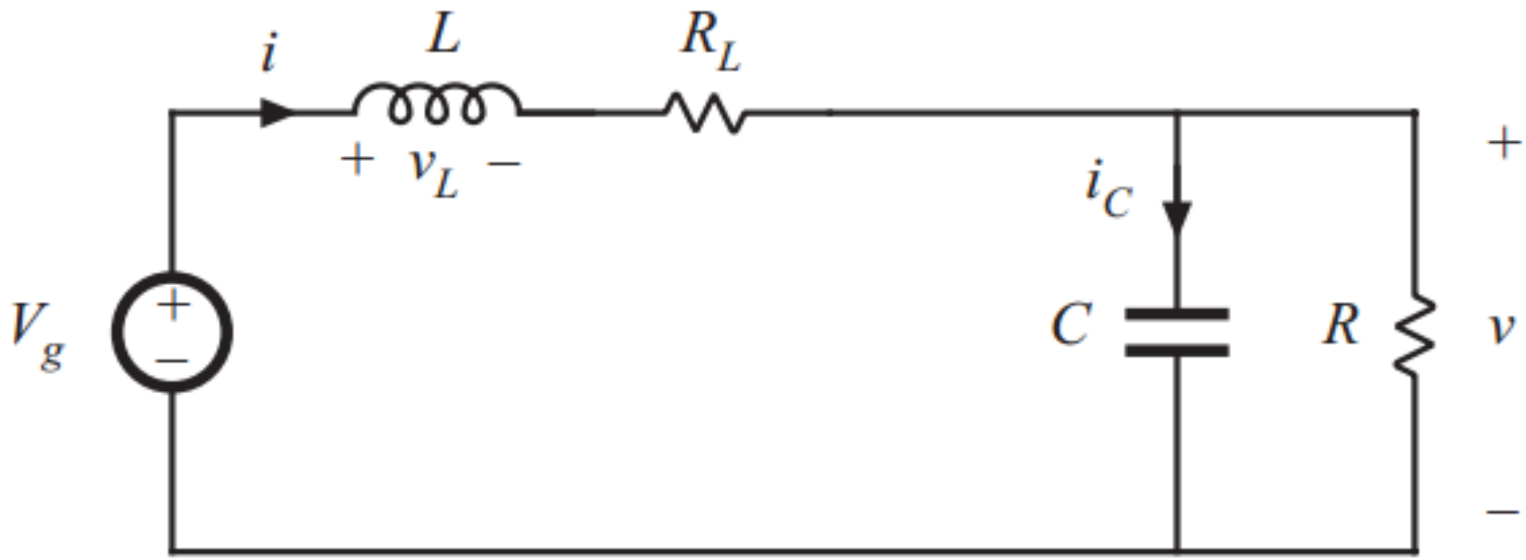
$$v_L(t) = V_g - i(t)R_L$$
$$i_C(t) = -v(t)/R$$



- **Small ripple approximation:**

$$v_L(t) = V_g - IR_L$$
$$i_C(t) = -V/R$$

Circuit Equations, Switch in Position 2



- $v_L(t) = V_g - i(t)R_L - v(t) \approx V_g - IR_L - V$
- $i_C(t) = i(t) - v(t)/R \approx I - V/R$

Inductor Voltage & Capacitor Current Waveforms

- **Average inductor voltage:**

$$\langle v_L(t)\rangle = \frac{1}{T_s}\int_0^{T_s} v_L(t)dt$$

$$= D\left(V_g - IR_L\right) + D'\left(V_g - IR_L - V\right)$$

- **Inductor volt-second balance:**

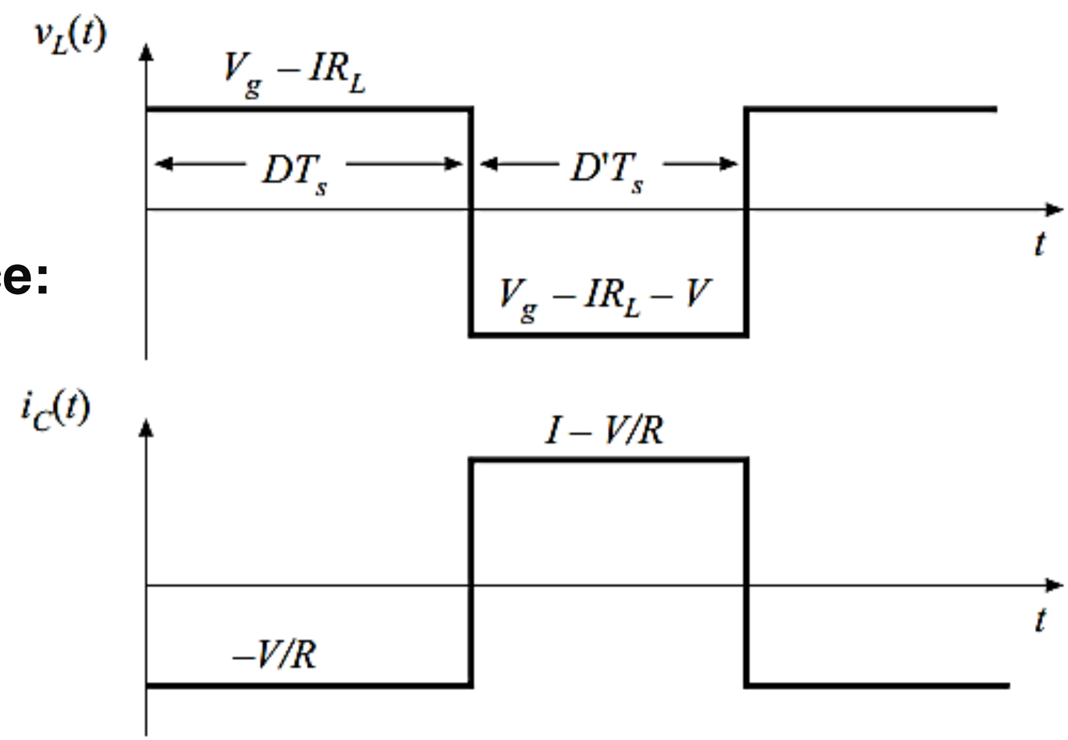
$$0 = V_g - IR_L - D'V$$

- **Average capacitor current:**

$$\langle i_C(t)\rangle = D(-V/R) + D'(I - V/R)$$

- **Capacitor charge balance:**

$$0 = D'I - V/R$$



Solution for Output Voltage

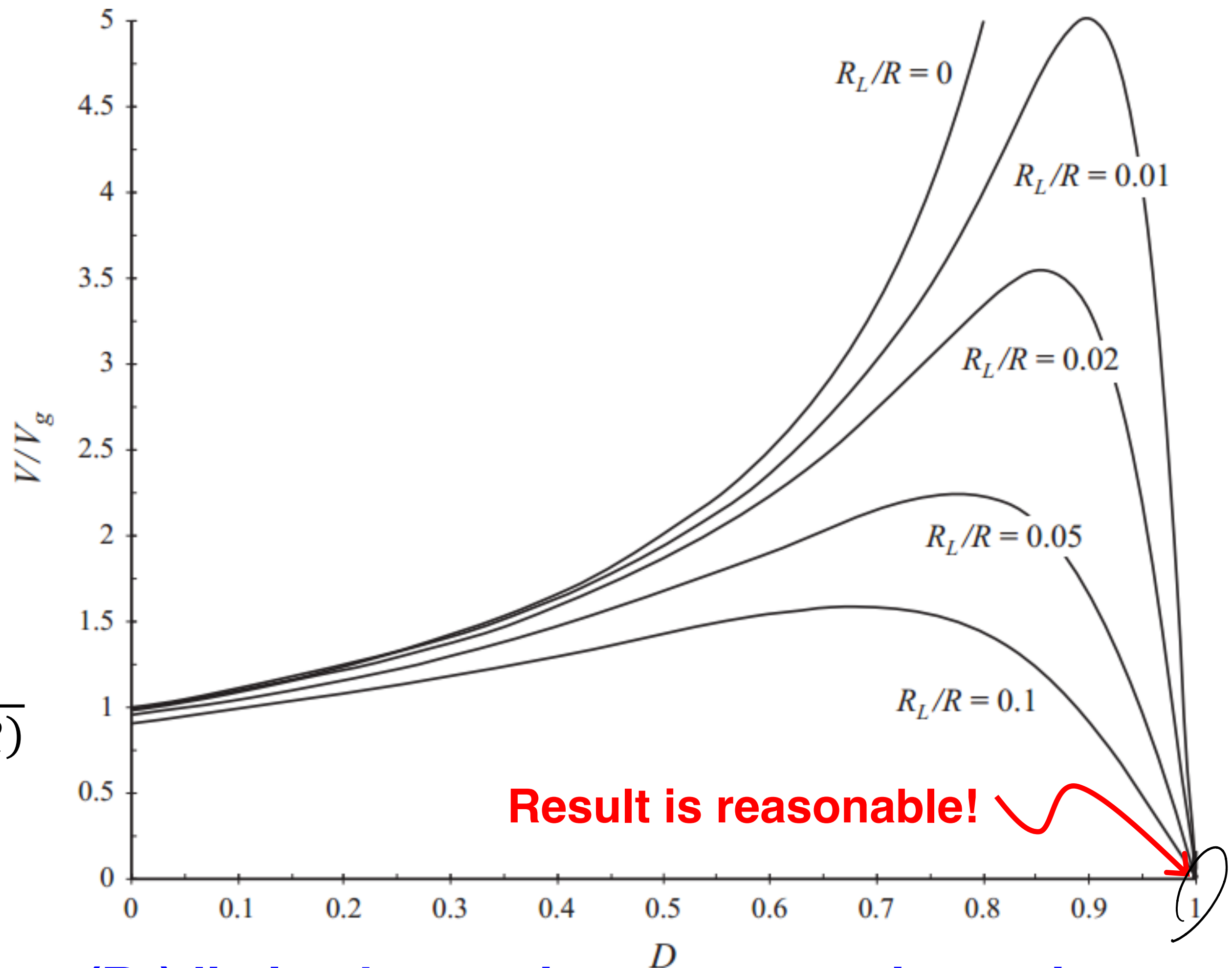
We now have two equations and two unknowns:

$$0 = V_g - IR_L - D'V$$

$$0 = D'I - V/R$$

Eliminate *I* and solve for *V*:

$$\frac{V}{V_g} = \frac{1}{D'}\frac{1}{(1 + R_L/D'^2 R)}$$



The ESR of inductor (R_L) limits the maximum conversion ratio.

Construction of Equivalent Circuit Model

- Results of previous section (derived via inductor volt-sec balance and capacitor charge balance):

$$\langle v_L \rangle = 0 = V_g - IR_L - D'V$$

$$\langle i_C \rangle = 0 = D'I - V/R$$

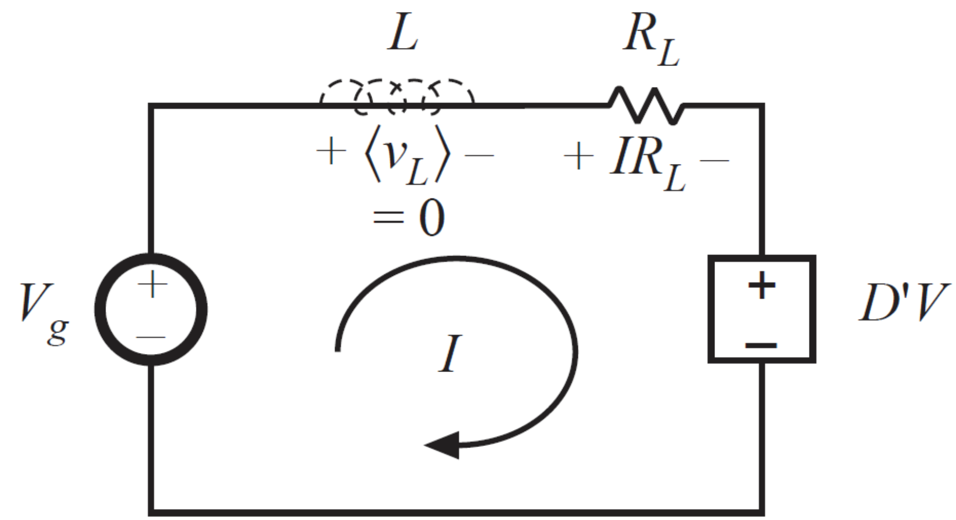
- View these as loop and node equations of the equivalent circuit.

Reconstruct a circuit model based on these equations, which describes the dc behavior of the boost converter with inductor copper loss.

Inductor Voltage Equation

$$\boxed{\langle v_L \rangle = 0 = V_g - IR_L - D'V}$$

- Derived via Kirchhoff's voltage law, to find the inductor voltage during each subinterval
- Average inductor voltage then set to zero
- This is a loop equation: the dc components of voltage around a loop containing the inductor sum to zero

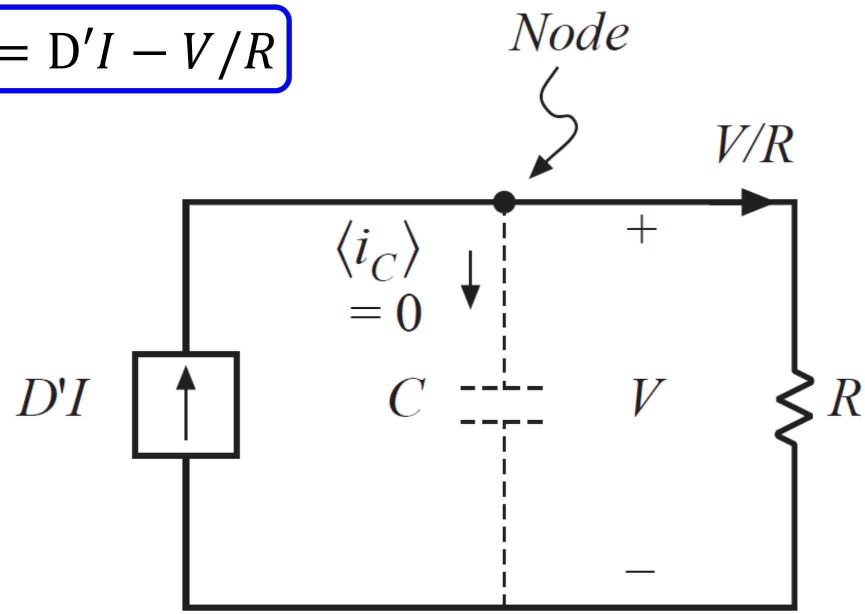


- IR_L term: voltage across resistor of value R_L having current I
- $D'V$ term: for now, leave as dependent source

Capacitor Current Equation

$$\boxed{\langle i_C \rangle = 0 = \mathrm{D}'I - V/R}$$

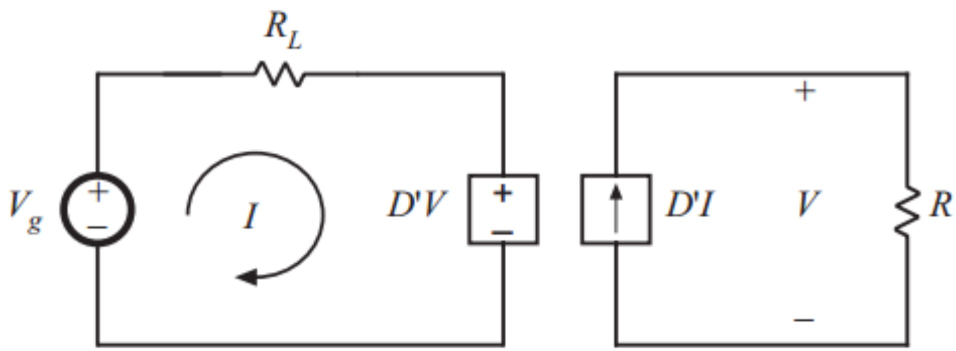
- Derived via Kirchoff's current law, to find the capacitor current during each subinterval
- Average capacitor current then set to zero
- This is a node equation: the dc components of current flowing into a node connected to the capacitor sum to zero



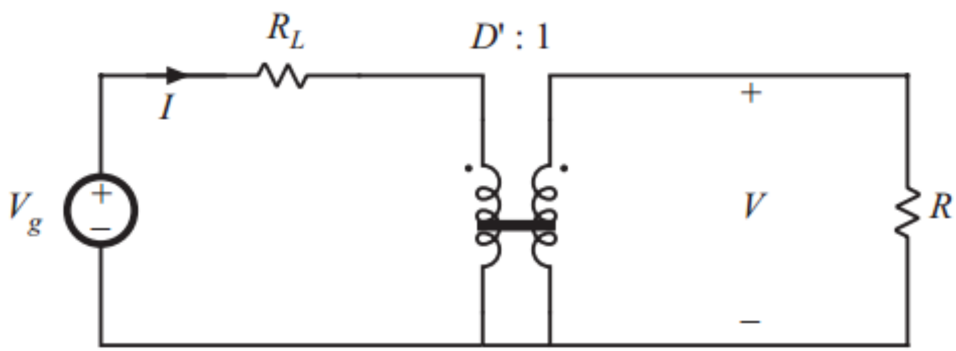
- V/R term: current through load resistor of value R having voltage V
- $D'I$ term: for now, leave as dependent source

Complete Equivalent Circuit

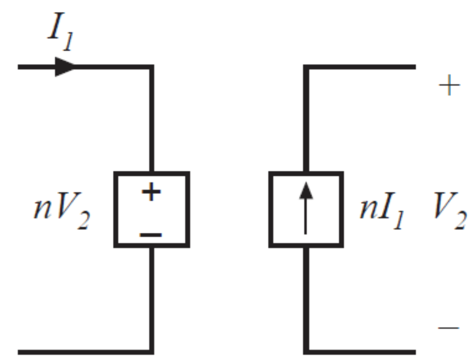
- The two circuits, drawn together:

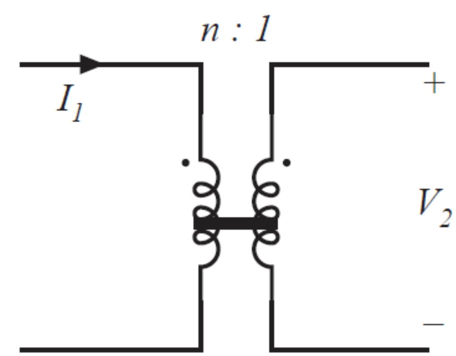


- The dependent sources are equivalent to a D′ : 1 transformer:



Dependent sources and transformers

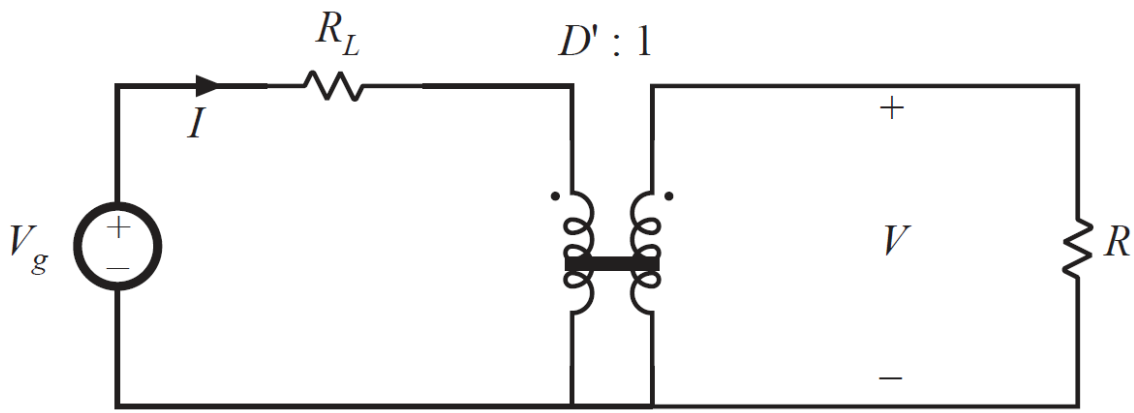




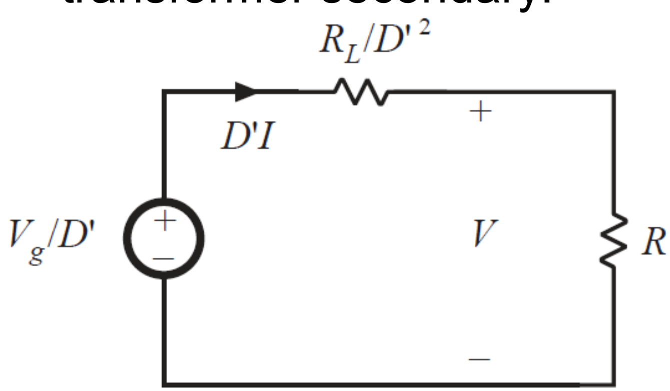
- sources have same coefficient
- reciprocal voltage/current dependence

Solution of Equivalent Circuit

- Converter equivalent circuit



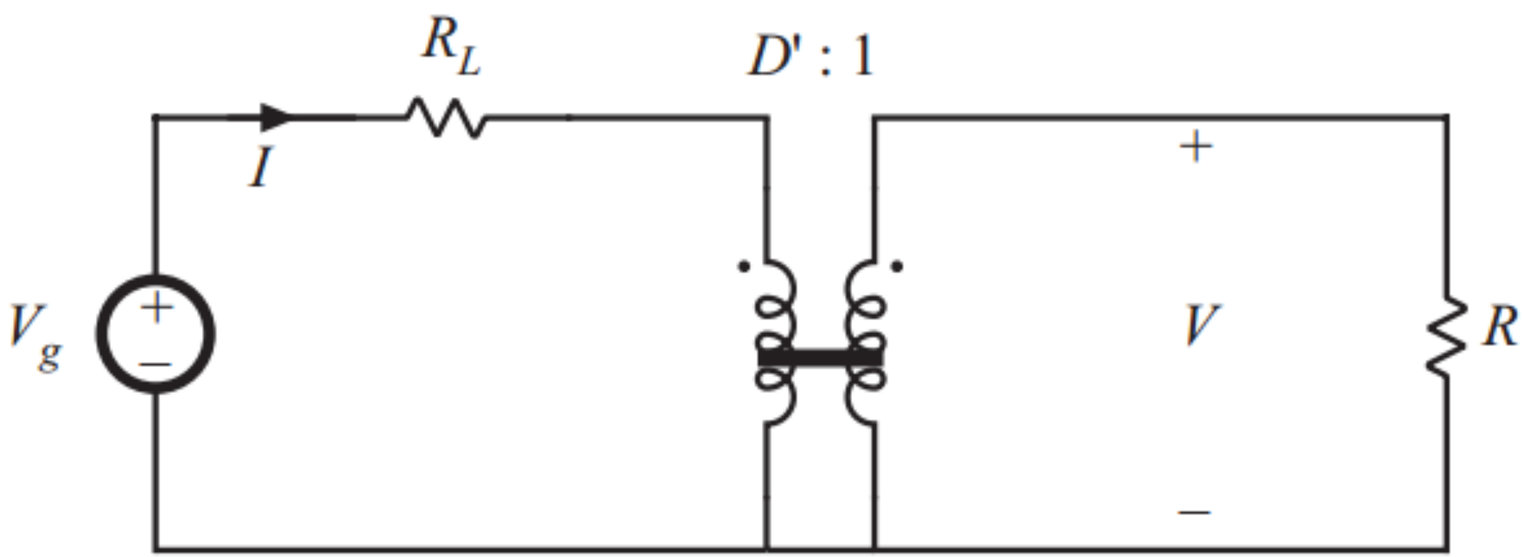
- Refer all elements to transformer secondary:



- Solution for output voltage using voltage divider formula:

$$V = \frac{V_g}{D'} \frac{R}{R + \frac{R_L}{D'^2}} = \frac{V_g}{D'} \frac{1}{1 + \frac{R_L}{D'^2 R}}$$

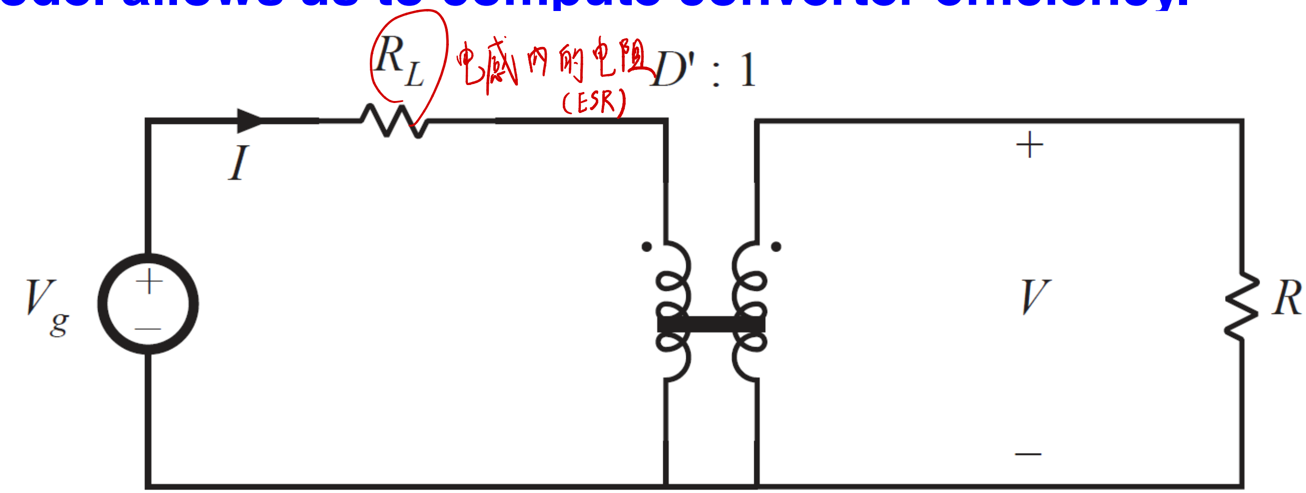
Solution for Input (Inductor) Current



$$I = \frac{V_g}{D'^2 R + R_L} = \frac{V_g}{D'^2 \boxed{R}} \frac{1}{1 + \frac{R_L}{D'^2 R}}$$

Solution for Converter Efficiency

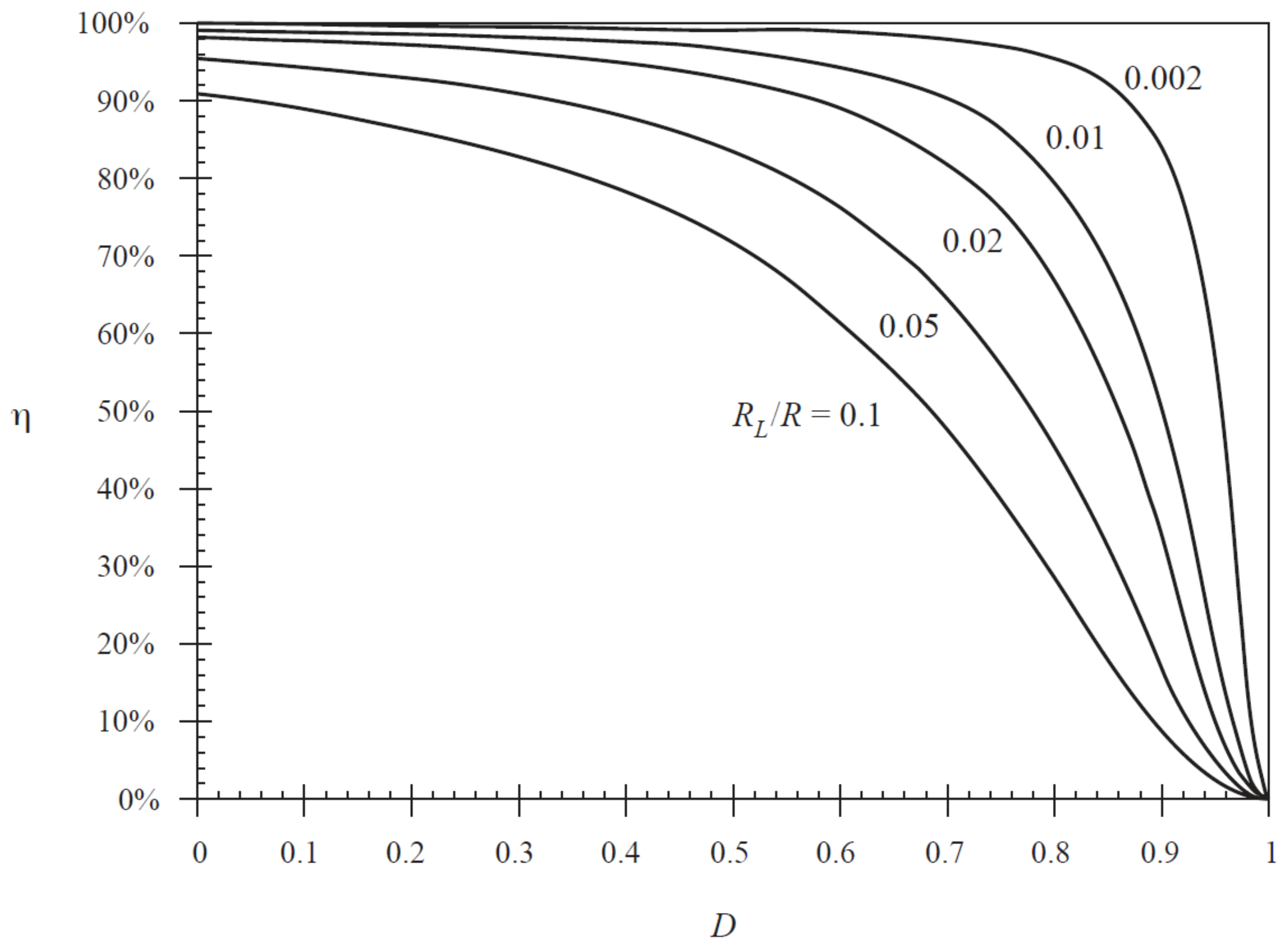
Equivalent circuit model allows us to compute converter efficiency.



- $P_{in} = (V_g)(I)$
- $P_{out} = (V)(D'I)$
- $\eta = \dfrac{P_{out}}{P_{in}} = \dfrac{(V)(D'I)}{(V_g)(I)} = \dfrac{V}{V_g} D'$
- $V = \dfrac{V_g}{D'} \dfrac{R}{R + \dfrac{R_L}{D'^2}} = \dfrac{V_g}{D'} \dfrac{1}{1 + \dfrac{R_L}{D'^2 R}}$
- $\eta = \dfrac{1}{1+\dfrac{R_L}{D'^2 R}}$ $= \dfrac{1}{1 + \dfrac{I^2 R_L^2}{D'^2 I^2 R}} = \dfrac{1}{1 + \dfrac{\text{耗在 } R_L \text{ 的 power}}{\text{耗在 } R \text{ 的 power}}}$

Efficiency for Various R_L

$$\eta = \frac{1}{1+\frac{R_L}{D'^2 R}}$$

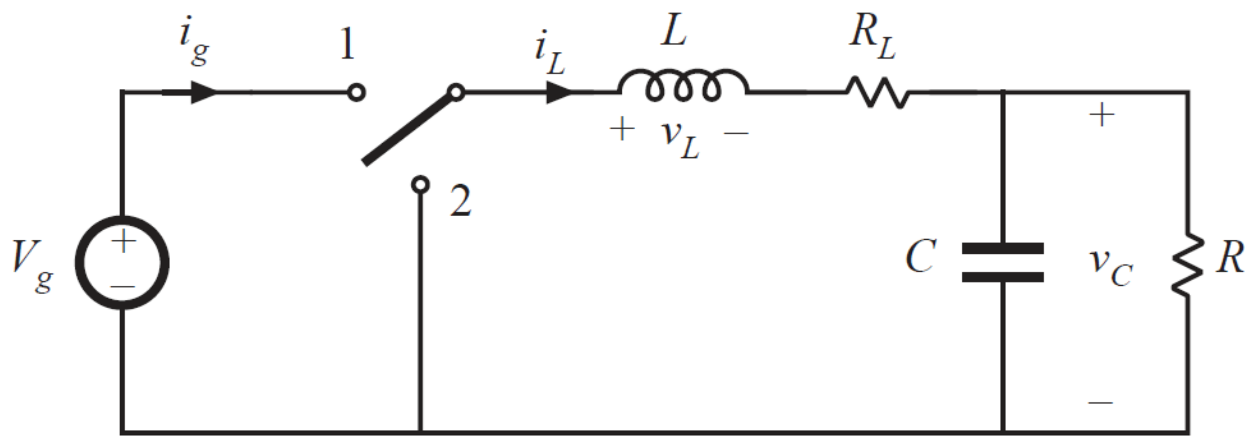


To obtain high efficiency, ESR of inductor (R_L) should be much smaller than D'^2R.

Derive the Model of Buck Converter

Buck converter : Need to obtain the input port of the model.

- Buck converter example —use procedure of previous section to derive equivalent circuit



- Average inductor voltage and capacitor current:

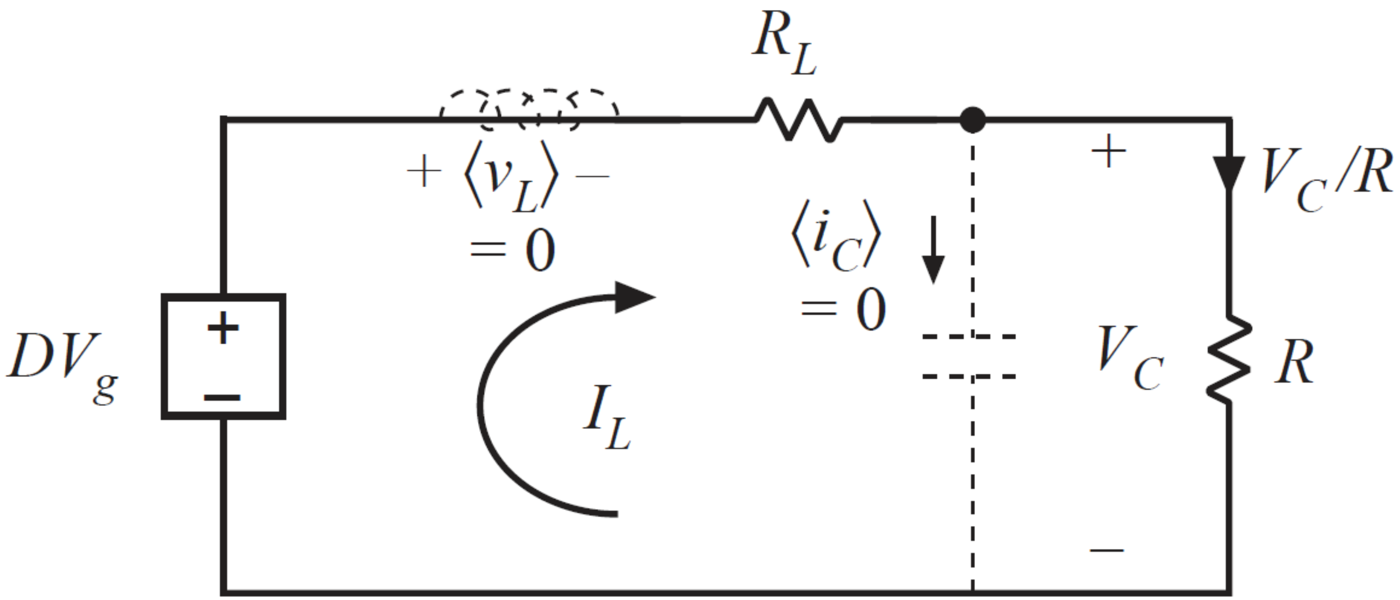
$$\boxed{\langle v_L \rangle = 0 = DV_g - I_L R_L - V_C}$$

$$\boxed{\langle i_C \rangle = 0 = I_L - V_C / R}$$

Construct Equivalent Circuit as Usual

$\boxed{\langle v_L \rangle = 0 = DV_g - I_L R_L - V_C}$

$\boxed{\langle i_C \rangle = 0 = I_L - V_C/R}$



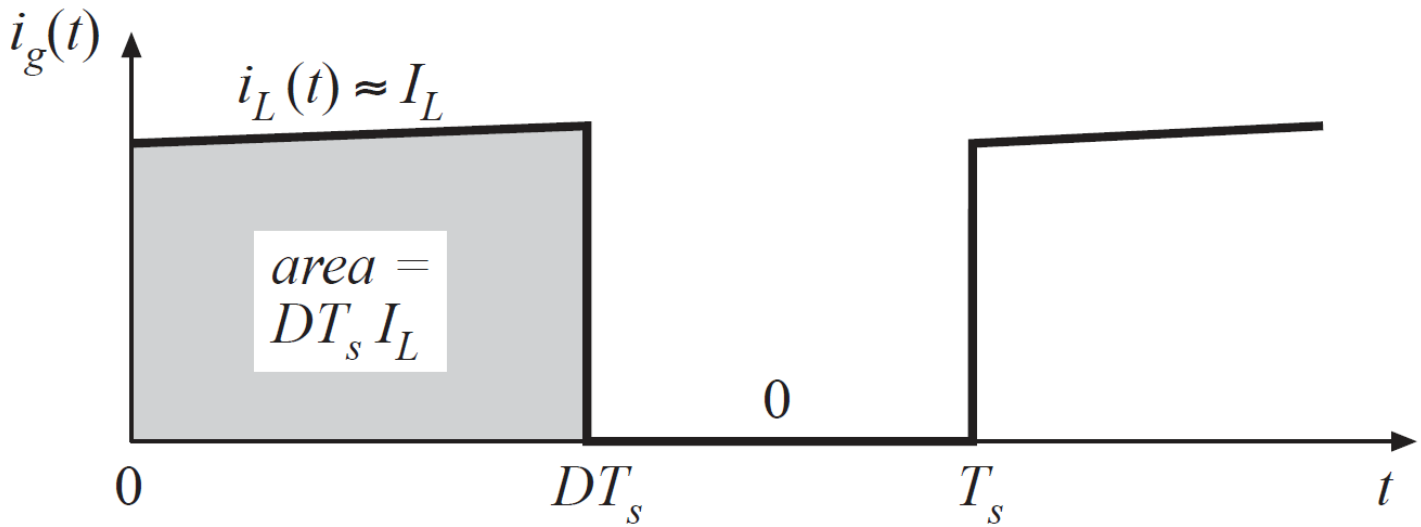
What happened to the transformer?

→ Need another equation.

Need to find the dc component of the input current $i_g(t)$.

Modeling the Converter Input Port

Input current waveform $i_g(t)$:



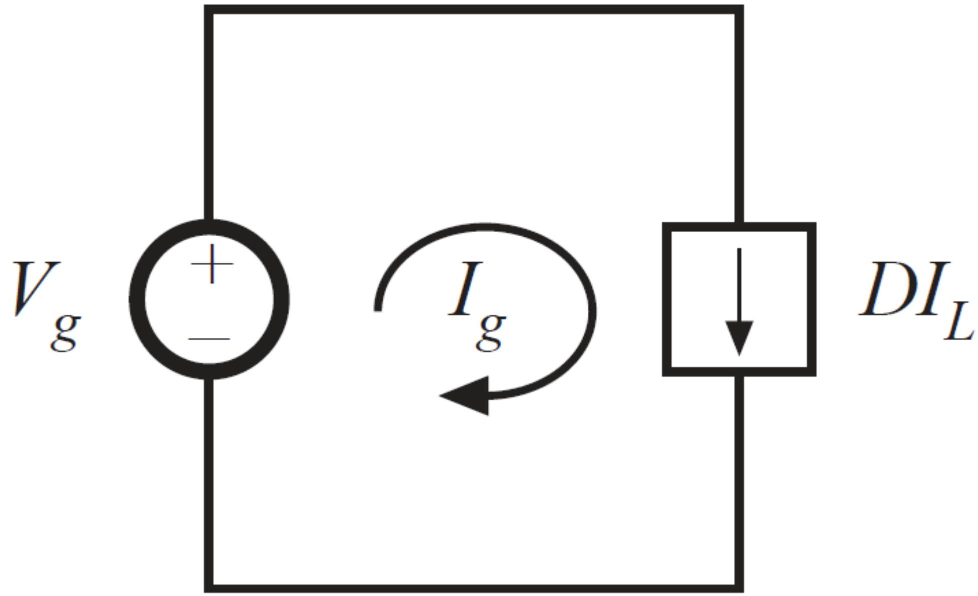
Dc component (average value) of $i_g(t)$ is

$$\boxed{I_g = \frac{1}{T_s}\int_0^{T_s} i_g(t)dt = DI_L}$$

Input Port Equivalent Circuit

非連續 input current

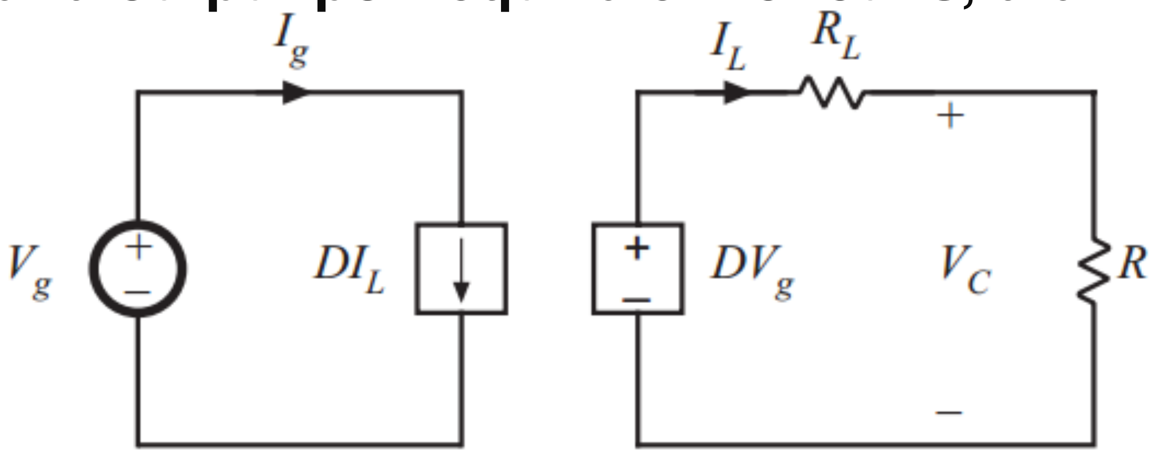
$$I_g = \frac{1}{T_s}\int_0^{T_s} i_g(t)dt = DI_L$$



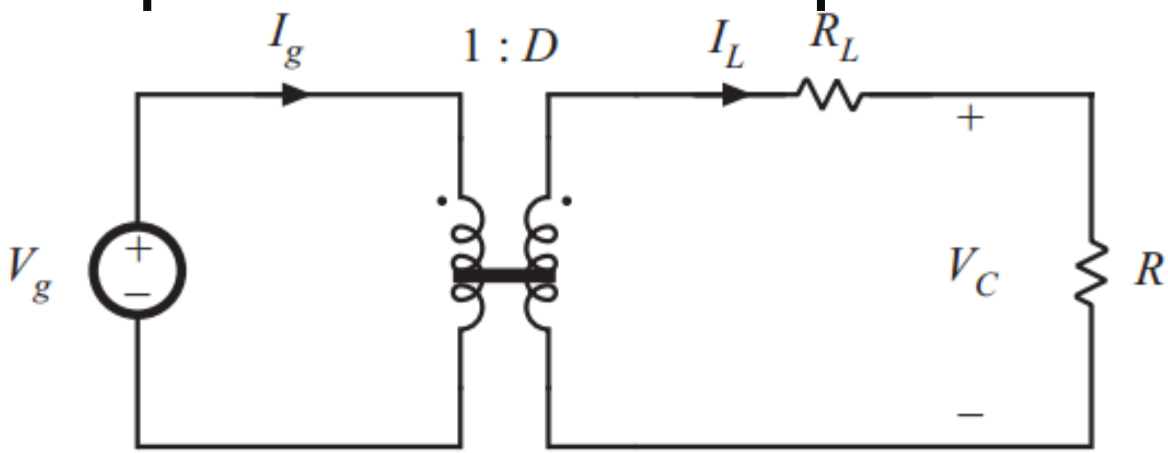
In buck converter, as well as in other converters having pulsating input current , this equivalent circuit contains a dependent current source which becomes the primary of a dc transformer model.

Complete Equivalent Circuit : Buck Converter

- Input and output port equivalent circuits, drawn together:



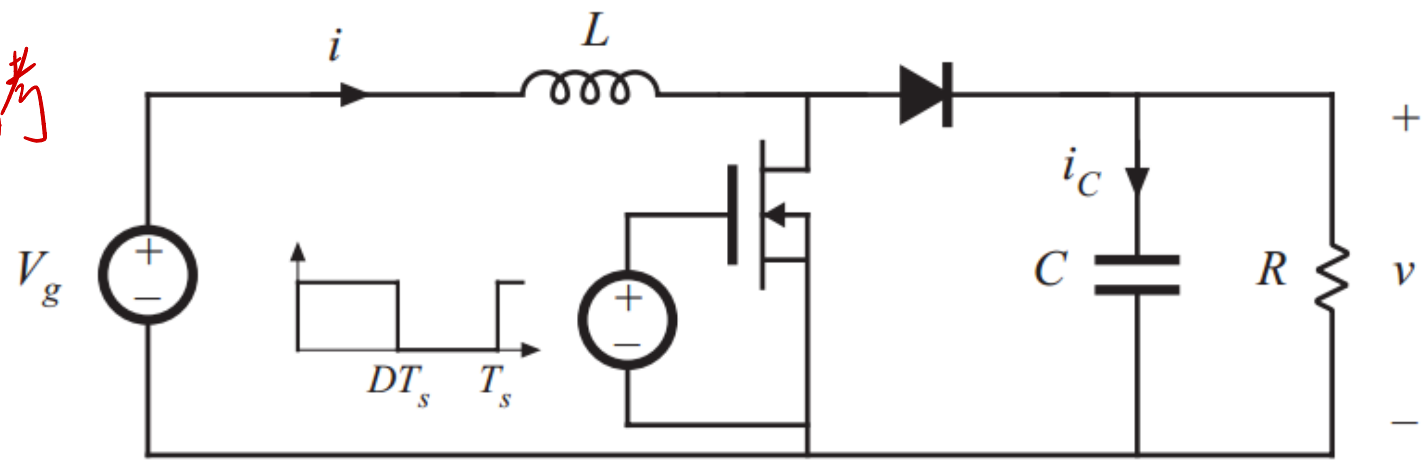
- Replace dependent sources with equivalent dc transformer:



Inclusion of Conduction Losses

- Boost converter example

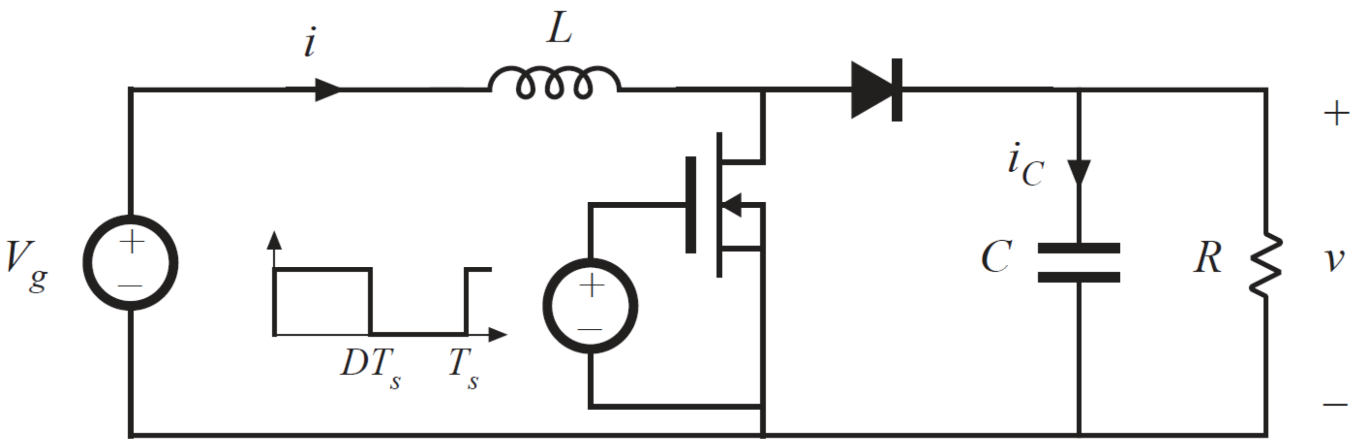
會考



- Models of on-state semiconductor devices: MOSFET: on-resistance R_{on}

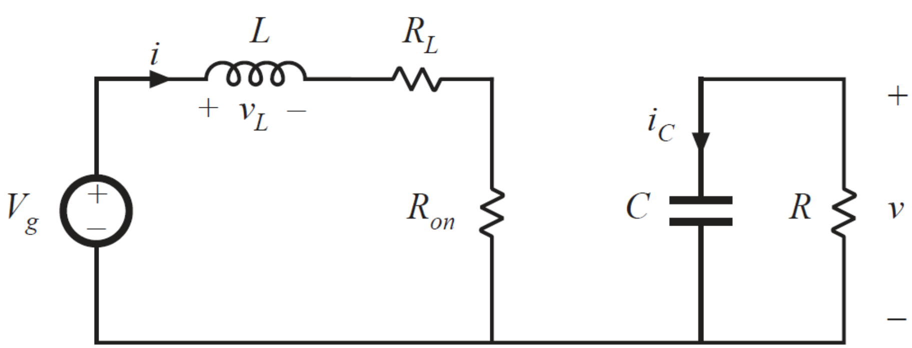
  Diode: constant forward voltage V_D plus on-resistance R_D
- Insert these models into subinterval circuits

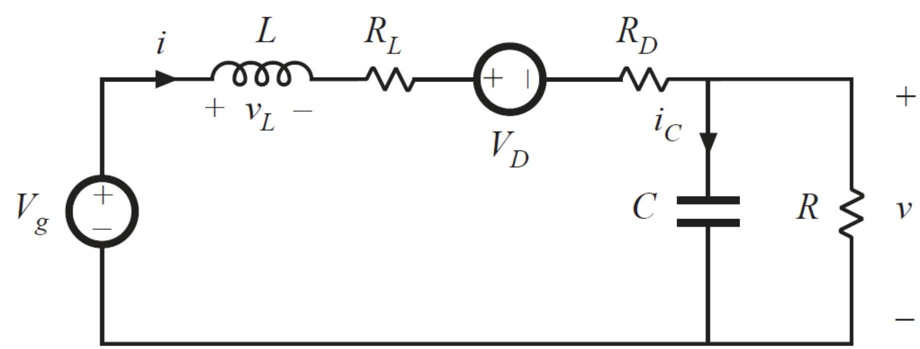
Equivalent Circuit in Different Subintervals



switch in position 1

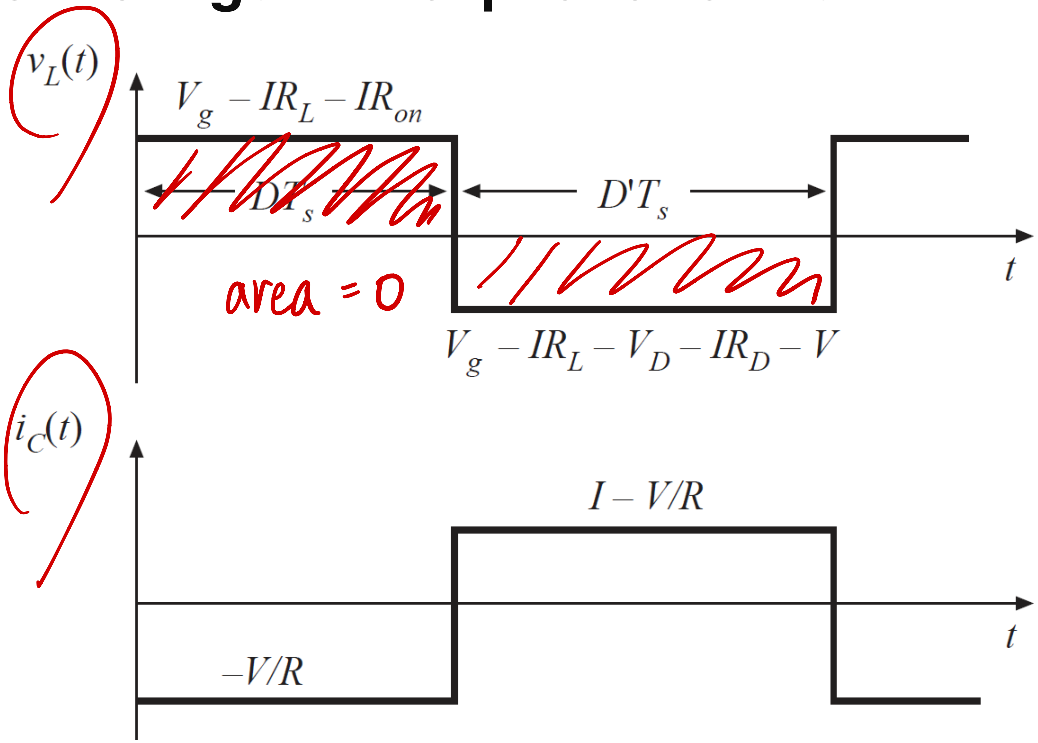
switch in position 2





Inductor Voltage and Capacitor Current

- Inductor voltage and capacitor current waveform

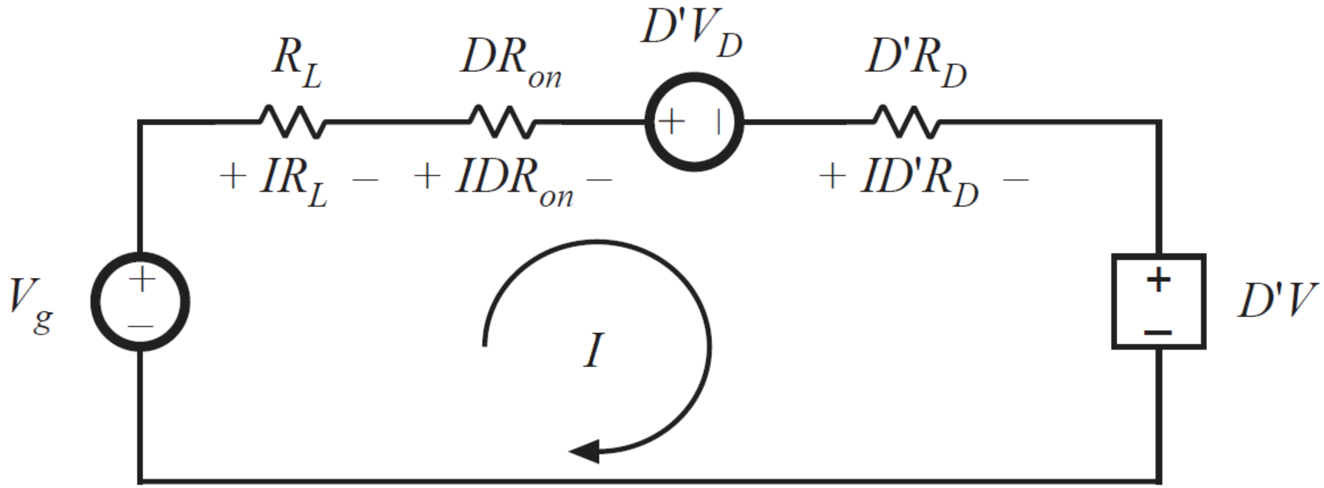


$$\langle v_L \rangle = D\left(V_g - IR_L - IR_{on}\right) + D'\left(V_g - IR_L - V_D - IR_D - V\right) = 0$$

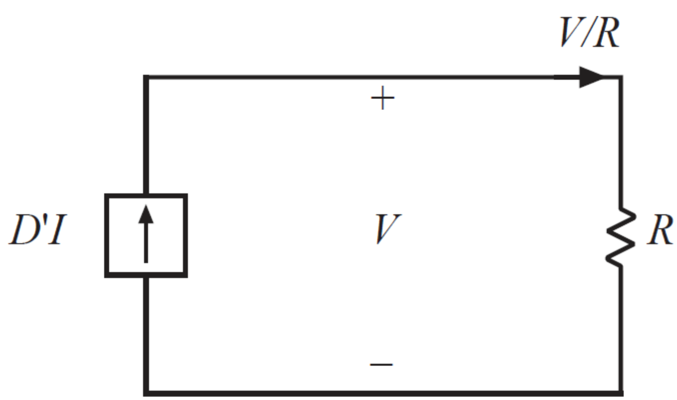
$$\langle i_C \rangle = D(-V/R) + D'(I - V/R) = 0$$

Construction of Equivalent Circuits

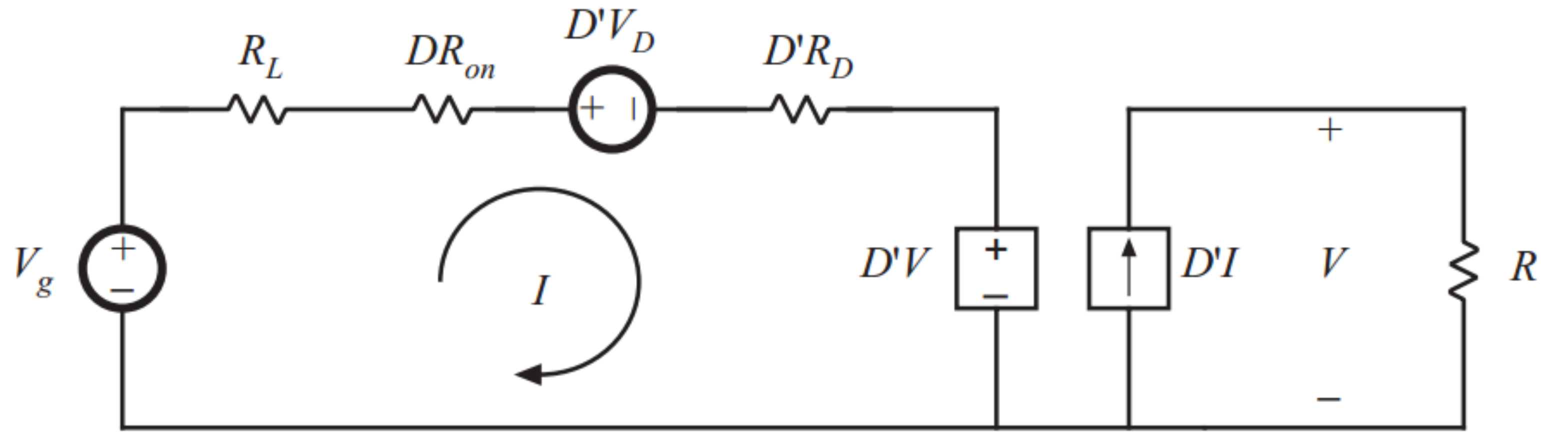
$$\boxed{V_g - IR_L - IDR_{on} - D'V_D - ID'R_D - D'V = 0}$$

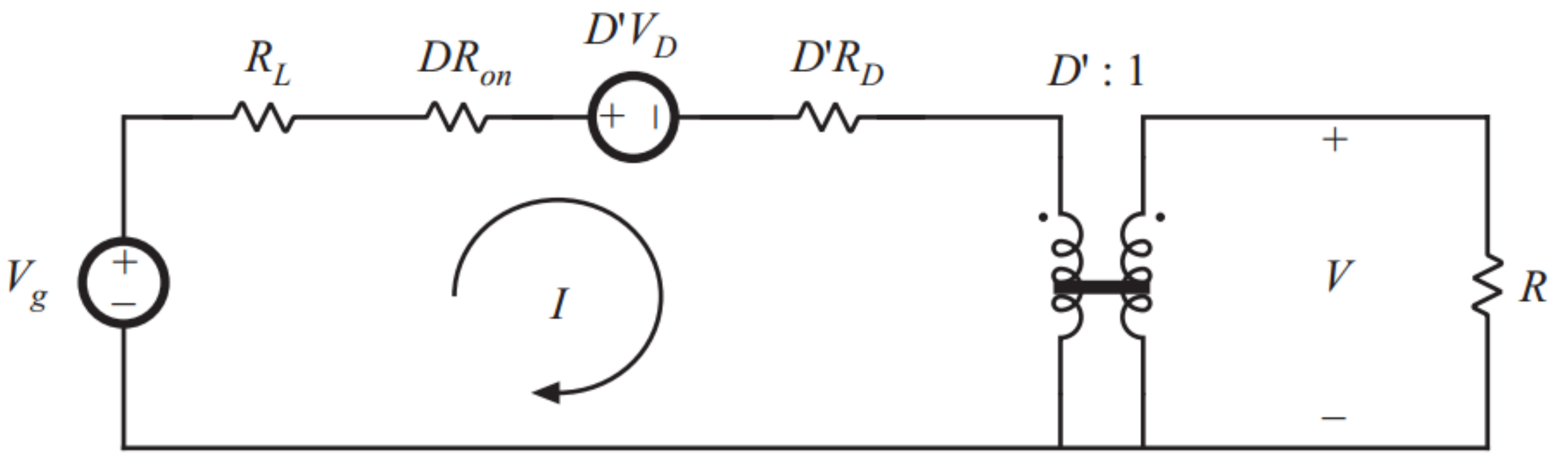


$$\boxed{D'I - V/R = 0}$$

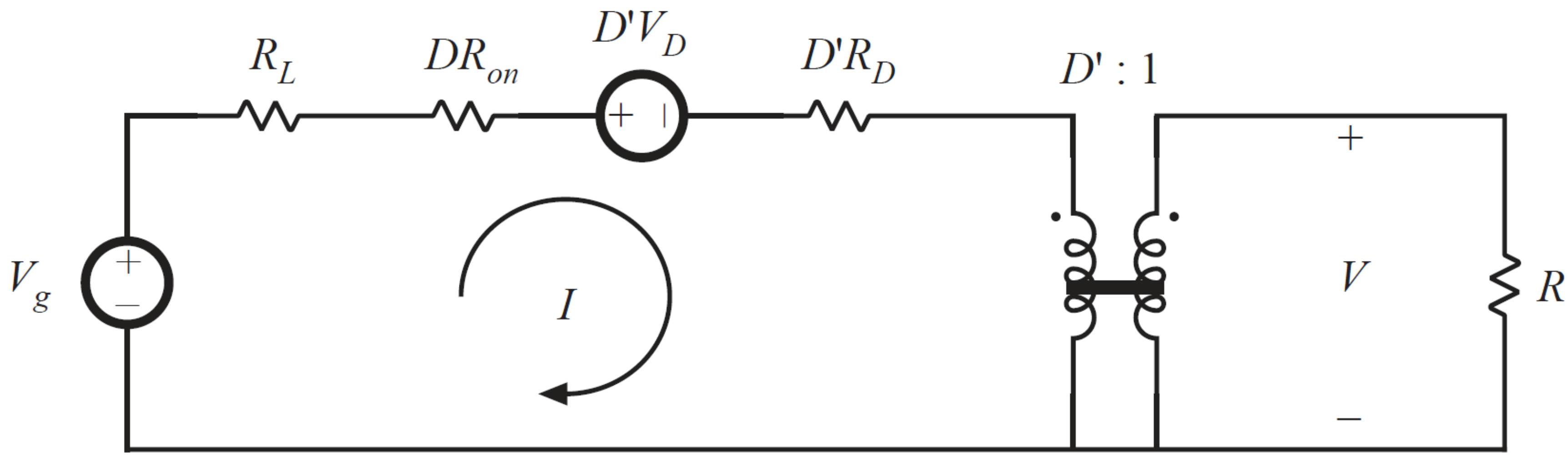


Complete Equivalent Circuit





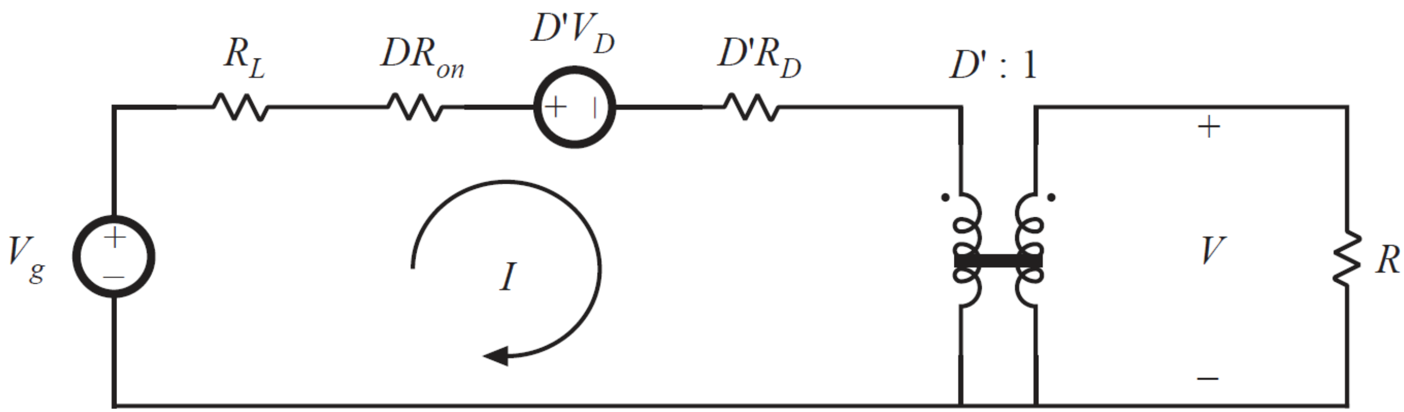
Solution for Output Voltage



- $V = \left(\frac{1}{D'}\right)\left(V_g - D'V_D\right)\left(\frac{D'^2R}{D'^2R+R_L+DR_{on}+D'R_D}\right)$

- $\frac{V}{V_g} = \left(\frac{1}{D'}\right)\left(1 - \frac{D'V_D}{V_g}\right)\left(\frac{1}{1+\frac{R_L+DR_{on}+D'R_D}{D'^2R}}\right)$

Solution for Converter Efficiency



$$P_{in} = (V_g)(I)$$

$$P_{out} = (V)(D'I)$$

$$\eta = D'\frac{V}{V_g} = \frac{\left(1 - \frac{D'V_D}{V_g}\right)}{\left(1 + \frac{R_L + DR_{on} + D'R_D}{D'^2 R}\right)}$$

Conditions for high efficiency:

$\boldsymbol{V_g/D' \gg V_D}$ 低壓不適合用diode做，沒辦法遠大於十倍以上

降壓後之電壓 diode voltage drop

↳ Ideal output voltage >> V_D

$\boldsymbol{D'^2 R \gg R_L + DR_{on} + D'R_D}$

（不同時間點）

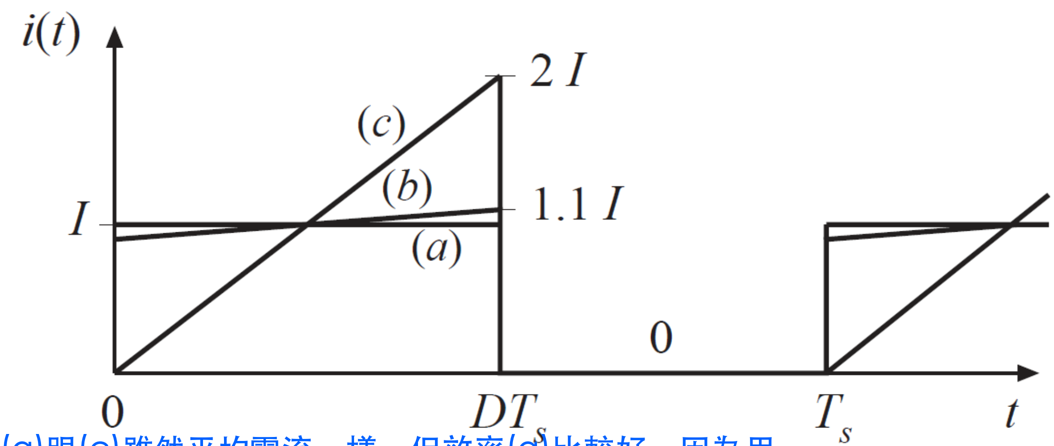
↳ Ideal output power >> power loss

算power loss只要算導通的時間

Considering Current Ripple

- Model uses average currents and voltages
- To correctly predict power loss in a resistor, use rms values
- Result is the same, provided ripple is small

MOSFET current waveforms, for various ripple magnitudes:



power loss用i^2R來算，因此(a)跟(c)雖然平均電流一樣，但效率(a)比較好，因為用i^2R積分去算之後，(c)的power loss是較大的

Inductor current ripple	*MOSFET rms current*	*Average power loss in R_{on}*
(a) $\Delta i = 0$	$I\sqrt{D}$	$D\,I^2\,R_{on}$
(b) $\Delta i = 0.1\,I$	$(1.00167)\,I\sqrt{D}$	$(1.0033)\,D\,I^2\,R_{on}$
(c) $\Delta i = I$	$(1.155)\,I\sqrt{D}$	$(1.3333)\,D\,I^2\,R_{on}$

Summary

1. The dc transformer model represents the primary functions of any dc-dc converter: transformation of dc voltage and current levels, ideally with 100% efficiency, and control of the conversion ratio M via the duty cycle D. This model can be easily manipulated and solved using familiar techniques of conventional circuit analysis.
2. The model can be refined to account for loss elements such as inductor winding resistance and semiconductor on-resistances and forward voltage drops. The refined model predicts the voltages, currents, and efficiency of practical non-ideal converters.
3. In general, the dc equivalent circuit for a converter can be derived from the inductor volt-second balance and capacitor charge balance equations. Equivalent circuits are constructed whose loop and node equations coincide with the volt-second and charge balance equations. In converters having a pulsating input current, an additional equation is needed to model the converter input port; this equation may be obtained by averaging the converter input current.