電源管理晶片設計與實作

Power Management IC Design and Implementation

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Controller Design

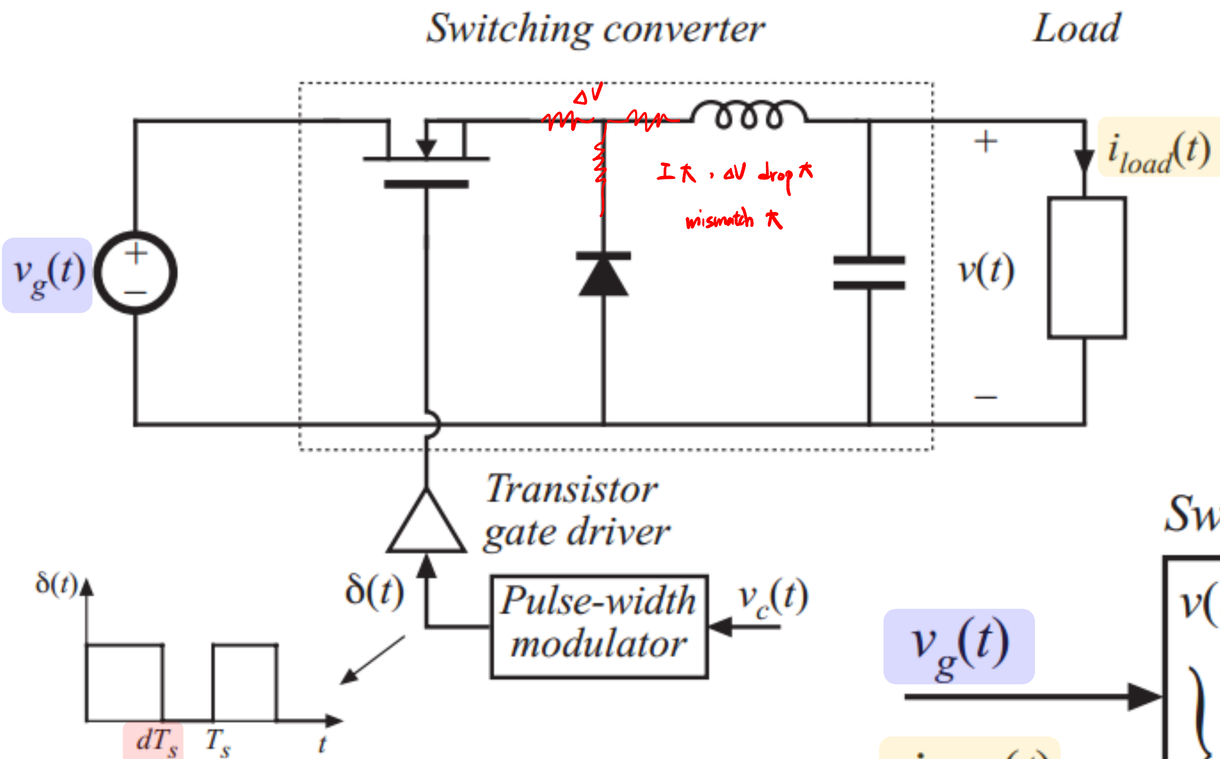
- Effect of negative feedback on the network transfer functions
  - Feedback reduces the transfer function from disturbances to the output
  - Feedback causes the transfer function from the reference input to the output to be insensitive to variations in the gains in the forward path of the loop
- Construction of the important quantities 1/(1+T) and T/(1+T) and the closed-loop transfer functions
- Stability
  - Phase margin test
  - The relation between phase margin and closed-loop damping factor
  - Transient response vs. damping factor

Controller Design

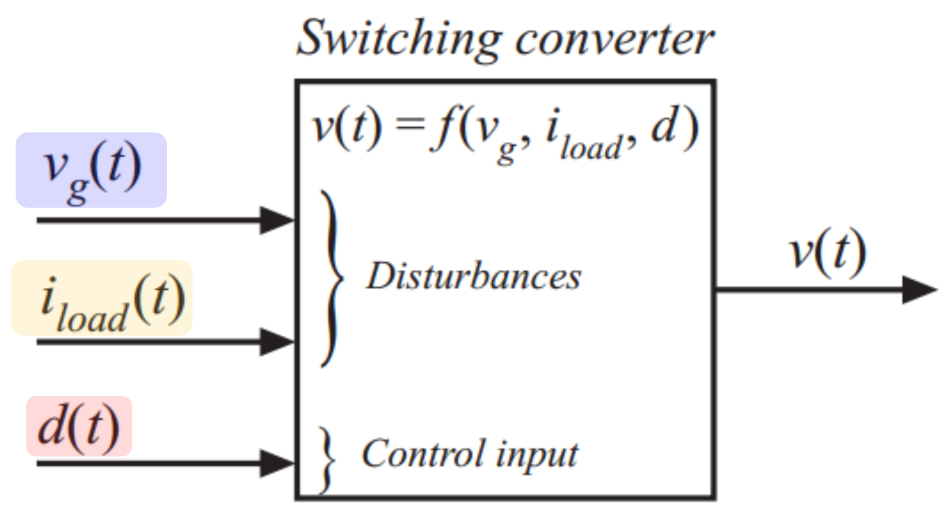
- Regulator design
  - Lead (PD) compensator
  - Lag (PI) compensator
  - Combined (PID) compensator
  - Design example

Partial

Introduction

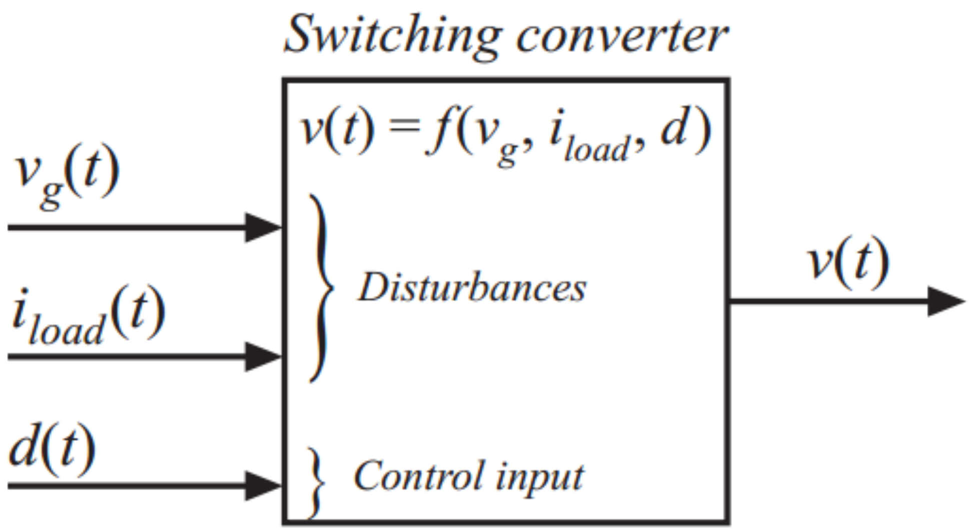


Output voltage of a switching converter depends on **duty cycle *d***, **input voltage v_g**, and **load current i_{load}**.



The DC Regulator Application

- Objective: maintain constant output voltage $v(t) = V$, in spite of disturbances in $v_g(t)$ and $i_{load}(t)$.
- Typical variation in $v_g(t)$: 100Hz or 120Hz ripple, produced by rectifier circuit.

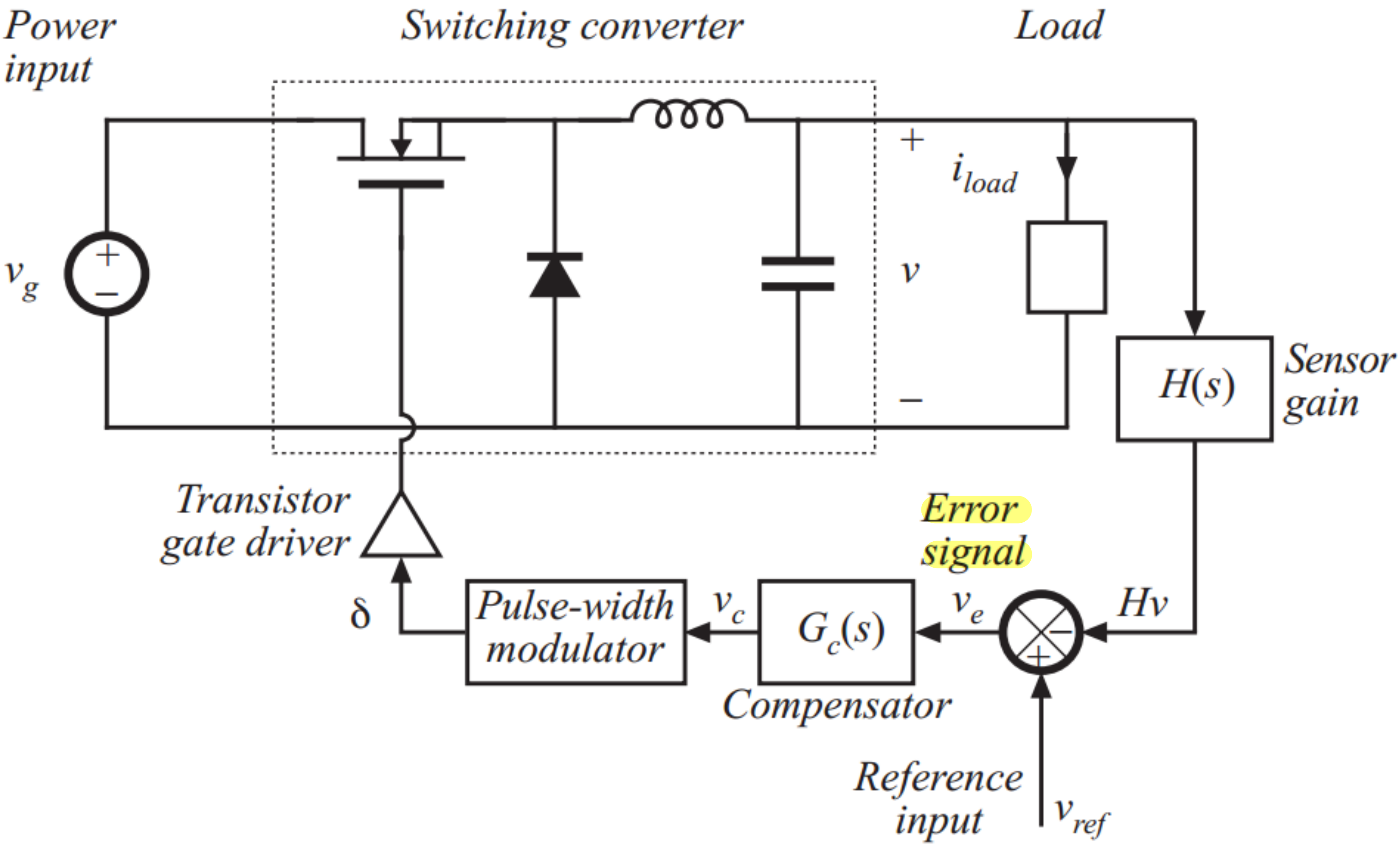


- Load current variations: a significant step-change in load current, such as from 50% to 100% of rated value, may be applied.
- A typical output voltage regulation specification: 5V ± 0.1V.
- Circuit elements are constructed to some specified tolerance. In high volume manufacturing of converters, all output voltages must meet specifications.

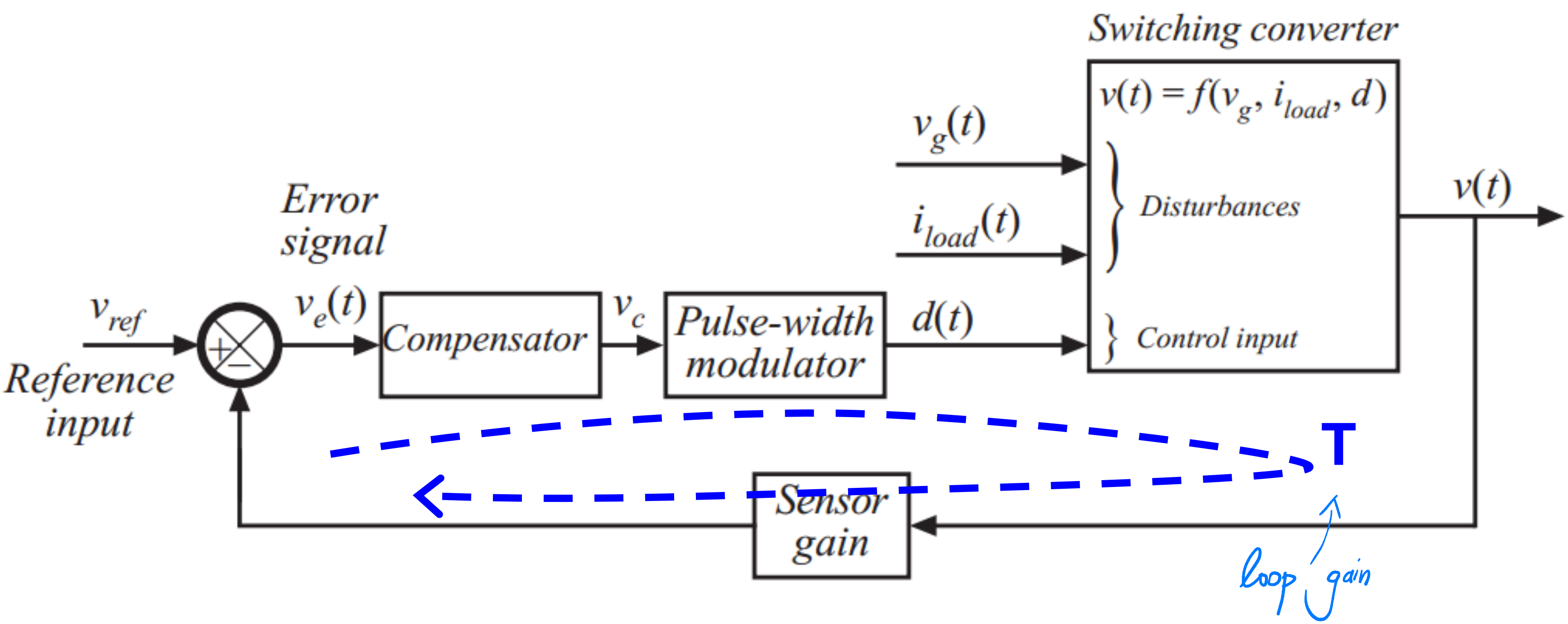
The DC Regulator Application

- So we cannot expect to set the duty cycle to a single value, and obtain a given constant output voltage under all conditions.
- Negative feedback: build a circuit that automatically adjusts the duty cycle as necessary, to obtain the specified output voltage with high accuracy, regardless of disturbances or component tolerances.

Negative Feedback: Switching Regulator System



Negative Feedback



Equivalent Circuit Model for Converter

- For common converters

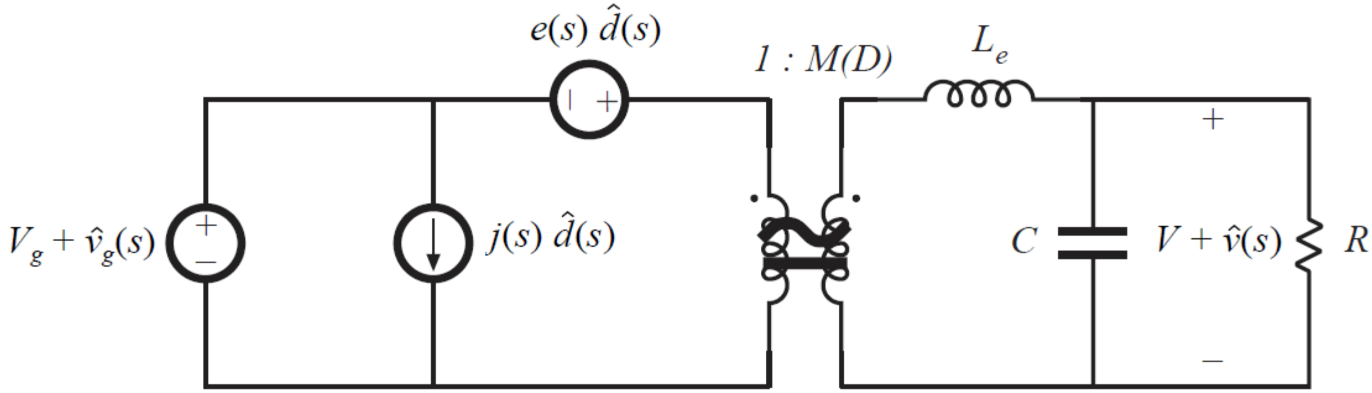
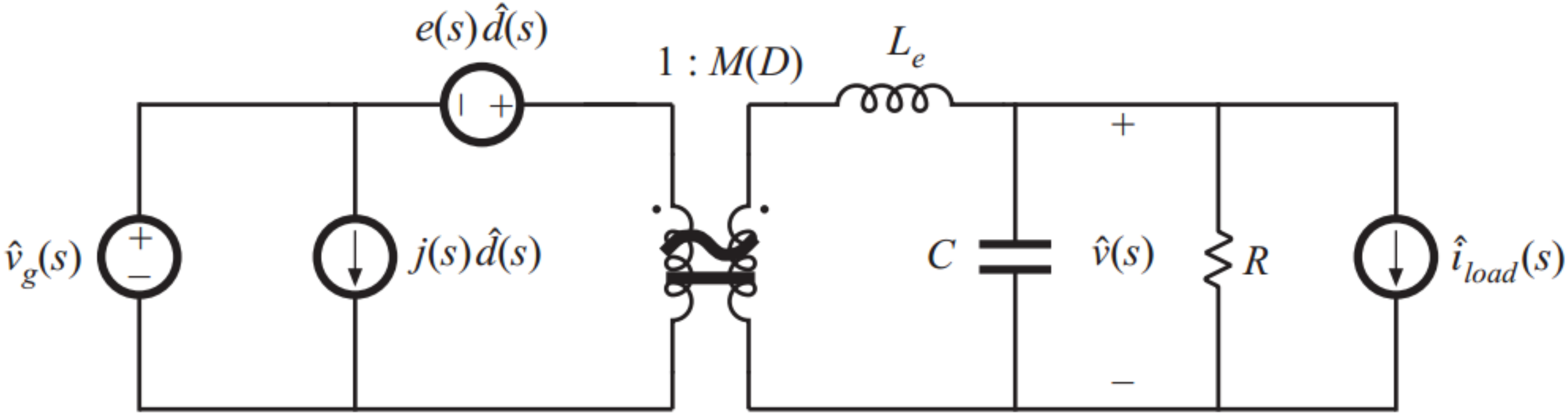


Table 7.1. Canonical model parameters for the ideal buck, boost, and buck-boost converters

Converter	$M(D)$		L_e	$e(s)$	$j(s)$
Buck	D	Duty cycle變化	L	$\frac{V}{D^2}$	$\frac{V}{R}$
Boost	$\frac{1}{D'}$		$\frac{L}{D'^2}$	$V\left(1 - \frac{s\,L}{D'^2\,R}\right)$	$\frac{V}{D'^2\,R}$
Buck-boost	$-\frac{D}{D'}$		$\frac{L}{D'^2}$	$-\frac{V}{D^2}\left(1 - \frac{s\,DL}{D'^2\,R}\right)$	$-\frac{V}{D'^2\,R}$

Effect of Negative Feedback on the Network Transfer Functions

- Small signal model: open-loop converter



- Output voltage can be expressed as

$$\hat{v}(s) = G_{vd}(s)\hat{d}(s) + G_{vg}(s)\hat{v}_g(s) - Z_{out}(s)\hat{i}_{load}(s)$$

ratio

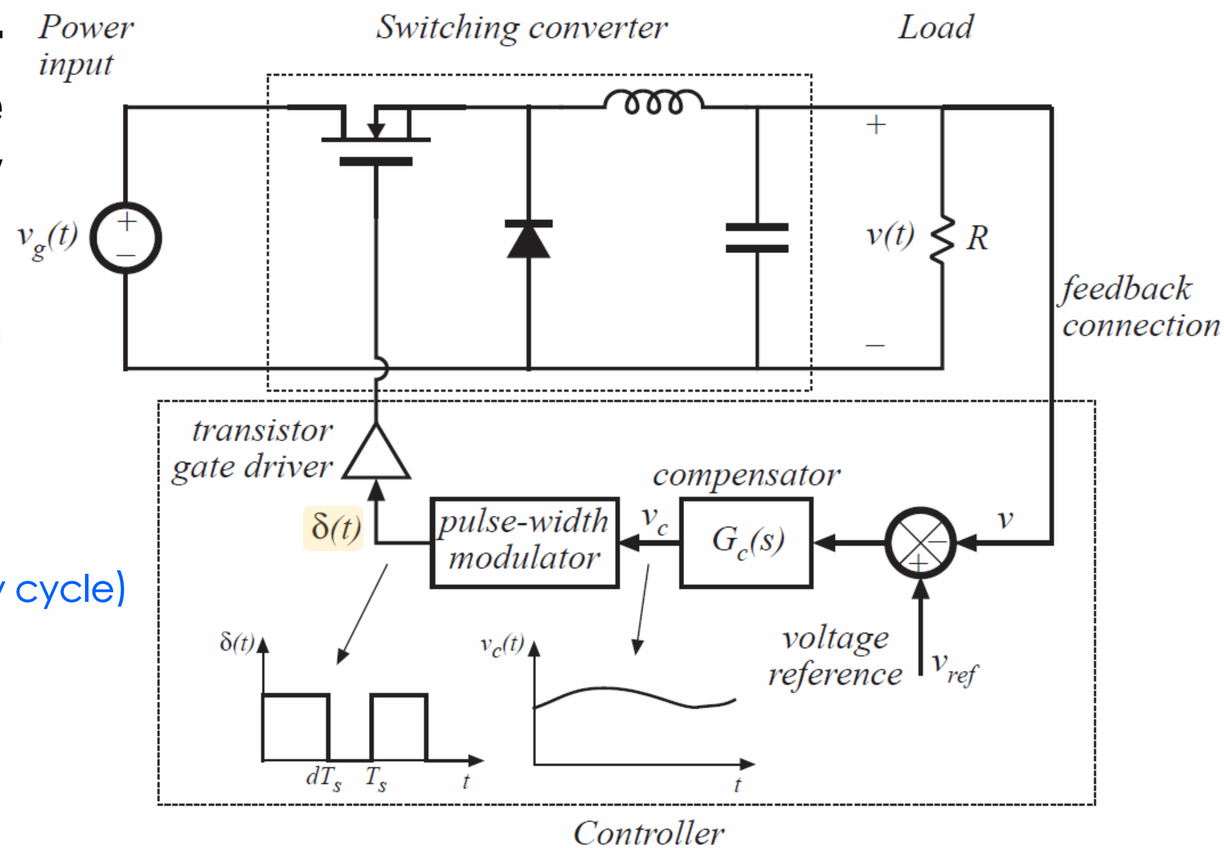
where

$$G_{vd}(s) = \left.\frac{\hat{v}(s)}{\hat{d}(s)}\right|_{\substack{\hat{v}_g=0 \\ \hat{i}_{load}=0}} \qquad G_{vg}(s) = \left.\frac{\hat{v}(s)}{\hat{v}_g(s)}\right|_{\substack{\hat{d}=0 \\ \hat{i}_{load}=0}} \qquad Z_{out}(s) = \left.\frac{\hat{v}(s)}{\hat{i}_{load}(s)}\right|_{\substack{\hat{d}=0 \\ \hat{v}_g=0}}$$

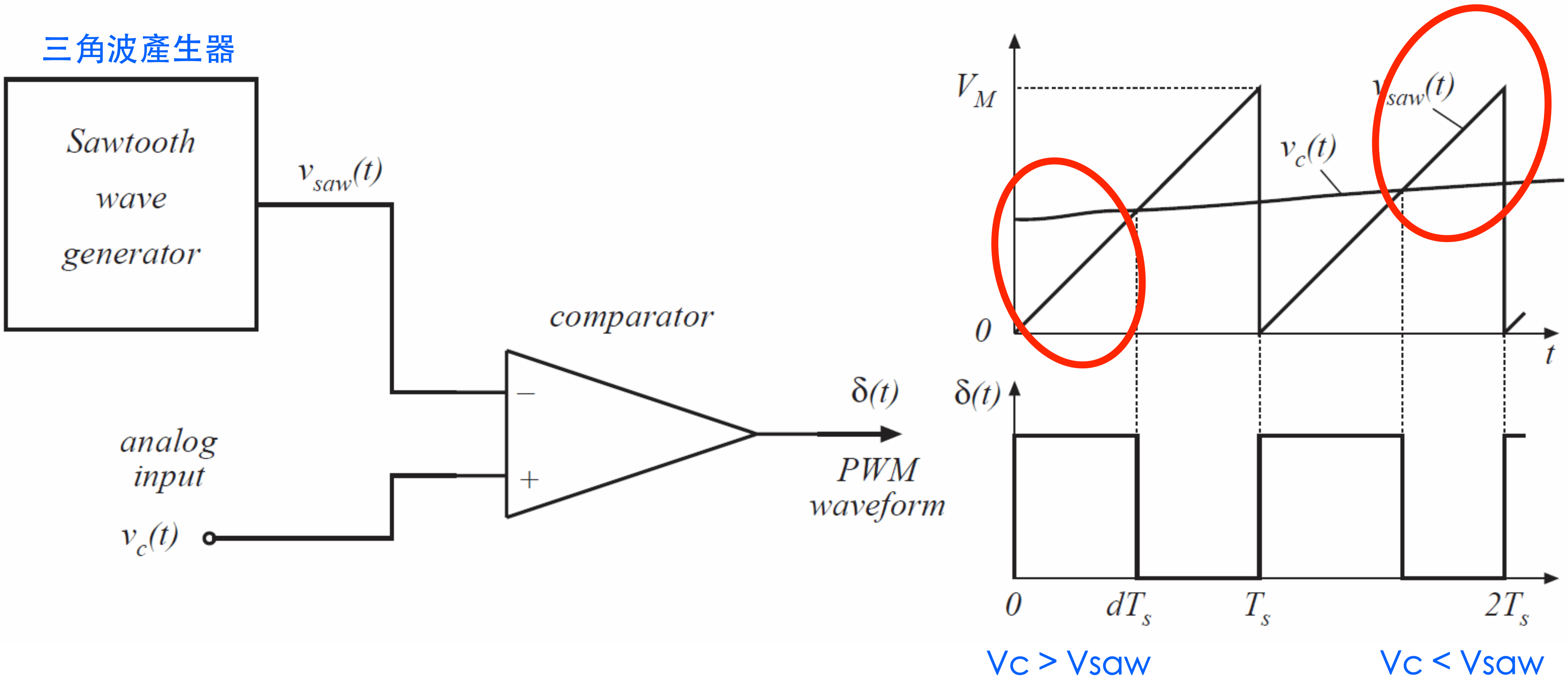
Modeling the Pulse-Width Modulator

- **Pulse-width modulator converts voltage signal $v_c(t)$ into duty cycle signal $d(t)$.**
- **What is the relation between $v_c(t)$ and $d(t)$?**

pulse-width modulator:
DC換成等效pulse width
誤差電壓值換成clock (有duty cycle)



A Simple Pulse-Width Modulator

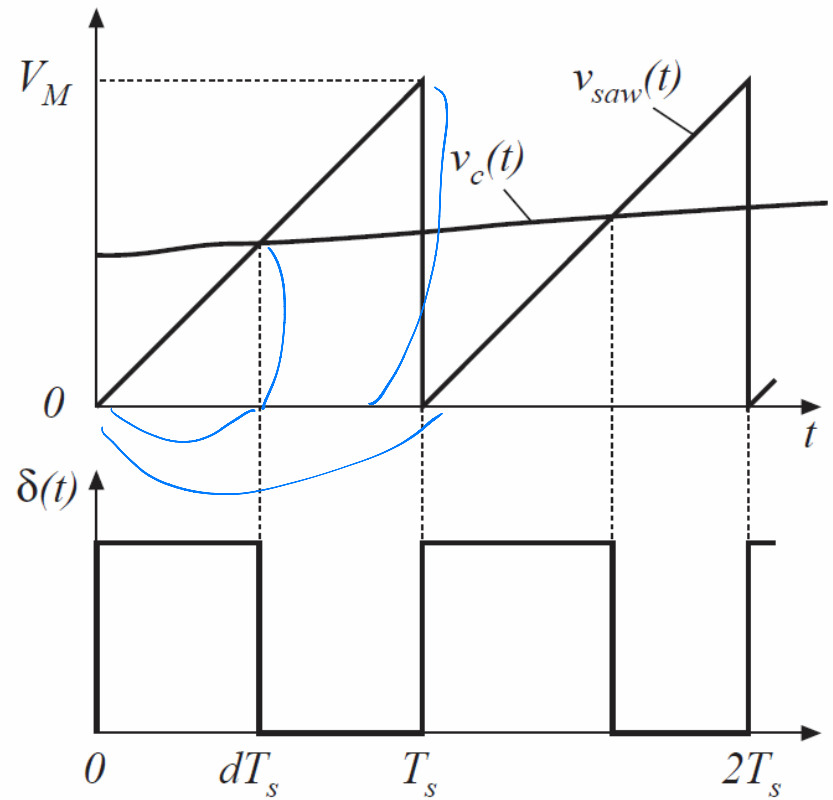


Equation of Pulse-Width Modulator

- For a linear sawtooth waveform:

$$d(t) = \frac{v_c(t)}{V_M} \qquad for\ 0 \le v_c(t) \le V_M$$

$V_M : V_c = T_s : DT_s$



- So ***d(t)* is a linear function of *$v_c(t)$*.**

Perturbed Equation of Pulse-Width Modulator

- **PWM equation:**

$$d(t) = \frac{v_c(t)}{V_M} \qquad for\ 0 \le v_c(t) \le V_M$$

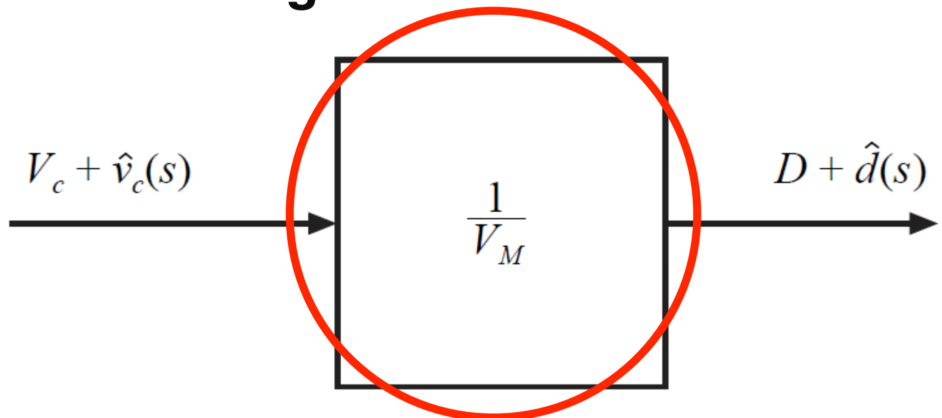
- **Perturb:**

$$v_c(t) = V_c + \hat{v}_c(t)$$
$$d(t) = D + \hat{d}(t)$$

- **Result:**

$$D + \hat{d}(t) = \frac{V_c + \hat{v}_c(t)}{V_M}$$

- **Block diagram:**



pulse-width modulator

上頁的等效block

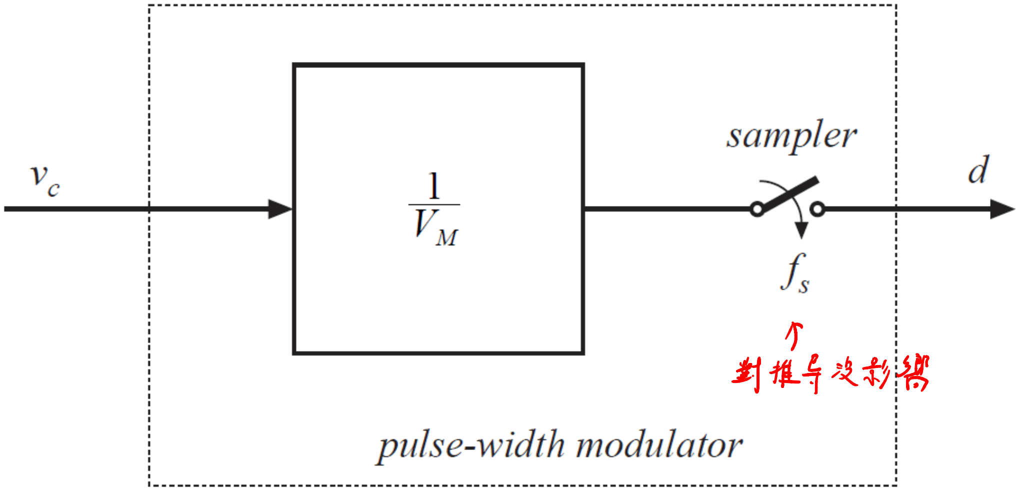
- **DC and AC relations:**

DC $$D = \frac{V_c}{V_M}$$

AC $$\hat{d}(t) = \frac{\hat{v}_c(t)}{V_M}$$

Sampling in Pulse-Width Modulator

- The input voltage is a continuous function of time, but there can be only **one discrete value of the duty cycle** for each switching period.
- Therefore, the pulse-width modulator samples the control waveform, **with sampling rate equal to switching frequency.**



上頁的更精確等效model

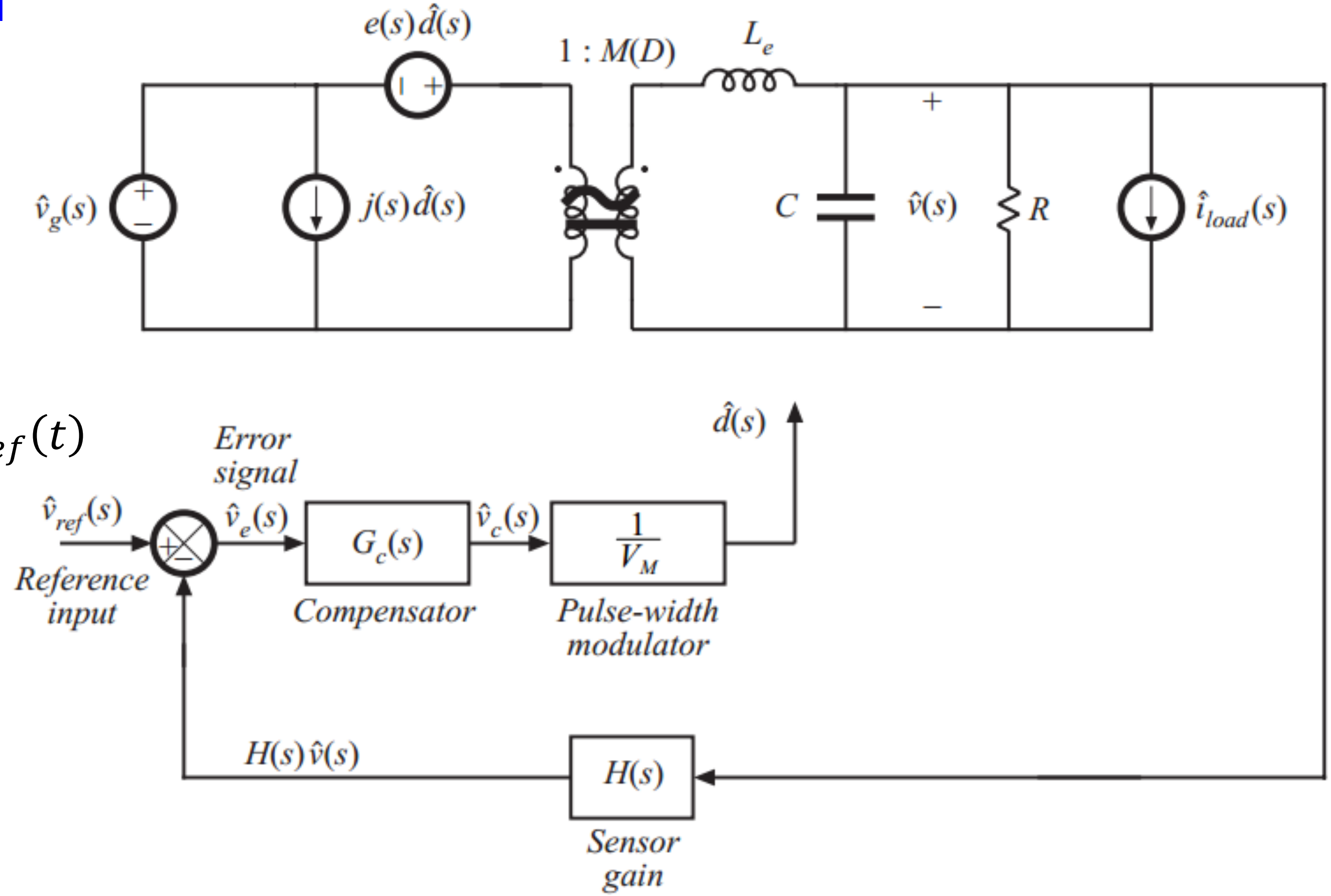
Voltage Regulator System Small-Signal Model

- Use small-signal converter model
- Perturb and linearize remainder of feedback loop:

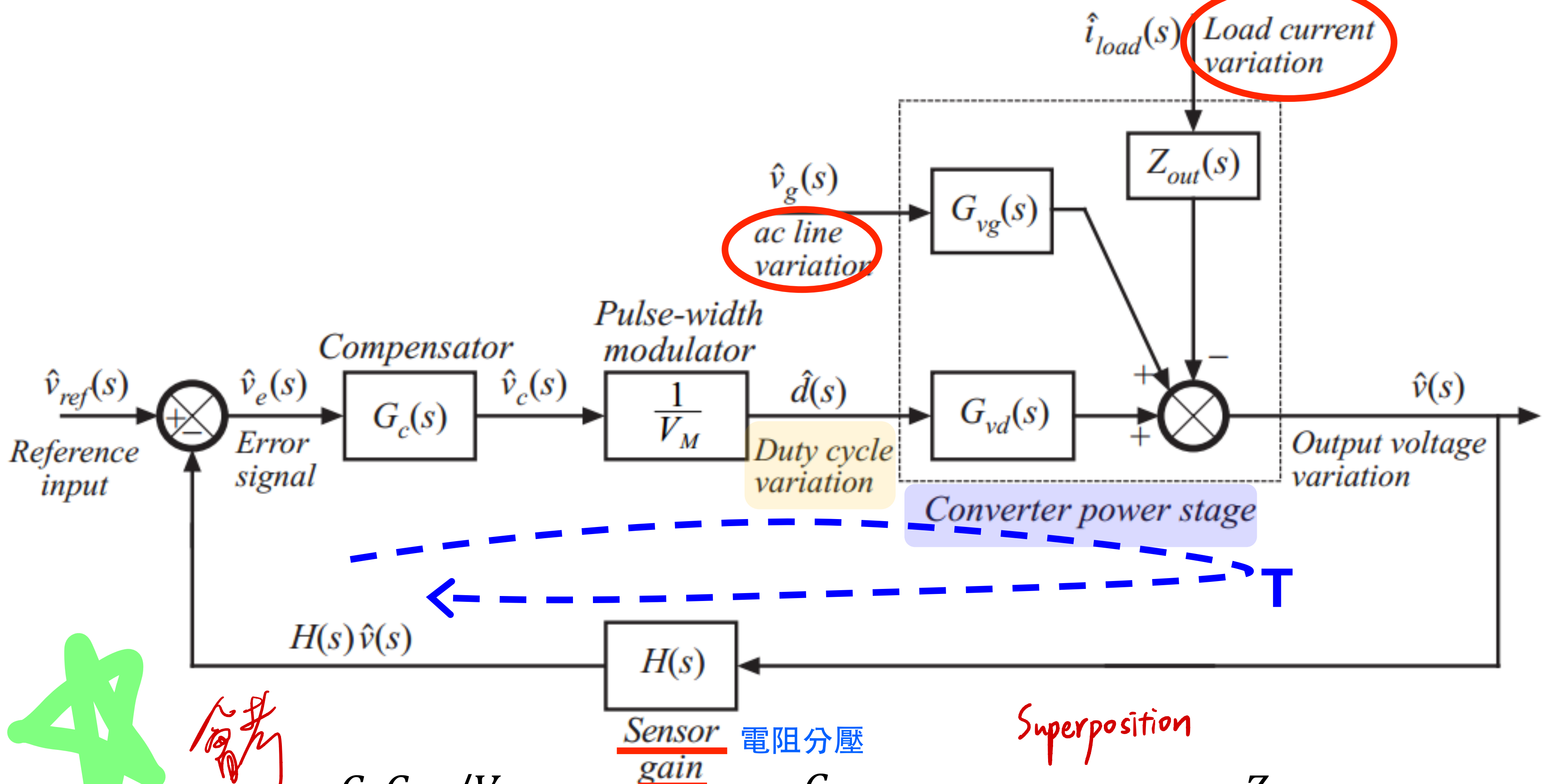
$$v_{ref}(t) = V_{ref} + \hat{v}_{ref}(t)$$

$$v_e(t) = V_e + \hat{v}_e(t)$$

$$etc.$$



Regulator System Small-Signal Block Diagram



會考

Superposition

$$\hat{v} = \hat{v}_{ref}\frac{G_cG_{vd}/V_M}{1+HG_cG_{vd}/V_M} + \hat{v}_g\frac{G_{vg}}{1+HG_cG_{vd}/V_M} - \hat{\imath}_{load}\frac{Z_{out}}{1+HG_cG_{vd}/V_M}$$

Solution of Block Diagram

- Manipulate block diagram to solve for $\hat{v}(s)$. Result is

$$\hat{v} = \hat{v}_{ref}\frac{G_c G_{vd}/V_M}{1 + HG_c G_{vd}/V_M} + \hat{v}_g \frac{G_{vg}}{1 + HG_c G_{vd}/V_M} \pm \hat{\imath}_{load}\frac{Z_{out}}{1 + HG_c G_{vd}/V_M}$$

which is of the form

↗趨近 1

$$\hat{v} = \hat{v}_{ref}\frac{1}{H}\frac{T}{1+T} + \hat{v}_g \frac{G_{vg}}{1+T} \pm \hat{\imath}_{load}\frac{Z_{out}}{1+T}$$

除以(1+T)倍

$$\text{with} \quad T(s) = \big(H(s)G_c(s)G_{vd}(s)\big)/V_M = \text{"}loop\ gain\text{"}$$

Open-loop transfer function

Loop gain *T(s)*= products of the gains around the negative feedback loop.

Feedback Reduces the Transfer Functions from Disturbances to the Output

- Original (open-loop) line-to-output transfer function:

$$G_{vg}(s) = \left.\frac{\hat{v}(s)}{\hat{v}_g(s)}\right|_{\substack{\hat{d}=0 \\ \hat{\imath}_{load}=0}}$$

- With addition of negative feedback, the line-to-output transfer function becomes:

$$\left.\frac{\hat{v}(s)}{\hat{v}_g(s)}\right|_{\substack{\hat{v}_{ref}=0 \\ \hat{\imath}_{load}=0}} = \frac{G_{vg}(s)}{1+T(s)}$$

- Feedback reduces the line-to-output transfer function by a factor of

$$\frac{1}{1+T(s)}$$

大，可降低ripple, variation

- If *T(s)* is large in magnitude, then the line-to-output transfer function becomes small.

Closed-Loop Output Impedance

- Original (open-loop) output impedance:

$$Z_{out}(s) = \left.\frac{\hat{v}(s)}{\hat{\imath}_{load}(s)}\right|_{\substack{\hat{d}=0 \\ \hat{v}_g=0}}$$

- With addition of negative feedback, the output impedance becomes:

$$\left.\frac{\hat{v}(s)}{-\hat{\imath}_{load}(s)}\right|_{\substack{\hat{v}_{ref}=0 \\ \hat{v}_g=0}} = \frac{Z_{out}(s)}{1+T(s)}$$

- Feedback reduces the output impedance by a factor of

$$\frac{1}{1+T(s)}$$

- If *T(s)* is large in magnitude, then the output impedance is greatly reduced in magnitude.

Feedback : Insensitive to Ref. Variations

- Closed-loop transfer function from

$$\left.\frac{\hat{v}(s)}{\hat{v}_{ref}(s)}\right|_{\substack{\hat{v}_g=0 \\ \hat{i}_{load}=0}} = \frac{1}{H(s)}\frac{T(s)}{1+T(s)}$$

- If the loop gain is large in magnitude, i.e., $\|T\| \gg 1$ then $(1+T) \approx T$ and $T/(1+T) \approx T/T = 1$. The transfer function then becomes

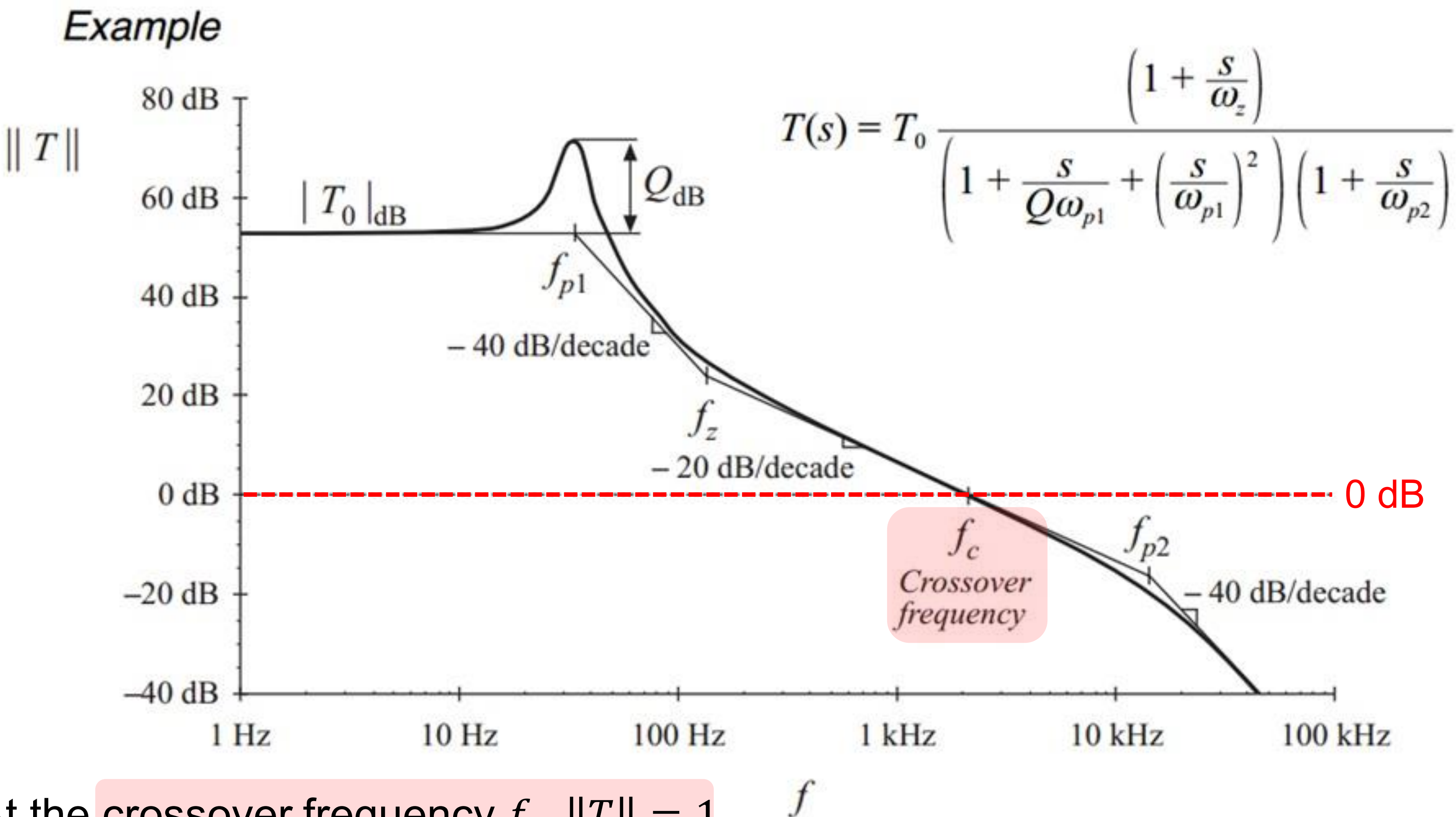
$$\frac{\hat{v}(s)}{\hat{v}_{ref}(s)} \approx \frac{1}{H(s)}$$

which is independent of the gains in the forward path of the loop.

- This result applies equally well to dc values:

$$\frac{V}{V_{ref}} = \frac{1}{H(0)}\frac{T(0)}{1+T(0)} \approx \frac{1}{H(0)}$$

Construction of the Important Quantities 1/(1+T) and T/(1+T)



At the crossover frequency f_c, $\|T\| = 1$

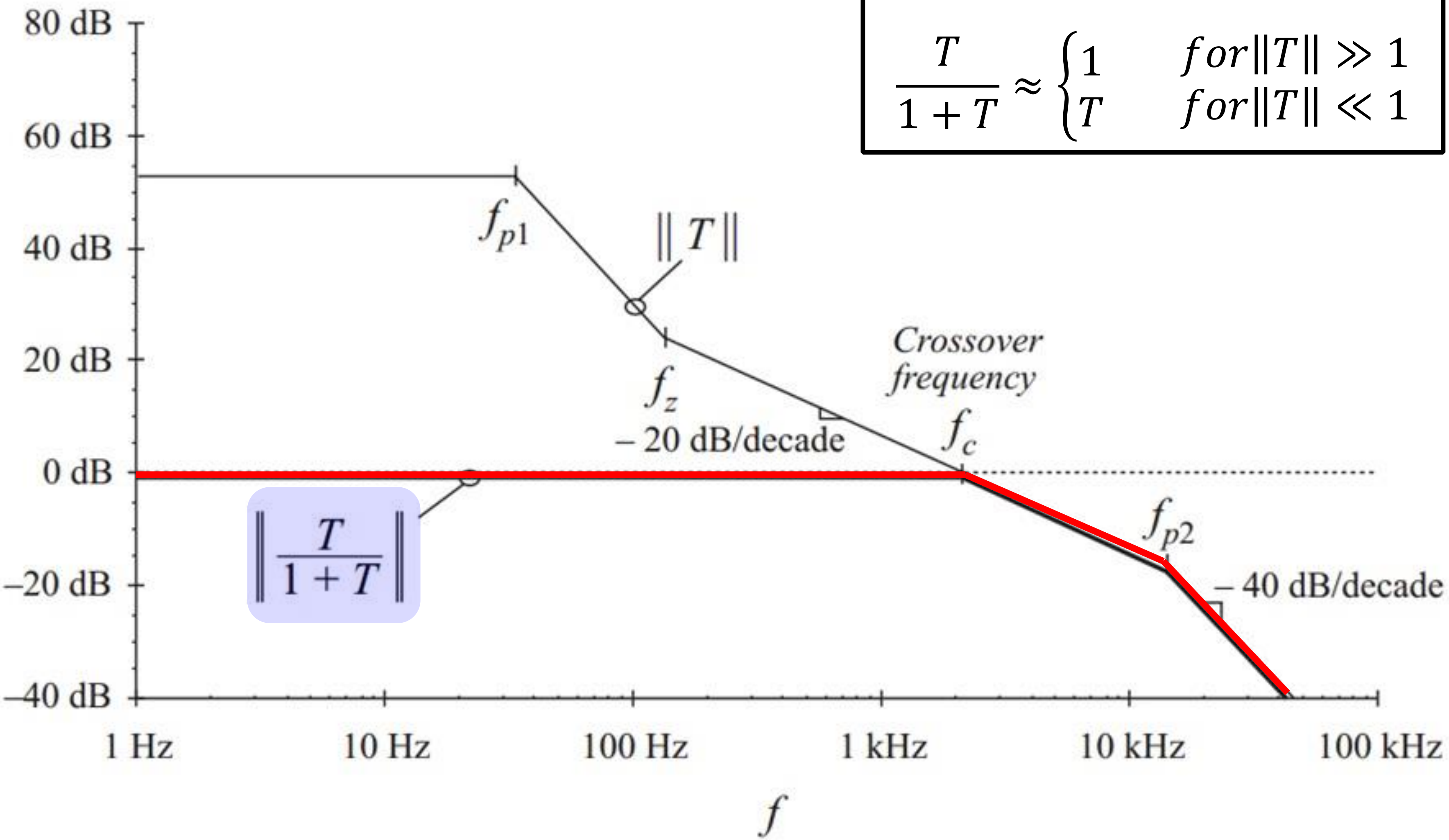
Approximating 1/(1+T) and T/(1+T)

$$\frac{T}{1+T} \approx \begin{cases} 1 & for \|T\| \gg 1 \\ T & for \|T\| \ll 1 \end{cases}$$

$$\frac{1}{1+T(s)} \approx \begin{cases} \dfrac{1}{T(s)} & for \|T\| \gg 1 \\ 1 & for \|T\| \ll 1 \end{cases}$$

Example: Construction of T/(1+T)

$$\frac{T}{1+T} \approx \begin{cases} 1 & for\|T\| \gg 1 \\ T & for\|T\| \ll 1 \end{cases}$$



Example: Analytical Expressions for Approximate Reference to Output Transfer Function

- At frequencies sufficiently less that the crossover frequency, the loop gain T(s) has large magnitude.
  - The transfer function from the reference to the output becomes

$$\frac{\hat{v}(s)}{\hat{v}_{ref}(s)} = \frac{1}{H(s)}\frac{T(s)}{1+T(s)} \approx \frac{1}{H(s)}$$

  - This is the desired behavior: the output follows the reference according to the ideal gain 1/H(s). The feedback loop works well at frequencies where the loop gain T(s) has large magnitude.

- At frequencies above that the crossover frequency, ||T||<1

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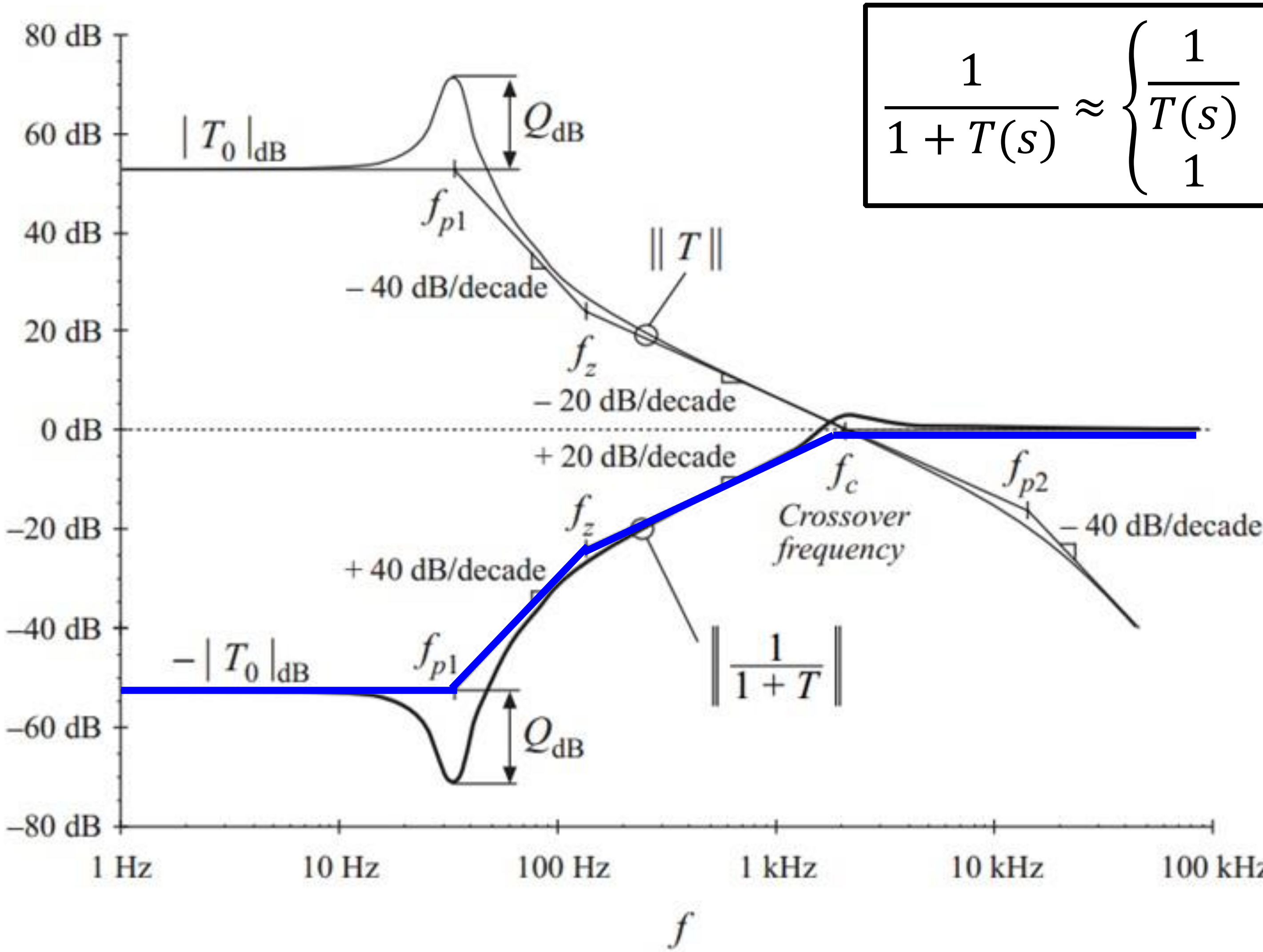
  - The quantity T/(1+T) then has magnitude approximately equal to T, and obtain

$$\frac{\hat{v}(s)}{\hat{v}_{ref}(s)} = \frac{1}{H(s)}\frac{T(s)}{1+T(s)} \approx \frac{T(s)}{H(s)} = \frac{G_c(s)G_{vd}(s)}{V_M}$$

  - This coincides with the open-loop transfer function from the reference to the output. At frequencies where ||T||<1, the loop has essentially no effect on the transfer function from the reference to the output.

Same Example: Construction of 1/(1+T)

$$\frac{1}{1+T(s)} \approx \begin{cases} \frac{1}{T(s)} & for\|T\| \gg 1 \\ 1 & for\|T\| \ll 1 \end{cases}$$



Interpretation: How the Loop Rejects Disturbances

$$\frac{1}{1+T(s)} \approx \begin{cases} \dfrac{1}{T(s)} & for \|T\| \gg 1 \\ 1 & for \|T\| \ll 1 \end{cases}$$

- Below the crossover frequency: $f < f_c$ and $\|T\| > 1$

  Then $1/(1+T) \approx 1/T$, and disturbances are reduced in magnitude by $1/\|T\|$

- Above the crossover frequency: $f > f_c$ and $\|T\| < 1$

  Then $1/(1+T) \approx 1$, and the feedback loop has essentially no effect on disturbances

Terminology: Open-Loop vs. Closed-Loop

- Original transfer functions, before introduction of feedback ("open-loop transfer functions"):

$$G_{vd}(s) \quad G_{vg}(s) \quad Z_{out}(s)$$

- Upon introduction of feedback, these transfer functions become ("closed-loop transfer functions"):

$$\frac{1}{H(s)}\frac{T(s)}{1+T(s)} \qquad \frac{G_{vg}(s)}{1+T(s)} \qquad \frac{Z_{out}(s)}{1+T(s)}$$

- The loop gain:

$$T(s)$$

Stability

- Even though the original open-loop system is stable, the closed-loop transfer functions can be unstable and contain right half-plane poles. Even when the closed-loop system is stable, the transient response can exhibit undesirable ringing and overshoot, due to the high Q -factor of the closed- loop poles in the vicinity of the crossover frequency.

- When feedback destabilizes the system, the denominator (1+$T(s)$) terms in the closed-loop transfer functions contain roots in the right half-plane (i.e., with positive real parts). If $T(s)$ is a rational fraction of the form $N(s)/D(s)$, where $N(s)$ and $D(s)$ are polynomials, then we can write

$$\frac{T(s)}{1+T(s)} = \frac{\frac{N(s)}{D(s)}}{1+\frac{N(s)}{D(s)}} = \frac{N(s)}{N(s)+D(s)}$$

$$\frac{1}{1+T(s)} = \frac{1}{1+\frac{N(s)}{D(s)}} = \frac{D(s)}{N(s)+D(s)}$$

- Could evaluate stability by evaluating $N(s) + D(s)$, then factoring to evaluate roots. This is a lot of work, and is not very illuminating.

Determination of Stability Directly from T(s)

- **Nyquist stability theorem: general result.**
  - **Number of right half-plane roots of (N(s)+D(s)) can be determined by testing T(s).**

- **A special case of the Nyquist stability theorem: the phase margin test.**
  - **Allows determination of closed-loop stability (i.e., whether *1/(1+T(s))* contains RHP poles) directly from the magnitude and phase of *T(s)*. A good design tool: yields insight into how T(s)should be shaped, to obtain good performance in transfer functions containing *1/(1+T(s))* terms.**

The Phase Margin Test

A test on $T(s)$, to determine whether $1/(1+T(s))$ contains RHP poles. The crossover frequency f_c is defined as the frequency where

$$\|T(j2\pi f_c)\| = 1 \Rightarrow 0\text{dB}$$

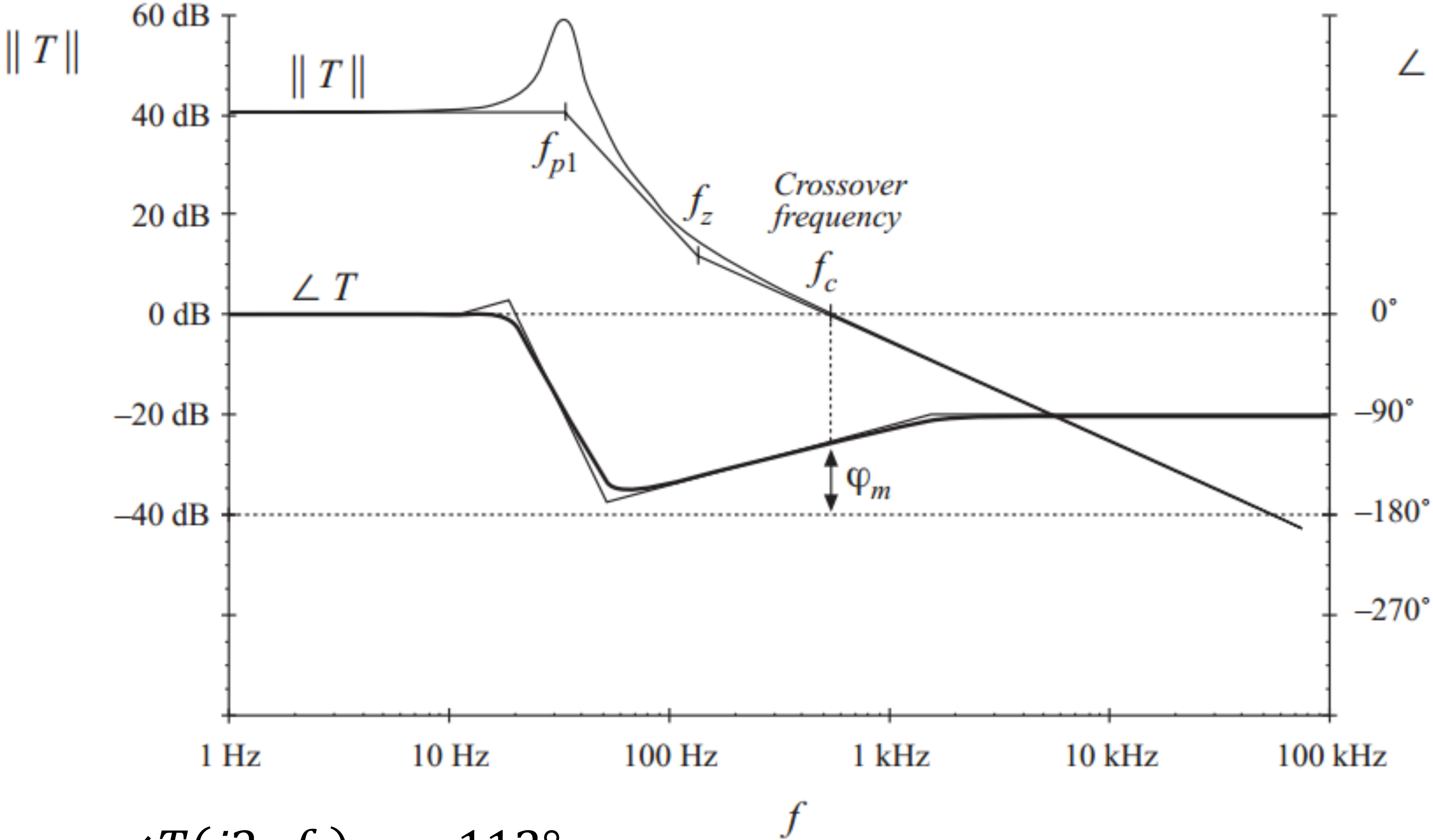
The phase margin φ_m is determined from the phase of $T(s)$ at f_c , as follows:

$$\varphi_m = 180° + \angle T(j2\pi f_c)$$

If there is exactly one crossover frequency, and if $T(s)$ contains no RHP poles, then

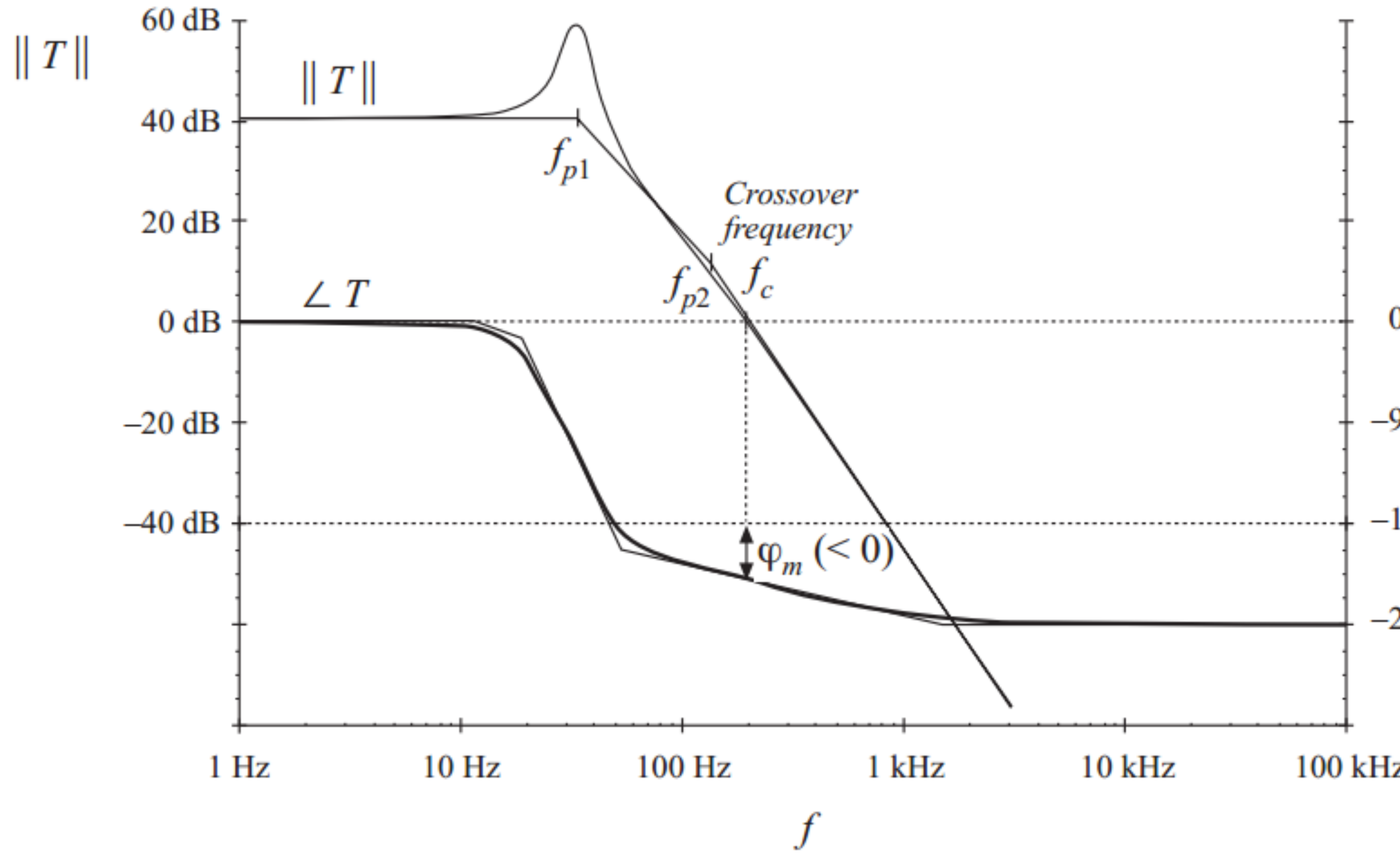
the quantities $T(s)/(1+T(s))$ and $1/(1+T(s))$ contain no RHP poles whenever the phase margin φ_m is positive.

Example: a Loop Gain Leading to a Stable Closed-Loop System



$$\angle T(j2\pi f_c) = -112°$$
$$\varphi_m = 180° - 112° = +68°$$

Example: a Loop Gain Leading to an Unstable Closed-Loop System



$$\angle T(j2\pi f_c) = -230°$$

$$\varphi_m = 180° - 230° = -50°$$

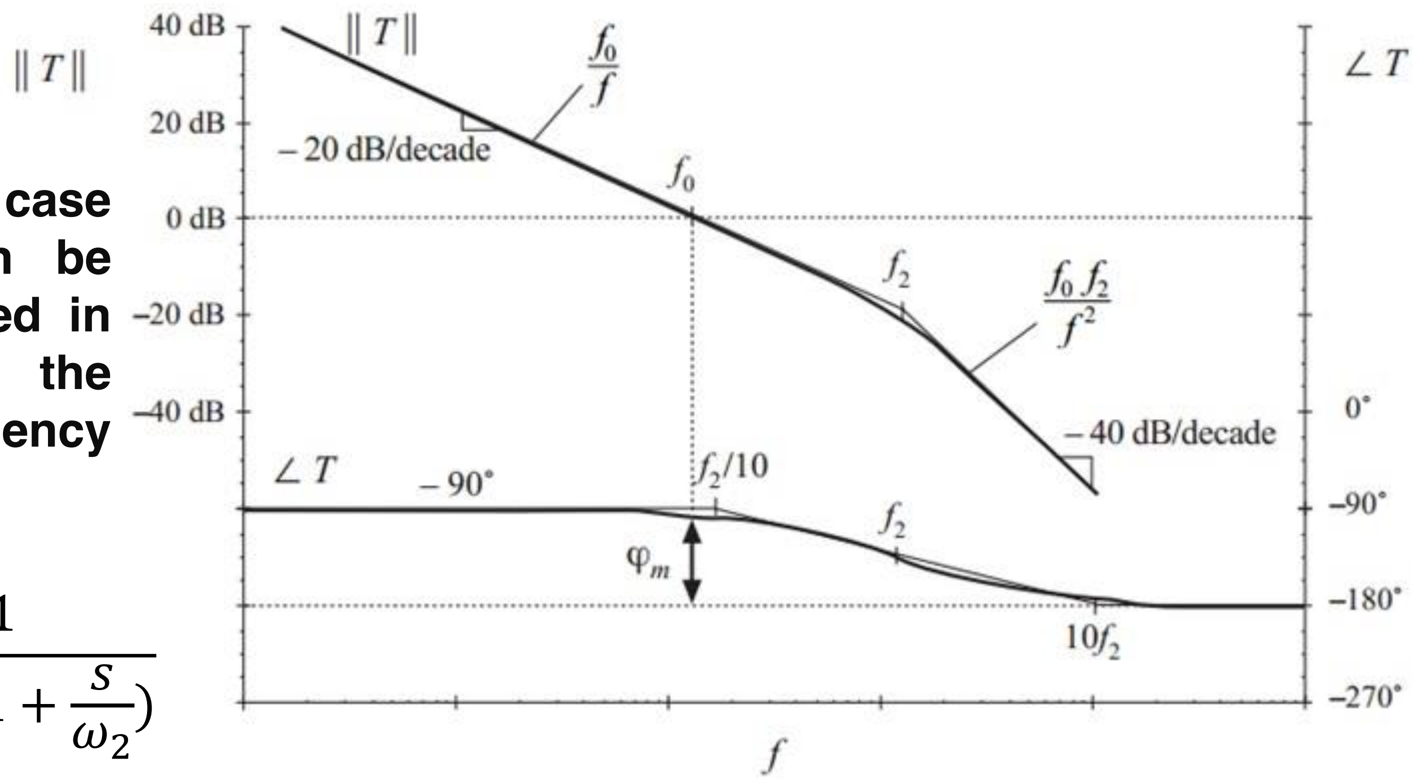
The Relation Between Phase Margin and Closed-loop Damping Factor

- How much phase margin is required?
  - A small positive phase margin leads to a stable closed-loop system having near the crossover frequency with high *Q*.
  - The transient response exhibits overshoot and ringing.
- Increasing the phase margin reduces the *Q*.
  - Obtaining real poles, with no overshoot and ringing, requires a large phase margin.
  - The relation between phase margin and closed-loop *Q* is quantified in this section.

A Simple Second-Order System

Consider the case where *T(s)* can be well-approximated in the vicinity of the crossover frequency as

$$T(s) = \frac{1}{(\frac{s}{\omega_0})(1+\frac{s}{\omega_2})}$$



Closed-Loop Response

If

$$T(s) = \frac{1}{(\frac{s}{\omega_0})(1 + \frac{s}{\omega_2})}$$

Then

$$\frac{T(s)}{1 + T(s)} = \frac{1}{1 + \frac{1}{T(s)}} = \frac{1}{1 + \frac{s}{\omega_0} + \frac{s^2}{\omega_0 \omega_2}}$$

or,

$$\frac{T(s)}{1 + T(s)} = \frac{1}{1 + \frac{s}{Q\omega_c} + \left(\frac{s}{\omega_c}\right)^2}$$

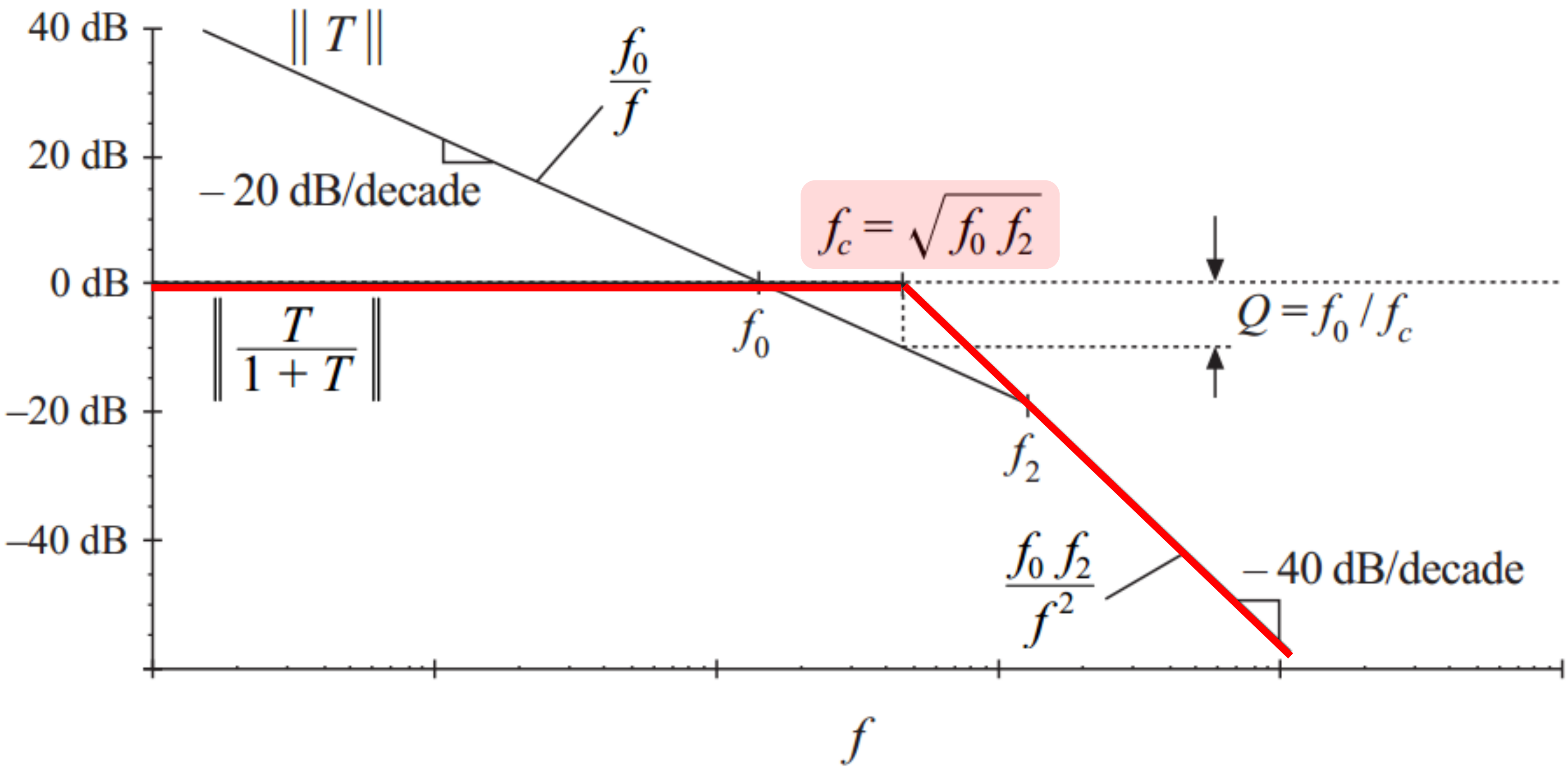
where

$$\omega_c = \sqrt{\omega_0 \omega_2} = 2\pi f_c \qquad Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}}$$

Low-Q Case

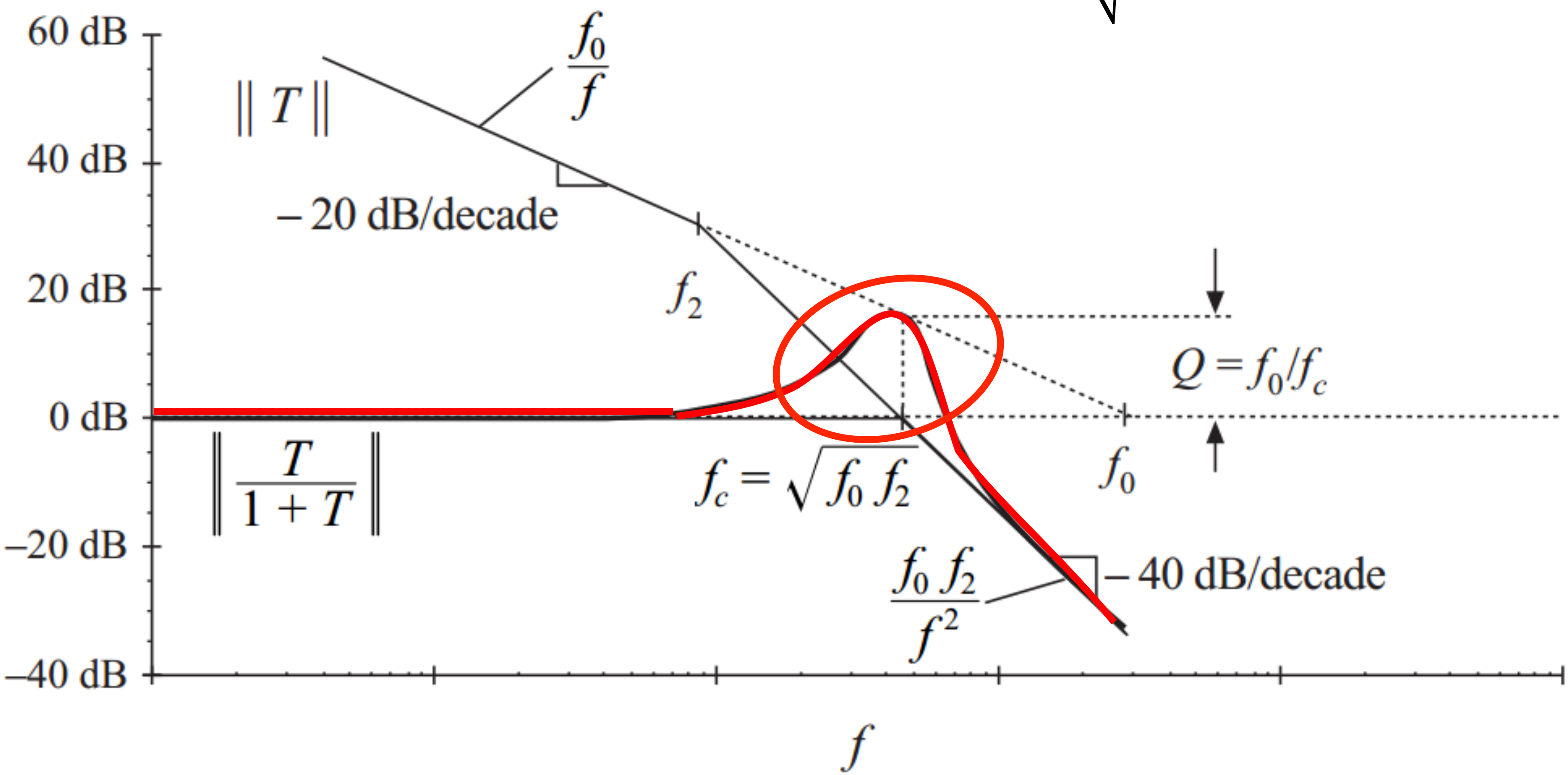
$$Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}}$$

low-Q approximation: $Q\omega_c = \omega_0 \qquad \frac{\omega_c}{Q} = \omega_2$



High-Q Case

$$\omega_c = \sqrt{\omega_0 \omega_2} = 2\pi f_c \qquad Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}}$$



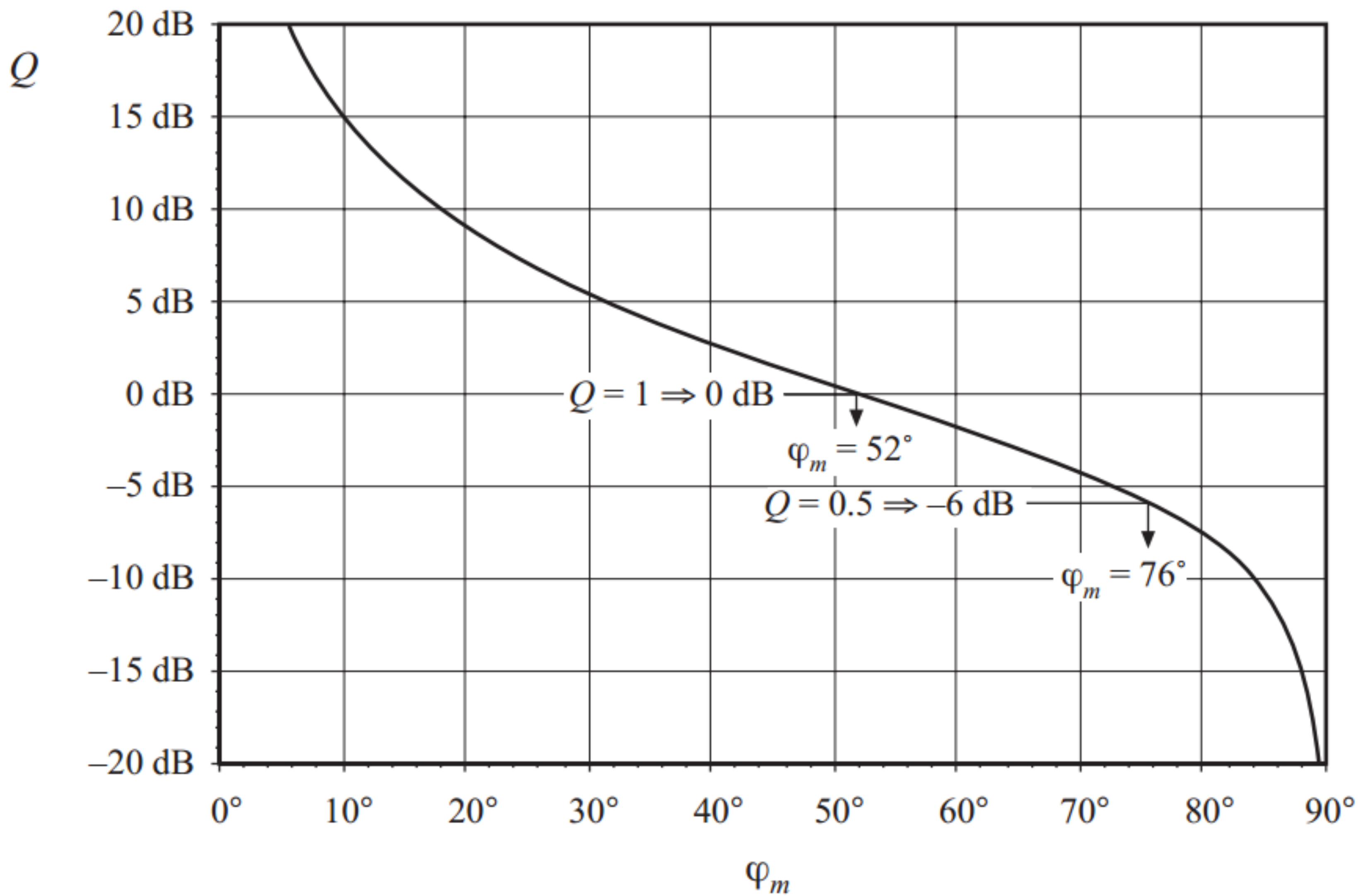
Q vs. ϕm

- Solve for exact crossover frequency, evaluate phase margin, express as function of ϕ_m.
- Result is:

$$Q = \frac{\sqrt{\cos \varphi_m}}{\sin \varphi_m}$$

$$\varphi_m = \tan^{-1}\sqrt{\frac{1+\sqrt{1+4Q^4}}{2Q^4}}$$

Q vs. ϕm (cont'd)



Q
20 dB
15 dB
10 dB
5 dB
0 dB
–5 dB
–10 dB
–15 dB
–20 dB
Q = 1 ⇒ 0 dB
φm = 52°
Q = 0.5 ⇒ –6 dB
φm = 76°
0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
φm

Transient Response vs. Damping Factor

- **Unit-step response of second-order system *T(s)/(1+T(s))***

$$\hat{v}(t) = 1 + \frac{2Qe^{-\omega_c t/2Q}}{\sqrt{4Q^2-1}} \sin\left[\frac{\sqrt{4Q^2-1}}{2Q}\omega_c t + \tan^{-1}\left(\sqrt{4Q^2-1}\right)\right] \quad Q > 0.5$$

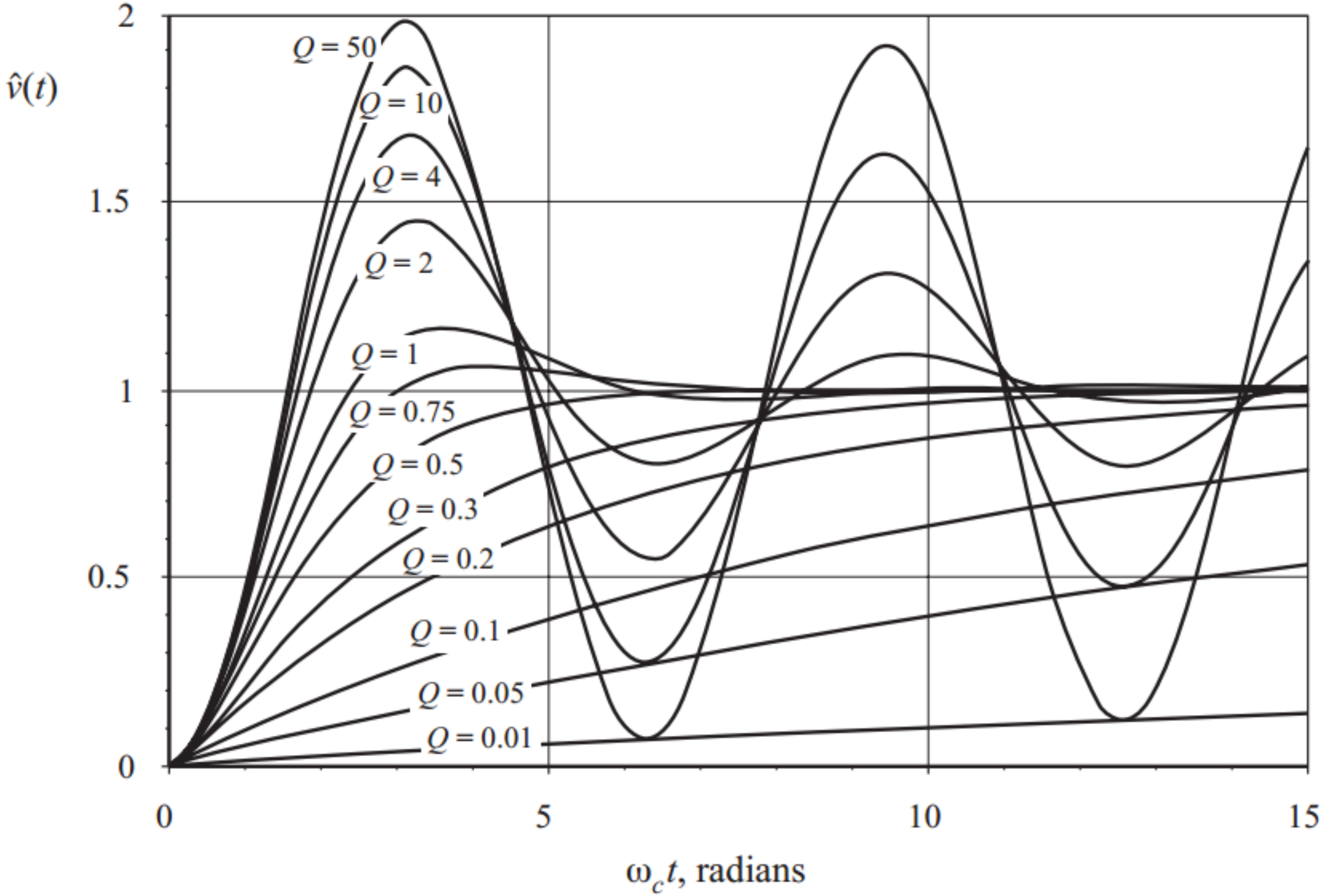
$$\hat{v}(t) = 1 - \frac{\omega_2}{\omega_2-\omega_1}e^{-\omega_1 t} - \frac{\omega_1}{\omega_1-\omega_2}e^{-\omega_2 t} \quad Q < 0.5$$

$$\omega_1, \omega_2 = \frac{\omega_c}{2Q}\left(1 \pm \sqrt{1-4Q^2}\right)$$

- **For $Q > 0.5$, the peak value is**

$$peak\ \hat{v}(t) = 1 + e^{-\pi/\sqrt{4Q^2-1}}$$

Transient Response vs. Damping Factor



Regulator Design

- **Typical specifications:** DC gain要夠高，Bandwidth要寬
  - Effect of load current variations on output voltage regulation
    - This is a limit on the maximum allowable output impedance
  - Effect of input voltage variations on the output voltage regulation
    - This limits the maximum allowable line-to-output transfer function
  - Transient response time
    - This requires a sufficiently high crossover frequency
  - Overshoot and ringing
    - An adequate phase margin must be obtained

$\frac{1}{1+T}$

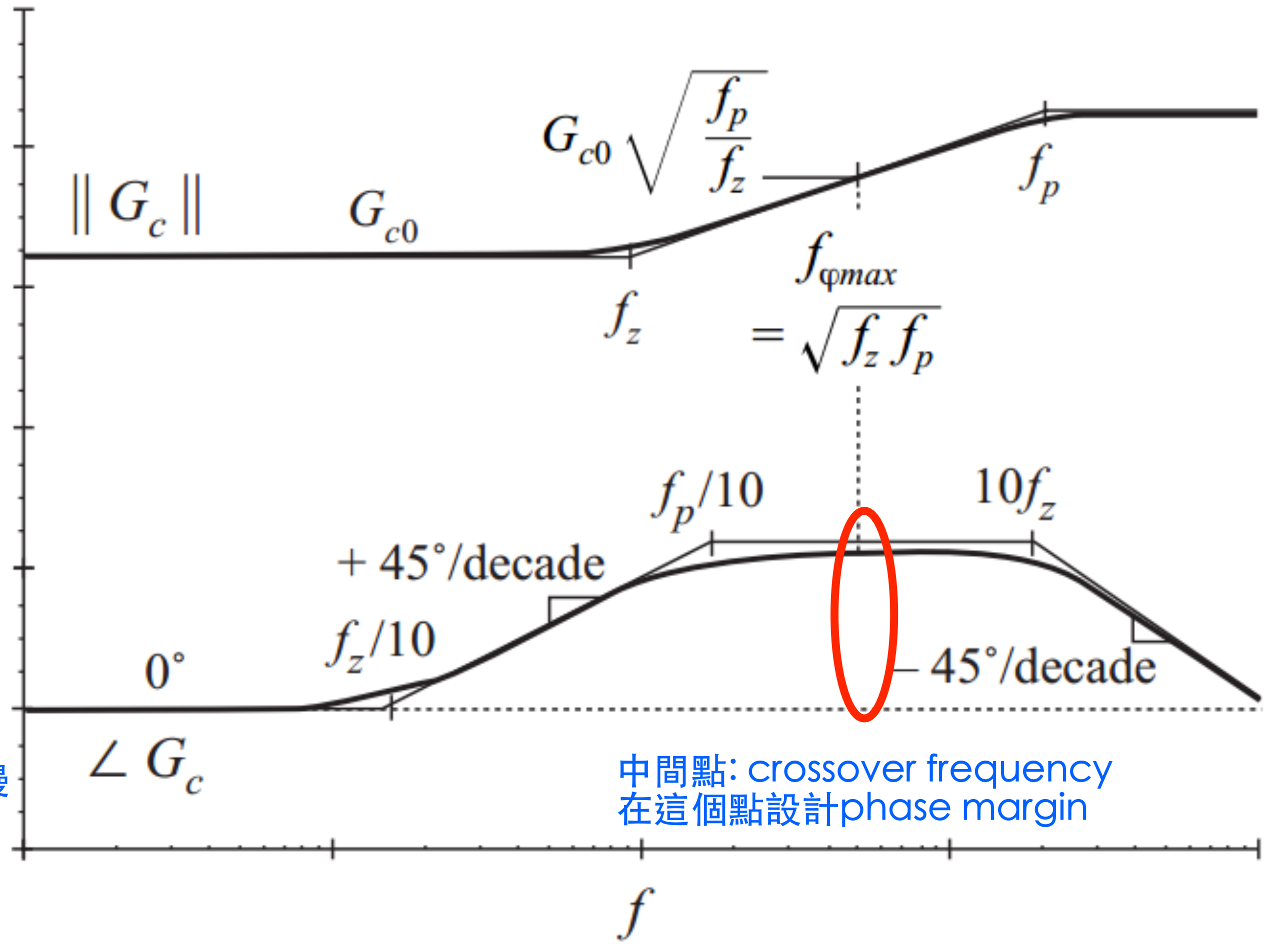
- **The regulator design problem: add compensator network $G_c(s)$ to modify $T(s)$ such that all specifications are met.**

Compensator Type

- **Lead (PD) compensator:**
  - A zero is added to the loop gain at a frequency f_Z sufficiently far below the crossover frequency f_C to improve the phase margin.
- **Lag (PI) compensator:**
  - An inverted zero is added to the loop gain at frequency f_L to improve low-frequency loop gain and regulation
- **Combined (PID) compensator:**
  - The advantages of the lead and lag compensators can be combined, to obtain both wide bandwidth and zero steady-state error.

Lead (PD) Compensator

$$G_c(s) = G_{c0}\frac{\left(1+\frac{s}{\omega_z}\right)}{\left(1+\frac{s}{\omega_p}\right)}$$



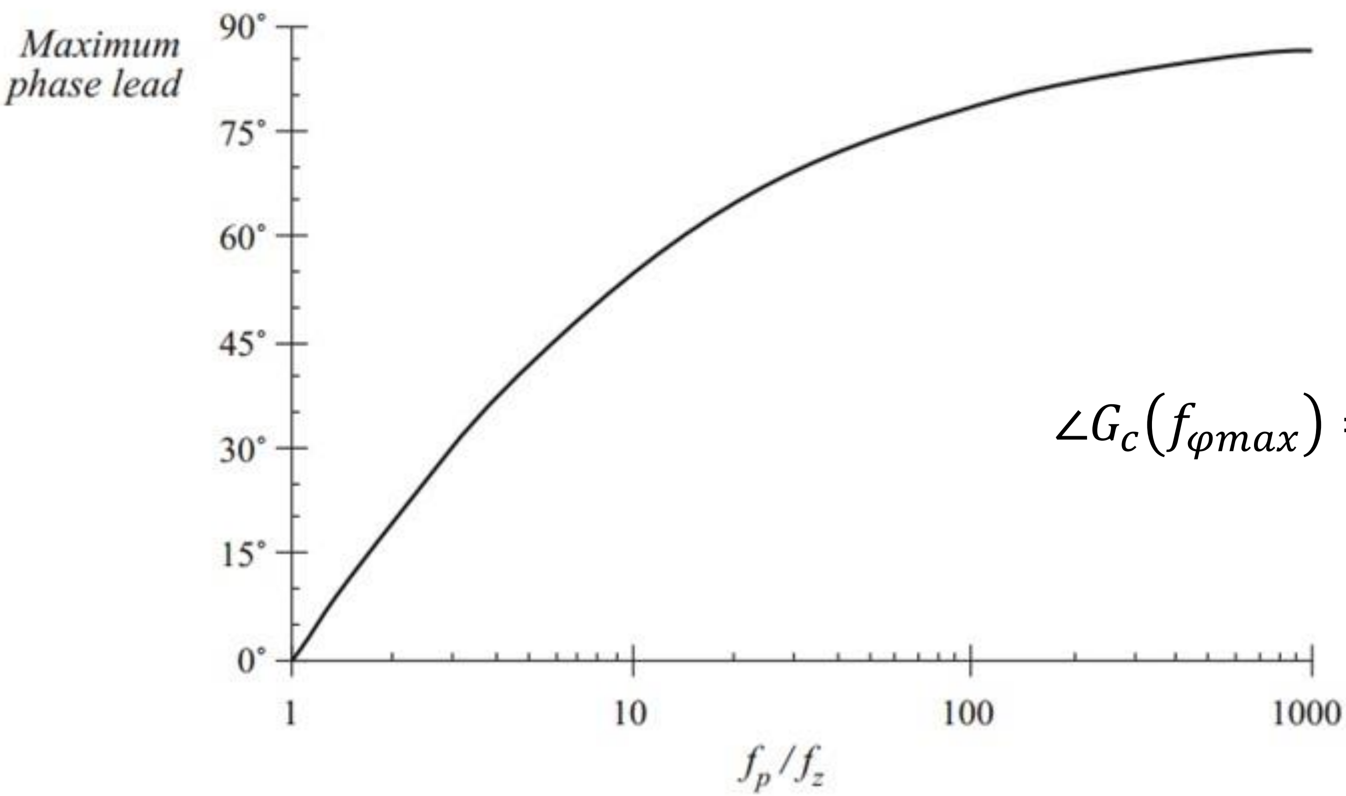
- A zero is added to the loop gain, at frequency far below f_C.

但可能造成較高頻的地方衰減變慢
藉此提高switching noise

中間點: crossover frequency
在這個點設計phase margin

Improves phase margin

Lead Compensator: Maximum Phase Lead



$$f_{\varphi max} = \sqrt{f_z f_p}$$

$$\angle G_c(f_{\varphi max}) = \tan^{-1}\left(\frac{\sqrt{\frac{f_p}{f_z}} - \sqrt{\frac{f_z}{f_p}}}{2}\right)$$

$$\frac{f_p}{f_z} = \frac{1 + \sin(\theta)}{1 - \sin(\theta)}$$

Lead Compensator Design

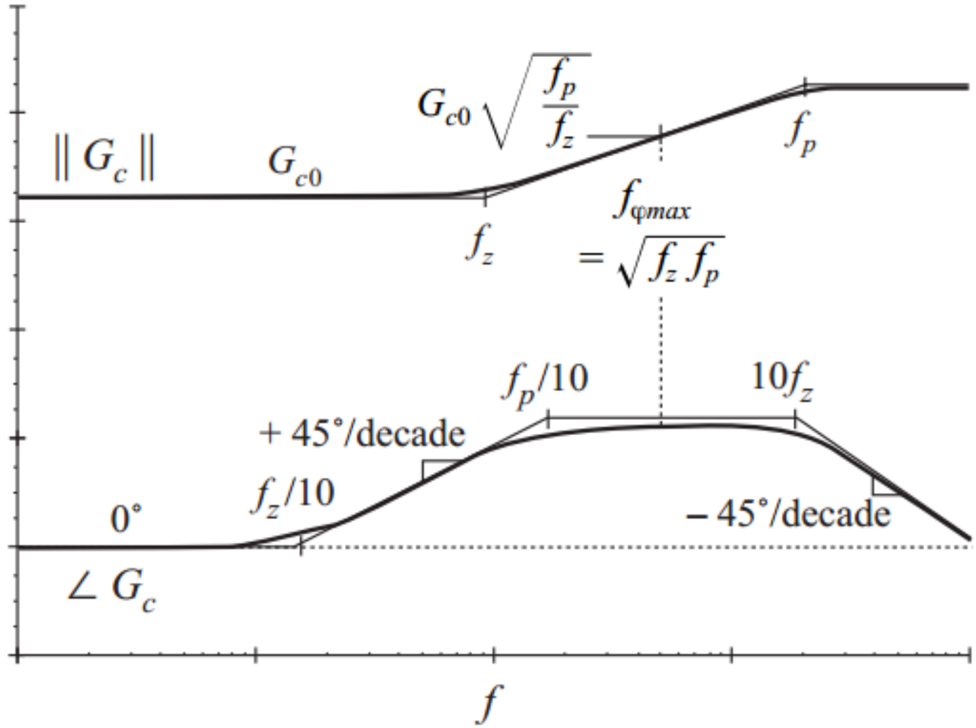
- To optimally obtain a compensator phase lead of θ at frequency f_c , the pole and zero frequencies should be chosen as follows:

$$f_z = f_c\sqrt{\frac{1-\sin(\theta)}{1+\sin(\theta)}}$$

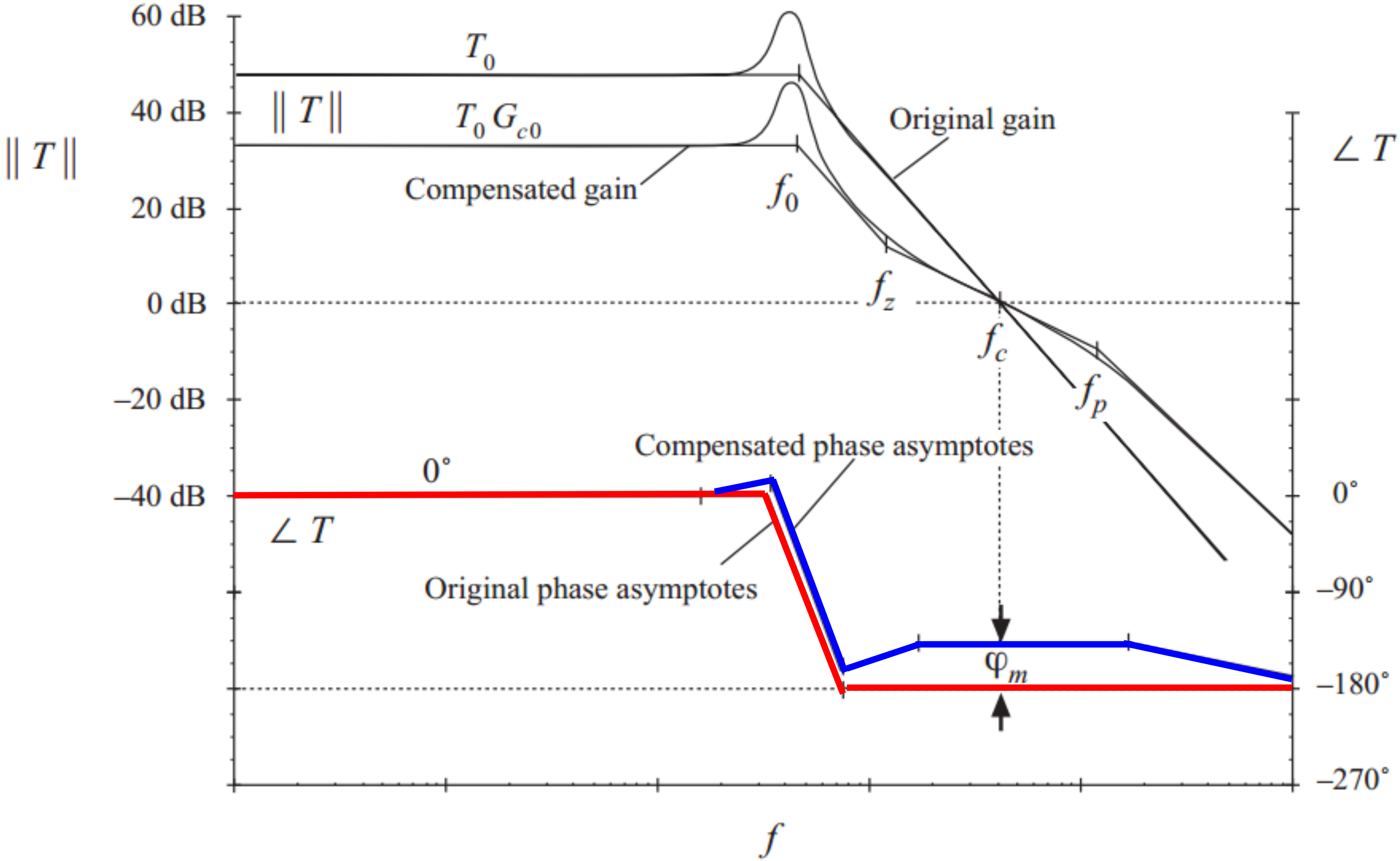
$$f_p = f_c\sqrt{\frac{1+\sin(\theta)}{1-\sin(\theta)}}$$

- If it is desired that the magnitude of the compensator gain at f_c be unity, then G_{c0} should be

$$G_{c0} = \sqrt{\frac{f_z}{f_p}}$$



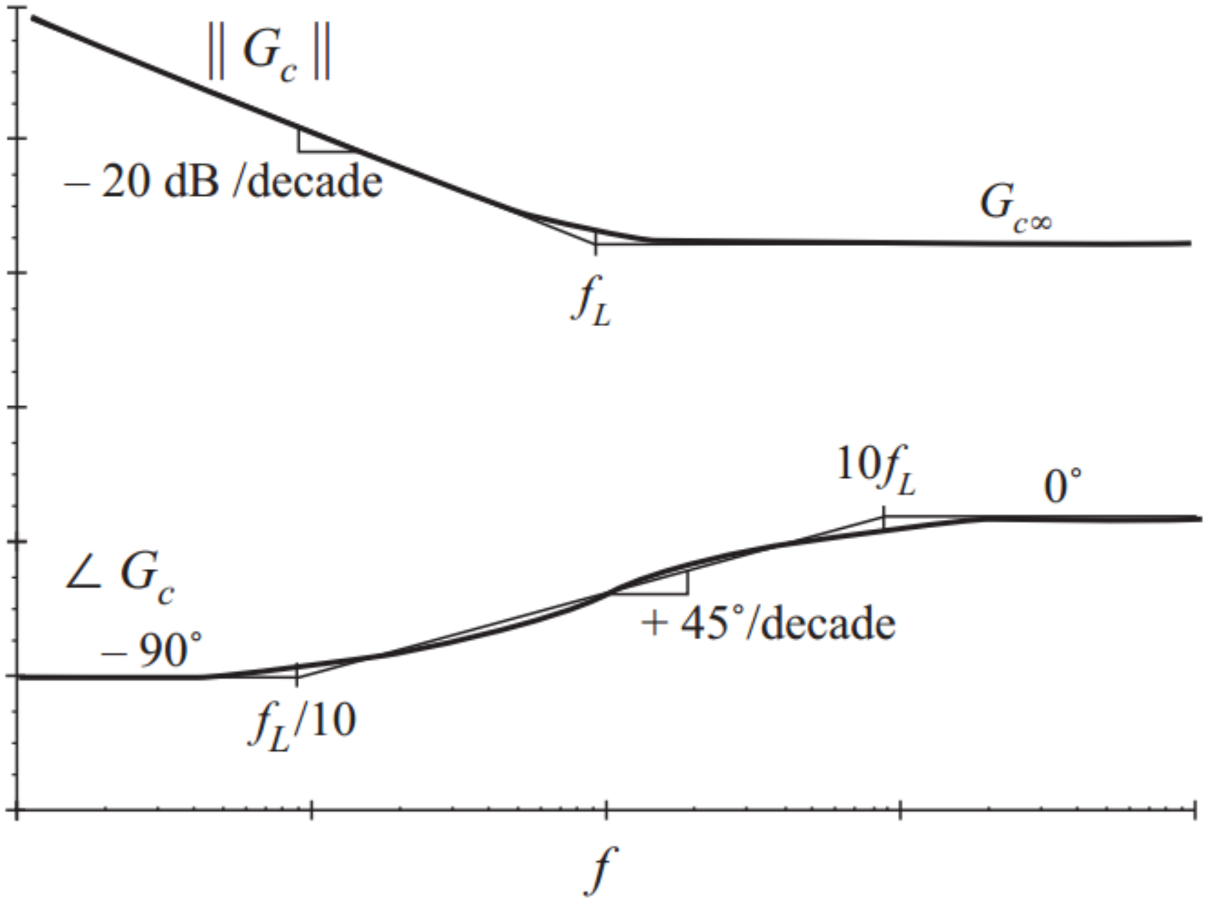
Example: Lead Compensation



Lag (PI) Compensation

$$G_c(s) = G_{c\infty}\left(1 + \frac{\omega_L}{s}\right)$$

- **An inverted zero is added to the loop gain at frequency f_L**



Improves low-frequency loop gain and regulation

Example: Lag Compensation

- Original loop gain (uncompensated) is

$$T_u(s) = \frac{T_{u0}}{\left(1 + \frac{s}{\omega_0}\right)}$$

- Compensator:

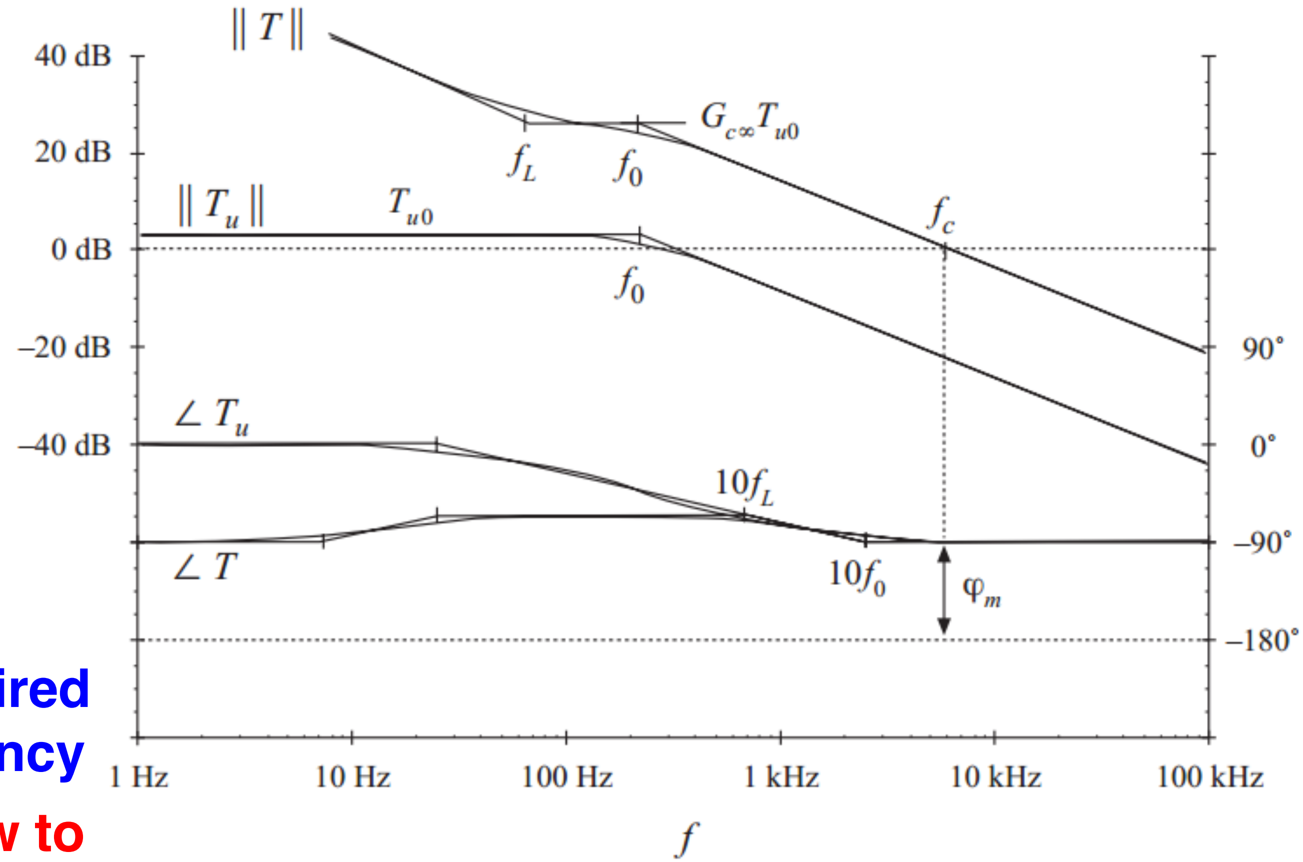
$$G_c(s) = G_{c\infty}\left(1 + \frac{\omega_L}{s}\right)$$

- Design strategy:

  Choose

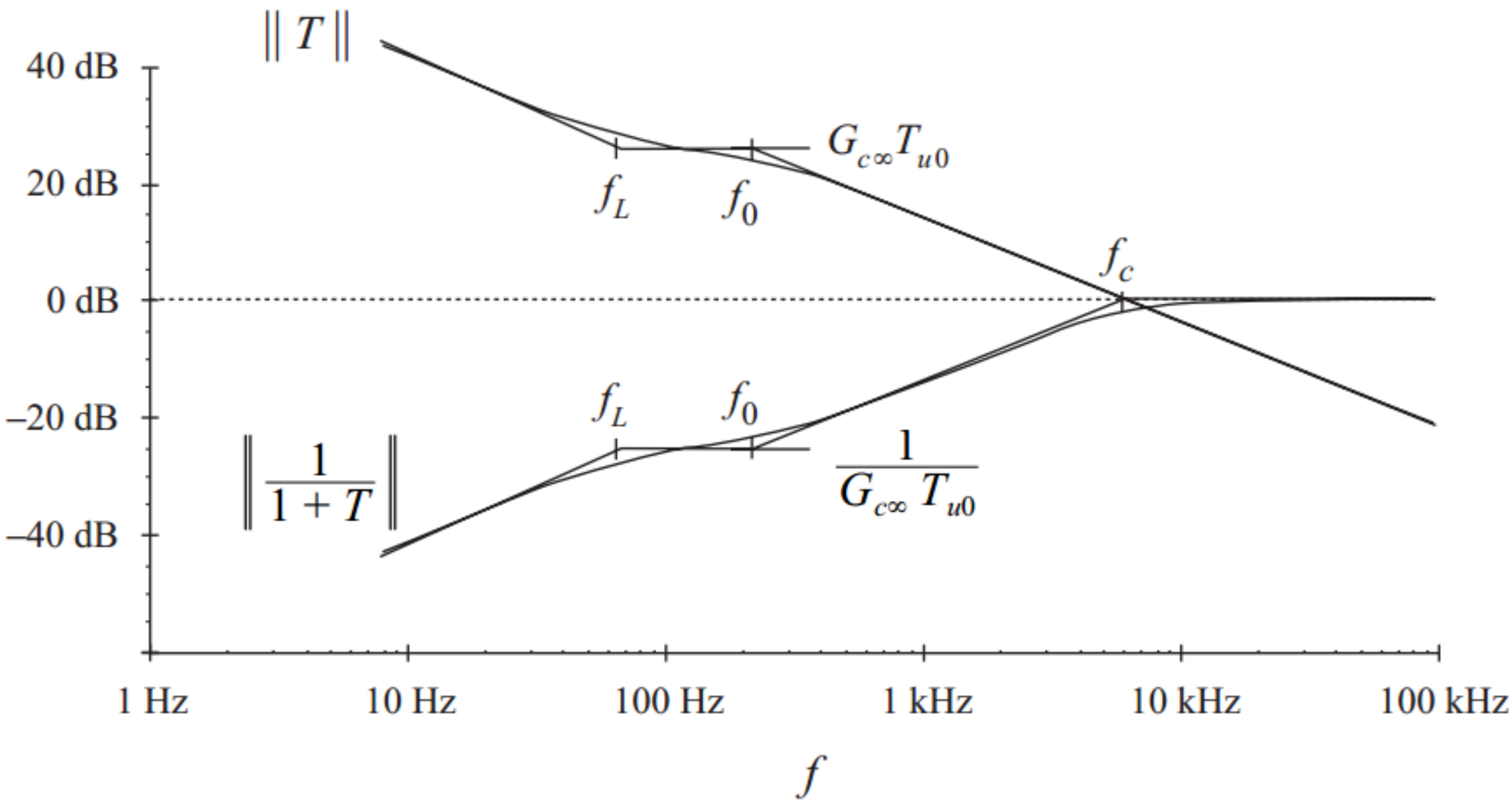
  $G_{c\infty}$ to obtain desired crossover frequency

  ω_L sufficiently low to maintain adequate phase margin



Example (cont'd)

- Construction of *1/(1+T)*, lag compensator example:

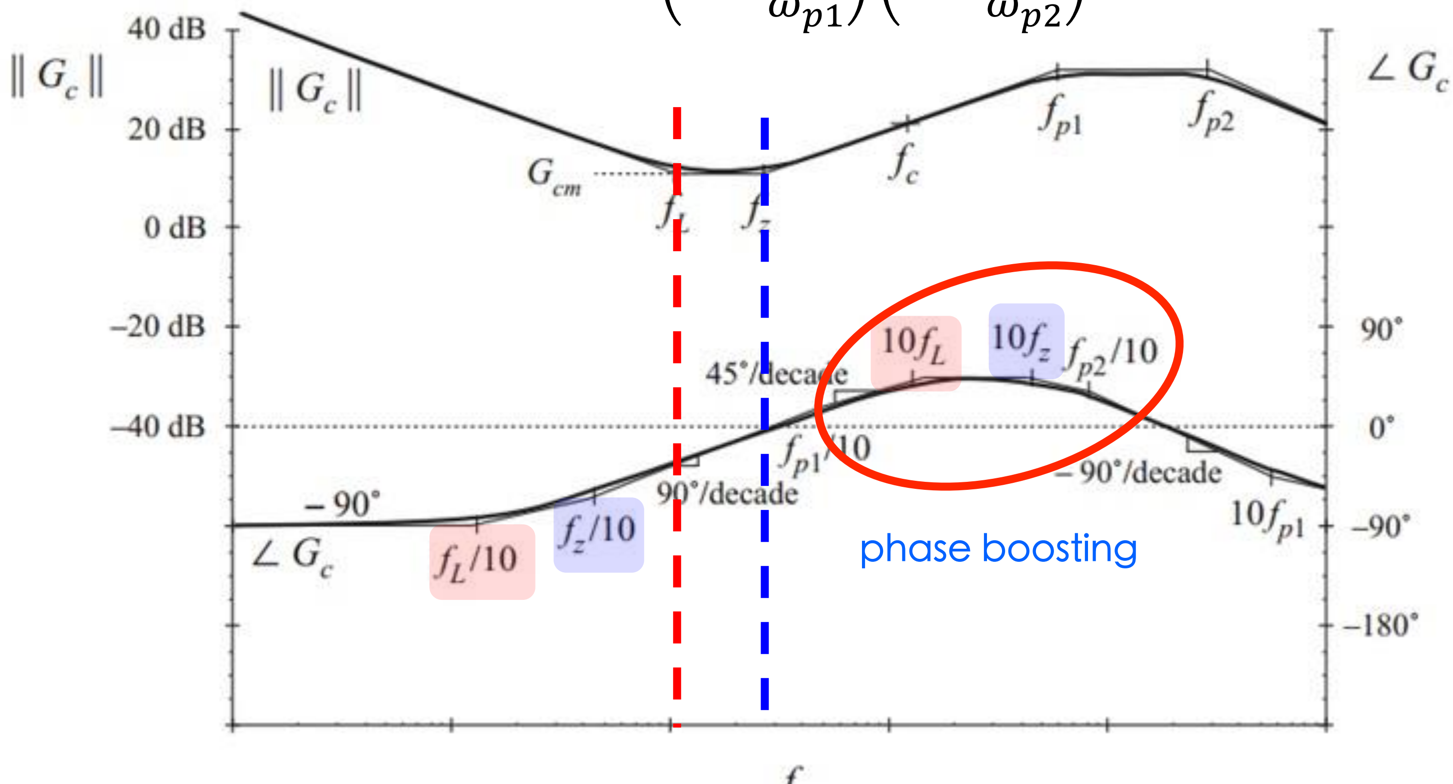


Combined (PID) Compensator

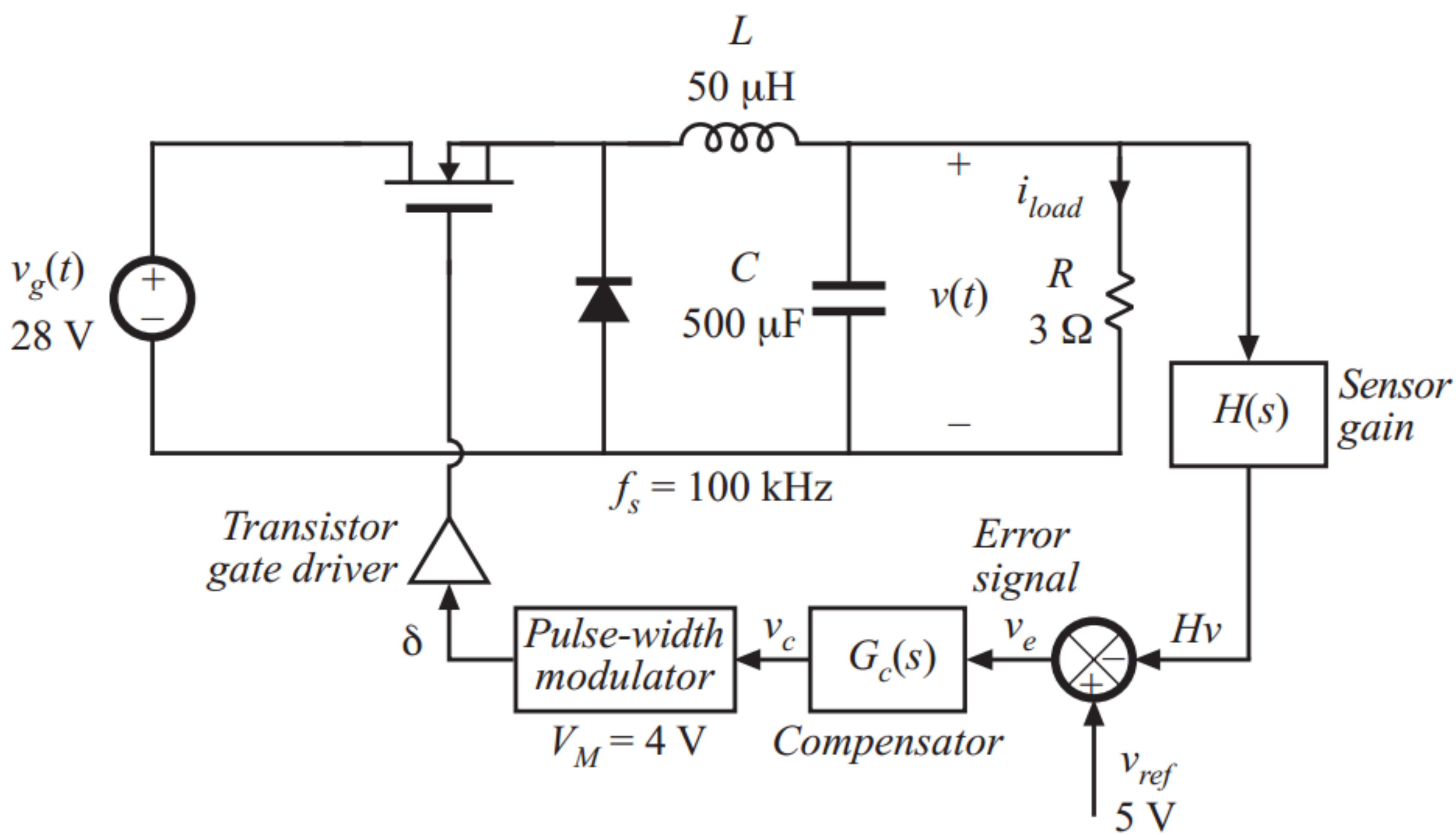
- The advantages of the **lead** and **lag** compensators can be combined, to obtain both **wide bandwidth** and **zero steady-state error.**

PI PD

$$G_c(s) = G_{cm}\frac{\left(1+\frac{\omega_L}{s}\right)\left(1+\frac{s}{\omega_z}\right)}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}$$



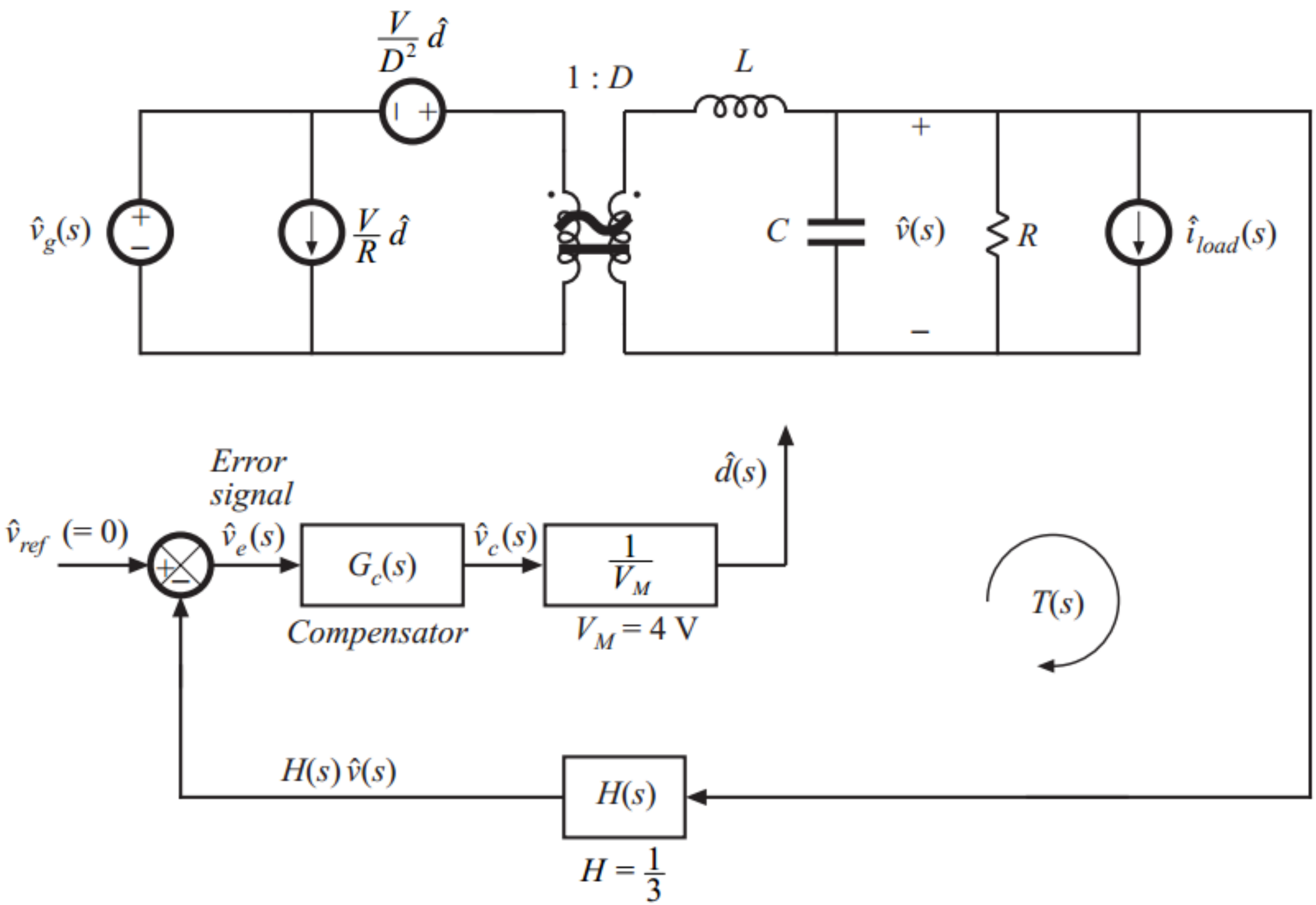
Design Example



Quiescent Operating Point

- Input voltage — $V_g = 28\text{V}$
- Output — $V = 15\text{V},\ I_{load} = 5\text{A} \rightarrow R = 3\Omega$
- Quiescent duty cycle — $D = 15/28 = 0.536$
- Reference voltage — $V_{ref} = 5\text{V}$
- Quiescent value of control voltage — $Vc = DV_M = 2.14\text{V}$
- Gain $H(s)$ — $H = V_{ref} / V = 5/15 = 1/3$

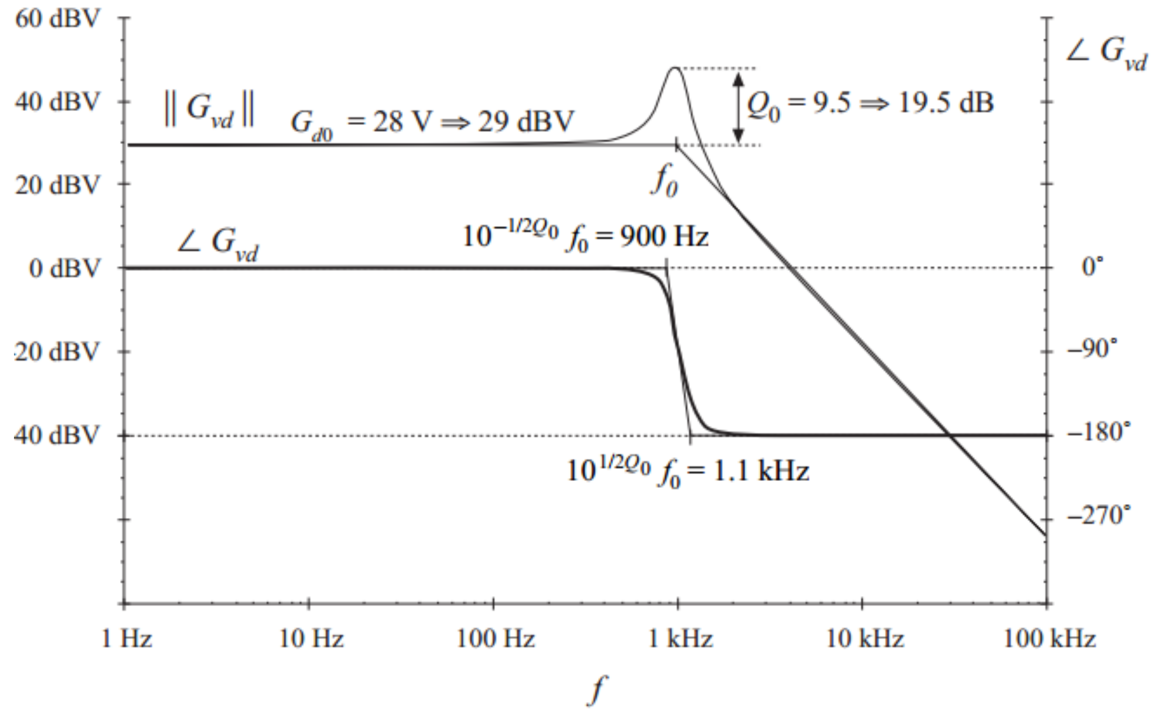
Small-Signal Model



Open-Loop Control-to-Output TF Gvd(s)

$\| G_{vd} \|$

$$G_{vd}(s) = \frac{V}{D} \frac{1}{1 + s\frac{L}{R} + s^2 LC}$$



- **Standard form:**

$$G_{vd}(s) = G_{d0} \frac{1}{1 + \frac{s}{Q_0 \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

- **Salient features:**

$$G_{d0} = \frac{V}{D} = 28V$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = 1\text{kHz}$$

$$Q_0 = R\sqrt{\frac{C}{L}} = 9.5 \Rightarrow 19.5\text{dB}$$

Open-Loop Line-to-Output TF and Output Impedance

$$G_{vg}(s) = D\frac{1}{1 + s\frac{L}{R} + s^2LC}$$

- **same poles as control-to-output transfer function standard form:**

$$G_{vg}(s) = G_{g0}\frac{1}{1 + \frac{s}{Q_0\omega_0} + (\frac{s}{\omega_0})^2}$$

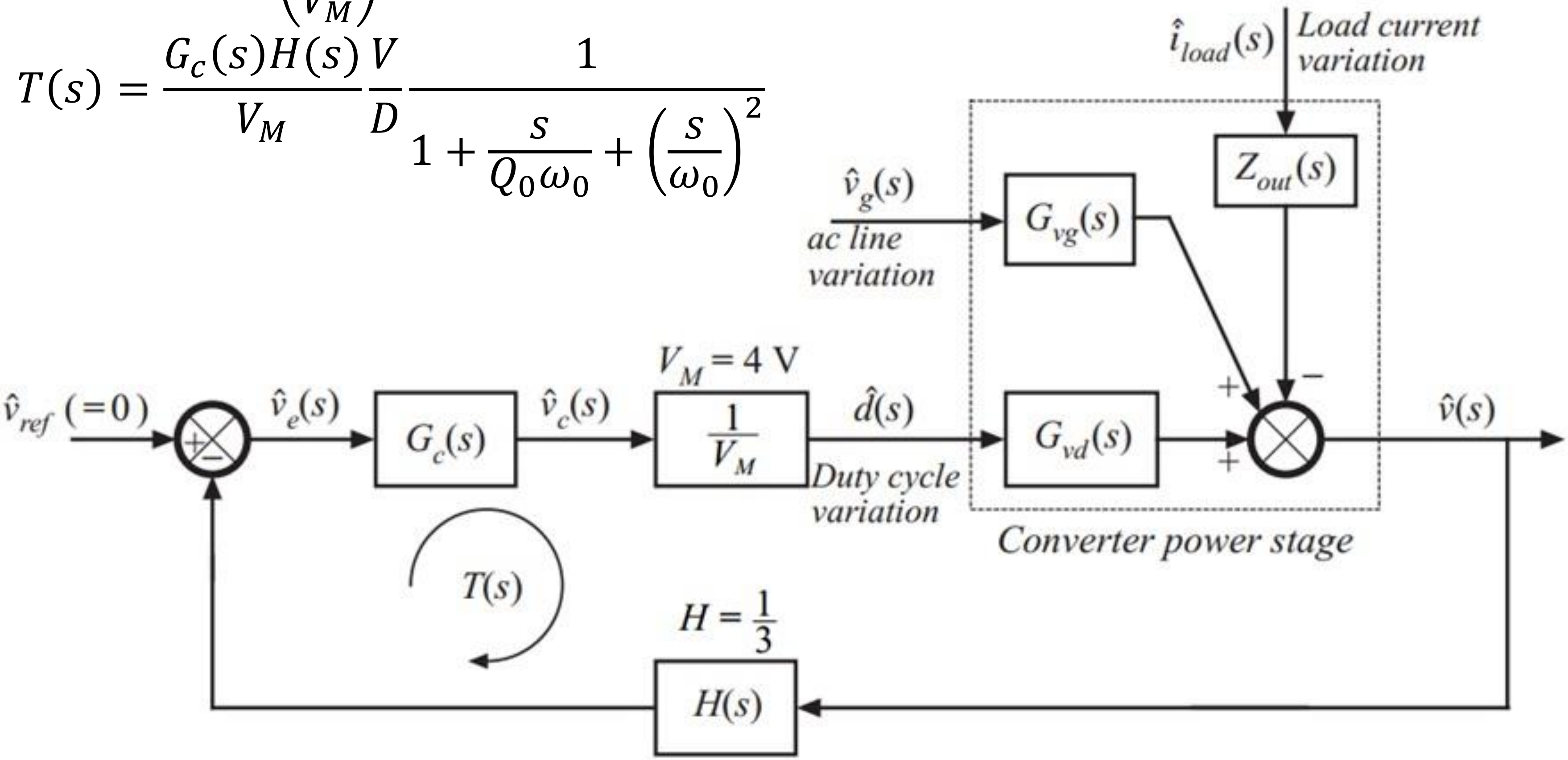
- **Output impedance:**

$$Z_{out}(s) = R\left\|\frac{1}{sC}\right\|sL = \frac{sL}{1 + s\frac{L}{R} + s^2LC}$$

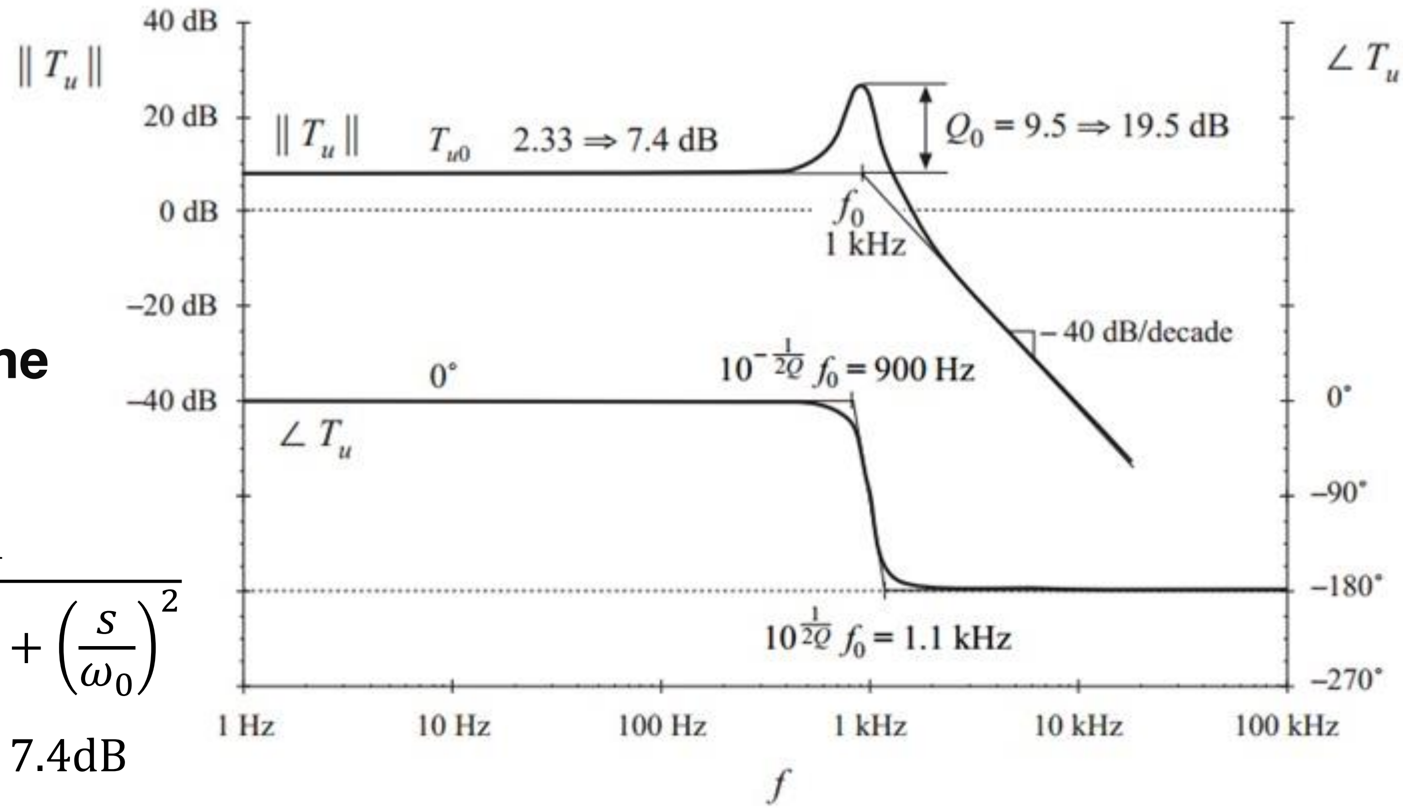
System Block Diagram

$$T(s) = G_c(s)\left(\frac{1}{V_M}\right)G_{vd}(s)H(s)$$

$$T(s) = \frac{G_c(s)H(s)}{V_M}\frac{V}{D}\frac{1}{1+\frac{s}{Q_0\omega_0}+\left(\frac{s}{\omega_0}\right)^2}$$



Uncompensated Loop Gain (with Gc = 1)



- With $G_c = 1$, the loop gain is

$$T_u(s) = T_{u0} \frac{1}{1 + \frac{s}{Q_0 \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

$$T_{u0} = \frac{H\,V}{D\,V_M} = 2.33 \Rightarrow 7.4\text{dB}$$

$$f_c = 1.8\ \text{kHz}, \varphi_m = 5°$$

Lead Compensator Design

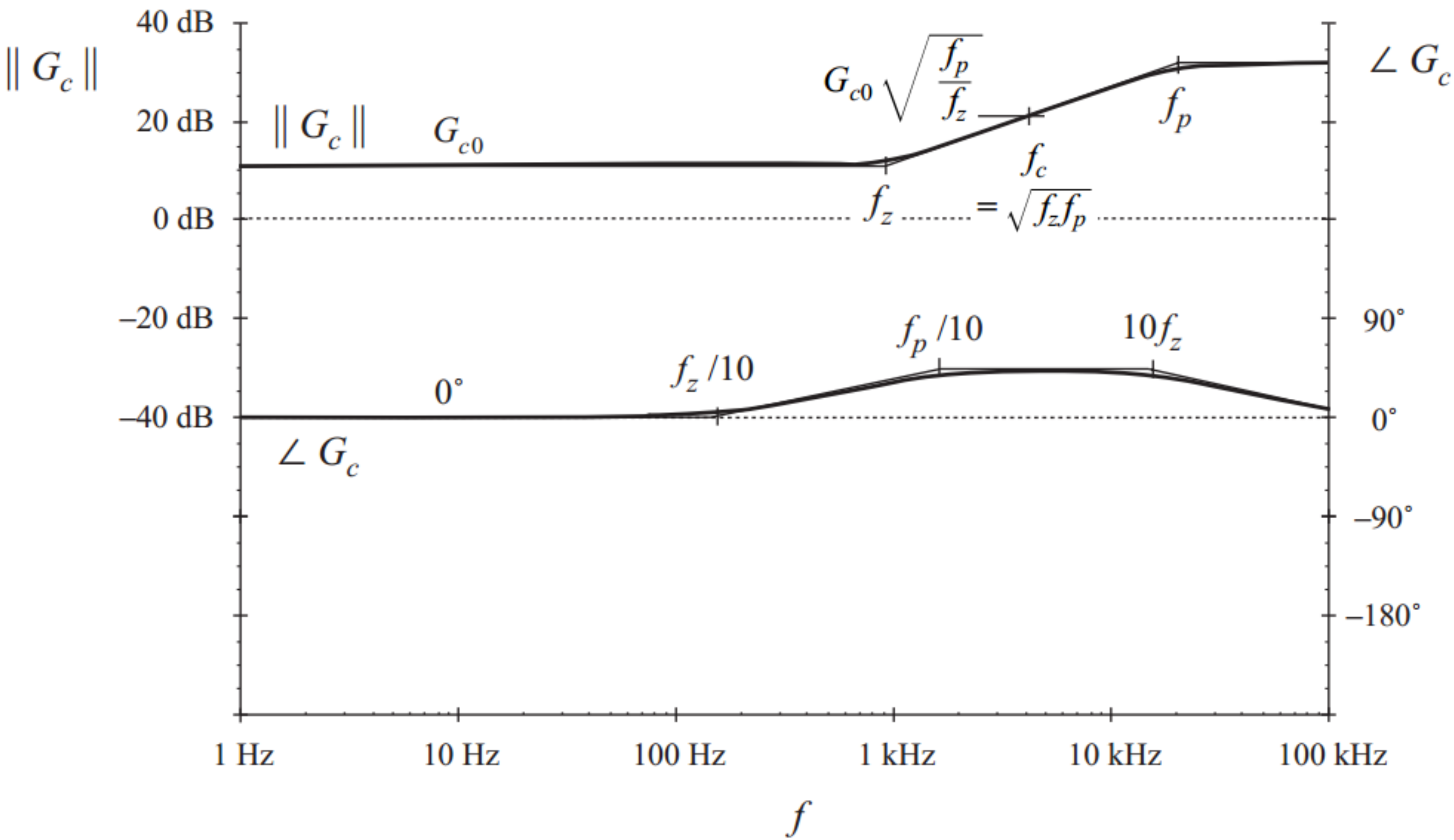
- Obtain a crossover frequency of 5 kHz, with phase margin of 52˚
- T_u has phase of approximately –180˚ at 5 kHz, hence lead (PD) compensator is needed to increase phase margin.
- Lead compensator should have phase of + 52˚ at 5 kHz
- T_u has magnitude of – 20.6 dB at 5 kHz
- Lead compensator gain should have magnitude of +20.6 dB at 5 kHz
- Lead compensator pole and zero frequencies should be

$$f_z = (5\text{kHz})\sqrt{\frac{1-\sin(52°)}{1+\sin(52°)}} = 1.7\text{kHz}$$

$$f_p = (5\text{kHz})\sqrt{\frac{1+\sin(52°)}{1-\sin(52°)}} = 14.5\text{kHz}$$

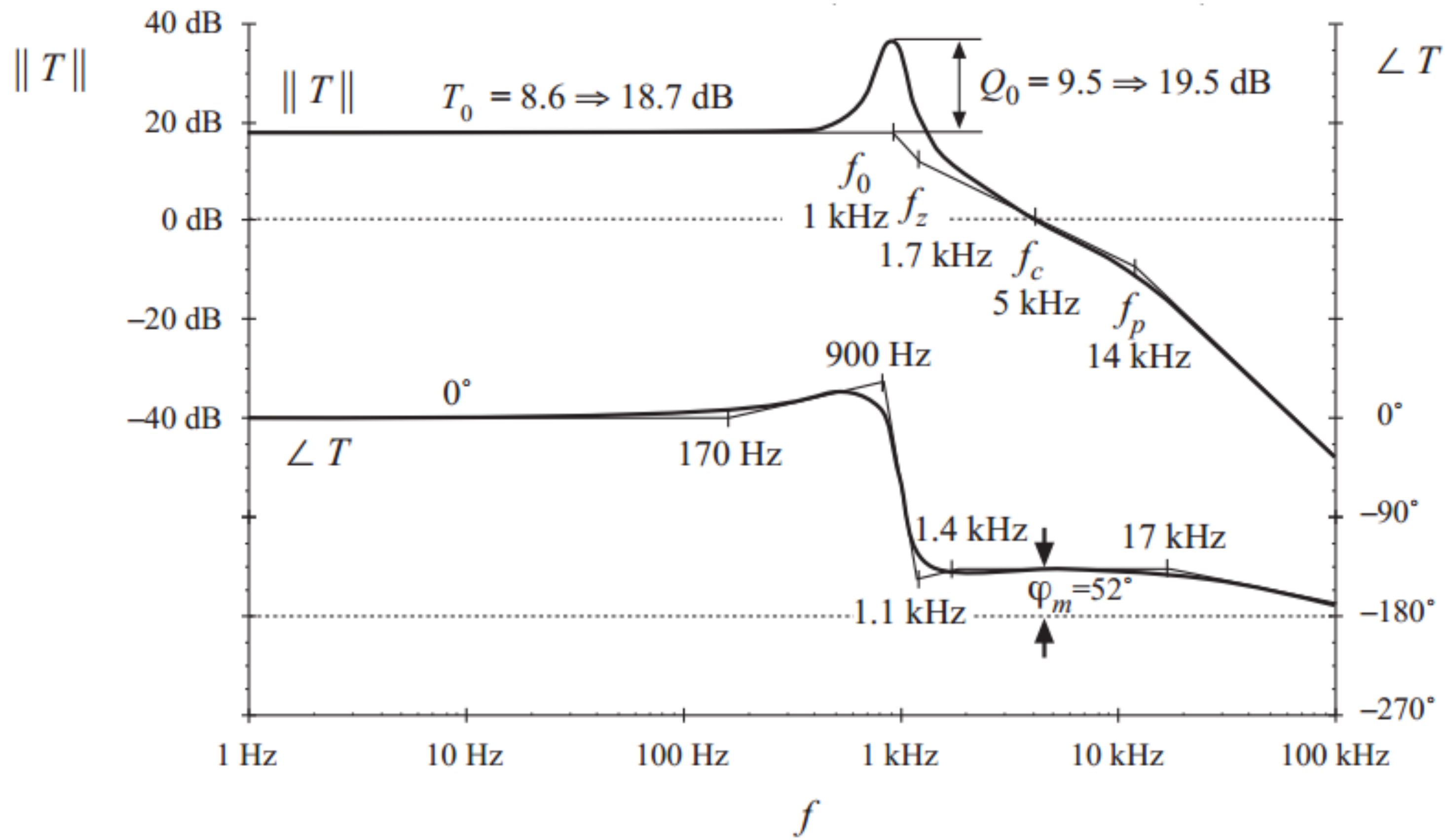
- Compensator dc gain should be $G_{c0} = (\frac{f_c}{f_0})^2 \frac{1}{T_{u0}} \sqrt{\frac{f_z}{f_p}} = 3.7 \Rightarrow 11.3\text{dB}$

Lead Compensator Bode Plot



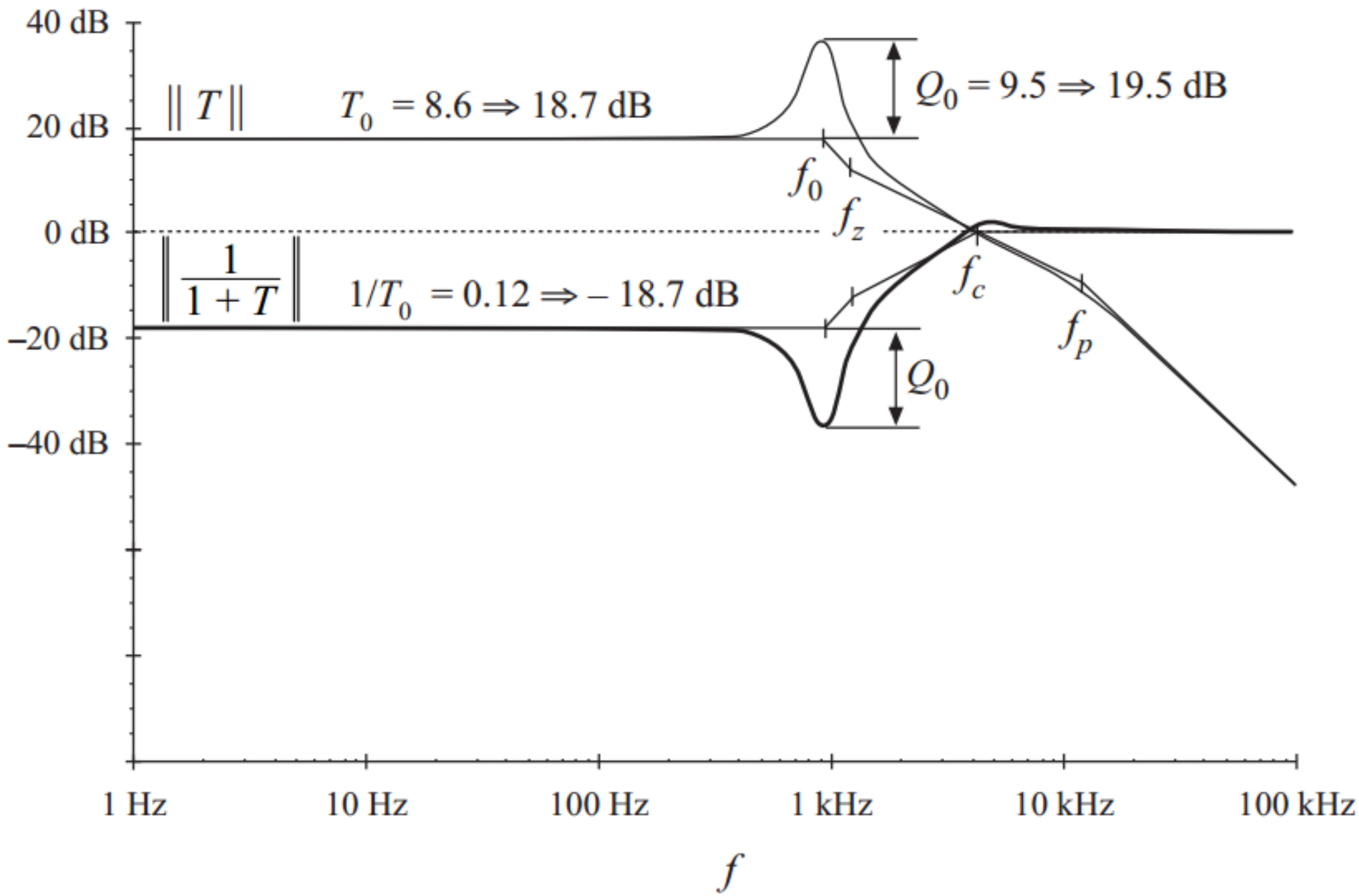
Loop Gain, with Lead Compensator

$$T(s) = T_{u0}\ G_{c0} \frac{\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_p}\right)\left(1 + \frac{s}{Q_0 \omega_0} + \left(\frac{s}{\omega_0}\right)^2\right)}$$



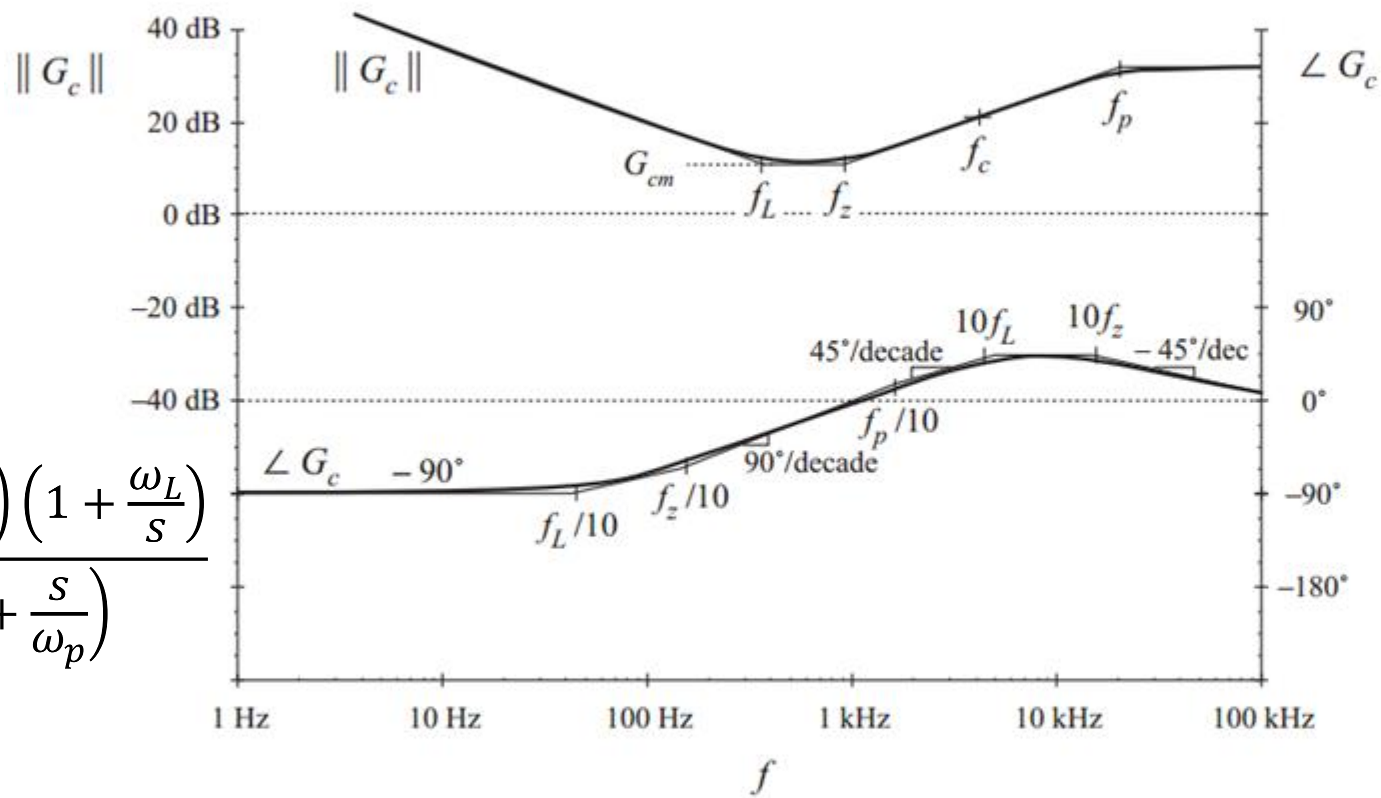
1/(1+T), with Lead Compensator

- Need more low-frequency loop gain
  → add inverted zero (PID controller)



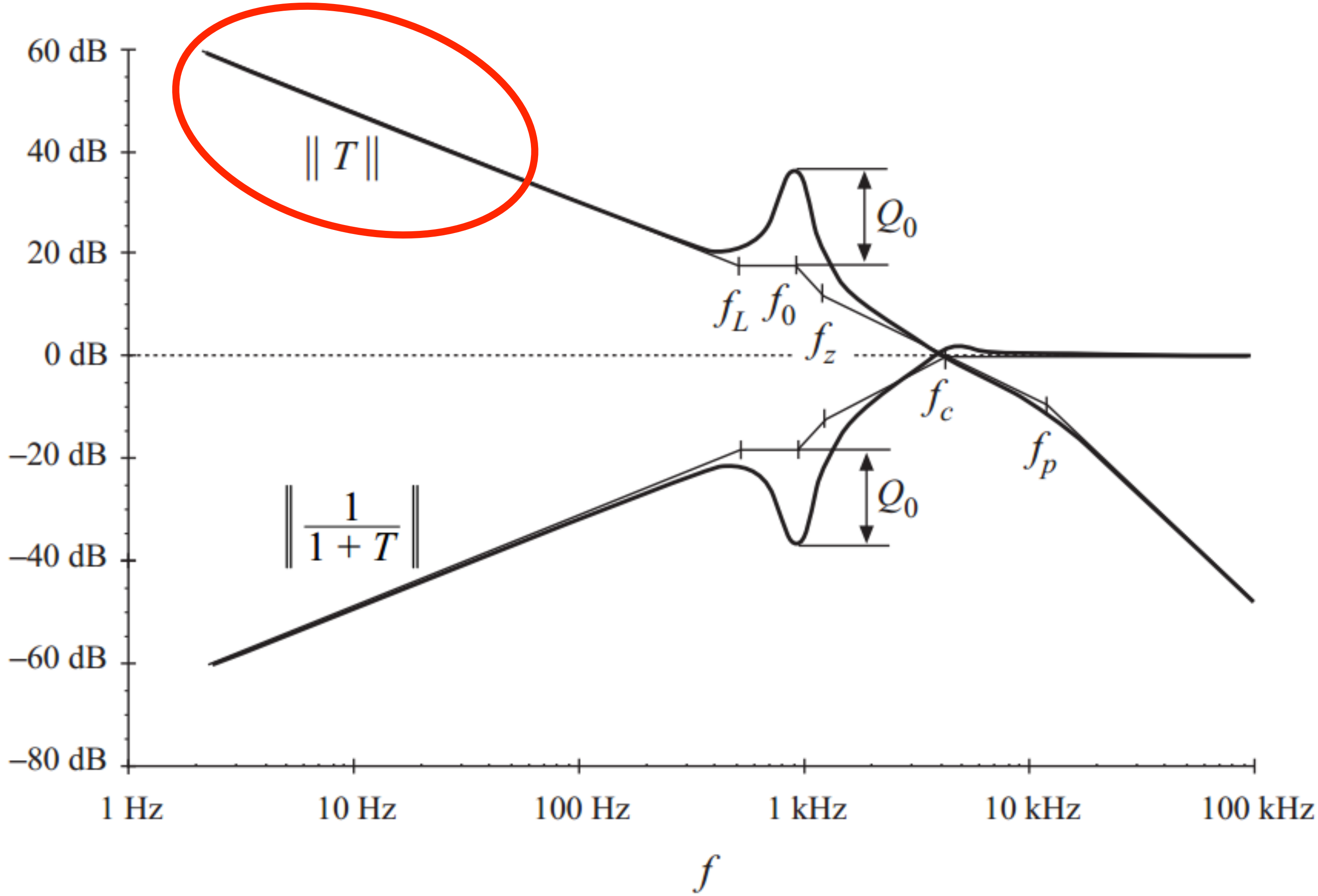
Improved Compensator (PID)

- Add inverted zero to PD compensator, without changing dc gain or corner frequencies
- Choose f_L to be $f_c/10$, so that phase margin is unchanged

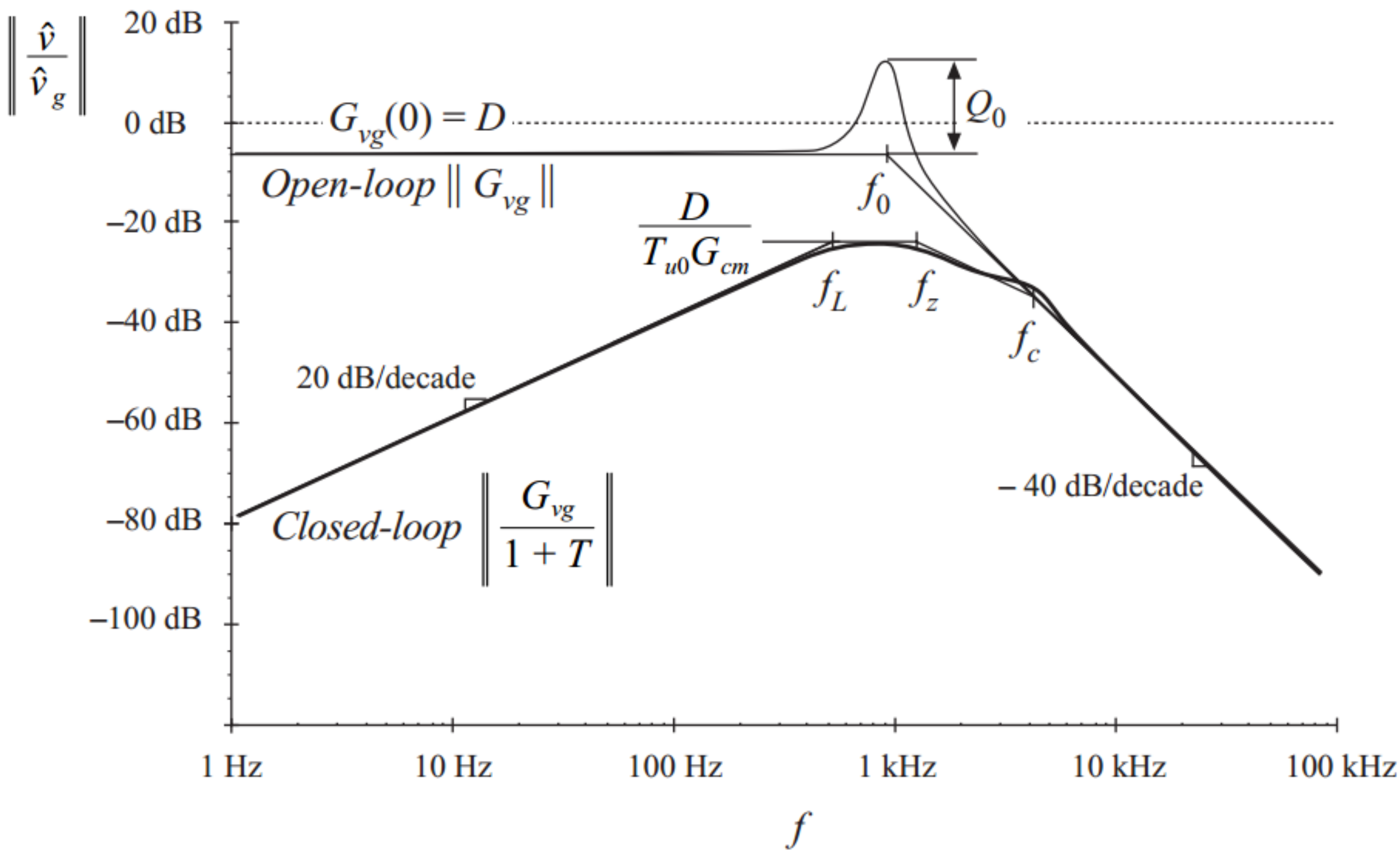


$$G_c(s) = G_{cm} \frac{\left(1 + \frac{s}{\omega_z}\right)\left(1 + \frac{\omega_L}{s}\right)}{\left(1 + \frac{s}{\omega_p}\right)}$$

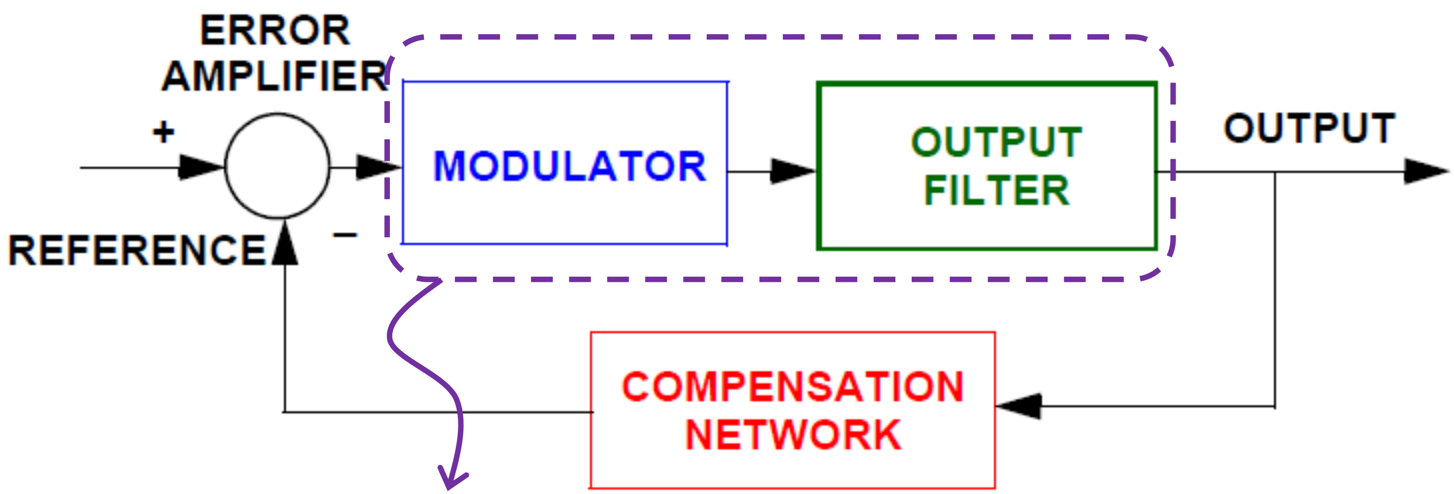
T(s) and 1/(1+T(s)), with PID Compensator



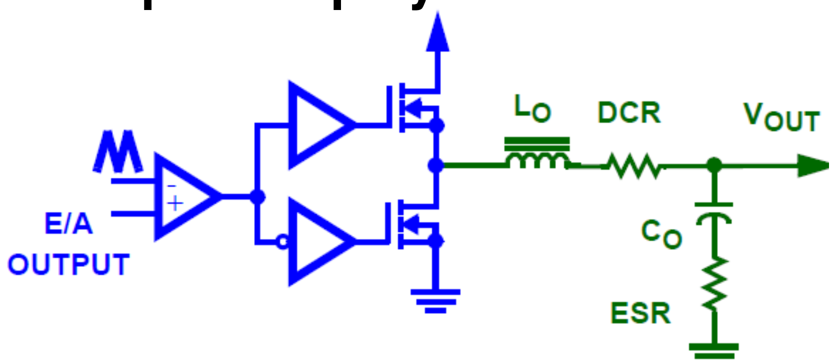
Line-to-Output Transfer Function

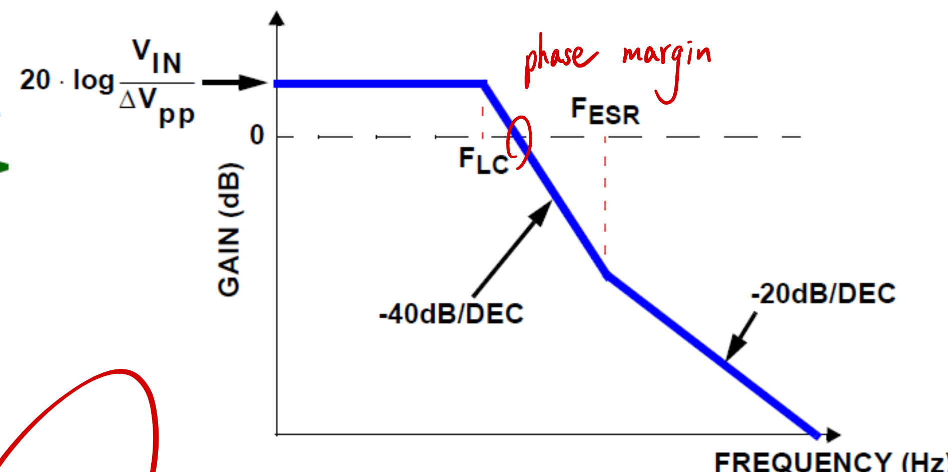


Basic Block Diagram of Buck Converter



- The open loop system:





$$GAIN_{OPENLOOP} = \frac{V_{IN}}{\Delta V_{OSC}} \cdot \frac{1 + s \cdot ESR \cdot C_{OUT}}{1 + s \cdot (ESR + DCR) \cdot C_{OUT} + s^2 \cdot L_{OUT} \cdot C_{OUT}}$$

LC double pole

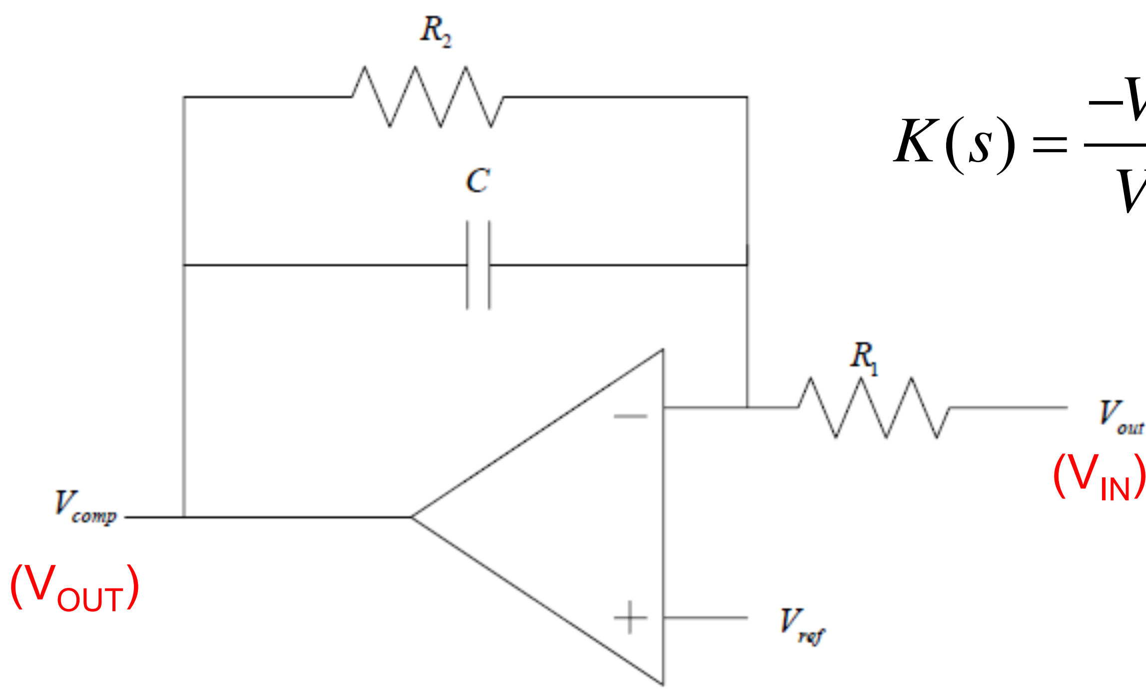
Compensator

- The original buck converter with LC filter has a very low phase margin (PM) which needs to be improved by adding a suitable controller.
- Proper compensation of the system will allow for a predictable bandwidth with unconditional stability.
- In most cases, a Type II or Type III compensated network will properly compensate the system.

考擇一推導

Type I Compensation

- Dominant pole compensation, single pole compensation.
- Poor transient response time because the gain crossover frequency occurs at a low frequency.
- Good load regulation due to high DC gain.



$$K(s)=\frac{-V_{out}(s)}{V_{in}(s)}=\frac{R_2}{R_1(1+R_2C_2s)}$$

若 pole 在低頻，改善 phase margin

but 降低 bandwidth

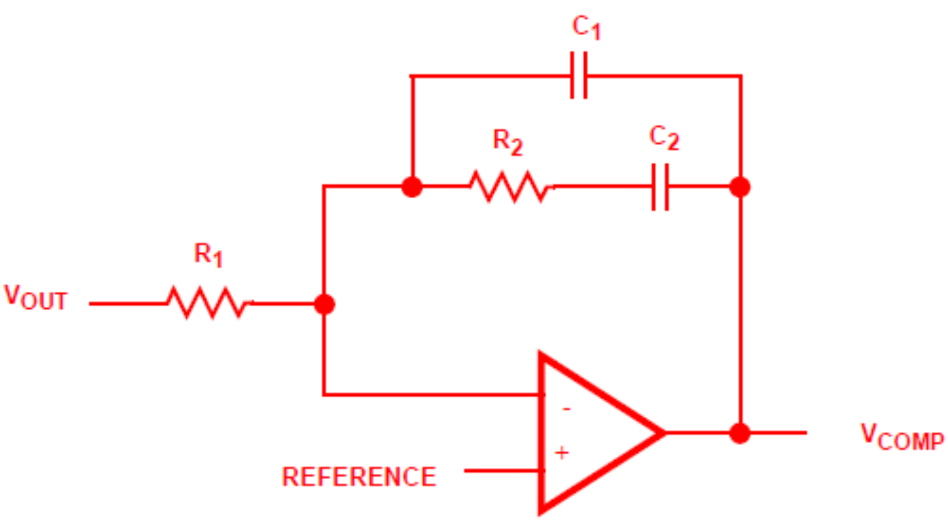
↳ power ic 不適用

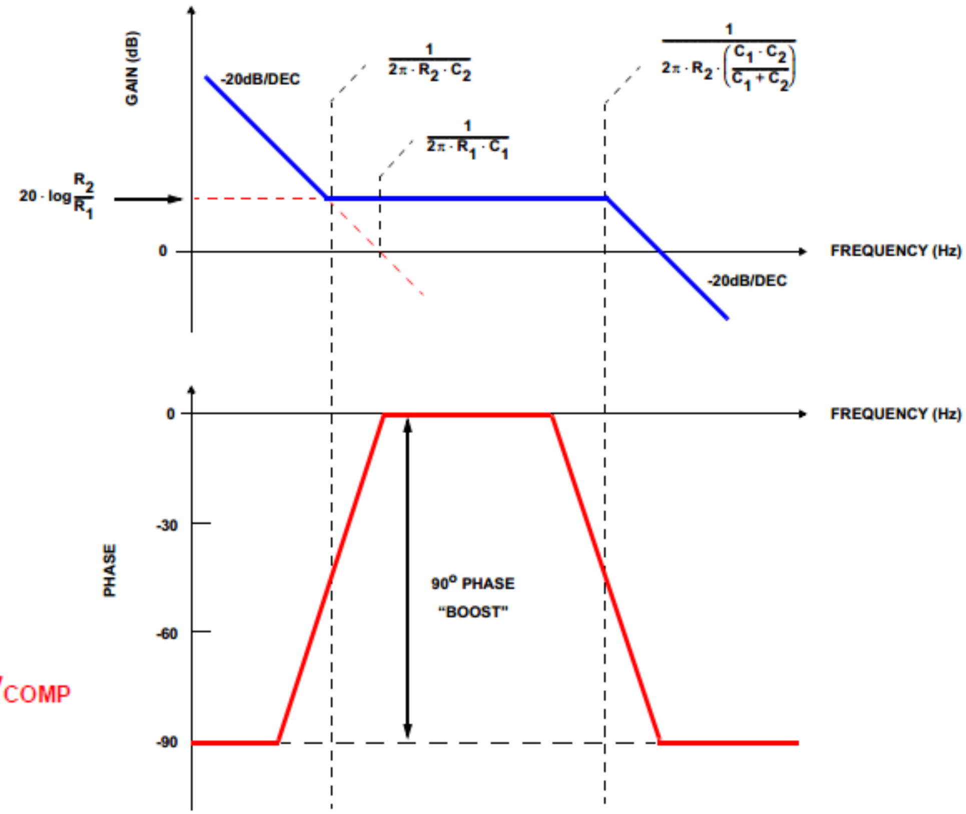
Type II Compensation

- 2 poles and 1 zero network.
- The maximum phase boost in type II is 90 degree.

phase margin 不變
(for 本來就穩定的 system)

$$Gain_{TYPEII} = \frac{1}{R_1C_1}\frac{(s+\frac{1}{R_2C_2})}{s(s+\frac{C_1+C_2}{R_2C_1C_2})}$$

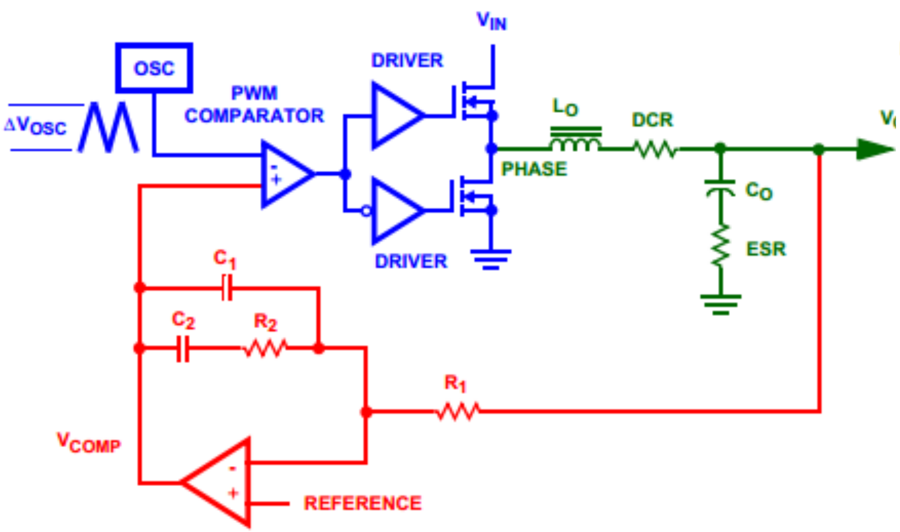


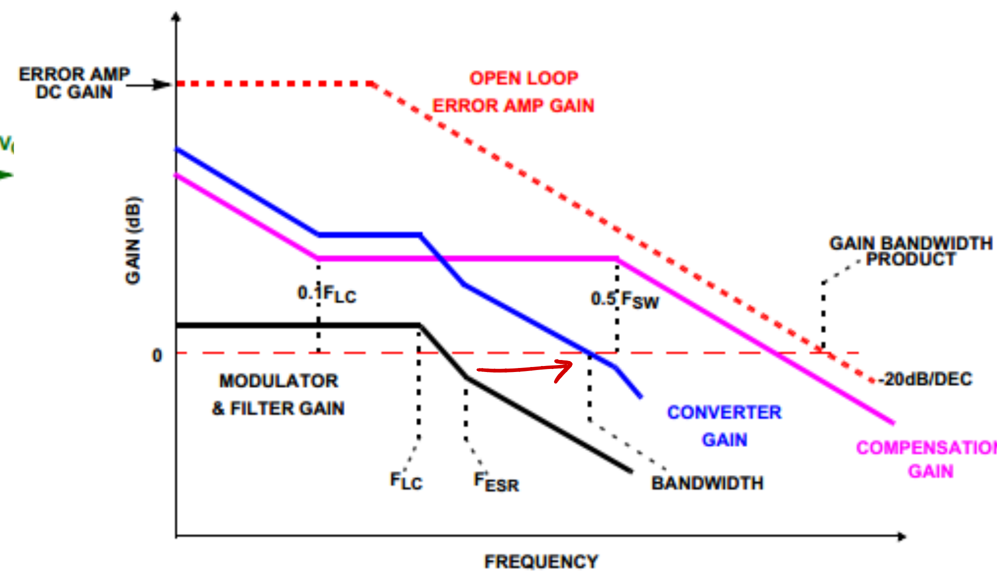


Closed Loop Compensation with Type II

For current mode

- Counteract the double pole caused by LC filter.
- Used in converters where output filter capacitor has a relatively high ESR. When output capacitor ESR is low, a type-III compensation is necessary.





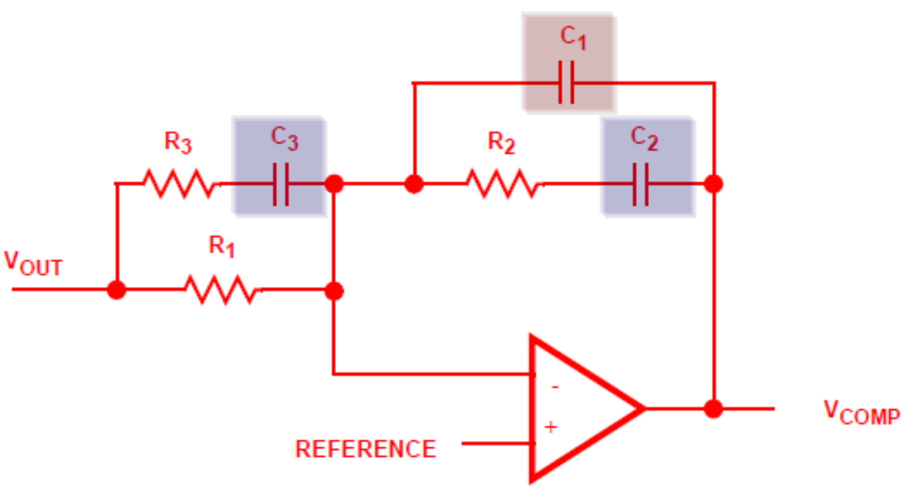
$$GAIN_{SYSTEM} = \frac{1}{R_1 \cdot C_1} \cdot \frac{\left(s + \frac{1}{R_2 \cdot C_2}\right)}{s \cdot \left(s + \frac{C_1 + C_2}{R_2 \cdot C_1 \cdot C_2}\right)} \cdot \frac{V_{IN}}{\Delta V_{OSC}} \cdot \frac{1 + s \cdot ESR \cdot C_{OUT}}{1 + s \cdot (ESR + DCR) \cdot C_{OUT} + s^2 \cdot L_{OUT} \cdot C_{OUT}}$$

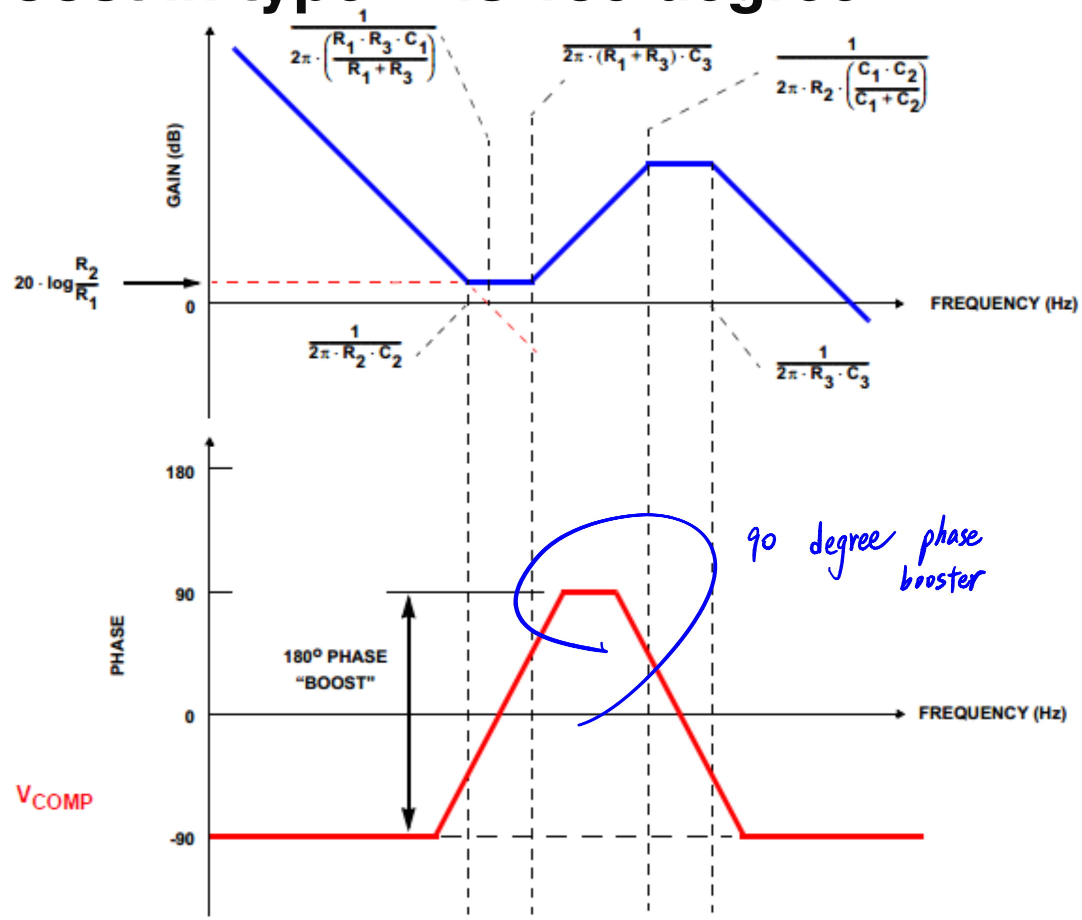
Type III Compensation

For voltage mode

- 3 poles and 2 zero network.
- The maximum phase boost in type III is 180 degree.

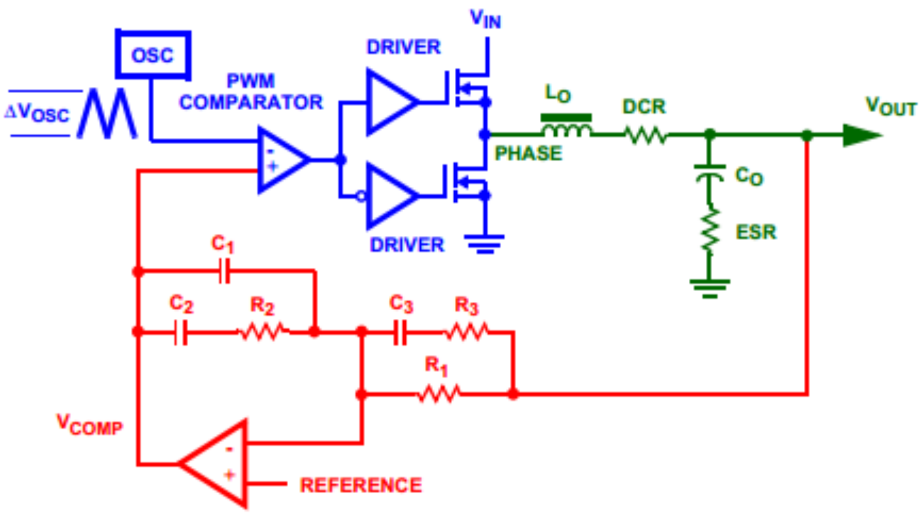
$$Gain_{TYPEIII} = \frac{R_1+R_3}{R_1R_3C_1}\frac{(s+\frac{1}{R_2C_2})(s+\frac{1}{(R_1+R_3)C_3})}{s(s+\frac{C_1+C_2}{R_2C_1C_2})(s+\frac{1}{R_3C_3})}$$

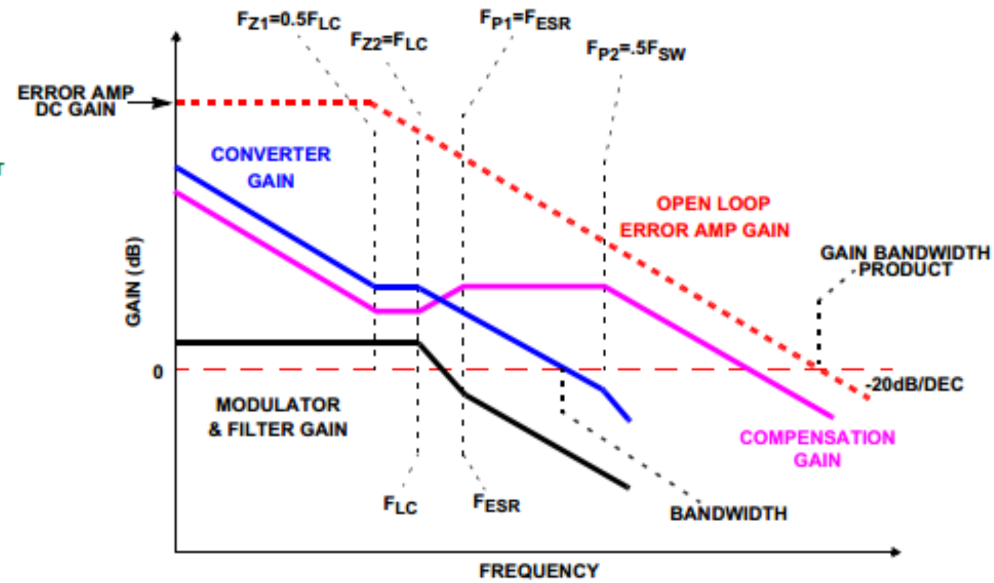




Closed Loop Compensation with Type III

- Type III compensator is sufficient to stabilize the synchronous buck converter for modes.





$$GAIN_{SYSTEM} = \frac{R_1 + R_3}{R_1 \cdot R_3 \cdot C_1} \cdot \frac{\left(s + \frac{1}{R_2 \cdot C_2}\right) \cdot \left(s + \frac{1}{(R_1 + R_3) \cdot C_3}\right)}{s \cdot \left(s + \frac{C_1 + C_2}{R_2 \cdot C_1 \cdot C_2}\right) \cdot \left(s + \frac{1}{R_3 \cdot C_3}\right)} \cdot \frac{V_{IN}}{\Delta V_{OSC}} \cdot \frac{1 + s \cdot ESR \cdot C_{OUT}}{1 + s \cdot (ESR + DCR) \cdot C_{OUT} + s^2 \cdot L_{OUT} \cdot C_{OUT}}$$

Summary of Key Points

1. Negative feedback causes the system output to closely follow the reference input, according to the gain $1/H(s)$. The influence on the output of disturbances and variation of gains in the forward path is reduced.

2. The loop gain $T(s)$ is equal to the products of the gains in the forward and feedback paths. The loop gain is a measure of how well the feedback system works: a large loop gain leads to better regulation of the output. The crossover frequency f_c is the frequency at which the loop gain T has unity magnitude, and is a measure of the bandwidth of the control system.

Summary of Key Points

3. The introduction of feedback causes the transfer functions from disturbances to the output to be multiplied by the factor $1/(1+T(s))$. At frequencies where Tis large in magnitude (i.e., below the crossover frequency), this factor is approximately equal to $1/T(s)$. Hence, the influence of low-frequency disturbances on the output is reduced by a factor of $1/T(s)$. At frequencies where Tis small in magnitude (i.e., above the crossover frequency), the factor is approximately equal to 1. The feedback loop then has no effect. Closed-loop disturbance-to-output transfer functions, such as the line-to-output transfer function or the output impedance, can easily be constructed using the algebra-on- the-graph method.

4. Stability can be assessed using the phase margin test. The phase of T is evaluated at the crossover frequency, and the stability of the important closed-loop quantities $T/(1+T)$ and $1/(1+T)$ is then deduced. Inadequate phase margin leads to ringing and overshoot in the system transient response, and peaking in the closed-loop transfer functions.

Summary of Key Points

5. Compensators are added in the forward paths of feedback loops to shape the loop gain, such that desired performance is obtained. Lead compensators, or *PD* controllers, are added to improve the phase margin and extend the control system bandwidth. *PI* controllers are used to increase the low-frequency loop gain, to improve the rejection of low-frequency disturbances and reduce the steady-state error.