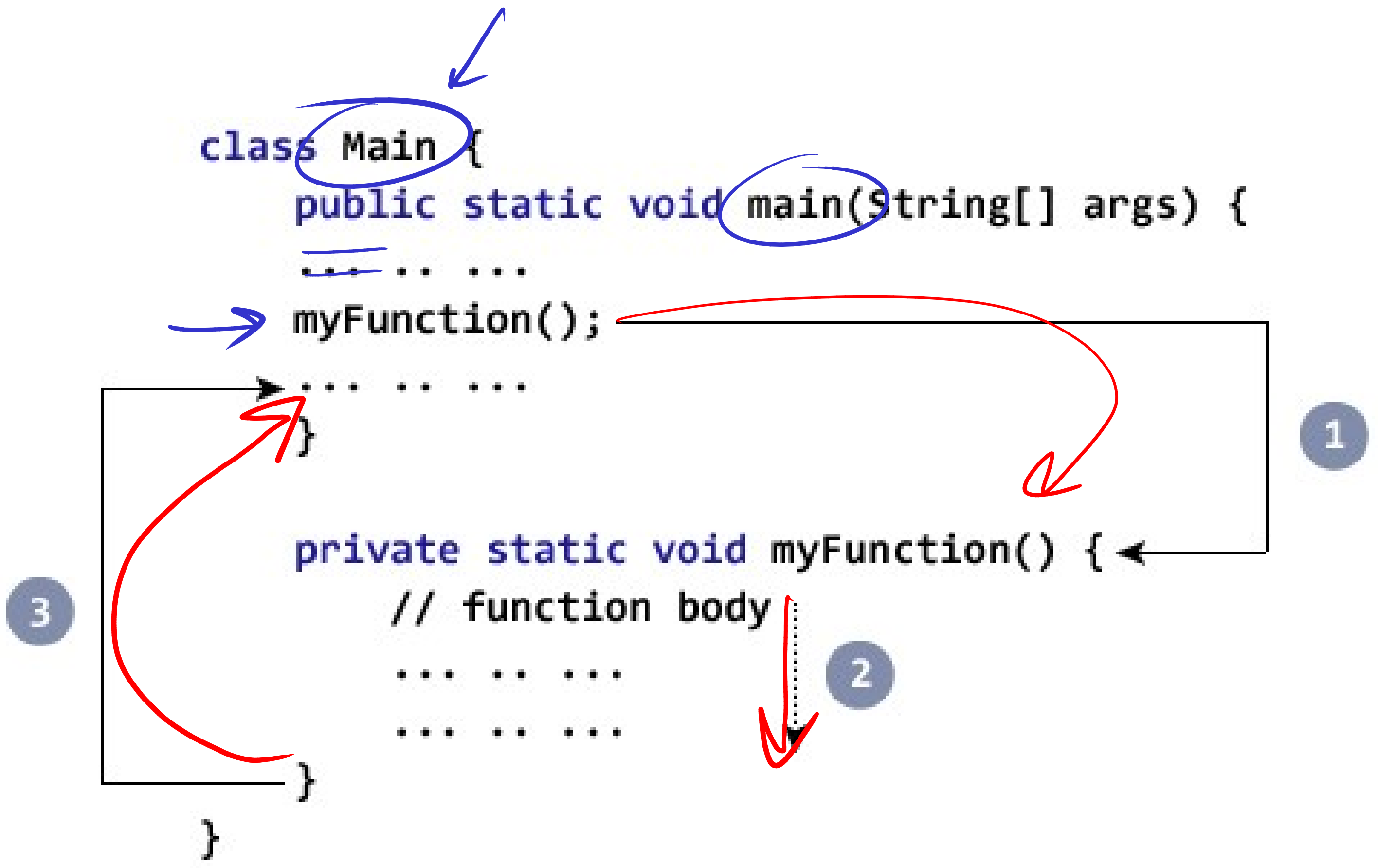
Computer Systems
Subroutines and Stacks

Lecture Objectives

To introduce the fundamentals concepts of subroutines and how they are implemented using stacks.

Lecture Outline

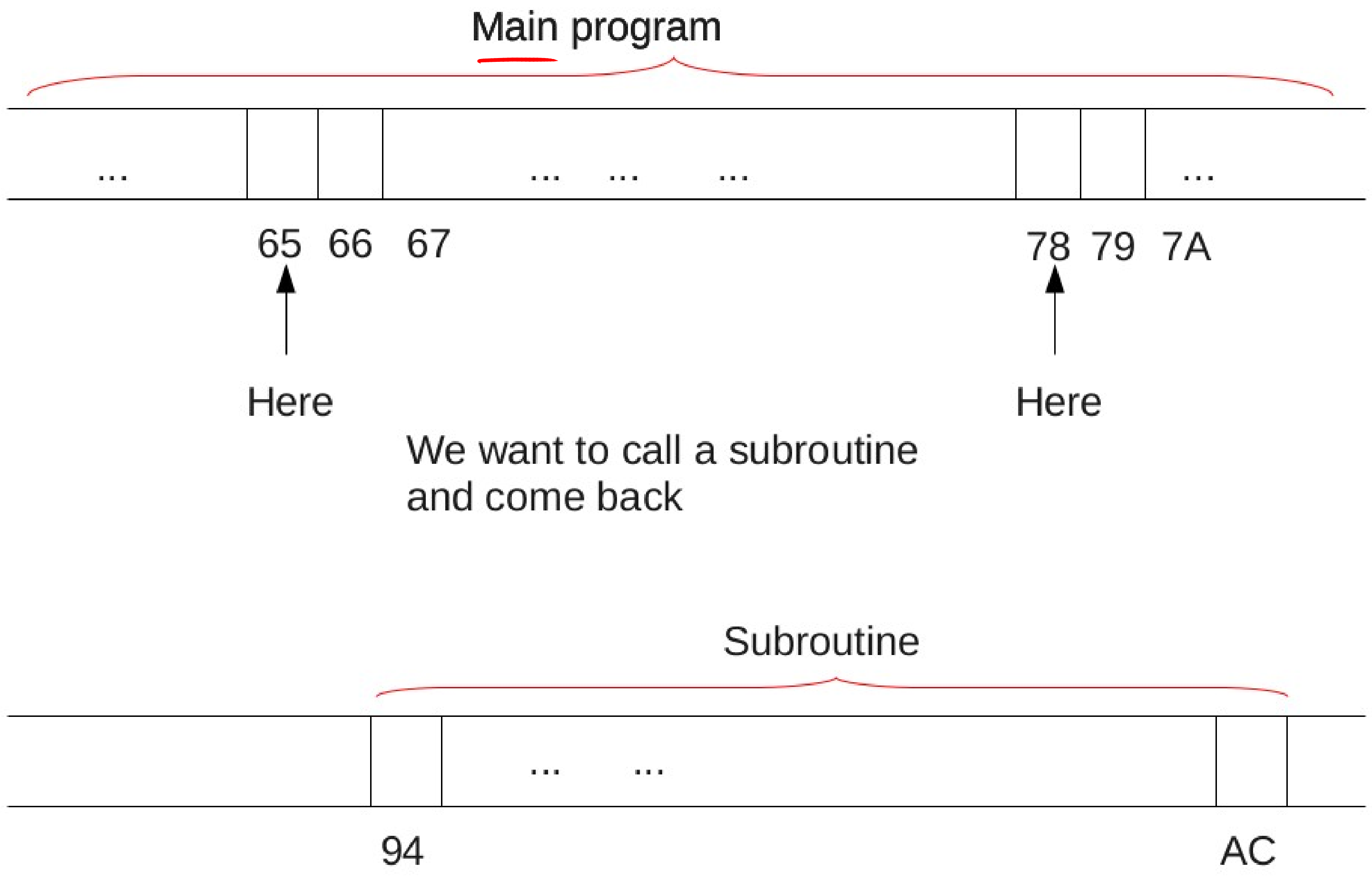
Subroutines (Methods) – Example



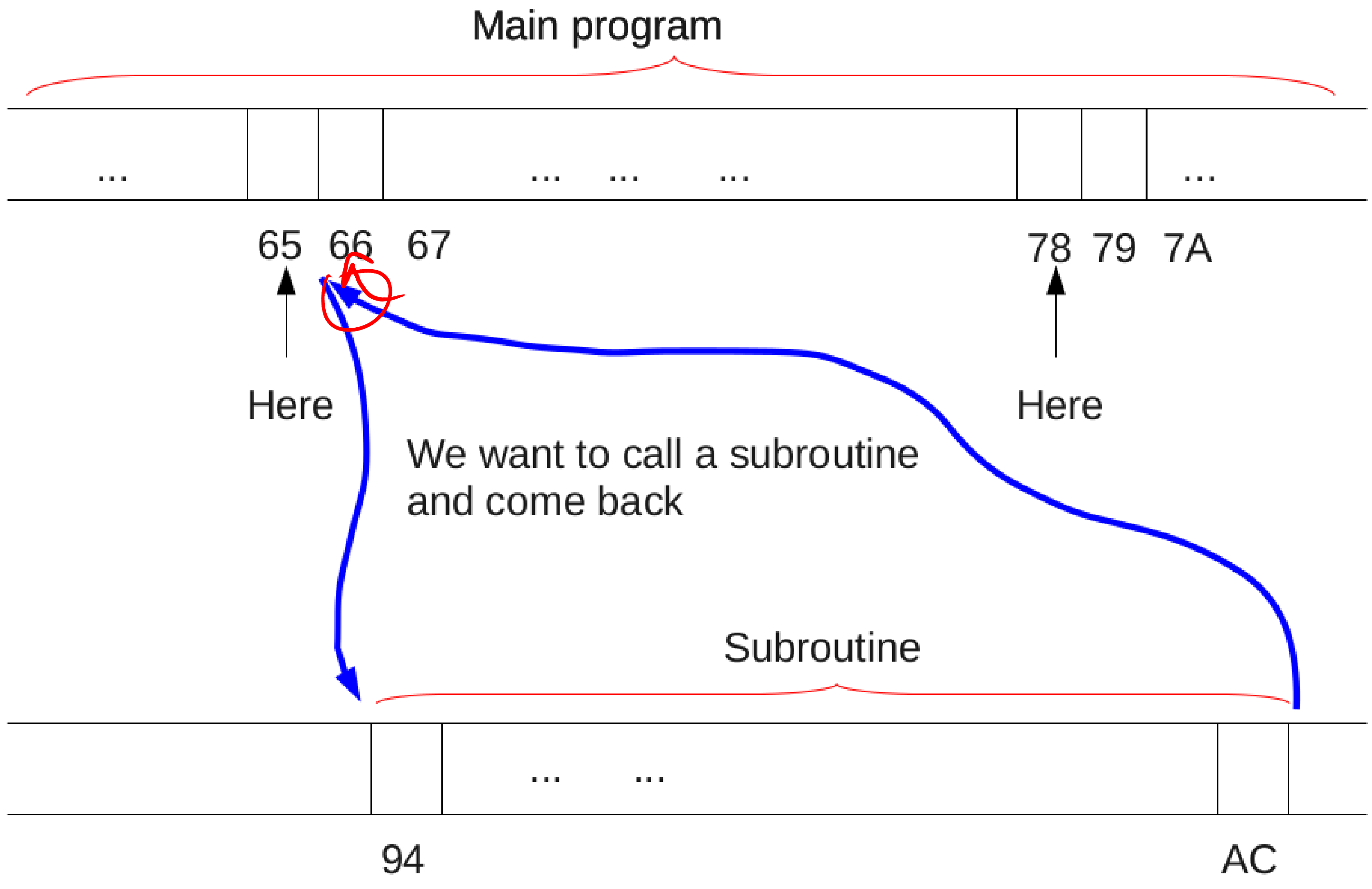
```
class Main {
    public static void main(String[] args) {
        ... .. ...
        myFunction();
        ... .. ...
    }

    private static void myFunction() {
        // function body
        ... .. ...
        ... .. ...
    }
}
```

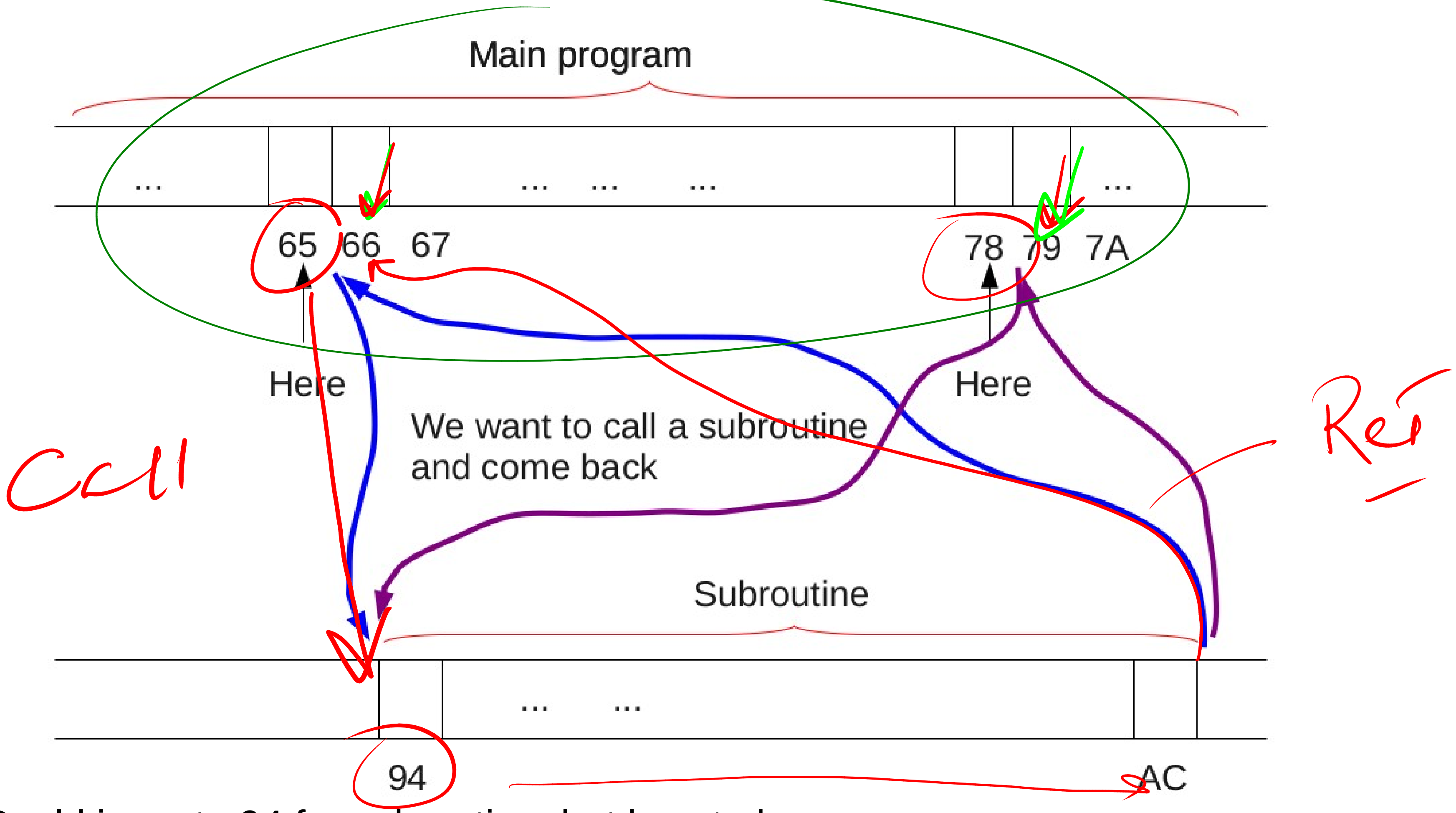
Subroutines (Methods)



Subroutines (Methods)



Subroutines (Methods)



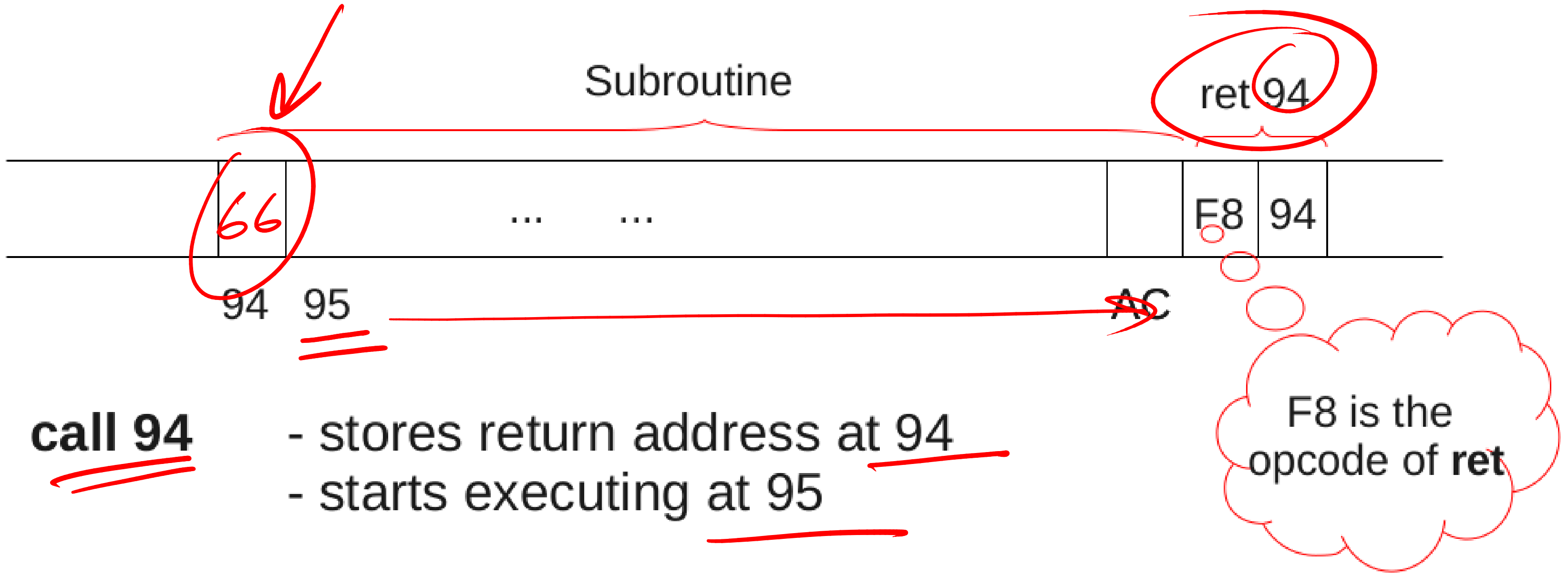
Could jump to 94 for subroutine, but how to know where to jump back afterwards?
Must store the return address somewhere.

Call / Return Instructions

- Two new operations:
  - **call** operand : Like jump, but stores current PC value (the return address) somewhere suitable.
  - **ret** : Read return address from where it was stored, and load it into PC.

Storing the Return Address

- **Idea #1**: In each subroutine, its first byte is used to store the return address

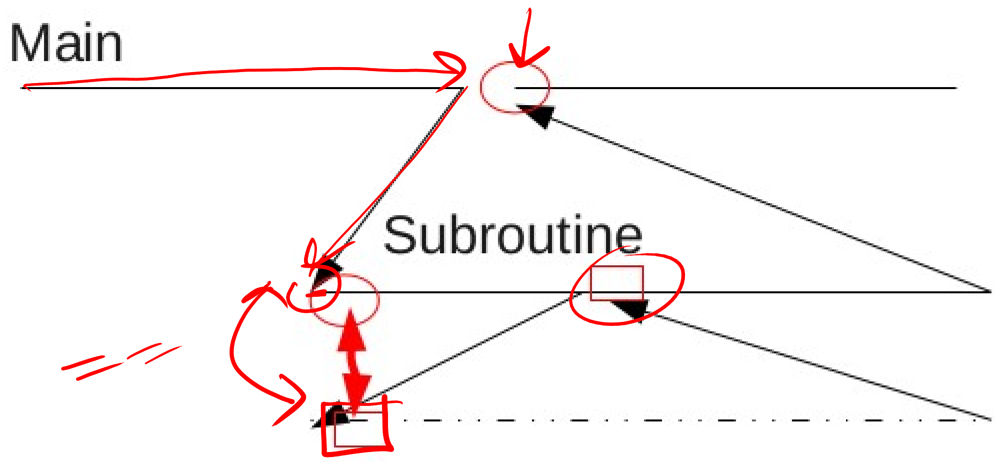


call 94 - stores return address at 94
- starts executing at 95

ret 94 - loads pc with return address at 94

Disadvantage of Idea #1

- Can only store one return address
- Consequence: subroutine cannot call itself
  - If it were to call itself it would need to store 2 return addresses



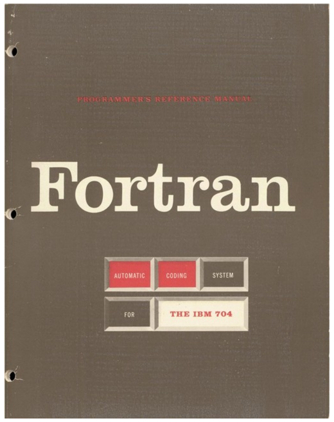
Main calls subroutine

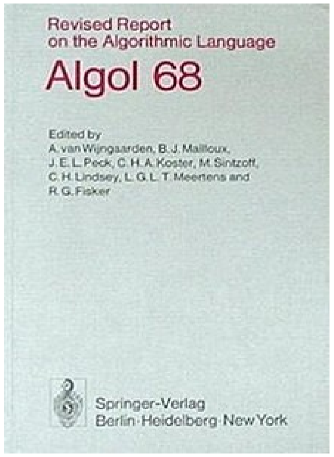
Subroutine calls itself

Dotted line = the same subroutine

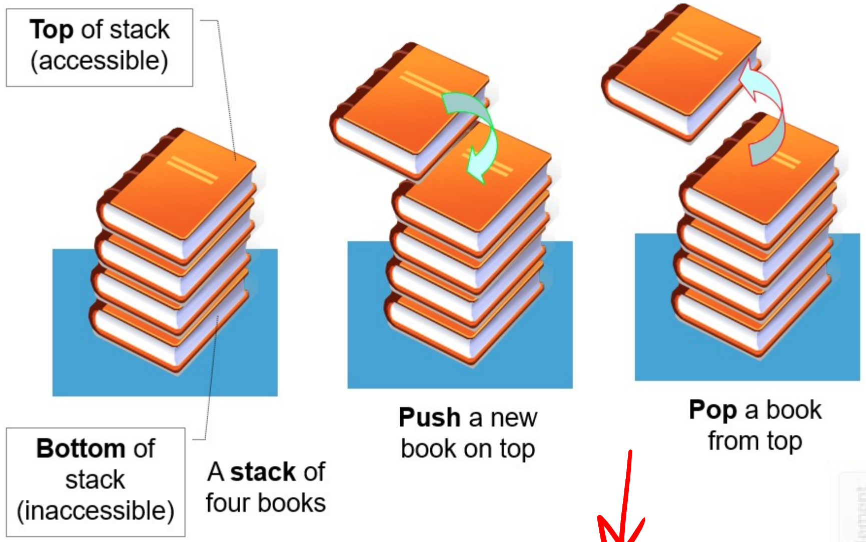
History: 2 Early Programming Languages

- FORTRAN ("FORmula TRANslation")
  - Used jumps (GOTO), conditional jumps
  - Banned recursion (i.e. for a method to call itself) to allow idea 1.
- Algol ("Algorithmic language")
  - structured programming (if .. then .. else, etc.)
  - Allowed recursion
- Initially, FORTRAN was more successful, but modern languages (e.g. C, Java) took forward the ideas from Algol.

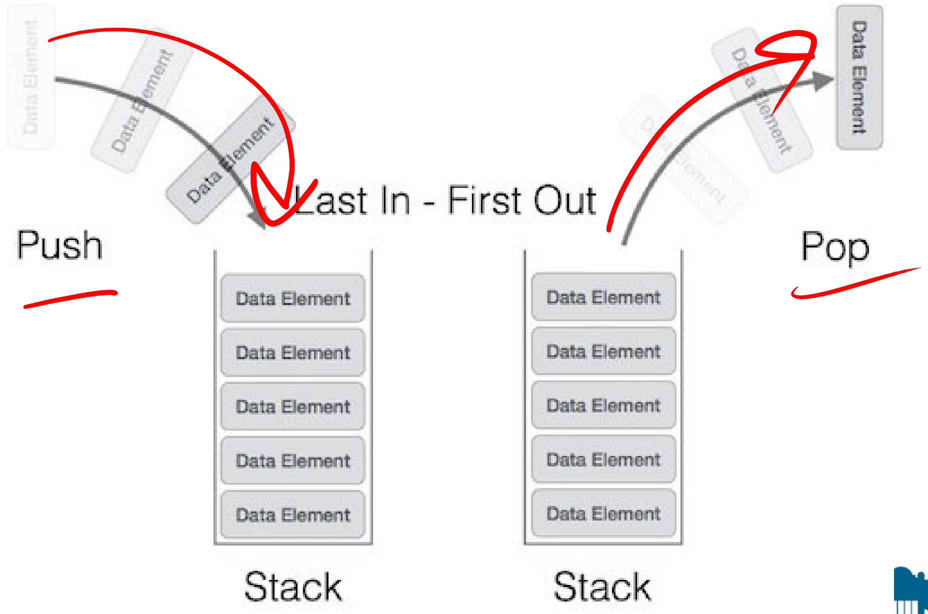



Revised Report on the Algorithmic Language
Algol 68
Edited by A. van Wijngaarden, B. J. Mailloux, J. E. L. Peck, C. H. A. Koster, M. Sintzoff, C. H. Lindsey, L. G. L. T. Meertens and R. G. Fisker
Springer-Verlag Berlin·Heidelberg·New York

What is a Stack?



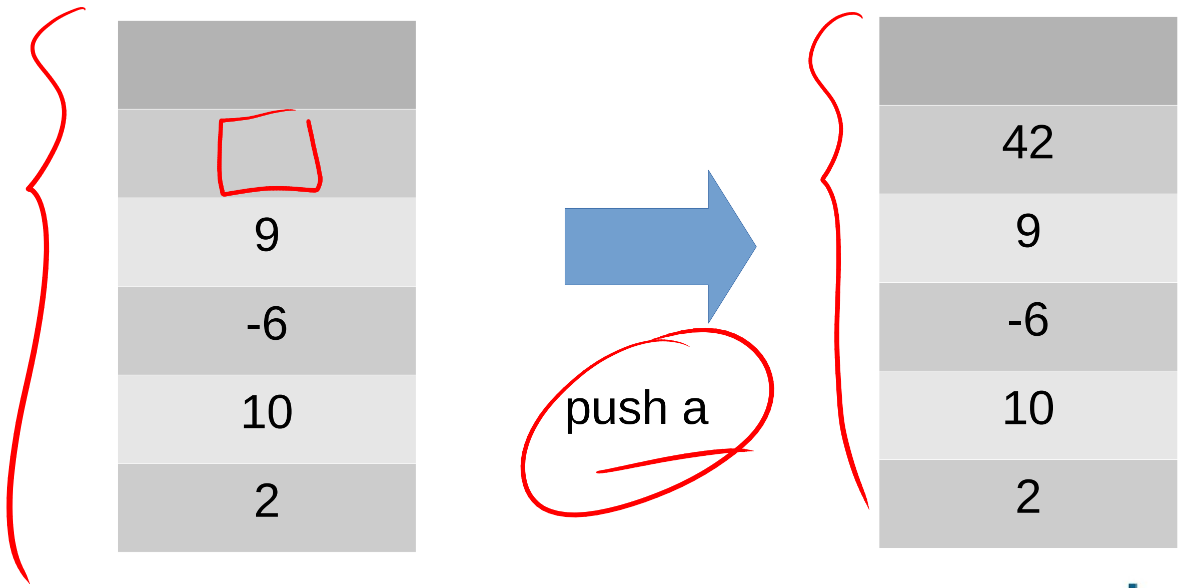
Top of stack (accessible)
Bottom of stack (inaccessible)
A **stack** of four books
Push a new book on top
Pop a book from top



How Stacks Work?

- A stack can flexibly store a variable number of bytes
- LIFO = Last In, First Out (aka FILO)

a	b	c	d
42	54	13	27



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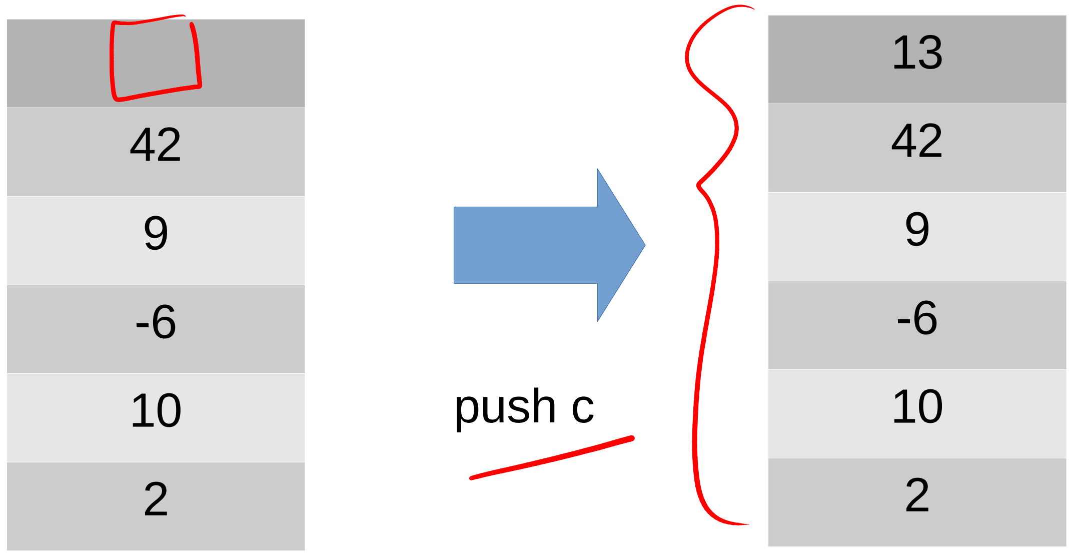

42
9
-6
10
2

push c

13
42
9
-6
10
2

b = pop();

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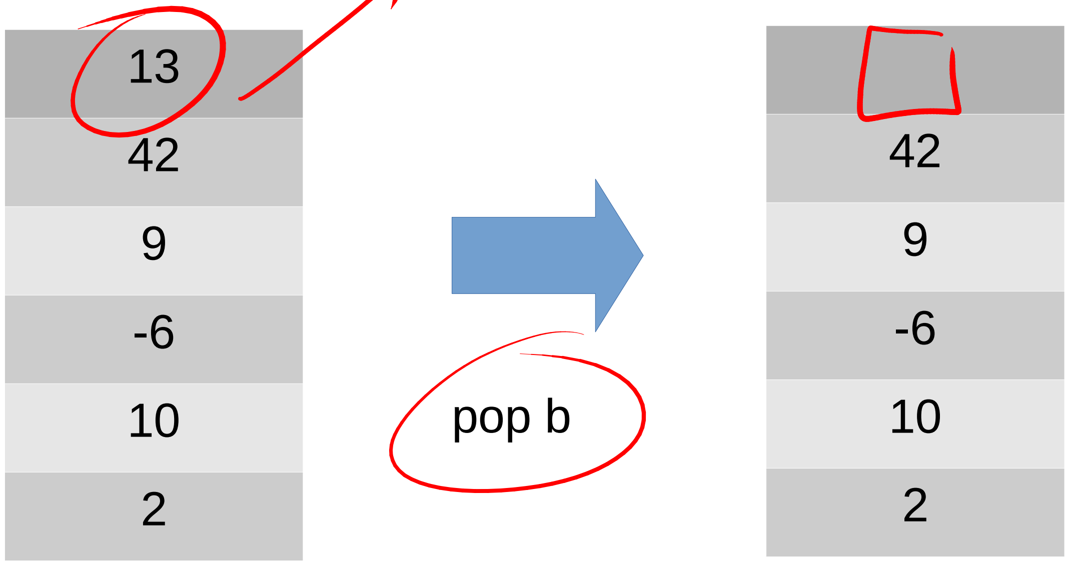
42	**13**	13	27



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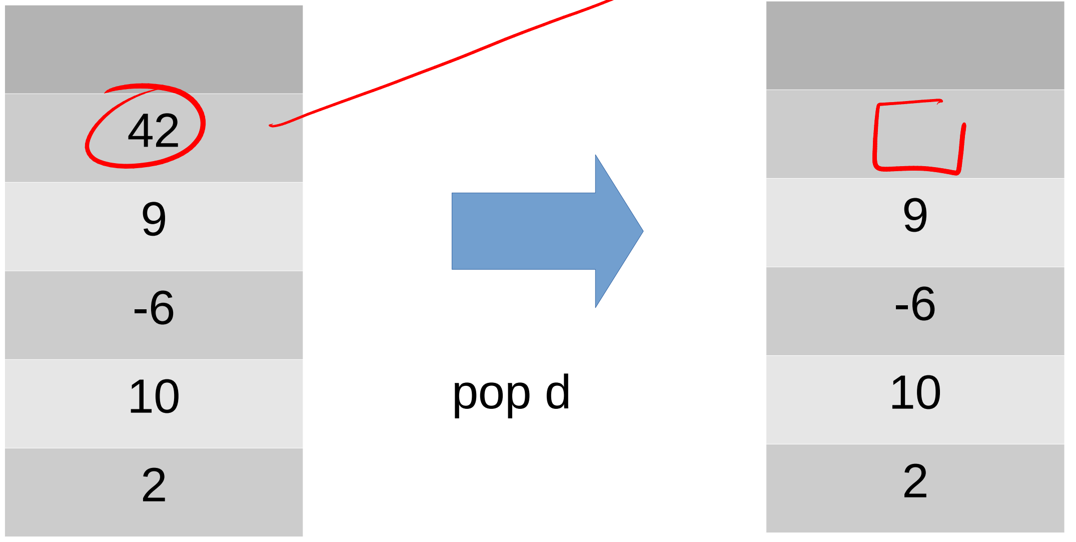
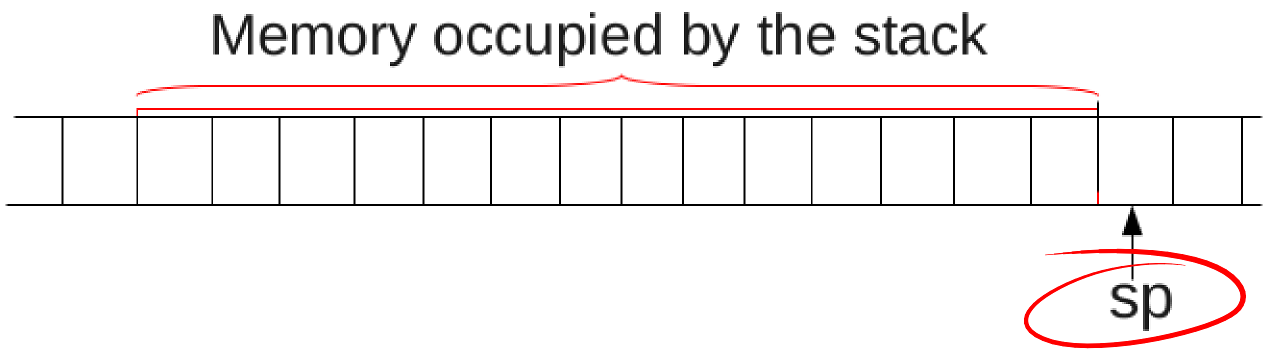


In Memory

- Give CPU another register
  - sp (stack pointer) – shows where top is



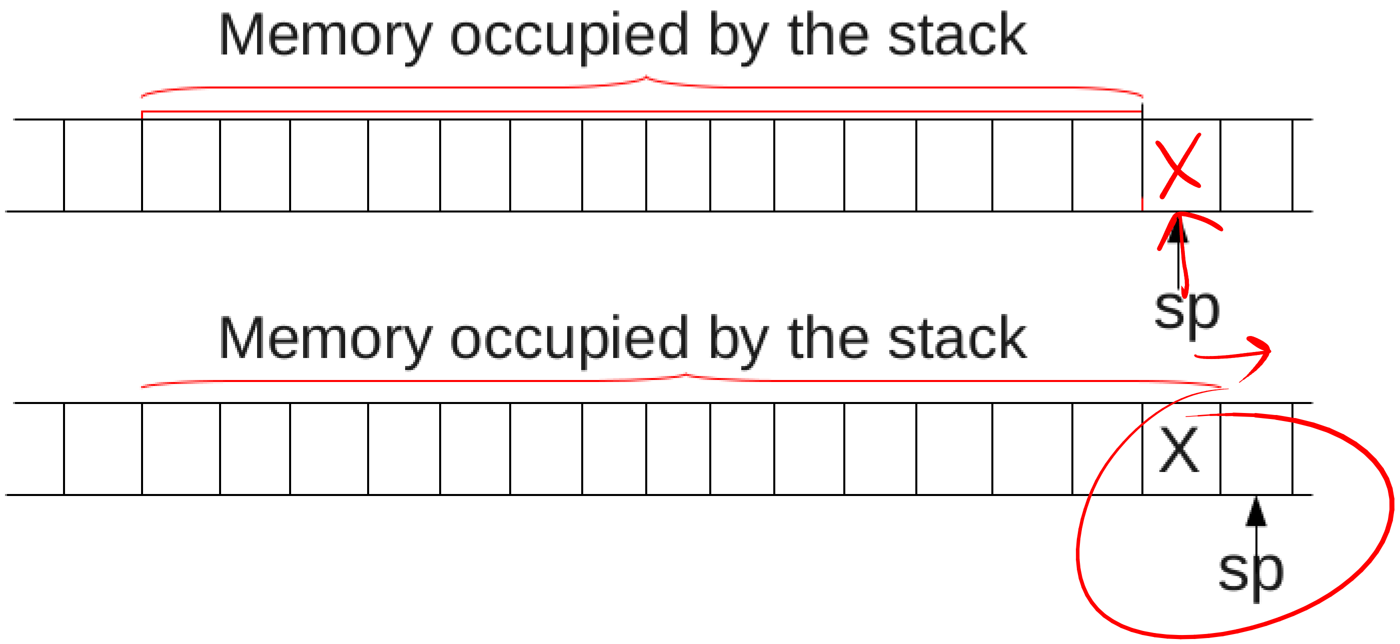
- Don't need to know where bottom is!
  - Provided that we are careful: only pop when you know you've pushed

In Memory

- To push a value X:
  - Write the value to memory at address sp
  - Add 1 to sp
- To pop a value:
  - Subtract 1 from sp
  - Read value from memory at address sp

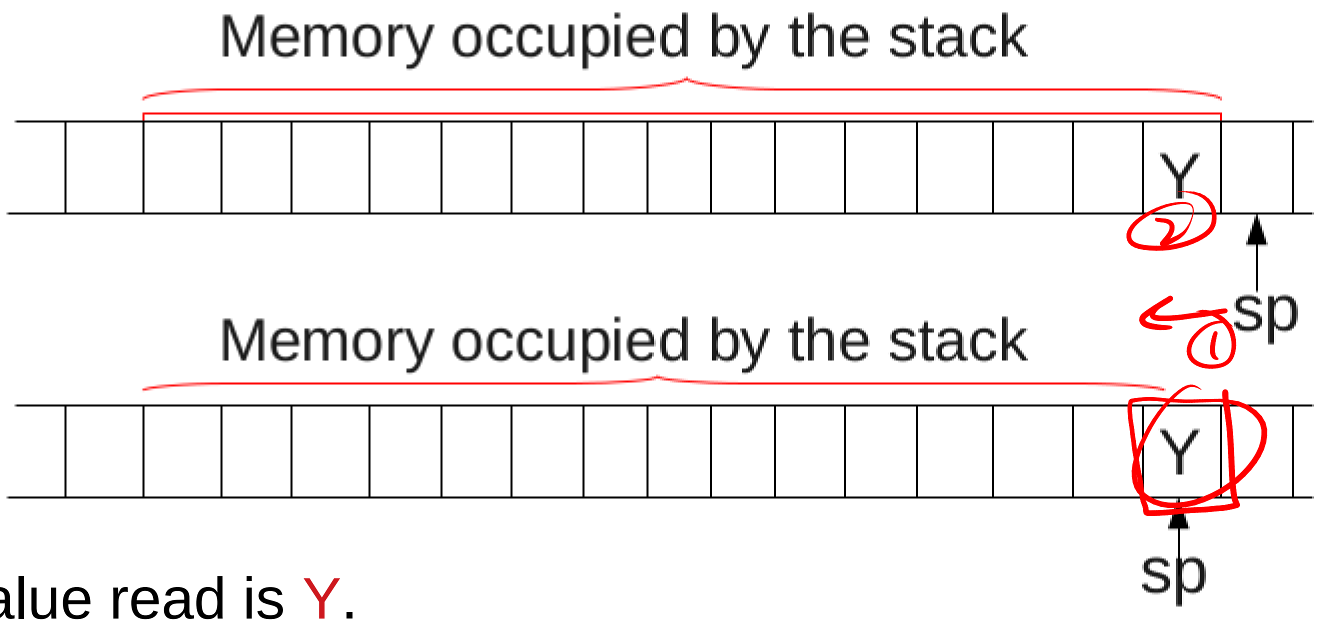
In Memory – Push a Value

- To push a value X:
  - Write the value to memory at address sp
  - Add 1 to sp



In Memory – Pop a Value

- To pop a value:
  - Subtract 1 from sp
  - Read value from memory at address sp



The value read is Y.

Still in memory but will be overwritten by a later push operation.

Storing the Return Address

- **Idea #2:** Store the return address on a stack.

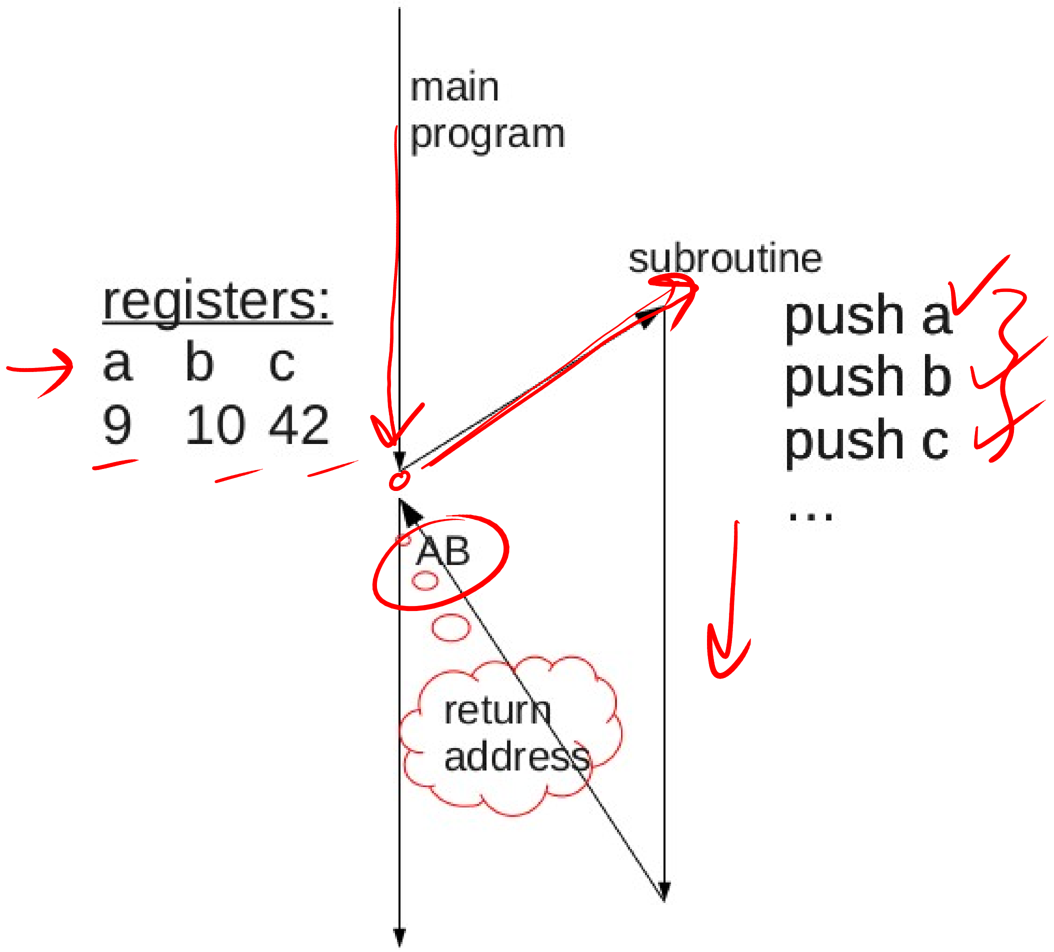
```
call N       works as if:
             push pc   // push program counter
             ld pc N   // jump to N

ret          works as if:
             pop pc    // pop return address
                       // & jump to it
```

(none of these are actual operations)

Why should we Save Registers?

- Subroutine may need to use registers for its calculations.
  - But previous register values are needed on return!
- Common pattern for subroutines:
  - Start by pushing all registers
  - Pop them back before return
- Return address & saved registers = **stack frame**
  Java method-calls develop this idea.

Saving Registers – Stack Frame



Stack:

..., AB

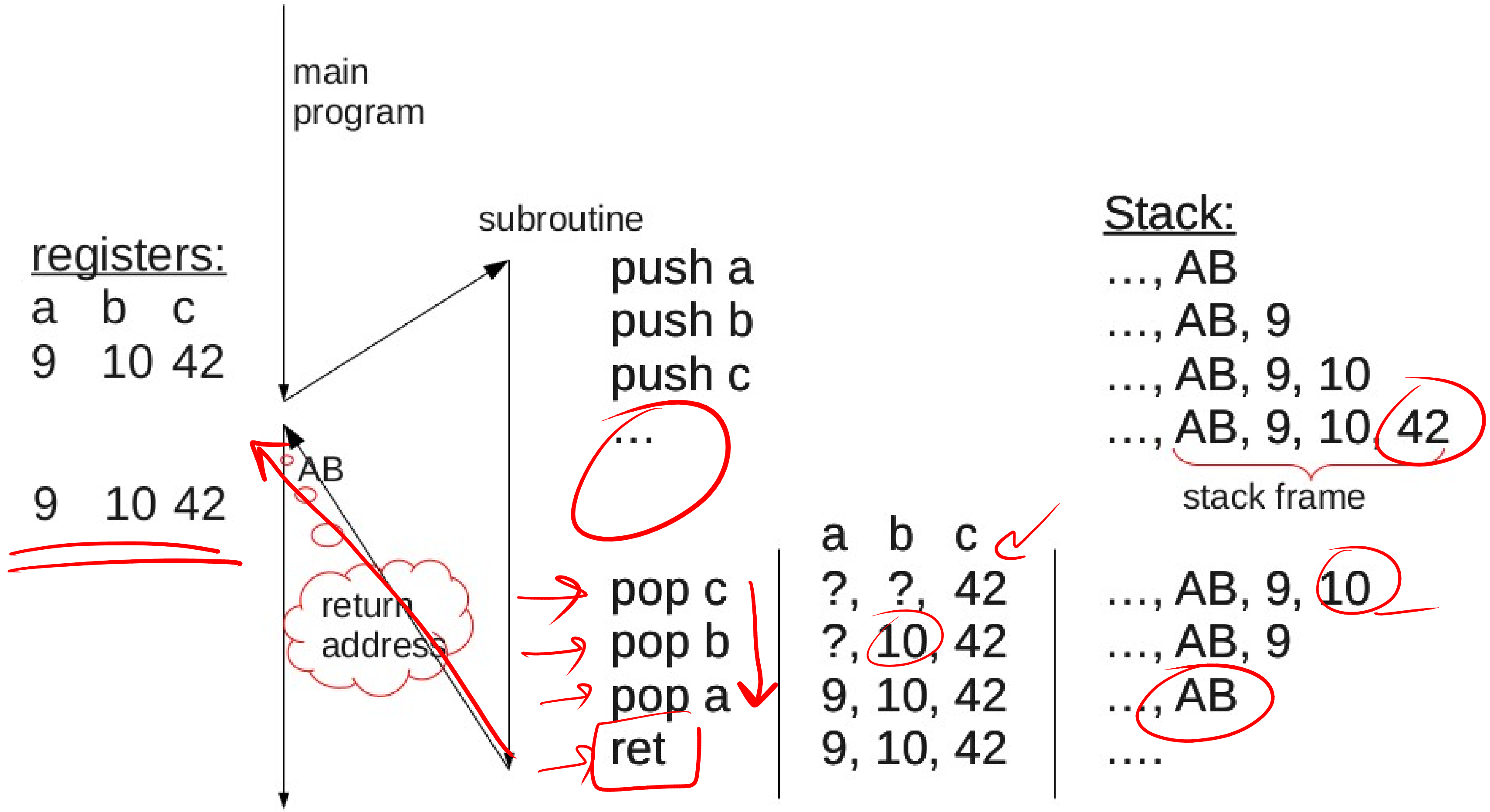
..., AB, 9

..., AB, 9, 10

..., AB, 9, 10, 42

stack frame

Saving Registers – Stack Frame



Stacks for Calculation

- Example:

$$\Rightarrow \quad (5+2)*\sqrt{x*x+y*y}+8$$

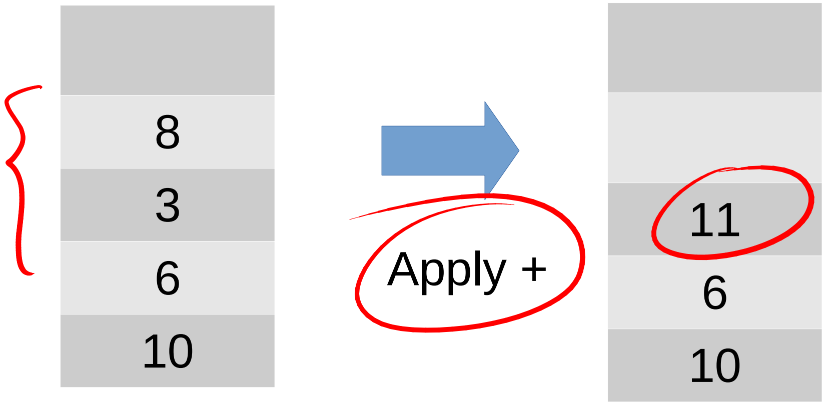
What order are operations applied in?

$$(5\underset{1}{+}2)\underset{6}{*}\underset{5}{\sqrt{x\underset{2}{*}x\underset{4}{+}y\underset{3}{*}y}}\underset{7}{+}8$$

$\sqrt{}$ means "square root of" (SQRT) e.g. SQRT(x*x + y*y)

Reverse Polish Notation (RPN)

- Order of operations is as written
- No brackets needed
- Powerful use of stack (= operand stack) to store intermediate results
- If its a Number or Variable: push it on the stack
- If its an Operation: apply to the top elements on stack & push result back on the stack



RPN for the Example

$$(5+2)*\sqrt{x*x+y*y}+8$$

$\underline{\quad 1 \quad\quad 6\ 5 \quad 2 \quad\quad 4 \quad\quad 3 \quad\quad 7 \quad}$

Reverse Polish: push operands, then operate.

We get:

$$\xrightarrow{\qquad\qquad\qquad\qquad}$$

$$\underline{5\ \ 2}\ \ \underset{\underline{1}}{+}\ \ \underline{x\ \ x}\ \ \underset{\underline{2}}{*}\ \ y\ \ y\ \ \underset{3}{*}\ \ \underset{4}{+}\ \ \underset{5}{\sqrt{\ }}\ \ \underset{6}{*}\ \ 8\ \ \underset{7}{+}$$

$$\xrightarrow{\qquad\qquad\qquad\qquad}$$

How to use RPN along with a Stack?

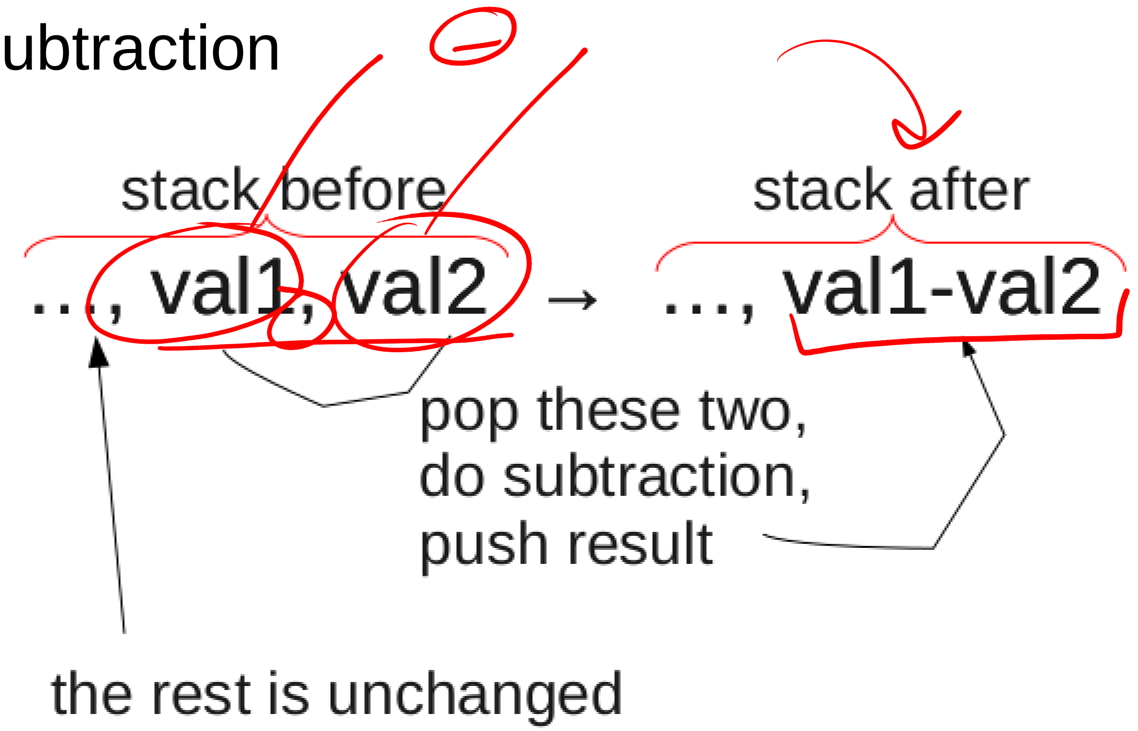
- Suppose that x has value 3, and y has value 4
- Now, evaluate the expression. (Top of stack is on right.)

5, 2

Operation	Stack
	empty
5	5
2	5,2
+	7
x	7,3
x	7,3,3
*	7,9
y	7,9,4
y	7,9,4,4
*	7,9,16
+	7,25
SQRT	7,5
*	35
8	35,8
+	43

Notation for Operand Stack

- To show what an operation does to the stack:
- e.g. subtraction



More on Reverse Polish Notation

- Any expression can be converted to Reverse Polish Notation and then its easy to execute with a stack.
- Applications
  - Humans use reverse Polish directly (See Details)
  - e.g. some pocket calculators – HP in 1970s (HP-41C)

  https://www.theregister.co.uk/Print/2014/01/03/ten_classic_calcutors/

- Forth programming language has two stacks:
  - → Operand stacks for calculations
  - → Return stack for module calls

  More details: https://en.wikipedia.org/wiki/Forth_(programming_language))

Applications of Reverse Polish Notation

- Compile to a reverse Polish form that is then executed.
  - e.g. Postscript format, for printable files
    - executed by printers
  - e.g. Java byte code
    - uses operand stacks for calculations

- In Java, each method call has its own operand stack.

Stack instead of Registers (Stack Machines)

- Use 2 stacks
  - **return stack** for subroutine return
  - **operand stack** for Reverse Polish calculations
- Don't need a,b,c registers
  - **Advantages**
    - ✓ More space for calculations
    - ✓ Opcodes don't need to specify registers
  - **Disadvantages**
    - → Harder to know where things are on the stack

What is an Operand?

- Underlying meaning:
  - Whatever an operator operates on?
- Two meanings here (don't confuse them):

1) Extra bytes after the instruction opcode in memory,

e.g.

$\rightarrow$ ld a $\boxed{42}$

2) Entries in the operand stack.

Machine Instructions as Stack Operators

- Arithmetic:

  e.g. **add** - adds top 2 stack entries
  …, val1, val2 → …, val1+val2

  e.g. **sub** - subtracts top 2 stack entries
  …, val1, val2 → …, val1-val2

  e.g. **neg** - negates top stack entry
  …, val → …, -val

- Similarity: **mul**, **div**, **rem**

remainder

Example of a Stack Machine (JVM mnemonics)

$5 \; 2 \; + \; x \; x \; * \; y \; y \; * \; + \; \sqrt{} \; * \; 8 \; +$

```
push 5
push 2
add
load x
load x
mul
load y
load y
mul
add
call √
mul
push 8
add
```

~~more next week~~

Operation	Stack
	empty
5	5
2	5,2
+	7
x	7,3
x	7,3,3
*	7,9
y	7,9,4
y	7,9,4,4
*	7,9,16
+	7,25
SQRT	7,5
*	35
8	35,8
+	43

Bitwise Boolean Operations

- Boolean operations on one bit
  0=false, 1=true

"eXclusive OR"

OR	0	1
0	0	1
1	1	1

AND	0	1
0	0	0
1	0	1

XOR	0	1
0	0	1
1	1	0

- Can do these bit-wise on binary values. Example for XOR:

```
0 0 1 1 1 0 1 1 1 1 ...............0 1 1 0 0 1 0 0 0
0 0 0 1 0 0 1 0 0 1 ...............1 1 1 1 0 1 1 0 0
------------------------------------------------------------
0 0 1 0 1 0 0 1 1 0 ...............1 0 0 1 0 0 1 0 0
```

XOR: done on top 2 stack entries (similar to: or, and):
…, val1, val2 → …, val1 XOR val2

Jumps

1 - Unconditional jumps
  - Operand stack is not used!
2 - Conditional jumps

0x1000

```
ifeq N            // jumps to N if val=0
    ..., val  →  ...

if_cmpeq N        // jumps to N if val1=val2
    ..., val1, val2  →  ...
```

compare

Conditional Jumps with Other Comparisons

$$\text{jump if} \quad \text{val} \left\{ \begin{array}{c} = \\ < \\ \leq \\ \neq \\ > \\ \geq \end{array} \right\} 0 \qquad \left. \begin{array}{l} \text{eq} \\ \text{lt} \\ \text{le} \\ \text{ne} \\ \text{gt} \\ \text{ge} \end{array} \right\}$$

$\neq$

6 operators: ifeq, iflt, ifle, etc.

also: if_cmpeq, if_cmplt, etc.

Summary

We have now seen:

- What is a subroutine and how it is implemented.
- What is a stack and how it is used for implementing subroutines.
- What is the importance of saving registers and how these can be used to perform computations.
- What is reverse polish notation and how it is used to to do calculations using an operand stack.
- What is a stack machine and how it operates using a stack instead of registers.
- What are Bitwise Boolean operations and Jump instructions.

Notes on Exercises

- You will need to use an algorithm to convert math expressions from the usual “infix” notation to reverse-Polish. This algo is called Dijkstra's Shunting-Yard Algorithm.

- Have a look at the following links:
  - https://brilliant.org/wiki/shunting-yard-algorithm/
  - https://en.wikipedia.org/wiki/Shunting-yard_algorithm