CS45 – Machine Learning for Signal Processing

Learning Time Series

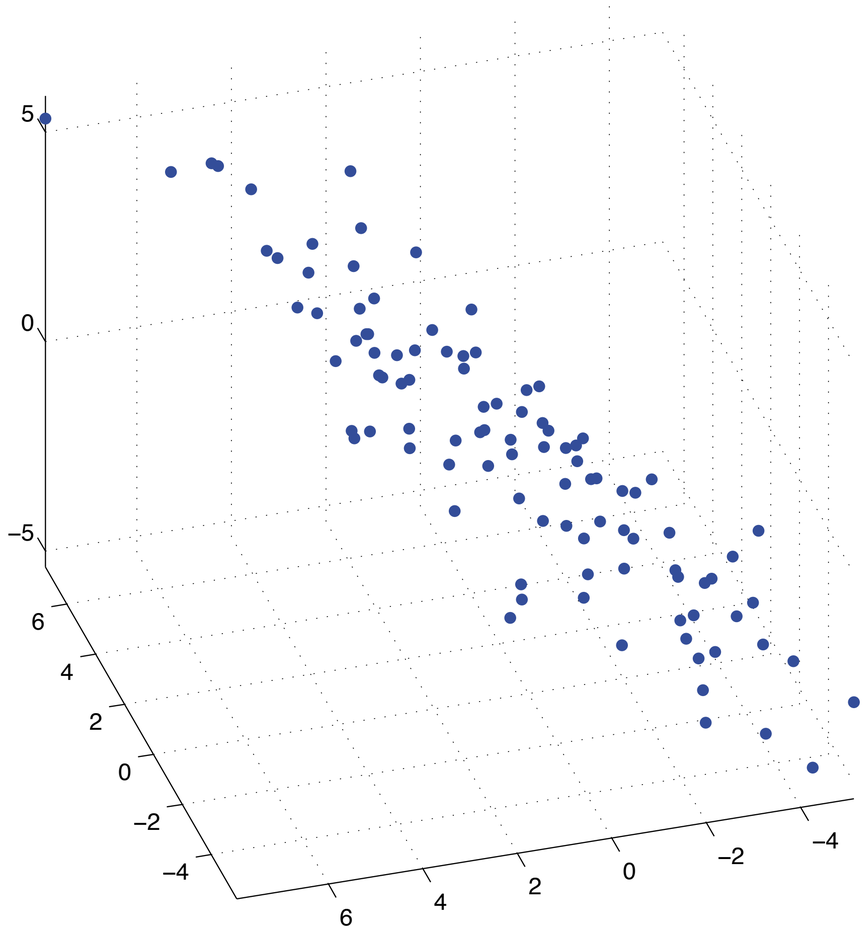
10 October 2023

Today's lecture

- Doing machine learning on time series (properly)
- Dynamic Time Warping
- Hidden Markov Models

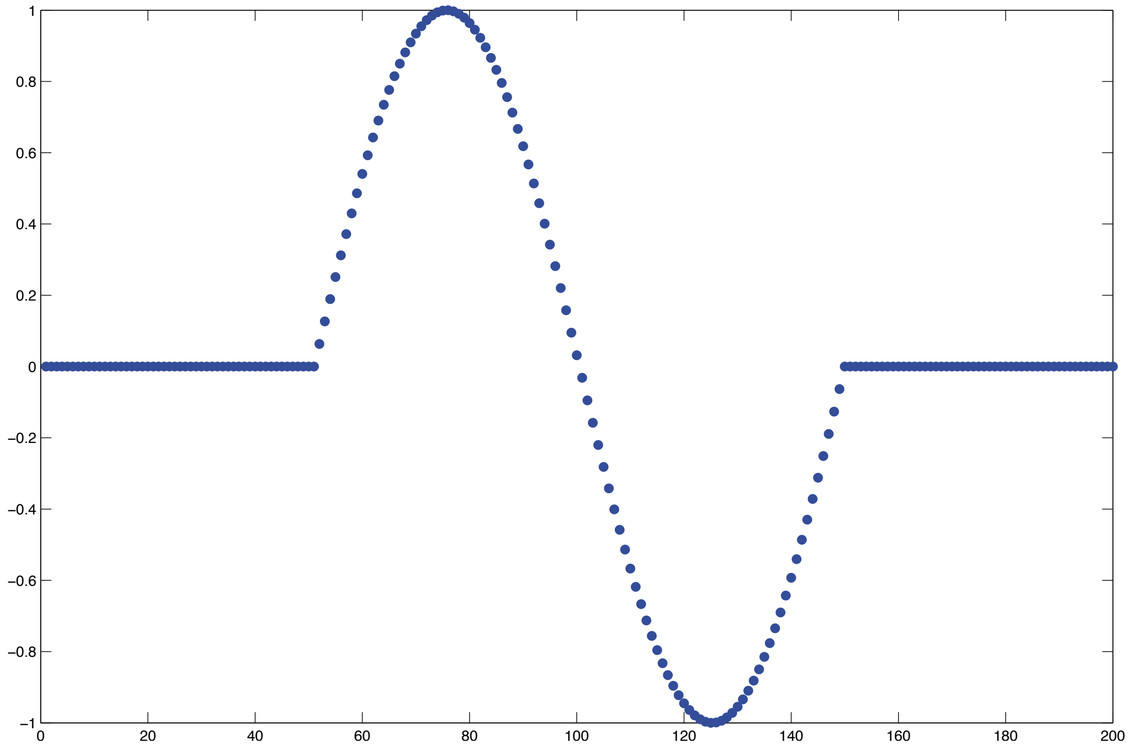
Our view of data so far

- Data are points in a high-dimensional space



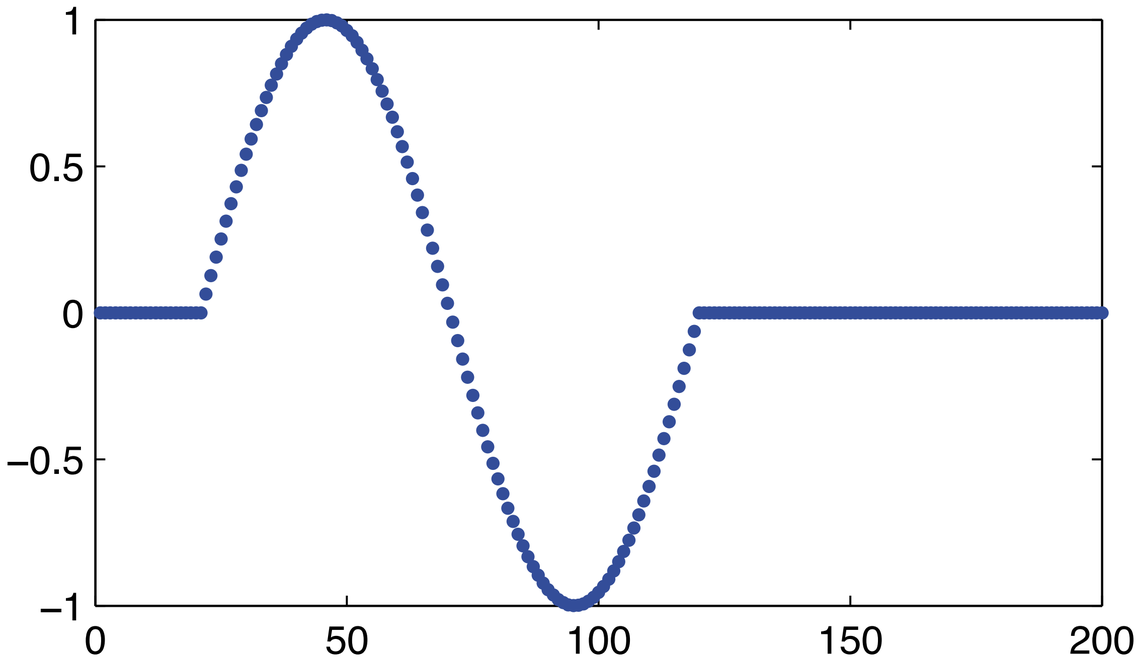
What about time series?

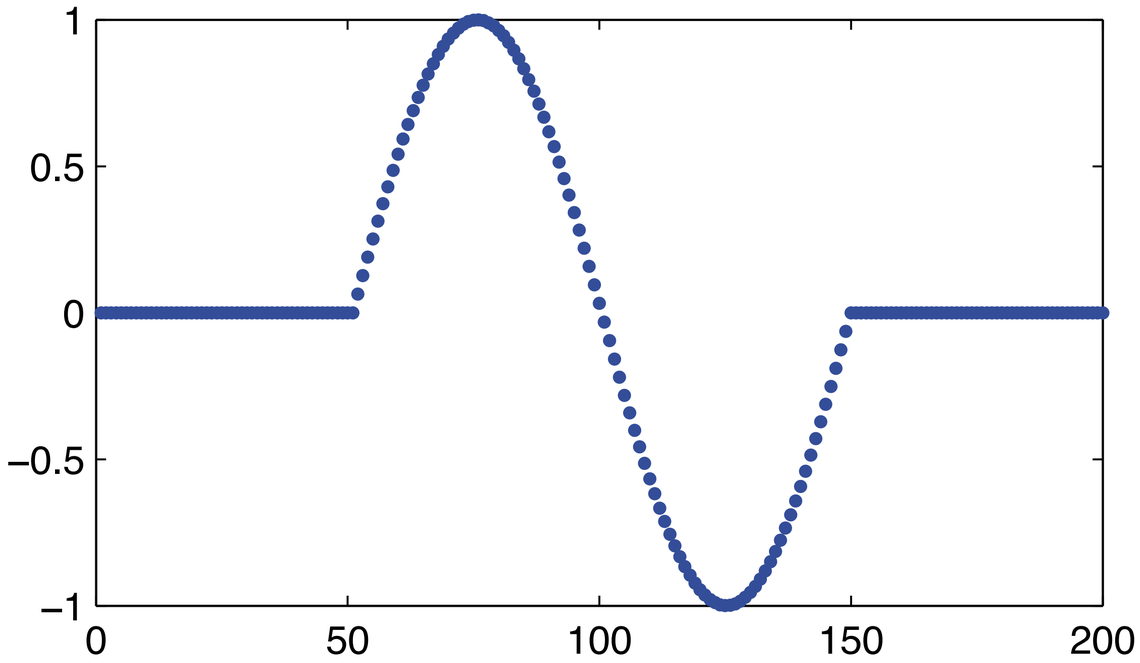
- A sequence of samples, can be thought of as a point in a very very high-D space
  - Often a bad idea
  - Remember the curse!!



Shift variance

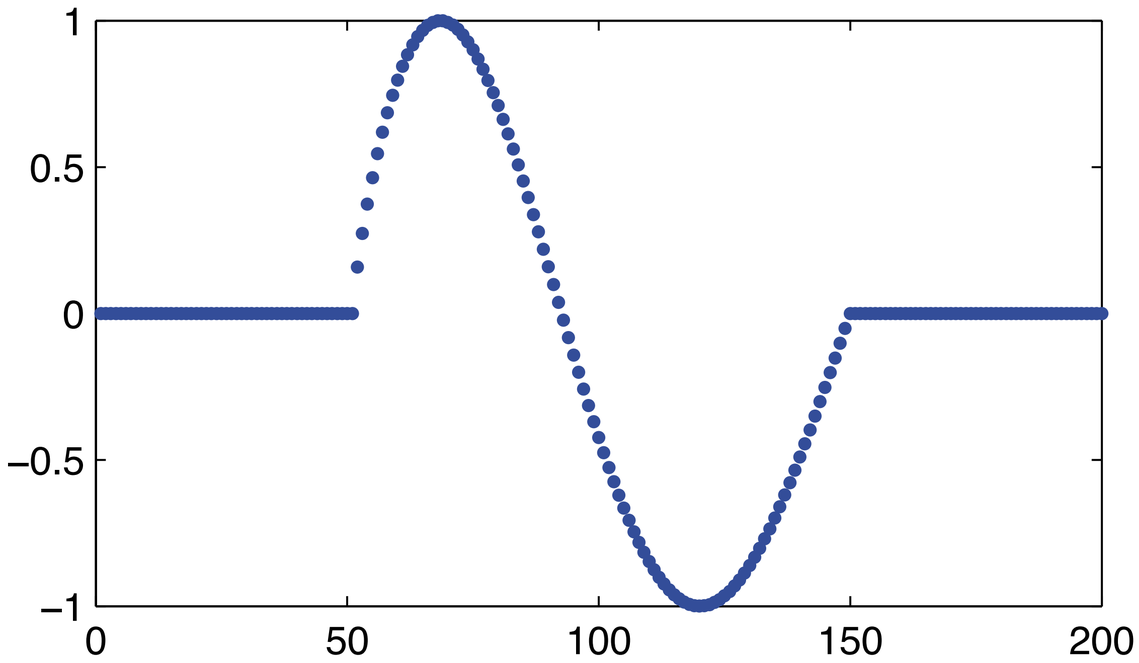
- Time series have shift variance
  - Are these two points close?

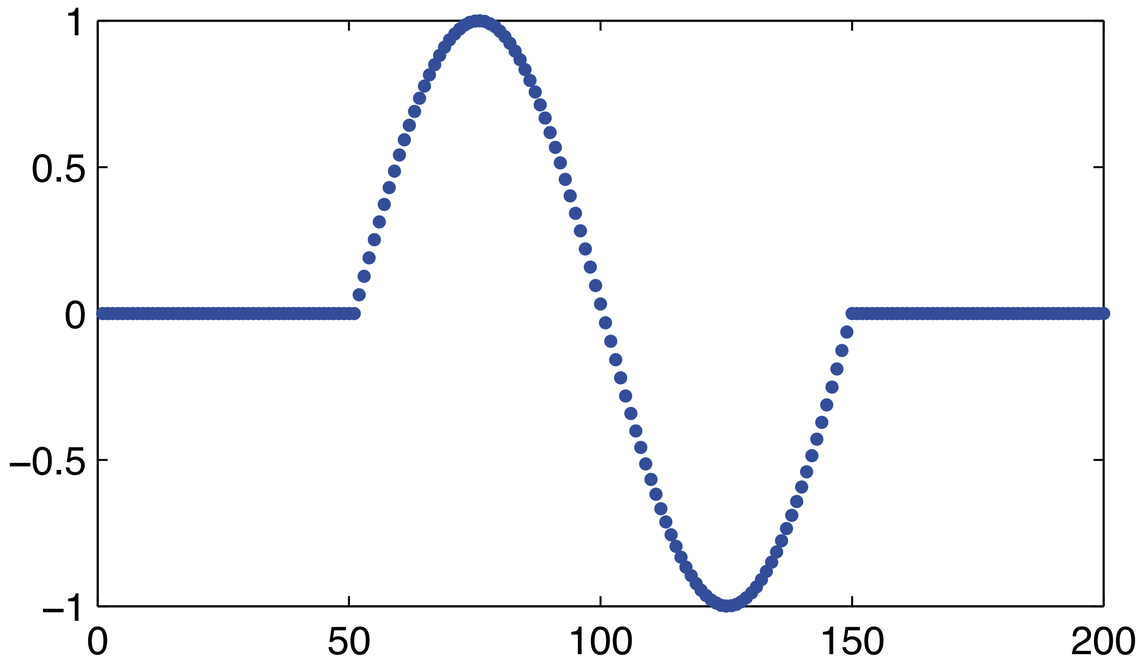




Time warping variance

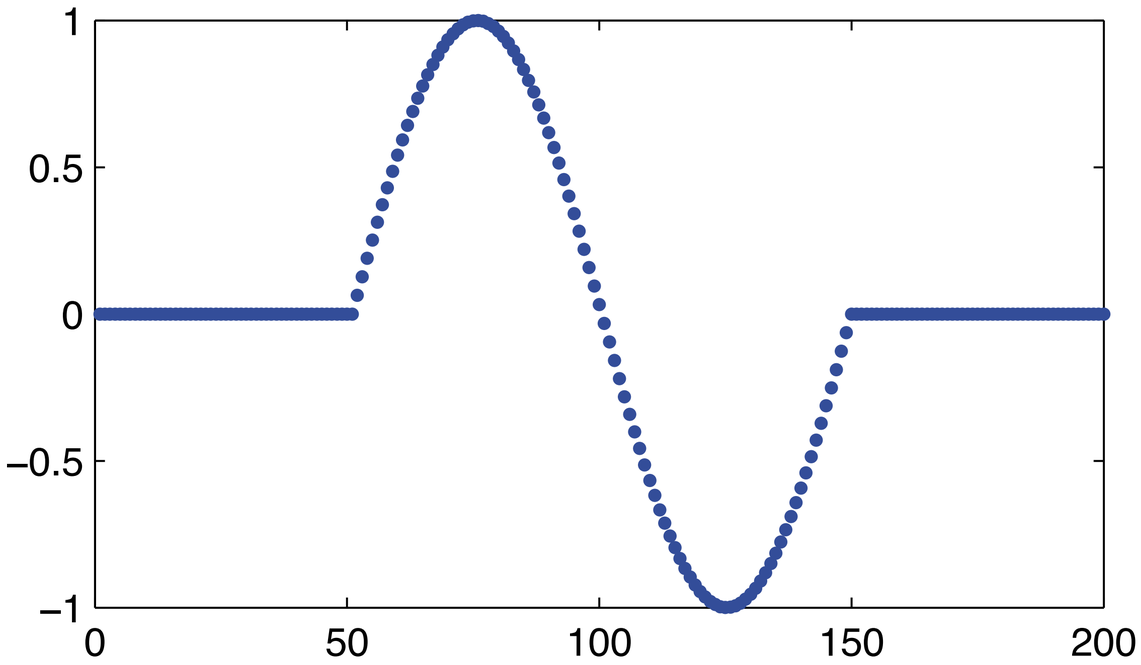
- Slight changes in timing are not relevant
  - Are these two points close?

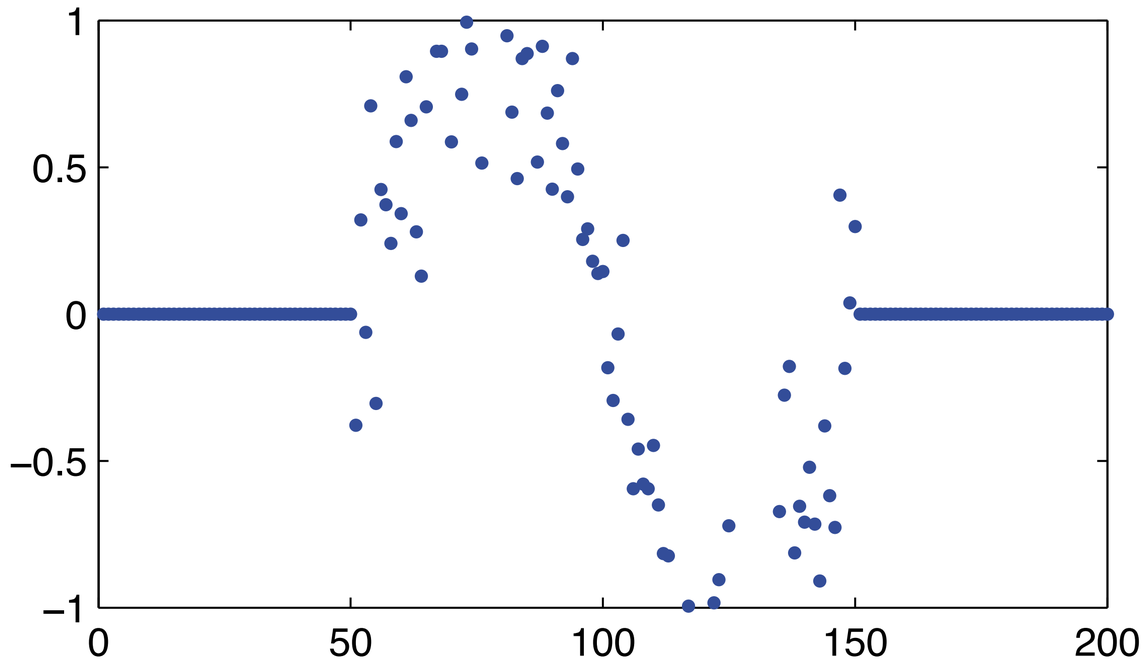




Noise/filtering variance

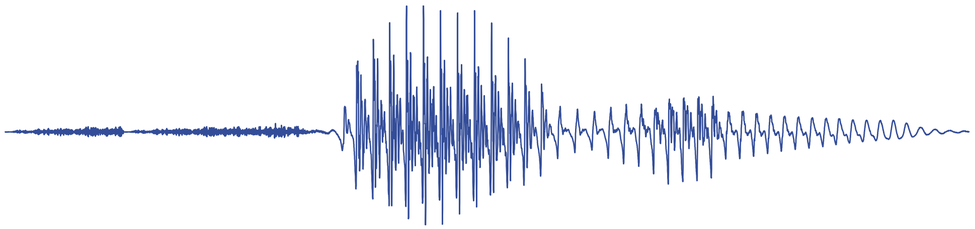
- Small changes can look serious
  - How about these two points?

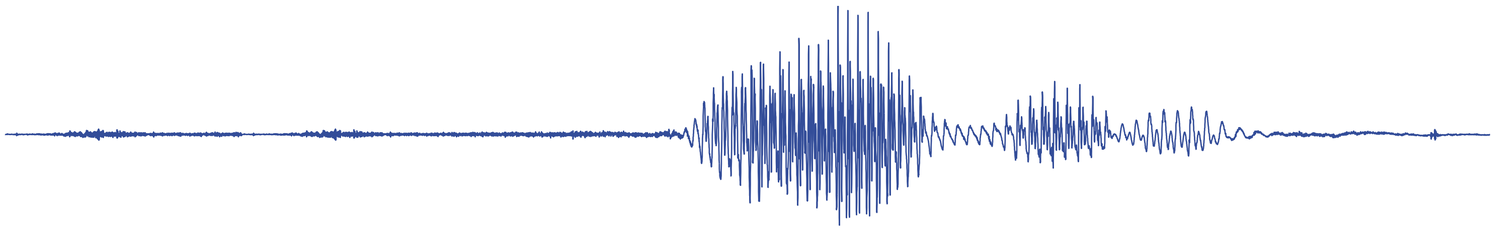


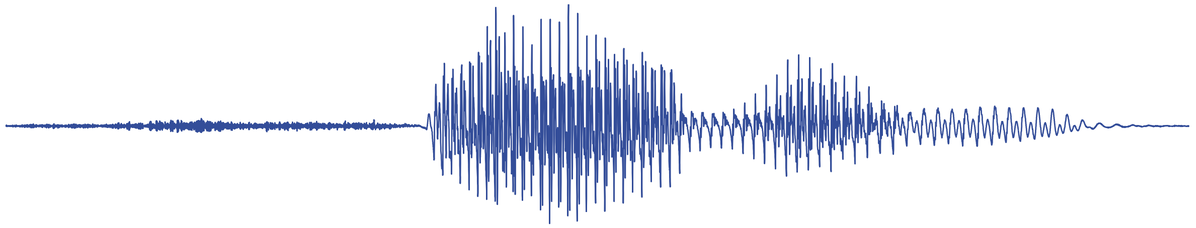


A real-world case

- Spoken digits







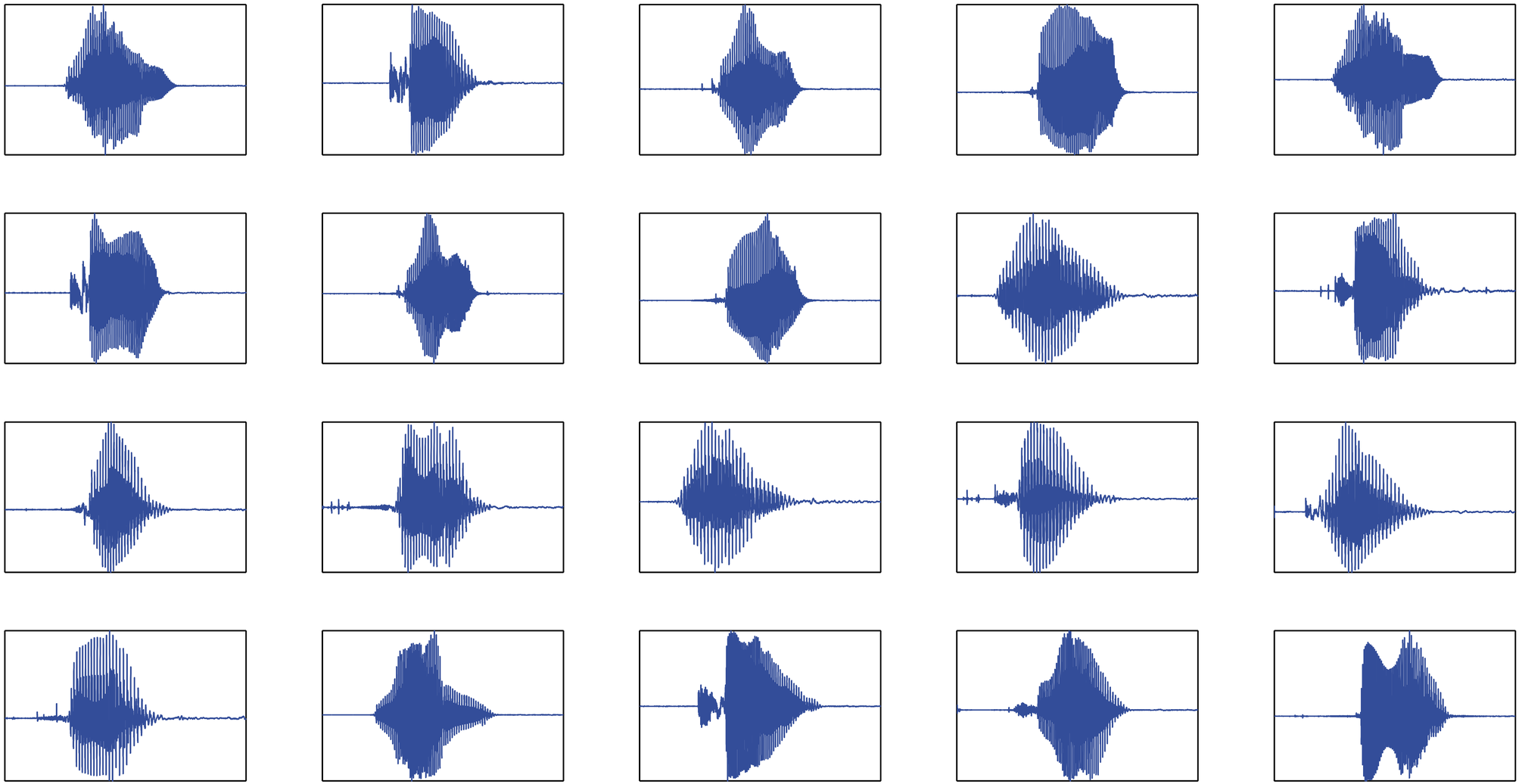
What now?

- Our models so far were too simple
- How do we incorporate time?
- How do we get around all these problems?

A motivating problem

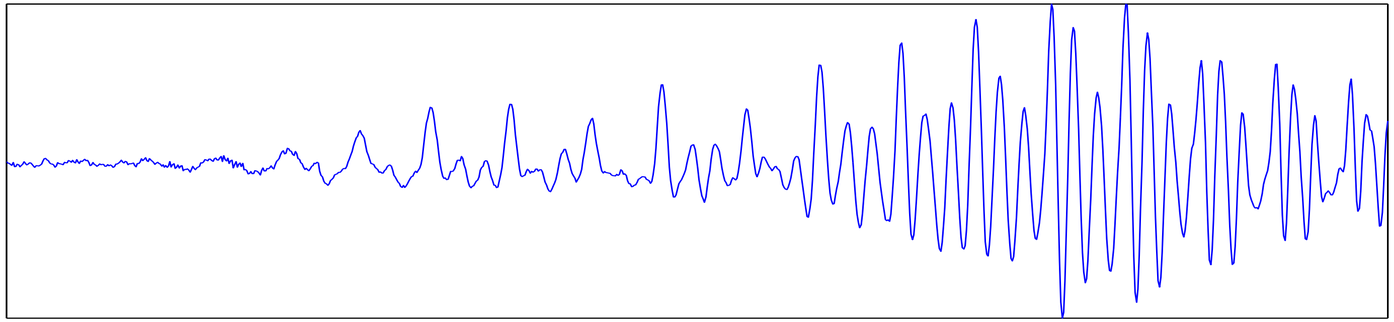
- How to recognize words
  - e.g. yes/no, or spoken digits
- We need reliable features
  - Invariant to minor differences in inputs
- We need a classifier that understands time
  - And is invariant to trivial temporal differences in inputs

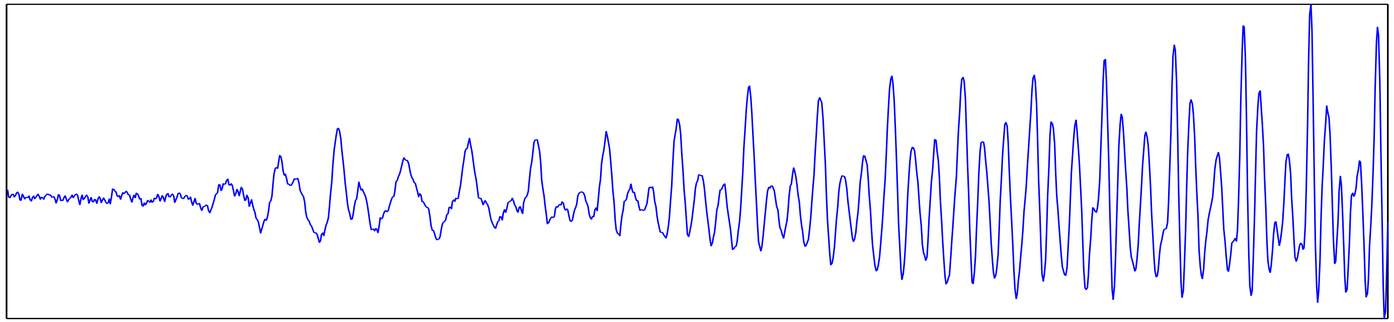
Example data



Going from fine to coarse

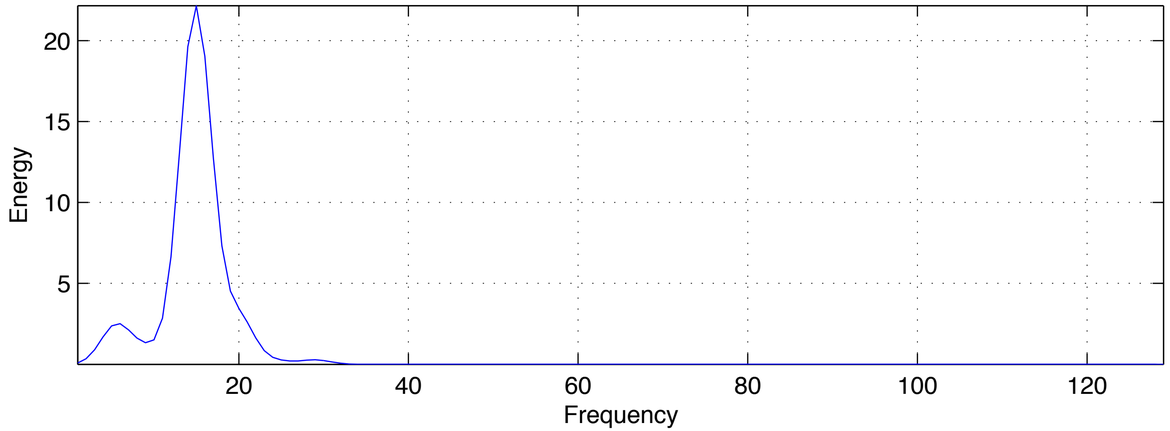
- Small differences are not important
  - Find features that obscure them

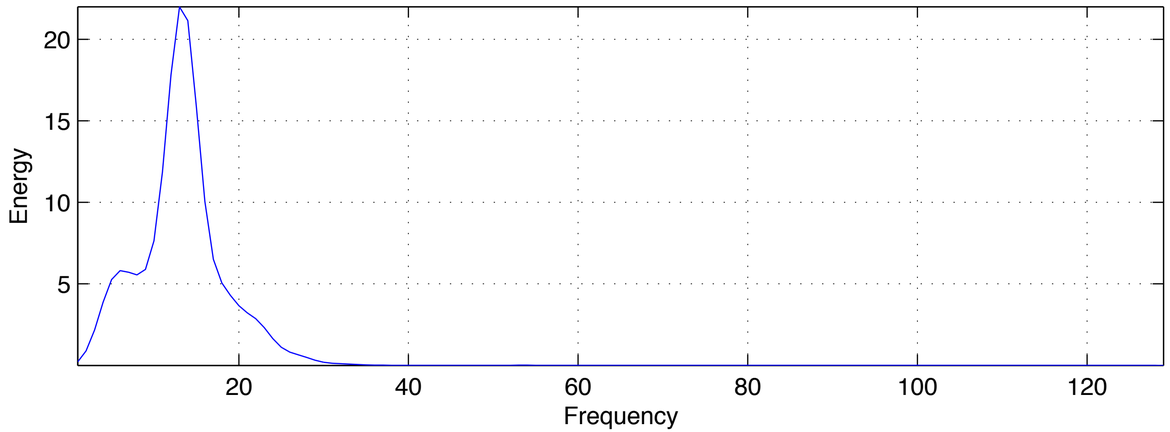




Frequency domain

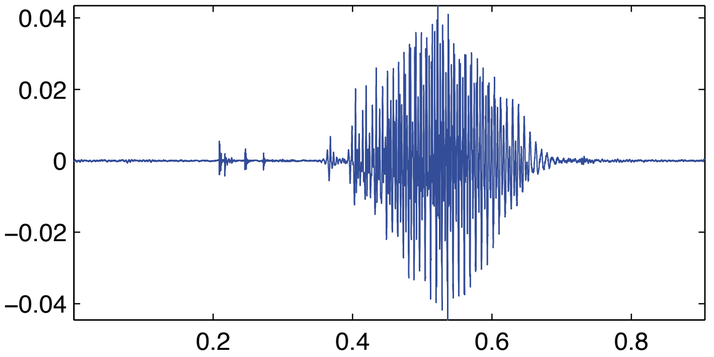
- Look at the magnitude Fourier transform

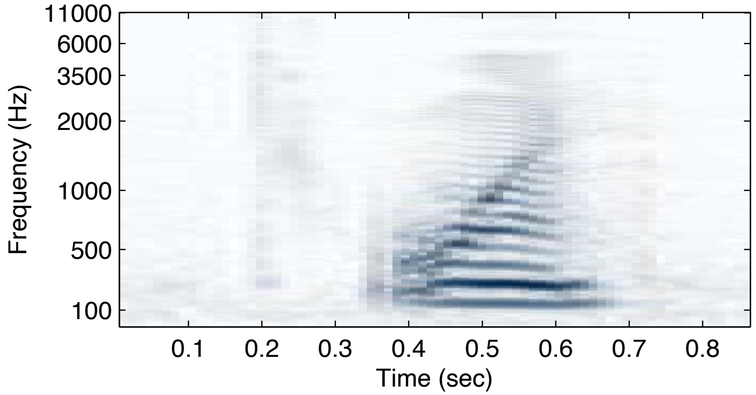


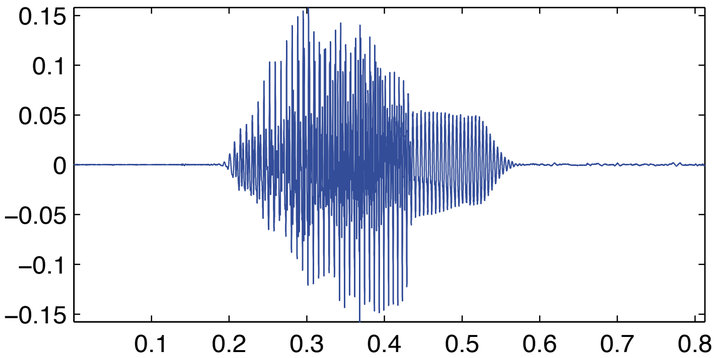


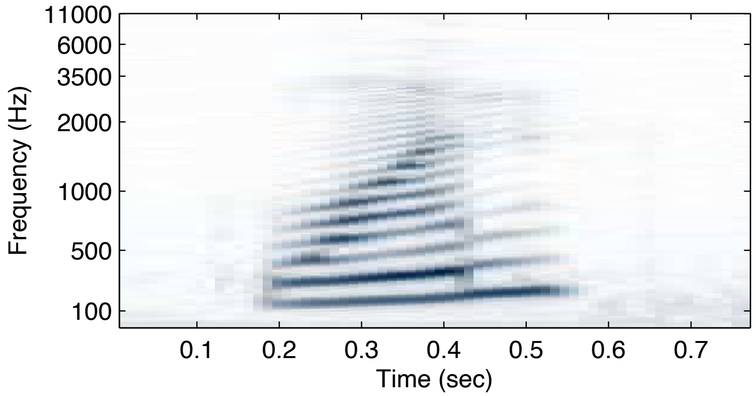
Time/Frequency features

- A more robust representation
  - Bypassing small waveform differences



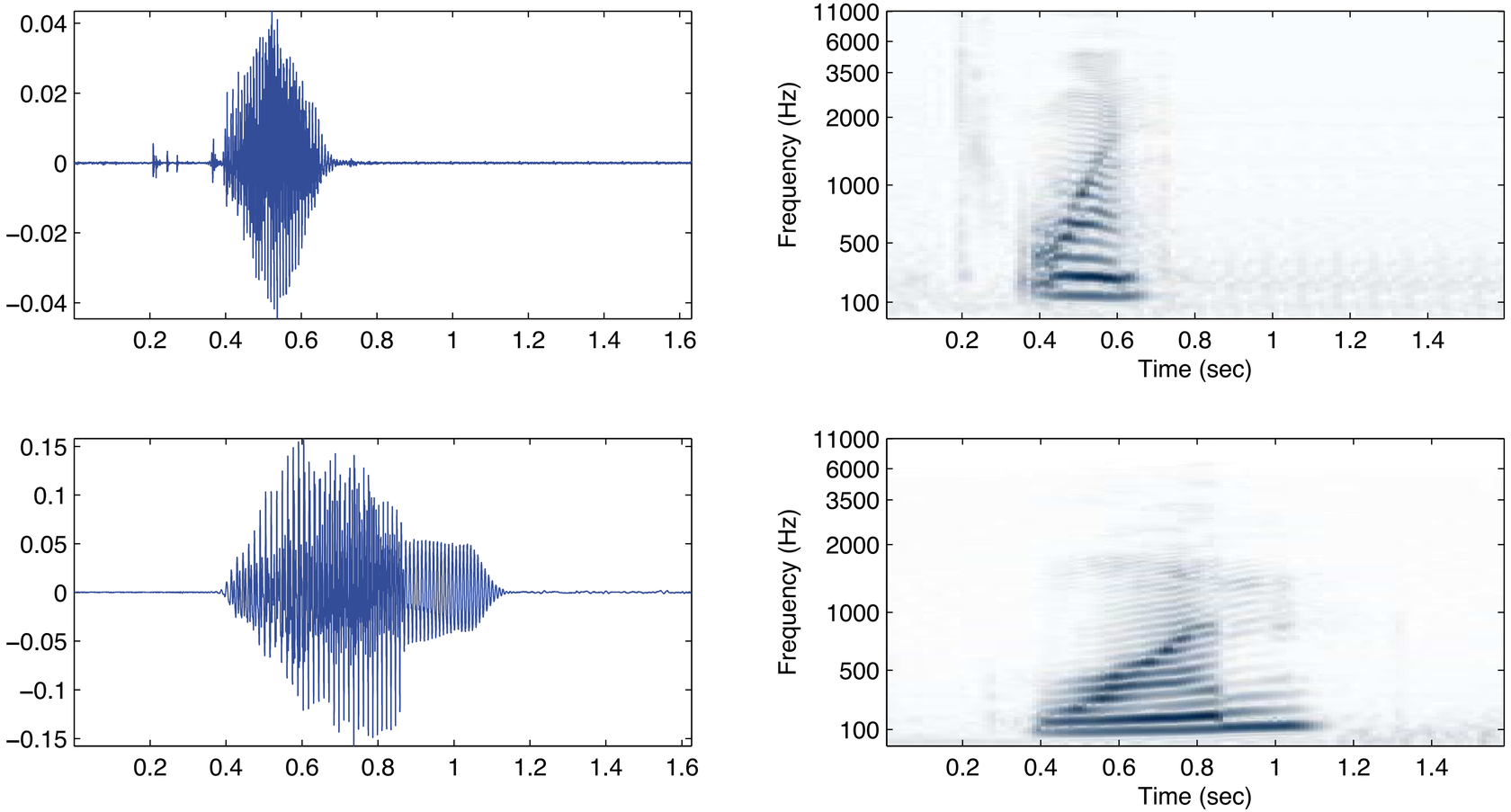






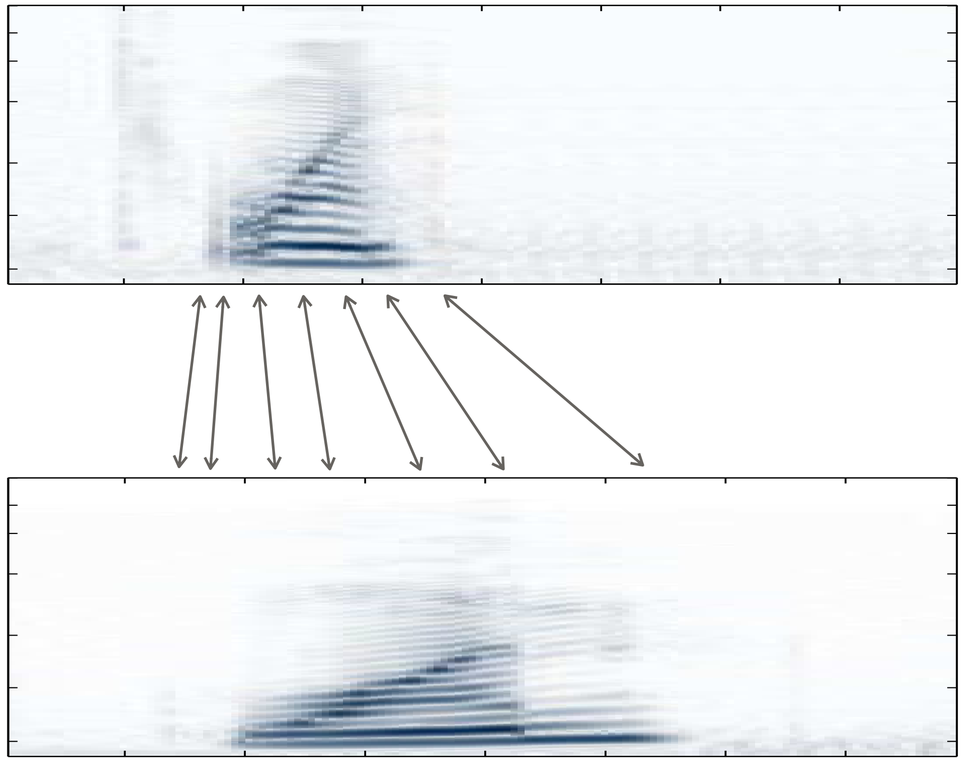
A new problem

- What about timing differences?



Time warping

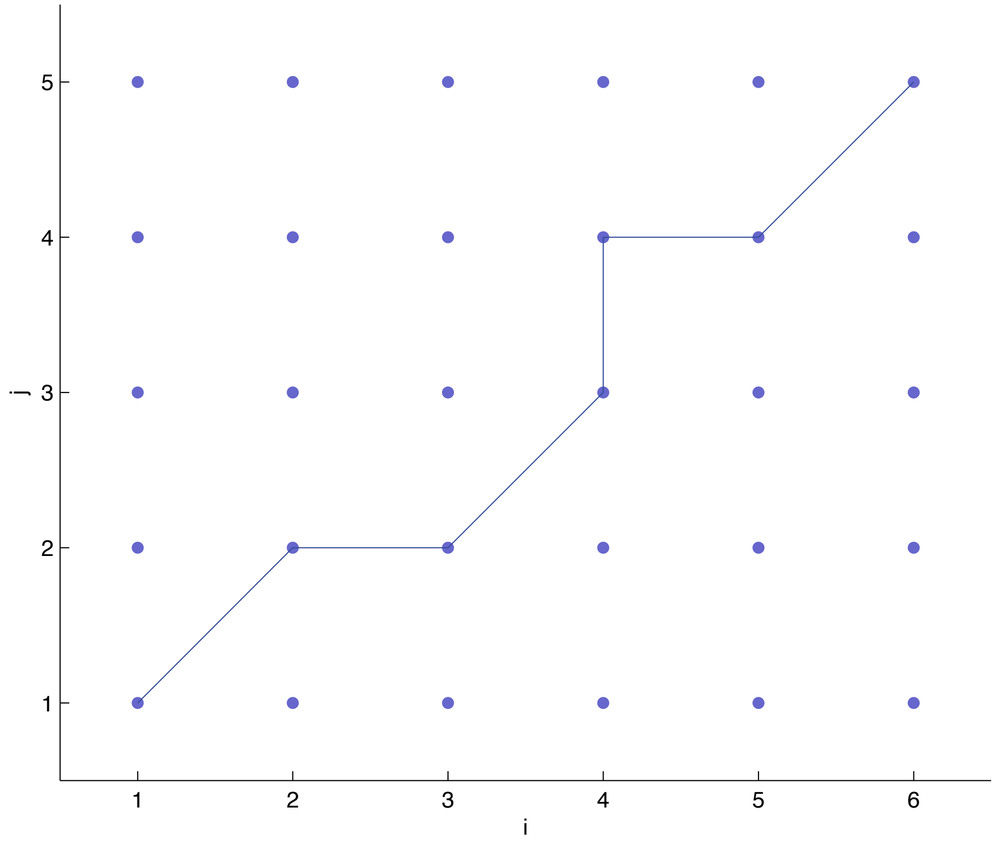
- We can *time warp* the sequences to line up
  - That will make the comparisons easier and more accurate
  - How do we find the warping map though?



Matching warped series

- Represent the warping with a path on a grid

$$\mathbf{r}(i), i = 1,2,\ldots,6 \quad \mathbf{t}(j), j = 1,2,\ldots,5$$

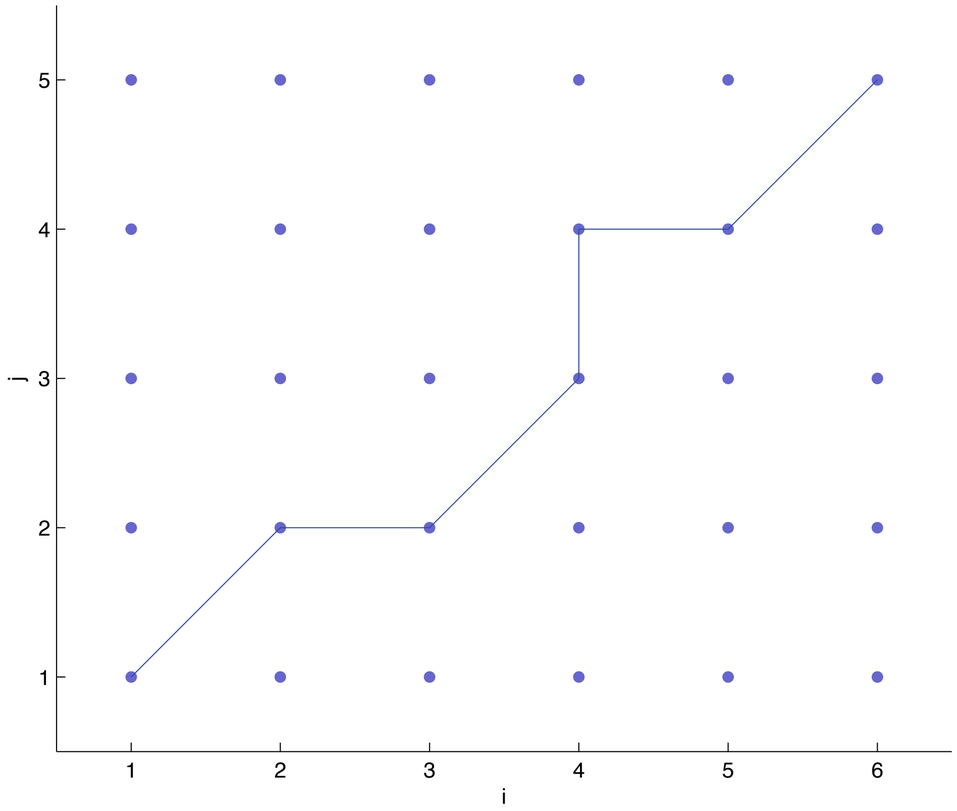


Finding the overall "distance"

- Each node will have a cost
  - e.g., $d(i,j) = ||\mathbf{r}(i) - \mathbf{t}(j)||$
- Overall path cost is:

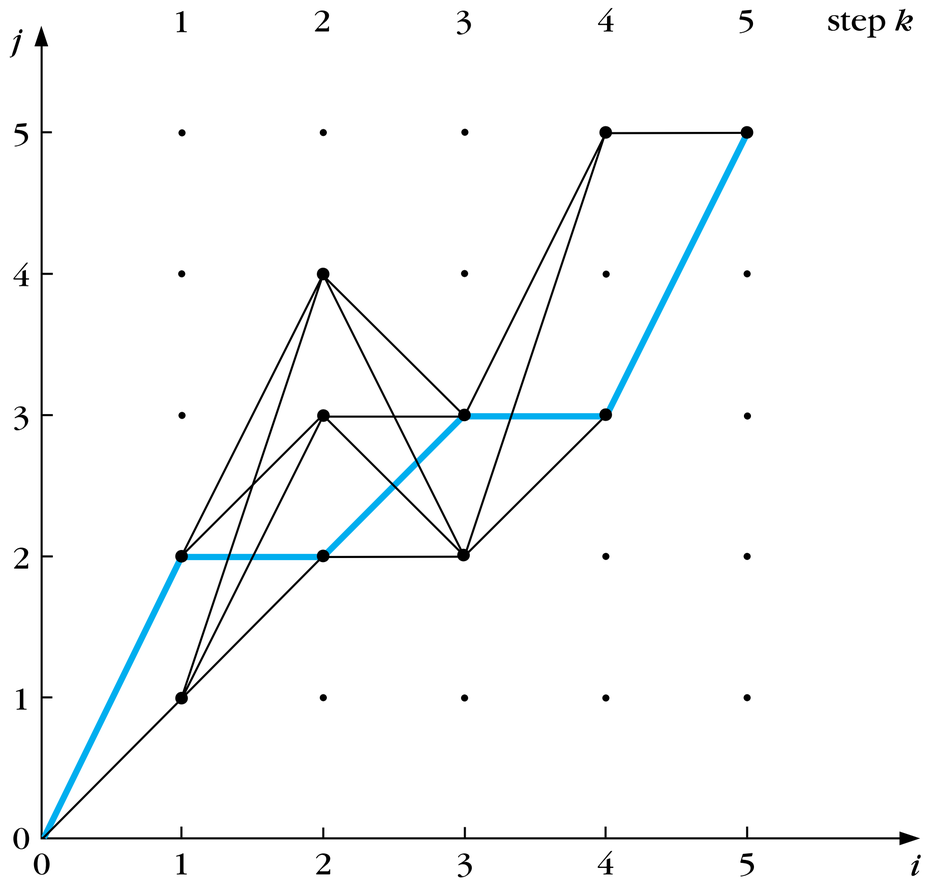
$$D = \sum_k d(i_k, j_k)$$

- Optimal path (i_k, j_k) defines the "distance" between given sequences
- But how do we find the optimal path?
  - Big search space …



Adding some constraints

- Global constraints
  - Can only visit these
    - Bold dots
- Local constraints
  - Can only transition using them
    - Black lines
- Optimal path
  - Blue line

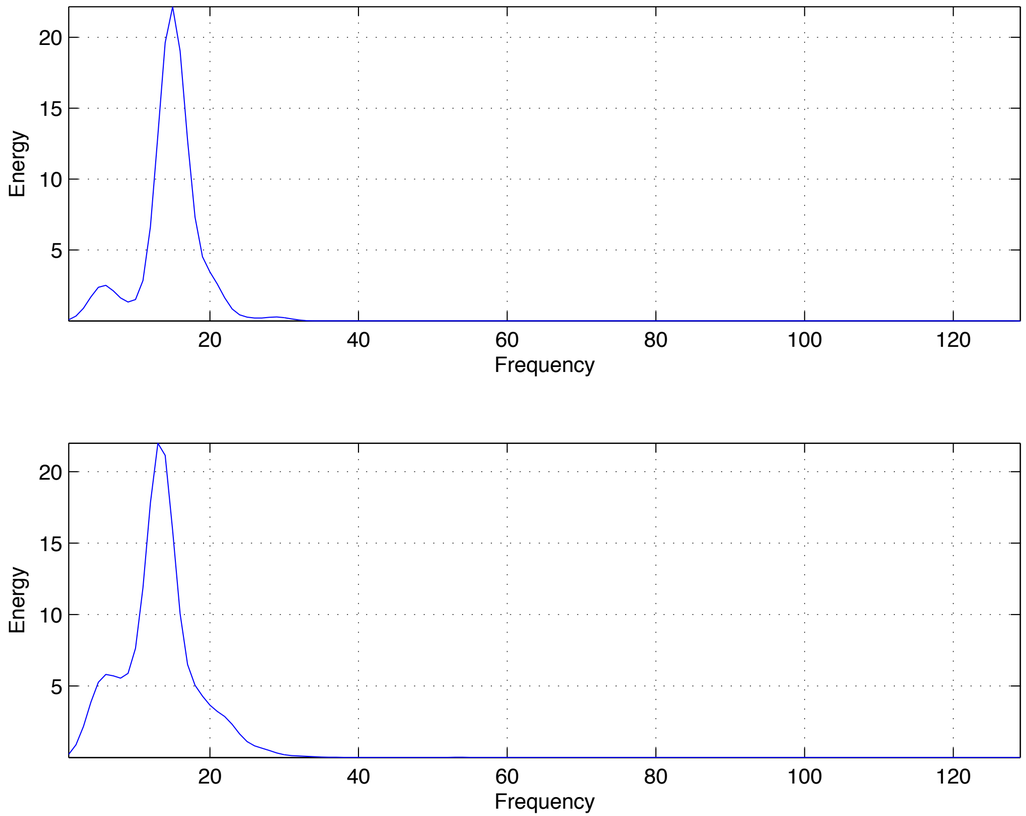


Making this work for speech recognition

- Define a distance function
- Define local constraints
- Define global constraints

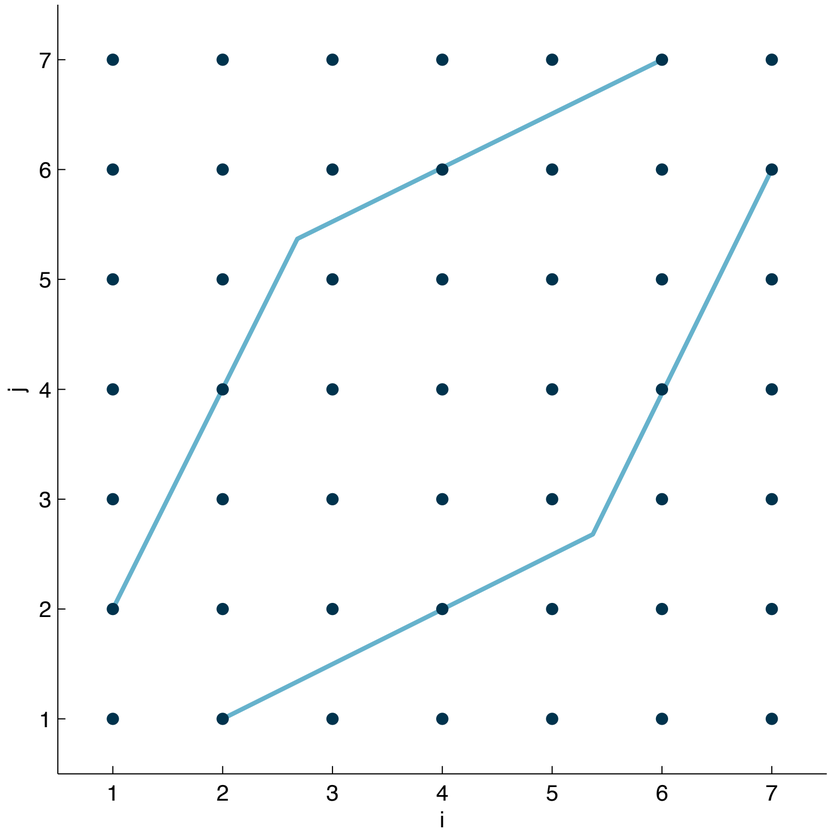
Distance function

- Given our robust feature we can use a simple measure like Euclidean distance: $d(i,j) = ||\mathbf{f}_1(i) - \mathbf{f}_2(j)||$



Global constraints

- Define time ratios that make sense
  - e.g. no sequence can be half as slow as the other one



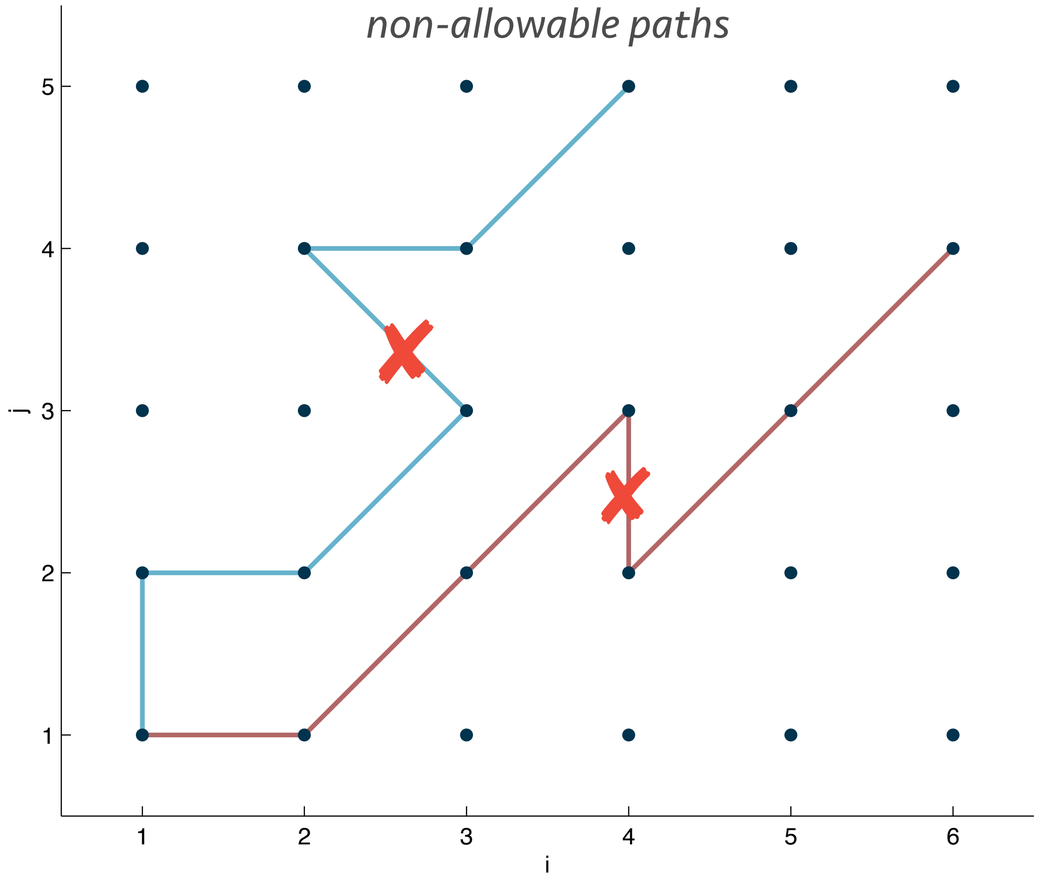
Local constraints

- Monotonicity:

$$i_{k-1} \leq i_k \quad j_{k-1} \leq j_k$$

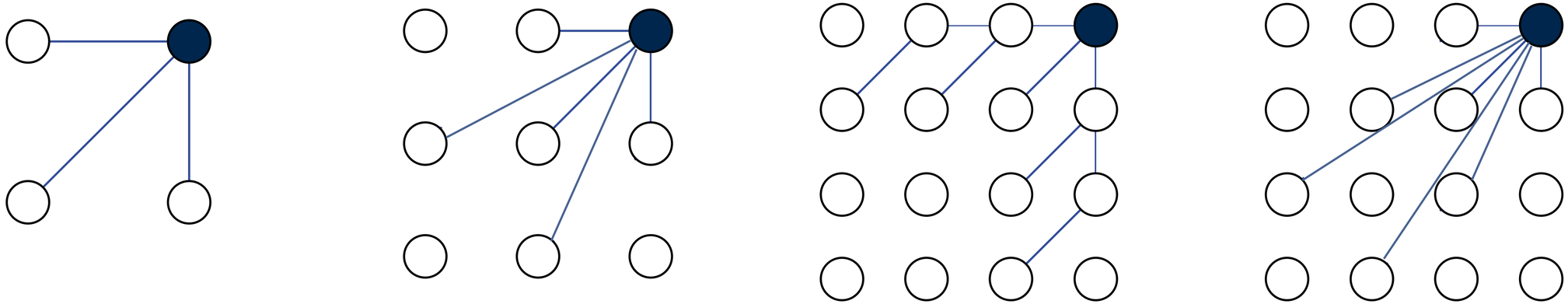
  - We can repeat a time point, but cannot go backwards in time

- This enforces time order
  - Don't match"ban" with "banana"
  - But can match "baan" with "ban"



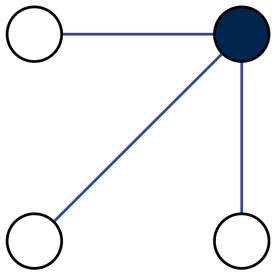
More local constraints

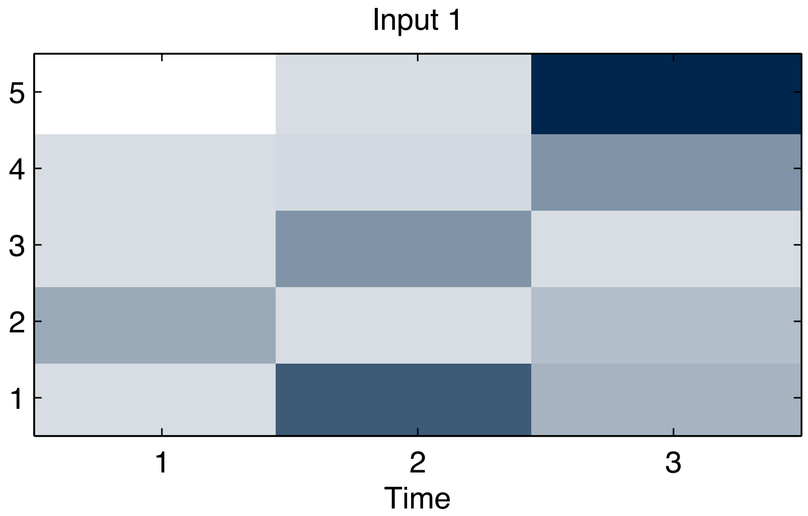
- Define acceptable local subpaths
  - Consider only these paths leading to current (dark) node
  - Application dependent

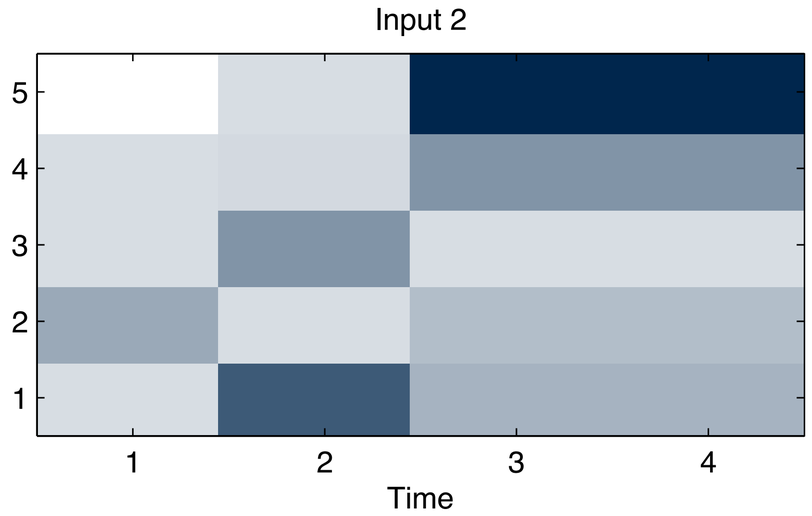


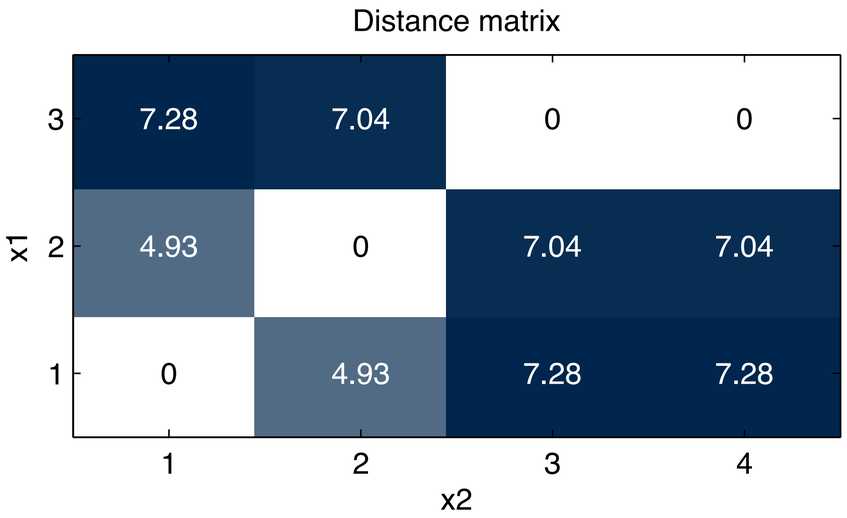
Toy data run

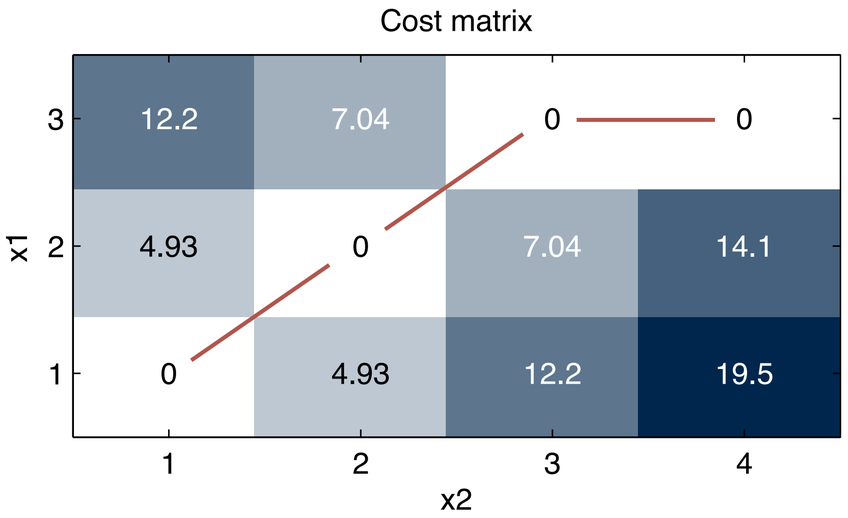
Local Constraint



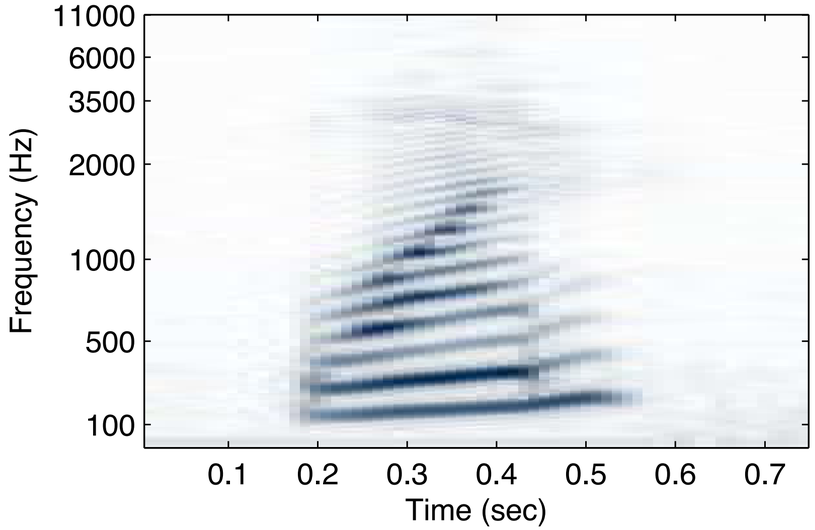


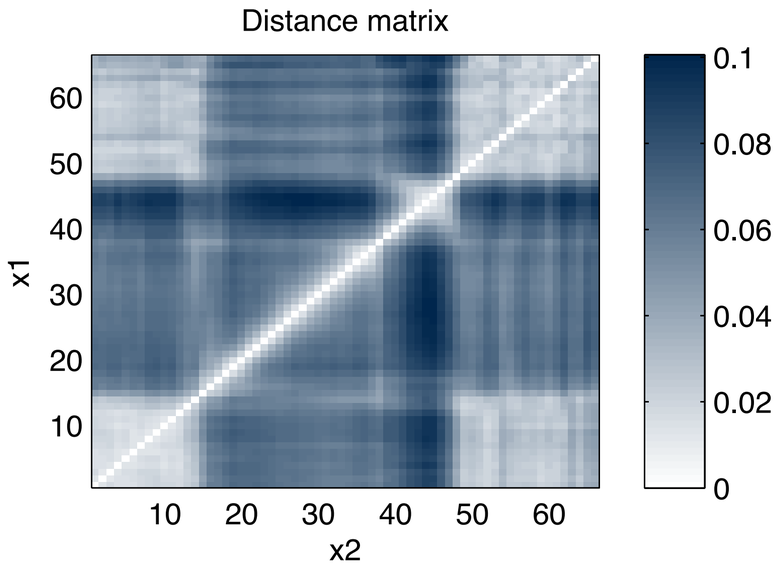


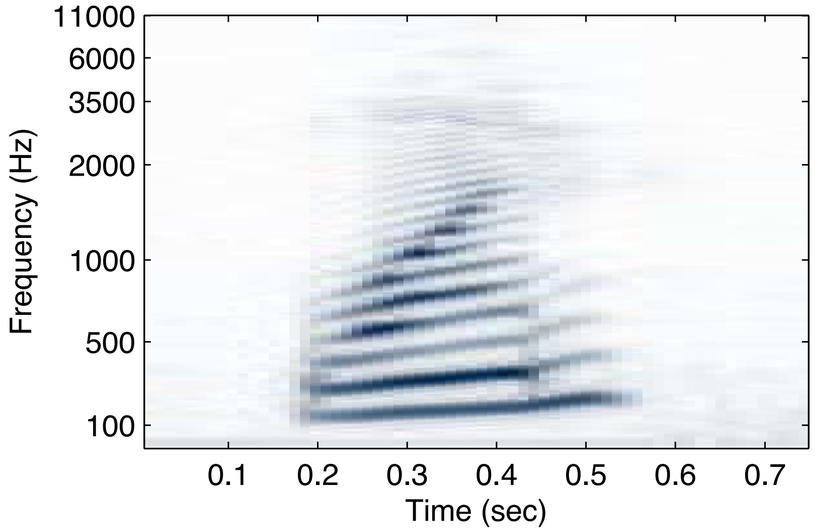


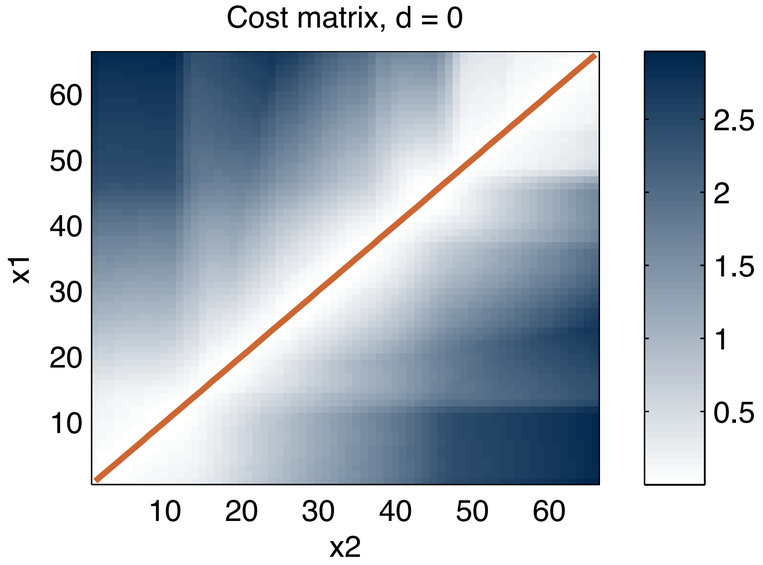


Speech example with identical utterance

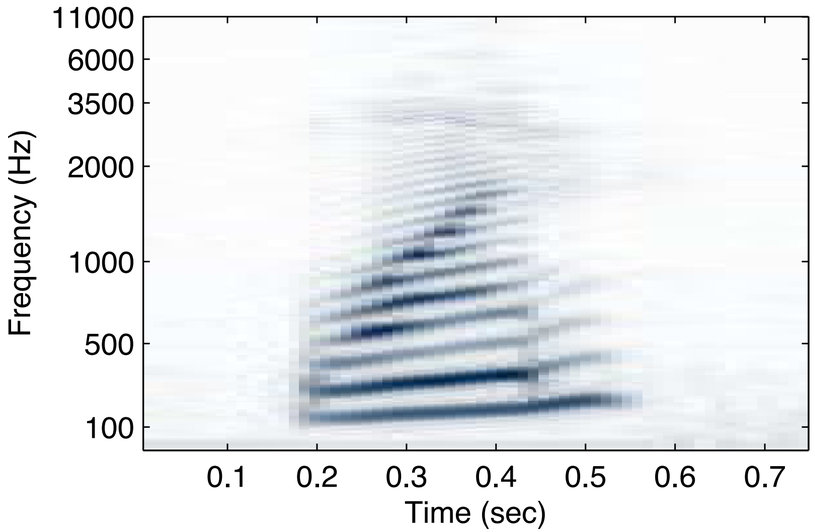


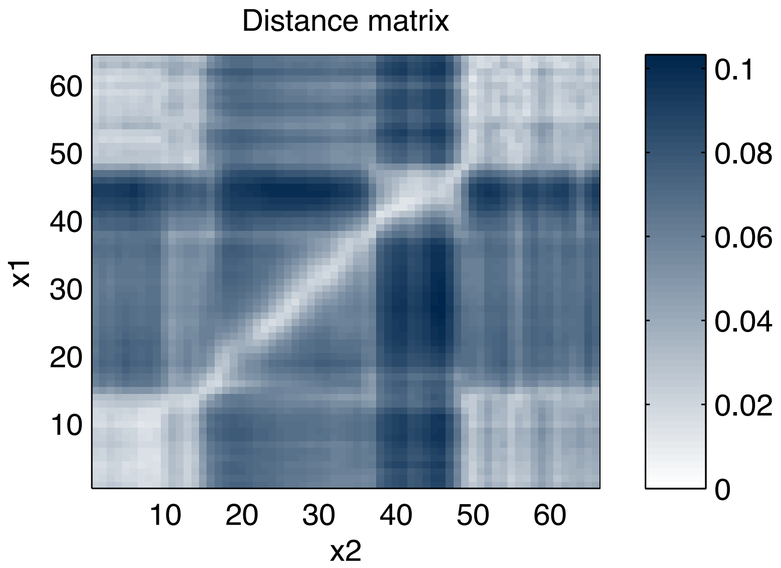


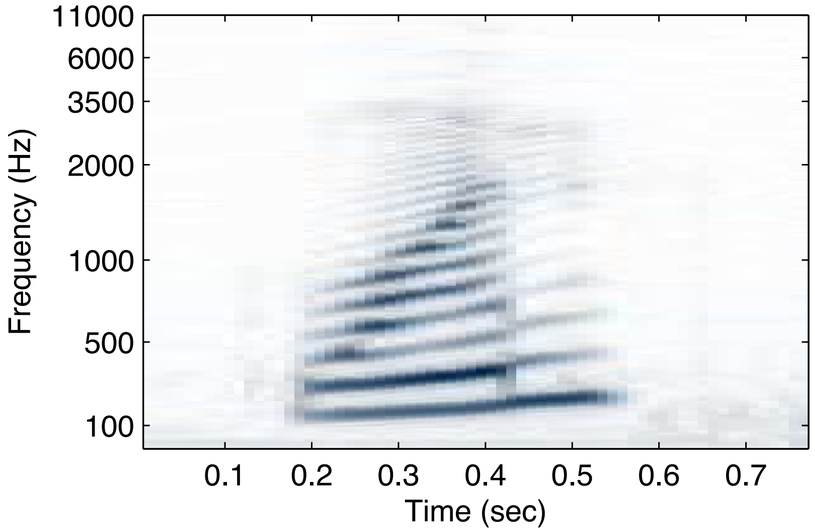


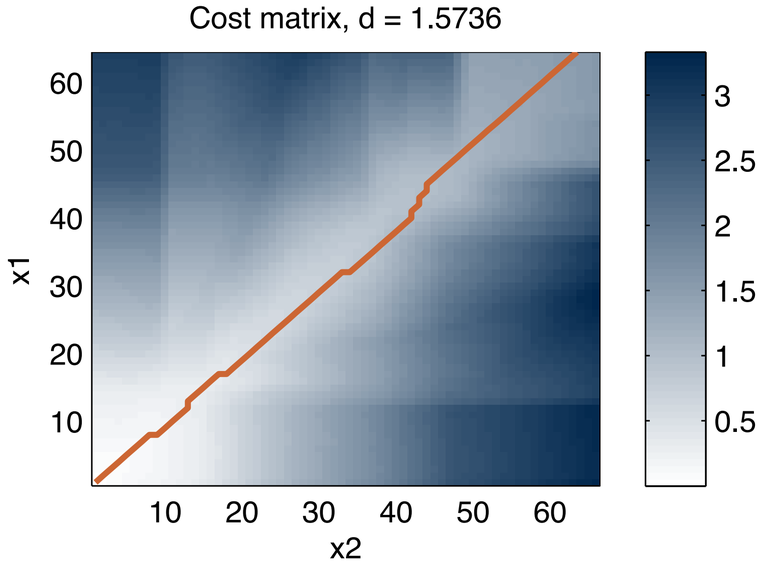


Ditto with similar utterance

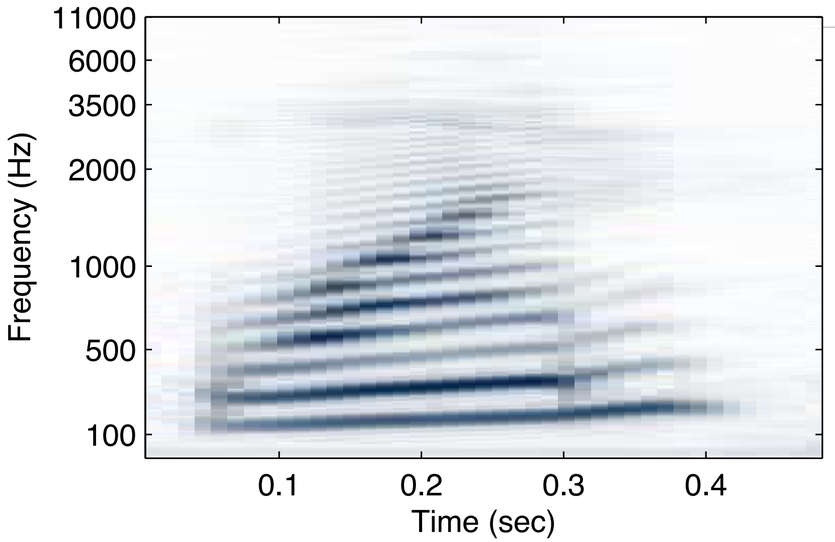


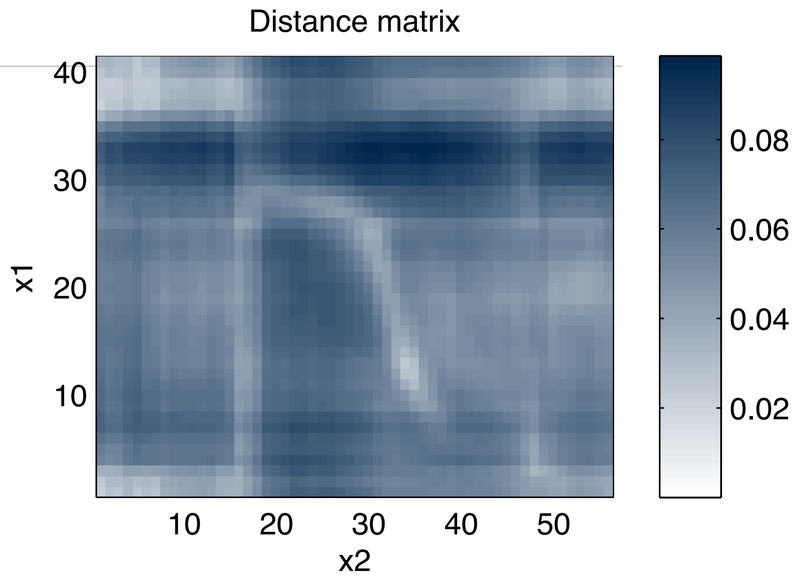


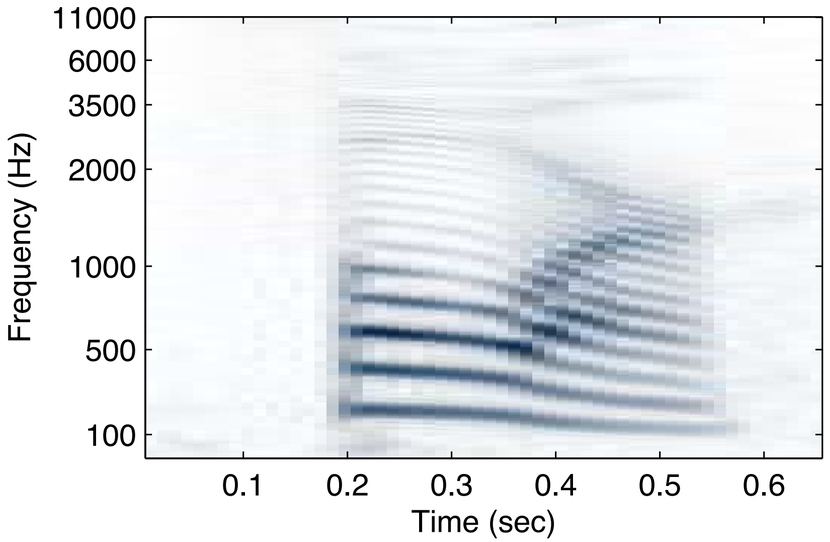


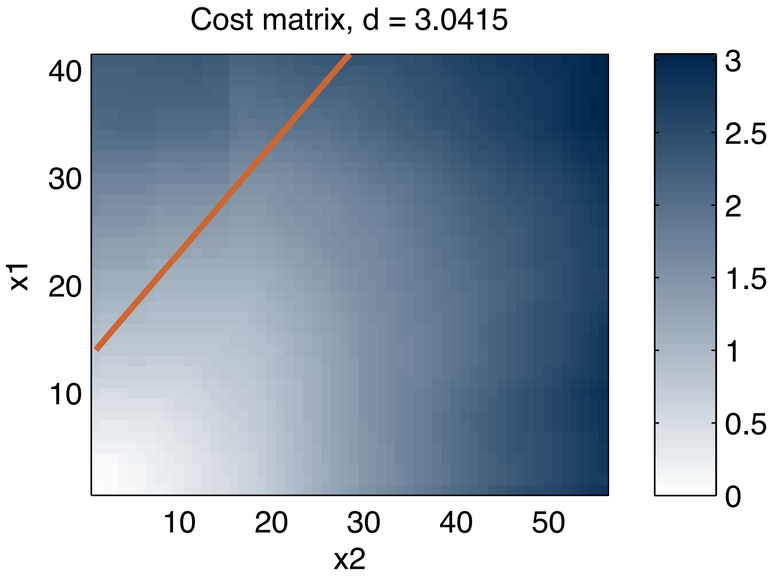


Ditto with different utterance



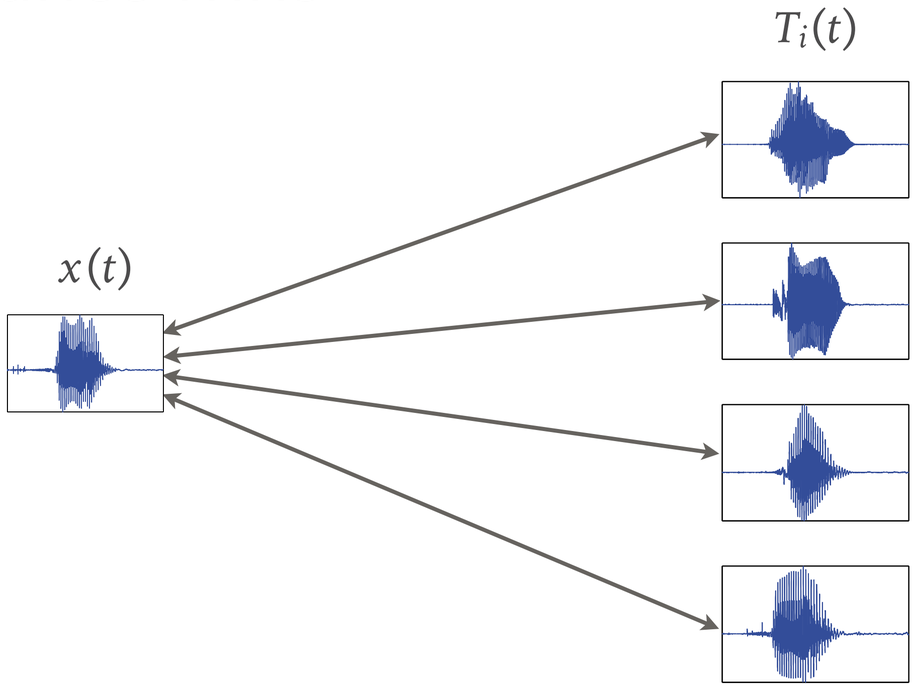




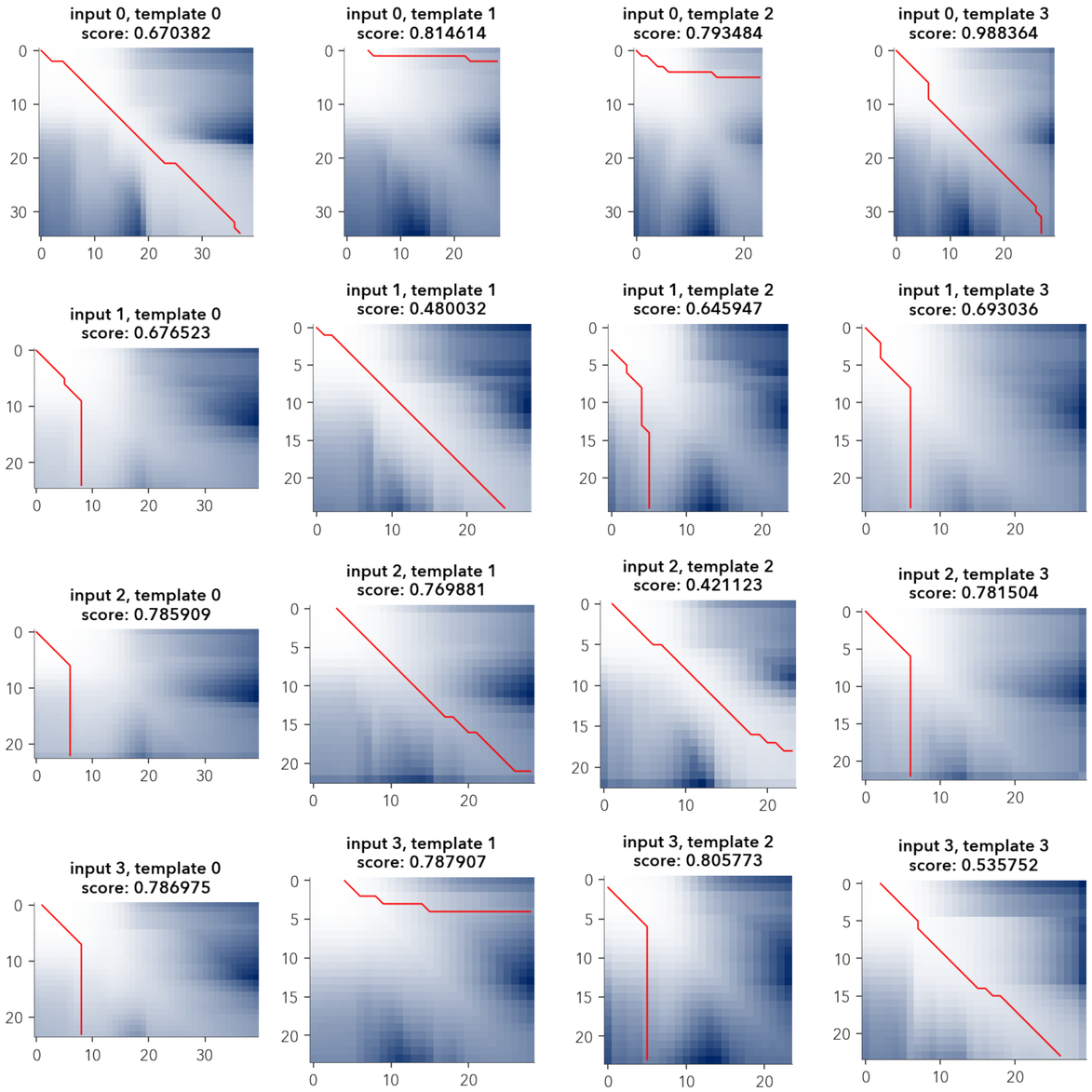


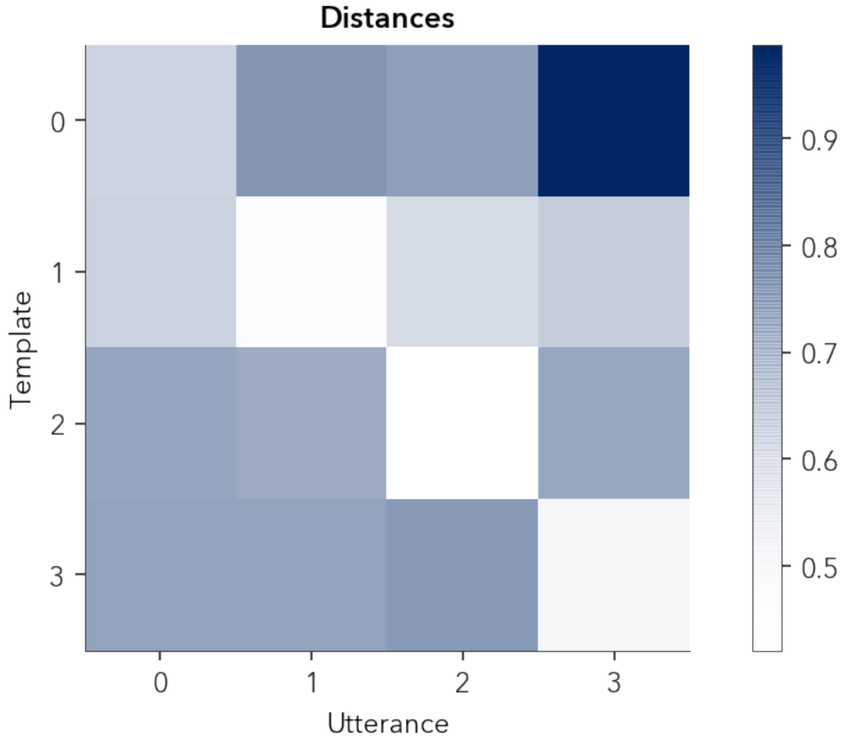
A basic speech recognizer

- Collect template spoken words $T_i(t)$
- Get their DTW distances from input $x(t)$
  - Smallest distance wins



Example case





Defining a distance for time series

- Dynamic Time Warping computes time series distances
  - Dependent on our stated constraints

- Having a time series distance gives us options
  - Nearest-neighbor classifiers (we just did that)
  - Clustering of time series
  - Manifold embeddings of time series
  - …

But that doesn’t make me happy …

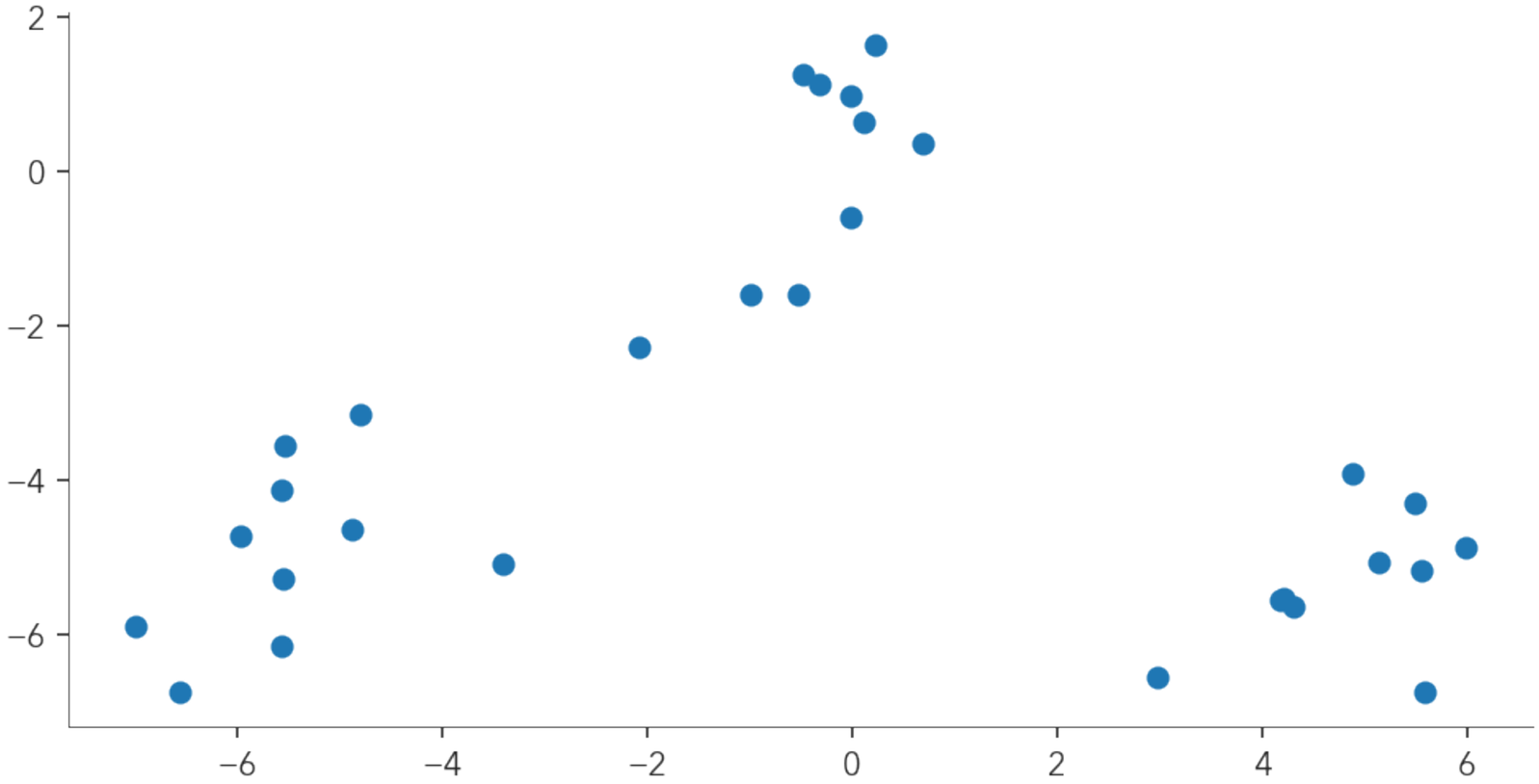
- What is my usual complaint at this point?

We prefer soft models

- The paths are hard decisions
- We'd rather have soft assignments
- Obtaining probabilities instead would also help

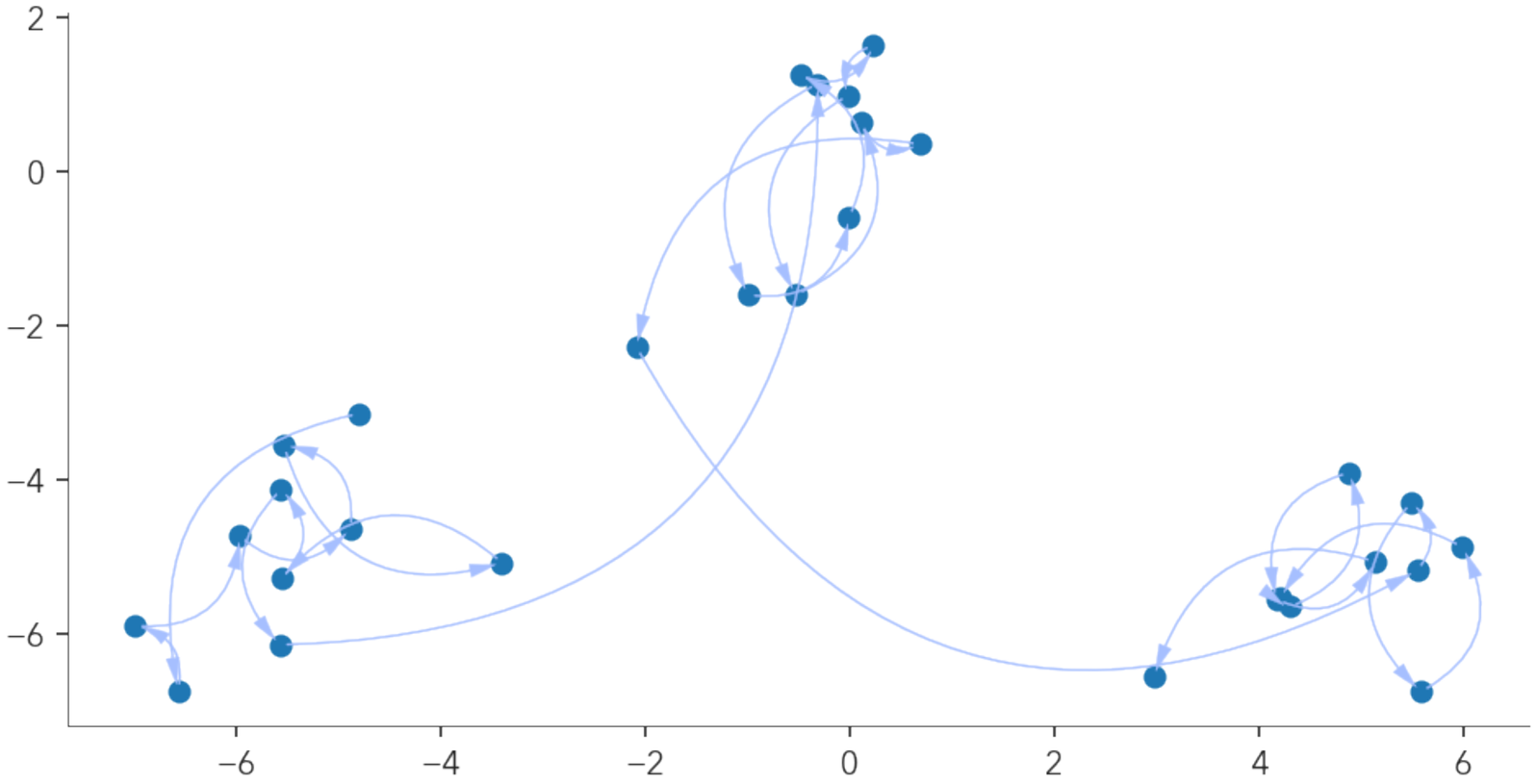
Starting with the GMM

- GMMs are “time agnostic”
  - We don’t have a notion of a sequence



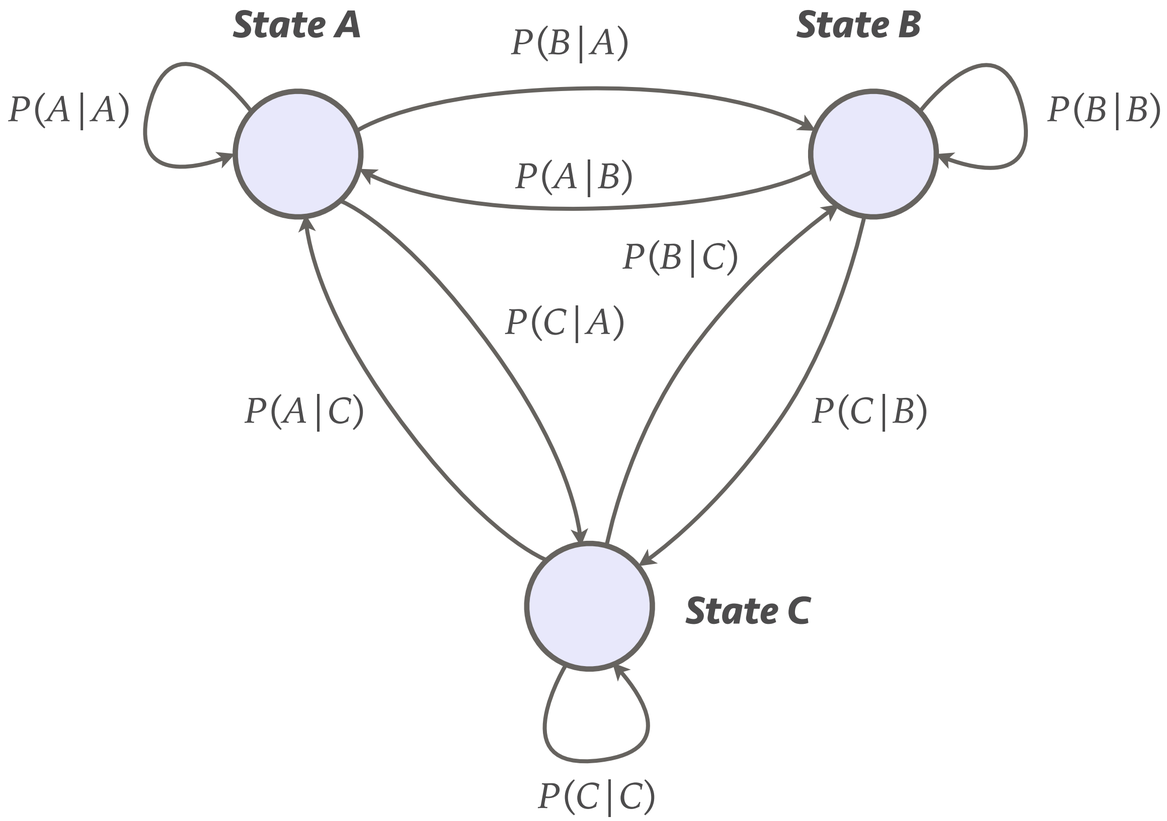
What if there is time order?

- We want to learn how the data progresses



Representing time

- Hidden Markov Model
  - Gaussian variant for now
- Each state is represented by a Gaussian distribution
- Transition probabilities between all states
  - Only the last state matters



Initial Probabilities: $P(A), P(B), P(C)$

Problems to solve

- Evaluation
  - Given a model, evaluate likelihood of an input
- Decoding
  - Given a model, find state sequences of an input
- Learning
  - Given an input, find the model parameters

Evaluation

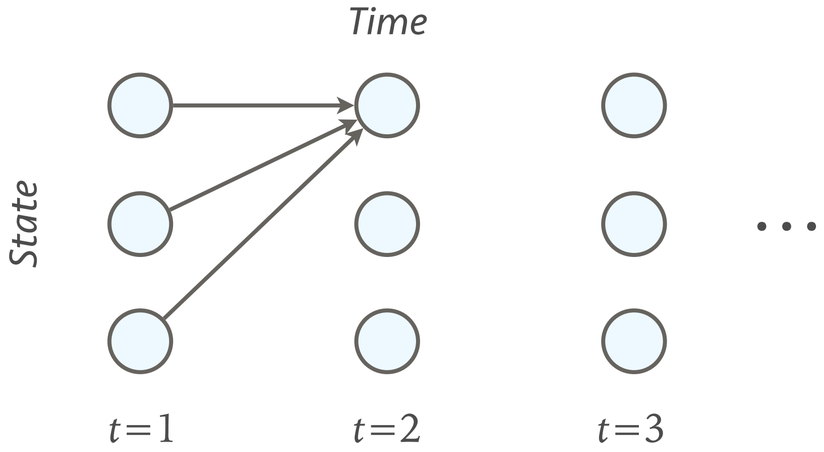
- Propagate likelihoods using the *forward algorithm*:

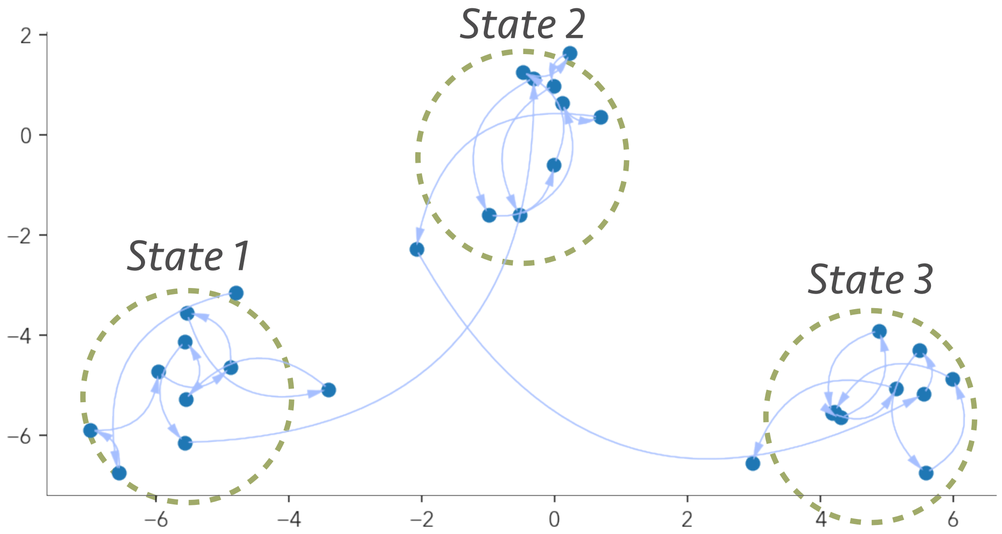
Probability of initial state *Likelihood of data given state*

$$\alpha_i(1) = P_1(i)P(x_1 \mid i)$$

$$\alpha_i(t+1) = P(x_{t+1} \mid i)\sum_j \alpha_j(t)P(i \mid j)$$

Probability of state i at time $t+1$





Evaluation

- Forward pass provides probability of a specific state at a specific time, given all past inputs

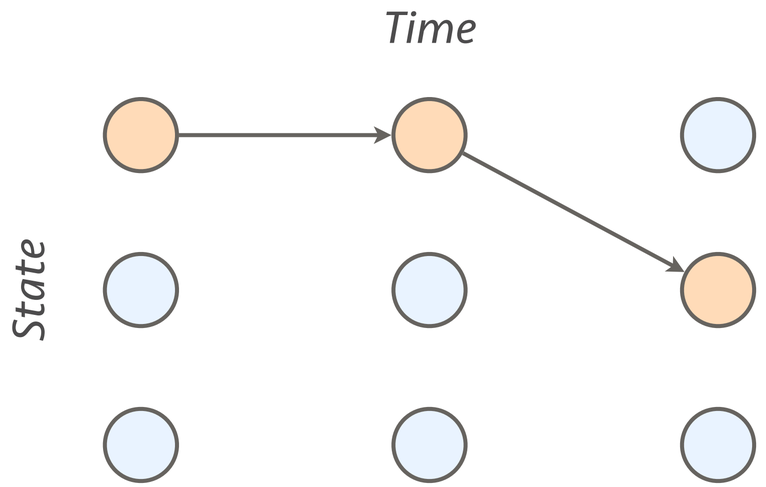
$$\alpha_i(t) = P(x_t \mid i) \sum_j \alpha_j(t-1) P(i \mid j)$$

- Terminal time point values provide the overall likelihood of model given an input

$$P(\mathbf{x}) = \sum_i \alpha_i(T)$$

Decoding

- The forward pass provides us with the state likelihoods
  - Similar to the DTW path costs
- With a backwards pass we can find most likely transitions
  - Just as we did with DTW
- The Viterbi algorithm



The Viterbi algorithm

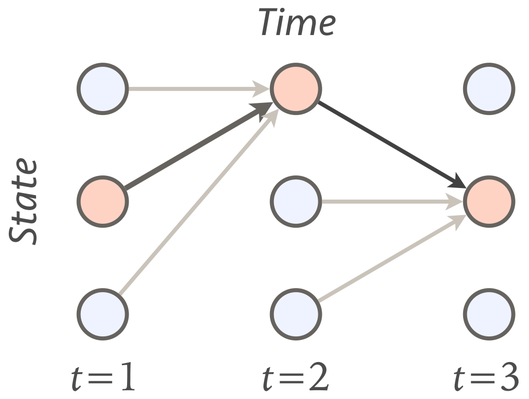
- Similar to forward algorithm, but with a hard decision:

$$v_i(1) = P_1(i)P(x_1 \mid i)$$

$$v_i(t+1) = P(x_{t+1} \mid i)\max\Big(P(i \mid j)v_j(t)\Big)$$

  - Probability of most likely path up to time $t+1$ that ends in state i
  - For every step remember most likely transition (as with DTW)

- Backwards pass
  - Get most likely terminal state
  - Work backwards using the most likely transitions



Learning

- An EM-type approach
  - Known as Baum-Welch training
- E-step
  - Find likelihood of each data point being associated with each state
- M-step
  - Estimate each state's parameters based on the associations from the E-step

E-step: Forward-backward pass

- Forward pass provides probability of a specific state and time given past inputs $a_i(t)$
- A similar backward pass will provide probability of given state and time given future inputs $\beta_i(t)$
  - Same as forward pass, but computed backwards in time
- The product of these will be the probability of a state and time given all inputs $\gamma_i(t) = a_i(t)\beta_i(t)$

M-step: Parameter estimation

- State models
  - Use $\gamma_i(t)$ as weights to compute state parameters, e.g.:

$$\mu_i = \frac{\sum_t x_t(t)\gamma_i(t)}{\sum_t \gamma_i(t)} \qquad \Sigma_i = \frac{\sum_t \gamma_i(t)\left(x_t - \mu_i\right)\left(x_t - \mu_i\right)^\top}{\sum_t \gamma_i(t)}$$

- Transition matrix and initial probabilities
  - Estimate by counting transitions and observing most likely start

A cheaper alternative

- Viterbi learning
  - Faster to run and easier to code but not as powerful
- E-step
  - Get state assignments using Viterbi
    - Yuk, hard assignments …
- M-step
  - Instead of using state likelihoods, use state assignments
    - So we won't use a weighted sum using γ, just the samples for each state

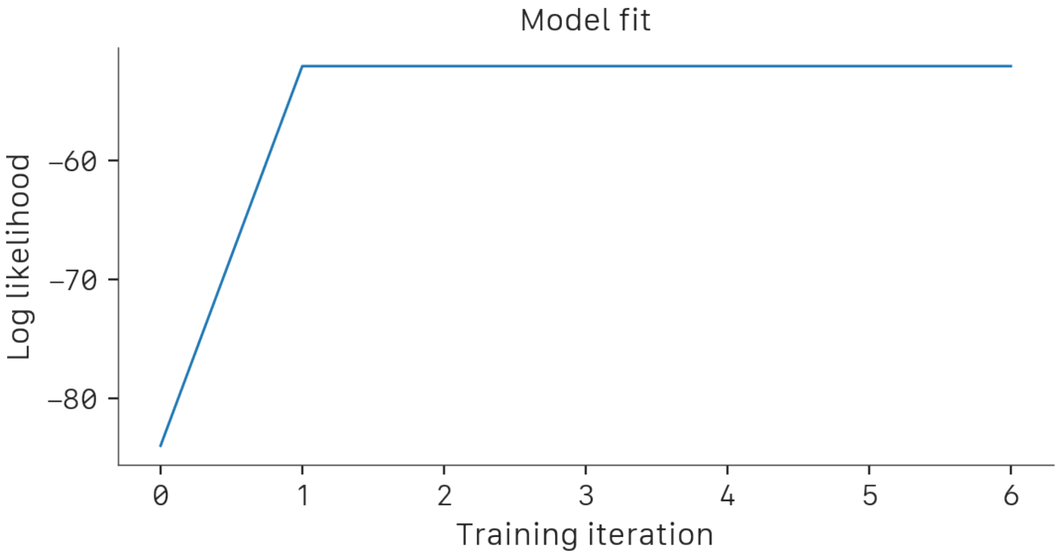
Noteworthy things

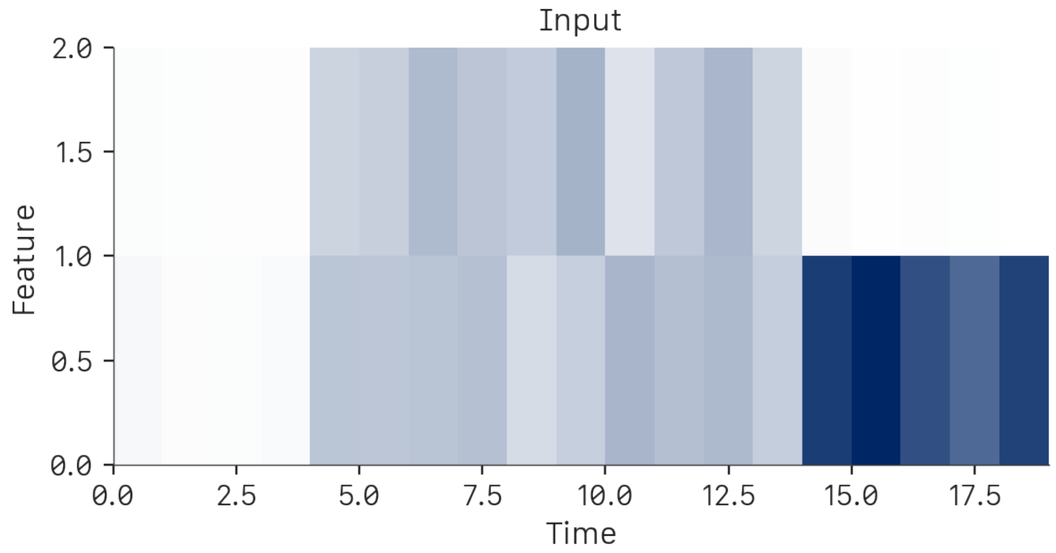
- HMM learning works with multiple training sequences
- You should use log probabilities
  - What's a state likelihood after a million time points? Can you represent it well numerically?

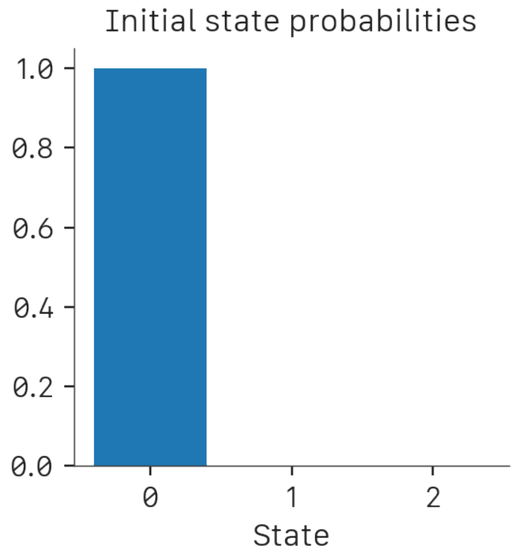
$$\alpha_i(t) = P(x_t \mid i) \sum_j \alpha_j(t-1) P(i \mid j)$$

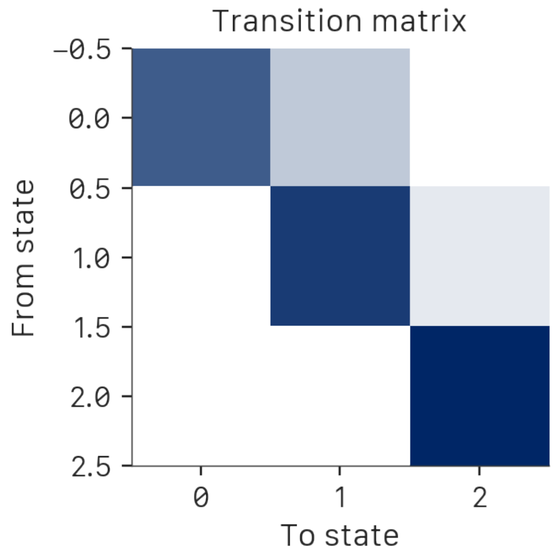
- You can use an arbitrary state model
  - e.g. a GMM per state, a neural net, or even an HMM!

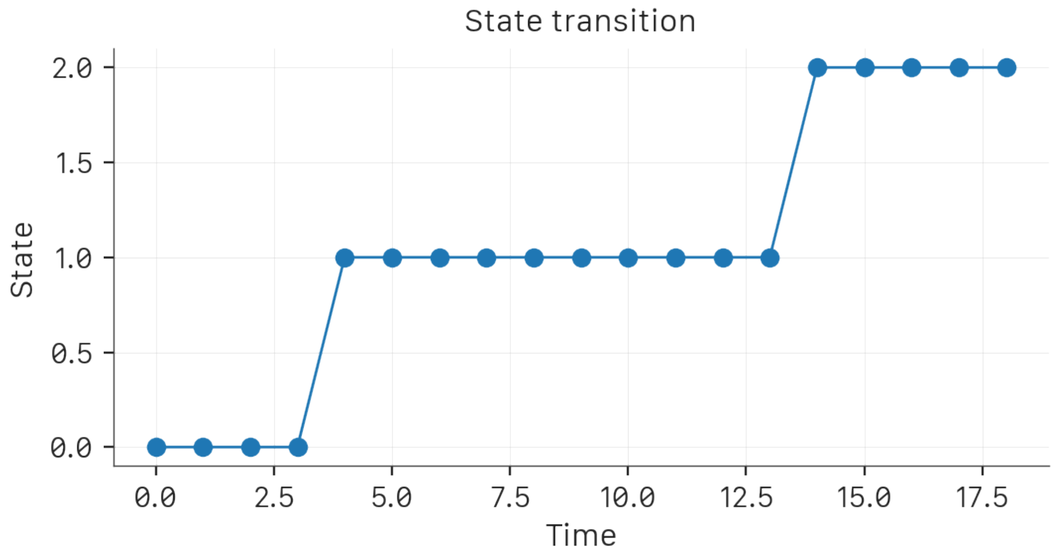
Learning an HMM model





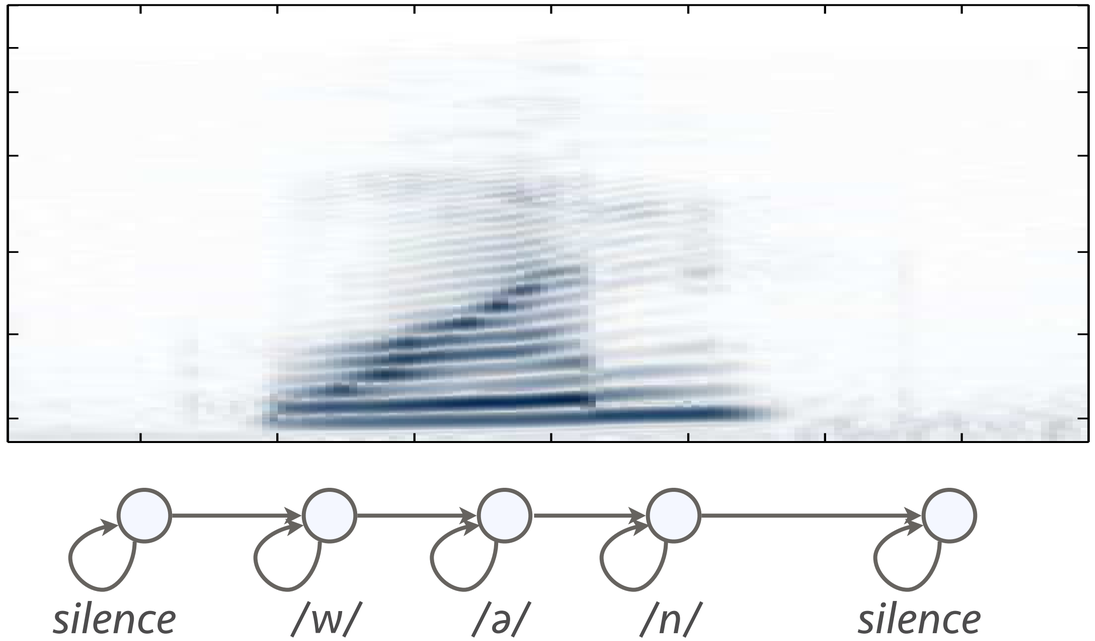




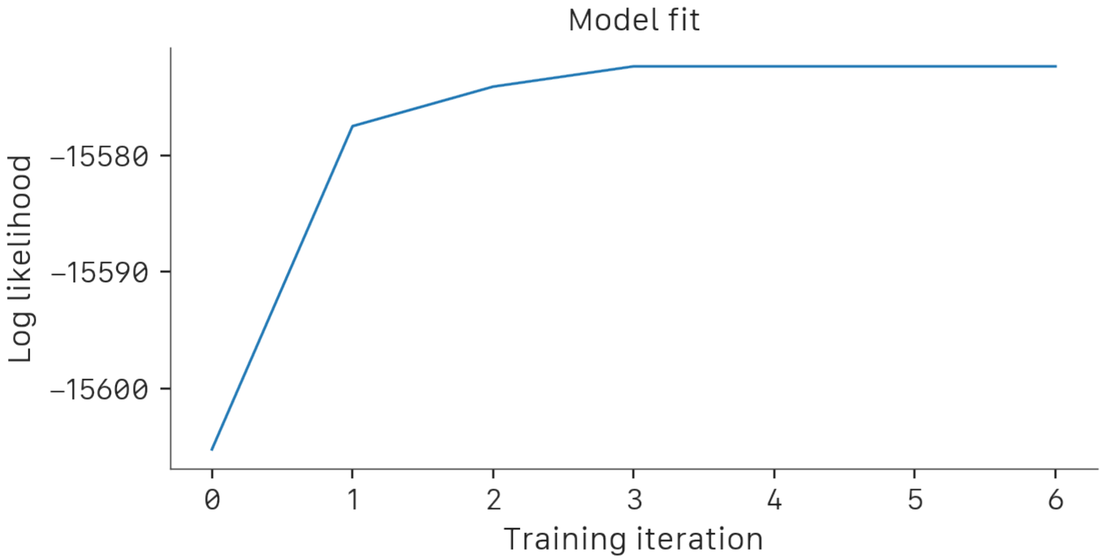


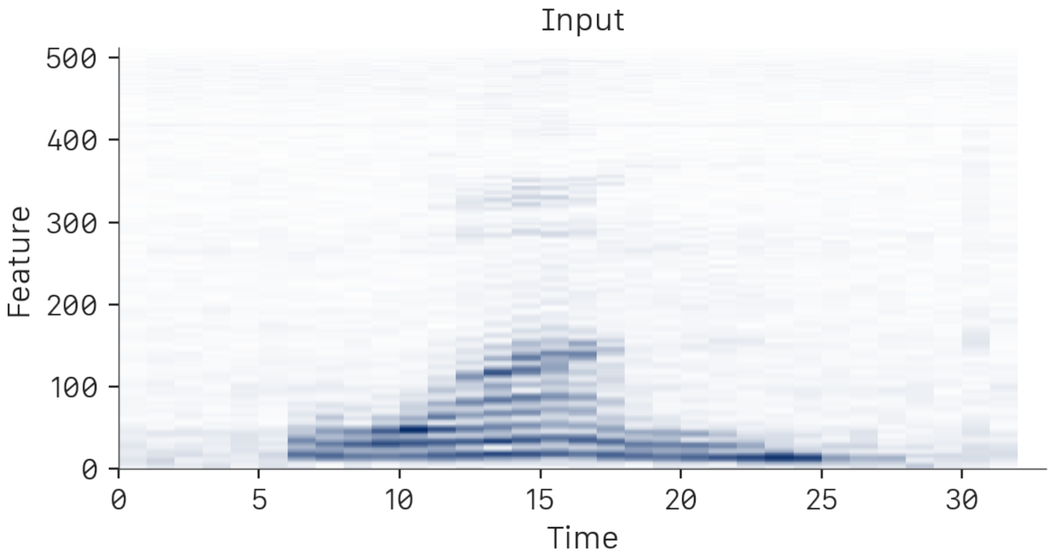
Back to speech

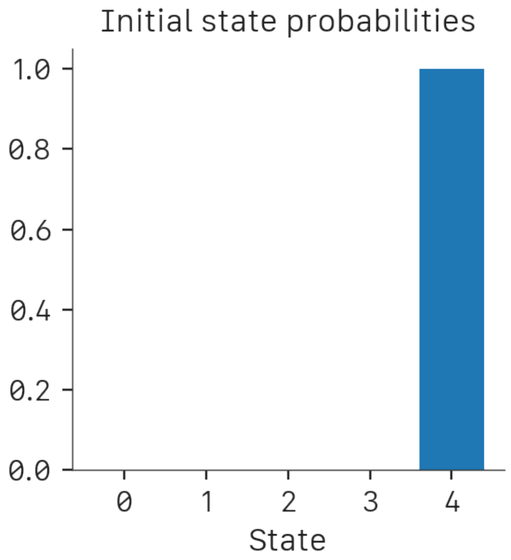
- Learning an HMM for a word
  - Each state should correspond to a coherent part
    - e.g. a syllable, a phoneme, etc

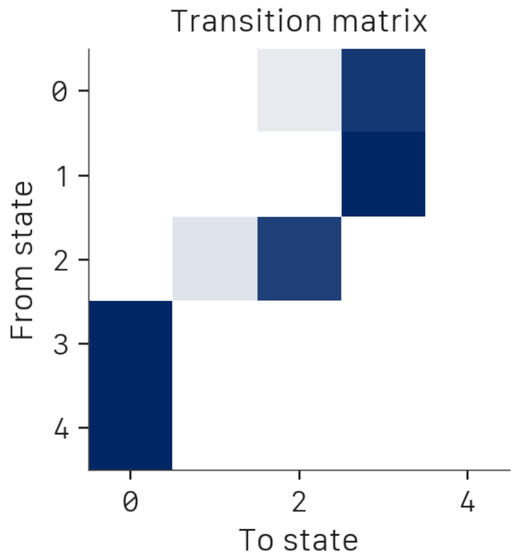


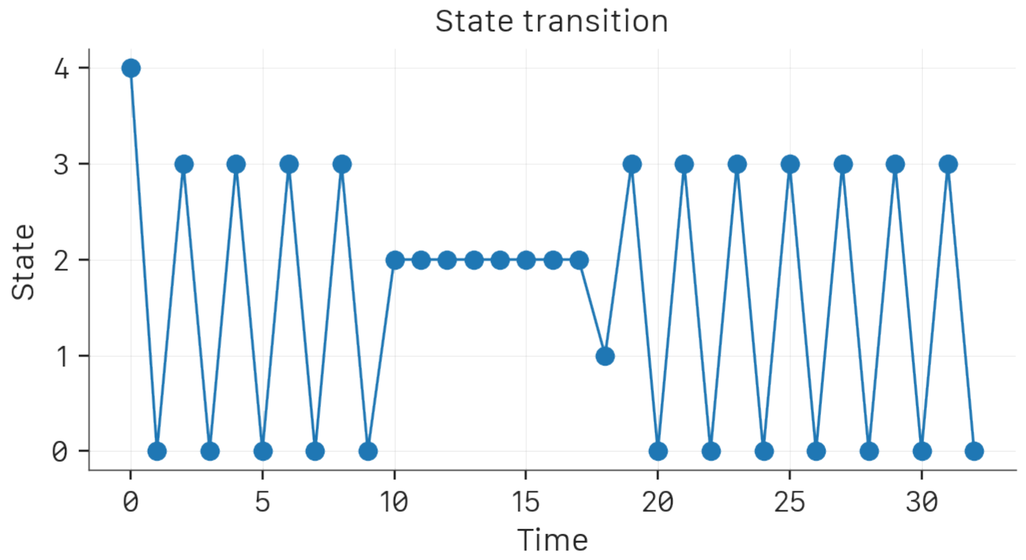
Fully-connected model







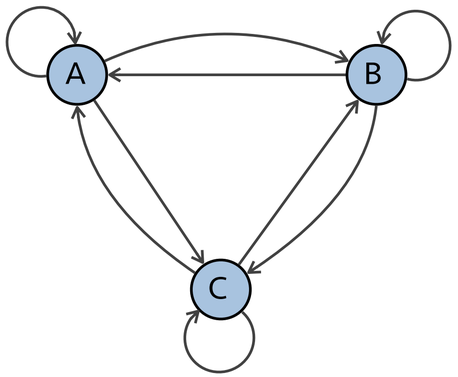




Limiting transitions

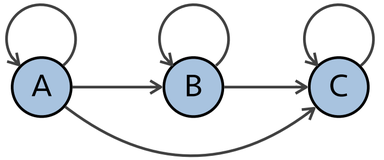
- Imposing structure through limiting transitions
  - In DTW we used local constraints for, e.g. time monotonicity
  - In HMMs we can do so by limiting the transition matrix
    - Set certain transitions to zero

Ergodic / Fully-connected HMM



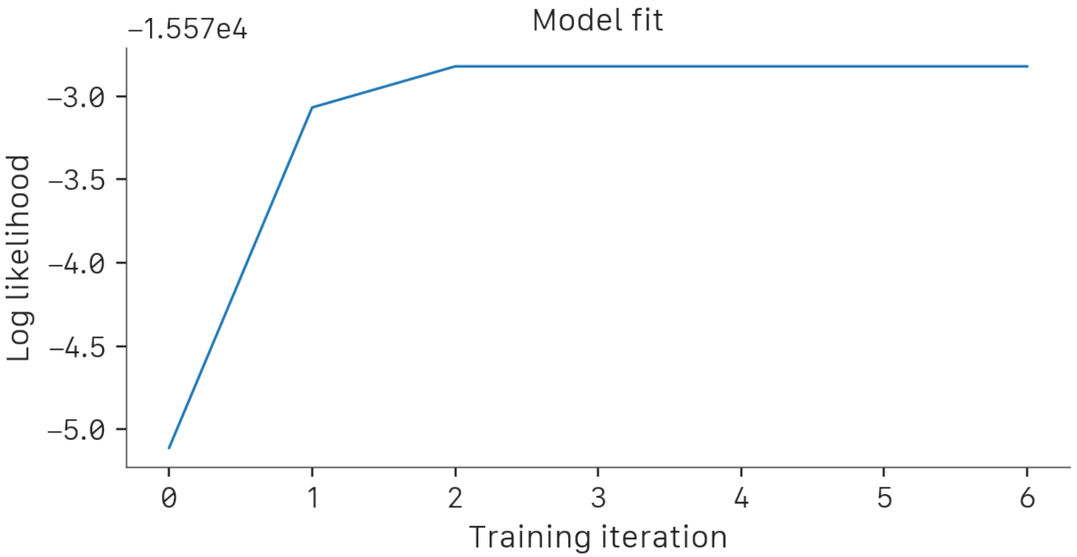
P(A|A)	P(B|A)	P(C|A)
P(A|B)	P(B|B)	P(C|B)
P(A|C)	P(B|C)	P(C|C)

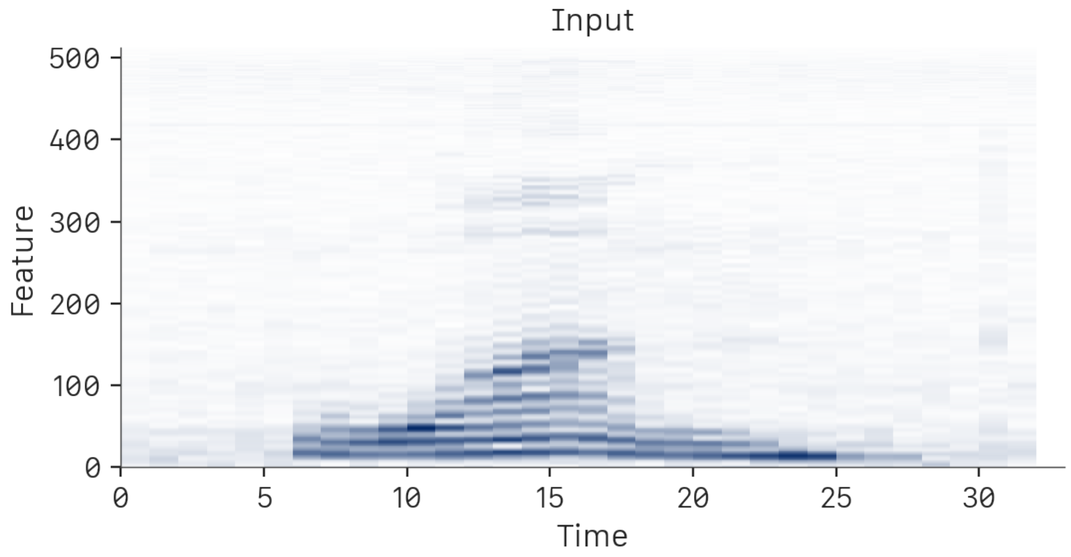
Left-to-Right HMM

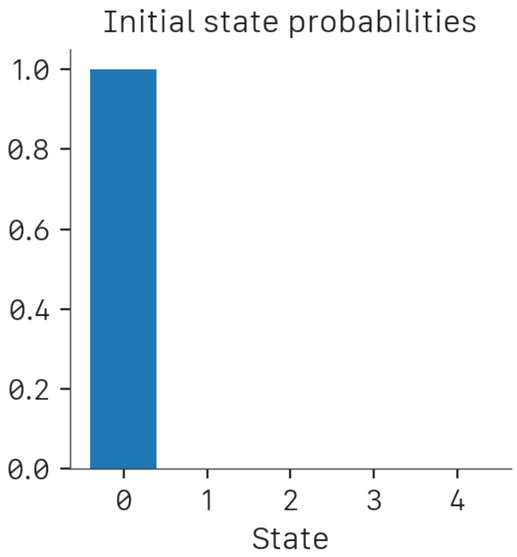


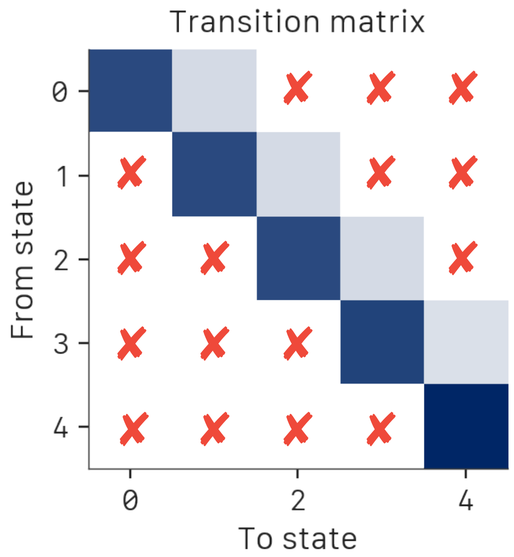
P(A|A)	P(B|A)	P(C|A)
0	P(B|B)	P(C|B)
0	0	P(C|C)

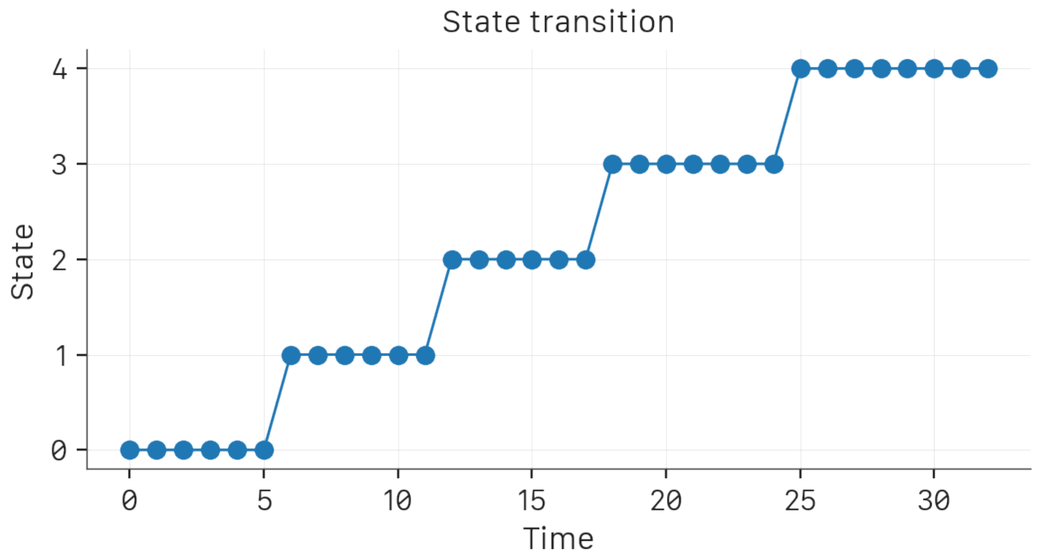
1st order left-to-right model (1st = only single step time jumps)



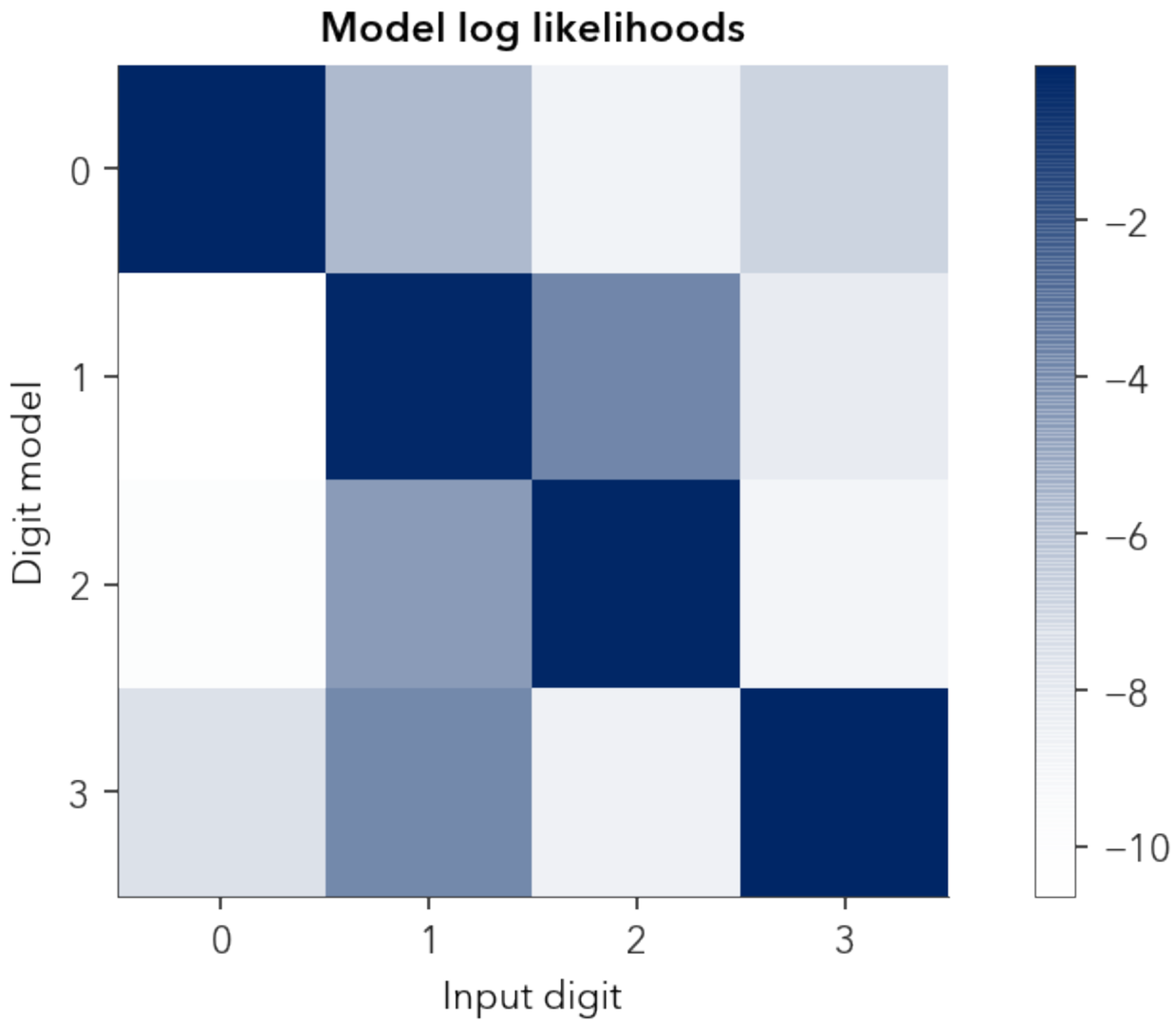






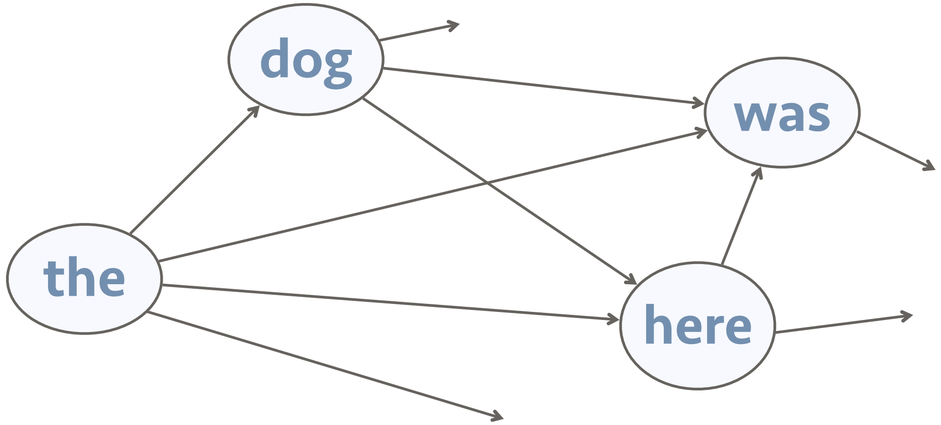


Digit recognition results



Speech recognition in a nutshell

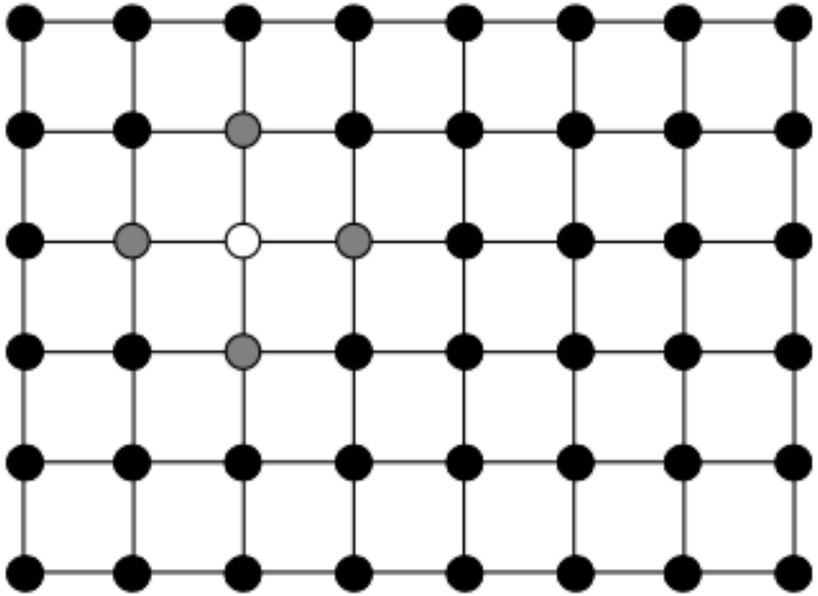
- Small scale speech recognition

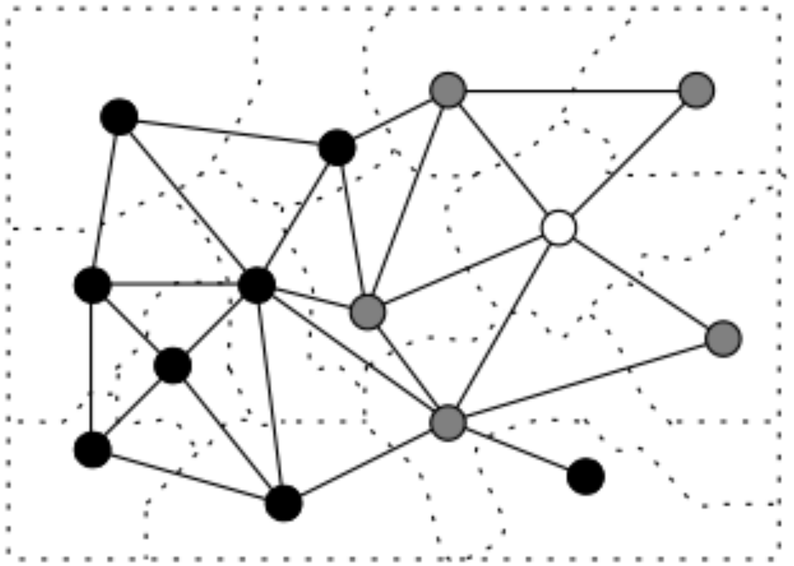


- Large scale systems are another beast …

Elaborations

- Denser interconnections
  - e.g., second order time ties
- Markov random fields





Recap

- Learning with time series
- Dynamic Time Warping
- Hidden Markov Models
- Some basic speech recognition

Next lecture

- Missing data
  - Using temporal dynamics to fill in missing values
- Dynamical systems
  - Learning continuous time-series

Reading

- Textbook chapters 8, 9

- A tutorial on HMMs
  - http://www.cs.ubc.ca/~murphyk/Bayes/rabiner.pdf

Some project advice

- Work on a domain you have expertise in (if any)
  - Bring your own (signal) data, don't stick to generic projects
- Show me that you learned something in class
  - Demonstrate proper use of the tools we cover, use some signals understanding
- Propose multiple objectives, see how far you can go
  - One objective will be really easy to do (you pass the class)
  - Another will be tough, but doable (you get a potential paper out of it)
  - Final should be an amazing result (here's your dissertation!)

Some samples of past projects

- Bird call identification
- Volumetric image segmentation
- Daily activity profiler
- EEG emotion prediction
- Brain-to-Action interface
- Fossil detection in shale samples
- Speech recognition through gyroscopes
- Hand gesture recognition from EMG
- Predicting football yardage from video
- Stock prediction
- Ecological acoustics
- Afro-cuban music style identification from drum beat
- Mechanical damage detection/assessment
- Wireless traffic prediction
- Face identification from flying drones
- Corporate earnings prediction from CEO speech
- Multimodal object detection
- Music transcriptions systems
- Embeddings of icons for quick searching
- Audio and Video style transfer
- Singer's voice separation from music recordings
- Online tensor decompositions
- Hearing loss assessment from speech queries
- ...