MTH 9893 Time Series HW 3

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1.
A $VAR(p)$ model is an extension of the $VAR(1)$ model to include p lags:

$$X_t = a + B_1 X_{t-1} + \cdots + B_p X_{t-p} + \varepsilon_t$$

Using the lag operator, we can write

$$\begin{aligned} X_t &= a + B_1 L X_t + \cdots + B_p L^p X_t + \varepsilon_t \\ \left(1 - B_1 L - \cdots - B_p L^p\right) X_t &= a + \varepsilon_t \end{aligned}$$

By introducing following two notations:

$$\begin{aligned} B &= B_1 + \cdots + B_p \\ \Gamma_j &= -\left(B_{j+1} + \cdots + B_p\right), \qquad \text{for } j = 1, \ldots, p-1 \end{aligned}$$

We could rewrite the previous equation as

$$\begin{aligned} \left(1 - BL - \left(B_2 L^2 - B_2 L\right) - \cdots - \left(B_p L^p - B_p L\right)\right) X_t &= a + \varepsilon_t \\ \left(1 - BL - B_2 L (L-1) - B_3 L (L-1)(L+1) - \cdots - B_p L (L-1)\left(L^{p-2} + L^{p-3} + \cdots + 1\right)\right) X_t &= a + \varepsilon_t \\ \left(1 - BL - (1-L)\left(-B_2 L - B_3 L (L+1) - \cdots - B_p L\left(L^{p-2} + L^{p-3} + \cdots + 1\right)\right)\right) X_t &= a + \varepsilon_t \\ \left(1 - BL - (1-L)\left(-\left(B_2 + \cdots + B_p\right) L - \left(B_3 + \cdots + B_p\right) L^2 - \cdots - B_p L^{p-1}\right)\right) X_t &= a + \varepsilon_t \\ \left(1 - BL - (1-L)\left(\Gamma_1 L + \Gamma_2 L^2 + \cdots + \Gamma_{p-1} L^{p-1}\right)\right) X_t &= a + \varepsilon_t \end{aligned}$$

which is equation (35) in Lecture Note #3.
Since $\Delta = 1 - L$, we have:

$$\begin{aligned} \left(1 - BL - \Delta\left(\Gamma_1 L + \Gamma_2 L^2 + \cdots + \Gamma_{p-1} L^{p-1}\right)\right) X_t &= a + \varepsilon_t \\ X_t - BLX_t - \Delta X_t\left(\Gamma_1 L + \Gamma_2 L^2 + \cdots + \Gamma_{p-1} L^{p-1}\right) &= a + \varepsilon_t \\ X_t - BX_{t-1} - \Delta\left(\Gamma_1 X_{t-1} + \Gamma_2 X_{t-2} + \cdots + \Gamma_{p-1} X_{t-p+1}\right) &= a + \varepsilon_t \end{aligned}$$

This could be written as

$$\begin{aligned} X_t &= a + BX_{t-1} + \Delta\left(\Gamma_1 X_{t-1} + \Gamma_2 X_{t-2} + \cdots + \Gamma_{p-1} X_{t-p+1}\right) + \varepsilon_t \\ &= a + BX_{t-1} + \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \cdots + \Gamma_{p-1} \Delta X_{t-p+1} + \varepsilon_t \end{aligned}$$

which is equation (36) in Lecture Note #3.