

# RL Adventure

## Distributional RL

이의령



C51

Distributional RL





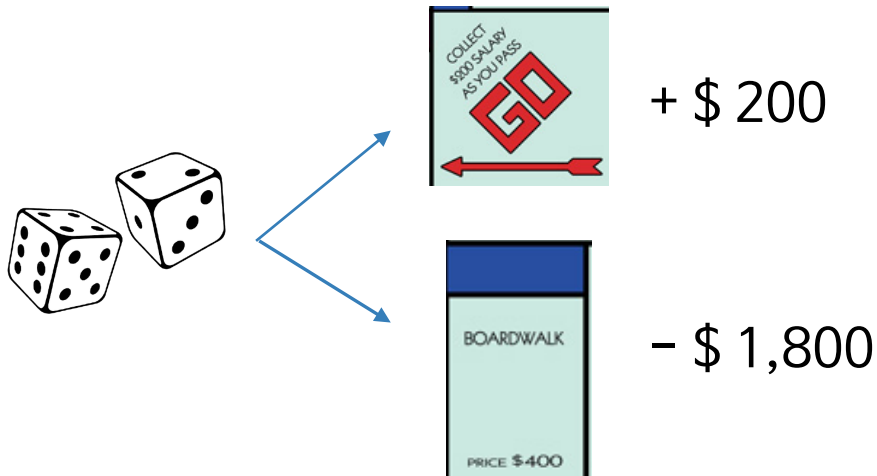
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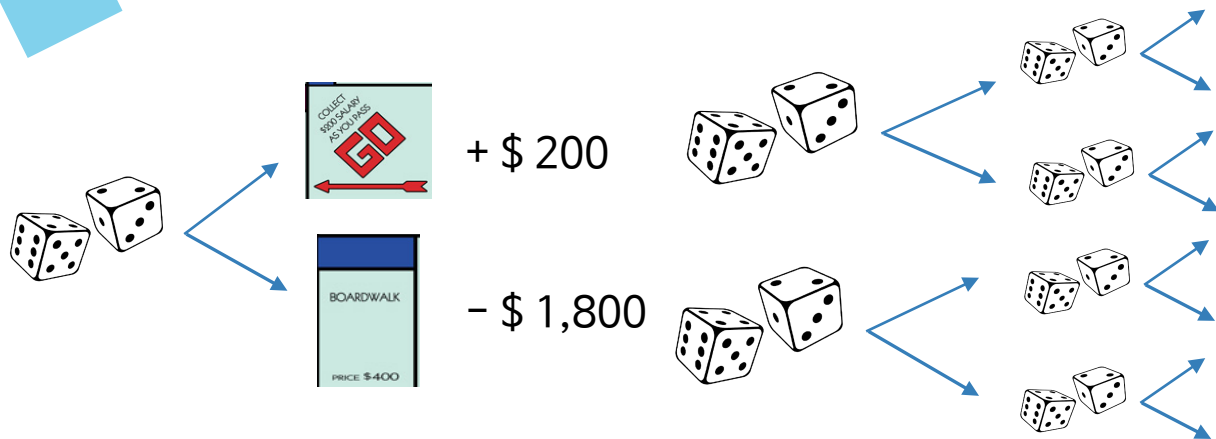
# 1. Motivation

# Motivation



$$\begin{aligned}
 E[R(x)] &= \frac{35}{36} \times 200 - \frac{1}{36} \times 1,800 \\
 &= 144
 \end{aligned}$$

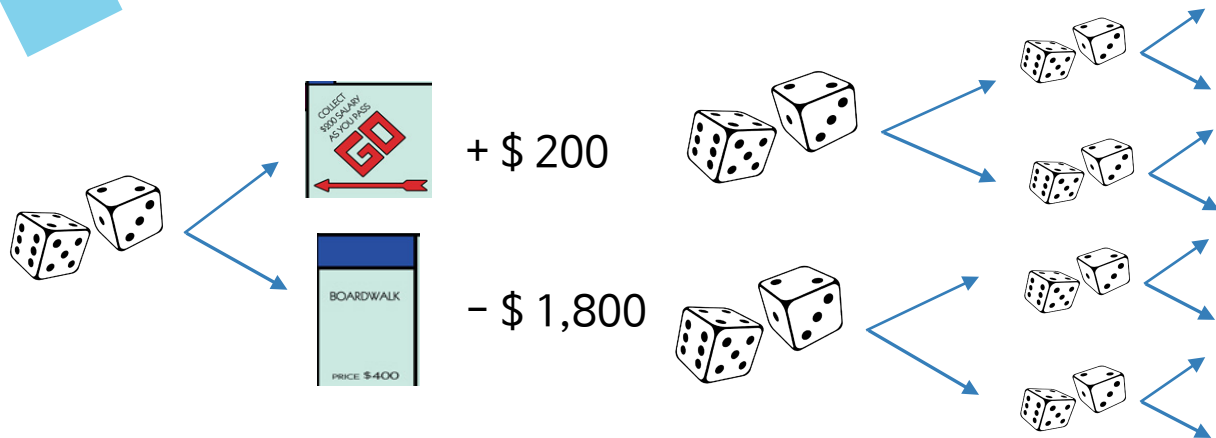
# Motivation



보상의 합

$$R_{t+1} + \gamma R_{t+2} + \cdots + \gamma^{T-t-1} R_T$$

# Expected RL



벨만 방정식

$$\begin{aligned}
 v(\mathcal{X}) &= E [R_{t+1} + \gamma R_{t+2} + \dots \mid S_t = \mathcal{X}] \\
 &= E [R_{t+1} + \gamma v(x) \mid S_t = \mathcal{X}] \\
 &= E R(x) + \gamma E v(x)
 \end{aligned}$$

# Expected RL

Reward를 Random Variable 관점에서 바라보면...

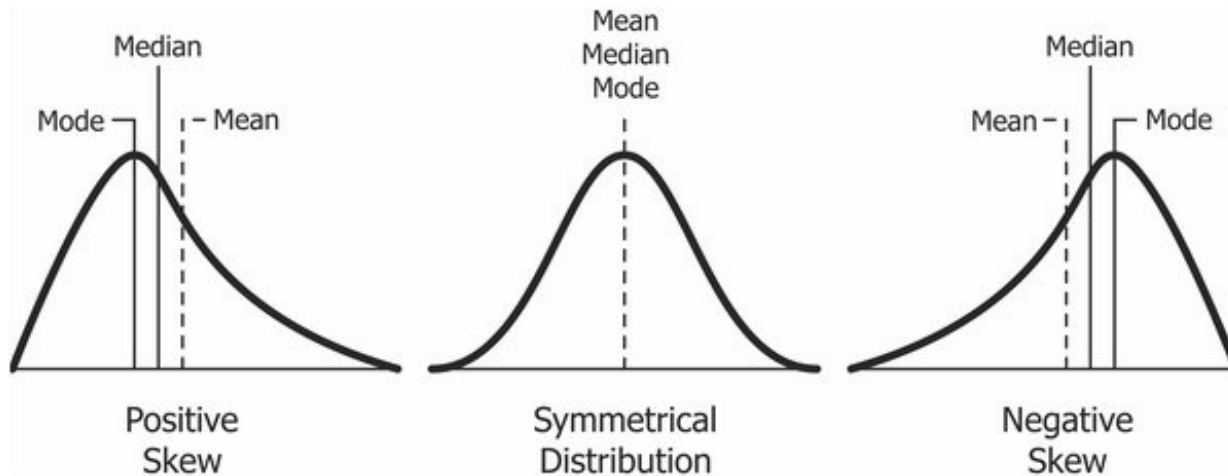
- 가치함수는 discount된 미래 보상에 대한 기댓값을 리턴한다.
- 기댓값 =  $\text{Scalar}(o) / \text{Distribution}(x)$
- 미래 보상 값들은 **complex, Multimodal**의 특성을 가진다.
- 기댓값은 각 보상들이 가지는 **intrinsic**(본질적인)한 특성을 담아내지 못한다.

$$\begin{aligned} E[R(x)] &= \frac{35}{36} \times 200 - \frac{1}{36} \times 1,800 \\ &= 144 \end{aligned}$$



# Expected RL

Reward를 Random Variable 관점에서 바라보면...





이러한 Expected RL의 한계점을 보완책

→ A Distributional Perspective on RL (C51)





Return을 Distribution으로 만들어

Randomness한 특성과 정보를 최대한 반영해보자


$$V^{\pi} = E[ Z^{\pi}(x) ] = E[ R(x) ] + E[ Z^{\pi}(X') ]$$




Return을 **Distribution**으로 만들어

**Randomness**한 특성과 정보를 최대한 반영해보자

$$V^{\pi} = \cancel{E}[Z^{\pi}(x)] = \cancel{E}[R(x)] + \cancel{E}[Z^{\pi}(X')]$$

$$Z^{\pi}(x) = R(x) + Z^{\pi}(X')$$


## 2. Distributional RL

# Distributional RL

## A Distributional Perspective on Reinforcement Learning (C51)

<https://arxiv.org/abs/1707.06887>

- Expected RL → Distributional RL
- Return에 대한 Value Distribution을 만들자.
- C51 = Categorical / 이산형 분포
- 51개의 bin을 이용하여 분포를 만든다.

# Distributional RL

## A Distributional Perspective on Reinforcement Learning (C51)

- Distributional Bellman Equation

$$Z(x, a) \stackrel{D}{=} R(x, a) + \gamma Z(X', A')$$

- Cf) Bellman Equation

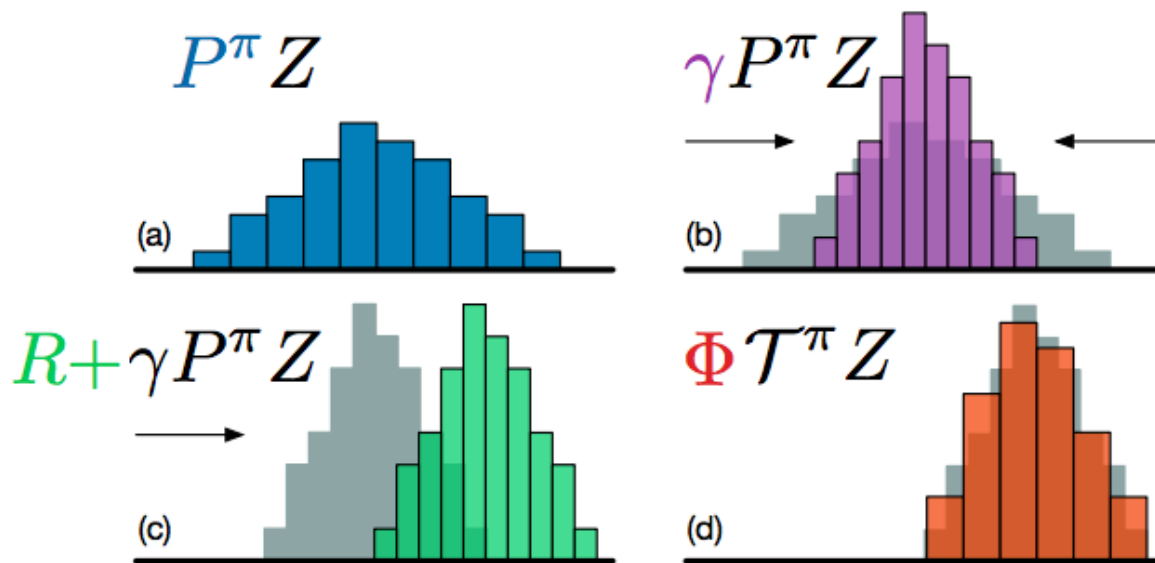
$$Q(x, a) = R(x, a) + \gamma Q_{\pi}(x', a')$$

- $Z(s, a)$  는 Distribution을 의미, 이를 이용하여 Distribution을 생성

$$Q(s, a) = E[Z(s, a)] = \sum_{i=1}^N p_i x_i$$

# Distributional RL

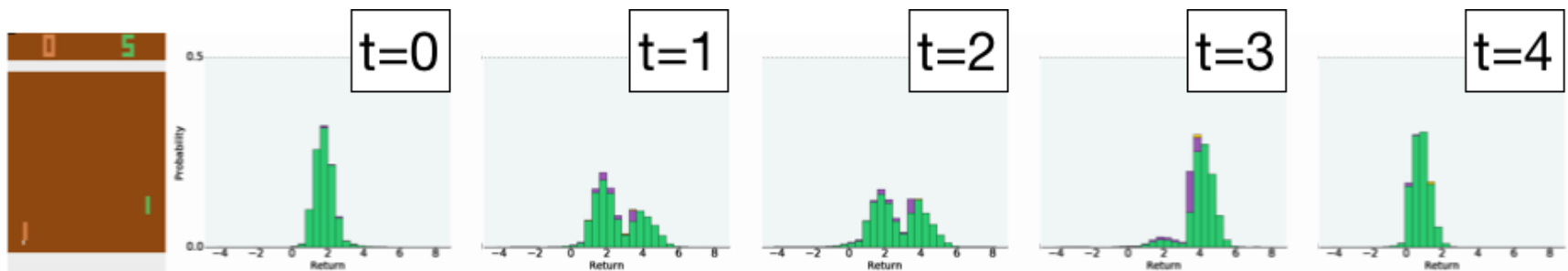
## A Distributional Perspective on Reinforcement Learning (C51)





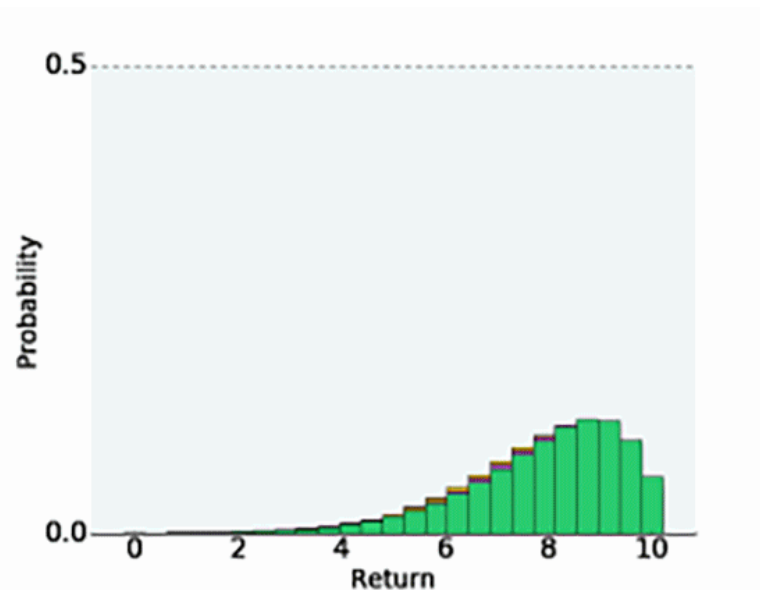
# Distributional RL

## A Distributional Perspective on Reinforcement Learning (C51)



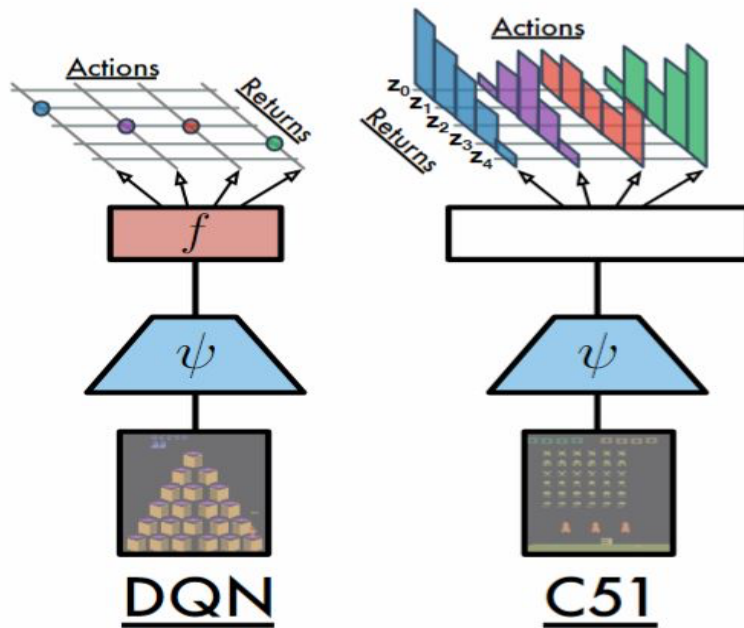
# Distributional RL

## A Distributional Perspective on Reinforcement Learning (C51)



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# Distributional RL

A Distributional Perspective on Reinforcement Learning (C51)

C51 = DQN + Projection Distribution  
(분포 만들기)

# Distributional RL

## A Distributional Perspective on Reinforcement Learning (C51)

### Distributional DQN

1. Return에 대한 Value Distribution(51개 bin)을 만든다.
2. 각 스텝마다 만든 Value Distribution 들간의 거리를 구한다.  
→ 논문에서 이론상 Wasserstein distance로 정의했지만  
실험에서 KL-divergence로 계산
3. Cross entropy로 분포간의 Loss 계산

# Distributional RL

## A Distributional Perspective on Reinforcement Learning (C51)

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### Algorithm 1 Categorical Algorithm

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**input** A transition  $x_t, a_t, r_t, x_{t+1}, \gamma_t \in [0, 1]$

$$Q(x_{t+1}, a) := \sum_i z_i p_i(x_{t+1}, a)$$

$$a^* \leftarrow \arg \max_a Q(x_{t+1}, a)$$

$$m_i = 0, \quad i \in 0, \dots, N-1$$

**for**  $j \in 0, \dots, N-1$  **do**

# Compute the projection of  $\hat{\mathcal{T}} z_j$  onto the support  $\{z_i\}$

$$\hat{\mathcal{T}} z_j \leftarrow [r_t + \gamma_t z_j]_{V_{\min}}^{V_{\max}}$$

$$b_j \leftarrow (\hat{\mathcal{T}} z_j - V_{\min}) / \Delta z \quad \# b_j \in [0, N-1]$$

$$l \leftarrow \lfloor b_j \rfloor, u \leftarrow \lceil b_j \rceil$$

# Distribute probability of  $\hat{\mathcal{T}} z_j$

$$m_l \leftarrow m_l + p_j(x_{t+1}, a^*)(u - b_j)$$

$$m_u \leftarrow m_u + p_j(x_{t+1}, a^*)(b_j - l)$$

**end for**

**output**  $-\sum_i m_i \log p_i(x_t, a_t)$  # Cross-entropy loss

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# Distributional RL

## A Distributional Perspective on Reinforcement Learning (C51)

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Replay Buffer에서 Batch size만큼 추출

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Projection Distribution  
(분포 만들기)



# Distributional RL

## A Distributional Perspective on Reinforcement Learning (C51)

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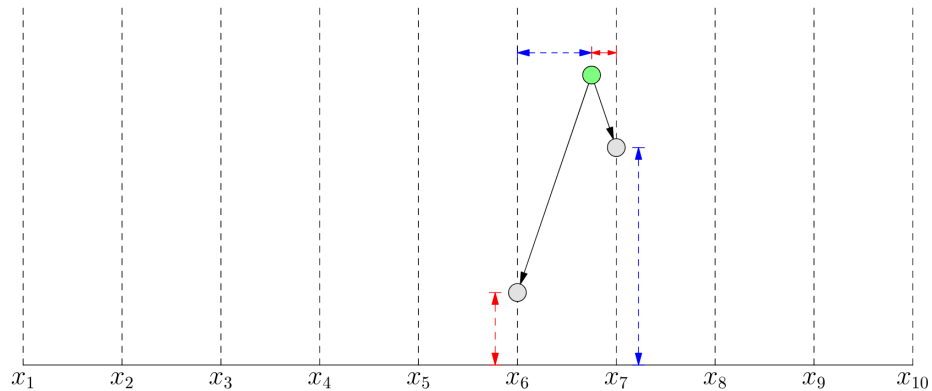
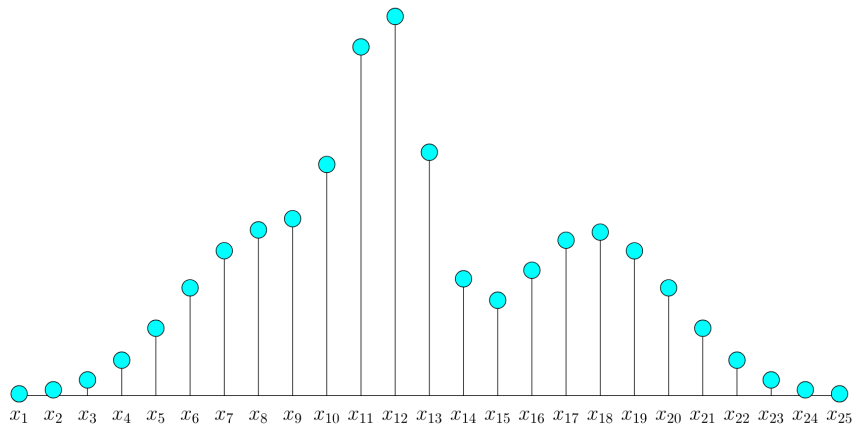
*Bellman distributional operator*

$$V_{\max} = 10$$

$$V_{\min} = -10$$

# Distributional RL

## A Distributional Perspective on Reinforcement Learning (C51)



# Distributional RL

## A Distributional Perspective on Reinforcement Learning (C51)

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**end for**

**output**  $-\sum_i m_i \log p_i(x_t, a_t)$  # Cross-entropy loss

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KL-divergence(cross entropy)로

Loss 구하기

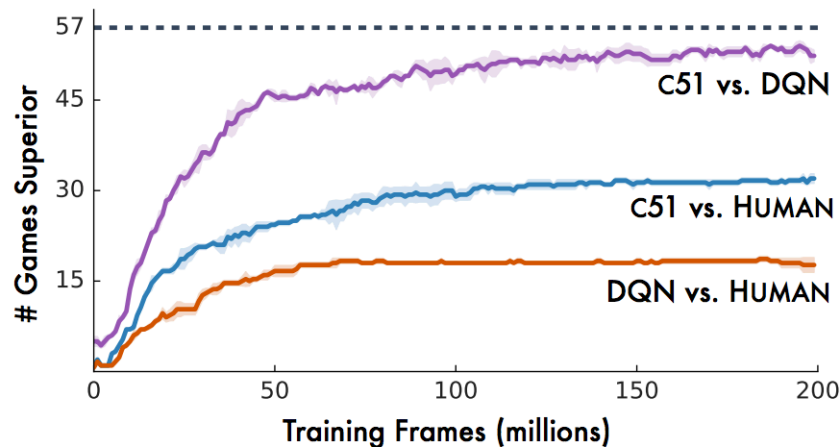
# Performance

## A Distributional Perspective on Reinforcement Learning (C51)

### Comparison

	Mean	Median	> H.B.	> DQN
DQN	228%	79%	24	0
DDQN	307%	118%	33	43
DUEL.	373%	151%	37	50
PRIOR.	434%	124%	39	48
PR. DUEL.	592%	172%	39	44
C51	<b>701%</b>	<b>178%</b>	<b>40</b>	<b>50</b>
UNREAL <sup>†</sup>	880%	250%	-	-

### Relative Performance



### 3. 코드 구현체 분석



# 감사합니다.

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