Spatial Index Structures

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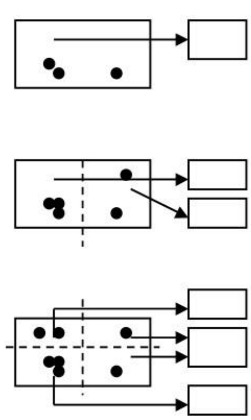
Outline

- Grid File
- Z-ordering
- Hilbert Curve
- Quad Tree
- PM
- PR
- R Tree
- R* Tree
- R+ Tree

Grid File

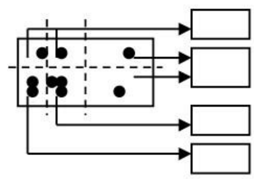
- Hashing methods for multidimensional points (extension of Extensible hashing)
- Idea: Use a grid to partition the space → each cell is associated with one page
- Two disk access principle (exact match)

Grid File



- Start with one bucket for the whole space.
- Select dividers along each dimension.
  Partition space into cells
- Dividers cut all the way.

Grid File



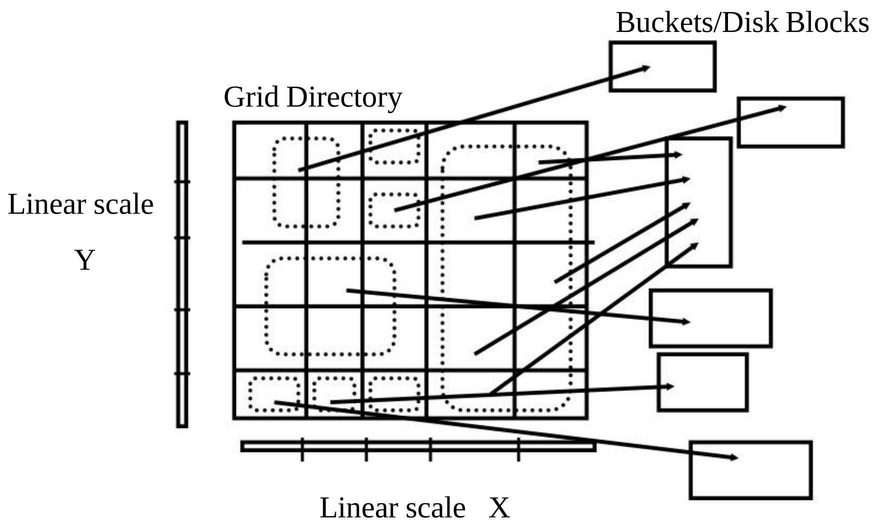
- Each cell corresponds to 1 disk page.
- Many cells can point to the same page.
- Cell directory potentially exponential in the number of dimensions

Grid File Implementation

Dynamic structure using a grid directory

- Grid array: a 2 dimensional array with pointers to buckets (this array can be large, disk resident) G (0,…, nx-1, 0, …, ny-1)
- Linear scales: Two 1 dimensional arrays that used to access the grid array (main memory) X(0, …, nx-1), Y(0, …, ny-1)

Example



Grid File Search

- Exact Match Search: at most 2 I/Os assuming linear scales fit in memory.
  - First use liner scales to determine the index into the cell directory
  - access the cell directory to retrieve the bucket address (may cause 1 I/O if cell directory does not fit in memory)
  - access the appropriate bucket (1 I/O)
- Range Queries:
  - use linear scales to determine the index into the cell directory.
  - Access the cell directory to retrieve the bucket addresses of buckets to visit.
  - Access the buckets.

Grid File insertion

- Determine the bucket into which insertion must occur.
- If space in bucket, insert.
- Else, split bucket
  - how to choose a good dimension to split?
  - ans: create convex regions for buckets.
- If bucket split causes a cell directory to split do so and adjust linear scales.
- insertion of these new entries potentially requires a complete reorganization of the cell directory--- expensive!!!

Grid File Deletion

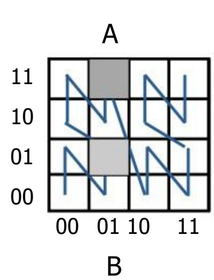
- Deletion may decrease the space utilization. Merge buckets
- We need to decide which cells to merge and a merging threshold
- Buddy system and neighbor system
  - A bucket can merge with only one *buddy* in each dimension
  - Merge adjacent regions if the result is a rectangle

Z-ordering

- Basic assumption: Finite precision in the representation of each coordinate, K bits (2^K values)
- The address space is a square (image) and represented as a 2^K x 2^K array
- Each element is called a pixel

Z-ordering

- Impose a linear ordering on the pixels of the image → 1 dimensional problem



$Z_A = \text{shuffle}(x_A, y_A) = \text{shuffle}(\text{“01”}, \text{“11”})$
$= 0111 = (7)_{10}$

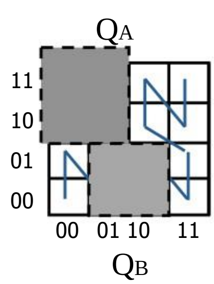
$Z_B = \text{shuffle}(\text{“01”}, \text{“01”}) = 0011$

Z-ordering

- Given a point (x, y) and the precision K find the pixel for the point and then compute the z-value
- Given a set of points, use a B+-tree to index the z-values
- A range (rectangular) query in 2-d is mapped to a set of ranges in 1-d

Queries

- Find the z-values that contained in the query and then the ranges



Q_A → range [4, 7]

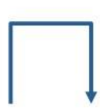
Q_B → ranges [2,3] and [8,9]

Hilbert Curve

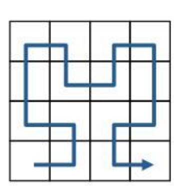
- We want points that are close in 2d to be close in the 1d
- Note that in 2d there are 4 neighbors for each point where in 1d only 2.
- Z-curve has some “jumps” that we would like to avoid
- Hilbert curve avoids the jumps : recursive definition

Hilbert Curve- example

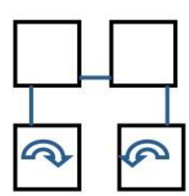
- It has been shown that in general Hilbert is beher than the other space filling curves for retrieval *
- Hi (order-i) Hilbert curve for 2_ix2_i array



H1



H2



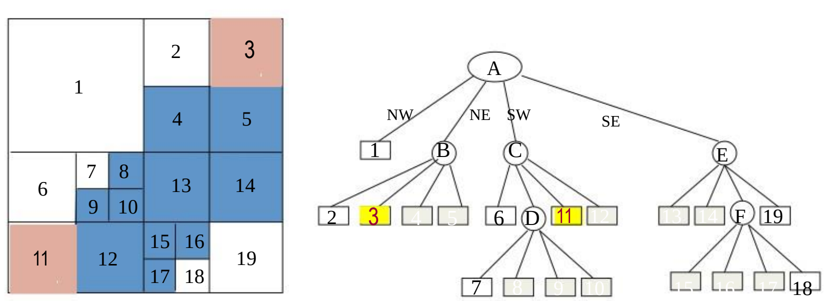
H(n+1)

* H. V. Jagadish: Linear Clustering of Objects with Multiple Atributes. ACM SIGMOD Conference 1990: 332-342

Quad Trees

- Region Quadtree
  - The blocks are required to be disjoint
  - Have standard sizes (squares whose sides are power of two)
  - At standard locations
  - Based on successive subdivision of image array into four equal-size quadrants
  - If the region does not cover the entire array, subdivide into quadrants, sub-quadrants, etc.
  - A variable resolution data structure

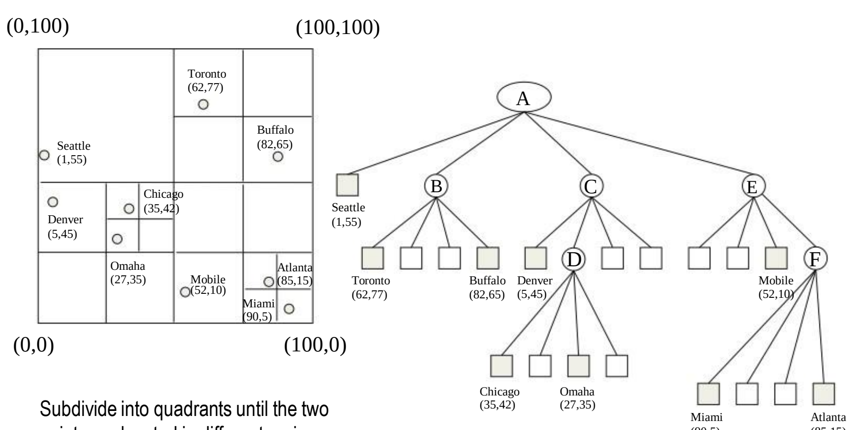
Example of Region Quadtree



PR Quadtree

- PR (Point-Region) quadtree
- Regular decomposition (similar to Region quadtree)
- Independent of the order in which data points are inserte into it
- L: if two points are very close, decomposition can be very deep

Example of PR Quadtree



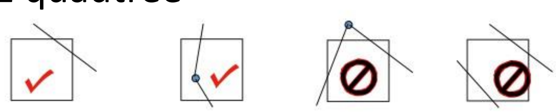
Subdivide into quadrants until the two points are located in different regions

PM Quadtree

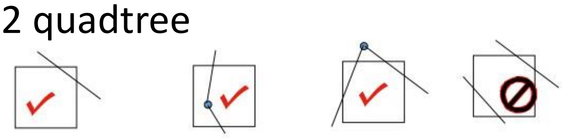
- PM (Polygonal-Map) quadtree family
  - PM1 quadtree, PM2 quadtree, PM3 quadtree, PMR quadtree, … etc.
- PM1 quadtree
  - Based on regular decomposition of space
  - Vertex-based implementaDon
  - Criteria
    - At most one vertex can lie in a region represented by a quadtree leaf
    - If a region contains a vertex, it can contain no partial-edge that does not include that vertex
    - If a region contains no vertices, it can contain at most one partial-edge

PM Quadtree

PM1 quadtree

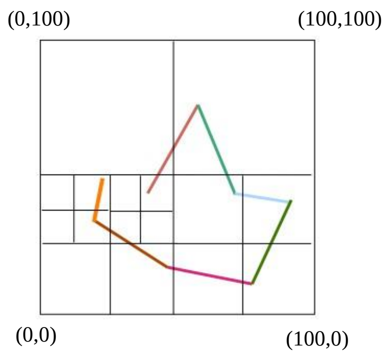


PM2 quadtree



PM3 quadtree

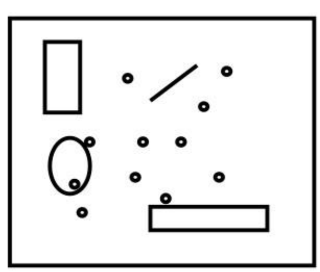
Example of PM1 Quadtree



- Each node in a PM quadtree is a collecDon of parDal edges (and a vertex)
- Each point record has two field (x,y)
- Each parDal edge has four field (starDng_point, ending_point, lel region, right region)

Remarks

- Given a collection of geometric objects (points, lines, polygons, ...)
- organize them on disk, to answer spatial queries (range, nn, etc)



R-trees

- [GuEman 84] Main idea: extend B+-tree to multidimensional spaces!

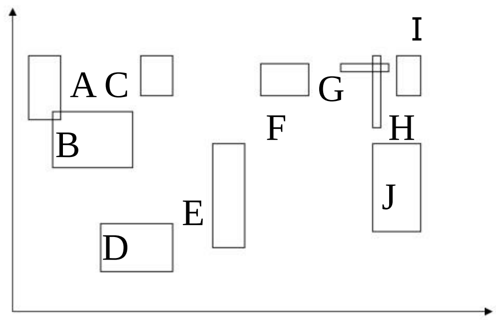
  - (only deal with Minimum Bounding Rectangles - **MBR**s)

R-trees

- A multi-way external memory tree
- Index nodes and data (leaf) nodes
- All leaf nodes appear on the same level
- Every node contains between m and M entries
- The root node has at least 2 entries (children)

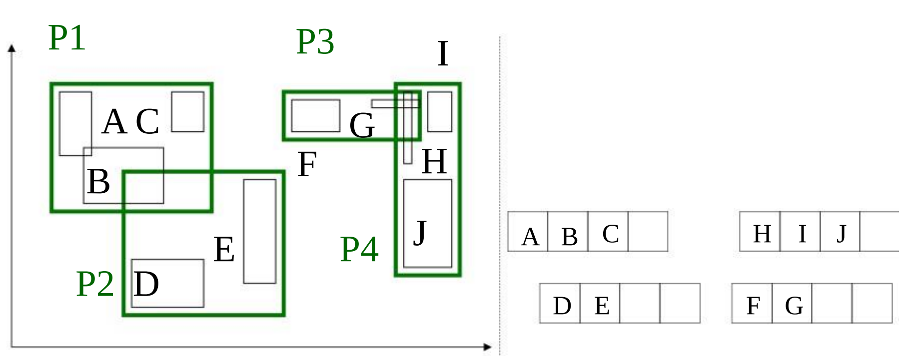
Example

- eg., w/ fanout 4: group nearby rectangles to parent MBRs; each group -> disk page



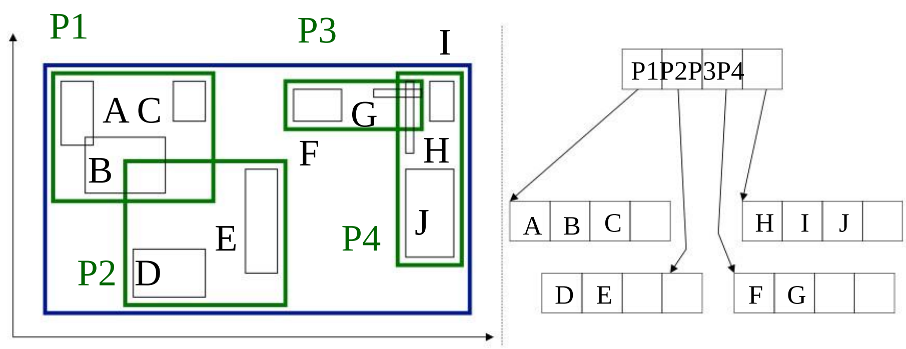
Example

- F=4



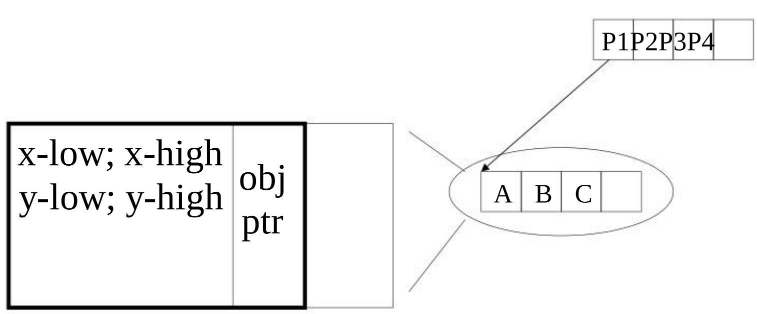
Example

- F=4



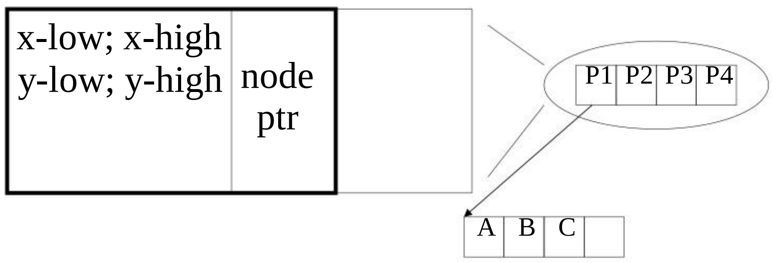
R-trees - format of nodes

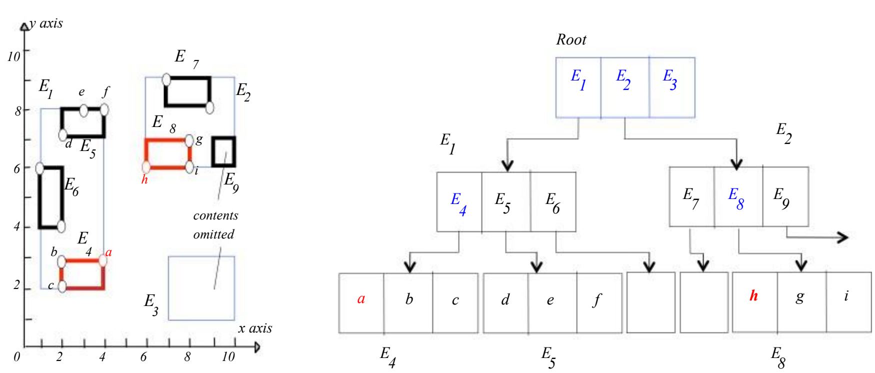
- {(MBR; obj_ptr)} for leaf nodes



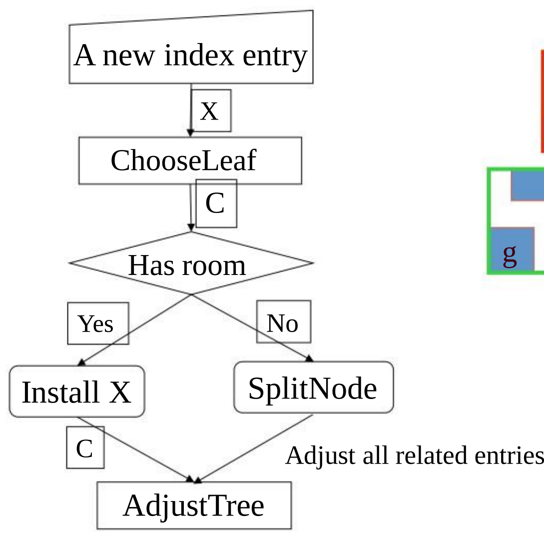
R-trees - format of nodes

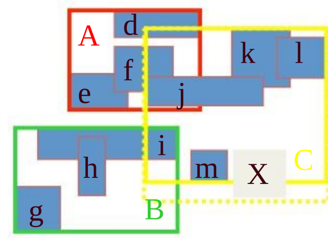
- {(MBR; node_ptr)} for non-leaf nodes





Insertion Processes



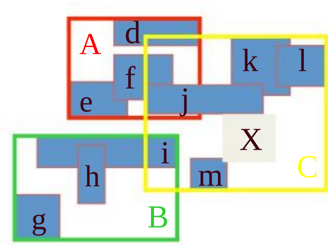


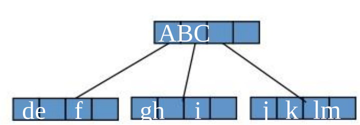
Different variant:
- Exhaustive
- Quadratic
- Linear
- Packed
- Hilbert Packed
- …etc.

Processes of Quadratic Spilt

(page 52 in GuEman’s paper)

Pick first entry for each group
Run PickSeeds





Processes of Quadratic Spilt

(page 52 in GuEman's paper)

PickSeeds

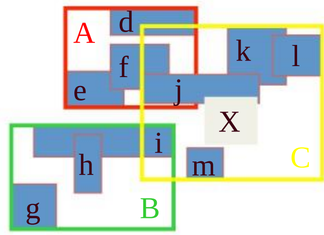
PS1 [Calculate inefficiency of grouping entries together]

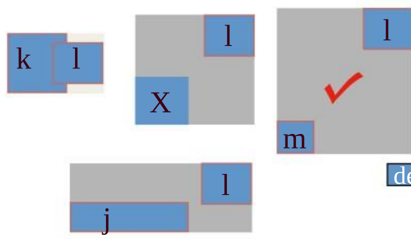
For each pair of E1 and E2, compose a rectangle R including E1 and E2

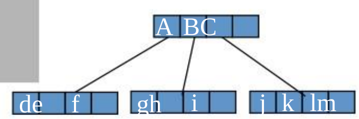
Calculate d = area(R) - area(E1) - area(E2)

PS2 [Choose the most wasteful pair]

Choose the pair with the largest d







Processes of Quadratic Spilt

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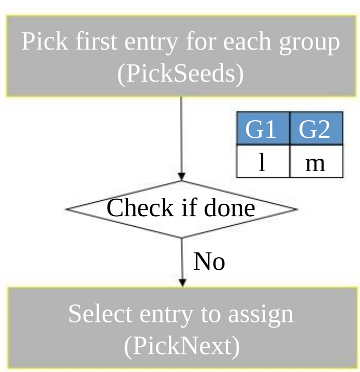

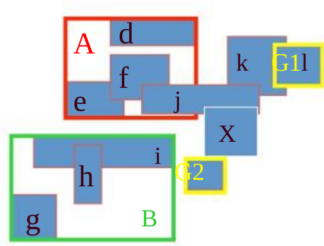

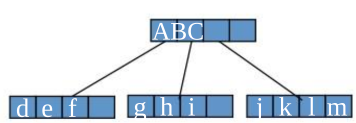

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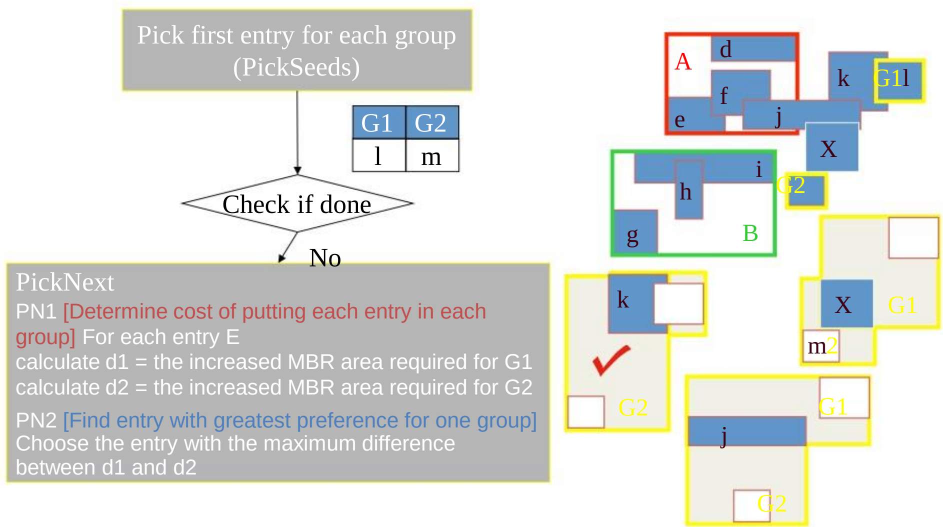

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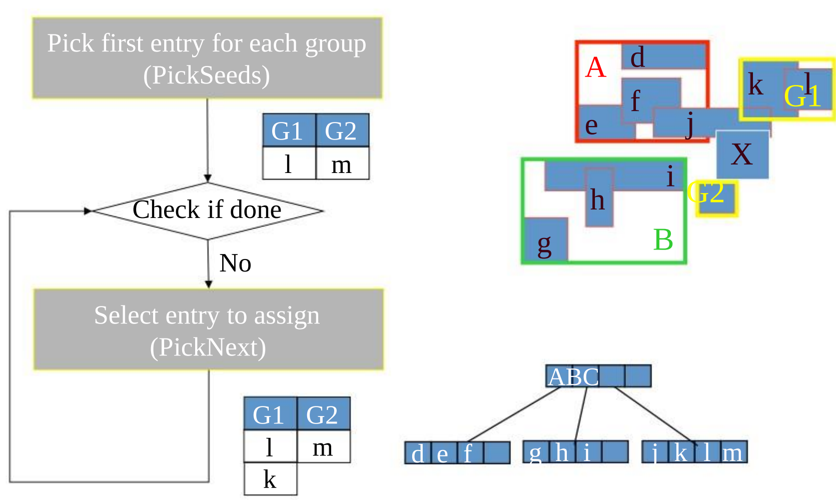

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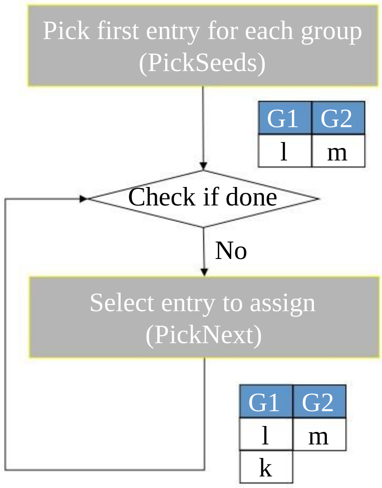

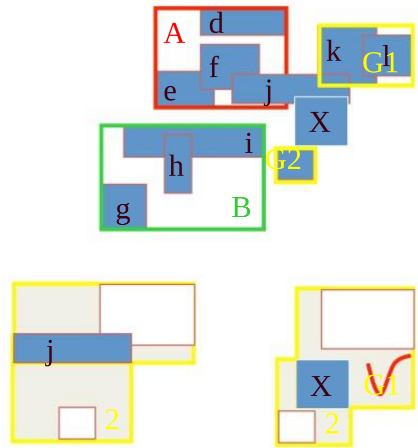

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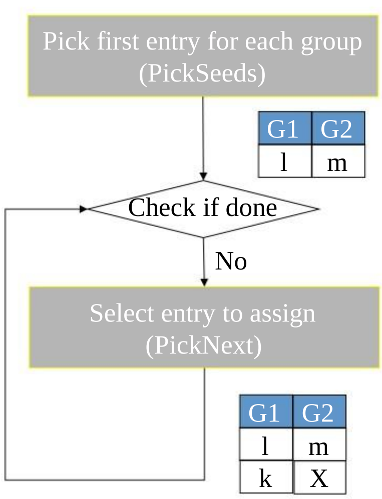

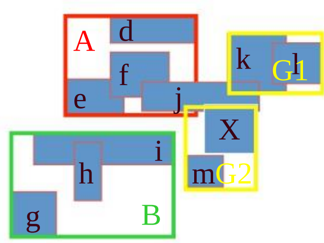

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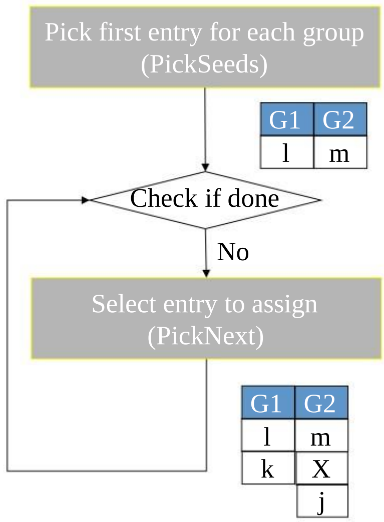
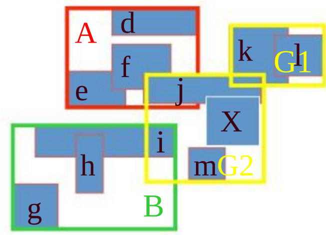
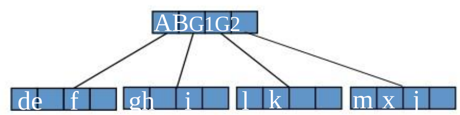
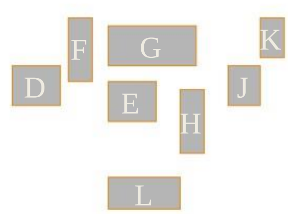






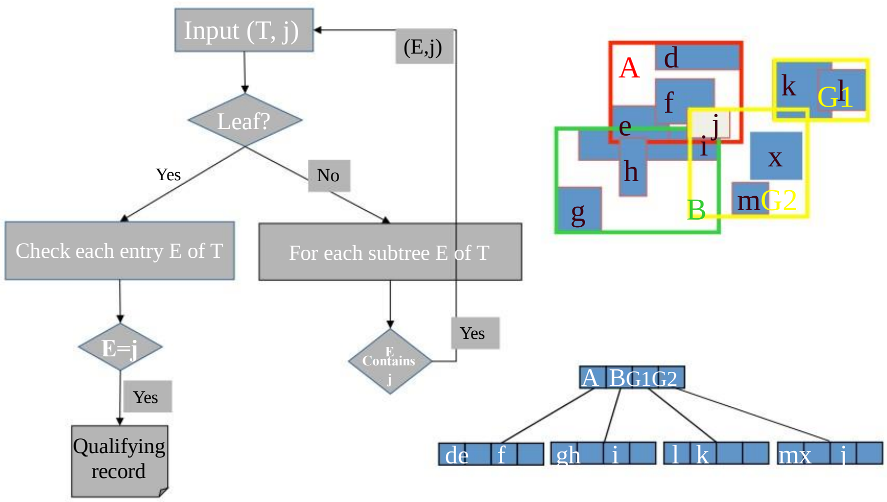
Excercise

- (m,M)=(2,4)

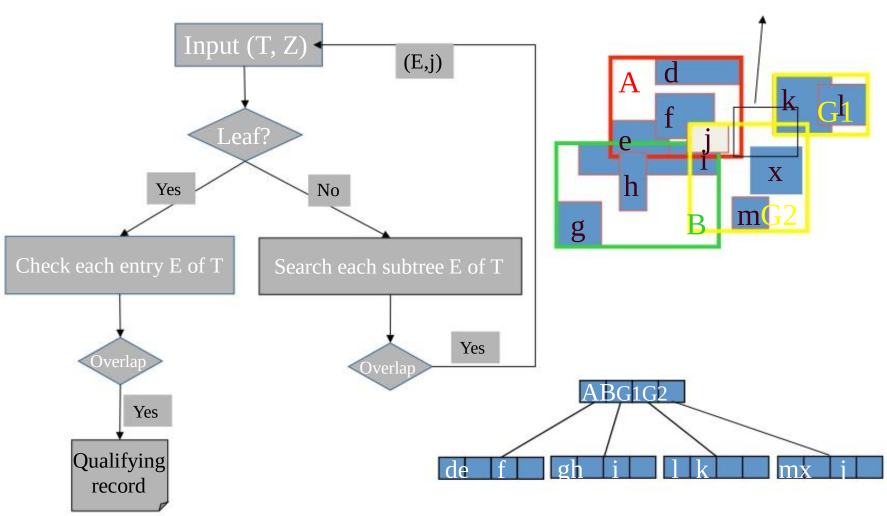


- Build a R-Tree for these spatial data
- Hint: You could use the Spatial index structures demo application step by step

Search Object in R-Tree



(find objects that overlap with Z)

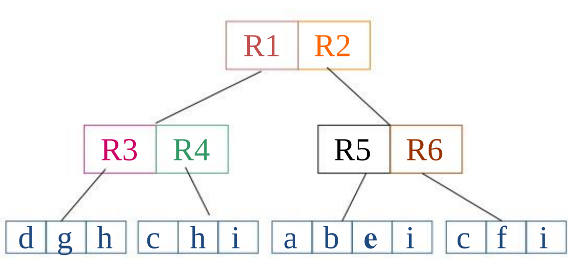


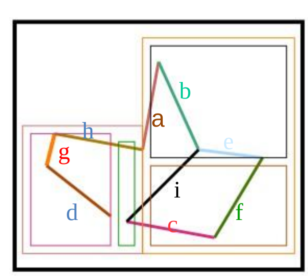
Main Drawbacks of R-Tree

- R-tree is not unique, rectangles depend on how objects are inserted and deleted from the tree.
- In order to search some object you might have to go through several rectangles or the whole database
  - Why?
  - Solution?

R+-Tree

- Overcome problems with R-Tree
- If node overlaps with several rectangles insert the node in all
- Decompose the space into disjoint cells





R+-Tree Properties

- R+-tree and cell-trees used approach of discomposing space into cells
  - R+-trees deals with collection of objects bounded by rectangles
  - Cell tree deals with collection of objects bounded by convex polyhedron
- R+-trees is extension of k-d-B-tree
- Retrieval /mes are smaller
- When summing the objects, needs eliminate duplicates
- Again, it is data-dependent

R-tree

- The original R-tree tries to minimize the area of each enclosing rectangle in the index nodes.
- Is there any other property that can be optimized?

R*-tree → Yes!

R*-tree

- Optimization Criteria:
  - (O1) Area covered by an index MBR
  - (O2) Overlap between index MBRs
  - (O3) Margin of an index rectangle
  - (O4) Storage utilization
- Sometimes it is impossible to optimize all the above criteria at the same time!

R*-tree

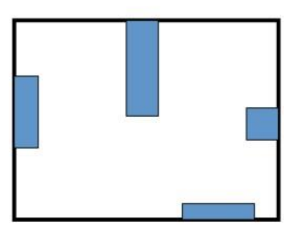
- ChooseSubtree:
  - If next node is a leaf node, choose the node using the following criteria:
    - Least overlap enlargement
    - Least area enlargement
    - Smaller area
  - Else
    - Least area enlargement
    - Smaller area

R*-tree

- SplitNode
  - Choose the axis to split
  - Choose the two groups along the chosen axis
- ChooseSplitAxis
  - Along each axis, sort rectangles and break them into two groups (M-2m+2 possible ways where one group contains at least m rectangles). Compute the sum S of all margin-values (perimeters) of each pair of groups. Choose the one that minimizes S
- ChooseSplitIndex
  - Along the chosen axis, choose the grouping that gives the minimum overlap-value

R*-tree

- Forced Reinsert:
  - defer splits, by forced-reinsert, i.e.: instead of splinng, temporarily delete some entries, shrink overflowing MBR, and re-insert those entries
- Which ones to re-insert?
- How many? A: 30%



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