Lecture #12

- Binary Tree Traversals
- Using Binary Trees to Evaluate Expressions

- Binary Search Trees
- Binary Search Tree Operations
  - Searching for an item
  - Inserting a new item
  - Finding the minimum and maximum items
  - Printing out the items in order
  - Deleting the whole tree

Binary Tree Traversals

When we process all the nodes in a tree, it's called a traversal.

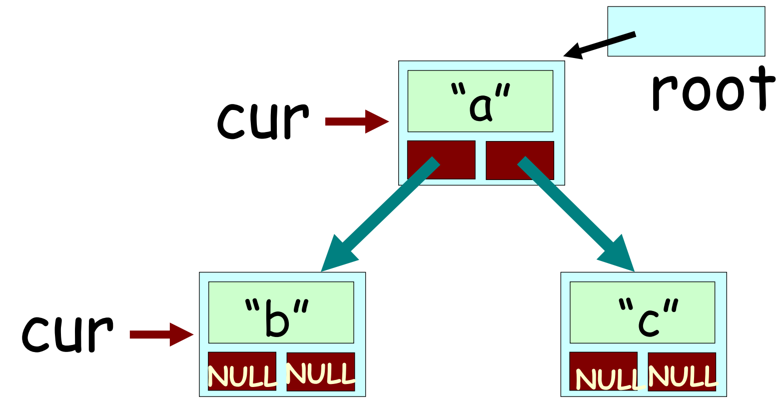
There are four common ways to traverse a tree.

1. Pre-order traversal (we did this last time)
2. In-order traversal
3. Post-order traversal
4. Level-order traversal

Let's see a pre-order traversal first!

The In-order Traversal

1. Process the nodes in the left sub-tree.
2. Process the current node.
3. Process the nodes in the right sub-tree.



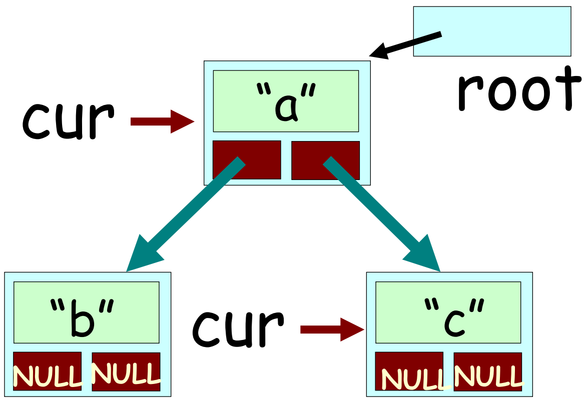

```
void InOrder(Node *cur)
{
    if (cur == NULL)           // if empty, return…
        return;

    InOrder(cur->left);      // Process nodes in left sub-tree.

    cout << cur->value;      // Process the current node.

    InOrder(cur-> right);  // Process nodes in left sub-tree.
}
```

Output:

b

The In-order Traversal

1. Process the nodes in the left sub-tree.
2. Process the current node.
3. Process the nodes in the right sub-tree.



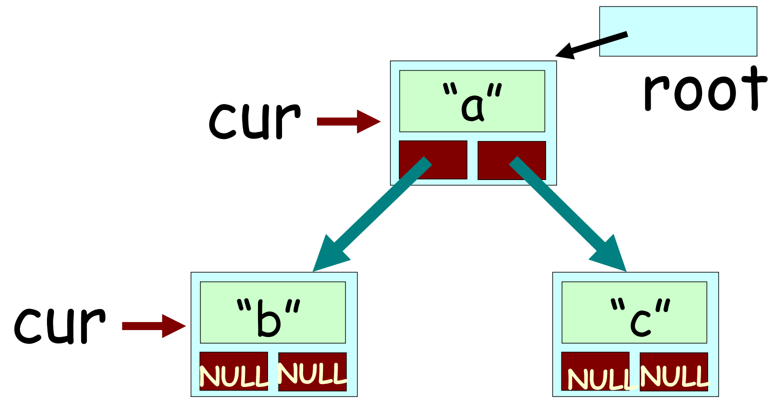

```
void InOrder(Node *cur)
{
    if (cur == NULL)            // if empty, return…
        return;

    InOrder(cur->left);      // Process nodes in left sub-tree.

    cout << cur->value;      // Process the current node.

    InOrder(cur-> right);  // Process nodes in left sub-tree.
}
```

Output:

b a c

The Post-order Traversal

1. Process the nodes in the left sub-tree.
2. Process the nodes in the right sub-tree.
3. Process the current node.



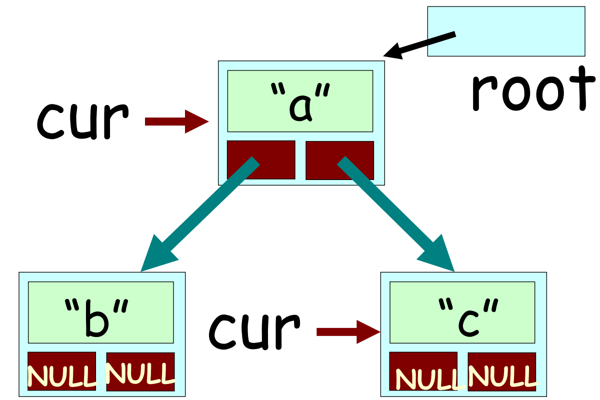

```
void PostOrder(Node *cur)
{
    if (cur == NULL)            // if empty, return…
        return;

    PostOrder(cur->left);      // Process nodes in left sub-tree.

    PostOrder(cur-> right);  // Process nodes in right sub-tree.

    cout << cur->value;       // Process the current node.
}
```

Output:

b

The Post-order Traversal

1. Process the nodes in the left sub-tree.
2. Process the nodes in the right sub-tree.
3. Process the current node.




```
void PostOrder(Node *cur)
{
   if (cur == NULL)           // if empty, return…
        return;

   PostOrder(cur->left);      // Process nodes in left sub-tree.

   PostOrder(cur-> right);  // Process nodes in right sub-tree.

   cout << cur->value;       // Process the current node.
}
```

Output:

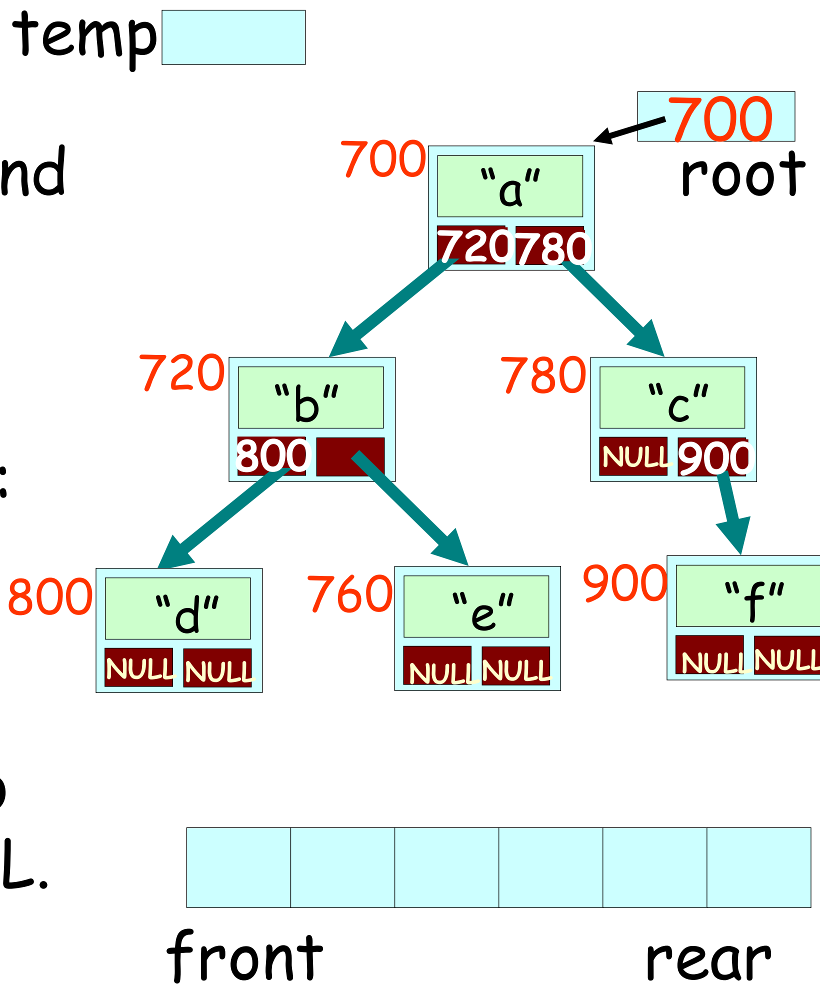
b c a

The Level Order Traversal

In a *level order traversal* we visit each level's nodes, from left to right, before visiting nodes in the next level.

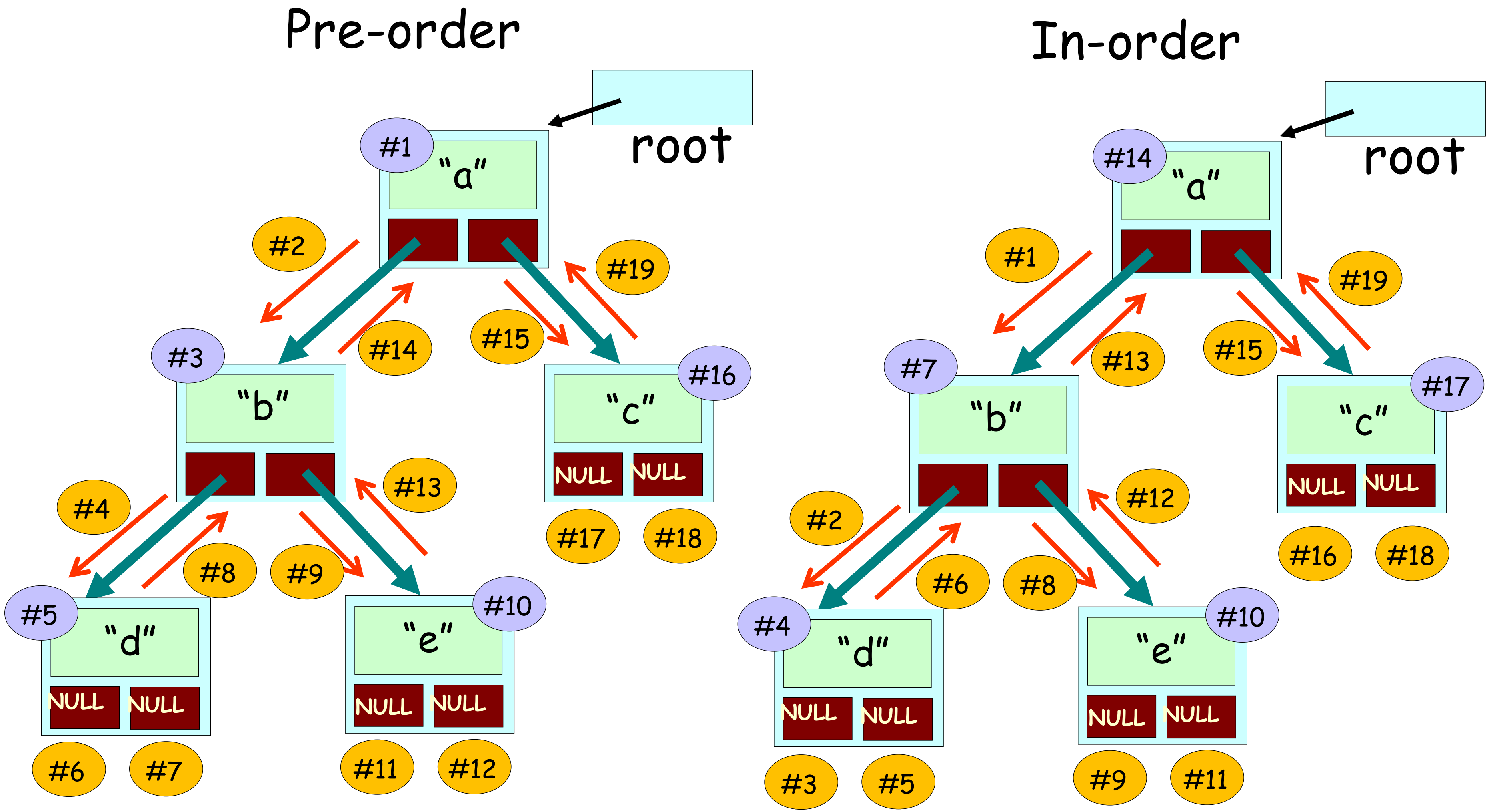
Here's the algorithm:

1. Use a temp pointer variable and a queue of node pointers.
2. Insert the root node pointer into the queue.
3. While the queue is not empty:
   A. Dequeue the top node pointer and put it in temp.
   B. Process the node.
   C. Add the node's children to queue if they are not NULL.



a b c d Etc…

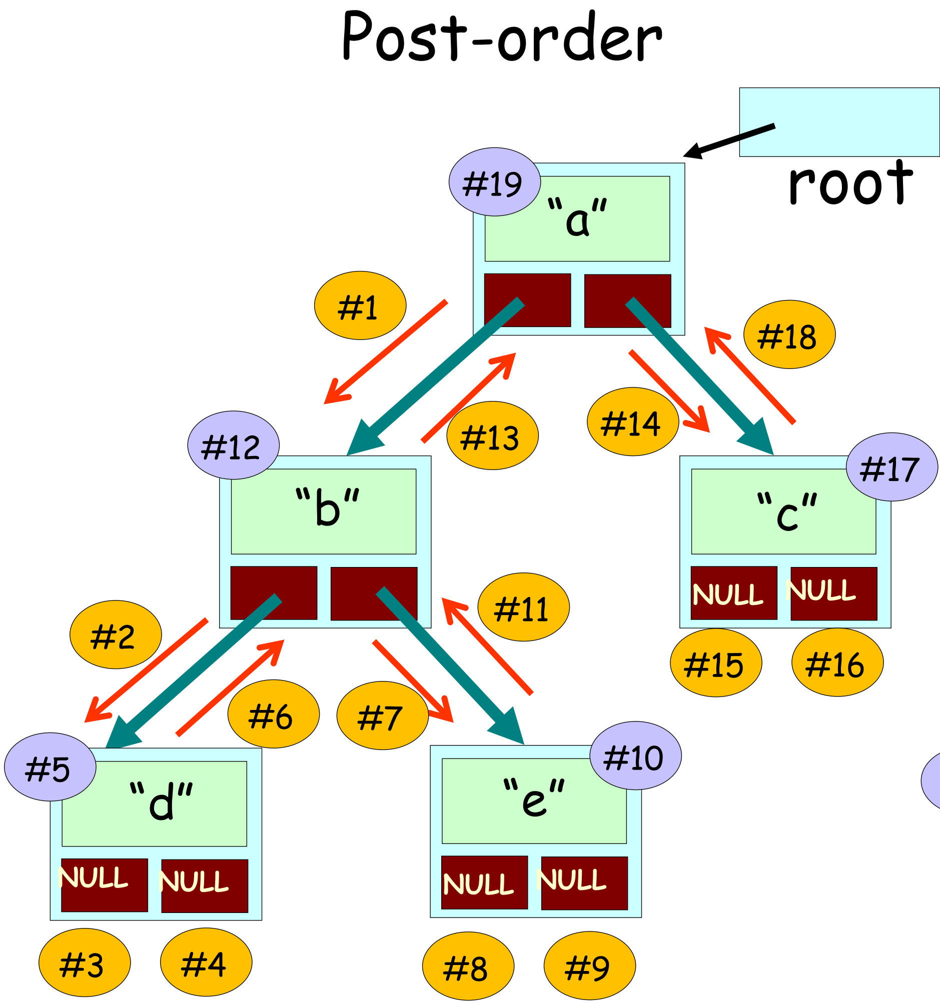
Traversal Overview, Part 1

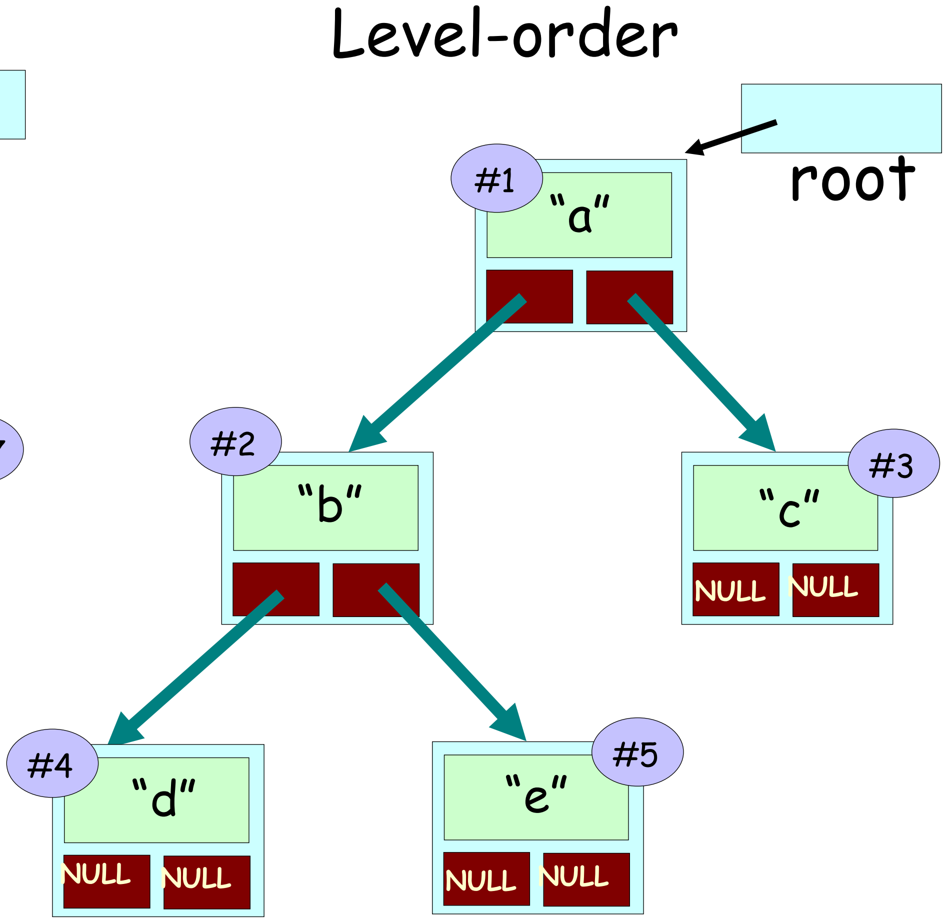


1. Process current node
2. Traverse left
3. Traverse right

1. Traverse left
2. Process current node
3. Traverse right

Traversal Overview, Part 2





1. Traverse left
2. Traverse right
3. Process current node

Big-Oh of Traversals?

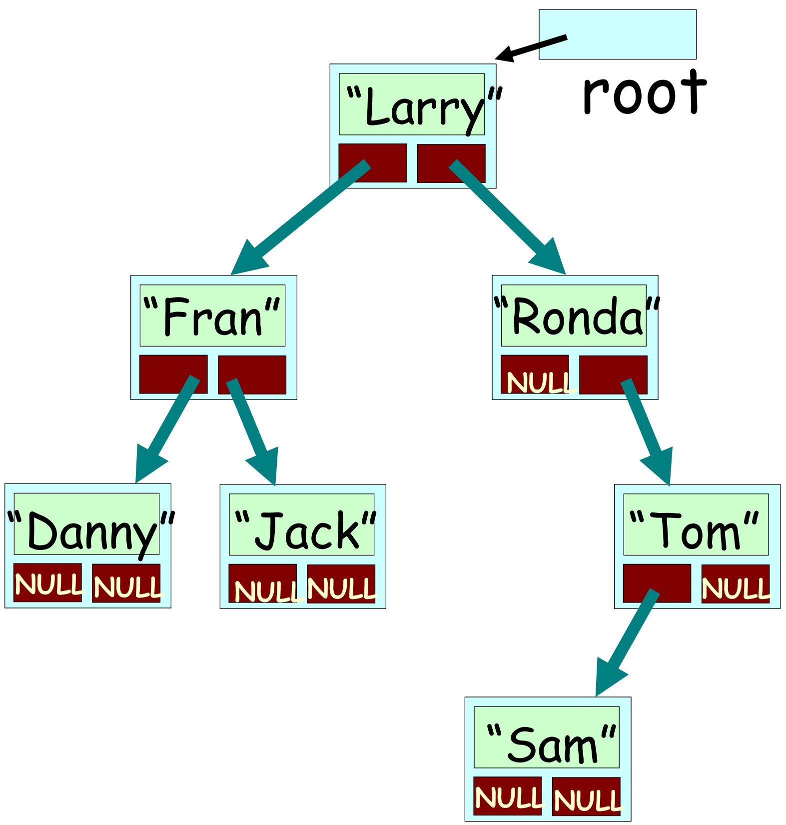
Question: What're the big-ohs of each of our traversals?

Traversal Challenge

RULES

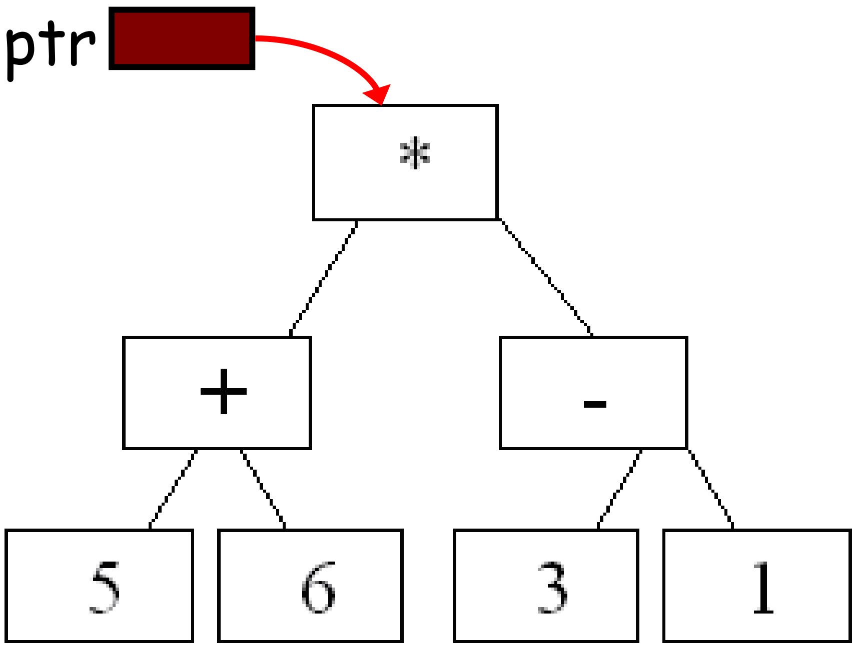
- The class will split into left and right teams
- One student from each team will come up to the board
- Each student can either
  - write one new item or
  - fix a single error in their teammates solution
- Then the next two people come up, etc.
- The team that completes their program first wins!

Challenge: What order will the following nodes be printed out if we use an in-order traversal?



Expression Evaluation

We can represent arithmetic expressions using a binary tree.



For example, the tree on the left represents the expression: (5+6)*(3-1)

Once you have an expression in a tree, its easy to evaluate it and get the result.

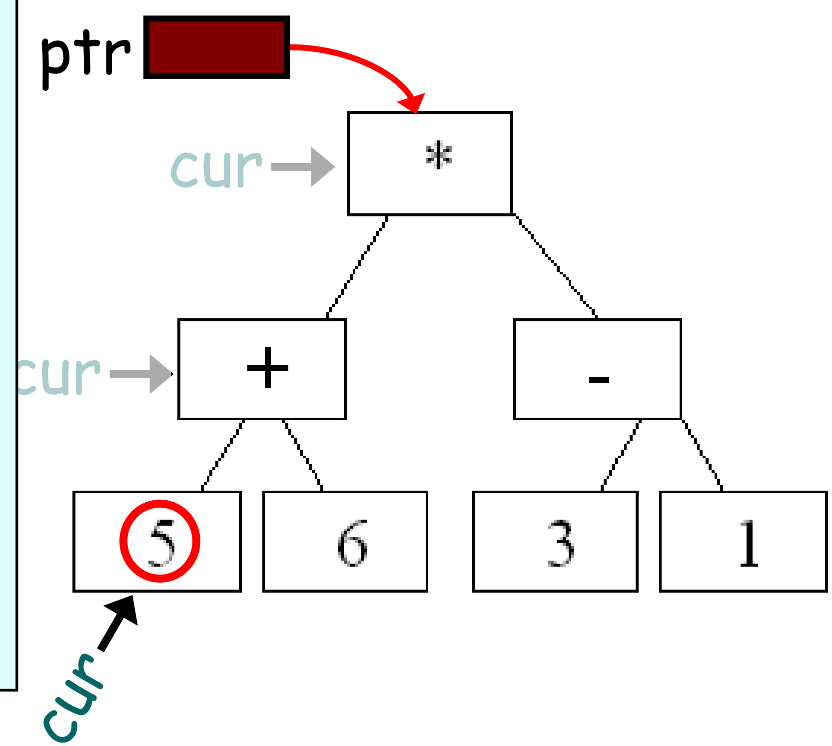
Let's see how!

Expression Evaluation

Here's our evaluation function. We start by passing in a pointer to the root of the tree.

1. If the current node is a number, return its value.
2. Recursively evaluate the left subtree and get the result.
3. Recursively evaluate the right subtree and get the result.
4. Apply the operator in the current node to the left and right results; return the result.

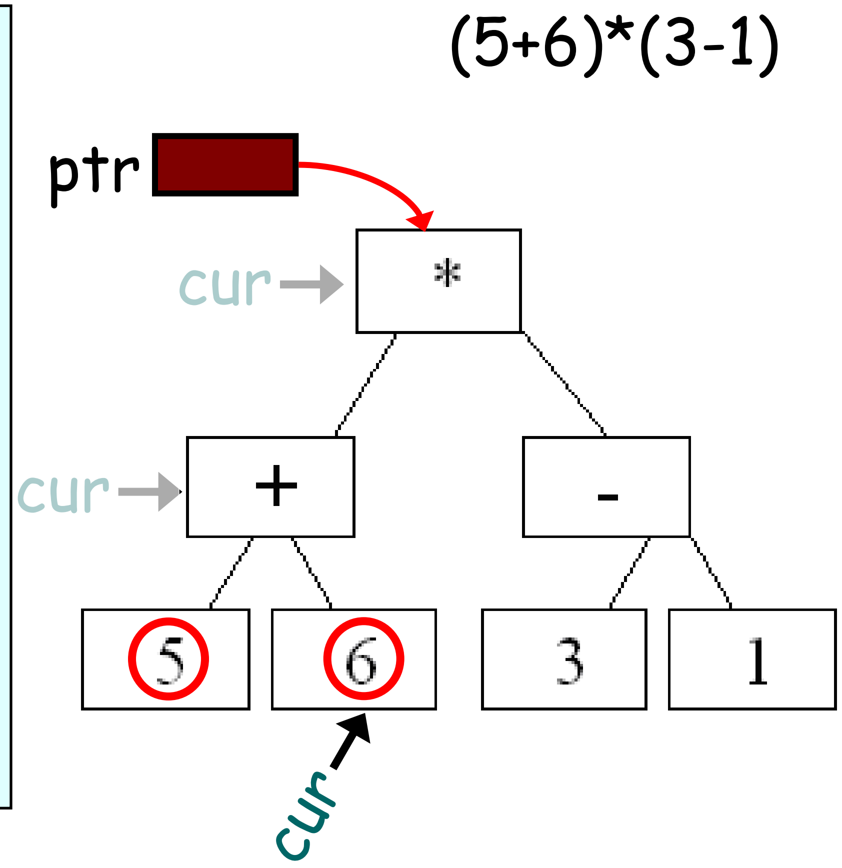
(5+6)*(3-1)



Expression Evaluation

Here's our evaluation function. We start by passing in a pointer to the root of the tree.

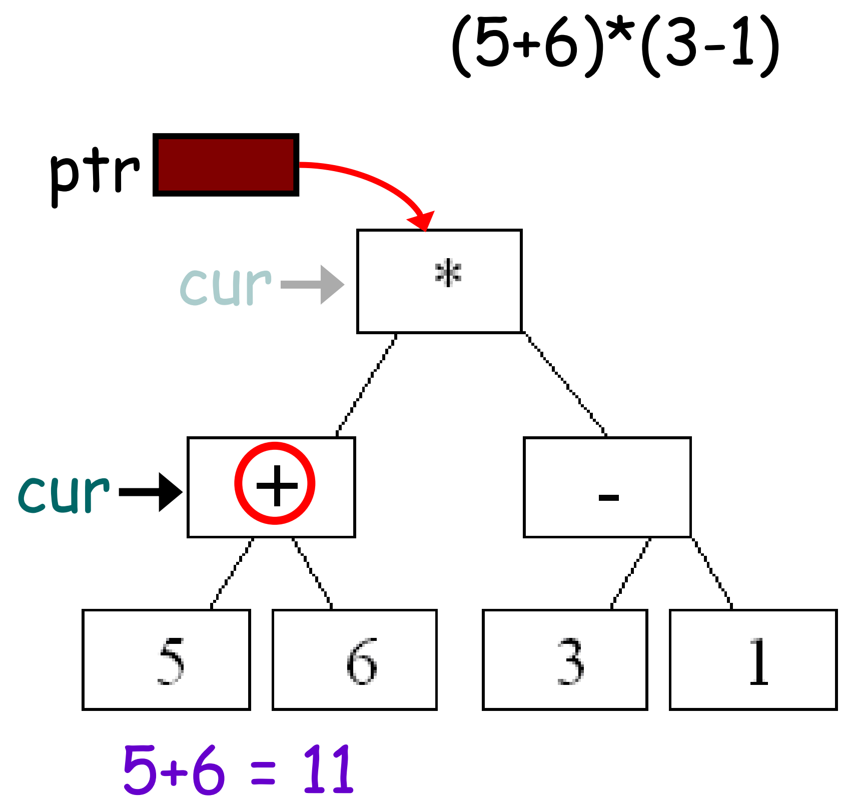
1. If the current node is a number, return its value.
2. Recursively evaluate the left subtree and get the result.
3. Recursively evaluate the right subtree and get the result.
4. Apply the operator in the current node to the left and right results; return the result.



Expression Evaluation

Here's our evaluation function. We start by passing in a pointer to the root of the tree.

1. If the current node is a number, return its value.
2. Recursively evaluate the left subtree and get the result.
   Result = 5
3. Recursively evaluate the right subtree and get the result.
   Result = 6
4. Apply the operator in the current node to the left and right results; return the result.



Expression Evaluation

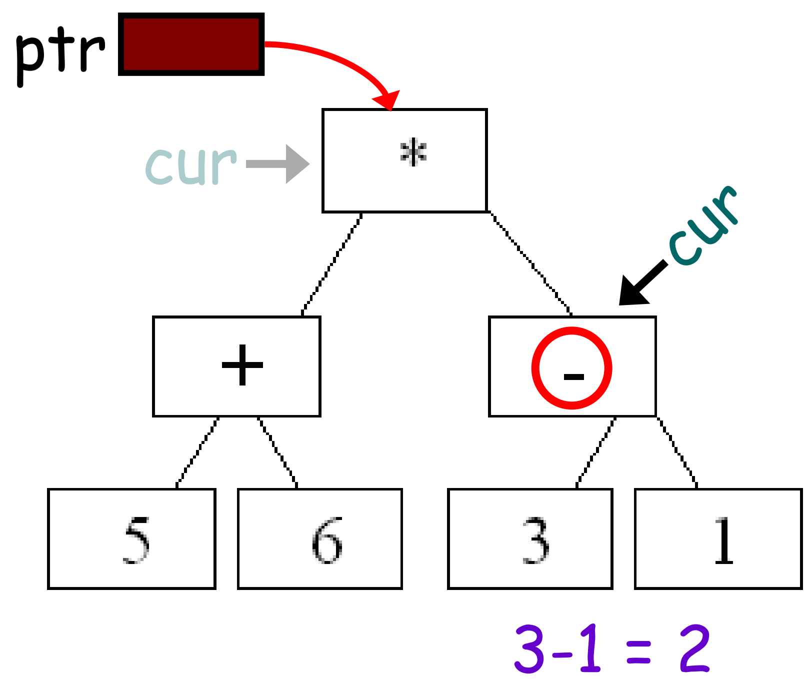
Here's our evaluation function. We start by passing in a pointer to the root of the tree.

1. If the current node is a number, return its value.
2. Recursively evaluate the left subtree and get the result.

   Result = 3
3. Recursively evaluate the right subtree and get the result.

   Result = 1
4. Apply the operator in the current node to the left and right results; return the result.

(5+6)*(3-1)



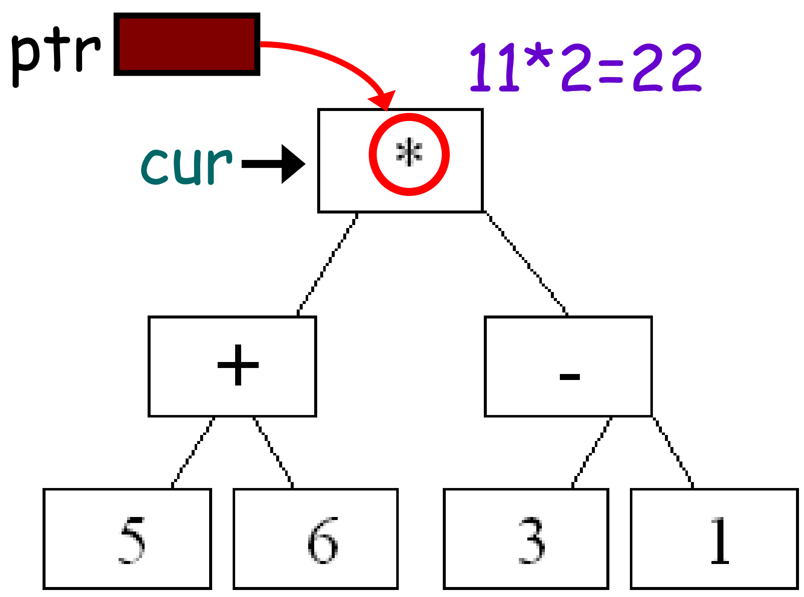
Expression Evaluation

Here's our evaluation function. We start by passing in a pointer to the root of the tree.

The result is 22.

(5+6)*(3-1)

1. If the current node is a number, return its value.
2. Recursively evaluate the left subtree and get the result.
   Result = 11
3. Recursively evaluate the right subtree and get the result.
   Result = 2
4. Apply the operator in the current node to the left and right results; return the result.



Expression Evaluation

Here's our evaluation function. We start by passing in a pointer to the root of the tree.

1. If the current node is a number, return its value.
2. Recursively evaluate the left subtree and get the result.
3. Recursively evaluate the right subtree and get the result.
4. Apply the operator in the current node to the left and right results; return the result.

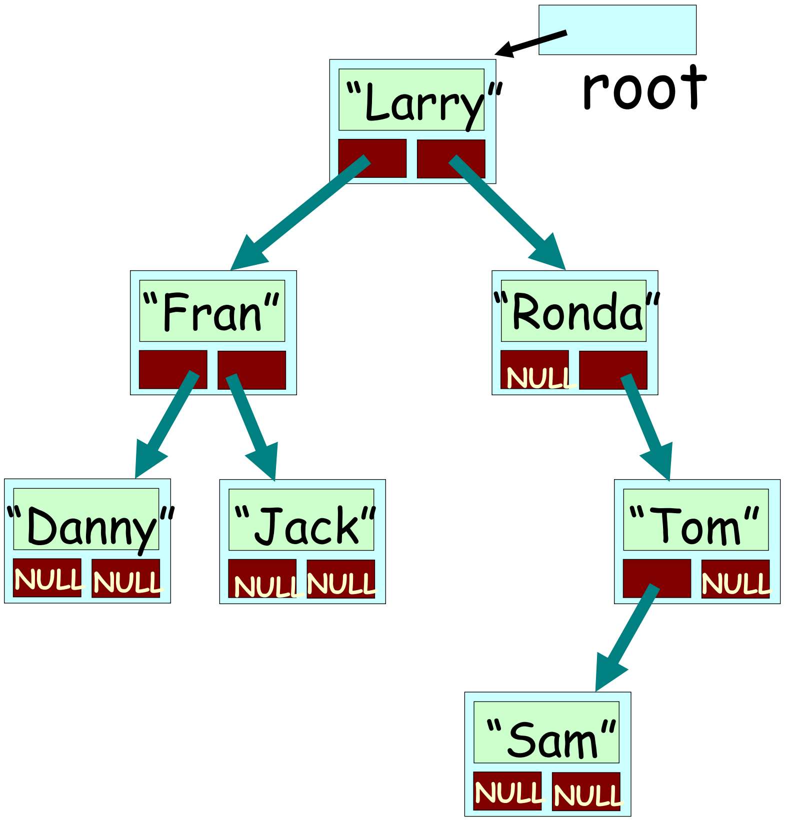
Question: Which other algorithm does this remind you of?

Binary Search Trees

Binary Search Trees are a type of binary tree with specific properties that make them very efficient to search for a value in the tree.

Like regular Binary Trees, we store and search for values in Binary Search Trees...

Here's an example BST...



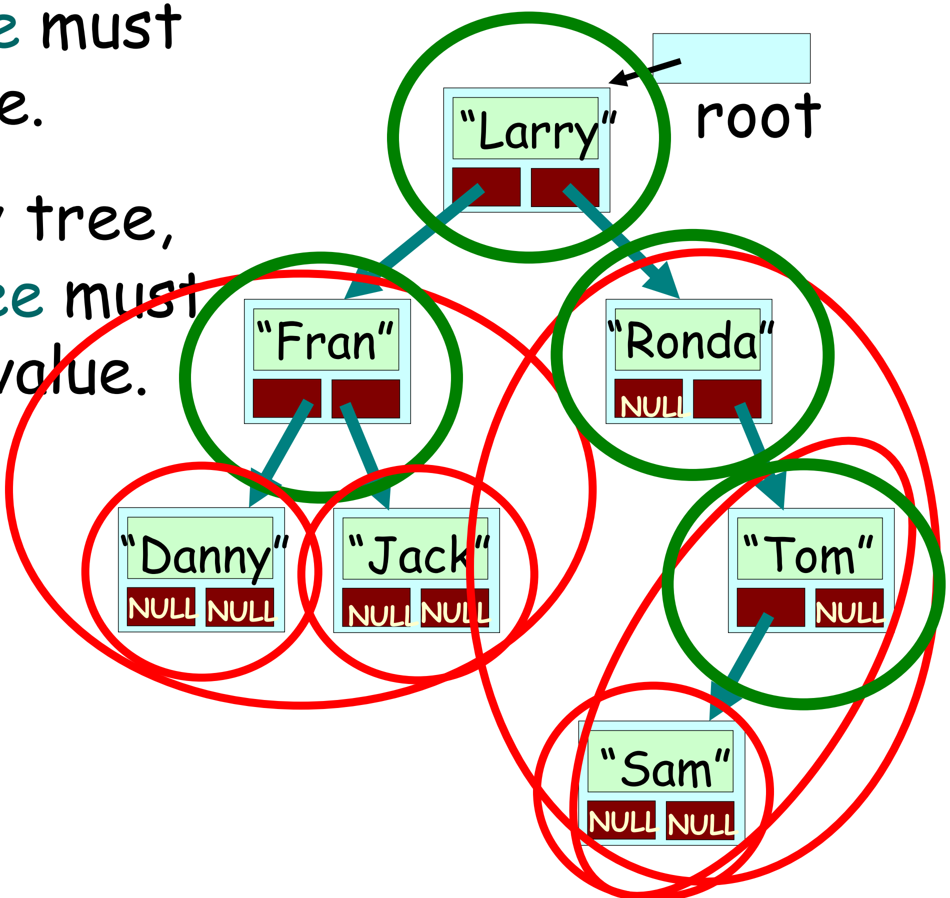
Binary Search Trees

BST Definition: A Binary Search Tree is a binary tree with the following two properties:

Given any node in the binary tree, all nodes in its left sub-tree must be less than the node's value.

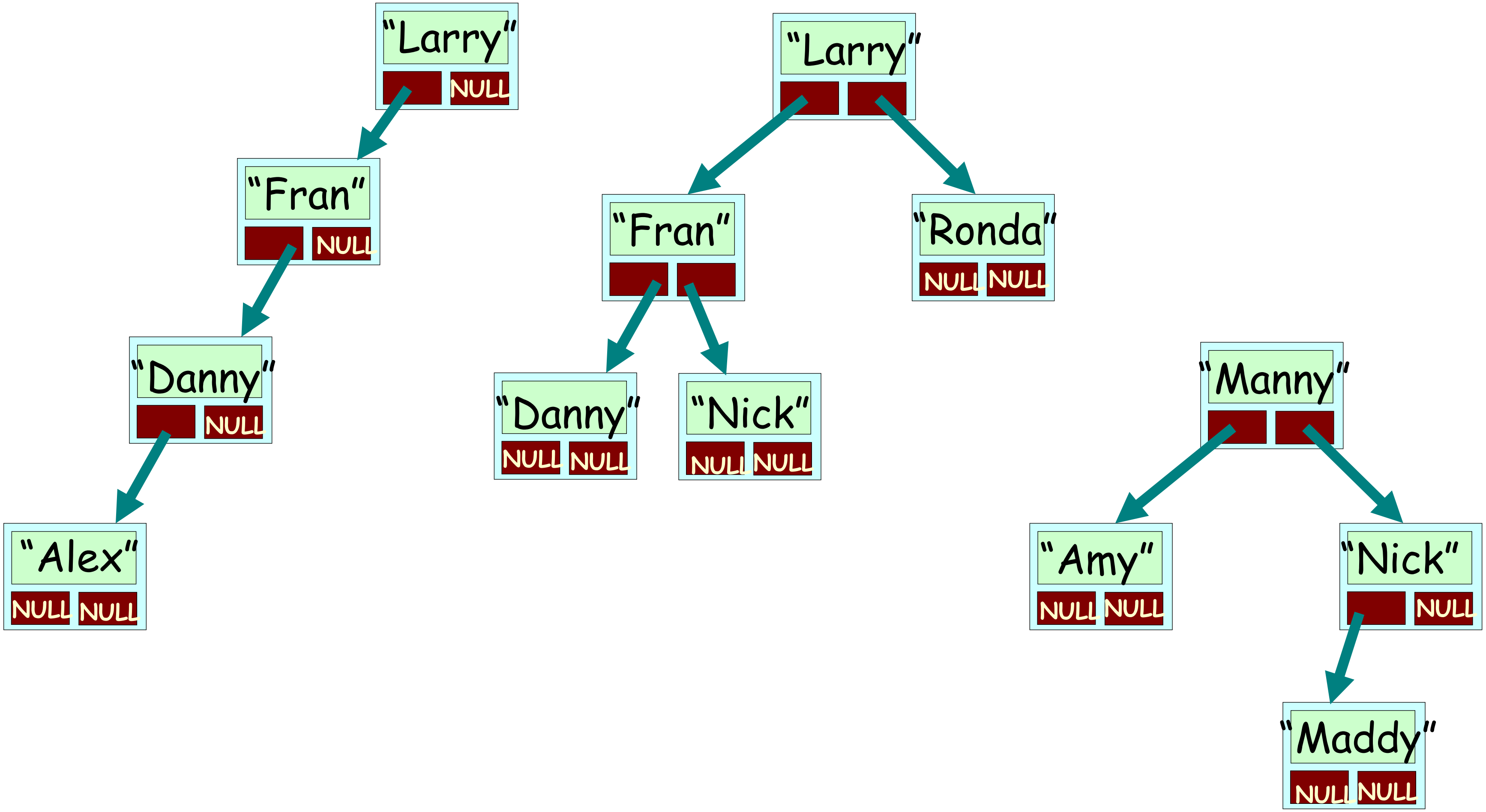
Given any node in the binary tree, all nodes in its right sub-tree must be greater than the node's value.

Let's validate that this is a valid BST...



Binary Search Trees

Question: Which of the following are valid BSTs?



Operations on a Binary Search Tree

Here's what we can do to a BST:

- Determine if the binary search tree is empty
- Search the binary search tree for a value
- Insert an item in the binary search tree
- Delete an item from the binary search tree
- Find the height of the binary search tree
- Find the number of nodes and leaves in the binary search tree
- Traverse the binary search tree
- Free the memory used by the binary search tree

Searching a BST

Input: A value V to search for
Output: TRUE if found, FALSE otherwise

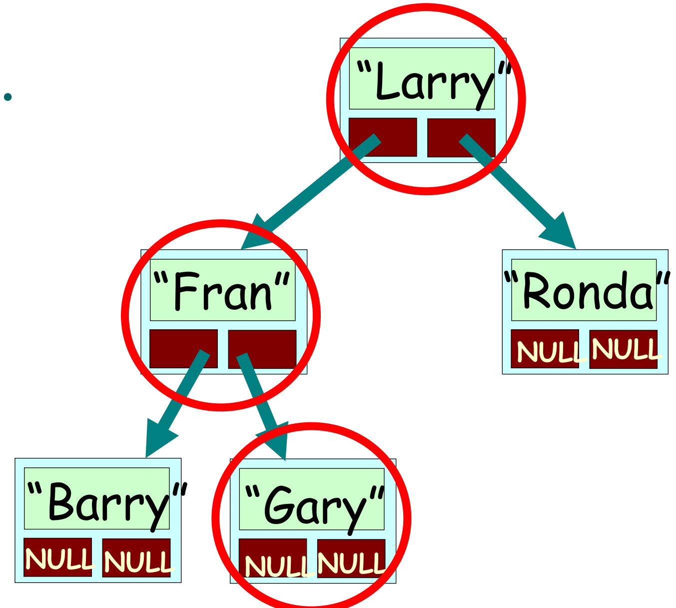
Start at the root of the tree
Keep going until we hit the NULL pointer

- If V is equal to current node's value, then found!
- If V is less than current node's value, go left
- If V is greater than current node's value, go right

If we hit a NULL pointer, not found.

Let's search for Gary.

Gary == Larry??
Gary < Larry??
Gary == Fran??
Gary < Fran??
Gary > Fran??
Gary == Gary??



Searching a BST

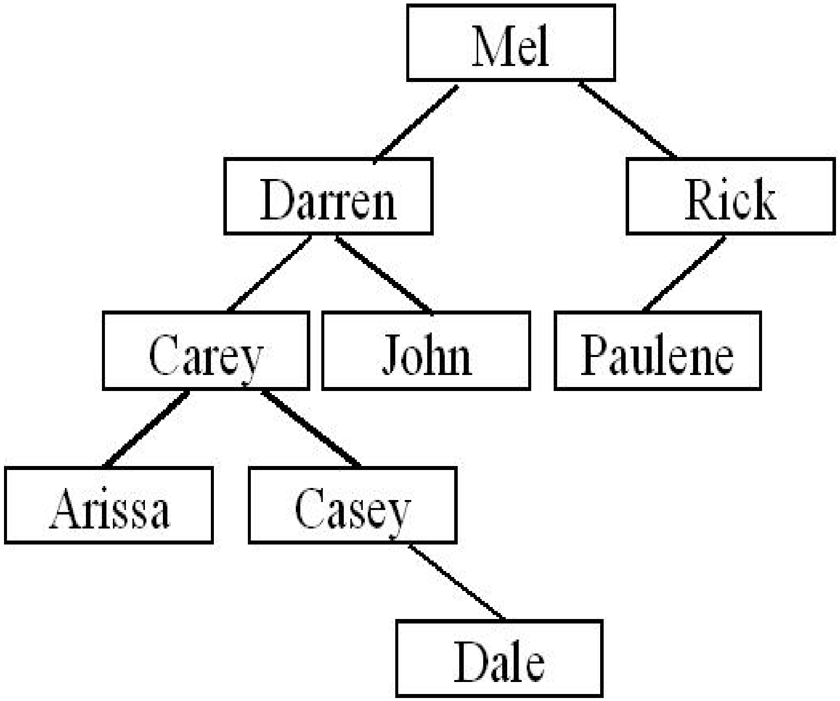
Start at the root of the tree
Keep going until we hit the NULL pointer

- If V is equal to current node's value, then found!
- If V is less than current node's value, go left
- If V is greater than current node's value, go right

If we hit a NULL pointer, not found.

Show how to search for:

1. Khang
2. Dale
3. Sam



Searching a BST

Here are two different BST search algorithms in C++, one recursive and one iterative:

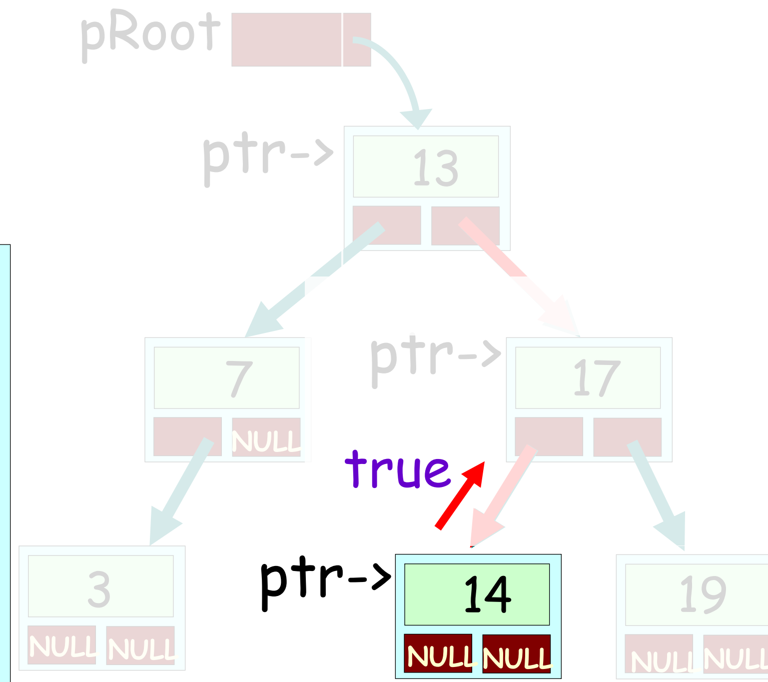
```
bool Search(int V, Node *ptr)
{
  if (ptr == NULL)
    return(false);   // nope
  else if (V == ptr->value)
    return(true);   // found!!!
  else if (V < ptr->value)
    return(Search(V,ptr->left));
  else
    return(Search(V,ptr->right));
}
```

```
bool Search(int V,Node *ptr)
{
  while (ptr != NULL)
  {
    if (V == ptr->value)
      return(true);
    else if (V < ptr->value)
      ptr = ptr->left;
    else
      ptr = ptr->right;
  }
  return(false); // nope
}
```

Let's trace through the recursive version…

Recursive BST Search

Lets search for 14.



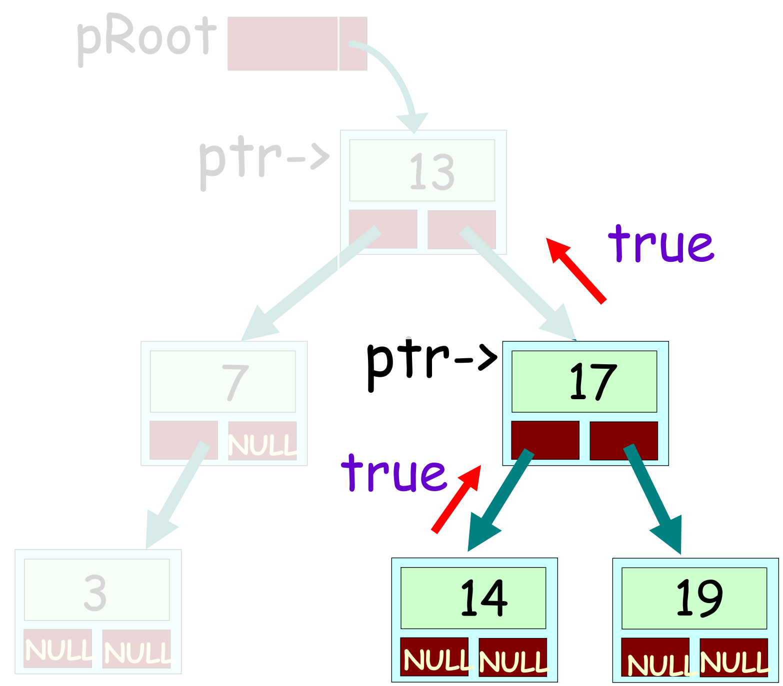

```
bool Search(int V, Node *ptr)
{
   if (ptr == NULL)
      return(false);   // nope
   else if (V == ptr->value)
      return(true);   // found!!!
   else if (V < ptr->value)
      return(Search(V,ptr->left));
   else
      return(Search(V,ptr->right));
}
```

return(Search(V,ptr->right));
}

```
void main(void)

{
   bool bFnd;
   bFnd = Search(14,pRoot);
}
```

Recursive BST Search

Lets search for 14.




```
bool Search(int V, Node *ptr)
{
   if (ptr == NULL)
      return(false);   // nope
   else if (V == ptr->value)
      return(true);   // found!!!
   else if (V < ptr->value)
      return(Search(V,ptr->left));
   else
      return(Search(V,ptr->right));
}
```

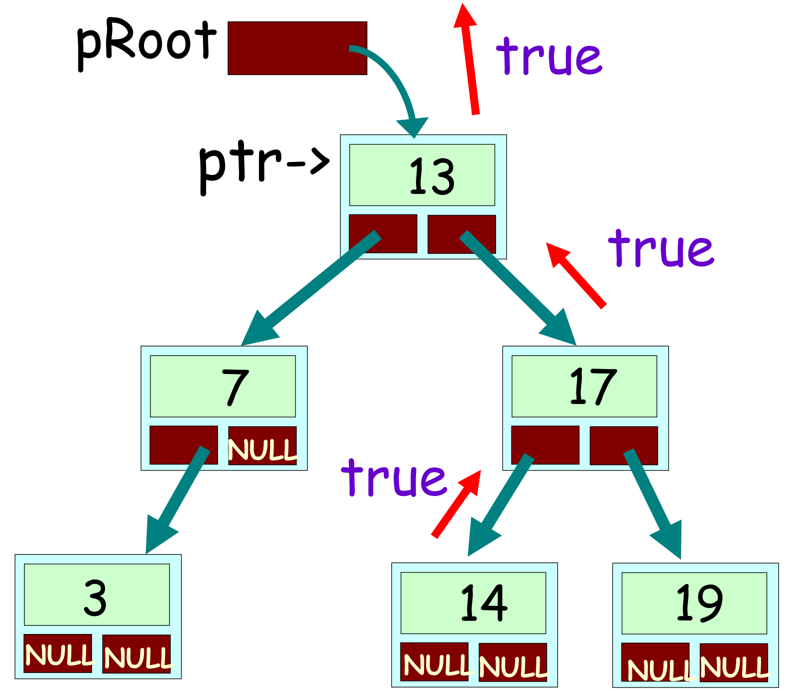
true

```
      return(Search(V,ptr->right));
}
```

```
void main(void)

{
   bool bFnd;
   bFnd = Search(14,pRoot);
}
```

Recursive BST Search

Lets search for 14.




```
bool Search(int V, Node *ptr)
{
  if (ptr == NULL)
    return(false);  // nope
  else if (V == ptr->value)
    return(true);  // found!!!
  else if (V < ptr->value)
    return(Search(V,ptr->left));
  else
    return(Search(V,ptr->right));
}
```

true

```
int main(void)

{
  bool bFnd;
  bFnd = Search(14,pRoot);
}
```

true

Big Oh of BST Search

Question:
In the average BST with N values, how many steps are required to find our value?

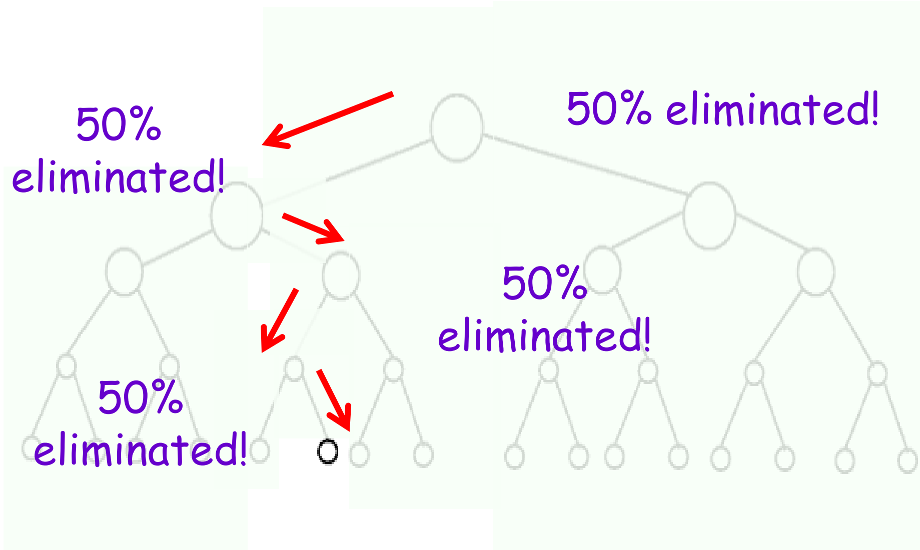
Right! $\log_2(N)$ steps

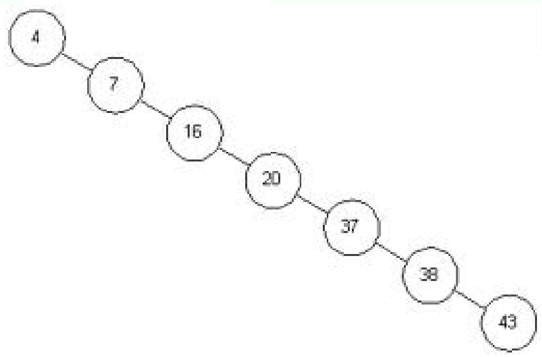
Question:
In the worst case BST with N values, how many steps are required find our value?

Right! N steps

Question:
If there are 4 billion nodes in a BST, how many steps will it take to perform a search?

Just 32!





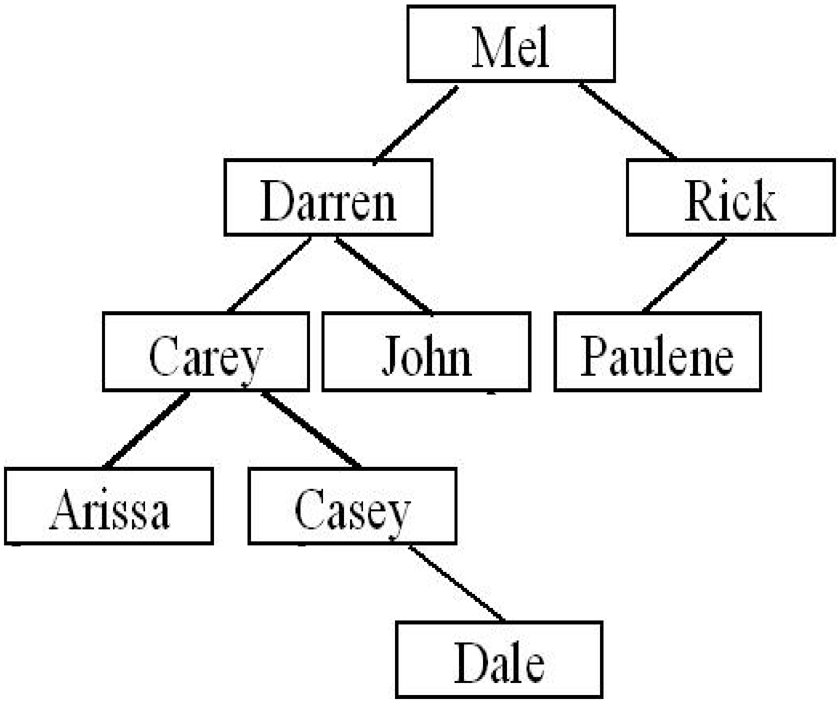
WOW!
Now that's PIMP!

Inserting A New Value Into A BST

To insert a new node in our BST, we must place the new node so that the resulting tree is still a valid BST!

Where would the following new values go?

Carly
Ken
Alice



Inserting A New Value Into A BST

Input: A value V to insert

If the tree is empty
- Allocate a new node and put V into it
- Point the root pointer to our new node. DONE!

Start at the root of the tree

While we're not done…

- If V is equal to current node's value, DONE! (nothing to do…)
- If V is less than current node's value
  - If there is a left child, then go left
  - ELSE allocate a new node and put V into it, and set current node's left pointer to new node. DONE!
- If V is greater than current node's value
  - If there is a right child, then go right
  - ELSE allocate a new node and put V into it, set current node's right pointer to new node. DONE!

```
struct Node
{

   Node(const std::string &myVal)
    {
      value = myVal;
      left = right = NULL;
    }
  std::string   value;
  Node          *left,*right;
};
```

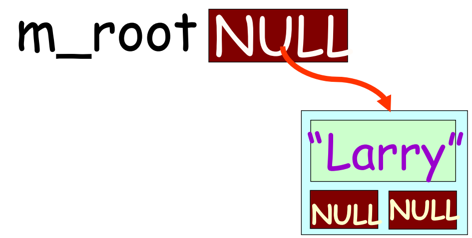
```
class BinarySearchTree
{
public:

   BinarySearchTree()
   {
     m_root = NULL;
   }

   void insert(const std::string &value)
   {
     …
   }

private:

   Node *m_root;
};
```

And our constructor initializes that root pointer to NULL when we create a new tree. (This indicates the tree is empty)

Our BST class has a single member variable – the root pointer to the tree.

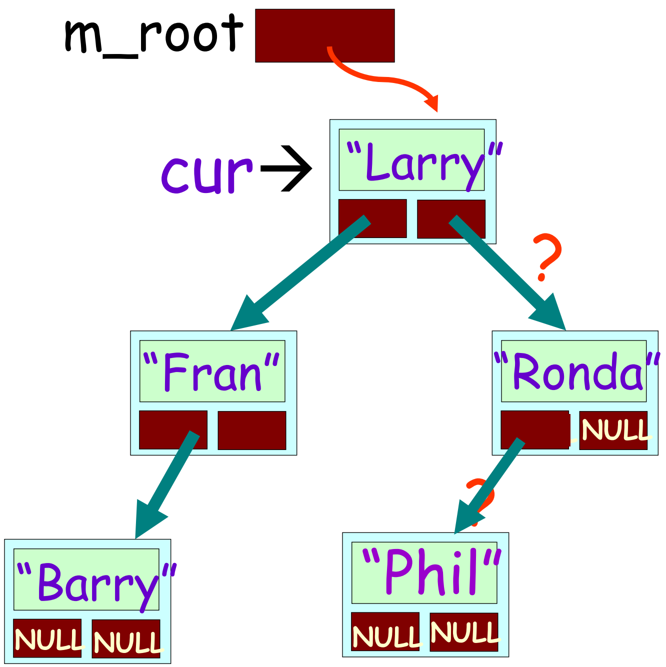
```
void insert(const std::string &value)
{
   if (m_root == NULL)
      {   m_root = new Node(value);   return; }

   Node *cur = m_root;
   for (;;)
    {
      if (value == cur->value)   return;

      if (value < cur->value)
      {
         if (cur->left != NULL)
            cur = cur->left;
         else
         {
            cur->left = new Node(value);
            return;
         }
      }
      else if (value > cur->value)
      {
         if (cur->right != NULL)
            cur = cur->right;
         else
         {
            cur->right = new Node(value);
            return;
         }
      }
    }
}
```

```
void insert(const std::string &value)
{
   if (m_root == NULL)
      {   m_root = new Node(value);    return; }

   Node *cur = m_root;
   for (;;)
    {
       if (value == cur->value)    return;

       if (value < cur->value)
       {
           if (cur->left != NULL)
               cur = cur->left;
           else
           {
               cur->left = new Node(value);
               return;
           }
       }
        else if (value > cur->value)
        {
            if (cur->right != NULL)
                cur = cur->right;
            else
            {
                cur->right = new Node(value);
                return;
            }
        }
    }
}
```




```
void main(void)
{
    BinarySearchTree bst;

    bst.insert("Larry");

    ...

    bst.insert("Phil");
}
```

```
void insert(const std::string &value)
{
   if (m_root == NULL)
      {   m_root = new Node(value);    return; }

   Node *cur = m_root;
   for (;;)
    {
       if (value == cur->value)    return;

       if (value < cur->value)
       {
          if (cur->left != NULL)
             cur = cur->left;
          else
          {
             cur->left = new Node(value);
             return;
          }
        }
        else if (value > cur->value)
        {
           if (cur->right != NULL)
              cur = cur->right;
           else
           {
              cur->right = new Node(value);
              return;
           }
        }
    }
}
```




```
void main(void)
{
   BinarySearchTree bst;
   bst.insert("Larry");
   ...
   bst.insert("Phil");
}
```

Inserting A New Value Into A BST

As with BST Search, there is a recursive version of the Insertion algorithm too. Be familiar with it!

Question:

Given a random array of numbers if you insert them one at a time into a BST, what will the BST look like?

Question:

Given a ordered array of numbers if you insert them one at a time into a BST, what will the BST look like?

Big Oh of BST Insertion

So, what's the big-oh of BST Insertion?

Right! It's also $O(\log_2 n)$

Why? Because we have to first use a binary search to find where to insert our node and binary search is $O(\log_2 n)$.

Once we've found the right spot, we can insert our new node in $O(1)$ time.

Groovy Baby!

Finding Min & Max of a BST

How do we find the minimum and maximum values in a BST?

```
int GetMin(node *pRoot)
{
  if (pRoot == NULL)
    return(-1);  // empty

  while (pRoot->left != NULL)
     pRoot = pRoot->left;

  return(pRoot->value);
}
```

```
int GetMax(node *pRoot)
{
  if (pRoot == NULL)
    return(-1);  // empty

  while (pRoot->right != NULL)
     pRoot = pRoot->right;

  return(pRoot->value);
}
```

Question: What's the big-oh to find the minimum or maximum element?

Finding Min & Max of a BST

And here are recursive versions for you…

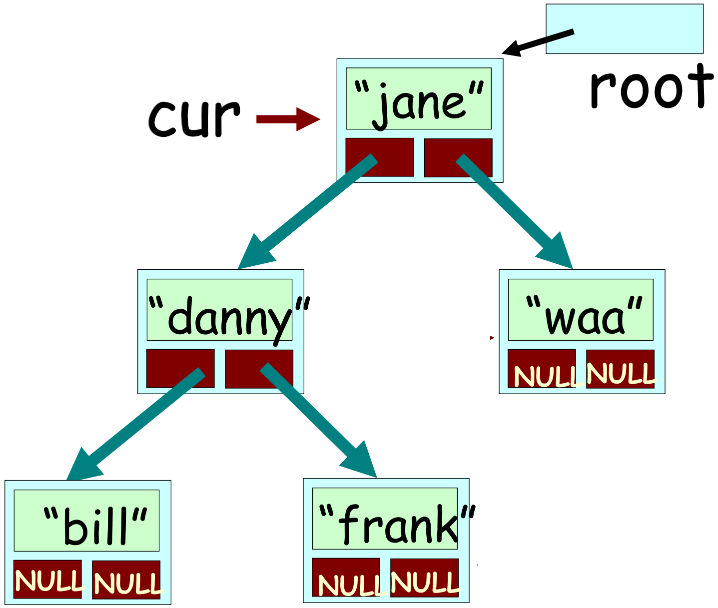
```
int GetMin(node *pRoot)
{
   if (pRoot == NULL)
      return(-1);   // empty

   if (pRoot->left == NULL)
      return(pRoot->value);

   return(GetMin(pRoot->left));
}
```

```
int GetMax(node *pRoot)
{
   if (pRoot == NULL)
      return(-1);   // empty

   if (pRoot->right == NULL)
      return(pRoot->value);

   return(GetMax(pRoot->right));
}
```

Hopefully you're getting the idea that most tree functions can be done recursively…

Printing a BST In Alphabetical Order

Can anyone guess what algorithm we use to print out a BST in alphabetical order?



Big-oh Alert!

So what's the big-Oh of printing all the items in the tree?

Right! O(n) since we have to visit and print all n items.

Output:

bill
danny
frank
jane
waa

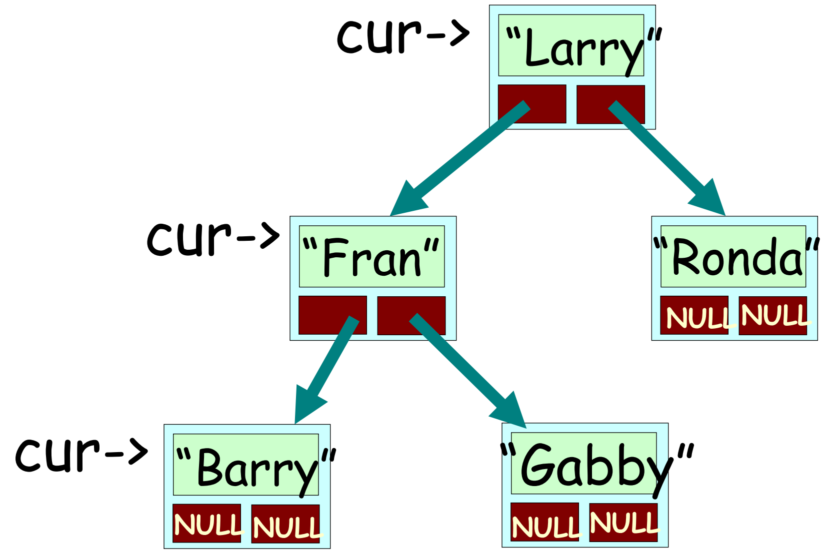
Freeing The Whole Tree

When we are done with our BST, we have to free every node in the tree, one at a time.

Question: Can anyone think of an algorithm for this?

```
void FreeTree(Node *cur)
{
   if (cur == NULL)              // if empty, return...
       return;

   FreeTree(cur->left);      // Delete nodes in left sub-tree.

   FreeTree (cur-> right);  // Delete nodes in left sub-tree.

   delete cur;                      // Free the current node
}
```

Freeing The Whole Tree



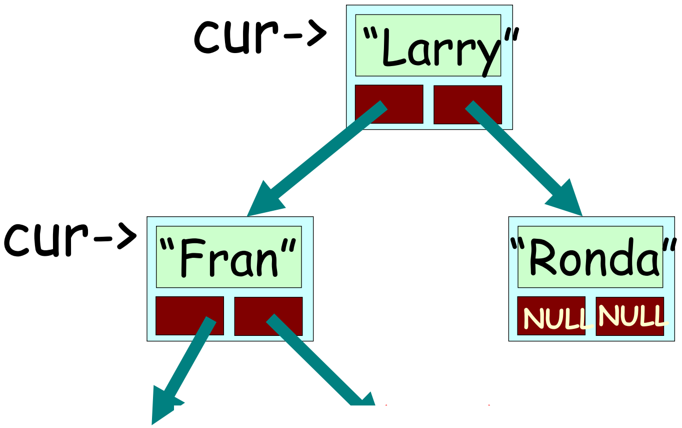
cur = NULL

```
void FreeTree(Node *cur)
{
    if (cur == NULL)
        return;

    FreeTree(cur->left);

    FreeTree (cur-> right);

   delete cur;
}
```

Freeing The Whole Tree

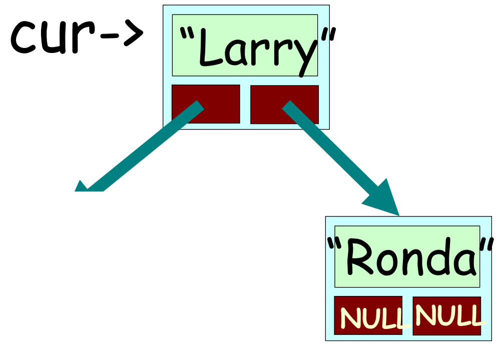

```
void FreeTree(Node *cur)
{
    if (cur == NULL)
        return;

    FreeTree(cur->left);

    FreeTree (cur-> right);

   delete cur;
}
```

Freeing The Whole Tree




```
void FreeTree(Node *cur)
{
    if (cur == NULL)
        return;

    FreeTree(cur->left);

    FreeTree (cur-> right);

    delete cur;
}
```

Freeing The Whole Tree




```
void FreeTree(Node *cur)
{
   if (cur == NULL)
       return;

   FreeTree(cur->left);

   FreeTree (cur-> right);


  delete cur;
}
```

Big-oh Alert!

So what's the big-Oh of freeing all the items in the tree?

It's still O(n) since we have to visit all n items.