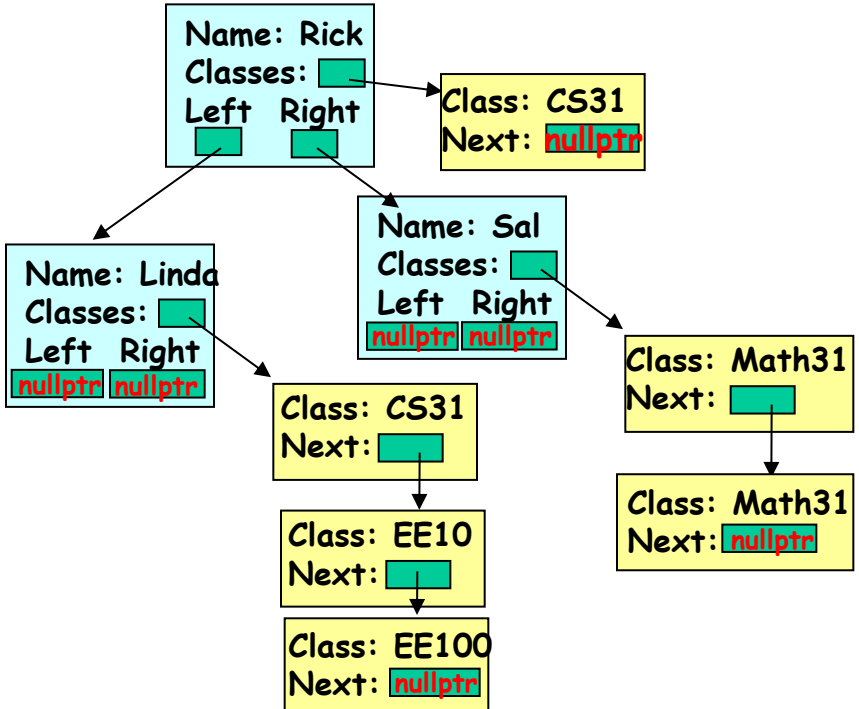
# Lecture #14

- The Modulus Operator
- Hash Tables
  - Closed hash tables
    - Inserting, Searching, Deleting
  - Open hash tables
  - Hash table efficiency and "load factor"
  - Hashing non-numeric values
  - Binary search trees vs. hash tables
- Tables

# Big-OH Craziness

Consider a *binary search tree* that holds N student records, all indexed by their name.

Each student record contains a linked-list of the L classes that they have taken while at UCLA.




What is the big-oh to determine if a student has taken a class?

```
bool HasTakenClass(
        BTree &b,
        string &name,
        string &class
        )
```

# The Modulus Operator

In C++, the % operator is used to divide two numbers and obtain the remainder.

For example, if we compute:

int x = 1234 % 100;

the value of x will be 34.

```
   12 R 34
  ---------
100 ) 1234
    100
  -------
     234
     200
  -------
      34
```

Now, as it turns out, the modulo operator has an interesting property!

Let's see if you can figure out what it is…

# The Modulus O

Let's modulus-divide a b
by 5 and see what th

Let's just store that interesting fact away in your brain for later…

0 % 5 = 0
1 % 5 = 1
2 % 5 = 2
3 % 5 = 3
4 % 5 = 4
5 % 5 = 0
6 % 5 = 1
7 % 5 = 2
8 % 5 = 3
9 % 5 = 4
10 % 5 = 0
11 % 5 = 1

What do you notice?

When we divide numbers by 5, all of the remainders are less than 5 (between 0-4)!

Let's try again with 3 for fun!

When we divide numbers by 3, all of the remainders are less than 3 (between 0-2)!

And as you'd guess, if you divided a bunch of numbers by 100,000, the remainders would all be less than 100,000 (between 0-99,999)!

Rule: When you divide by a given value N, all of your remainders are guaranteed to be between 0 and N-1!

# The "Hash Table"

OK… So far, what's the most efficient ADT we know of to insert and search for data?

Right!

Can we do any better? If so, how much better?

Challenge:

Build an ADT that holds a bunch of 9-digit student ID#s such that the user can add new ID#s or determine if the ADT holds an existing ID# in just 1 step – not $O(N)$ or $O(\log_2 N)$ but $O(1)$.

# The (Almost) Hash Table

How can we create an ADT where we can insert the 9-digit student ID#s for all 50,000 UCLA students…

and then find if our ADT holds a given ID# in just one algorithmic step?!?!?

That can't be done… can it?

It can, and let's see how!

Let's use a really, really large array to hold our #s.

# The (Almost) Hash Table

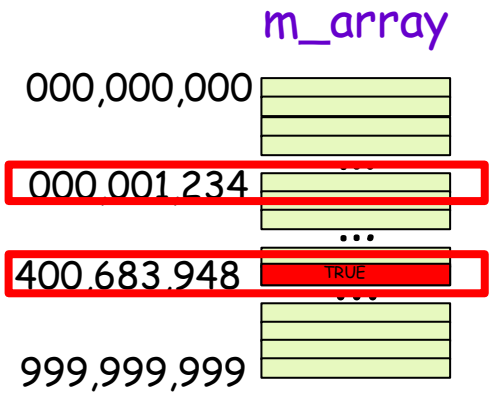
```
class AlmostHashTable
{
public:
   void addItem(int n)
   {
      m_array[n] = true;
   }
   bool holdsItem(int q)
   {
      return m_array[q] == true;
   }
private:
   bool m_array[1000000000]; // big!
};

int main()
{
    AlmostHashTable x;

    x.addItem(400683948);
    if (x.holdsItem(1234) != true)
       cout<< "Couldn't find it!";
}
```

Idea:

Let's create an array with 1 billion slots - one slot for each valid ID#.

To add a new ID# with a value of N, we'll simply set array[N] to true.

To determine if our array holds a previously-added value Q, simply check if array[Q] is true.



# The (Almost) Hash Table

OK – so now we know how to build an O(1) search!
But what's the problem with our ADT?

It's really, really inefficient:
Our array has 1 billion slots
yet there are only 50,000 UCLA student IDs
we could possibly add to it,
so we're wasting 999,950,000 of the slots…

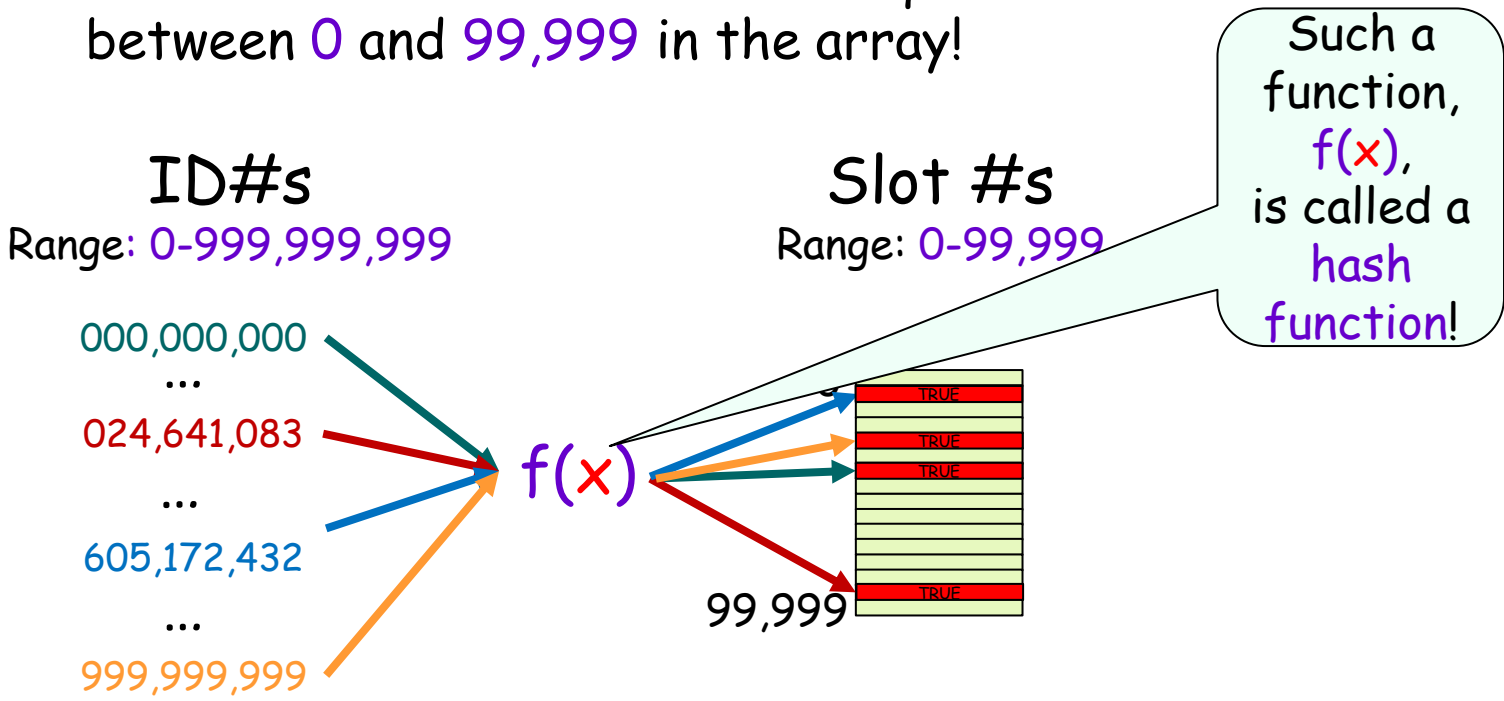
It would be great if we could use the same algorithm but with a smaller array, say one with 100,000 slots instead of 1 billion!

# The (Almost) Hash Table

Lets say we want to keep track of our 50,000 ID#s in an array with just 100,000 slots.

If we just try to use our 9-digit number to index the array, there won't be room!

What we need is some cool mathematical function that takes in a 9-digit ID# and somehow converts it to a unique slot number between 0 and 99,999 in the array!



# The (Almost) Hash Table

```
class AlmostHashTable
{
public:
   void addItem(int n)
   {
       int slot = hashFunc(n);
       m_array[slot] = true;
   }
   bool containsItem(int q)
   {
       int slot = hashFunc(q);
       return m_array[slot] == true;
   }
private:
   int hashFunc(int idNum)
     { /* ??? */ }

   bool m_array[100000]; // not so big!
};
```

This converts our 9-digit ID# into a slot # between 0 and 99,999.

Assuming we can come up with such a hash function…

Then we track our ID# in that slot by setting it to true.

By the way, the official CS lingo for a "slot" in the array is a "bucket." So that's what we'll call our slots from now on! ☺

We can use a (small) 100,000 element array to hold our data…

And to add a new item in one step, we can do this…

And to search in one step…

# The Hash Function

How can we write a hashFunc that converts our large ID# into a bucket # that falls within our 100,000 element array?

```
int hashFunc(int idNum)
{
  const int ARRAY_SIZE = 100000;

  int bucket = idNum % ARRAY_SIZE;
  return bucket;
}
```

This line takes an input value idNum and returns an output value between 0 and ARRAY_SIZE - 1. (0 to 99,999)

RIGHT! The C++ % operator (aka the modulus division operator) does exactly what we want!!!

So now for each input ID# we can compute a corresponding value between 0-99,999!

And this corresponding value can be used to pick a bucket in our 100,000 element array!

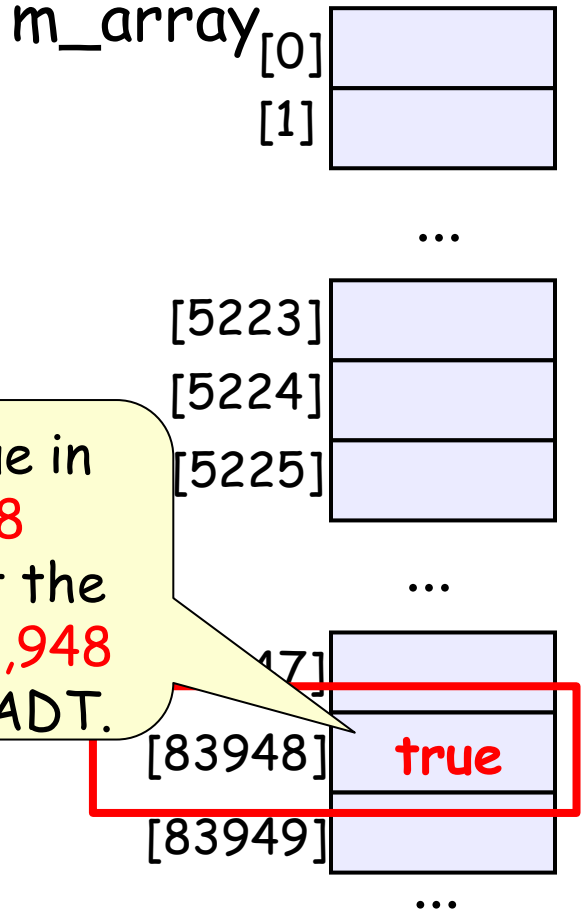
# The (Almost) Hash Table

Let's see how it works.

```
class AlmostHashTable2
{
public:
   void addItem(int n)
   {
      int bucket = hashFunc(n);
      m_array[bucket] = true;
   }

private:
   int hashFunc(int idNum)
   {
      return idNum % 100000;
   }
   bool m_array[100000]; // no s
};
```

400,683,948 % 100,000
= 83,948

```
int main()
{
    AlmostHashTable2 x;
    x.addItem(400683948);
    x.addItem(111105224);
    x.addItem(222205224);
}
```




The true value in slot 83,948 indicates that the value 400,683,948 is held in our ADT.

# The (Almost) Hash Table

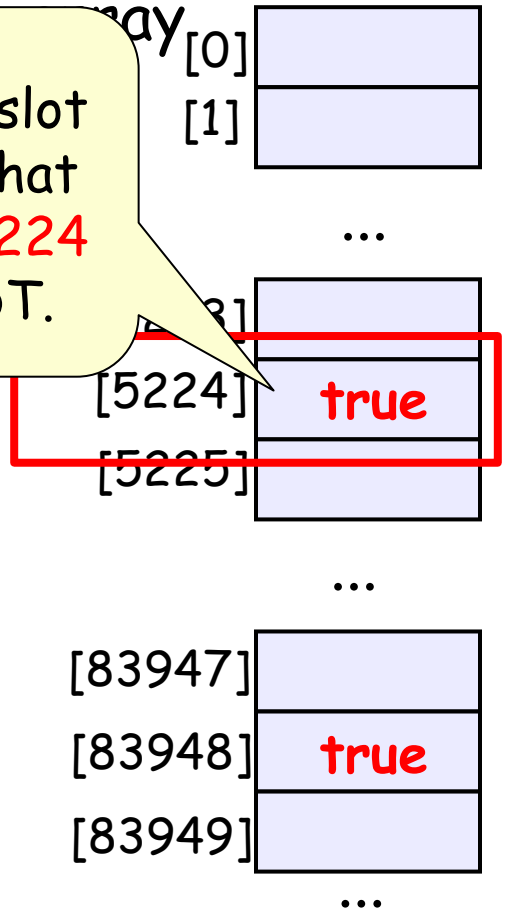
```
class AlmostHashTable2
{
public:
   void addItem(int
   {
       int bucket = hashFunc(n);
       m_array[bucket] = true;
   }

private:
   int hashFunc(int idNum)
   {
       return idNum % 100000;
   }
   bool m_array[100000]; // no so big!
};
```

```
int main()
{
    AlmostHashTable2 x;
    x.addItem(400683948);
    x.addItem(111105224);
    x.addItem(222205224);
}
```

111,105,224 % 100,000 = 5,224

Let's see how it works.

The true value in slot 5,224 indicates that the value 111,105,224 is held in our ADT.



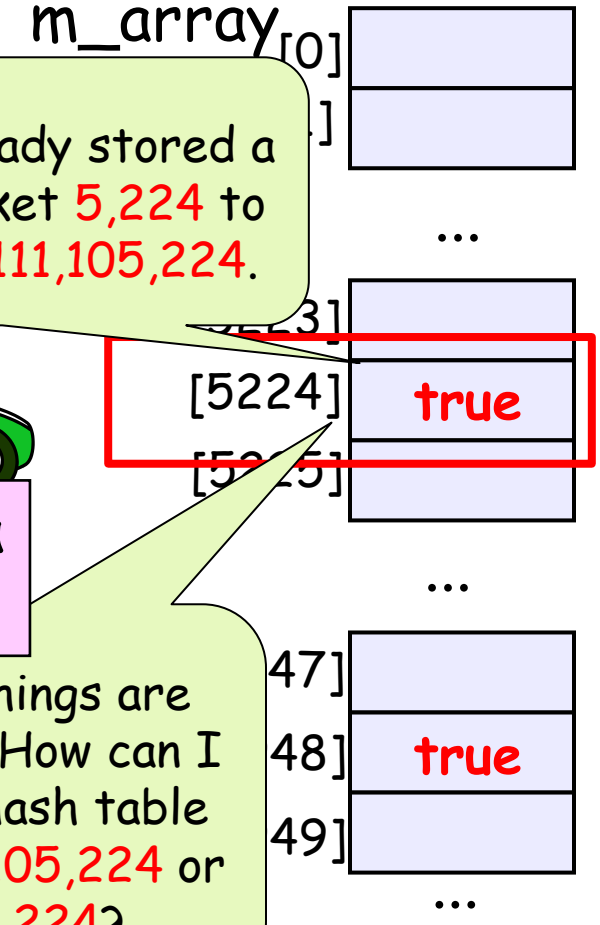
# sh Table

But our hash function wants to also put a true value in slot 5,224 to represent 222,205,224!

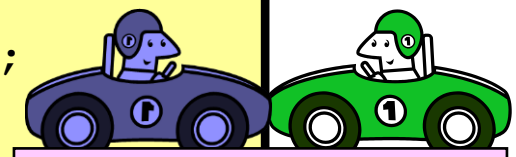
```
class AlmostHashTable2
{
public:
   void addItem(int n)
   {
      int bucket = hashFunc(n);
      m_array[bucket] = true;
   }

private:
   int hashFunc(int idNum)
   {
      return idNum % 100000;
   }

   bool m_array[100000]; // r
};
```

222,205,224 % 100,000 = 5,224

Ok, let's add the last ID# to our table…




But wait! We already stored a true value in bucket 5,224 to represent value 111,105,224.



This is called a collision!

But now things are ambiguous! How can I tell if my hash table holds 222,205,224 or 111,105,224?

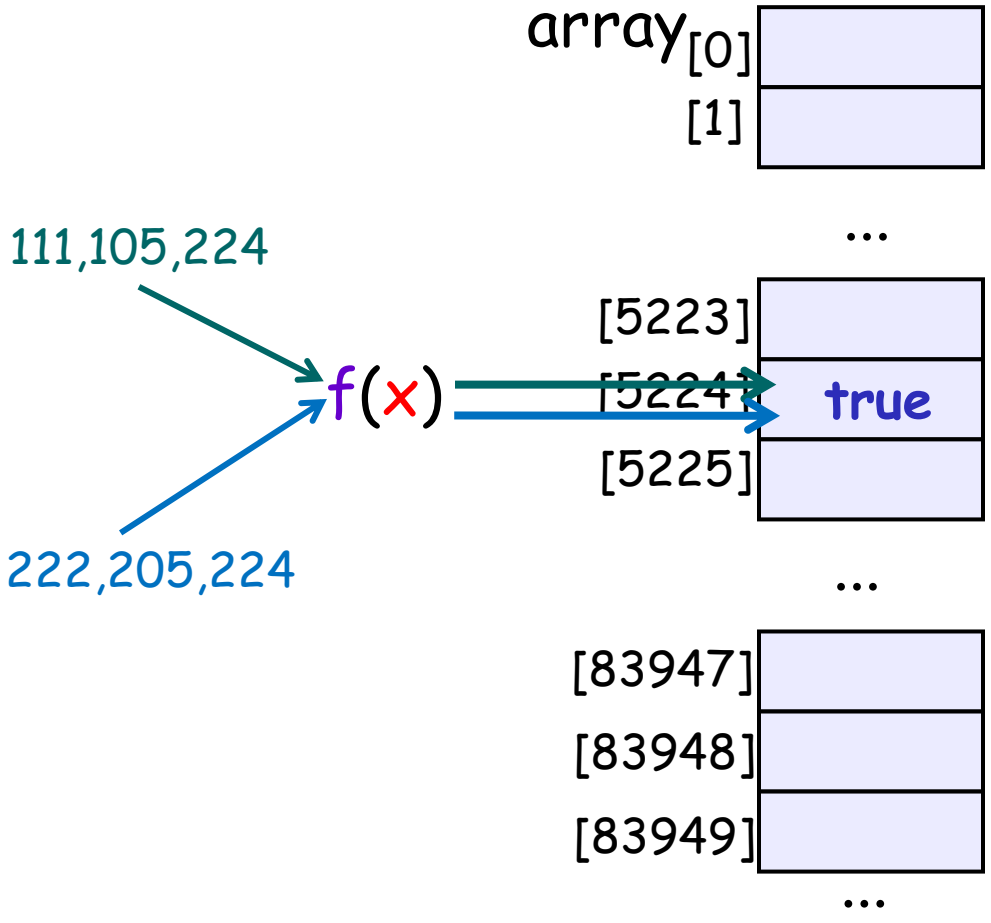
```
int main()
{
    AlmostHashTable2 x;
    x.addItem(400683948);
    x.addItem(111105224);
    x.addItem(222205224);
}
```

# The (Almost) Hash Table: A problem!

A collision is a condition where two or more values both "hash" to the same bucket in the array.

This causes ambiguity, and we can't tell what value was actually stored in the array!

Let's see how to fix this problem!



# REAL Hash Tables

There are many schemes for dealing with collisions, and today we'll learn two of the most popular…

The Closed Hash Table with "Linear Probing"

The "Open Hash Table"

# Closed Hash Table with Linear Probing: Insertion

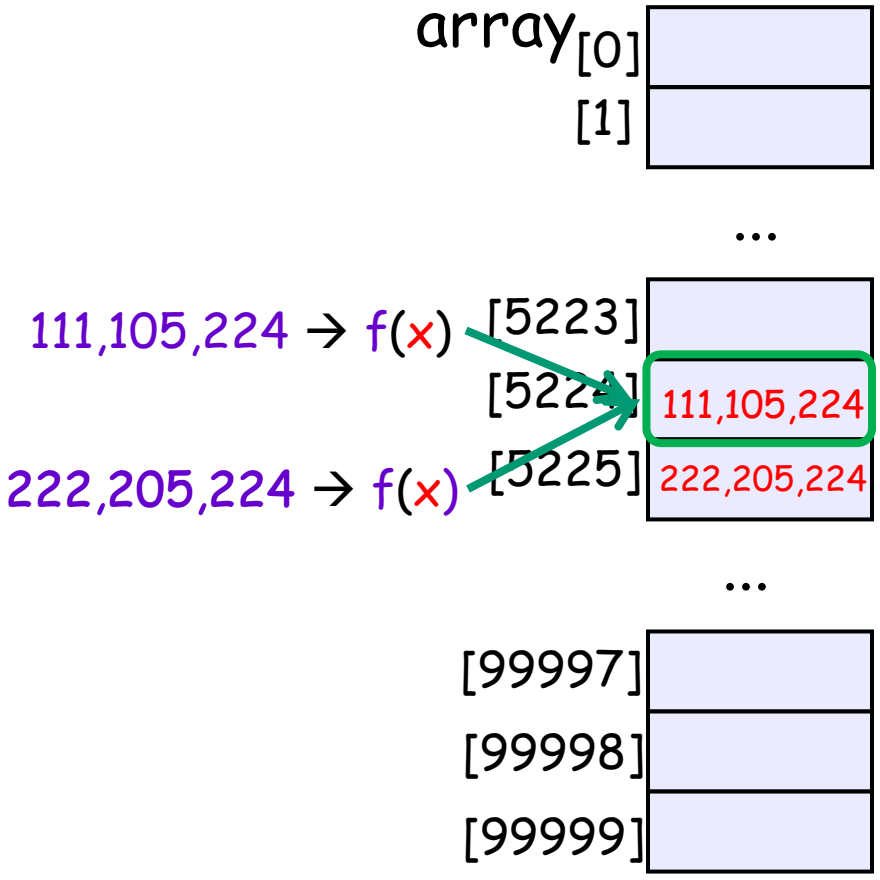
## Linear Probing Insertion:

As before, we use our hash function to locate the right bucket in our array.

If the target bucket is empty, we can store our value there.

However, instead of storing true in the bucket, we store our full original value – this prevents ambiguity!

If the bucket is occupied, scan down from that bucket until we hit the first open bucket. Put the new value there.



# Closed Hash Table with Linear Probing: Insertion

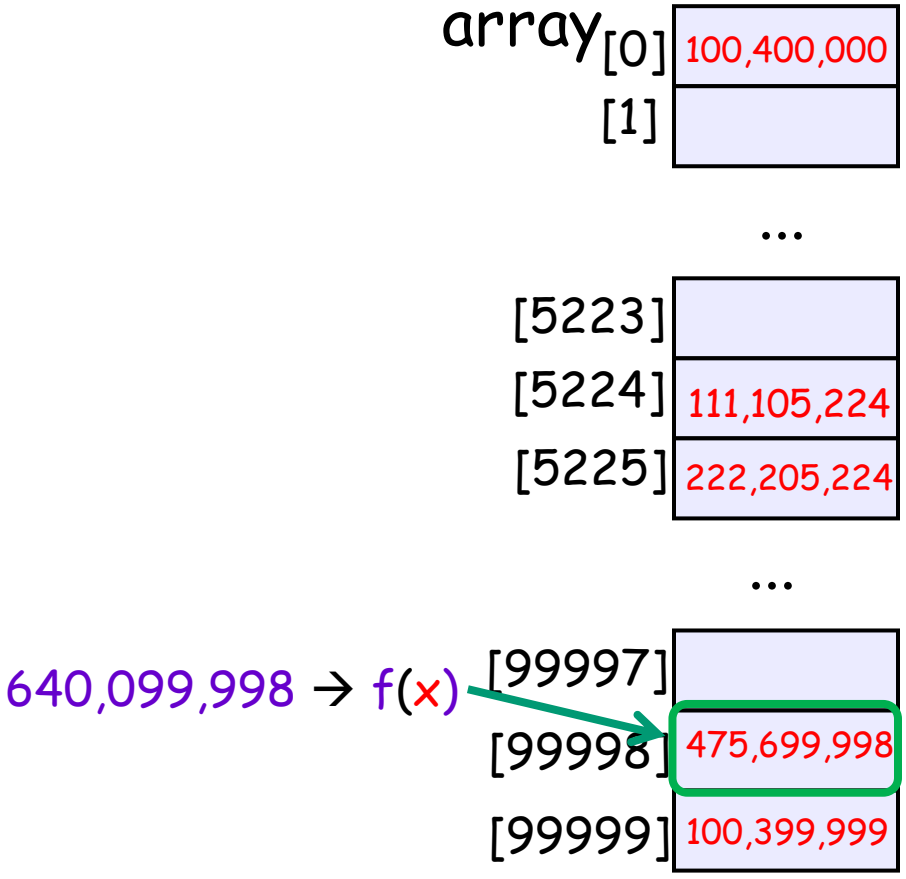
## Linear Probing Insertion:

Sometimes, you'll need to insert an item near the end of the table…

For instance, let's say we want to insert a new value of 640,099,998 into our hash table.

If you run into a collision on the last bucket, and go past the end…

You simply wrap back around the top!



# Closed Hash Table with Linear Probing: Searching

## Linear Probing Searching:

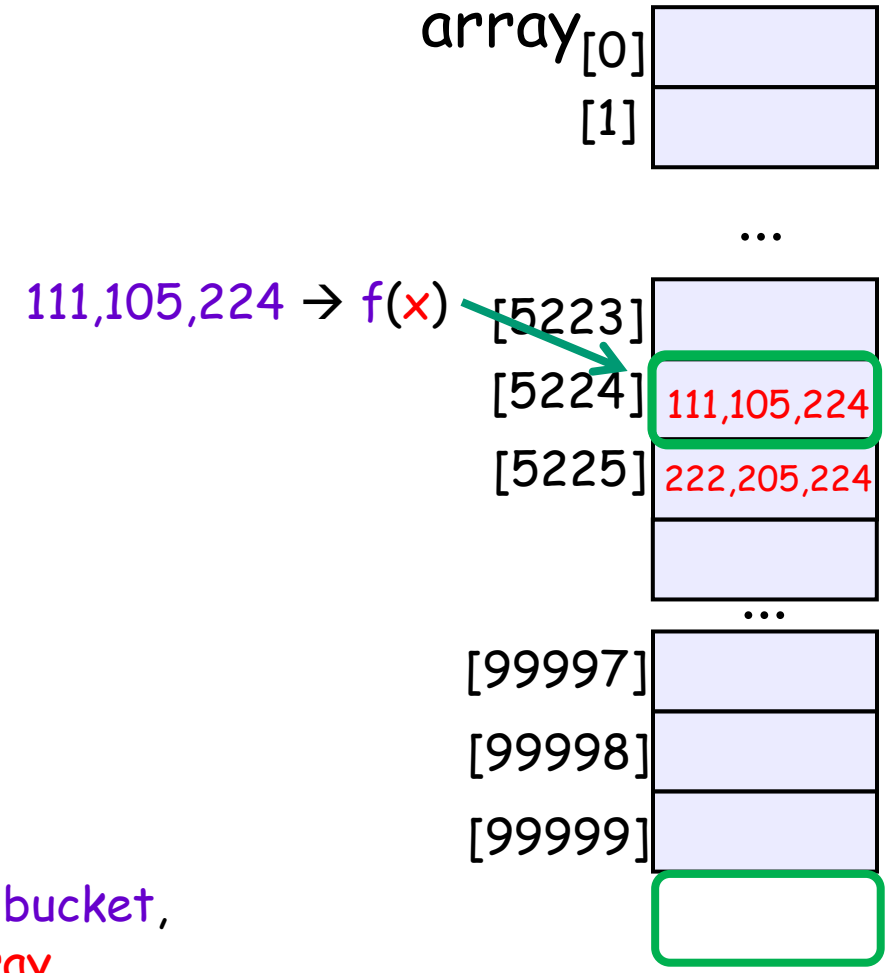
To search our hash table, we use a similar approach.

We compute a target bucket number with our hash function.

We then look in that bucket for our value. If we find it, great!

If we don't find our value, we probe linearly down the array until we either find our value or hit an empty bucket.

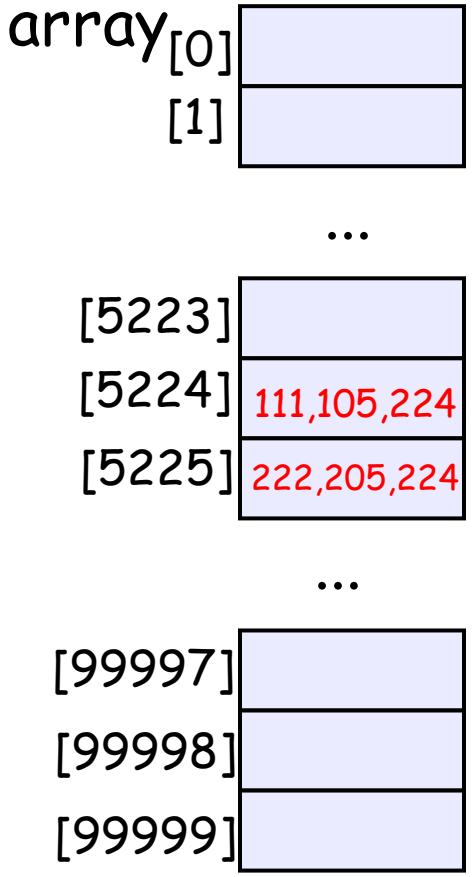
If while probing, you run into an empty bucket, it means: your value isn't in the array.



# Closed Hash Table with Linear Probing

This approach addresses collisions by putting each value as close as possible to its intended bucket.

Since we store every original value (e.g., 111,105,224) in the array, there is no chance of ambiguity.



# Closed Hash Table with Linear Probing

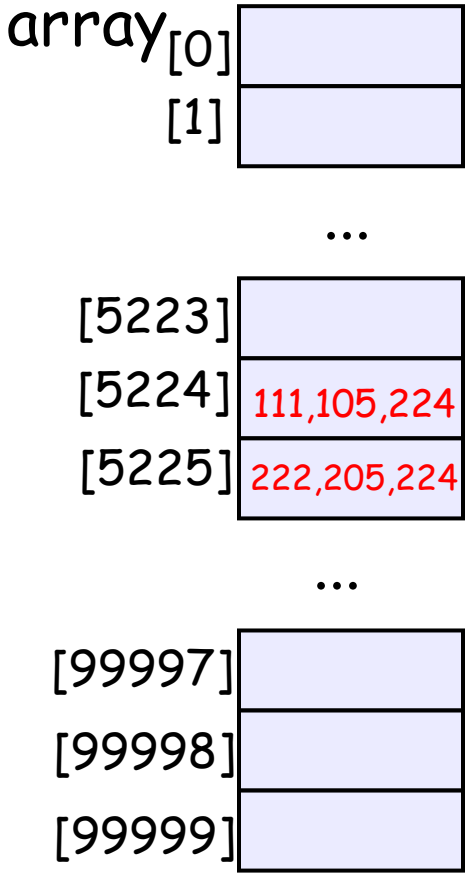
So why do we call this a "Closed" hash table???

Since our data is stored in a fixed-size array, there are a fixed (closed) number of buckets for us to put values.

Once we run out of empty buckets, we can't add new values...

Linked lists and binary search trees don't have this problem!

Ok, let's see the C++ code now!



# Linear Probing Hash Table: The Details

In a Linear Probing Hash Table, each bucket in the array is just a C++ struct.

Each bucket holds two items:

1. A variable to hold your value (e.g., an int for an ID#)
2. A "used" field that indicates if this bucket in the hash table has been filled or not.

```
struct BUCKET
{
   // a bucket stores a value (e.g. an ID#)
   int idNum;

   bool           used;  // is bucket in-use?
};
```

If this field is false, it means that this Bucket in the array is empty. If the field is true, then it means this Bucket is already filled with valid data.

```
#define NUM_BUCK    10

class HashTable
{
public:
   void insert(int idNum)
   {
      int bucket = hashFunc(idNum);

      for (int tries=0;tries<NUM_BUCK;tries++)
      {
         if (m_buckets[bucket].used == false)
         {
            m_buckets[bucket].idNum = idNum;
            m_buckets[bucket].used = true;
            return;
         }
         bucket = (bucket + 1) % NUM_BUCK;
      }
      // no room left in hash table!!!
   }

private:
   int hashFunc(int idNum) const
        {  return idNum % NUM_BUCK;  }

   BUCKET m_buckets[NUM_BUCK];
};
```

First we compute the starting bucket number.

Since our array has 10 slots, we will loop up to 10 times looking for an empty space. If we don't find an empty space after 10 tries, our table is full!

We'll store our new item in the first unused bucket that we find, starting with the bucket selected by our hash function.

If the current bucket is already occupied by an item, advance to the next bucket (wrapping around from slot 9 back to slot 0 when we hit the end).

Here's our hash function. As before, we compute our bucket number by dividing the ID number by the total # of buckets and then taking the remainder (%).

Our hash table has 10 slots, aka "buckets."

```
#define NUM_BUCK   10

class HashTable
{
public:
   void insert(int idNum)
   {
      int bucket = hashFunc(idNum);

      for (int tries=0;tries<NUM_BUCK;tries++)
      {
         if (m_buckets[bucket].used == false)
         {
            m_buckets[bucket].idNum = idNum;
            m_buckets[bucket].used = true;
            return;
         }
         bucket = (bucket + 1) % NUM_BUCK;
      }
      // no room left in hash table!!!
   }

private:
    int hashFunc(int idNum) const
        {  return idNum % NUM_BUCK;  }

    BUCKET m_buckets[NUM_BUCK];
};
```

29

bucket 9

# Linear Probing: Inserting

| | | | | |
|---|---|---|---|---|
| 0 | idNum: | | used: | f |
| 1 | idNum: | | used: | f |
| 2 | idNum: | | used: | f |
| 3 | idNum: | | used: | f |
| 4 | idNum: | | used: | f |
| 5 | idNum: | | used: | f |
| 6 | idNum: | | used: | f |
| 7 | idNum: | | used: | f |
| 8 | idNum: | | used: | f |
| 9 | idNum: | 29 | used: | f |

```
main()
{
   HashTable ht;

   ht.insert(29);
   ht.insert(65);
   ht.insert(79);

}
```

```
#define NUM_BUCK   10

class HashTable
{
public:
   void insert(int idNum)
   {
      int bucket = hashFunc(idNum);

      for (int tries=0;tries<NUM_BUCK;tries++)
      {
         if (m_buckets[bucket].used == false)
         {
            m_buckets[bucket].idNum = idNum;
            m_buckets[bucket].used = true;
            return;
         }
         bucket = (bucket + 1) % NUM_BUCK;
      }
      // no room left in hash table!!!
   }

private:
    int hashFunc(int idNum) const
        {  return idNum % NUM_BUCK;  }

    BUCKET m_buckets[NUM_BUCK];
};
```

65

bucket 5

# Linear Probing: Inserting

| | | | | |
|---|---|---|---|---|
| 0 | idNum: | | used: | f |
| 1 | idNum: | | used: | f |
| 2 | idNum: | | used: | f |
| 3 | idNum: | | used: | f |
| 4 | idNum: | | used: | f |
| 5 | idNum: | 65 | used: | f |
| 6 | idNum: | | used: | f |
| 7 | idNum: | | used: | f |
| 8 | idNum: | | used: | f |
| 9 | idNum: | 29 | used: | T |

```
main()
{
   HashTable ht;

   ht.insert(29);
   ht.insert(65);
   ht.insert(79);

}
```

```
#define NUM_BUCK   10

class HashTable
{
public:
   void insert(int idNum)
   {
      int bucket = hashFunc(idNum);

      for (int tries=0;tries<NUM_BUCK;tries++)
      {
         if (m_buckets[bucket].used == false)
         {
            m_buckets[bucket].idNum = idNum;
            m_buckets[bucket].used = true;
            return;
         }
         bucket = (bucket + 1) % NUM_BUCK;
      }
      // no room left in hash table!!!
   }

private:
    int hashFunc(int idNum) const
        {  return idNum % NUM_BUCK;  }

    BUCKET m_buckets[NUM_BUCK];
};
```

79

bucket 9

# Linear Probing: Inserting

| | | | | |
|---|---|---|---|---|
| 0 | idNum: | 79 | used: | f |
| 1 | idNum: | | used: | f |
| 2 | idNum: | | used: | f |
| 3 | idNum: | | used: | f |
| 4 | idNum: | | used: | f |
| 5 | idNum: | 65 | used: | T |
| 6 | idNum: | | used: | f |
| 7 | idNum: | | used: | f |
| 8 | idNum: | | used: | f |
| 9 | idNum: | 29 | used: | T |

```
main()
{
   HashTable ht;

   ht.insert(29);
   ht.insert(65);
   ht.insert(79);

}
```

```
#define NUM_BUCK  10

class HashTable
{
public:

  bool search(int idNum)
  {
    int bucket = hashFunc(idNum);

    for (int tries=0;tries<NUM_BUCK;tries++)
    {
      if (m_buckets[bucket].used == false)
        return false;
      if (m_buckets[bucket].idNum == idNum)
        return true;

      bucket = (bucket + 1) % NUM_BUCK;
    }
    return false;// not in the hash table
  }

private:
    int hashFunc(int idNum) const
        {  return idNum % NUM_BUCK;  }

    BUCKET m_buckets[NUM_BUCK];
};
```

Compute the starting bucket where we expect to find our item.

Since we may have collisions, in the worst case, we may need to check the entire table! (10 slots)

If we reach an empty bucket (and haven't yet found our item) then we know our item is not in the table!

Otherwise, the bucket is in-use. If it also holds our ID# then we've found our item and we're done.

If we didn't find our item, advance to the next bucket in search of it. Wrap around when we reach the end of the array.

If we went through every bucket and didn't find our item, then it's not in the hash table! Tell the user.

```
#define NUM_BUCK     10

class HashTable
{
public:
  bool search(int idNum)
  {
     int bucket = hashFunc(idNum);

     for (int tries=0;tries<NUM_BUCK;tries++)
     {
       if (m_buckets[bucket].used == false)
         return false;
       if (m_buckets[bucket].idNum == idNum)
         return true;

       bucket = (bucket + 1) % NUM_BUCK;
     }
     return false;// not in the hash table
  }

private:
    int hashFunc(int idNum) const
        {  return idNum % NUM_BUCK;  }

    BUCKET m_buckets[NUM_BUCK];
};
```

29

bucket 9

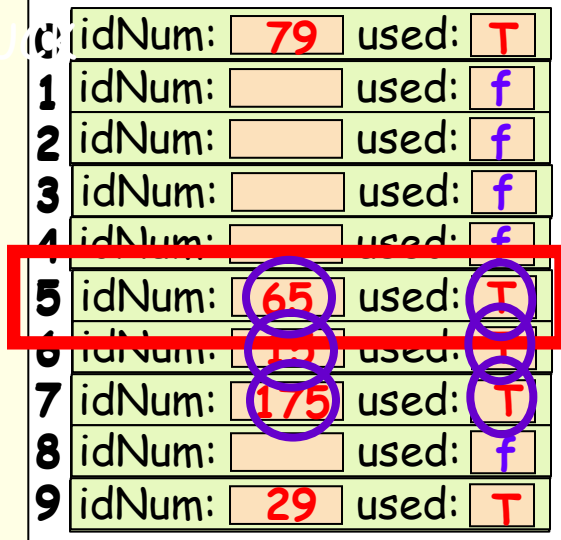
| | | |
|---|---|---|
| 0 | idNum: 79 | used: T |
| 1 | idNum: | used: f |
| 2 | idNum: | used: f |
| 3 | idNum: | used: f |
| 4 | idNum: | used: f |
| 5 | idNum: 65 | used: T |
| 6 | idNum: 15 | used: T |
| 7 | idNum: 175 | used: T |
| 9 | idNum: 29 | used: T |

```
main()
{
   HashTable ht;
   …
   bool x;
   x = ht.search(29);
   x = ht.search(175);
   x = ht.search(20);
}
```

```
#define NUM_BUCK     10

class HashTable
{
public:
  bool search(int idNum)
  {
     int bucket = hashFunc(idNum);

     for (int tries=0;tries<NUM_BUCK;tries++)
     {
       if (m_buckets[bucket].used == false)
         return false;
       if (m_buckets[bucket].idNum == idNum)
         return true;

       bucket = (bucket + 1) % NUM_BUCK;
     }
     return false;// not in the hash table
  }

private:
    int hashFunc(int idNum) const
        {  return idNum % NUM_BUCK;  }

    BUCKET m_buckets[NUM_BUCK];
};
```

175

bucket = 175 % NUM_BUC
bucket = 175 % 10
bucket = 5




```
main()
{
   HashTable ht;
   …
   bool x;
   x = ht.search(29);
   x = ht.search(175);
   x = ht.search(20);
}
```

```
#define NUM_BUCK     10

class HashTable
{
public:

  bool search(int idNum)
  {
     int bucket = hashFunc(idNum);

     for (int tries=0;tries<NUM_BUCK;tries++)
     {
       if (m_buckets[bucket].used == false)
         return false;
       if (m_buckets[bucket].idNum == idNum)
         return true;

       bucket = (bucket + 1) % NUM_BUCK;
     }
     return false;// not in the hash table
  }

private:
    int hashFunc(int idNum) const
        {  return idNum % NUM_BUCK;  }

    BUCKET m_buckets[NUM_BUCK];
};
```

20

bucket = 20 % NUM_BUCK
bucket = 20 % 10
bucket = 0

| | | |
|---|---|---|
| 0 | idNum: 79 | used: T |
| 1 | idNum: | used: T |
| 2 | idNum: | used: f |
| 3 | idNum: | used: f |
| 4 | idNum: | used: f |
| 5 | idNum: 65 | used: T |
| 6 | idNum: 15 | used: T |
| 7 | idNum: 175 | used: T |
| 8 | idNum: | used: f |
| 9 | idNum: 29 | used: T |

```
main()
{
   HashTable ht;
   …
   bool x;
   x = ht.search(29);
   x = ht.search(175);
   x = ht.search(20);
}
```

# What Can you Store in your Hash Table?

Oh, and if you like, you can include additional associated values (e.g., a name, GPA) in each bucket!

For instance, what if I want to also store the student's name and GPA in each bucket along with their ID#?

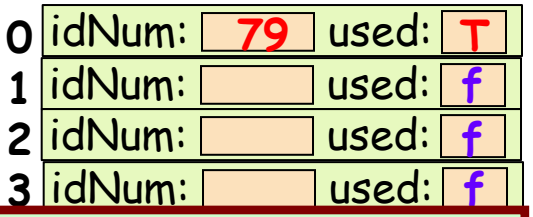
You can do that!

Now when you look up a student by their ID# you can ALSO get their name and GPA!

```
struct Bucket
{
    int             idNum;
    string          name;
    float           GPA;
    bool            used;
}
```

```
bool search(int id, string &name, float &GPA)
{
  int bucket = hashFunc(idNum);

  for (int tries=0;tries<NUM_BUCK;tries++)
  {
    if (m_buckets[bucket].used == false)
      return false;
    if (m_buckets[bucket].idNum == idNum)
    {
      name = m_buckets[bucket].name;
      GPA = m_buckets[bucket].GPA;
      return true;
    }
    bucket = (bucket + 1) % NUM_BUCK;
  }
  return false;// not in the hash table
}
```

# Linear Probing: Deleting?




So far, we've seen how to insert items into our Linear Probe hash table.

What if we want to delete a value from our hash table?

Let's take a naïve approach and see what happens…

To delete the value, let's just zero out our value and set the used field to false…

If we delete a value where a collision happened…

When we try to search again, we may prematurely abort our search, failing to find the sought-for value.

So, as you can see, if we simply delete an item from our hash table, we have problems!

There are ways to solve this problem with a Linear Probing hash table, but they're not recommended!

So, in summary, only use Closed/Linear Probing hash tables when you don't intend to delete items from your hash table.

Like if you're building a hash table that holds words for a dictionary…

You'll just add words, never delete any, right?

# The "Open Hash Table"

We just saw how to use linear probing to deal with collisions in our closed hash table.

Our closed hash table + linear probing works just fine, but it still has a few problems:

It's difficult to delete items

It has a cap on the number of items it can hold… That's a bummer.

It'd be nice if we could find a way to avoid both of these problems, yet still have an O(1) table!

We can! And it's called the "Open Hash Table."
Let's see how it works!

# The "Open" Hash Table

Idea: Instead of storing our values directly in the array, each array bucket points to a linked list of values.

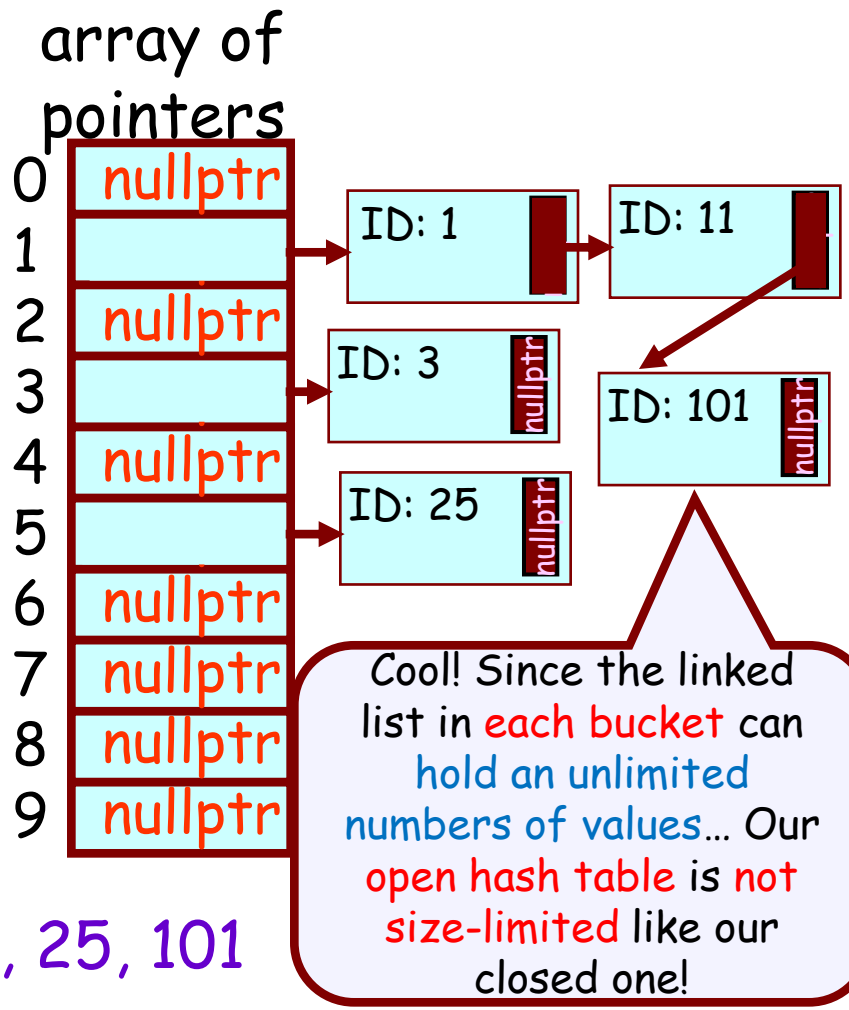
How about searching our Open hash table?

## To search for an item:

1. As before, compute a bucket # with your hash function:

```
bucket = hashFunc(idNum);
```

2. Search the linked list at array[bucket] for your item
3. If we reach the end of the list without finding our item, it's not in the table!




Insert the following values: 1, 3, 11, 25, 101

# The "Open" Hash Table: Deletions

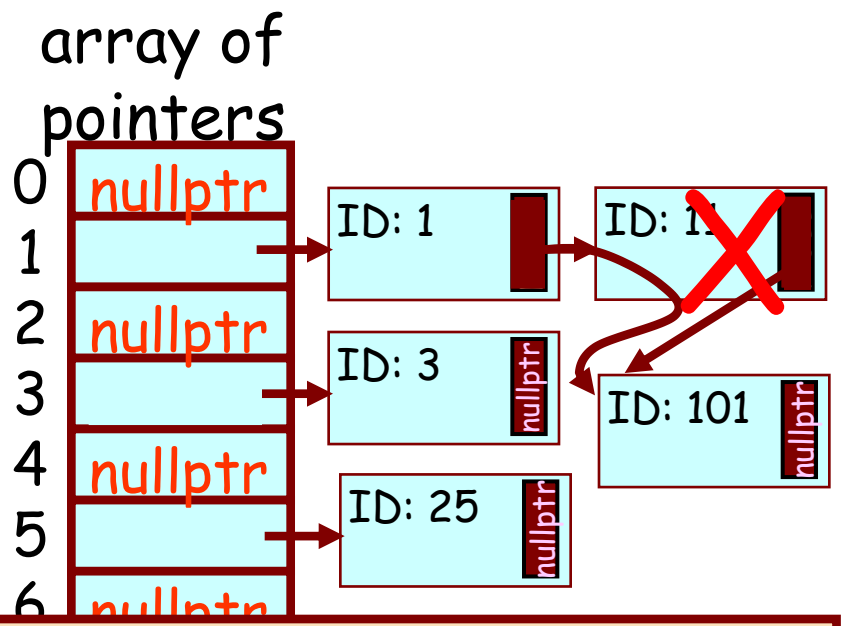
Oh – and there's no reason why we have to use a linked-list to deal with collisions…

**Question:**
How do you delete an item from an open hash table?

**Answer:**
You just remove the value from the linked list.

Let's delete the student with ID=11 and see what happens…

Cool! Unlike a closed hash table, you can easily delete items from an open hash table!




If you plan to repeatedly insert and delete values into the hash table, then the Open table is your best bet!

*Also, you can insert more than N items into your table and still have great performance!*

# Hash Table Efficiency

Question: How efficient is the hash table ADT?
How long does it take to locate an item?
How long does it take to insert an item?

Answer:

It depends upon:

(a) The type of hash table (e.g., closed vs. open),

(b) how full your hash table is, and

(c) how many collisions you have in the hash table.

# Hash Table Efficiency

Question: How efficient is the hash table ADT?
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How long does it take to insert an item?

Answer:

It depends upon:

(a) The type of hash table (e.g., closed vs. open),

(b) how full your hash table is, and

(c) how many collisions you have in the hash table.

# Hash Table Efficiency

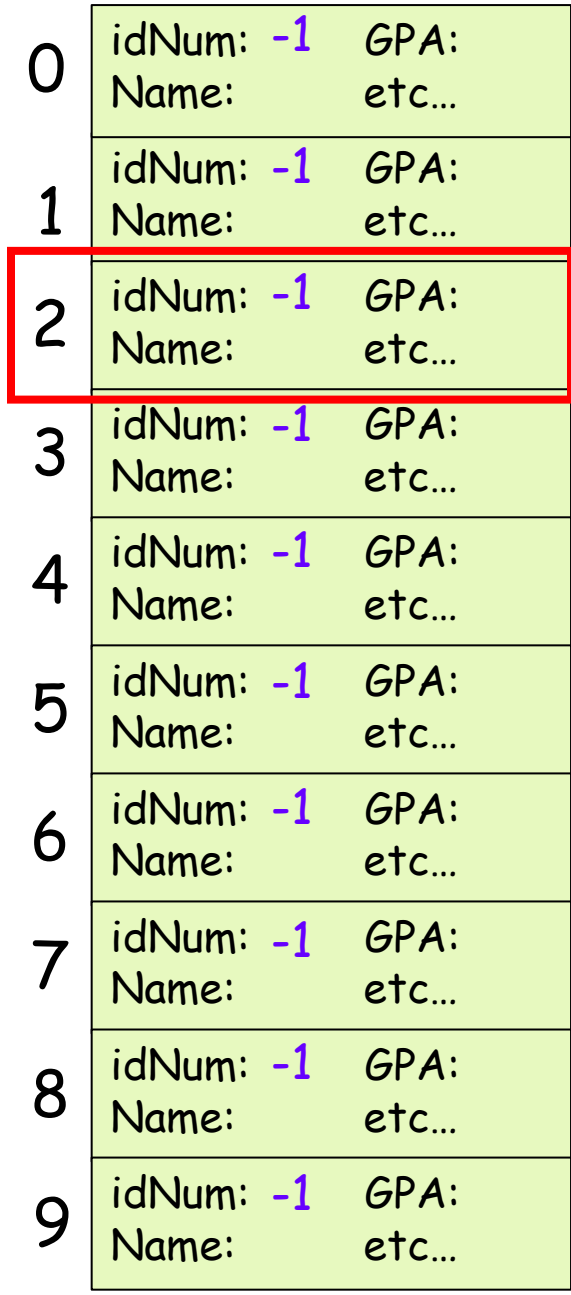
Let's assume we have a completely (or nearly) empty hash table…

What's the maximum number of steps required to insert a new value ?

Right! There's zero chance of collision, so we can add our new value in one step!

And finding an item in a nearly-empty hash table is just as fast!

We have no collisions so either we find an item right away or we know it's not in the hash table…



# Hash Table Efficiency

Ok, but what if our hash table is nearly full?

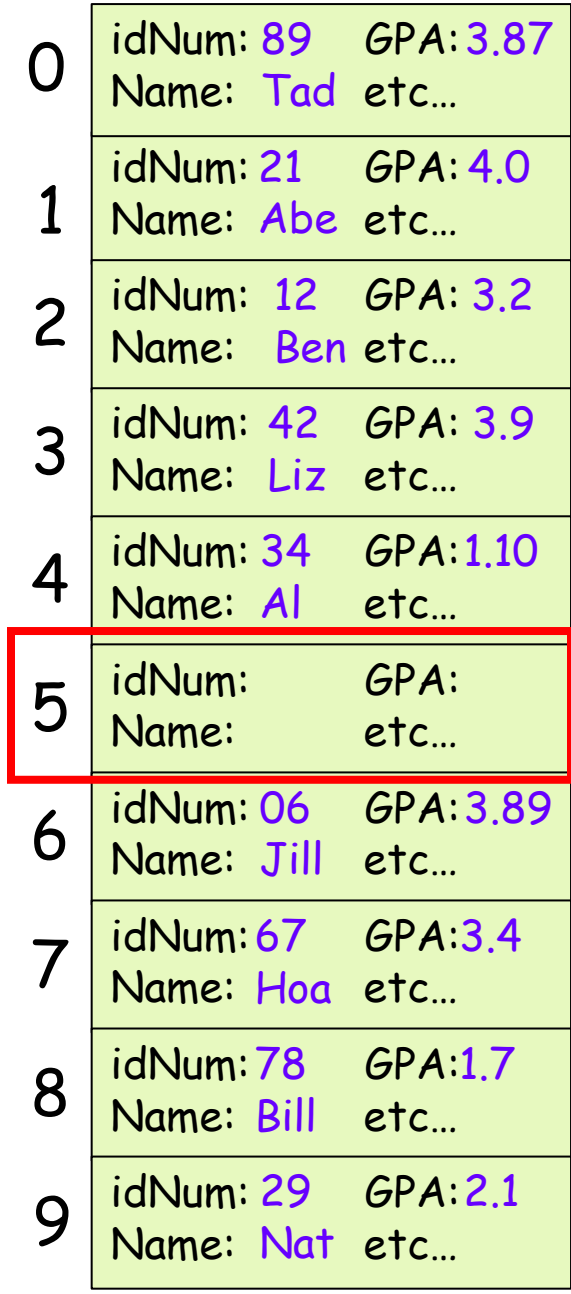
What's the maximum number of steps required to insert a new value ?

Right! It could take up to N steps!

And searching can take just as long in the worst case…

So technically, a hash table can be up to O(N) when it's nearly full!

So how big must we make our hash table so it runs quickly? To figure this out, we first need to learn about the "load" concept…

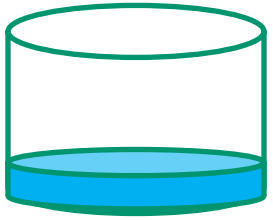


# Hash Table Efficiency: The Load Factor

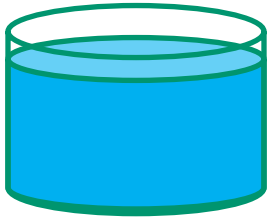
The "load" of a hash table is the
maximum number of values you intend to add
divided by
the number of buckets in the array.

$$L = \frac{\text{Max \# of values to insert}}{\text{Total buckets in the array}}$$

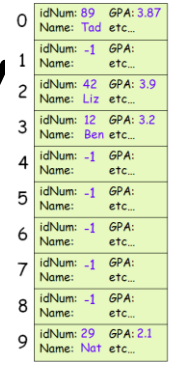
Example: A load of L=.1 means your array has 10X more buckets than you need (you'll only fill 10% of the buckets).



Example: A load of L=.9 means your array has 10% more buckets than you need (you'll fill 90% of the buckets).



# Closed Hash w/Linear Probing Efficiency



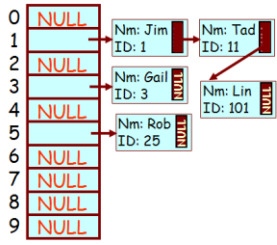

Given a particular load L for a Closed Hash Table w LP, it's easy to compute the average # of tries it'll take you to insert/find an item:

$$\text{Average \# of Tries} = \tfrac{1}{2}(1 + 1/(1-L)) \quad \text{for } L < 1.0$$

So, if your closed hash table has a

| load factor of | your search will take |
|---|---|
| .10 (your array is 10x bigger than required) | ~1.05 searches |
| .20 (your array is 5x bigger than required) | ~1.12 searches |
| .30 (your array is 3x bigger than required) | ~1.21 searches |
| ... | |
| .70 (your array is 30% bigger than required) | ~2.16 searches |
| .80 (your array is 20% bigger than required) | ~3.00 searches |
| .90 (your array is 10% bigger than required) | ~5.50 searches |

# Open Hash Table Efficiency



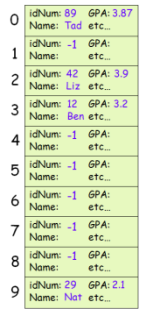

Given a particular load L for an Open Hash Table, it's also easy to compute the average # of tries to insert/find an item:

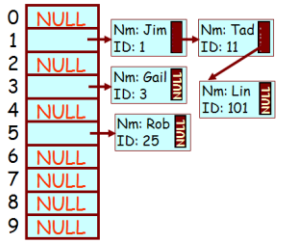
Average # of Checks = 1 + L/2

So, if your open hash table has a

| load factor of | your search will take |
|---|---|
| .10 (your array is 10x bigger than required) | ~1.05 searches |
| .20 (your array is 5x bigger than required) | ~1.10 searches |
| .30 (your array is 3x bigger than required) | ~1.15 searches |
| ... | |
| .70 (your array is 30% bigger than required) | ~1.35 searches |
| .80 (your array is 20% bigger than required) | ~1.40 searches |
| .90 (your array is 10% bigger than required) | ~1.45 searches |

# Closed vs. Open Hash Table




## Closed Hash w/L.P.

| Load | Avg Steps |
|---|---|
| .10 | ~1.05 searches |
| .20 | ~1.12 searches |
| .30 | ~1.21 searches |
| | ... |
| .70 | ~2.16 searches |
| .80 | ~3.00 searches |
| .90 | ~5.50 searches |

## Open Hash

| Load | Avg Steps |
|---|---|
| .10 | ~1.05 searches |
| .20 | ~1.10 searches |
| .30 | ~1.15 searches |
| | ... |
| .70 | ~1.35 searches |
| .80 | ~1.40 searches |
| .90 | ~1.45 searches |

Moral: Open hash tables are almost ALWAYS more efficient than Closed hash tables!

# Sizing your Hash Table

## Challenge:

If you want to store up to 1000 items in an Open Hash Table and be able to find any item in roughly 1.25 searches, how many buckets must your hash table have?

Remember: Expected # of Checks = 1 + L/2

If our hash table has 2000 buckets and we're inserting a maximum of 1000 values, we are guaranteed to have an average of 1.25 steps per insert/search!

This result means:
"If you want to be able to find/insert items nto your open hash table in an average of 1.25 steps, you need a load of .5, or roughly 2x more buckets than the maximum number of values you'll put into your table."

## Answer:

Part 1: Set the equation above equal to 1.25 and solve for L:

$1.25 = \rightarrow .25 = L/2 \rightarrow .5 = L$

Part 2: Use the load formula to solve for "Required size":

$$L = \frac{\text{\# of items to insert}}{\text{Required hash table size}} \rightarrow .5 = \frac{1000}{\text{Required hash table size}} \rightarrow \text{Required hash table size} = \frac{1000}{.5}$$

= 2000 buckets

# So basically it's a tradeoff!

You could always use a really big hash table with way-too-many buckets and ensure really fast searches…

But then you'll end up wasting lots of memory…

On the other hand, if you have a really small hash table (with just barely enough room), it'll be slower.

Finally, when choosing the exact size of your hash table (the number of buckets)…

Always try to choose a prime number of buckets…

Instead of 2000 buckets,
give your hash table 2021 buckets.

This causes more even distribution and fewer collisions!

# What Happens If…

What happens if we want to allow the user to search by the student's name instead of their ID number?

Well, our original hash function function won't quite work:

```
int hashFunc(int ID)
{
   return(ID % 100000)
}
```

```
int hashFunc(string &name)
{
   // what do we do?
}
```

Now we need a hash function that can convert from a string of letters to a number between 0 and N-1.

# A Hash Function for Strings

Here's one possibility for a hash function that can convert a string into a number between 0 and N-1.

```
int hashFunc(string &name)
{
   int i, total=0;

   for (i=0;i<name.length(); i++)
          total = total + name[i];

   total = total % HASH_TABLE_SIZE;

   return(total);
}
```

Hint:

What happens if we hash "BAT"?

What happens if we hash "TAB"?

But this hash function isn't so good. Why not?

How can we fix it?

# A Better Hash Function for Strings

Here's better version of our string hashing function – while not perfect, it disperses items more uniformly in the table.

```
int hashFunc(string &name)
{
   int i, total=0;

   for (i=0;i<name.length(); i++)
          total = total + (i+1) * name[i];

   total = total % HASH_TABLE_SIZE;

   return(total);
}
```

Now "BAT" and "TAB" hash to different slots in our array since this version takes character position into account.

# Choosing a Hash Function: Tips

1. The hash function must always give us the same bucket # for a given input value:

   Today: hashFunc(400683948) → bucket 83,948

   Tomorrow: hashFunc(400683948) → *still* bucket 83,948

2. The hash function should disperse items throughout the hash array as randomly as possible.

   Hash("abc") = 294

   Hash("cba") = 294

   Not good!

3. When coming up with a new hash function, always measure how well it disperses items (do some experiments!)

Hint: Use C++'s built in hash function for strings:

```
#include <functional>

void someFunc(const std::string &hashMe)
{
  std::hash<std::string> str_hash;                  // creates a string hasher!
  unsigned int hashValue = str_hash(hashMe);   // now hash our string!
  unsigned int bucket = hashValue % NUM_BUCKETS;
}
```

Notice that you have to add your own modulo based on your table size. C++'s hash function won't do this for you!

# Hash Tables vs. Binary Search Trees

| | Hash Tables | Binary Search Trees |
|---|---|---|
| Speed | O(1) regardless of # of items | $O(\log_2 N)$ |
| Simplicity | Easy to implement | More complex to implement |
| Max Size | **Closed:** Limited by array size<br>**Open:** Not limited, but high load impacts performance | Unlimited size |
| Space Efficiency | Wastes a lot of space if you have a large hash table holding few items | Only uses as much memory is needed (one node per item inserted) |
| Ordering | No ordering (random) | Alphabetical ordering |

# "Tables"

Let's say you want to want to write a program to keep track of all your BFFs…

Of course, you want to remember all the important dirt about each BFF:

And you want to quickly be able to search for a BFF in one or more ways…

" Find all the dirt on my BFF 'David Johansen' "

" Find all the dirt on the BFF whose number is 867-5309 "

Name: Carey Nash
Phone number: 867-5309
Birthday: July 28
iPhone or 'droid: iPhone
Social Security #: 111222333
Favorite food: …

# "Tables"

In CS lingo, a group of related data is called a "record."

Each record has a bunch of "fields" like Name, Phone #, Birthday, etc. that can be filled in with values.

If we have a bunch of records, we call this a "table." Simple!

While you may have many records with the same Name field value (e.g., John Smith) or the same Birthday field value (e.g., Jan 1st)…

Some fields, like Social Security Number, will have unique values across all records - this type of field is useful for searching and finding a unique record!

**A BFF Record**

Name: Carey Nash
Phone number: 867-5309
Birthday: July 28
iPhone or 'droid: iPhone
Social Security #: 111222333
Favorite food: …

**Table of BFF Records**

Name: David Small

Name: John Rohr
Phone number: 999-9191
Birthday: Jan 1
iPhone or 'droid: Droid
Social Security #: 47372727
Favorite food: …

A field (like the SSN) that has unique values across all records is called a "key field."

# Implementing Tables

## How could you create a record in C++?

Answer: Just use a struct or class to represent a record of data!

```
struct Student
{
   string name;
   int IDNum;
   float GPA;
   string phone;
   ...
};
```

## How can you create a table in C++?

Answer: You can simply create an array or vector of your struct!

```
vector<Student> table;
```

```
// algorithm to search by the name field
int SearchByName(vector<Student> &table, string &findName)
{
   for (int s = 0; s < table.size(); s++ )
      if (findName == table[ s ].name)
               return( s );// the student you're looking for is in slot s
   return( -1 );                    // didn't find that student in your table
}
```

```
// algorithm to search by the phone field
int SearchByPhone(vector<Student> &table, string &findPhone)
{
   for (int s = 0; s < table.size(); s++ )
      if (findPhone == table[ s ].phone)
               return( s );// the student you're looking for is in slot s
   return( -1 );                    // didn't find that student in your table
}
```

# Implementing Tables

Heck, why not just create a whole C++ class for our table?

```
struct Student
{
   string name;
   int IDNum;
   float GPA;
   string phone;
   ...
};
```

```
class TableOfStudents
{
public:
   TableOfStudents();      // construct a new table
   ~TableOfStudents();     // destruct our table
   void addStudent(Student &stud);  // add a new Student
   Student getStudent(int s);   // retrieve Students from slot s
   int searchByName(string &name);  // name is a searchable field
   int searchByPhone(int phone);  // phone is a searchable field
   ...
private:

   vector<Student> m_students;
};
```

# Tables

In the TableOfStudents class, we used a vector to hold our table and a linear search to find Students by their name or phone.

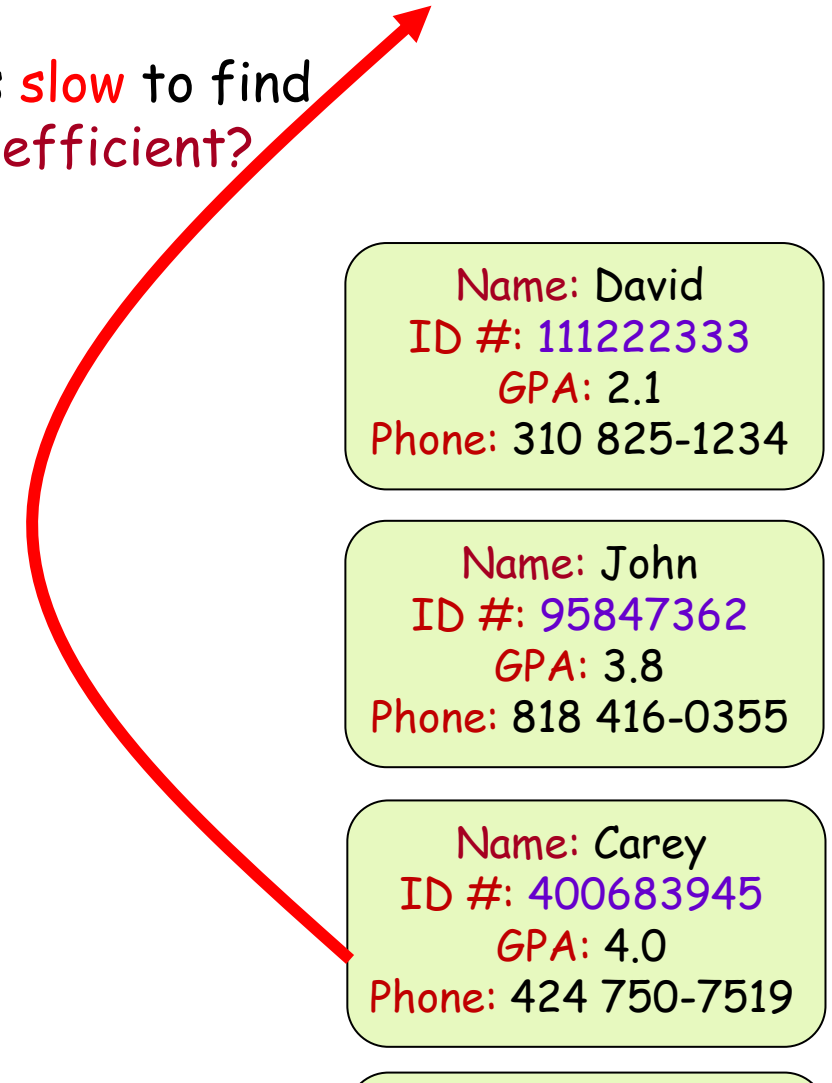
This is a perfectly valid table – but it's slow to find a student! How can we make it more efficient?

Well, we could alphabetically sort our vector of records by their names…

Then we could use a binary search to quickly locate a record based on a person's name.

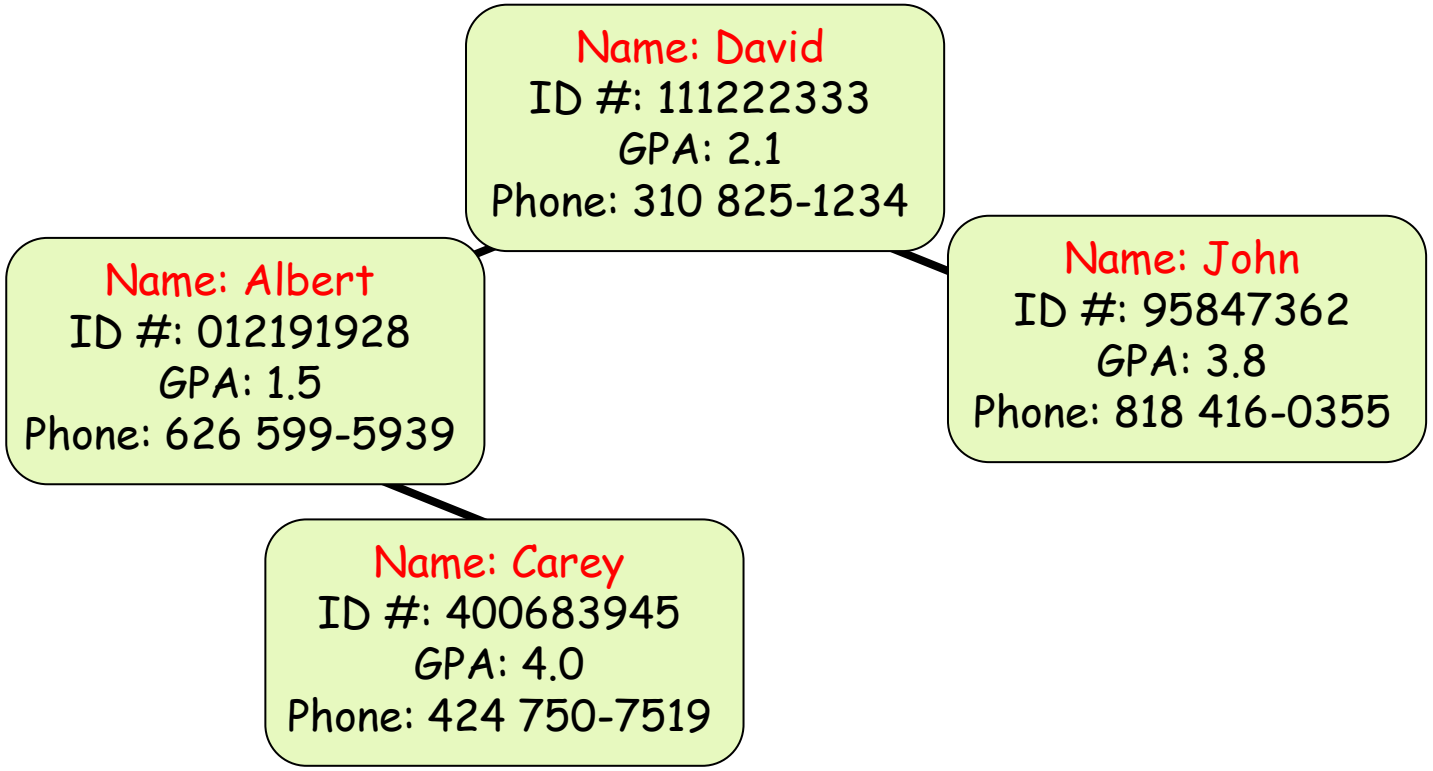
But then every time we add a new record, we have to re-sort the whole table. Yuck!

And if we sort by name, we can't search efficiently by other fields like phone # or ID #!



# Tables

Hmmm… What if we stored our records in a binary search tree (e.g., a map) organized by name? Would that fix things?

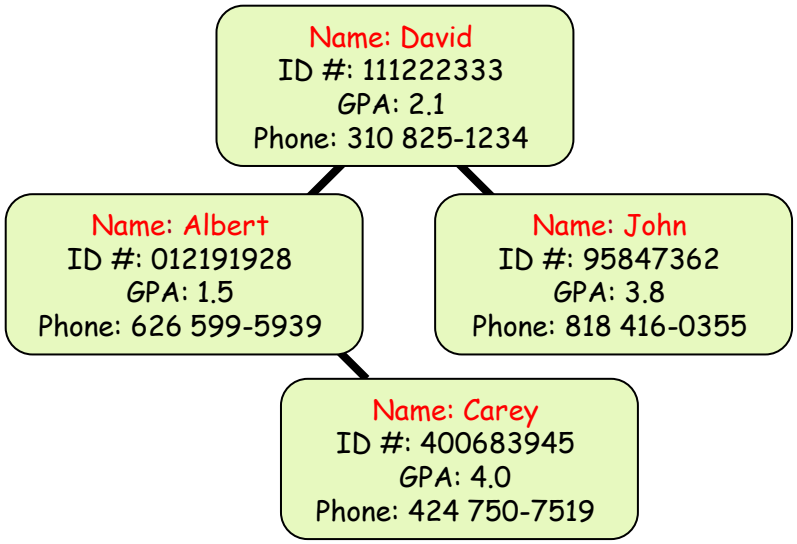


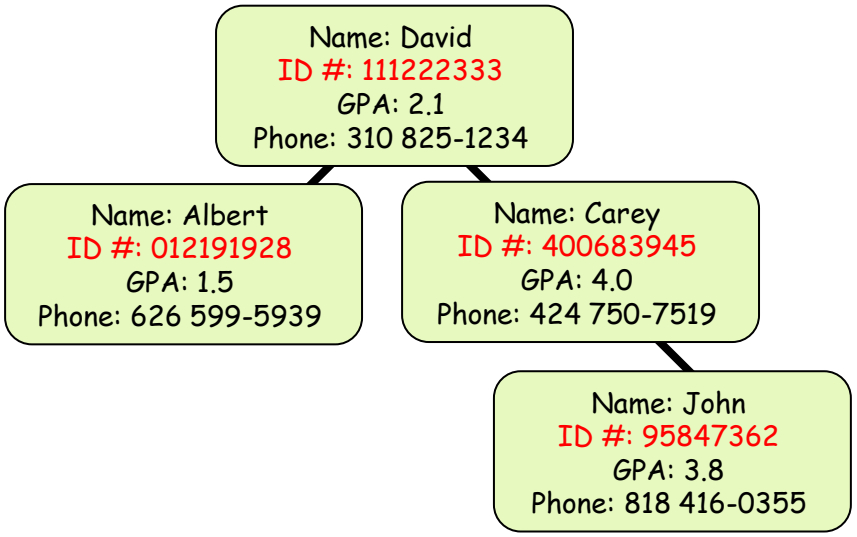

Well, now we can search the table efficiently by name…

But we still can't search efficiently by ID# or Phone #….

# Tables

Hmmm… What if we create two tables,
ordering the first by name and the second by ID#?




That works… Now I can quickly find people by name or ID#!

But now we have two copies of every record, one in each tree!
If the records are big, that's a waste of space!

So what can we do? Let's see!

# Making an Efficient Table

1. We'll still use a vector to store all of our records…
2. Let's also add a data structure that lets us associate each person's name with their slot # in the vector…
3. And we can add another data structure to associate each person's ID # with their slot # too!

m_students

| | |
|---|---|
| 0 | name: Alex<br>GPA: 2.05<br>ID: 7124<br>… |
| 1 | name: Linda<br>GPA: 3.99<br>ID: 0003<br>… |
| 2 | name: Jason<br>GPA: 1.55 |

```
class TableOfStudents
{
public:
   TableOfStudents();
   ~TableOfStudents();
   void addStudent(Student &stud);
   Student getStudent(int s);
   int searchByName(string &name);
   int searchByPhone(int phone);

private:
   vector<Student> m_students;
   map<string,int>  m_nameToSlot;
   map<int,int>     m_idToSlot;
   map<int,int>     m_phoneToSlot;
};
```

Our second data structure lets us quickly look up a name and find out which slot in the vector holds the related record.

Our third data structure lets us quickly look up an ID# and find out which slot in the vector holds the related record.

These secondary data structures are called "indexes."
Each index lets us efficiently find a record based on a particular field.
We may have as many indexes as we need for our application.

# Making an Efficient Table

So what does
method lo

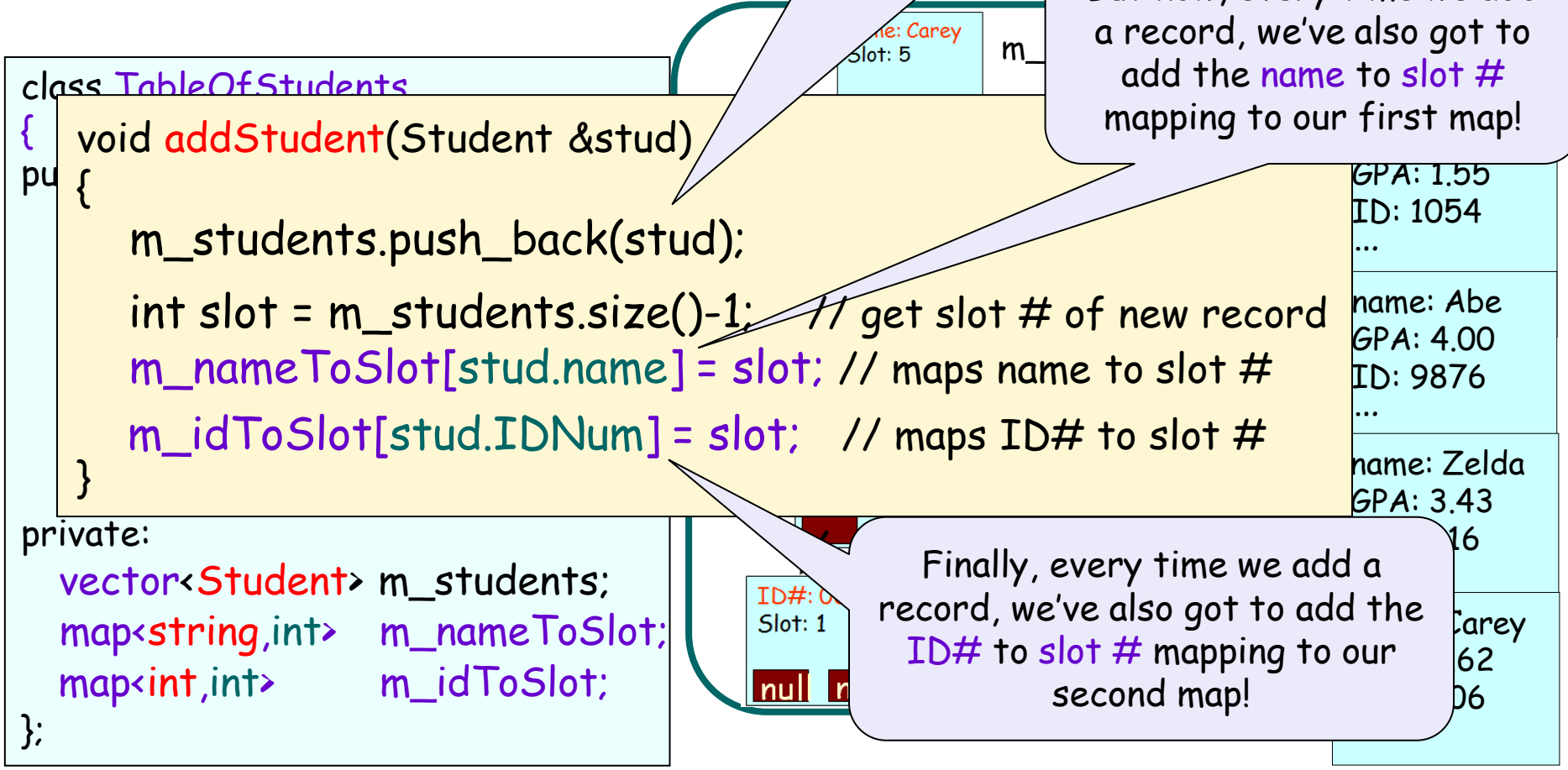
Well, we have to add our new student record to our vector just like before.

But now, every time we add a record, we've also got to add the name to slot # mapping to our first map!

Finally, every time we add a record, we've also got to add the ID# to slot # mapping to our second map!

```
class TableOfStudents
{
pu
    void addStudent(Student &stud)
    {
        m_students.push_back(stud);
        int slot = m_students.size()-1;   // get slot # of new record
        m_nameToSlot[stud.name] = slot; // maps name to slot #
        m_idToSlot[stud.IDNum] = slot;   // maps ID# to slot #
    }

private:
    vector<Student> m_students;
    map<string,int>   m_nameToSlot;
    map<int,int>      m_idToSlot;
};
```

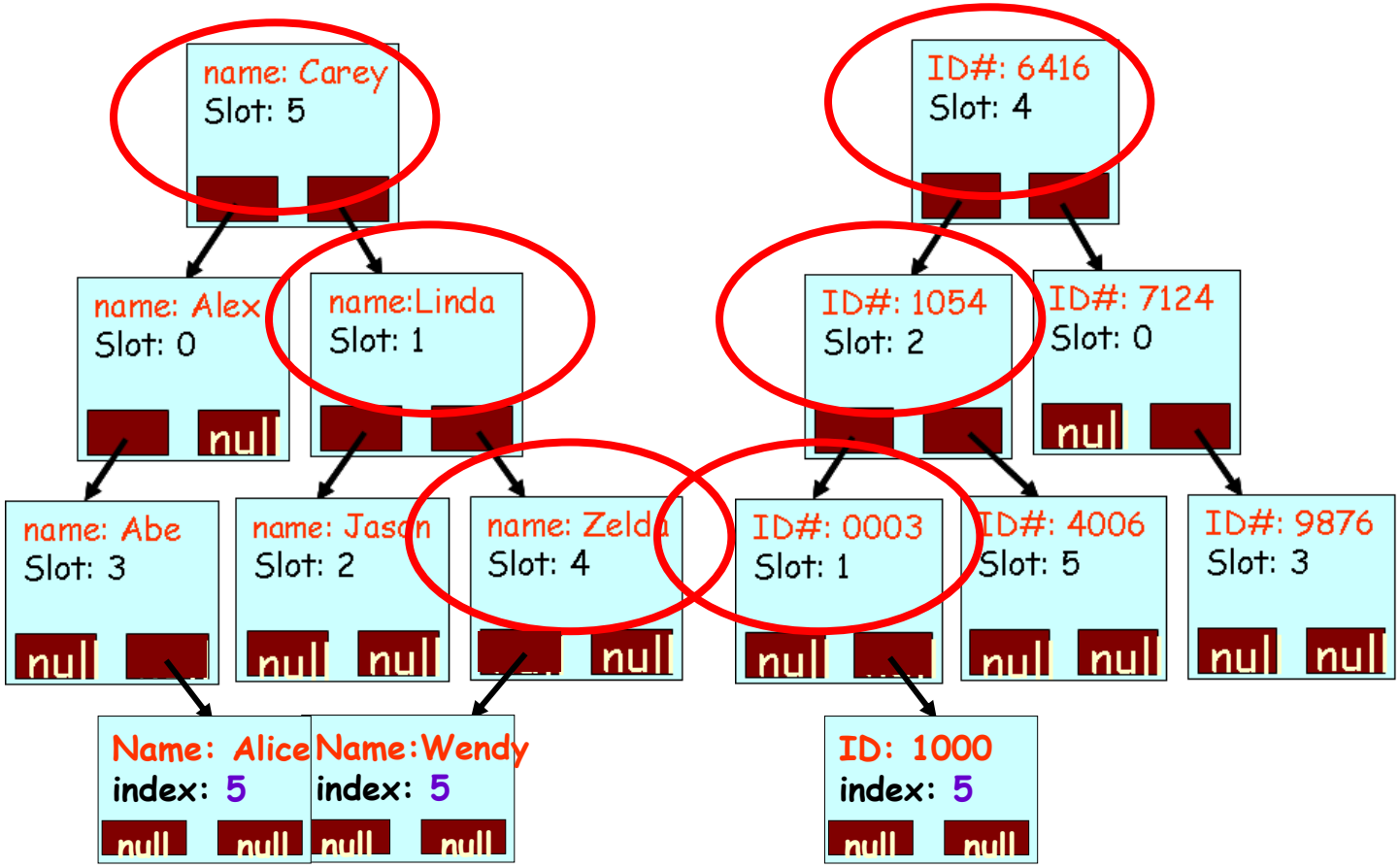


# Compl

But wait!!!! - Any time you delete a record or update a record's searchable fields, you also have to update your indexes!

So to review, what do we have to do to insert a new record into our table?

Let's add: Wendy, ID=1000, GPA=3.9




1
2
3
4
5

ID: 7124
...

name: Linda
GPA: 3.99
ID: 0003
...

name: Jason
GPA: 1.55
ID: 1054
...

name: Abe
GPA: 4.00
ID: 9876
...

name: Zelda
GPA: 3.43
ID: 6416
...

name: Carey
GPA: 3.62
ID: 4006
...

name: Alice
GPA: 3.9
ID: 1000

# Tables

As it turns out, databases like "Oracle" use exactly this approach to store and index data!

(The only difference is they usually store their data on disk rather than in memory)

And by the way… While my example used binary search trees to index our table's fields…

You could use any efficient data structure you like!

For example, you could use a hash table!

# Using Hashing to Speed Up Tables

Can we use hash tables to index our data instead of binary search trees?

Of course!

Now we can have O(1) searches by name! Cool! But in that case why not just always use hash tables to index all of our key fields?

Answer: Because hash tables store the data in an essentially random order.

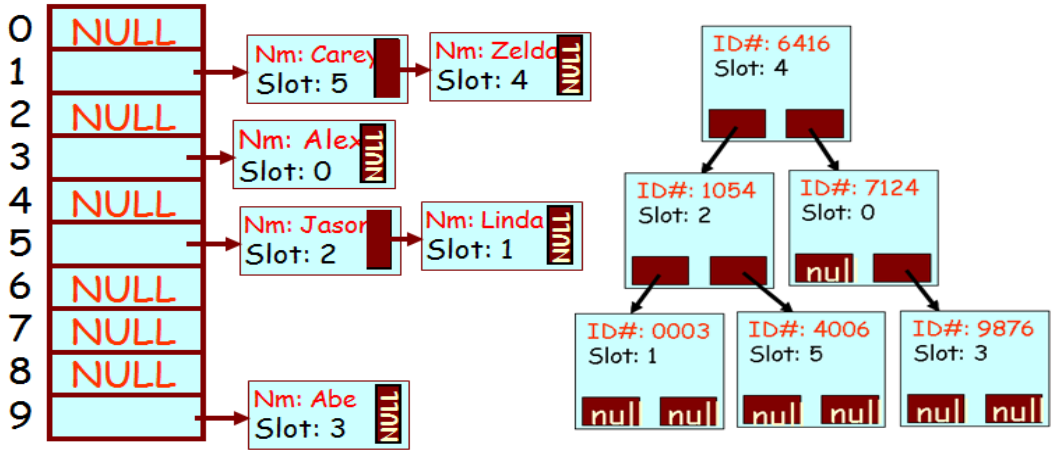
While a BST is slower, it does order the key fields in alphabetical order…

For instance, what if we want to be able to print out all students alphabetically by their name.

If our index data structure is a binary search tree, that's easy!

If we indexed with a hash table, we'd have to do a lot more work to do the same thing…

| | |
|---|---|
| 0 | name: Alex<br>GPA: 2.05<br>ID: 7124<br>… |
| 1 | name: Linda<br>GPA: 3.99<br>ID: 0003<br>… |
| 2 | name: Jason<br>GPA: 1.55<br>ID: 1054<br>… |
| 3 | name: Abe<br>GPA: 4.00<br>ID: 9876 |




Moral: You need to understand how your table will be used to determine how to best index each field.

For example:

I'd use a BST for the name field so I can print people's names in alphabetical order. But I'd use a hash table for the phone field, cause I just need to search quickly but I don't need to order records by their phone #.

# Challenges

Question: What is the big-oh of traversing all of the elements in a hash table?

Question: I have two hash tables: the first has 10 buckets, and the second has 20 buckets. If I insert each of the following IDs into each hash table, where will each ID number end up (which bucket #s)?

ID = 5
ID = 15
ID = 25
ID = 100

Question: How can you print out the items in a hash-table in alphabetical/numerical order.