

# Time-dependent multi-depot green vehicle routing problem with time windows considering temporal-spatial distance

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## ABSTRACT

Reducing distribution costs is one of the effective ways for logistics enterprises to improve their core competitiveness. Aiming at the **multi-depot vehicle routing problem** under the **time-varying** road network, this paper proposes an integer programming model with the **minimum total costs** by comprehensively considering the **fixed costs** of vehicles, **penalty costs** on earliness and tardiness, **fuel costs** and the effects of vehicle speed, load and road gradient on **fuel consumption**. A hybrid genetic algorithm with variable neighborhood search is developed to solve the problem. In the algorithm, the temporal-spatial distance is introduced to cluster the customers to generate an initial population to improve the quality of the initial solution. Adaptive neighborhood search times strategy and simulated annealing inferior solution acceptance mechanism are used to balance the diversification and exploitation in the algorithm iteration process. Numerical results show that the model and algorithm we proposed are rather effective. The research results not only deepen and expand the vehicle routing problem (VRP) theory research, but also provide a scientific and reasonable method for logistics enterprises to make the vehicle scheduling plan.

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## 1. Introduction

With the rapid development of the logistics industry, sharing distribution resources and reducing vehicle energy consumption has become the main trend of logistics distribution. At the same time, the development of an intelligent transportation system provides the possibility to obtain traffic information about road networks. Therefore, some researchers have researched the multi-depot vehicle routing problem (MDVRP), green vehicle routing problem (GVRP), and time-dependent vehicle routing problem (TDVRP). We focus on the **multi-depot green vehicle routing problem (MDGVRP)** with the **time-varying vehicle speed** and **soft time windows**, which is related to the MDVRP, GVRP and TDVRP.

With the continuous expansion of the scale of logistics enterprises, they have multiple depots in the service area, and information and orders can be shared among the depots. The traditional research on the vehicle routing problem is no longer applicable to this distribution model, prompting researchers to study MDVRP (Salhi et al., 2014; Sadati et al., 2020). The multi-depot distribution mode can optimize the distribution scheme through global information, which can effectively solve the problems of high trans-

portation costs and low service levels for a single depot to serve customers (Fan et al., 2019). In recent years, energy depletion and the greenhouse effect have become increasingly prominent. Researchers have begun to consider factors such as fuel consumption and carbon emissions that harm the environment in VRP, and research GVRP (Yu et al., 2019; Poonthaler and Nadarajan, 2018). In existing GVRP literature, the models for calculating fuel consumption can be divided into two categories. One assumes that it is only related to vehicle type, load, and distance between two nodes, and has nothing to do with vehicle speed (Rezaei et al., 2019). The other is to consider the impact of vehicle speed on fuel consumption (Liu et al., 2019; Xiao and Konak, 2016). Compared with MDVRP, research on MDGVRP needs to consider more factors, which increases the complexity of the problem and the difficulty of solving it. Therefore, researchers have successively proposed effective methods to solve MDGVRP (Jabir et al., 2017; Li et al., 2018).

In previous researches on VRP, researchers usually assumed that the vehicle speed was constant. However, the traffic environment is dynamically changing, which limits the speed of vehicles, so that vehicles can only travel slowly during rush hours. In 1981, Beasley (1981) proposed the TDVRP and studied the VRP in which the travel time changes with the departure time. Subsequent research on MDVRP mostly used the time-dependent function based on travel speed proposed by Ichoua et al (2003), and

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express the vehicle speed as a piecewise function (Mu et al., 2015; Figliozzi, 2012). The Federal Highway Administration (2015) mentioned that the speed of vehicles changes smoothly, rather than a step-change at a certain moment. Xu et al. (2019) proposed to use a **trigonometric function** to describe the change of vehicle speed, which is closer to the reality of distribution production activities.

**The time-dependent multi-depot green vehicle routing problem with time windows** (TMDGVRPTW) considering **temporal-spatial distance** proposed in this paper is based on the historical traffic information of the distribution network, and considering the impact of the fixed costs of vehicles and the temporal-spatial distance between customers on the total costs, which is a VRP expansion problem that optimizes vehicle routing in the multi-depot distribution network. The contribution of this paper can be summarized as:

Considering the **constraint of time-dependent travel time, fixed costs of vehicles, time window, fuel consumption, and the impact of vehicle type, speed, load, and road gradient on fuel consumption**, an integer programming model is proposed for this problem.

A developed hybrid genetic algorithm with variable neighborhood search is applied to solve the presented problem. In the algorithm, the initial population is generated by the temporal-spatial clustering of customers, and an adaptive neighborhood search strategy and a simulated annealing inferior solution acceptance mechanism are introduced.

The rest of this article is as follows: **Section 2** briefly reviews the relevant literature. **Section 3** describes the problem and formulates a mathematical model. In **Section 4**, the hybrid genetic algorithm with variable neighborhood search is developed. **Section 5** tests the algorithm and summarizes the experimental results. **Section 6** presents the conclusions.

## 2. Literature review

The research of vehicle routing problem with time windows (VRPTW) is relatively mature. In this paper, the research on MDVRP, GVRP, TDVRP, and time-dependent green vehicle routing problem (TDGVRP) is mainly reviewed.

### 2.1. Research on MDVRP

Researchers have done extensive research on the optimization model and solution approach of the MDVRP. Karakatić and Podgorelec (2015) summarized various genetic algorithms (GA) that are designed for solving MDVRP and evaluated their performance. Bezerra et al. (2018) developed a general variable neighborhood search (GVNS) algorithm to solve the proposed MDVRP, its goal is to minimize the costs of optimized vehicle routing. Oliveira et al. (2016) proposed a decomposition method for MDVRP and designed a cooperative coevolutionary algorithm to solve it. Ray et al. (2014) developed a heuristic algorithm to solve the established ILP optimization model of the MDVRP including depot selection and route optimization. Afshar-Nadjafi and Afshar-Nadjafi (2014) considered the influence of departure time on travel time between nodes and developed a heuristic algorithm to solve the formulated mixed integer programming (MIP) model. Soto et al. (2017) developed a general multiple neighborhood search hybridized with a tabu search (MNS-TS) strategy to solve the multi-depot open vehicle routing problem (MDOVRP). Liu and Ma (2013) proposed a MIP model for MDOVRP with time window based on vehicle leasing and sharing and developed a hybrid genetic algorithm (HGA) to solve the problem. Li et al. (2016) considered the constraints of time window to propose a model with the minimum travel costs and developed a HGA with adaptive local

search to solve it. Fan et al. (2019) considered the timeliness requirements of fresh product distribution and proposed a MDOVRP based on joint distribution mode of fresh food, constructed an optimization model aiming at minimizing the total distribution costs, and designed an ant colony optimization (ACO) to solve it. Contardo and Martinelli (2014) proposed an optimization model for MDVRP under capacity and route length constraints, and a new exact algorithm was adopted to minimize the total travel time. Salhi et al. (2014) developed a variable neighborhood search to solve the heterogeneous fleet MDVRP.

### 2.2. Research on GVRP

In recent years, GVRP has attracted the attention of researchers. Rezaei et al. (2019) proposed a mixed ILP model considering the impact of vehicle type, load, and distance on fuel consumption. A GA and population-based simulated annealing were developed to minimize distribution costs and carbon emissions. Aiming at the integrated scheduling of production and distribution, Ganji et al. (2020) proposed a multi-objective MIP model considering the heterogeneous fleet, time window, and the relationship between fuel consumption and vehicle type, speed and load, and solved it with three heuristic algorithms. Some researchers have begun to study the GVRP of alternative fuel-powered vehicles (AFVs) that may need to visit alternative fuel stations during delivery (Bruglieri et al., 2019; Andelmin and Bartolini, 2017; Zhang et al., 2020; Koç and Karaoglan, 2016). Bruglieri et al. (2019) considered the linear relationship between fuel consumption and distance, and developed a path-based solution approach to solve the proposed problem. Andelmin and Bartolini (2017) modeled GVRP as a set partitioning problem and designed an exact algorithm to solve it. Zhang et al. (2020) considered the capacity limit of the fuel tank and the relationship between the travel distance and carbon emissions, and a two-stage ant colony system was adopted to solve the MDGVRP to minimize carbon emissions. Koç and Karaoglan (2016) developed a heuristic-based exact solution approach to solve the proposed GVRP.

### 2.3. Research on TDVRP

Considering the time dependence of vehicle speed in VRP can effectively avoid traffic congestion and shorten delivery time. Haghani and Jung (2005) considered the dynamic changes of travel time between nodes and customer demand, formulated a TDVRP optimization model with the minimum delivery time as the goal, and developed a GA to solve it. Wu and Ma (2017) considered the time-varying characteristics of the distribution network, proposed an optimization model for the TDVRP of perishable food with minimum total costs, and designed a hybrid genetic algorithm to solve it. Taniguchi and Shimamoto (2004) studied the advanced intelligent transportation system used in urban distribution, proposed an optimization model for VRP with the time-varying vehicle speed, and dynamically updated travel time using dynamic traffic simulation data. Mancini (2014) considered traffic congestion during rush hours, proposed to use multiple functional expressions to express the change of road speed in one day, and developed greedy randomized adaptive search procedures (GRASP) to solve time-dependent vehicle routing and scheduling problem (VRSP). Zhang and Zhang (2017) considered the time dependence of vehicle travel time, proposed a heterogeneous fleet TDVRP, and adopted an improved genetic algorithm (IGA) to solve the problem to minimize the total costs. Duan et al. (2019) considered both the stochastic and time-varying of the distribution network and designed a non-dominated sorting ant colony optimization (NSACO) to solve the formulated optimization model of stochastic TDVRP. Considering that the travel time of nodes is affected by

traffic congestion and other factors, [Sabar et al. \(2019\)](#) designed a self-adaptive evolutionary algorithm for solving the proposed dynamic VRP with traffic congestion.

#### 2.4. Research on TDGVRP

With further research on GVRP and TDVRP, researchers began to study TDGVRP. [Soysal and Çimen \(2017\)](#) formulated a model for TDGVRP, which assumes that fuel consumption depends on distance and time-dependent speed. A simulation based restricted dynamic programming approach (RDP) was proposed to optimize the vehicle routing with minimum costs. [Liu et al. \(2020\)](#) proposed a model for VRP under time-varying distribution network, which assumes that carbon emissions depend on vehicle speed, vehicle load, and other factors. An improved ACO was developed to solve the proposed problem. [Poonthaler and Nadarajan \(2018\)](#) adopted the objective planning method for the GVRP and designed a particle swarm optimization to minimize the route costs and fuel consumption. [Xu et al. \(2019\)](#) proposed a GVRPTW with time-varying speed, formulated an optimization model considering the impact of vehicle load and speed on fuel consumption, and designed a heuristic algorithm for the solution. [Xiao and Konak \(2016\)](#) proposed a MIP model for time-dependent green VRSP in which emissions were assumed to depend on vehicle type, speed, and load. A hybrid algorithm was developed to solve it. [Çimen and Soysal \(2017\)](#) developed a heuristic algorithm to solve the proposed TDGVRP with stochastic vehicle speeds. [Qian and Eglese \(2015\)](#) formulated an optimization model of VRP under time-varying speed to minimize fuel consumption and solved problems with a

tabu search algorithm. [Wang et al. \(2019\)](#) proposed a bi-objective model aiming to minimize total carbon emission and operating costs for the MDGVRP with shared transportation resources and developed a hybrid heuristic algorithm to optimize vehicle routing.

The above-mentioned related literature is of great reference value for the research of this paper. [Table 1](#) summarizes the main contributions already proposed in the literature. Through combing the above-related literature and [Table 1](#), we can summarize: (1) Most of the related researches on TDVRP express the vehicle speed in a certain period time by the average speed, ignoring the smooth increase or decrease of vehicle speed, which is inconsistent with the actual distribution production activities. (2) Most of the researches on MDVRPTW, GVRPTW, and TDVRPTW generate the initial solution randomly or cluster customers in space without considering the influence of time, which leads to poor quality of initial solution and poor performance of algorithm optimization. (3) There is a lack of research on the optimization of vehicle routing considering four aspects: time-dependent vehicle travel time, multiple depots, time windows, and fuel consumption. In the existing research, we only found that [Wang et al. \(2019\)](#) considered these four features at the same time, but it assumed that the speed of the vehicle follows a normal distribution, and the average speed at the beginning and end of a certain period is used as the speed of the vehicle during the period, without considering the smooth increase or decrease of the vehicle speed ([Federal Highway Administration, 2015; Xu et al., 2019](#)), and ignore the temporal and spatial differences of vehicle speed caused by the different types of roads in the distribution network. Besides, external envi-

**Table 1**  
A summary of the selected literature.

Reference	Problem features				Objectives	Solution
	TW	MD	TD	Factors in fuel consumption/carbon emissions		
<a href="#">Liu and Ma (2013)</a>	*	*	×		Total costs	HGA
<a href="#">Afshar-Nadjafi and Afshar-Nadjafi (2014)</a>	*	*	*	×	Total costs	Constructive heuristic algorithm
<a href="#">Qian and Eglese (2015)</a>	*		*	Speed and waiting time	Fuel emissions	Column generation based tabu search
<a href="#">Li et al. (2016)</a>	*	*		×	Distance	HGA with adaptive local search
<a href="#">Mancini (2014)</a>	*		*	×	–	GRASP
<a href="#">Xiao and Konak (2016)</a>	*		*	Vehicle type, load, and speed	CO <sub>2</sub> emissions	Hybrid optimization
<a href="#">Wu and Ma (2017)</a>	*		*	×	Total costs	HGA
<a href="#">Soysal, and Çimen (2017)</a>			*	Vehicle type, speed, and distance	Total costs	Simulation based RDP
<a href="#">Çimen, and Soysal (2017)</a>			*	Vehicle type, speed, and distance	CO <sub>2</sub> emissions	ADP-based heuristic
<a href="#">Poonthaler and Nadarajan (2018)</a>			*	Speed and distance	Route costs and fuel consumption	Particle swarm optimization
<a href="#">Fan et al. (2019)</a>	*	*	×		Total costs	ACO
<a href="#">Rezaei et al. (2019)</a>	*			Load and distance	transportation costs and CO <sub>2</sub> emissions	GA and population based simulated annealing
<a href="#">Bruglieri et al. (2019)</a>				AFVs: distance	Distance	Path-based solution
<a href="#">Duan et al. (2019)</a>	*		*	×	Number of vehicles and travel time	NSACO
<a href="#">Sabar et al. (2019)</a>			*	×	Distance	Self-adaptive evolutionary algorithm
<a href="#">Liu et al. (2019)</a>	*		*	Speed, load and gradient	Carbon emissions	Improved ACO
<a href="#">Xu et al. (2019)</a>	*		*	Speed, load, the road, and the vehicle conditions	Fuel consumption and satisfaction	Improved NSGA-II
<a href="#">Wang et al. (2019)</a>	*	*	*	Speed, vehicle parameters, distance	Carbon emissions and operating costs	Multi-phase hybrid heuristic algorithm
<a href="#">Ganji et al. (2020)</a>	*			Speed, distance, vehicle type	Costs, tardiness of orders and customer's dissatisfaction	NSGAI, MOPSO, and MOACO
<a href="#">Zhang et al. (2020)</a>		*		Alternative fuel-powered vehicles: distance	Carbon emissions	Two-stage ant colony system
This paper	*	*	*	Speed, load, vehicle type, and gradient	Total costs: fixed vehicle costs, fuel costs, and penalty costs	Hybrid genetic algorithm with variable neighborhood search considering temporal-spatial distance

ronmental factors such as aerodynamic drag coefficient and air density, and many vehicle parameters such as engine friction factor and efficiency of the vehicle drive train are considered in the calculation model of carbon emissions. For different types of vehicles, these parameters are different and will change constantly in the process of distribution, which is difficult to determine and has poor practicability in distribution and production activities.

### 3. Problem and mathematical model

#### 3.1. Problem description

This paper is a **multi-depot vehicle routing problem**. The depot can be open to all vehicles and accept the final stop of vehicles. The customer node only accepts the delivery service provided by the vehicle and does not accept the final stop of the vehicle. The problem can be described as: The road network constitutes a complete **directed graph**  $G = (V, E)$ , there are different types of roads, and the **vehicle speed of each type of road changes continuously**. At a certain moment, the vehicle leaves the depot to visit the customer and **provides service** to the customer **within the service time window** required by the customer. There will be **penalty costs** for vehicles arriving **early or late**. Vehicles can return to any depot after visiting the last customer, and there is **no path connection between the depots**. The fuel consumption during the delivery process varies with factors such as vehicle speed and load. **Table 2** summarizes the sets, parameters, and variables used in the model.

#### 3.2. Determination of time-dependent function of vehicle speed

Most of the existing researches on TDVRP used a **step function** to **express the vehicle speed of the road section**, and the vehicle speed is abrupt at a certain time. In reality, the **vehicle speed** varies **continuously**, such as the trigonometric function relation  $v(t) = \varphi \sin(\gamma t) + \delta$  (which  $\varphi, \gamma, \delta$  are related to **road conditions**) between the vehicle speed ( $v$ ) and the time ( $t$ ) (Xu et al., 2019). In this paper, the variation of vehicle speed on all-day roads is approximately expressed by a plurality of trigonometric function

**Table 2**  
Nomenclature of sets, parameters, and variables.

Notation	Meaning
$V$	$V = D \cup C$ is nodes set, where $D$ is the set of depots and $C$ represents the set of all customers
$E$	$E = \{(i, j)   i, j \in V, i \neq j\}$ is edge set
$l_{ij}$	The distance between nodes $i$ and $j$ (Unit: km)
$v$	The speed of the vehicle (Unit: km/h)
$F_{ij}$	The fuel consumption of the vehicle from nodes $i$ to $j$ (Unit: liter)
$K$	The set of all vehicles
$k$	Any vehicle in the vehicle set $K$
$Q$	The capacity of the vehicle (Unit: ton)
$[T_s, T_f]$	Time window of the depot, where $T_s$ indicates the time when the vehicle leaves the depot and $T_f$ is the latest time the vehicle returns to the depot
$[ET_i, LT_i]$	Time window of customer $i$
$d_i$	The demand of the customer $i$ (Unit: ton)
$\tau_{ik}$	Handle time of vehicle $k$ at node $i$
$T_{ik}$	The time when the vehicle $k$ arrives at the node $i$
$c_1$	The fuel price per liter (Unit: CNY/liter)
$c_2$	The fixed costs of a vehicle (Unit: CNY)
$c_3$	Penalty cost per unit time for vehicles arriving earlier than $ET_i$ (Unit: CNY/h)
$c_4$	Penalty cost per unit time for vehicles arriving later than $LT_i$ (Unit: CNY/h)
$x_{ijk}$	If the vehicle $k$ reaches the node $j$ from the node $i$ , $x_{ijk} = 1$ , else $x_{ijk} = 0$
$y_{ij}$	If the customer $i$ is served by the depot $j$ , $y_{ij} = 1$ , else $y_{ij} = 0$

relations. The trigonometric expression between speed ( $v$ ) and time ( $t$ ) can be expressed as follows:

$$v = \begin{cases} a_0 \sin[b_0(t - c_0)] + d_0, & t \in [0, T_1] \\ \vdots \\ a_\beta \sin[b_\beta(t - c_\beta)] + d_\beta, & t \in [T_\beta, T_{\beta+1}] \\ \vdots \\ a_n \sin[b_n(t - c_n)] + d_n, & t \in [T_n, 24] \end{cases} \quad (1)$$

The parameters  $a_\beta, b_\beta, c_\beta, d_\beta, \beta \in \{1, 2, \dots, n\}$  are determined by road conditions.

According to Yang's (2018) research on the evolution of urban recurrent congestion, the trend of increasing or decreasing vehicle speed on weekdays can be obtained, as presented in Fig. 1.

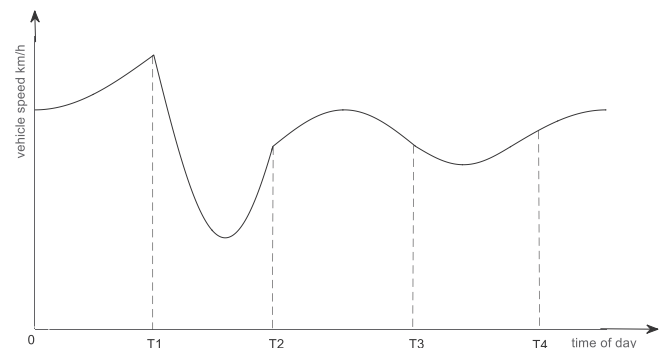
The whole **day** is **divided** into a plurality of **periods** according to the changing trend of vehicle speed, and the functional relationship between vehicle speed ( $v$ ) and the time ( $t$ ) is different in each period. Assuming that the time  $T_i$  when the vehicle leaves node  $i$  is within  $[T_\beta, T_{\beta+1}]$ , there are two possibilities for the vehicle to travel from node  $i$  to node  $j$ , i.e. across the period or not. If  $l_{ij} \leq \int_{T_i}^{T_{\beta+1}} v(t) dt$ , the vehicle arrives at the node  $j$  within  $[T_i, T_{\beta+1}]$  without across the period, the travel time  $t_{ij}$  can be obtained by calculating the upper limit of integration according to the speed function relation of the period; if  $l_{ij} > \int_{T_i}^{T_{\beta+1}} v(t) dt$ , the vehicle needs to across periods, assuming that the vehicle travels from node  $i$  to node  $j$ , the travel time is  $t_{ij} = (T_{\beta+M-1} - T_i) + t_{ij}^{\beta+M}$  and across  $M$  periods, the distance traveled in each period is  $l_{ij}^\beta, l_{ij}^{\beta+1}, \dots, l_{ij}^{\beta+M}$ , and the time  $t_{ij}^{\beta+M}$  traveled in period  $M$  can be obtained by calculating the upper limit of integration according to the speed function relation of the period.

#### 3.3. Calculation of the amount of fuel consumption

This paper uses the MEET model including the correction coefficients for the road gradient and vehicle load given by Hickman et al. (1999) to **calculate vehicle carbon emissions**. According to the relevant research of Alinaghian and Naderipour (2016) on 3.5 t–7.5 t vehicles, combined with the above-mentioned speed time-dependence function, can use the integral idea to obtain the **fuel consumption**  $F_{ij}$  of vehicles traveling from the node  $i$  to node  $j$ .

$$F_{ij} = \lambda \times \int_{T_i}^{T_j} [(110 + 0.000375v^3 + 8702/v) \times GC \times LC] v dt \quad (2)$$

In this equation,  $\lambda = 0.00043L/g$ ,  $GC$  is the road gradient correction coefficient,  $LC$  is the load gradient correction coefficient. The calculation formula of  $GC$  and  $LC$  is:



**Fig. 1.** Vehicle speed-time dependence function.



$$GC = \exp((0.0059v^2 - 0.0775v + 11.936)\xi) \quad (3)$$

$$LC = 0.27\omega + 1 + 0.0614\xi\omega - 0.0011\xi^3\omega - 0.00235v\omega - (0.33/v)\omega \quad (4)$$

In Eq. (3),  $\xi$  is the **road gradient** in terms of percentage, In Eq. (4),  $\omega$  is the ratio of vehicle load to vehicle capacity which is a digit between [0, 1].

### 3.4. Mathematical model

The mathematical optimization model of this paper is formulated to minimize the total costs, as follows:

$$\begin{aligned} \min Z = & c_1 \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} x_{ijk} F_{ij} + c_2 \sum_{i \in D} \sum_{j \in C} \sum_{k \in K} x_{ijk} \\ & + c_3 \sum_{i \in V} \sum_{j \in C} \sum_{k \in K} x_{ijk} \max\{(ET_j - T_{jk}), 0\} \\ & + c_4 \sum_{i \in V} \sum_{j \in C} \sum_{k \in K} x_{ijk} \max\{(T_{jk} - LT_j), 0\} \end{aligned} \quad (5)$$

s.t

$$\sum_{i \in V} \sum_{j \in C} x_{ijk} d_j \leq Q, \forall k \in K \quad (6)$$

$$\sum_{i \in D} \sum_{j \in C} \sum_{k \in K} x_{ijk} \leq |K| \quad (7)$$

$$\sum_{i \in V} \sum_{k \in K} x_{ijk} = \sum_{i \in V} \sum_{k \in K} x_{jik} = 1, \forall j \in C \quad (8)$$

$$\sum_{i \in D} \sum_{j \in C} x_{ijk} = \sum_{i \in D} \sum_{j \in C} x_{jik} \leq 1, \forall k \in K \quad (9)$$

$$\sum_{i \in D} y_{ij} = 1, \forall j \in C \quad (10)$$

$$\sum_{i \in S} \sum_{j \in S} x_{ijk} \leq |S| - 1, \forall k \in K, |S| = \sum_{j \in C} x_{ijk}, \forall i \in D, k \in K \quad (11)$$

$$T_s + \sum_{i \in V} \sum_{j \in V} t_{ij} x_{ijk} + \sum_{i \in V} \sum_{j \in C} \tau_{ik} x_{ijk} \leq T_f, \quad \forall k \in K \quad (12)$$

$$x_{ijk} \in \{0, 1\}, \forall i \in V, \forall j \in V, \forall k \in K, \text{ and } \{i \in D\} \cap \{j \in D\} = \emptyset \quad (13)$$

$$y_{ij} = \{0, 1\}, \forall i \in D, \forall j \in C \quad (14)$$

Eq. (5) represents the objective function, which aims at minimizing the total costs including the fixed costs of vehicles, fuel costs, and time window penalty costs. Eq. (6) represents the capacity limit of the vehicle. In reality, the number of vehicles owned by depots is limited. In this paper, the number of vehicles is considered when optimizing vehicle routing. Eq. (7) indicates that the number of vehicles leaving the depot does not exceed the total number of vehicles that can be used to ensure that the feasible solution meets the actual requirements of logistics enterprises. Eq. (8) indicates that each customer is visited only once by one vehicle, and it is the vehicle arrival and departure balance constraint. Eq. (9) indicates that the vehicle  $k$  may not be selected to leave the depot to visit customers. If the vehicle  $k$  is selected to perform the delivery task, the vehicle  $k$  can only leave a certain depot  $i$  ( $i \in D$ ) to visit customers and can return to any depot. That is, the vehicle  $k$  can return to a depot different from the initial one. Eq. (10) indicates that each customer can only be served by one depot. Eq. (11) is to eliminate the sub-loop constraint. Eq. (12)

restricts the time when the vehicle returns to the depot from exceeding the working deadline of the depot. Eqs. (13) and (14) are decision variables.

### 4. Solution method

The **genetic algorithm** searches for satisfactory solutions by simulating the natural evolution process in biology. It can evaluate multiple feasible solutions in the search space at the same time and has better robustness and global search performance. However, the algorithm has the disadvantages of **slow convergence** speed and easy falling into **local optimization**. The **variable neighborhood search algorithm** (VNS) uses multiple different neighborhood structures for systematic search and has a strong local search capability. In this paper, **customers are clustered** according to the spatial-temporal distance between customers. Combining the advantages of GA and VNS, a hybrid genetic algorithm with variable neighborhood search considering the temporal-spatial distance (HGAVNS\_TS) is developed to solve the proposed problem. The pseudo-code of this algorithm is given below:

#### Algorithm (HGAVNS\_TS)

##### Parameters

- *pop\_size*: population size;
  - *MAX\_gen*: maximum number of iterations;
  - $N_k = \{N_1, N_2, \dots, N_l\}$ : neighborhood structure  $N_l$  is the *lth* neighborhood structure;
  - $S_n$ : adaptive neighborhood search times;
  - *max.N*: maximum number of neighborhood cycles;
1. Initialize  $P(t)$ ; % using a genetic algorithm to cluster customers in temporal-spatial
  2. *gen* = 0;
  3. **while** *gen* < *MAX\_gen* + là gì?
  4. evaluate  $P(t)$ ; %
  5. select  $P(t+1)$  from  $P(t)$ ; % elitist preservation + roulette wheel selection
  6. reproduce pairs in  $P(t+1)$ ; % order crossover
  7. **for** *i* = 1 : *pop\_size*
  8. **for** *j* = 1 : *max.N*
  9. Individual  $P_i(t+1)$  disturbed from the first neighborhood structure  $N_1$ , *iter* ← 1;
  10. **if**  $p \geq p_0$  %  $p_0$  is the acceptance probability of the new solution → 3 lần
  11.  $P_i(t+1) \leftarrow P'_i(t+1)$ ; why 11?
  12. **break**
  13. **else** *iter* ← *iter* + 1;
  14. **end**
  15. **until** (*iter* =  $S_n$ );
  16. The individual  $P_i(t+1)$  continues to be disturbed by the next neighborhood structure  $N_l$ , *iter* ← 1;
  17. **repeat**
  18. **until** (*iter* =  $S_n$ );
  19. **end**
  20. **end**
  21. *gen* = *gen* + 1;
  22. **end**
  23. Best solution

In this paper, the **genetic algorithm** is adopted to **generate the initial solution** by clustering customers in temporal-spatial, and then a hybrid genetic algorithm with variable neighborhood search (HGAVNS) is adopted to solve this problem. Taking an instance with

customer size of  $n$  as an example, if the average total travel time of each feasible solution is  $t\_time$  and the integration step is  $p$ . Then the complexity of each routine is estimated as follows: (1) In step 4, the number of times to invoke “evaluate  $p(t)$ ” is  $MAX\_gen$ , and its time complexity is  $O(T_1) = MAX\_gen \cdot pop\_size \cdot (t\_time/p)$ ; (2) The time complexity of “select” is  $O(T_2) = MAX\_gen$ ; (3) The time complexity of “order crossover” is  $O(T_3) = MAX\_gen \cdot pop\_size \cdot (t\_time/p)$ ; (4) The complexity of variable neighborhood search is  $O(T_4) = MAX\_gen \cdot pop\_size \cdot max\_N \cdot Sn \cdot (t\_time/p)$ . The time complexity of the proposed algorithm is  $O(T) = O(T_1) + O(T_2) + O(T_3) + O(T_4) = O([MAX\_gen \cdot pop\_size \cdot max\_N \cdot Sn \cdot (t\_time/p)])$ .

According to the pseudo-code of this algorithm and the above analysis, the complexity of variable neighborhood search is the main component of the complexity of the proposed algorithm.

#### 4.1. Generation of the initial population

##### (1) Temporal-spatial distance

The existing researches on VRPTW mostly consider the time window and spatial distance of customers separately when clustering customers, ignoring one of the factors, which leads to the higher penalty costs or transportation costs of the generated initial solution. This paper considers the **two-dimensional distribution** characteristics of the customer's **time window and spatial distance** when generating the initial population.

In this paper, Wang et al. (2018) method of measuring the temporal distance between customers is used to quantify the proximity of customers' handle time. Assume that the time windows of customers  $i$  and  $j$  are  $[ET_i, LT_i], [ET_j, LT_j]$ , and  $ET_i < ET_j$ . The time when the vehicle  $k$  arrives at the customer  $i$  is  $T_{ik} \in [ET_i, LT_i]$ , then  $T_{jk}$  is within  $[ET_i + t_{ik}^h + t_{ij}, LT_i + t_{ik}^h + t_{ij}]$ . Let  $[ET_i + t_{ik}^h + t_{ij}, LT_i + t_{ik}^h + t_{ij}]$  be  $[a, b]$ , there are four relationships between  $[ET_j, LT_j]$  and  $[a, b]$ , as shown in Fig. 2.

Eq. (15) is employed to calculate the temporal distance  $l_{ij}^T$  between customer  $i$  and customer  $j$  (Wang et al., 2018).

$$l_{ij}^T = \begin{cases} c_3(ET_j - b), & b < ET_j \\ c_4(a - LT_j), & a > LT_j \\ c_5(t_{ik}^h + l_{ij}/v), & a \leq ET_j < b \text{ or } a < LT_j < b \\ t_{ik}^h + l_{ij}/v, & ET_j < a < b < LT_j \end{cases} \quad (15)$$

In Eq. (15),  $c_5$  is the **deviation coefficient**. The temporal distance is directional, i.e.  $l_{ij}^T \neq l_{ji}^T$ , but the distance is not directional in the clustering process, so the larger of the two is taken as the time distance between customer  $i$  and customer  $j$ .

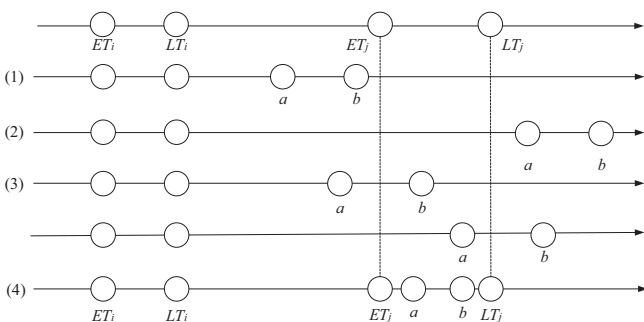


Fig. 2. Temporal distance diagram.

Eq. (16) is employed to calculate the temporal-spatial distance between customers (Qi et al., 2012).

$$l_{ij}^{ST} = \alpha_1 \left[ l_{ij}^S - \min_{\substack{m,n \in C \\ m \neq n}} (l_{mn}^S) \right] / \left[ \max_{\substack{m,n \in C \\ m \neq n}} (l_{mn}^S) - \min_{\substack{m,n \in C \\ m \neq n}} (l_{mn}^S) \right] + \alpha_2 \left[ l_{ij}^T - \min_{\substack{m,n \in C \\ m \neq n}} (l_{mn}^T) \right] / \left[ \max_{\substack{m,n \in C \\ m \neq n}} (l_{mn}^T) - \min_{\substack{m,n \in C \\ m \neq n}} (l_{mn}^T) \right], \quad \alpha_1 + \alpha_2 = 1, i, j \in C \quad (16)$$

##### (2) Customer clustering and initial solution construction

In this paper, a genetic algorithm is used to cluster customers according to the temporal-spatial distance to minimize the sum of the temporal-spatial distance from the customers in each cluster  $i$  ( $i = 1, 2, \dots, u$ ) to the cluster center (Qi et al., 2014). Eq. (17) is the objective function.

$$MinG = \sum_{i=1}^u \sum_{j \text{ assigned to } i} l_{ij}^{ST}, \quad \forall i, j \in C \quad (17)$$

The algorithm in this paper adopts the integer encoding method. **Set up a virtual depot 0**, and divide the vehicle routing according to the solution strategy of “routing first and then group” with the vehicle capacity as the constraint. The process is as follows:

- 1) **Select the customer closest to the virtual depot and assign available vehicles.** After visiting this customer, visit other customers in the cluster in turn according to the principle of temporal-spatial distance from near to far. For customers that exceed the vehicle capacity constraint, the residual capacity of vehicles delivered by adjacent clusters is sequentially detected according to the principle of temporal-spatial distance from near to far to meet the customer demand, and if there is a cluster that meets the customer demand, it is added to the cluster; otherwise, divide the customer into a new cluster.
- 2) **Check whether there are any unassigned clusters and if so, go to step 1).**
- 3) After the assignment task division of all clusters is completed, calculate the distance between the first and last customers and each depot in the routing of each vehicle.
- 4) According to the principle of minimum distance, the depot is inserted between the first and last customers and the virtual depot to obtain the initial solution.

#### 4.2. Fitness function

The fitness function of chromosomes can be constructed according to Eq. (5). The fitness function of the chromosome  $R$  can be expressed as follows:

$$f_R = \frac{1}{Z_R} \quad (18)$$

In Eq. (18),  $Z_R$  is the objective function value of the chromosome  $R$ . The smaller the objective value of a chromosome, the greater its fitness, and the greater the probability of being selected.

#### 4.3. Select operation

The selection operation adopts the strategy of combining elite reservations with the roulette wheel selection. Specific steps are as follows: using the roulette wheel selection method, the probability of each individual being selected is proportional to its fitness value, i.e. the higher the fitness value, the higher the probability of the individual being selected. And the improved method of the

elite reservation is adopted. Firstly, an individual  $A$  with the highest fitness value in the population is stored in the temporary matrix. After the neighborhood search is completed, if the optimal solution has not changed, the similarity between the individual  $A$  and the sub-generation individuals is calculated and the sub-generation  $A_1$  with the highest similarity with the individual  $A$  is replaced. The variable neighborhood search strategy adopted by the algorithm in this paper may still achieve the level of the individual  $A$  for the sub-generation  $A_1$  by further depth search. In contrast, although the fitness value of the optimal individual  $B$  in the sub-generation is lower than that of the individual  $A$ , it has some good gene sequences and can provide better genes in the population crossing of the next generation. Compared with the sub-generation  $A_1$ , it has more value of continuous iteration.

#### 4.4. Evolutionary operation

The evolutionary operation in this paper adopts an order crossover operator. In accordance with Fig. 3, when order crossover is performed on the parent  $A$ , the parent  $B$  is randomly selected from the population. Firstly, points  $i_{11}, i_{12}, i_{21}$  and  $i_{22}$  are randomly generated respectively, the part between  $i_{11}$  and  $i_{12}$  of the parent  $A$  is taken as the first segment of the sub-generation  $A_1$  and the subsequent points of the sub-generation  $A_1$  are related to the parent  $B$ , i.e. the customer points between  $i_{11}$  and  $i_{12}$  in the parent  $B$  are eliminated first. In the elimination process, the position sequence of the customer points in the parent  $B$  is not changed, and then the eliminated customer points are arranged as the second segment of the sub-generation  $A_1$  to form the sub-generation  $A_1$  and the sub-generation  $B_1$ . The order crossover operator used in this paper is unidirectional, that is, it only accepts the operation in a good direction.

#### 4.5. Local search strategy

The local search strategy is one of the core functions of the algorithm, which determines the local search capability of the algo-

rithm. In this paper, a variable neighborhood search is applied to the local search strategy to strengthen the depth search of the population.

##### (1) Neighborhood structure

- 1) Insert: Randomly select a customer  $i$  from the chromosome and insert it randomly after the customer  $j$ . As shown in Fig. 4(a), customer 3 is inserted behind customer 6.
- 2) Exchange: Randomly select two customers  $i$  and  $j$ , then exchange their positions. As shown in Fig. 4(b), exchange the positions of customer 3 and customer 6.
- 3) 2-OPT: Randomly select customers  $i$  and  $j$ , and exchange the order of other customers among customer points. As shown in Fig. 4(c), the position of customer 3 is kept unchanged, and customers 4, 5, 7, and 6 are in reverse order.

##### (2) Adaptive mechanism and solution acceptance

In this paper, we design an adaptive neighborhood search times strategy and a new solution acceptance rule to balance the breadth and depth required for evolution, so that the algorithm jumps out of the local optimum. The disturbance intensity required by the population is different at different stages of the algorithm. The strategy of adaptive neighborhood search times in this paper is as follows:

$$S_n = \beta_1 + \left\lfloor \beta_2 \cdot \left( \frac{gen}{MAX\_gen} \right) \right\rfloor \quad (19)$$

In Eq. (19),  $S_n$  is the number of search times when the algorithm runs to the  $gen$  generation,  $\beta_1$  is the minimum number of searches,  $\beta_2$  is the adaptive number of searches,  $\lfloor \cdot \rfloor$  represents round down. It can be seen from Eq. (19) that the search times gradually increase with the increase of iteration number, which increases the depth search of the population.

To further improve the perturbation of the population and expand the search space, this paper uses the acceptance rule of

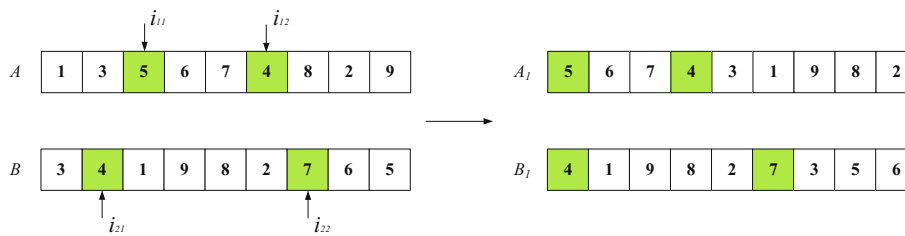


Fig. 3. The diagram of order crossover operator.

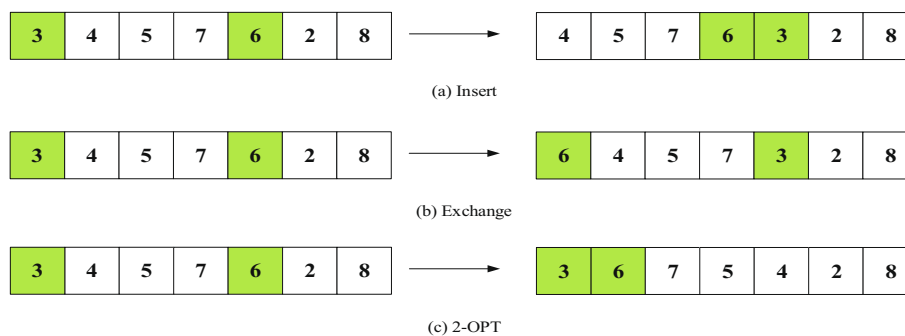


Fig. 4. Neighborhood structures.

the solution in simulated annealing algorithm to accept the poor solution with a certain probability to improve the algorithm's optimization ability. Eq. (20) is the probability of new solution acceptance.

$$p = \begin{cases} 1, & f_R(x) \leq f_R(x') \\ \exp\left(\frac{f_R(x) - f_R(x')}{\text{gen}}\right), & f_R(x) > f_R(x') \end{cases} \quad (20)$$

## 5. Computational experiment and analyses

To test the performance of the proposed algorithm, Section 5.1 uses HGAVNS\_TS to solve the basic problem of this paper, and the solution results are compared with other heuristics. Section 5.2 designs the instance of this problem and uses HGAVNS\_TS to solve it, and sets up five cases to analyze the influence of vehicle speed

and time windows on the solution result. The structure of Section 5 is shown in Table 3.

### 5.1. Test the performance of the algorithm

MDVRP and MDVRPTW are the basic problems of this paper, this paper selects MDVRP and MDVRPTW instances to test the algorithm. The algorithm is programmed with MATLAB2018b in the Windows10 system, the running memory of the computer is 4G, the CPU is Intel(R)Core(TM)i7-7700, and the dominant frequency is 3.60 GHz. Instances of testing algorithm performance cover different customer sizes, customer distributions, and time window sizes, which requires that different parameters should be set during algorithm operation. Firstly, the function of parameters in the algorithm is analyzed theoretically, and then the influence of parameter values on the running results is analyzed. Finally, the parameter values are determined through repeated

**Table 3**  
The structure of Section 5.

Section	Content
5.1 Test the performance of the algorithm	(1) Instance analysis of MDVRP HGAVNS_TS was used to solve MDVRP, and the solution results were compared with adaptive large neighborhood search (ALNS 50 K) (Pisinger and Ropke, 2007) and hybrid genetic search with adaptive diversity control (HGSADC) (Vidal et al., 2012) to verify the effectiveness of HGAVNS_TS in solving MDVRP. (2) Instance analysis of MDVRPTW HGAVNS_TS was used to solve MDVRPTW, and the solution results were compared with the variable neighborhood search algorithm (VNS) and hybrid genetic algorithm with variable neighborhood search (HGAVNS). Besides, HGAVNS_TS is used to solve the instance of MDVRPTW designed by Zhong et al. (2006), and compared with other heuristics to verify the effectiveness of HGAVNS_TS in solving MDVRPTW.
5.2 Case study	(1) Network description and parameters setting Design the case of the problems proposed in this paper and set the relevant parameters. (2) Optimization results Use HGAVNS_TS to solve the designed case. (3) Sensitivity analyses Five cases are set to analyze the influence of vehicle speed and time windows on the solution results

**Table 4**  
Comparison of HGAVNS\_TS with other heuristics on MDVRP instances.

Instance	n	d	Pre-best	ALNS			HGSADC			HGAVNS_TS			
				best	Avg	t/min	best	Avg	t/min	best	Avg	t/min	Gap(%)
P01	50	4	576.87 <sup>a</sup>	576.87	576.87	0.48	576.87	576.87	0.23	577.63	580.60	0.22	0.13
P02	50	4	473.53 <sup>a</sup>	473.53	473.53	0.47	473.53	473.53	0.21	473.53	475.86	0.19	0.00
P03	75	5	641.19 <sup>b</sup>	641.19	641.19	1.07	641.19	641.19	0.43	646.12	652.36	0.87	0.77
P04	100	2	1001.04 <sup>d</sup>	1001.04	1006.09	1.47	1001.04	1001.23	1.94	1016.31	1027.06	1.92	1.53
P05	100	2	750.03 <sup>b</sup>	751.26	752.34	2.00	750.03	750.03	1.06	754.76	758.81	1.42	0.63
P06	100	3	876.50 <sup>c</sup>	876.70	883.01	1.55	876.50	876.50	1.14	895.91	897.11	1.57	2.21
P07	100	4	881.97 <sup>d</sup>	881.97	889.36	1.47	881.97	884.43	1.55	901.36	907.87	2.04	2.20
P08	249	2	4372.78 <sup>e</sup>	4390.80	4421.03	5.55	4372.78	4387.38	10.00	4465.30	4547.15	15.36	2.12
P09	249	3	3858.66 <sup>e</sup>	3873.64	3892.50	6.02	3858.66	3873.64	9.50	3921.52	4094.31	13.83	1.63
P10	249	4	3631.11 <sup>e</sup>	3650.04	3666.85	6.05	3631.11	3650.04	9.82	3687.84	3744.58	14.22	1.56
P11	249	5	3546.06 <sup>d</sup>	3546.06	3573.23	5.95	3546.06	3546.06	7.14	3622.31	3689.53	13.56	2.15
P12	80	2	1318.95 <sup>c</sup>	1318.95	1319.13	1.25	1318.95	1318.95	0.52	1319.46	1324.06	0.83	0.04
P13	80	2	1318.95 <sup>c</sup>	1318.95	1318.95	1.00	1318.95	1318.95	0.57	1325.63	1331.28	1.37	0.51
P14	80	2	1360.12 <sup>b</sup>	1360.12	1360.12	0.97	1360.12	1360.12	0.55	1376.32	1381.61	1.31	1.19
P15	160	4	2505.42 <sup>b</sup>	2505.42	2519.64	4.22	2505.42	2505.42	1.92	2515.87	2543.17	6.14	0.42
Pr01	48	4	861.32 <sup>b</sup>	861.32	861.32	0.50	861.32	861.32	0.17	864.63	869.25	0.20	0.38
Pr02	96	4	1307.34 <sup>d</sup>	1307.34	1308.17	1.72	1307.34	1307.34	0.76	1310.95	1316.50	1.44	0.28
Pr03	144	4	1803.80 <sup>e</sup>	1806.60	1810.66	3.57	1803.80	1803.80	1.91	1836.32	1890.12	3.97	1.80
Pr04	192	4	2058.31 <sup>e</sup>	2060.93	2073.16	4.93	2058.31	2060.93	5.22	2088.10	2103.33	7.89	1.45
Pr05	240	4	2331.20 <sup>e</sup>	2337.84	2350.31	6.20	2331.20	2337.84	9.56	2396.50	2419.33	12.66	2.80
Pr06	288	4	2676.30 <sup>e</sup>	2687.60	2695.74	7.75	2676.30	2685.35	10.00	2793.31	2866.03	17.75	4.37
Pr07	72	6	1089.56 <sup>b</sup>	1089.56	1089.56	0.97	1089.56	1089.56	0.34	1103.63	1109.22	0.76	1.29
Pr08	144	6	1664.85 <sup>d</sup>	1664.85	1675.74	3.45	1664.85	1665.05	2.05	1679.03	1697.42	3.51	0.85
Avg. Gap				0.11	0.44		0.00	0.10		1.32	2.47		

Source of instances: <http://neo.lcc.uma.es/vrp/vrp-instances/multiple-depot-vrp-instances/>.

Computer configurations used to test the algorithms: the ALNS runs on a Pentium IV 3.0 GHz; the HGSADC runs on an Opteron 2.4 GHz scaled for a Pentium IV 3.0 GHz.

<sup>a</sup> Optimality proved.

<sup>b</sup> Results of Cordeau et al. (1997).

<sup>c</sup> Results of Renaud et al. (1996).

<sup>d</sup> Results of Pisinger and Ropke (2007).

<sup>e</sup> Results of Vidal et al. (2012).



experiments. After testing, the parameter settings are as follows: the maximum number of iterations  $max\_gen = 50$  when the genetic algorithm generates an initial solution by temporal-spatial clustering of customers,  $\alpha_1 = 0.6$ ,  $\alpha_2 = 0.4$ ,  $pop\_size = 50 \sim 150$ ,  $MAX\_gen = 100 \sim 500$ ,  $max\_N = 30 \sim 100$ ,  $\beta_1 = 1$ ,  $\beta_2 = MAX\_gen/10 \sim MAX\_gen/5$ , the new solution acceptance probability  $p = 0.98 \sim 1$ . The larger the number of customers, the larger the settings of  $MAX\_gen$ ,  $pop\_size$  and  $max\_N$ .

### (1) Instance analysis of MDVRP

Table 4 shows the results of the ALNS 50 K, HGSADC and HGAVNS\_TS. The results of some instances obtained by ALNS and HGSADC are current the best solution.  $n$  is the number of customers,  $d$  is the number of depots,  $Pre\_best$  is the best-known solutions,  $best$  is the best solution obtained by the algorithm,  $Avg$  is the average result of 10 runs of the algorithm,  $t$  is the time spent in one experiment (Unit: minutes),  $Gap$  is the gap between the best solution obtained by HGAVNS\_TS and the  $Pre\_best$ , and  $Avg\_gap$  is the average gap between the average solution and the  $Pre\_best$ . Table 4 shows that in the results of the benchmark instances solved by HGAVNS\_TS. The minimum value of  $Gap$  is 0.00% and the maximum is 4.73%. The average gap between the best solution and the average solution relative to the best-known solution is 1.32% and 2.47% respectively. In terms of algorithm running time, when the number of customers is less than 100, the solution time of the proposed algorithm is similar to that of ALNS and HGSADC, but when the number of customers exceeds 100, the solution time of the proposed algorithm increases, but it is acceptable. The effectiveness of the HGAVNS\_TS in solving the MDVRP problem can be verified by comparing the calculation results and the running time of the algorithm.

### (2) Instance analysis of MDVRPTW

To verify the performance of the proposed algorithm for solving MDVRP with soft time windows, this paper adopts the VNS, HGAVNS and HGAVNS\_TS to solve 12 MDVRPTW instances, and compares and analyzes the results. PrA\_B represents the first B customers of PrA (for example, Pr04\_48 represents the first 48 customers of Pr04), VNS and HGAVNS randomly generate the initial population, and VNS adopts three neighborhood structures: insert, exchange and 2-opt. Allow vehicles to return to the nearest depot, set vehicle speed  $v = 1$ , the transportation cost per unit distance is 1.5,  $c_3 = 1$ ,  $c_4 = 3$ ,  $c_5 = 2$ . Table 5 summarizes the results of the algorithm running 10 times, and  $Best$  is the best solution obtained by the three algorithms. Fig. 5 is a quality comparison diagram of

randomly generating an initial population and initializing the population by clustering customers in temporal-spatial when solving Pr02. Fig. 6 shows the iterative process of three algorithms to solve Pr02.

Compared with the results of the MDVRP example, the time window constraint is considered in MDVRPTW, which increases the time spent to obtain the optimal solution for the same customer size. Fig. 5 shows that the quality of the initial solution can be improved by initializing the population by spatiotemporal clustering of customers. It can be seen from Table 5 and Fig. 6 that HGAVNS\_TS has good initial solution quality and strong local searchability, and the improved algorithm has better solution performance.

Zhong et al. (2006), Duan and Fu (2008), and Ling and Gu (2017) studied the multi-depot open vehicle routing problem with time windows, which is the basic problem of this paper. To further verify the performance of the algorithm for MDVRP with soft time window, HGAVNS\_TS is adopted to solve the instance with 3 depots and 24 customers. The algorithm parameters are set as follows:  $max\_gen = 50$ ,  $pop\_size = 50$ ,  $max\_N = 60$ ,  $\beta_1 = 1$ ,  $\beta_2 = MAX\_gen/5$ ,  $p = 1$ . Table 6 summarizes the best solutions under different time window penalty costs obtained by genetic algorithm (GA) (Zhong et al., 2006), tabu search algorithm (TS)

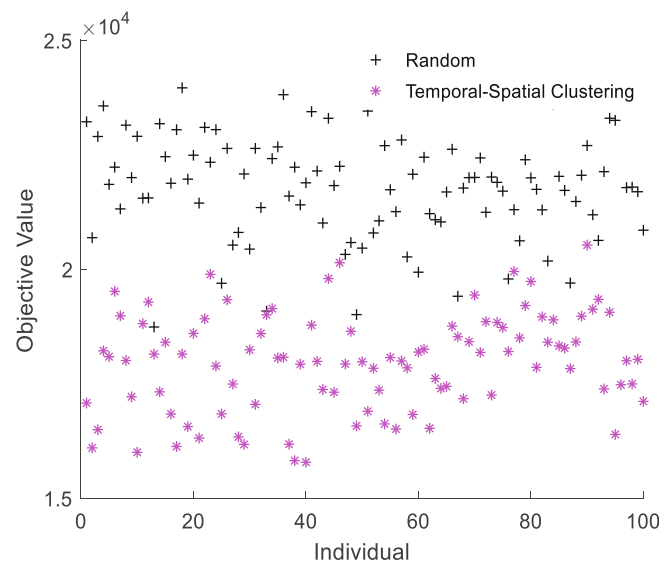


Fig. 5. The objective value of the initial solution of Pr02.

**Table 5**  
Comparison of HGAVNS\_TS with other heuristics on MDVRPTW instances.

Instance	$n$	$d$	$Best$	VNS			HGAVNS			HGAVNS_TS			
				$best$	$Avg$	$t/min$	$best$	$Avg$	$t/min$	$best$	$Avg$	$t/min$	$Gap$ (%)
Pr01	48	4	<b>4343.3</b>	4612.8	5044.0	0.41	4423.0	4574.7	0.73	<b>4343.3</b>	4504.8	0.48	0
Pr11	48	4	<b>2441.0</b>	2657.7	2910.3	0.41	2450.2	2514.8	0.69	<b>2441.0</b>	2467.0	0.45	0
Pr05_48	48	4	<b>5575.4</b>	5931.1	5994.4	0.44	5768.7	5848.2	0.80	<b>5575.4</b>	5683.2	0.58	0
Pr08_48	48	6	<b>4436.9</b>	5273.7	5388.2	0.39	4648.1	4999.3	0.74	<b>4436.9</b>	4478.9	0.54	0
Pr07	72	6	<b>5788.9</b>	5967.0	6156.2	2.02	5970.0	6199.3	3.67	<b>5788.9</b>	5940.1	2.15	0
Pr17	72	6	<b>2988.7</b>	3910.4	4058.8	1.97	3034.9	3365.4	3.41	<b>2988.7</b>	3093.7	1.99	0
Pr06_72	72	4	<b>7892.5</b>	9307.2	9369.7	2.18	8063.6	8527.9	4.02	<b>7892.5</b>	8022.2	2.06	0
Pr08_72	72	6	<b>5276.6</b>	5580.6	5738.4	2.01	5368.4	5615.4	3.50	<b>5276.6</b>	5375.0	2.60	0
Pr02	96	4	<b>7853.1</b>	7902.3	8078.4	4.51	8073.1	8295.7	6.62	<b>7853.1</b>	7972.7	5.15	0
Pr12	96	4	<b>3923.2</b>	4726.3	4841.6	4.31	4363.9	4453.9	6.16	<b>3923.2</b>	4024.9	5.34	0
Pr06_96	96	4	<b>11187.7</b>	11716.9	11916.5	5.02	11214.0	11514.7	6.24	<b>11187.7</b>	11251.9	5.57	0
Pr03	144	4	<b>12139.3</b>	13679.4	13808.6	12.37	12857.4	13206.5	30.04	<b>12139.3</b>	12299.7	20.35	0
Avg_Gap (%)p				11.37	15.04		3.27	7.58		0	1.94		

Source of instances: <https://neo.lcc.uma.es/vrp/vrp-instances/multiple-depot-vrp-with-time-windows-instances/>.

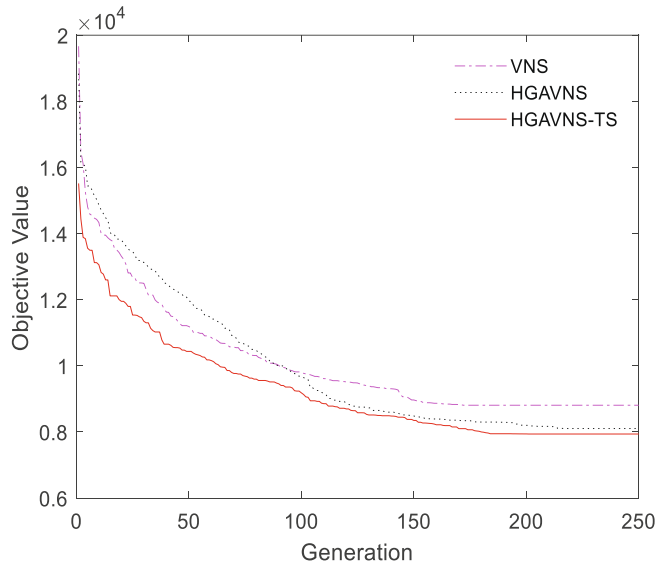


Fig. 6. Iterative process for solving Pr02.

(Duan and Fu, 2008), improved ant colony algorithm (IACO) (Ling and Gu, 2017) and HGAVNS\_TS. It can be seen from Table 6 that the minimum gap between the results obtained by GA, IACO, and TS and the best solution obtained by HGAVNS\_TS is 0.06%, and the maximum gap is 8.54%.

## 5.2. Case study

With the development of business, some supermarket chains, express companies and fast-moving consumer goods companies have chosen to establish multiple depots in the service areas for joint distribution in order to reduce the high costs caused by long-distance delivery and improve the delivery timeliness. In reality, the traffic environment is dynamic, and traffic congestion may make it impossible to provide services to customers within the expected time. Besides, considering the fuel consumption in the

delivery process can effectively reduce the negative impact of cargo transportation on the environment. The integer planning model developed in this paper takes into account factors such as multi-depots, time-dependent vehicle speed, time windows and fuel consumption, which is closer to the actual delivery production activities and ensures the sustainable transportation of goods. In this paper, the proximity of temporal distance between customers is quantified and combined with spatial distance to cluster customers in the temporal-spatial distance. At the same time, the variable neighborhood search strategy is introduced into the genetic algorithm to enhance the searchability and improve the performance of the algorithm. The performance of the proposed mathematical model and heuristic algorithm to solve this problem is verified and analyzed by numerical experiments.

### (1) Network description and parameters setting

The benchmark instance Pr01 is selected and modified as the instance of the proposed problem. The instance includes 4 depots and 48 customers, and the node coordinate position is the same as Pr01. The rest information of depots and customers is shown in Table 7. Parameter settings are as follows:  $Q = 5t$ ,  $\tau_{ik} = 0.5d_i$ ,  $c_1 = 6.9$ ,  $c_2 = 500$ ,  $c_3 = 20$ ,  $c_4 = 30$ ,  $c_5 = 25$ ,  $\xi = 0$ . There are three types of roads in the distribution network: urban main roads, secondary roads and branch roads. As shown in Fig. 7, red represents the main roads, black represents secondary roads and undrawn routes are branch roads. Fig. 8 shows vehicle speeds for three types of roads.

### (2) optimization results

The proposed algorithm is used to solve the problem in this paper. The parameters of the algorithm are set as follows:  $max\_gen = 50$ ,  $pop\_size = 100$ ,  $MAX\_gen = 150$ ,  $max\_N = 60$ ,  $\beta_1 = 1$ ,  $\beta_2 = MAX\_gen/5$ ,  $p = 0.99$ . The distribution scheme is shown in Table 8, and vehicle routing is shown in Fig. 9. To verify the effectiveness of considering temporal-spatial distance in this experiment, HGAVNS is used to solve the problem under the same conditions. The iterative process of the two algorithms is shown in

Table 6  
Results of comparison between HGAVNS\_TS and GA, TS, IACO.

Parameters	Best	GA	Gap (%)	IACO	Gap (%)	TS	Gap (%)	HGAVNS_TS	Gap (%)	t/s
$c_3 = 0.2, c_4 = 2$	<b>665.4</b>	665.8	0.06	675.1	1.46	–	–	<b>665.4</b>	0	3.05
$c_3 = 0, c_4 = 2$	<b>612.5</b>	–	–	641.3	4.70	664.8	<b>8.54</b>	<b>612.5</b>	0	3.27

Source of instances: Zhong et al. (2006).

Table 7  
Information about depots and customers.

Node	$d_i$	$ET_i$	$LT_i$	Node	$d_i$	$ET_i$	$LT_i$	Node	$d_i$	$ET_i$	$LT_i$	Node	$d_i$	$ET_i$	$LT_i$
D1		6	18	10	0.9	7.5	16.5	23	0.7	7.5	16.5	36	0.5	8.5	14
D2		6	18	11	0.35	8.5	13.5	24	0.95	6.5	16.5	37	0.55	7.5	15.5
D3		6	18	12	0.3	7.5	16.5	25	0.7	8.5	13.5	38	0.75	7.5	16.5
D4		6	18	13	0.45	7.5	15.5	26	0.3	6.5	12	39	0.65	6.5	12
1	0.6	6.5	10	14	0.45	6.5	10.5	27	0.8	7.5	16.5	40	0.75	7.5	16.5
2	0.4	7.5	16.5	15	0.2	6.5	9.5	28	0.45	8.5	13.5	41	0.4	7.5	16.5
3	0.8	8.5	15.5	16	1.25	7.5	16.5	29	1	7.5	16.5	42	1.1	7.5	11.5
4	0.25	7.5	16.5	17	0.25	7.5	15.5	30	0.65	6.5	12	43	1.2	7.5	16.5
5	0.6	8.5	13.5	18	0.85	6.5	9.5	31	0.5	7.5	15.5	44	0.15	7.5	16.5
6	0.25	7.5	16.5	19	0.15	7.5	16.5	32	0.8	6.5	10.5	45	1.25	7.5	11.5
7	0.65	6.5	9.5	20	0.8	6.5	12	33	0.95	6.5	9.5	46	0.95	6.5	13.5
8	1	8.5	13.5	21	1.25	7.5	12	34	1.1	7.5	12	47	1.05	7.5	12
9	0.65	6.5	10	22	1.05	7.5	16.5	35	0.7	7.5	16.5	48	0.5	7.5	16.5

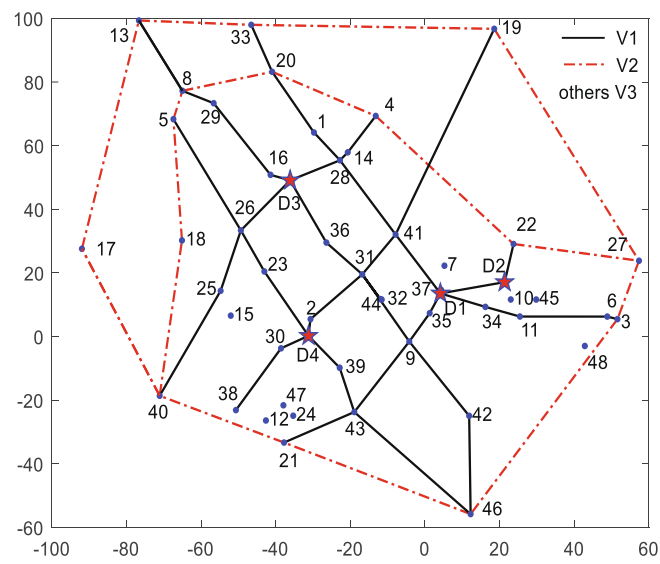


Fig. 7. Distribution network.

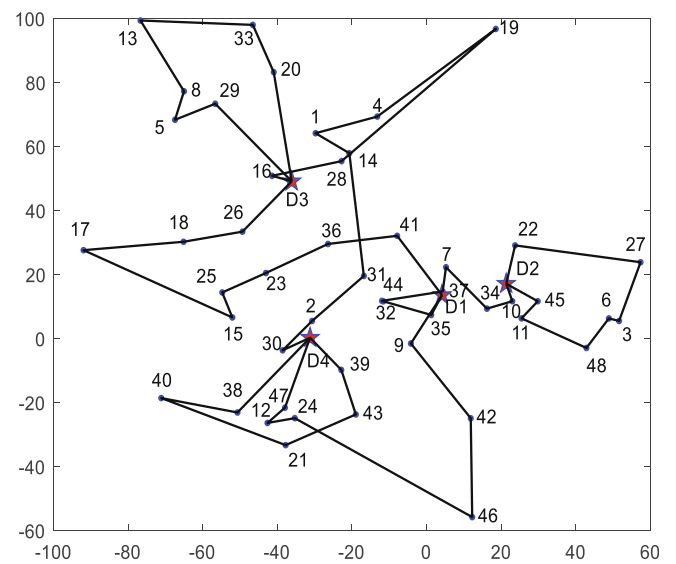


Fig. 9. Vehicle routing corresponding to the best solution.

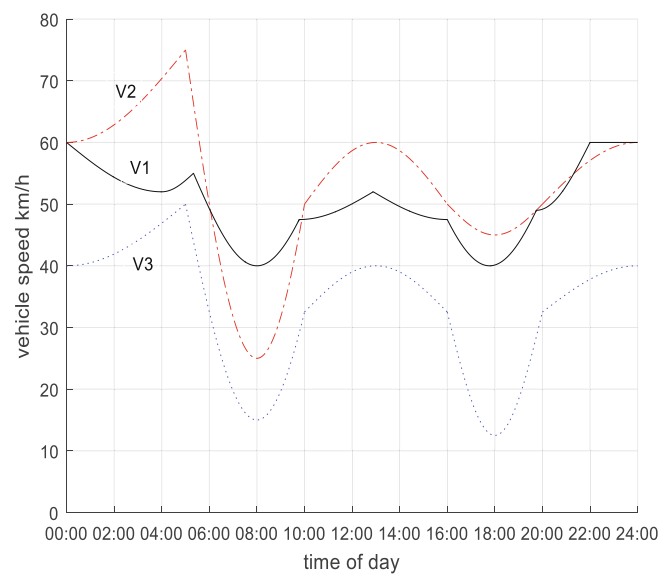


Fig. 8. The time-dependent function of speed.

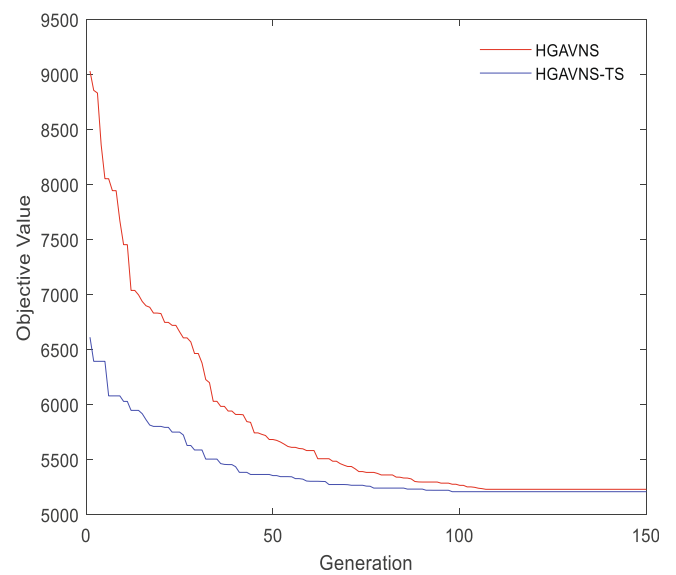


Fig. 10. Comparison of the iterative process.

Fig. 10. It can be seen from Fig. 10 that the objective value of the best individual in the initial population generated by the spatiotemporal clustering of customers is better than that of the initial population generated randomly, and the convergence algebra is reduced, thus accelerating the convergence speed. It is further proved that the algorithm has a strong optimization ability.

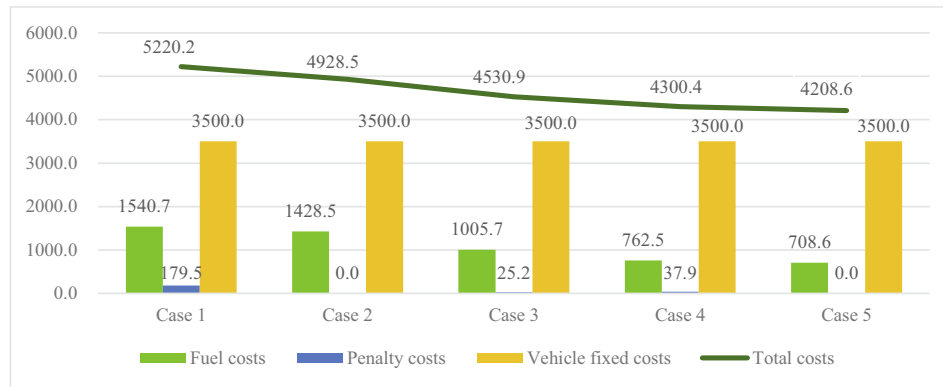
It can be seen from Table 8 that if the time window penalty cost is ignored, the total cost is 5040.66, that is, the opportunity loss cost of the optimized scheme is 179.49. If the traveling speed of vehicles is constant at 30 km/h, 40 km/h and 50 km/h, the corresponding total costs of the delivery scheme are 5493.31, 4926.12 and 4642.15 respectively, and the deviations from those when con-

Table 8  
The best distribution scheme.

Vehicle	Distribution route	Distance	Costs	Fuel costs	Penalty costs
1	D1-9-42-46-24-12-47-D4	170.25	741.36	219.86	21.50
2	D3-20-33-13-8-5-29-D3	158.28	726.70	216.70	10.00
3	D4-39-43-21-40-38-D4	136.44	711.14	185.65	25.49
4	D4-30-2-31-14-1-4-19-28-16-D3	232.23	852.93	317.93	35.00
5	D2-22-27-3-6-48-11-45-D2	116.05	710.23	168.73	41.50
6	D1-35-32-44-37-7-34-10-D2	74.80	670.20	140.20	30.00
7	D3-26-18-17-15-25-23-26-41-D1	189.91	807.59	291.59	16.00
Summation		1077.96	5220.15	1540.66	179.49

**Table 9**  
Parameter setting of sensitivity analysis.

	Case1	Case2	Case3	Case4	Case5
Time windows	$[ET_i, LT_i]$	–	$[ET_i, LT_i]$	$[ET_i, LT_i]$	–
Vehicle speed	Time-dependent	Time-dependent	30	40	40
Parameters	$c_3 = 20, c_4 = 30$	–	$c_3 = 20, c_4 = 30$	$c_3 = 20, c_4 = 30$	–



**Fig. 11.** Comparison of sensitivity analysis results.

sidering all features are 5.23%, –5.63% and –11.07% respectively. Therefore, it is very important to consider these features when formulating optimization models.

### (3) Sensitivity analyses

In the previous MDVRP related studies, some researchers ignored the time dependence of vehicle speed or did not consider the time window, and there may be some deviation between the solution result of the corresponding model and the actual cost. To analyze the influence of time window and the change of vehicle speed on the formulation of the delivery scheme, five cases are set up and the results of the delivery schemes solved under different conditions are compared and analyzed. Among them, Case1 is the numerical experiment of this paper, Case2 ignores the time window constraint, Case3 and Case4 ignore the time dependence of vehicle speed, Case5 ignores the time window constraint and the vehicle speed is 40 km/h, as shown in Table 9. The proposed algorithm is used to solve the models in five cases, and the delivery schemes and costs in different cases are obtained. The best solution in 10 runs is shown in Fig. 11.

It can be seen from Fig. 11 that the cost of the scheme obtained when ignoring the time window constraint is 4828.5, which is 291.7 different from the delivery scheme obtained under Case1. When the time dependence of vehicle travel time is ignored and the vehicle travel speed is constant at 30 km/h and 40 km/h, the cost of the delivery scheme obtained by the corresponding model is different from the optimization result considering all features by –13.20% and –17.62%, respectively. The total cost difference between case3 and Case4 is 230.5, and the difference in fuel costs is 243.3. Comprehensive analysis shows that the change of vehicle speed and customer time window will have a great impact on the optimization results. Therefore, it is of great significance to study the proposed problem in this paper.

## 6. Conclusions

In reality, more and more enterprises have multiple depots due to the integration of resources, and the dynamic changes of the traffic environment also affect the distribution of goods all the

time. Seeking new optimization strategies and obtaining better distribution schemes can effectively reduce distribution costs. In this paper, the influence of the fixed costs of vehicles, traffic information of distribution network and the temporal-spatial between customers on the total costs are considered comprehensively, an integer programming model to minimize the distribution costs is constructed and develops a hybrid genetic algorithm with variable neighborhood search considering the temporal-spatial to solve the proposed problem. Numerical results show that the model and metaheuristic we proposed are rather effective. Obtains the following conclusions:

- (1) The formulated model has fully considered the sharing of customer and vehicle resources among multiple depots, effectively improved the distribution efficiency, and further expanded and deepened the VRP research.
- (2) The model considers the smooth increase or decrease of the vehicle speed and the influence of vehicle speed, load, and road gradient on fuel consumption. Although it increases the complexity of the problem and the difficulty of solving it, it is closer to the reality of distribution production activities.
- (3) The designed hybrid genetic algorithm with variable neighborhood search considering the temporal-spatial effectively improves the quality of the solution by generating initial solutions through the temporal-spatial clustering of customers.
- (4) The selection operation adopts the strategy of combining elite reservations with the roulette wheel selection ensures the effective convergence of the algorithm. The designed adaptive neighborhood search times strategy and simulated annealing inferior solution acceptance mechanism can balance the diversification and exploitation in the algorithm iteration process.

The research in this paper applies to the time-dependent multi-depot green vehicle routing problem with time windows, which can further reduce the distribution costs of logistics enterprises and provide a theoretical solution for the multi-region joint distribution of logistics enterprises.



## CRediT authorship contribution statement

**Houming Fan:** Conceptualization, Methodology, Investigation, Supervision. **Yueguang Zhang:** Writing - original draft, Data curation, Software. **Panjun Tian:** Writing - review & editing, Validation. **Yingchun Lv:** Writing - review & editing, Validation. **Hao Fan:** Writing - review & editing, Validation.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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