ELSEVIER

Int. J. Production Economics 77 (2002) 113–130

international journal of production economics

www.elsevier.com/locate/dsw

Incorporating transportation costs into inventory replenishment decisions

Scott R. Swenseth[a], Michael R. Godfrey[b,*]

[a] Department of Management, University of Nebraska-Lincoln, Lincoln, NE 68508, USA
[b] College of Business Administration, University of Wisconsin Oshkosh, Oshkosh, WI 54901-8675, USA

Accepted 27 November 2001

Abstract

Dating to the origination of economic order quantity (EOQ) models, the objective of inventory replenishment decisions has centered on the minimization of total annual logistics cost. Accurate solutions require that all relevant costs be appropriately incorporated into the total annual logistics cost function to determine purchase quantities. Depending on the estimates used, upwards of 50% of the total annual logistics cost of a product can be attributed to transportation. Any consideration of purchase quantities should therefore consider transportation costs. To appropriately incorporate transportation cost into the total annual logistics cost function, it must first be possible to identify transportation cost functions that emulate reality and simultaneously provide a straightforward representation of actual freight rates. This study demonstrates that straightforward freight rate functions presented in the literature can be incorporated into inventory replenishment decisions without compromising the accuracy of the decision. Equally important, these functions can be incorporated without adding undue complexity to the decision process. © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Logistics; Purchasing; Inventory models; Transportation; Freight rates

1. Introduction

Attention to inbound material transportation has received increased impetus with the relaxation of regulations in the transportation industry. For example, although Just-in-Time (JIT) was discussed as a concept prior to deregulation, economic regulation of transportation within the United States had acted as a significant barrier to JIT logistics implementation [1]. Deregulation has led to an increasingly competitive environment where shipping rates and services must address innovative shipping strategies [2]. Today, with the increased emphasis on supply chain management and enterprise resource planning (ERP), the need to develop models with appropriate representation of transportation considerations is further enhanced.

Although some motor carriers provide software for rate lookups (and thus simplified rate determination), this software is limited in that it cannot be used by a logistics decision maker to determine an optimal lot size based upon simultaneous consideration of holding, order, and transportation costs. Actual shipping decisions fall into three

*Corresponding author. Tel.: +1-920-424-1232.
E-mail address: godfrey@uwosh.edu (M.R. Godfrey).

PII: S0925-5273(01)00230-4

categories: (1) shipments that result in true truckload (TL) shipping quantities, (2) shipments that are likely to be over-declared as TL, and (3) shipments that are not likely to be over-declared as TL and are therefore shipped at less-than-truckload (LTL) rates. Not all shippers move large enough shipments to rely exclusively upon TL shipments. Therefore, the purpose of this study was to extend the consideration of transportation costs in inventory replenishment decisions to include environments where it was not so clear whether it would be appropriate to over-declare to TL shipments.

In the model presented in this study, two freight rate functions, the inverse and the adjusted inverse, were incorporated into the total annual logistics cost function to determine their impact on purchasing decisions. The inverse function was used in this study because of its ability to model freight rates exactly when TL shipping weights are transported. The adjusted inverse function was used because it takes on the same characteristics as the inverse function and because it emulates LTL freight rates particularly well.

The model presented here can be used to determine the following: (1) the minimum weight at which a particular shipment should be over-declared to a TL, and therefore, whether a particular shipment should be over-declared to a TL shipment or shipped LTL, (2) an estimate of the unknown parameters necessary for calculating purchase order quantities; and (3) purchase order quantities based on annual ordering, holding, and transportation costs. A complete range of shipping weights (including both LTL and TL weights) was incorporated into the model presented in this paper because it was not apparent until after the shipping weight was determined whether the shipment should have been over-declared as a TL shipment or shipped LTL.

Section 2 provides a discussion of freight rates in practice. Literature related to freight rate functions and inventory models is discussed in Section 3. A complete inventory replenishment model is presented in Section 4. Included in that section is a discussion of the development of (1) the inverse and the adjusted inverse logistics functions and (2) the procedure for determining which function to use to estimate freight rates when solving for the optimal purchase order quantity. Experimentation and results are presented in Section 5, and conclusions are drawn in Section 6.

2. Discussion of freight rates in practice

Motor carrier freight rates are a function of the total weight in a given shipment. In the current competitive environment, freight rates decrease at a decreasing rate as shipping weight increases. Truckload rates are normally stated on a per-mile basis. The LTL rates generally are stated per hundredweight (CWT) and rely on three components: the class rate system, base rates, and discounts. Assuming that the published rates are the same for all carriers, the shipper needs only to negotiate the discount structure [3]. This discount is a flat percentage deducted from the published (base) rate. Regardless of the negotiation process, freight rates take the form of a step function decreasing at a decreasing rate as shipping weight increases. This reflects the economies of scale that accrue for larger shipping weights and the additional consolidation costs involved when determining load priorities for small shipments. Table 1 provides an example of the stated and discounted freight rates (assuming a 20% discount) for a particular shipping route.

In addition, actual transportation rates must account for the temporal nature of transportation

Table 1
Nominal freight rate schedule for example problem

Weight break	Freight rate	Discounted rate[a]
Minimum charge	\$50.00	\$40.00
1–499 pounds	\$22.00/CWT	\$17.60/CWT
500–999 pounds	\$18.50/CWT	\$14.80/CWT
1000–1999 pounds	\$17.25/CWT	\$13.80/CWT
2000–4999 pounds	\$16.00/CWT	\$12.80/CWT
5000–9999 pounds	\$15.50/CWT	\$12.40/CWT
10,000–19,999 pounds	\$7.60/CWT	\$6.08/CWT
Truckload (20,000 pounds or more)[b]	\$1,110.00	

[a] Freight class = 77.5; Discount = 20%.
[b] TL charge = 600 miles at \$1.85/mile.

rates, which gives rise to the practice of over-declared shipments [4,5] and what Ferrin and Carter [6] refer to as anomalous weight breaks in LTL pricing. Shippers use over-declared shipments to achieve a lower total tariff. This is accomplished by artificially inflating the shipping weight to a higher weight-break point and a lower marginal tariff. Motor carriers are, in essence, prevented from charging more per shipment for a smaller weight than they do for a larger weight. For each LTL weight-break level there may exist an indifference point beyond which shipping weights will be over-declared as a TL. That is, there may exist a weight, which when multiplied by its corresponding freight rate, will yield the same total charge as that for a TL. For example, given the information presented in Table 1, the shipper would never agree to pay more than the stated \$1,110.00 for a TL shipment, regardless of the amount being shipped. If the stated freight rate per CWT for 10,000–19,999 pounds costs resulted in a per shipment cost of more than \$1,110.00, the shipper would declare a full TL shipment and pay the \$1,110.00 TL charge. In the example given in Table 1, dividing the TL shipping charge of \$1,110.00 by the freight rate of \$6.08/CWT yields a shipping weight (indifference point) of 182.5657 CWT or 18,257 pounds. This means that any shipping weight of 18,257 pounds or more would be over-declared to a full TL shipment. This change is shown in Table 2.

As is the case for determining when to over-declare to a TL, each LTL weight-break level must also be considered for over-declared points. That is, each LTL weight-break level must be checked to determine if there exists a weight that, when multiplied by its corresponding freight rate, will yield the same total charge as the rate breakpoint of the next LTL weight-break level multiplied by its corresponding freight rate. So far it has been determined for this example that anything $\geqslant$18,257 pounds will be shipped as a TL shipment. As a result, anything between 10,000 and 18,256 pounds will be shipped at the \$6.08/CWT freight rate. What follows then is the determination of whether some shipments of $<$10,000 pounds should be over-declared as 10,000-pound shipments. Applying the same calculation as before, it can be determined, as shown in Table 2, that any shipment of 4,750 pounds or more should be shipped at the 10,000-pound rate. This is referred to as an anomalous weight break since it completely skips over the 5,000–9,999 pounds shipping weight range. Shipping 4,750 pounds at a rate of \$12.80/CWT results in a charge of \$608.00 for the shipment, the same charge as shipping 10,000 pounds at a rate of \$6.08/CWT. Therefore, any shipment of this size or more would be shipped as a 10,000 pound shipment. In this instance, the freight rate for the entire shipping range from 5,000 to 9,999 pounds would be ignored because the 10,000-pound rate completely eliminates the need for this freight rate category in this example.

Table 2
Actual freight rate schedule for example problem

Weight break	Freight rate
Minimum charge (up to 227 pounds)	\$40.00
228–420 pounds	\$17.60/CWT
421–499 pounds	\$74.00
500–932 pounds	\$14.80/CWT
933–999 pounds	\$138.00
1,000–1,855 pounds	\$13.80/CWT
1,856–1,999 pounds	\$256.00
2,000–4,749 pounds	\$12.80/CWT
4,750–9,999 pounds	\$608.00
10,000–18,256 pounds	\$6.08/CWT
18,257 pounds or more	\$1,110.00

As demonstrated in Table 2, any shipping weight between 1,856 and 1,999 pounds would be over-declared as a 2,000-pound shipment, any shipping weight between 933 and 999 pounds would be over-declared as a 1,000-pound shipment, and any shipping weight between 421 and 499 pounds would be over-declared as 500 pounds. Further, any shipping weight of 227 pounds or less would pay the minimum shipping charge of \$40.00 per shipment.

In summary, to consider over-declared weights and anomalous weight breaks, the actual freight rate schedule is transformed into alternating ranges of a constant charge per CWT followed by a constant fixed charge, which results when an LTL shipment is over-declared to another LTL weight break or as a TL shipment. When the weight at which an LTL shipment is over-declared as a TL is reached, the TL fixed charge is applied to all shipment weights beyond that weight.

3. Related research

Total annual logistics cost models have been presented in various forms for many decades. For each model there was a different determination of the 'optimal' order quantity. Baumol and Vinod [7] first introduced the integration of transportation and inventory costs into what they called inventory-theoretic models. Others later used their inventory-theoretic approach as a basis for further development.

Langley [8] was one of the first researchers to demonstrate the inclusion of freight rates into the lot sizing decision using either actual freight rates or functions to estimate freight rates. Some researchers (e.g., [9–13] have developed lot-sizing models using enumeration techniques that explicitly consider actual freight rate schedules in the determination of the optimal purchase order quantity. Other studies (e.g., [14–19]) have relied upon complex algorithms to incorporate actual freight rate schedules in the determination of optimal purchase order quantities.

Other researchers, including Ballou [20], Buffa [2], and Swenseth and Buffa [21,22] have proposed using freight rate functions to estimate freight rates as part of the lot sizing decision. Ballou [20] argued that practical considerations such as time, cost, and effort dictate that logistics decision makers use estimated rather than actual freight rates. Some companies, for example, General Motors (GM), have simultaneously incorporated transportation and inventory considerations into models using functions to estimate freight rates. The research group at GM developed a model called TRANSPART [23]. Initial application of this tool resulted in a 26% reduction in total annual logistics cost. This translated into savings of $2.9 million/year in a single division alone, Delco Electronics, of GM. Use of TRANSPART within GM was expanded later into 40 plants.

Several freight rate functions have been presented in the literature. These functions were intended to provide the best possible emulation of actual freight rates. The TRANSPART model implemented by GM was a simple model that merely considered all shipments to be full truckload (TL) shipments regardless of their shipping weights. Given the nature of the operating environment at GM, it was likely that nearly all shipments would either be full TL shipments or would be over-declared as TL shipments. As a result, this method of treating all shipments as TL shipments would provide the best means of determining order quantities for most every purchase decision. The model used at GM incorporated an inverse function to estimate freight rates.

Swenseth and Godfrey [24] determined that a simple straight-line freight rate function outperformed other, more complex, functions in terms of its ability to emulate actual freight rates. The straight-line function used to emulate freight rates in their study, referred to as the proportional function, was most accurate over the entire range of shipping weights from 100 to 46,000 pounds. This accuracy was based on the minimum squared difference (MSD) between the actual freight rates for given shipping routes and weights and those freight rates that would have been generated by the alternative freight rate functions considered in their study. However, this MSD was determined over the entire range of potential shipping weights, giving no regard for the potential that some functions may perform better over subsections of the potential range of shipping weights. Preliminary analysis using the same MSD approach indicated that when separating TL and LTL shipments, the inverse function combined with the adjusted inverse function significantly improved the emulation of freight rates as compared to the single straight line, proportional, function considered in their study. Furthermore, there was no consideration of the effects of the proportional function on actual logistics decisions.

4. Inventory replenishment model

The two freight rate functions considered in this study were the adjusted inverse and the inverse.

Both of these functions were incorporated into total logistics cost models that extend the basic economic order quantity (EOQ) model. Several recent studies (e.g., [25,26] have noted the continued use of the EOQ model, particularly at smaller manufacturing concerns that are not requesting JIT deliveries from their suppliers [27]. The inventory replenishment models presented in this section required the following assumptions:

1. Purchased items are classified in freight class 60, 65, 70 or 77.5, classes for which it is expected that 46,000 pounds can be legally loaded onto a trailer.
2. Quantity discounts are not available.
3. Annual demand for a purchased item is known and constant.
4. Ordering cost is fixed per order.
5. Annual holding cost is linearly related to the average inventory.
6. Shortages are not allowed.
7. All items are purchased F.O.B. Origin. The buyer incurs all transportation charges.

All freight rates are expressed on a per pound basis to simplify formulas presented later in this paper.

4.1. The economic order quantity model

The most basic total annual logistics cost (inventory-theoretic) model is the simple EOQ model that has been around for nearly a century. This model is composed of the inventory holding cost and the ordering cost of an individual item. Incorporating a fixed rate per pound for shipping (F_y) provides the resulting total annual logistics cost function (L):

$$L = \frac{QC_h}{2} + \frac{RC_o}{Q} + F_y R w, \tag{1}$$

where Q is the order quantity (units), C_h the cost to hold one unit in inventory for one year (calculated by taking unit cost, C, multiplied by holding cost rate, i), R the annual requirements (units), C_o the cost to place one order, F_y the freight rate per pound for a given shipping weight (y) over a given route, and w the weight per unit.

Assuming that no quantity discounts exist and taking the derivative of L with respect to Q, setting the result equal to zero, and solving for Q provides the optimal order quantity for the EOQ model:

$$Q = \sqrt{\frac{2RC_o}{C_h}}. \tag{2}$$

This has the same effect as not incorporating freight rates into the model. Further, because actual freight rates are not constant over all weights but instead decrease at a decreasing rate as shipment weight increases, the incorporation of actual freight rates into this model would result in benefits for larger shipments. Therefore, the EOQ model would provide the lowest realistic order quantity (and shipment weight). Table 3 contains information, which along with the actual freight rates from Table 2, was used to demonstrate the impact of using this model to calculate the order quantity. In this example the EOQ was 115.47 units. The total cost for ordering and holding EOQ units was $5,196.16. The shipping weight for EOQ units was 2,540.34 pounds, which resulted in a freight rate of $12.80/CWT or $0.1280 per pound. Shipping 2,540.34 pounds at $12.80/CWT resulted in a transportation cost of $325.12 per shipment or $28,160 total for the 86.6 shipments that would be required on an annual basis. This resulted in a total annual logistics cost of $33,356.16. The total annual logistics cost of the EOQ model must then be compared with the cost of the true optimal order quantity.

Table 3
Parameter values for example problem

Parameter	Value
Unit weight (w)	22 pounds per unit
Annual requirements (R)	10,000 units per year
Holding cost rate (i)	90%
Unit cost (C)	$50.00 per unit
Holding cost per unit per year (C_h)	$45.00 per unit per year
Order cost (C_o)	$30.00 per order
LTL discount (d)	20%
Freight class	77.5
TL freight rate per pound at 46,000 pounds (F_x)	$0.0241 per pound

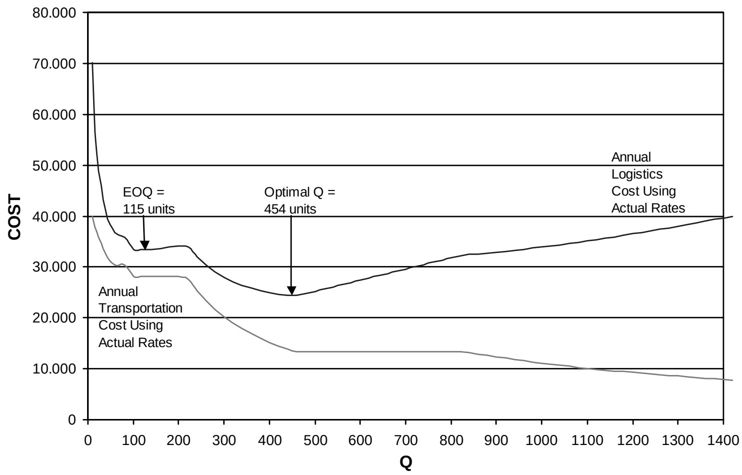

Fig. 1. Annual costs for EOQ and optimal order quantity

The optimal order quantity, which was determined using enumeration, was 454 units (9,988 pounds) with annual order cost equal to \$660.79, annual holding cost equal to \$10,215.00, annual transportation cost equal to \$13,392.07, and total annual logistics cost equal to \$24,267.86. Fig. 1 shows the annual transportation cost and total annual logistics cost for this example. It can be readily seen that the total annual logistics cost could be dramatically reduced by shipping at the optimal level of 454 units rather than at the EOQ value of 115.47 units. At the optimal level the total annual logistics cost was reduced to \$24,267.86. As a result, the cost at EOQ was \$9,088.30 more or approximately 37.5% greater than the optimal total annual logistics cost.

4.2. Inverse model

The inverse function provides a constant charge per shipment as compared to the constant charge per unit (or per unit weight) of the EOQ model discussed above. This results in the greatest charge per shipment and the largest order quantities, rather than the lowest order quantities that were provided by the EOQ model. Essentially, the inverse function assumes that all shipments are over-declared as TL shipments. For the inverse function, the determination of F_y and the resulting cost function and order quantity equations are as follows:

$$F_y = \frac{F_x W_x}{W_y}, \tag{3}$$

where F_x is the TL freight rate per pound (at the full TL shipping weight) for a given route, W_x the full TL shipping weight (assumed to equal 46,000 pounds), and W_y the actual shipping weight.

Here, $F_x W_x$ is the total charge for a truckload shipment for a given route. This truckload charge could also be calculated by taking the freight rate per mile multiplied by the distance (miles). Substituting the inverse function into the total logistics cost formula yields

$$L = \frac{QC_h}{2} + \frac{RC_o}{Q} + \left[\frac{F_x W_x}{W_y}\right] Rw. \tag{4}$$

Because shipping weight, W_y, is a function of the order quantity ($W_y = Qw$), the formula for L must be modified to the following:

$$L = \frac{QC_h}{2} + \frac{RC_o}{Q} + \left[\frac{F_x W_x}{Qw}\right] Rw. \tag{5}$$

Assuming that no quantity discounts exist and taking the derivative of L with respect to Q, setting the result equal to zero, and solving for Q provides the optimal order quantity for the

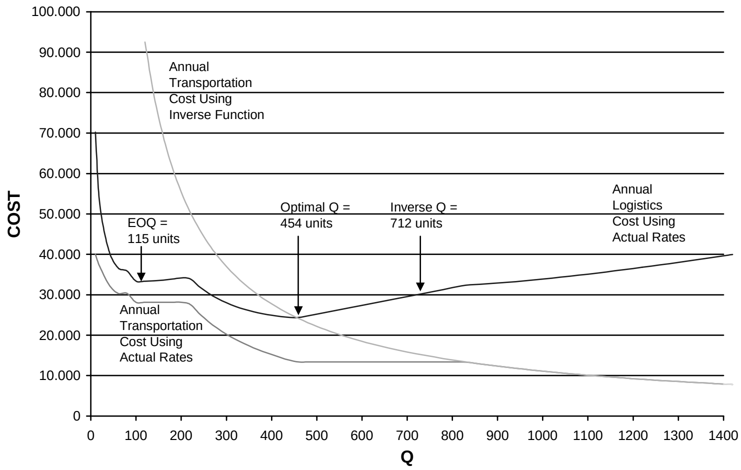

Fig. 2. Annual costs for EOQ, inverse quantity, and optimal order quantity

inverse function

$$Q = \sqrt{\frac{2R(C_o + F_x W_x)}{C_h}}. \tag{6}$$

This model essentially adds the TL charge to the cost of placing an order in the order quantity determination. When the optimal solution is an order quantity for which the shipping weight would be a TL or would be over-declared as a TL, the inverse function provides the true optimal solution. Fig. 2 demonstrates the effect of the inverse function as applied to the example introduced in Section 4.1.

Because the optimal shipping quantity of 454 units (9,988 pounds) would not be over-declared as a TL shipment, the inverse function did not provide the optimal solution in this example. Here, the inverse function provided a solution of 711.81 units (15,659.82 pounds per shipment). Ordering in quantities of 711.81 units would result in an annual order cost of \$421.46, an annual holding cost of \$16,015.73, an annual transportation cost of \$13,376.00, and a total annual logistics cost of \$29,813.19 (approximately 22.9% more than the true optimal solution).

4.3. Adjusted inverse model

The functions analyzed by Swenseth and Godfrey [24] provide a means of emulating freight rates without further complicating the order quantity decision through explicit treatment of freight rate quantity discounts. Of those functions studied, the one that was most effective when only considering the LTL shipments was the adjusted inverse function. The adjusted inverse function has the added advantage of taking on the form of the inverse function discussed in Section 4.2. For the adjusted inverse function, the determination of F_y and the resulting cost function and order quantity equations are as follows:

$$F_y = F_x + \alpha F_x \left[\frac{(W_x - W_y)}{W_y}\right], \tag{7}$$

where α (alpha) is a constant between 0 and 1.

Substituting the adjusted inverse function into the total logistics cost formula yields

$$L = \frac{QC_h}{2} + \frac{RC_o}{Q} + \left[F_x + \alpha F_x \left[\frac{(W_x - W_y)}{W_y}\right]\right] Rw. \tag{8}$$

Because shipping weight, W_y, is a function of the order quantity ($W_y = Qw$), the formula for L must be modified to the following:

$$L = \frac{QC_h}{2} + \frac{RC_o}{Q} + \left[F_x + \alpha F_x \left[\frac{(W_x - Qw)}{Qw}\right]\right] Rw. \tag{9}$$

Assuming that no quantity discounts exist and taking the derivative of L with respect to Q, setting the result equal to zero, and solving for Q provides the optimal order quantity for the adjusted inverse function

$$Q = \sqrt{\frac{2R(C_o + \alpha F_x W_x)}{C_h}}. \tag{10}$$

The value for the predicted α was determined with Eq. (A.1) from Appendix A. Using the calculated value for α of 0.11246 and the data presented in Section 4.1 to calculate the adjusted inverse quantity provided a solution of 262.33 units (5,771.26 pounds). Ordering in quantities of 262.33 units would result in annual order cost of \$1,143.60 and an annual holding cost of \$5,902.43. This quantity would be over-declared as 10,000 pounds at a fixed charge of \$608.00 per shipment. Shipping 262.33 units at this rate would result in an annual transportation cost of \$23,176.91. The total annual logistics cost would be \$30,222.94 (approximately 24.5% more than the true optimal cost of \$24,267.86). This difference, as it relates to the EOQ and inverse models, is presented in Fig. 3.

Incorporating the adjusted inverse function into the order quantity decision as shown in Eq. (10) generally (on average) results in an order quantity and shipping weight slightly less than the true optimal solution. This occurs because Eq. (A.1) is based solely on freight rates. Other logistics cost factors, namely ordering and carrying costs are incorporated in the actual determination of the optimal order quantity, thus increasing the likelihood of over-declaring shipments.

4.4. The solution procedure

Preliminary study, as depicted by the above example, indicated that there would be a significant improvement in cost if the appropriate freight rates were applied. Because it is not realistic to incorporate the true optimal freight rates into practical situations, the next best alternative is to develop a simple heuristic that will emulate reality as close as possible. Because preliminary study indicated that the adjusted inverse function would perform well for LTL shipments and that the inverse function would provide the optimal solution for TL shipments, a new heuristic solution procedure was developed. The proposed solution procedure is demonstrated in Fig. 4. This solution procedure combines two freight rate functions, the inverse and the adjusted inverse, that have previously been studied, with practical application scenarios.

The logic for this solution procedure is as follows. The inverse function will always provide a result that is greater than or equal to the optimal solution. The adjusted inverse function tends to fall close to the optimal solution, but as discussed

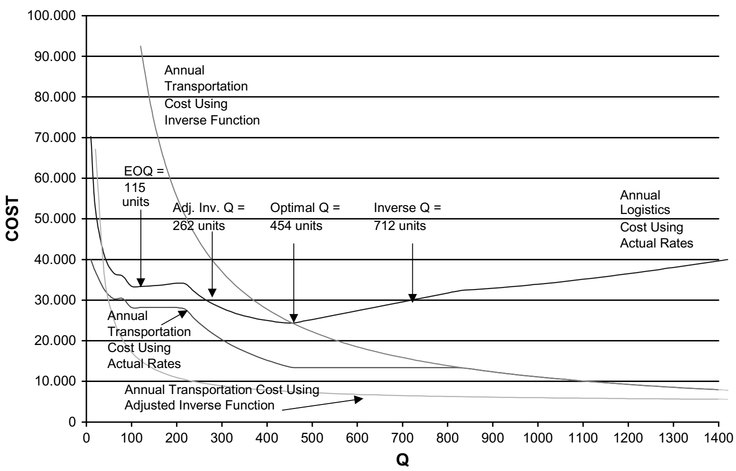

Fig. 3. Annual costs for EOQ, inverse quantity, adjusted inverse quantity, and optimal order quantity

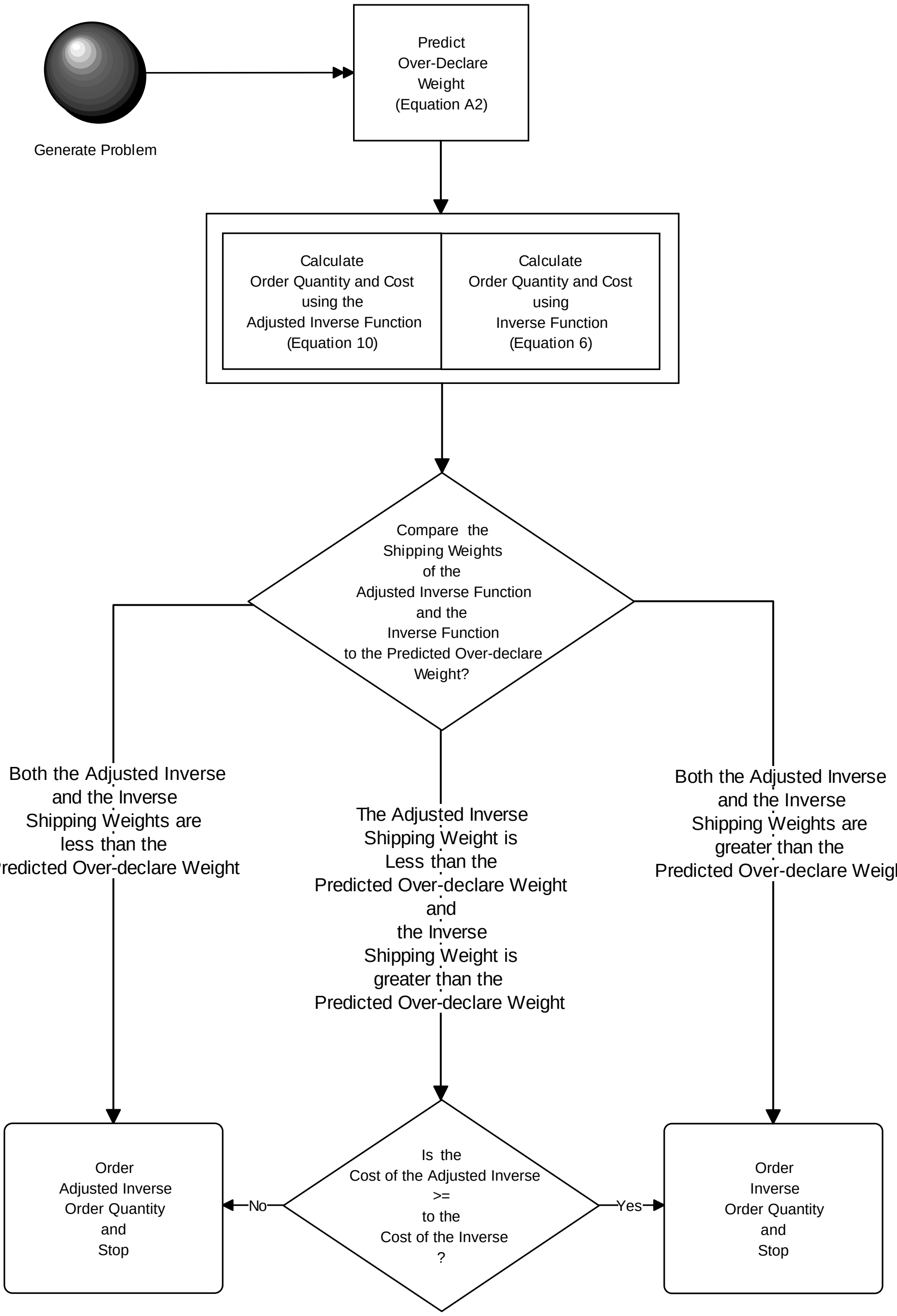

Fig. 4. Processing sequence for the proposed model.

above, has a slight bias toward lower shipping weights. The adjusted inverse function will always provide a shipping weight that is less than that of the inverse function. Ultimately, this procedure identifies whether to over-declare a shipment as a TL, thus using the inverse function, or to proceed with an LTL shipment, using the adjusted inverse function.

There are three possible outcomes that must be considered. Both the adjusted inverse and the inverse functions suggest that the shipping weight should be less than the over-declare weight, both the adjusted inverse and the inverse functions suggest that the shipment should be over-declared as a TL shipment; or the adjusted inverse function suggests that the shipment should not be over-declared, but the inverse function suggests that the shipment should be over-declared as a TL shipment. It cannot happen that the adjusted inverse function would suggest over-declaring a shipment as a full TL shipment, but that the inverse function does not. Once it has been determined whether to over-declare a shipment as a TL, the appropriate function is applied and the solution is selected.

Before beginning the solution procedure, a decision maker must gather all relevant information. The information required for each item ordered includes the following: unit weight (w), annual requirements (R), holding cost rate (i), unit cost (C), holding cost/unit/year (C_{h}), order cost (C_{o}), the estimated or actual discount (d) offered by an LTL carrier, and the TL freight rate per pound at the TL shipping weight (F_x). The solution procedure is shown in Fig. 4 and discussed throughout the remainder of this section.

First, the weight at which shipments should be over-declared to a TL for a given route is calculated using Eq. (A.2) from Appendix A. Second, the predicted α value used in the adjusted inverse function is calculated using Eq. (A.1) from Appendix A and then plugged into Eq. (10) to determine the adjusted inverse quantity. The shipping weight for this quantity is calculated by multiplying the order quantity by unit weight. The annual logistics cost for the adjusted inverse quantity is determined with Eq. (9). Simultaneously, the inverse quantity is calculated using Eq. (6), with total annual logistics cost calculated using Eq. (5). Next, the weights determined with the adjusted inverse and inverse functions are compared to the predicted over-declare weight. These weights will (a) both be less than or equal to the predicted over-declare weight; (b) be split with the adjusted inverse weight being less than the predicted over-declare weight and the inverse weight being greater than the over-declare weight; or (c) both be greater than the predicted over-declare weight. The solution procedure branches three different ways depending upon this result.

If the shipping weights for both the adjusted inverse and inverse order quantities are less than or equal to the predicted over-declare weight, then the adjusted inverse function (Eq. (10)) is used to determine the optimal order quantity and Eq. (9) is used to estimate total annual logistics cost.

If the shipping weight calculated with the adjusted inverse function is less than the predicted over-declare weight, but the inverse function provides a weight greater than the over-declare point, then it is still unknown whether it would be better to over-declare the shipment as a TL. When this occurs, the solution procedure proposes to choose the function that provides the lower total annual logistics cost. The inverse function logistics cost is calculated using Eq. (5) and the adjusted inverse function logistics cost is calculated with Eq. (9). The solution procedure determines which of these two functions gives the lower annual logistics cost, and order quantities are given by the function with the lower cost.

If the shipping weights for both the adjusted inverse and inverse order quantities are greater than or equal to the predicted over-declare weight, then the inverse function (Eq. (6)) is used to determine the optimal order quantity and Eq. (5) is used to estimate total annual logistics cost.

Underlying this solution procedure is the relationship that the minimum cost quantity provided by EOQ will always be lower than the other order quantities because the EOQ model ignores transportation costs. Further, the inverse quantity will always be higher than the adjusted inverse quantity because the inverse function, which assumes that all shipments move as TL, always provides the highest estimate of freight rates (and transportation costs), thereby creating incentives for larger shipments. Likewise, for all LTL shipments the inverse function over-estimates freight rates compared to actual, thereby providing the same incentive for larger quantities. However, for TL shipments the inverse and optimal quantities must be equal because their freight rates are equal.

The relationship between the adjusted inverse and inverse functions remains the same on a percentage basis, but as shipping weight increases, the dollar value of the difference decreases. Therefore, as shipping weight increases, the resulting solutions become closer together, decreasing the likelihood that the adjusted inverse function provides a better solution than the inverse function. Further, because the adjusted inverse function tends to provide a solution slightly lower (on average) than the true optimal solution, if the adjusted inverse function leads to an over-declared shipment, then it is highly likely that over-declaring to a full TL shipment is the best alternative. The implication here is that if the adjusted inverse function indicates a shipment should be over-declared, then it is logical to recalculate the order quantity using the function that ensures the best possible (optimal) solution for over-declared shipments. As a result, if the shipping weight calculated with the adjusted inverse function is greater than the weight predicted as the over-declared point, the actual optimal solution will almost always recommend over-declaring the shipment as a TL. Therefore, the recommendation would be to over-declare the shipment as a TL and to use the inverse function to determine the shipping weight and order quantity.

4.5. Example using the solution procedure

First, applying Eq. (A.2) to the example data gave a predicted over-declare weight of 5,419.78 pounds (approximately 246 units at 22 pounds each). Second, the predicted α was calculated as 0.11246 (determined with Eq. (A.1)), and the adjusted inverse shipping quantity was calculated as 262.33 units (using Eq. (10)), with a shipping weight of 5,771.26 pounds. Third, Eq. (6) was used to calculate the inverse quantity of 711.81 units, with a weight of 15,659.82 pounds. Fourth, because the adjusted inverse weight was greater than the predicted over-declare weight, it was determined that the inverse function would likely provide a better result than would the adjusted inverse function. Therefore, the inverse function quantity of 711.81 units was identified as the "best" order quantity with a shipping weight of 15,659.82 pounds. Actually, the cost for the order quantity determined by using the adjusted inverse weight ($30,222.94) was higher than the cost determined by using the inverse function ($29,813.19). While these costs do not differ by a substantial amount, they do indicate that the solution procedure worked as intended.

5. Experimentation and results

The effectiveness of the proposed model was determined by experimenting with a wide range of inventory replenishment decision scenarios. Essentially, the decision process reduces to whether to over-declare a shipment as a TL by using the inverse function or not to over-declare a shipment as a TL by using the adjusted inverse function. Two possible errors can arise in this procedure. These errors can be interpreted along the same lines as Type I and Type II errors in a statistical analysis. A Type I error would be to over-declare a LTL shipment as a TL shipment when it should not be over-declared. A Type II error would be to fail to over-declare a shipment as a TL when, in fact, it should be over-declared as a TL shipment.

Ranges were considered for all of the input variables used in the models. These ranges were selected to provide a broad-based comparison of the models. While it is possible to have higher unit weights, order costs, etc., increasing these variables has the general effect of increasing order sizes. By further increasing order sizes, the models, and therefore the comparisons, would be biased toward TL shipments. As a result, it was assumed that the ranges incorporated were sufficient.

The solution procedure presented in Section 4.4 requires only two additional pieces of information beyond that required for a basic EOQ determination. These are discussed in Appendix A. Table 4 summarizes the information required for the solution procedure in the proposed model. In addition to annual demand or requirements, ordering cost, holding cost rate, and unit cost, a logistics decision maker needs the TL freight rate per pound at 46,000 pounds for a given route and the applicable LTL discount negotiated with the LTL carrier. The TL freight rate and the LTL discounts are likely to be known with greater confidence than the other parameters required for EOQ analysis. The TL freight rate is used directly in the determination of the order quantity. Both the TL freight rate and the LTL discount are used to estimate the α parameter for the adjusted inverse freight rate function and the point at which LTL shipments should be over-declared as TL shipments. LTL rates were obtained from a Class I LTL motor carrier and TL rates were obtained from a Class I TL motor carrier serving all of the contiguous US.

Table 4
Parameter ranges for randomly generated problems

Parameter	Range
Unit weight (w)	0–1,000 pounds per unit
Annual requirements (R)	0–100,000 units per year
Holding cost rate (i)	0–100%
Unit cost (C)	\$0–\$1,000 per unit
Holding cost per unit per year (C_h)	\$0–\$1,000 per unit per year
Order cost (C_o)	\$0–\$1,000 per order
LTL discount (d)	20%, 30%, 40%, 50%
Freight class	60, 65, 70, 77.5
TL freight rate per pound at 46,000 pounds (F_x)	Dependent on route

Following the procedure outlined in Fig. 4, 6000 problems were randomly generated and solved using the parameters and ranges presented in Table 4. Actual freight rates were used to determine the true optimal purchase order quantities. Actual freight rates and optimal purchase order quantities were found with a search routine coded in Fortran. This search routine was designed to consider LTL discounts, over-declaring of shipments, and anomalous weight breaks. Any problems that resulted in lowest possible shipping weights greater than maximum TL shipping weights (46,000 pounds) were eliminated from the study because clearly these weights should be shipped as TL. Including these problems in the experimentation would only serve to bias the results of the study. The solution procedure can be viewed from three perspectives: (1) overall model comparisons, (2) model comparisons when the solution procedure selected the correct cost function, and (3) model comparisons when the solution procedure did not select the correct cost function, i.e., when either of the Type I or Type II error possibilities discussed above occurred.

5.1. Overall model comparisons

The overall results for all 6000 problems are shown in Table 5(a). Table 5(a) lists the mean

Table 5
(a) Analysis of all 6,000 problems

Method	Mean cost (\$)	Mean weight (lb)	Mean quantity (units)
Adjusted inverse	182,676.96	14,549.30	362.52
Proposed	164,752.91	21,498.95	509.84
Optimal	162,521.93	20,808.46	478.12
Inverse	171,611.84	22,436.66	638.28

(b) Multiple comparisons of absolute differences in mean costs for all 6000 problems

Method	Proposed	Optimal	Inverse
Adjusted inverse	\$17,924.05[a]	\$20,155.03[a]	\$11,065.12[a]
Proposed		\$ 2,230.98	\$ 6,858.93[a]
Optimal			\$ 9,089.91[a]

[a] Significant at the 0.05 level. Tukey's critical number $\omega = 6{,}579.69$.

costs, mean weights, and mean quantities for the adjusted inverse function, the proposed solution procedure, the optimal solution, and the inverse function. Analysis of variance (ANOVA) was used to test the null hypothesis that the mean costs were all equal (with $\alpha = 0.05$), and this null hypothesis was rejected, $F(3, 23996) = 24.97$, $p < 0.00001$, indicating that at least one of the functions provided a solution different from the others. As shown in Table 5(b), multiple comparisons using Tukey's method (with $\alpha = 0.05$) indicated that (a) the adjusted inverse function produced a higher total annual logistics cost than did the other three methods, (b) the inverse function produced a higher total annual logistics cost than did the optimal and proposed methods, and (c) the total annual logistics costs of the proposed and optimal methods did not differ significantly.

5.2. Model comparisons when the solution procedure selected the correct cost function

Next, the results for the 5,262/6,000 problems (87.7%) for which the solution procedure selected the appropriate cost function are listed in Table 6(a). ANOVA was used to test the null hypothesis that the mean costs were all equal (with $\alpha = 0.05$), and this null hypothesis was rejected, $F(3, 21044) = 27.19$, $p < 0.00001$, indicating that at least one of the means was different. As shown in Table 6(b), multiple comparisons using Tukey's method (with $\alpha = 0.05$) indicated that (a) the adjusted inverse function produced a higher total annual logistics cost than did the other three methods, (b) the inverse function produced a higher total annual logistics cost than did the optimal and proposed methods, and (c) the total annual logistics costs of the proposed and optimal methods did not differ significantly.

The results of the 5,262 problems solved correctly were then further sub-divided into 2,279 LTL problems and 2,983 TL problems solved correctly. The results for the 2,279 LTL problems solved correctly are listed in Table 7(a). ANOVA was used to test the null hypothesis that the mean costs were all equal (with $\alpha = 0.05$), and this null hypothesis was rejected, $F(3, 9112) = 17.64$, $p < 0.00001$, indicating that at least one of the means was different. As shown in Table 7(b), multiple comparisons using Tukey's method (with $\alpha = 0.05$) indicated that (a) there were no significant differences between the total annual logistics costs of the proposed, adjusted inverse, and optimal methods and (b) the inverse function produced a higher total annual logistics cost than did the other three methods.

The results for the 2,983 TL problems solved correctly are listed in Table 8(a). ANOVA was used to test the null hypothesis that the mean costs were all equal (with $\alpha = 0.05$), and this null hypothesis was rejected, $F(3, 11928) = 42.25$, $p < 0.00001$, indicating that at least one of the means was different. As shown in Table 8(b), multiple comparisons using Tukey's method (with $\alpha = 0.05$) indicated that (a) there were no significant differences between the total annual logistics costs of the proposed, adjusted inverse, and optimal methods and (b) the inverse function produced a higher total annual logistics cost than did the other three methods.

5.3. Model comparisons when the solution procedure did not select the correct cost function

Finally, the results for the 738/6,000 problems (12.3%) where the solution procedure did not select the appropriate cost function are listed in Table 9. ANOVA was used to test the null hypothesis that the mean costs were all equal (with $\alpha = 0.05$), and this null hypothesis was not rejected, $F(3, 2948) = 1.26$. This indicates that there was no statistical evidence to indicate that selecting a different cost function would have resulted in a significantly different solution for these problems.

Next, these 738 problems were further subdivided into 721 LTL and 17 TL problems. That is, there were 721 problems that should have been solved using the adjusted inverse function that were actually solved using the inverse function. Likewise, there were 17 problems that were solved using the inverse function that should have been solved using the adjusted inverse function.

Table 6
(a) Analysis of 5,262 problems for which correct function was selected

Method	Mean cost ($)	Mean weight (lb)	Mean quantity (units)
Adjusted inverse	183,850.73	15,702.03	368.53
Proposed	162,816.45	22,612.98	489.68
Optimal	161,937.58	22,677.97	492.98
Inverse	170,648.72	23,674.62	635.45

(b) Multiple comparisons of absolute differences in mean costs for 5262 problems

Method	Proposed	Optimal	Inverse
Adjusted inverse	$21,034.28[a]	$21,913.15[a]	$13,202.01[a]
Proposed		$ 8,78.87	$ 7,832.27[a]
Optimal			$ 8,711.14[a]

[a] Significant at the 0.05 level. Tukey's critical number $\omega = 7,800.70$.

Table 7
(a) Analysis of 2,279 LTL problems for which correct function was selected

Method	Mean cost ($)	Mean weight (lb)	Mean quantity (units)
Adjusted inverse	128,414.34	2,526.15	379.34
Proposed	128,414.34	2,526.15	379.34
Optimal	126,385.10	2,676.19	386.96
Inverse	146,498.32	4,977.37	715.90

(b) Multiple comparisons of absolute differences in mean costs for 2,279 LTL problems

Method	Proposed	Optimal	Inverse
Adjusted inverse	$0	$2,029.24	$18,083.98[a]
Proposed		$2,029.24	$18,083.98[a]
Optimal			$20,113.22[a]

[a] Significant at the 0.05 level. Tukey's critical number $\omega = 10,730.28$.

Table 8
(a) Analysis of 2,983 TL problems for which correct function was selected

Method	Mean cost ($)	Mean weight (lb)	Mean quantity (units)
Adjusted inverse	226,203.92	25,768.34	360.28
Proposed	189,099.53	37,959.24	573.98
Optimal	189,099.53	37,959.24	573.98
Inverse	189,099.53	37,959.24	573.98

(b) Multiple comparisons of absolute differences in mean costs for 2,983 TL problems

Method	Proposed	Optimal	Inverse
Adjusted inverse	$37,104.39[a]	$37,104.39[a]	$37,104.39[a]
Proposed		$0	$0
Optimal			$0

[a] Significant at the 0.05 level. Tukey's critical number $\omega = 7,123.58$.

Table 9
Analysis of 738 problems for which correct function was not selected[a]

Method	Mean cost ($)	Mean weight (lb)	Mean quantity (units)
Adjusted inverse	174,307.89	6,330.24	319.66
Proposed	178,560.00	13,555.81	653.55
Optimal	166,686.17	7,478.74	372.16
Inverse	178,478.97	13,609.88	658.43

[a] Multiple comparisons are not included because no significant difference was identified.

Table 10
Analysis of 721 LTL problems for which correct function was not selected[a]

Method	Mean cost ($)	Mean weight (lb)	Mean quantity (units)
Adjusted inverse	175,690.59	6,368.22	316.77
Proposed	180,042.95	13,764.17	658.53
Optimal	167,972.10	7,488.47	365.51
Inverse	180,042.95	13,764.17	658.53

[a] Multiple comparisons are not included because no significant difference was identified.

Table 11
Analysis of 17 TL problems for which correct function was not selected[a]

Method	Mean cost ($)	Mean weight (lb)	Mean quantity (units)
Adjusted inverse	115,665.24	4,719.09	442.04
Proposed	115,665.24	4,719.09	442.04
Optimal	112,147.84	7,066.22	654.16
Inverse	112,147.84	7,066.22	654.16

[a] Multiple comparisons are not included because no significant difference was identified.

Table 10 lists the results for the 721 LTL problems in this group. ANOVA was used to test the null hypothesis that the mean costs were all equal (with $\alpha = 0.05$), and this null hypothesis was not rejected, $F(3, 2880) = 1.28$. This indicates that there was no statistical evidence to indicate that selecting a different cost function would have resulted in a significantly different solution for these problems.

Table 11 lists the results for the 17 TL problems in this group. ANOVA was used to test the null hypothesis that the mean costs were all equal (with $\alpha = 0.05$), and this null hypothesis was not rejected, $F(3, 64) = 1.26$. This indicates that there was no statistical evidence to indicate that selecting a different cost function would have resulted in a significantly different solution for these problems.

6. Conclusions

Considerable changes have occurred over the past two decades that have substantially changed the treatment of inventory replenishment decisions. Many researchers have stated that the analysis of these decisions is less important now because many decisions are made based on strategic intent instead of optimization. JIT is one area in particular where this justification is often made without consideration for traditional costing models. JIT, however, with its lower prescribed inventory levels, has increased the need to look at the implications of transportation in the system. Smaller, more frequent shipments must be folded into the decision process. Further, a solution procedure was provided that systematically solves inventory replenishment problems, and the procedure does so without incorporating any practically intractable optimization routines.

The procedure first predicts the shipping weight at which a shipment should be over-declared as a TL shipment. Regression was used to specify a function for predicting the over-declared point. This function, significant at a level of 0.0001, explained approximately 75% of the variance in actual over-declared points. The procedure then determines, for the specific parameters of the decision being considered, the best identifiable solution.

The results indicate that for shipments that should have been over-declared, the model was correct more than 99% of the time and for shipments that should not have been over-declared, the model was correct more than 75% of the time. For those 99% TL problems, if the model had incorrectly selected the adjusted inverse function, the annual logistics costs would have been on average \$37,104 (19.6%) higher than the true optimal. Alternatively, for those 75% LTL problems, if the model had incorrectly selected the inverse function, the annual logistics costs would have been on average \$20,113 (15.9%) higher than the true optimal. In short, it is better to commit a Type I error (over-declaring a LTL shipment as a TL) than a Type II error (not over-declaring a TL shipment as a TL). This is explained by the insensitivity of the total cost equation near the optimal point and the fact that the solution procedure fails to identify the correct solution when all of the solutions are essentially the same.

As applied by GM in the TRANSPART model [23], the inverse function does indeed provide a simple method of reducing transportation cost when it is appropriate to over-declare LTL shipments as TL shipments. When the inverse function is applied to shipments that should not be over-declared, the result is an average total logistics cost of \$18,084 (14.1%) more than the average cost when the adjusted inverse function is used. This translates into further savings that should not be ignored.

While previous studies have indicated that alternative freight rate functions can emulate actual freight rates, no previous work has linked these alternative freight rate functions into a heuristic procedure that can accurately incorporate freight rates into the inventory-theoretic model without adding undue complexity. Using the supplied regression equations for predicting the over-declared point and predicting the appropriate α value for the adjusted inverse function necessitate little added complexity and allow different freight rate functions to be used for the shipping weights where they are most applicable.

Knowing the impact of incorporating the appropriate freight rate function into the inventory replenishment decision makes it possible for practitioners and researchers to now incorporate similar functions into alternative models that may be a better fit with their particular application. This can have an impact on supply chain configurations, supplier selection decisions, and even on make-versus-buy and vertical integration decisions, with the most impact on the virtual organization.

Appendix A

Predicting α and the over-declare weight

As indicated, the proposed model requires two data inputs that must be determined before proceeding. These are the values to be used for α and the weight at which shipments should be over-declared as TL shipments.

The adjusted inverse function is determined directly from the inverse function using very straightforward logic. There are two known parameters for all freight rates. One is that no shipper would be willing to pay more (in total) for shipping a smaller quantity than would be paid for shipping a full TL shipment (as is the case when the inverse function is used). The other is that no carrier would charge less per pound (or per CWT) than would be charged per pound (or per CWT) for a full TL shipment. Therefore, some value less than or equal to the inverse function, but greater than the rate for a full TL shipment represents the actual rate that will be charged for LTL shipments. Further, both endpoints for this range can be determined by knowing only the TL freight rate. The purpose of the α value is to identify the best point between the two limits on this range of potential freight rates. This works much like the manner in which the α value in exponential smoothing controls the movement of the forecast between the actual and predicted values.

A process was developed to provide representative values for these factors. Data were collected for all major shipping routes in the continental United States. Data included all TL and LTL rates for four freight classes (60, 65, 70, and 77.5). A Fortran program was created that randomly selected 2,228 routes, randomly chose a freight class for each route, and randomly chose a discount factor. Discount factors included were 20%, 30%, 40%, and 50%. This resulted in a data set that included 35,648 data points. Regression was then used to predict the α value and the over-declare weight.

The regression equation for predicting the α value in the adjusted inverse function is given by

$$\begin{aligned}\text{Predicted}\,\alpha = {} & 0.173050 \\ & - 1.460799\,(\text{TL rate per lb at } 46{,}000\text{ LBS.}) \\ & - 0.126689\,(\text{LTL discount}).\end{aligned} \tag{A.1}$$

Together, the TL rate per pound at 46,000 pounds and the LTL discount explained 69.57% of the variation in the actual over-declare weights ($R^2 = 0.6957$, $p = 0.0001$).

The regression equation for predicting the weight at which a shipment should be over-declared as a TL is given by

$$\begin{aligned}\text{Predicted over-declare weight} = {} & -2487.67 \\ & + 169{,}108\,(\text{TL rate per lb at } 46{,}000\text{ LBS.}) \\ & + 19{,}134\,(\text{LTL discount}).\end{aligned} \tag{A.2}$$

Together, the TL rate per pound at 46,000 pounds and the LTL discount explained 74.85% of the variation in the actual over-declare weights ($R^2 = 0.7485$, $p = 0.0001$).

A logical explanation for these two factors (TL rate and LTL discount factor) explaining such a large portion of the variance is the time taken by carriers in establishing their rate structures. TL rates and LTL discount factors are established for given routes based on many factors, including distance, likelihood of a backhaul, desirability of destination (including safety concerns), total volume for the route, etc.

References

[1] P.J. Daugherty, M.S. Spencer, Just-in-time concepts: Applicability to logistics/transportation, International Journal of Physical Distribution & Logistics Management 20 (7) (1990) 12–18.

[2] F.P. Buffa, Inbound consolidation strategy: The effect of inventory cost changes, International Journal of Physical Distribution & Materials Management 18 (7) (1988) 3–14.

[3] J. Baker, Emergent pricing structures in LTL transportation, Journal of Business Logistics 2 (1) (1991) 91–202.

[4] R.M. Russell, Optimal purchase and transportation cost lot sizing for a single item, Decision Sciences Institute 1989 Proceedings, Atlanta, 1989, pp. 1109–1111.

[5] R.M. Russell, L.J. Krajewski, Optimal purchase and transportation cost lot sizing for a single item, Decision Sciences 22 (4) (1991) 940–951.

[6] B.G. Ferrin, J.R. Carter, The effect of less-than-truckload rates on the purchase order lot size decision, Transportation Journal 34 (3) (1995) 35–44.

[7] W.J. Baumol, H.D. Vinod, An inventory theoretic model of freight transport demand, Management Science 16 (7) (1970) 413–421.

[8] J.C. Langley, The inclusion of transportation costs in inventory models: some considerations, Journal of Business Logistics 2 (1) (1980) 106–125.

[9] J.R. Carter, B.G. Ferrin, Transportation costs and inventory management: Why transportation costs matter, Production and Inventory Management Journal 37 (3) (1996) 58–62.

[10] N. Gaither, Using computer simulation to develop optimal inventory policies, Simulation 39 (3) (1982) 81–87.

[11] P.D. Larson, The economic transportation quantity, Transportation Journal 28 (2) (1988) 43–48.

[12] J.E. Tyworth, Transport selection: computer modelling in a spreadsheet environment, International Journal of Physical Distribution & Logistics Management 21 (7) (1991) 28–36.

[13] J.C. Wehrman, Evaluating the total cost of a purchase decision, Production and Inventory Management 25 (4) (1984) 86–90.

[14] T.H. Burwell, D.S. Dave, K.E. Fitzpatrick, M.R. Roy, Economic lot size model for price dependent demand

under quantity and freight discounts, International Journal of Production Economics 48 (2) (1997) 141–155.

[15] H. Hwan, D.H. Moon, S.W. Shin, An EOQ model with quantity discounts for both purchasing and freight costs, Computers and Operations Research 17 (1) (1990) 73–78.

[16] C. Lee, The economic order quantity for freight discount costs, IIE Transactions 18 (3) (1986) 318–320.

[17] R.V. Ramasesh, A logistics-based inventory model for JIT procurement, International Journal of Operations & Production Management 13 (6) (1993) 44–58.

[18] R.J. Tersine, S. Barman, Lot size optimization with quantity and freight rate discounts, Logistics and Transportation Review 27 (4) (1991) 319–332.

[19] R.J. Tersine, P.D. Larson, S. Barman, An economic inventory/transport model with freight rate discounts, Logistics and Transportation Review 25 (4) (1989) 291–306.

[20] R.H. Ballou, The accuracy in estimating truck class rates for logistical planning, Transportation Research-A 25A (6) (1991) 327–337.

[21] S.R. Swenseth, F.P. Buffa, Just-in-Time: Some effects on the logistics function, The International Journal of Logistics Management 1 (2) (1990) 25–34.

[22] S.R. Swenseth, F.P. Buffa, Implications of inbound lead time variability for just-in-time manufacturing, International Journal of Operations & Production Management 11 (7) (1991) 37–48.

[23] D.E. Blumenfeld, L.D. Burns, C.F. Daganzo, M.C. Frick, R.W. Hall, Reducing logistics costs at General Motors, Interfaces 17 (1) (1987) 26–47.

[24] S.R. Swenseth, M.R. Godfrey, Estimating freight rates for logistics decisions, Journal of Business Logistics 17 (1) (1996) 213–231.

[25] F. Fazel, K.P. Fischer, E.W. Gilbert, JIT purchasing vs. EOQ with a price discount: An analytical comparison of inventory costs, International Journal of Production Economics 54 (1) (1998) 101–109.

[26] A.C. Pan, L. Ching-Jong, An inventory model under Just-in-Time purchasing agreements, Production and Inventory Management Journal 30 (1) (1989) 49–52.

[27] C. Temponi, S.Y. Pandya, Implementation of two JIT elements in small-sized manufacturing firms, Production and Inventory Management Journal 36 (3) (1995) 23–29.