Pergamon

0360-8352(94)00192-8

Computers ind. Engng Vol. 27, Nos 1-4, pp. 229-232, 1994
Elsevier Science Ltd. Printed in Great Britain

A MATHEMATICAL MODEL TO MINIMIZE THE INVENTORY AND TRANSPORTATION COSTS IN THE LOGISTICS SYSTEMS

Beumjun AHN[1], Norio WATANABE[2] and Shusaku HIRAKI[1]

[1]Faculty of Economics, Hiroshima University,
1-1-89 Higashisenda-machi, Naka-ku, Hiroshima 730,
Japan. Tel: +81-82-241-1221, Fax: +81-82-245-2905
[2]Hiroshima Prefectural University, 562 Nanatsuka,
Shobara, Hiroshima-ken 727, Japan.
Tel: +81-8247-4-1000, Fax: +81-8247-4-0191

ABSTRACT

In this paper we consider a logistics system for parts manufacturer distribution center(depot) to supply the parts to the parent company. And we formulate the mathematical model to minimize the sum of inventory holding costs at the depot, and the transportation and inventory costs at parts manufacturer.

We apply the model to an actual automobile parts manufacturer to demonstrate the effectiveness of the proposed model.

Key words :Logistics, Mathematical Model, JIT production system

1. INTRODUCTION

Recently, a great deal of attention has been focused on logistics by managers and researchers because of need to consider total material flow.

Logistics management is the process of planning, implementing and controlling the efficient, cost-effective flow and storage of raw materials, in-process inventory, finished goods, and related information from point-of-origin to point-of-consumption for the purpose of confirming to customer requirements[1]. A parts manufacturer may use a distribution center(depot) to supply the parts to the parent company, if the transportation cost is cheaper than the cost to supply them to the parent company directly. The manufacturer's objective is to obtain optimum performance of these functions at minimum total cost.

In this paper, we present a mathematical model for pull type ordering systems based on JIT manufacturing systems[2], [3].

As to mathematical programming approaches to the JIT production system, Bitran and Chang[4] first proposed a model to determine the number of Kanbans. Originally, they provided a nonlinear programming model. Next, they transformed the resulting model to a mixed integer programming model and then to a linear programming model. Bard and Golany[5] proposed another mathematical programming model considering the setup and lead time for the multi-product production system to determine the number of Kanban. Hong and Fukukawa[6] described the model to determine the number of Kanbans in JIT production systems with the mixed integer goal programming. And they considered the following factors: stock on hand process, inventory and labor costs, vendors supplying capacity, work load.

On the other hand, Moeeni and Chang[7] proposed a simple heuristic model for computing the number of Kanbans in the system which has a multi-stage, uncapacitied, assembly tree structure, with every stage producing only one item at a time.

But these studies developed the mathematical model for pull type production systems only inside the firms.

Therefore, we will formulate the mathematical model to minimize the sum of inventory holding cost at the depot and the inventory and transportation costs in the parts manufacturer on JIT production systems. The objective of the model is to determine how many produce the parts and when supply the parts to the depot at parts manufacturer.

2. MODEL DESCRIPTION

In this paper, we consider a multi-stage production process which produces multiple parts. And the parts are supplied from the final inventory point to the depot. And the presented model consists of N stages at the parts manufacturer. Let $n \in \{1,2,\ldots,N\}$ be an index of the stages with the condition that n=1 stands for the final stage. Each stage $n \in \{1,2,\ldots,N\}$ means a production process and includes an immediately succeeding inventory point. Let $t \in \{0,1,\ldots,T\}$ be an index of the time periods with the condition that the planning horizon starts at the beginning of period 1 and finishes at the end of period T.

We assume the model to satisfy the following conditions.

(1) Demand for final products in each period is suggested by customers(parent company) and deterministic.

(2) Each stage produces M types of items and let

$i \in \{1,2,\ldots,M\}$ be an index of items.

(3) The ordering quantity of production and transportation for each item in each stage is calculated at the end of each period.

(4) The processing time for each item at each stage is known and fixed in the planning horizon.

(5) The quantity of item i at each stage is exactly required to make the quantity of item i at the immediately succeeding stage.

(6) The transportation from the parts manufacturer to depot lead time is L1.

(7) A minimum shipping quantity for the transportation of parts to the depot is predetermined.

(8) There is a space limit at the depot for each parts manufacturer.

2. 1 Notation

J_t^n : operation time at production process of stage n in period t;

$J_t^n \geq 0$ $(n=1,2,\ldots,N;t=1,2,\ldots,T)$.

$a^{n(i)}$: processing time required to make one unit of item i at production process of stage n; $(i=1,2,\ldots,M;n=1,2,\ldots,N)$.

$D_t^{(i)}$: demand for the final product i in period t;

$D_t^{(i)} \in \{0,1,2,\ldots\}$ $(i=1,2,\ldots,M;t=1,2,\ldots,T)$.

$I_0^{0(i)}$: initial inventory quantity of item i in the depot;

$I_0^{0(i)} \in \{0,1,2,\ldots\}$ $(n=1,2,\ldots,N;i=1,2,\ldots,M)$.

$I_t^{0(i)}$: inventory quantity of item i in the depot at the end of period t;
$(i=1,2,\ldots,M;t=1,2,\ldots,T)$.

$I_0^{n(i)}$: initial inventory quantity of item i fabricated by stage n;

$I_0^{n(i)} \in \{0,1,2,\ldots\}$ $(n=1,2,\ldots,N;i=1,2,\ldots,M)$.

$I_t^{n(i)}$: inventory quantity of item i fabricated by stage n at the end of period t;
$(n=1,2,\ldots,N;i=1,2,\ldots,M;t=1,2,\ldots,T)$.

$S_t^{0(i)}$; $S_t^{n(i)}$: safety inventory level at the end of period t of item i in the depot and the parts manufacturer by stage n;
$(n=1,2,\ldots,N;i=1,2,\ldots,M;t=1,2,\ldots,T)$.

$C^{0(i)}$: holding cost including transportation ordering cost for item i in the depot;
$(i=1,2,\ldots,M)$.

$C^{n(i)}$: inventory cost including production ordering cost for item i fabricated by stage n;
$(i=1,2,\ldots,M;n=1,2,\ldots,N)$.

CT : transportation cost on one time between the parts manufacturer and the depot.

$TA^{(i)}$: size of item i; $(i=1,2,\ldots,M)$.

TI、TX : minimum and maximum shipping quantity to transport the parts from the parts manufacturer to the depot.

TD : inventory space for the parts manufacturer in the depot.

$Q^{(i)}_{-j:L1+1-j}$: transportation quantity in process of i based on the preordered quantity;
$(i=1,2,\ldots,M;j=1,2,\ldots,L1)$.

$Q^{(i)}_{t-1:t+L1}$: actual transportation quantity of item i based on the order calculated at the end of period t-1, which is placed into transportation during period t and completed in period t+L1 (decision variable); $(i=1,2,\ldots,M;t=1,2,\ldots,T)$.

$P^{n(i)}_{t-1:t}$: actual production quantity of item i for stage n based on the order calculated at the end of period t-1, which is produced in period t (decision variable);
$(n=1,2,\ldots,N;i=1,2,\ldots,M;t=1,2,\ldots,T)$.

Y_t: variable to represent transportation at period t (decision variable); $(t=1,2,\cdots,T)$.

$V_0^{(i)}$: transportation ordering quantity of item i which is presented at the beginning of the planning horizon (decision variable);
$(i=1,2,\ldots,M)$.

$V_{t-1}^{(i)}$: transportation ordering quantity of item i which is calculated at the end of period t-1, which is placed into transportation during period t and completed in period t+L1;
$(i=1,2,\ldots,M;t=1,2,\ldots,T)$.

$U_0^{n(i)}$: production ordering quantity of item i for stage n which is presented at the beginning of the planning horizon(decision variable);
$(n=1,2,\ldots,N;i=1,2,\ldots,M)$.

$U_{t-1}^{n(i)}$: production ordering quantity of item i for stage n which is calculated at the end of period t-1, which is produced in period t;
$(n=1,2,\ldots,N;i=1,2,\ldots,M;t=1,2,\ldots,T)$.

Fig.1 shows a conceptual diagram of the model presented in this paper. The presented model deals with a multi-stage multi-item production system including the depot.

2. 2 Formulation

Using the notation defined above, we formulate a

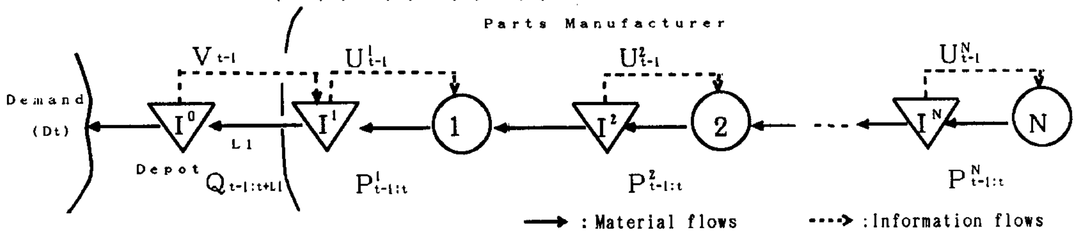

Fig. 1 A conceptual diagram of the presented model

mathematical model considering the depot on JIT manufacturing systems as follows.

$$V_t^{(i)} = V_{t-1}^{(i)} - Q_{t-1:t+L1}^{(i)} + D_t^{(i)}$$
$$(t=1,2,\dots,T、 i=1,2,\dots,M) \qquad (1)$$

$$U_t^{1(i)} = U_{t-1}^{1(i)} - P_{t-1:t}^{1(i)} + Q_{t-1:t+L1}^{(i)}$$
$$(t=1,2,\dots,T、 i=1,2,\dots,M) \qquad (2)$$

$$U_t^{n(i)} = U_{t-1}^{n(i)} - P_{t-1:t}^{n(i)} + P_{t-1:t}^{n-1(i)}$$
$$(t=1,2,\dots,T、 i=1,2,\dots,M、 n=2,3,\dots,N) \qquad (3)$$

$$I_t^{0(i)} = I_{t-1}^{0(i)} + Q_{t-L1-1:t}^{(i)} - D_t^{(i)}$$
$$(t=1,2,\dots,T、 i=1,2,\dots,M) \qquad (4)$$

$$I_t^{1(i)} = I_{t-1}^{1(i)} + P_{t-1:t}^{1(i)} - Q_{t-1:t+L1}^{(i)}$$
$$(t=1,2,\dots,T、 i=1,2,\dots,M) \qquad (5)$$

$$I_t^{n(i)} = I_{t-1}^{n(i)} + P_{t-1:t}^{n(i)} - P_{t-1:t}^{n-1(i)}$$
$$(t=1,2,\dots,T、 i=1,2,\dots,M、 n=2,3,\dots,N) \qquad (6)$$

$$\sum_{i=1}^{M} a^{n(i)} P_{t-1:t}^{n(i)} \leqq J_t^n$$
$$(t=1,2,\dots,T、 n=1,2,\dots,N) \qquad (7)$$

$$\begin{cases} Q_{t-1:t+L1}^{(i)} \leqq V_{t-1}^{(i)} \\ P_{t-1:t}^{n(i)} \leqq U_{t-1}^{n(i)} \end{cases}$$
$$(t=1,2,\dots,T、 i=1,2,\dots,M、 n=1,2,\dots,N) \qquad (8)$$

$$Y_t = \begin{cases} 0, & \text{if } \sum_{i=1}^{M} Q_{t-1:t+L1}^{(i)} = 0 \\ 1, & \text{if } \sum_{i=1}^{M} Q_{t-1:t+L1}^{(i)} > 0 \end{cases}$$
$$(t=1,2,\dots,T) \qquad (9)$$

$$TX > \sum_{i=1}^{M} TA^{(i)} \times Q_{t-1:t+L1}^{(i)} \geqq TI$$
$$, \text{ if } \sum_{i=1}^{M} Q_{t-1:t+L1}^{(i)} > 0 \qquad (t=1,2,\dots,T) \quad (10)$$

$$\sum_{i=1}^{M} TA^{(i)} \times I_t^{0(i)} \leqq TD \qquad (t=1,2,\dots,T) \quad (11)$$

$$I_t^{n(i)} \geqq S_t^{n(i)}$$
$$(t=1,2,\dots,T、 i=1,2,\dots,M、 n=0,1,2,\dots,) \quad (12)$$

$$V_0^{(i)},\ U_0^{n(i)},\ V_t^{n(i)},\ U_t^{n(i)},\ I_t^{n(i)},$$
$$I_t^{0(i)},\ P_{t-1:t}^{n(i)},\ Q_{t-1:t+L1}^{(i)} : \text{nonnegative integer}$$
$$(t=1,2,\dots,T、 i=1,2,\dots,M、 n=1,2,\dots,N) \quad (13)$$

We propose the following optimization model in a logistics system. In this model, the objective is to minimize the holding costs at the depot, inventory and transportation costs at the parts manufacturer.

Minimize

$$F = \sum_{i=1}^{M} C^{0(i)} \times (I_0^{0(i)} + V_0^{(i)}) + \sum_{i=1}^{M} \sum_{n=1}^{N} C^{n(i)} \times (I_0^{n(i)} + U_0^{n(i)}) + CT \times \sum_{t=1}^{T} Y_t \qquad (14)$$

s. t. (1)-(13)

The model we formulated above has the following characteristics;

(1) Transportation lead time is considered.
(2) Multi-product production system is adopted.
(3) The constraint for shipping quantities is considered.

3. APPLICATION TO AN AUTOMOBILE PARTS MANUFACTURER

In order to demonstrate the effectiveness of the model developed in the above section, we apply the model to an actual automobile parts manufacturer and make a numerical experiment using a mathematical programming package. The system consists of 2 stages. Each stage produces 3 types of the item. Each production process is as follows;

3.1 Numerical experiments

We assume that the planning horizon starts at the beginning of period 1 and finishes at the end of period T=5, each stage produces 3 types of item and transportation lead time is 1. Input data are as follows;

(1) Demand(Unit)

t=	1	2	3	4	5
$D_t^{(1)}$	30	30	35	35	30
$D_t^{(2)}$	30	25	30	30	25
$D_t^{(3)}$	40	45	40	45	40

(2) operation time at production processes (Min.)

$J_t^{(1)}=580$, $J_t^{(2)}=460$ $(t=1,\dots,5)$

(3) processing time required to make one unit of item i at production processes(Min.)

$a^{1(i)}=4$, $a^{2(i)}=3$ $(i=1,2,3)$

(4) initial inventory quantity in the depot and the parts manufacturer(Unit)

$I_0^{0(1)}=40$, $I_0^{0(2)}=40$, $I_0^{0(3)}=50$

$I_0^{1(i)}=10$, $I_0^{n(i)}=5$ $(n=1,\dots,N; i=1,2,3)$

(5) inventory level and transportation quantity in process(Unit)

$S_t^{1(i)}=10$, $S_t^{n(i)}=5 (n=2,\dots,N; i=1,2,3; t=1,\dots,5)$

$S_t^{0(i)}=10$ $(i=1,2,3; t=1,\dots,4)$

$S_t^{0(1)}=40$, $S_t^{0(2)}=40$, $S_t^{0(3)}=50$ $(t=5)$

$Q_{-11}^{(1)}=50$, $Q_{-11}^{(2)}=50$, $Q_{-11}^{(3)}=60$

(6) size of item (m^3)

$TA^{(i)}=1$ $(i=1,2,3)$

(7) minimum and maximum shipping quantity to transport the parts from the parts manufacturer to the depot(m^3)

$TI=160$, $TX=200$

(8) inventory space for the parts manufacturer in the depot(m^3)

$TD=300$

(9) holding cost in the depot, inventory and transportation costs at the parts manufacturer($)

$C^{0(i)}=5$, $C^{1(i)}=4$, $C^{2(i)}=3$, $CT=10$ $(i=1,2,3)$.

3.2 Computation results

By computing for the above mentioned input data, we obtained quantities of transportation order and actual transportation (Table 1).

Table 1. Quantities of transportation order and actual transportation

	i=1	i=2	i=3		i=1	i=2	i=3
$V_0^{(1)}$	33	25	37	$Q_{0:2}^{(1)}$	0	0	0
$V_1^{(1)}$	63	55	77	$Q_{1:3}^{(1)}$	63	50	77
$V_2^{(1)}$	30	30	45	$Q_{2:4}^{(1)}$	0	0	0
$V_3^{(1)}$	65	60	85	$Q_{3:5}^{(1)}$	47	40	73
$V_4^{(1)}$	53	50	57	$Q_{4:6}^{(1)}$	53	50	57

Table 2 and 3 show quantities of production order and actual production at the stage 1 and 2.

Table 2. Quantities of production order and actual production(process 1)

	i=1	i=2	i=3		i=1	i=2	i=3
$U_0^{(1)}$	63	50	77	$P_{0:1}^{1(1)}$	37	45	53
$U_1^{(1)}$	26	5	24	$P_{1:2}^{1(1)}$	26	5	24
$U_2^{(1)}$	63	50	77	$P_{2:3}^{1(1)}$	26	45	53
$U_3^{(1)}$	37	5	24	$P_{3:4}^{1(1)}$	37	0	24
$U_4^{(1)}$	47	45	73	$P_{4:5}^{1(1)}$	37	45	53

Table 3. Quantities of production order and actual production(process 2)

	i=1	i=2	i=3		i=1	i=2	i=3
$U_0^{(1)}$	37	45	53	$P_{0:1}^{2(1)}$	37	45	53
$U_1^{(1)}$	37	45	53	$P_{1:2}^{2(1)}$	26	45	24
$U_2^{(1)}$	37	5	53	$P_{2:3}^{2(1)}$	26	5	53
$U_3^{(1)}$	37	45	53	$P_{3:4}^{2(1)}$	37	45	53
$U_4^{(1)}$	37	0	24	$P_{4:5}^{2(1)}$	37	0	24

Table 4, 5 and 6 show the behaviour of inventory quantities of the depot, stage 1 and 2 at the parts manufacturer.

Table 4. Behaviour of inventory quantities at the depot

t=	0	1	2	3	4	5
$I_t^{0(1)}$	40	60	30	30	23	40
$I_t^{0(2)}$	40	60	35	55	25	40
$I_t^{0(3)}$	50	70	25	62	17	50

Table 5. Behaviour of inventory quantity at inventory 1

t=	0	1	2	3	4	5
$I_t^{1(1)}$	10	47	10	36	26	10
$I_t^{1(2)}$	10	55	10	55	15	10
$I_t^{1(3)}$	10	63	10	63	14	10

Table 6. Behaviour of inventory quantity at inventory 2

t=	0	1	2	3	4	5
$I_t^{2(1)}$	5	5	5	5	5	5
$I_t^{2(2)}$	5	5	45	5	50	5
$I_t^{2(3)}$	5	5	5	5	34	5

From these tables, we obtain the following.

(1) The actual quantities of transportation are more than 160 units and not more than 200 units. See table 1.

(2) The actual quantities of production are not more than the quantities of production order. See tables 2 and 3.

(3) Inventory levels at each stage in each period are satisfied the condition of input data. See tables 4, 5 and 6.

4. CONCLUSIONS

In this paper we developed an optimization model to minimize the holding cost at the depot and inventory and transportation costs at the parts manufacturer in the JIT production system.

Finally, we make a numerical experiment using a mathematical programming package.

By applying this model, we assist manufacturers in determining the quantity of production at each stage in each period and transportation to supply the parts to the depot in the view of total minimum cost in logistics system.

REFERENCES

[1] Douglas, M. L. and James, R. S.:" Strategic Logistics Management(Third edition)", Irwin, 1993.

[2] Kimura, O. and Terada, H.:" Design and Analysis of Pull System A Method of Multi-Stage Production Control", Int. J. Prod. Res., pp.241-253, Vol.19, No.3(1981).

[3] Moden, Y.:" Toyota Production System", Industrial Engineering and Management Press(1983).

[4] Bitran, G. R. and Chang, L.: " A Mathematical Programming Approach to a Deterministic Kanban System", Management Sci., pp.427-441, Vol.33, No.4(1987).

[5] Bard, J. F. and Golany, B.:" Determining the number of kanbans in a multiproduct, multistage production system", Int. J. Prod. Res., pp.881-895, Vol.29, No.5(1991).

[6] Hong, S. C. and Fukukawa, T.:" A Goal Programming Approach to a Deterministic Kanban system", Journal of Japan Industrial Management Association, pp.85-92, Vol.43, No.2(1992).

[7] Farhad, M. and Yih-Long, C.:" An Approximate Solution to Deterministic Kanban Systems", Decision Sci., pp.596-607, Vol. 21, No.3(1990)