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# Integration of inventory and transportation decisions in a logistics system

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ABSTRACT

This paper addresses some of the challenges faced by a company which is responsible for delivering coal to its four subsidiaries situated along a river, through river hired or self-owned vessels. We propose to adopt a vendor managed inventory concept that involves establishment of a central warehouse at the port, and apply the Markov Decision Process (MDP) to formulate both ordering and delivery problems, considering different transportation modes, costs, and inventory issues. An efficient algorithm is developed for solving the MDP models. Our computational tests show that the proposed strategy can significantly reduce the overall system costs while maintaining smooth Just-in-Time supplies of coal to the subsidiaries.



## 1. Introduction

Vendor managed inventory (VMI) is an important coordination mechanism in supply chain management where the supplier is given the responsibility of managing inventory in accordance with agreed-upon number of units at specified retail locations (Aviv and Federguen, 1998). The VMI system can reduce inventories and shortages by using advanced online messages and data-retrieval systems (Angulo et al., 2004; Zhao and Cheng, 2009). In addition, as the vendor is guided by mutual agreements on inventory levels, fill rates and transaction costs, both suppliers and customers can maximize their benefits (Andel, 1996; Wang, 2009). However, as most such systems are complicated, the vendor is usually faced with challenges of designing an integrated replenishment strategy (Silver et al., 1998; Nachiappan and Jawahar, 2007). In this paper, a VMI model is applied to a two-echelon logistics system comprising of a parent company (which is called SPC in this paper) and its subsidiaries located along the Yangtze River, China. The parent company is responsible for delivery of coal, the raw material for production, to its subsidiaries. Coal is first supplied from Huainan Mining Group Corporation (HMGC) to the Wu Hu port (WHP) by train, and is then shipped to the subsidiaries using river vessels.

In this paper, we propose establishment of a central warehouse at WHP, which should own a fleet of vessels, and administer the VMI system at the central warehouse for both ordering and delivery decisions. The objective is to integrate ordering and delivery decisions in the system such that the overall costs, including holding, shortage and transportation costs, can be minimized. We thus build a stochastic planning model based on the Markov Decision Process (MDP). An algorithm based on Modified Policy Iteration (MPI), with the action elimination procedure, is designed for the MDP model, such that an approximate optimal solution can be found within reasonable time.

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It is suggested by theoretical and computational analysis that the VMI mode of operations provides significant competitive advantages. In contrast to the existing VMI literature, where the suppliers and their downstream enterprises belong to different business entities, in this paper, the warehouse (acting as a supplier) and the subsidiaries (acting as customers) are all controlled by SPC, such that they have more incentives for information sharing. Moreover, the proposed model can still be considered as VMI for the following two reasons: (1) the warehouse and the subsidiaries are independent of each other in financial terms; and (2) the warehouse is responsible for inventory decisions of the subsidiaries in a VMI-like fashion.

In terms of supply and demand, the model and approach provided in this paper match the situations where a supply lead time may exist, and demands of customers are stochastic, but can be approximated to some discrete distributions. In addition, different from most literature about VMI, our study focuses on complicated logistics systems where different transportation modes are involved along the supply chain, and their capacities, fixed and variable costs, and other issues like penalty costs, lead time, are fully considered. The proposed method for the integration of inventory and transportation decisions is more comprehensive and realistic. We introduce some advanced supply chain notions such as VMI and Just-in-Time to a traditional industry in China, and also successfully apply a stochastic process MDP model for solving practical problems. Therefore, this work can provide insights to companies facing similarly complicated operations in reality.

The rest of this paper is organized as follows. In Section 2, the background of the problem is given. Some general strategies, including VMI, are proposed for overcoming problems in the existing logistics system. A review of relevant literature is provided in Section 3. Section 4 addresses notations and provides a detailed analysis for developing the MDP model, involving both delivery and ordering decisions at the warehouse. In Section 5, a MPI-based algorithm for solving the MDP models is presented. Comparative analysis of the proposed system with the current system is conducted in Section 6. Finally, conclusion is given in Section 7.

## 2. The background of the problem

SPC, which includes more than 80 subsidiaries, either wholly- or partially-owned, is one of the largest producers and suppliers of petrochemical products in China. Coal is one of the important raw materials for petrochemical products such as synthetic resins, synthetic fibers and chemical fertilizers. In 2007, SPC purchased more than 20 million tons of coal, of which nearly 0.25 million tons were consumed by its subsidiaries in the Yangtze River region. One of the coal suppliers to this region is HMGC. Each year, approximately 0.6 million tons of coal are transported through the midway Wu Hu port (WHP) to SPC's subsidiaries, by the supplier. WHP is the largest port for coal transshipment along the Yangtze River and plays a crucial role in coal distribution. In our case study, we focus on ordering and delivery decisions for subsidiaries along the Yangtze River, since the current logistics system has some special properties and potential for improvement.

### 2.1. Current system and its problems

YA, YI, JI and AN are the four main subsidiaries along the Yangtze River. According to their past ordering data, we find that monthly demand of each subsidiary closely follows a Poisson distribution. In the current delivery system, each subsidiary submits its monthly order to the parent company and then the SPC transfers these orders to the supplier (HMGC), without making any modifications. The ordered quantity of coal is delivered from the supplier to the WHP port by train, and is then transshipped to individual subsidiaries by vessels. See Fig. 1 for reference.

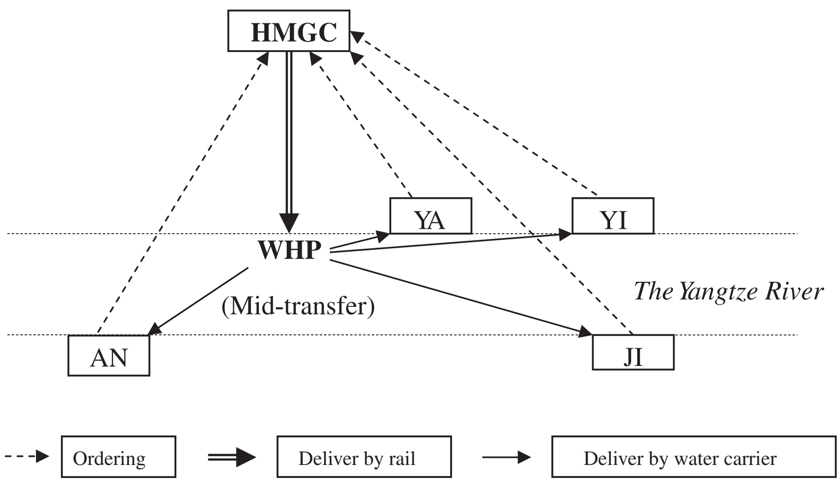


**Fig. 1.** The current logistics system.

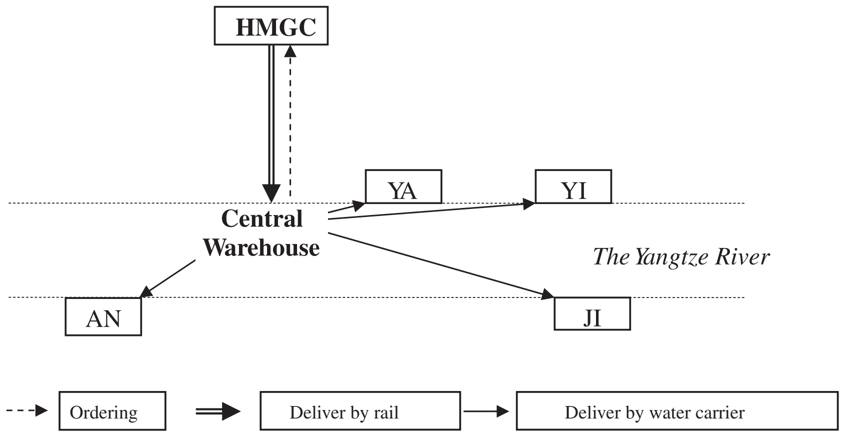


**Fig. 2.** The proposed logistics system.

However, rail carriers are exclusively owned by the government in China, while water carriers are owned mainly by private companies. In the current logistics system, SPC and its subsidiaries rent trains and vessels from different carriers for goods transportation and hence have limited control over transportation schedules and costs.

To make things worse, domestic consumption of coal has shot up in the past few years due to the rapid industrial development of China. Hence, demand for railway wagons and vessels to transport coal has increased as well, resulting in longer and uncertain lead times. Consequently, to ensure stable supplies of coal, each subsidiary has to maintain a large inventory of coal and incur considerable operational costs.

### *2.2. Proposed general solutions*

To solve the above problems, general solutions based on the VMI mechanism are proposed as follows.

Since HMGC, the railways and the water carriers usually pay more attention to larger orders, in order to make the supply of coal more stable, and to take advantage of economies of scale, integrating coal orders from the four subsidiaries is the first step towards a general solution.

Further, as the unit holding cost at WHP is lower than that at any of the subsidiaries, the second step towards the general solution is to establish a central warehouse at WHP, so that coal can be stored here as buffer stock, which can help reduce inventory levels and costs at subsidiaries; in the VMI mode, the central warehouse is responsible for ordering and delivery and acts as the vendor of the subsidiaries.

Finally, to achieve maximum decrease in inventory at the subsidiaries, the Just-in-Time (JIT) mode is introduced in the process of delivery from WHP to the subsidiaries. To realize this, it is suggested that a fleet of vessels should be owned by the central warehouse. These vessels can act as a backup in case the hired water carriers cannot handle and deliver the cargo in time.

The solution framework can now be summarized. A central warehouse, leased and operated by the parent company SPC, is proposed to be established at WHP. At the beginning of each decision period, the central warehouse sends an order to HMGC, considering demand from all subsidiaries, and makes delivery decisions for them. The objective of both delivery and ordering decisions is to minimize the overall costs of the logistics system, with availability of a private fleet of vessels. See Fig. 2 for reference.

In the proposed system, concrete strategies for ordering and delivery decisions need to be developed. A Markov Decision Process (MDP) model is adopted for this purpose, followed by an algorithm based on Modified Policy Iteration (MPI) with an action elimination procedure. Further, considering SPC pays for setting up the central warehouse and provides the investment required for owning the vessels, and bears the cost of maintaining them, a comparison of the proposed system with the current system is conducted.

## 3. Literature review

Literature on vendor managed inventory (VMI) system is quite rich. The advantages of VMI in centralized, as well as decentralized systems are shown by both empirical and quantitative analyses (Stevenson and Spring, 2007; Yao et al., 2007; Bichescu and Fry, 2009). VMI strategies with identified operational guidelines have been developed for different supply chain systems, some of which address classic problems such as inventory routing problem (Archetti et al., 2007; Zhao et al., 2007), while others address real-world problems (Ng et al., 2008; Mustaffa and Potter, 2009).

Managing inventory under uncertainty has received a lot of attention from academics and practitioners alike, and several kinds of reordering policies have been presented accordingly. It is shown that, in a serial multi-echelon inventory system, where there is no fixed ordering cost and demand occurs at the lowest echelon, the optimal inventory of the overall system follows a Base-Stock policy for every echelon (Clark and Scarf, 1960). For models with fixed ordering costs, the class of optimal policies are called as state dependent $(s, S)$ policies (Simchi-Levi and Zhao, 2004), where $s$ and $S$ are lower and upper thresholds that are determined by the given conditional distribution (at that point of time) of future demand. However, the rather simple forms of these policies do not always lead to efficient algorithms for computing optimal policies. In contrast, the corresponding dynamic programs are relatively straightforward to follow, but are usually very time consuming when the state space is large. To avoid "the curse of dimensionality", some researchers have attempted to construct computationally efficient (but suboptimal) heuristics for these problems, such as myopic policies, which attempt to minimize the expected cost for one period, ignoring the potential effect on the cost in future periods (Levi et al., 2007).

Models integrating both inventory and transportation decisions make optimal policies more intractable. Ganeshan (1999) presented a $(s, Q)$-type inventory policy for a network with multiple suppliers by replenishing a central depot, which in turn distributed to a large number of retailers. This paper considered transportation costs, but only as a function of the shipment size. Qu et al. (1999) dealt with an inbound material-collection problem, and provided for decisions for both inventory and transportation to be made simultaneously. However, vehicle capacity was assumed to be unlimited, and so it was solved as a traveling salesman problem (TSP). Cheung and Lee (2002) showed the benefit of shipment coordination and stock rebalancing, ensuring economies of scale in shipping. Yet the employed $(R, Q)$ strategy is not common for delivery of bulk commodities like coal and steel, etc. Mason et al. (2003) developed a discrete event simulation model of a multi-product supply chain to examine the potential benefits to be gained from global integration of warehousing and transportation functions. However, costs associated with such integrated systems were not explicitly assessed. Cardos and Garcia-Sabater (2006) studied an outbound delivery problem considering multiple items; they integrated inventory and transportation decisions into a single model where transportation costs were calculated on the basis of detailed delivery schedules, by VRP heuristic. Disney et al. (2003) used a system dynamics methodology to develop difference equation models of three scenarios (traditional, internal consolidation and VMI) to investigate the impact of a vendor managed inventory strategy on transportation operations in a supply chain. However, the developed models did not include any transport constraints, focusing mainly on the issue of batching for better use of vehicles. Kutanoglu and Lohiya (2008) built a single-echelon, multi-facility stochastic Base-Stock inventory model and integrated it with transportation options and service responsiveness that can be achieved using alternate modes (namely, slow, medium and fast). The capacities of the vehicles, the fixed cost and the penalty cost were not considered in their models. Pundoor and Chen (2009) studied an integrated production and distribution scheduling model in a supply chain consisting of one or more suppliers, a warehouse, and a customer. The problem was to find jointly a cyclic production schedule for each supplier, a cyclic delivery schedule from each supplier to the warehouse, and a cyclic delivery schedule from the warehouse to the customer. Unlike our approach, however, they assumed constant demand.

It is obvious from the above analysis that though quite a few papers consider integration of inventory and transportation decisions, applications of the developed models, in practice, especially for bulk goods delivery, are limited; either the models are too simplified, or they are otherwise unsuitable for real situations. So there is still a wide gap between theory and practice. This paper attempts to match theory and practice, providing a suitable method for integration of inventory and transportation decisions in a real world logistics system, which can not only be a guide for this particular company, but also for other logistics systems facing similar situations or problems. Moreover, different from most of the extant literature on VMI, our study focuses on a complicated logistics system where different capacities of different transportation modes, their respective fixed and variable costs, and other relevant issues like penalty costs and lead time, are fully considered. The proposed method for integration of inventory and transportation decisions is more comprehensive and realistic.

A Markov Decision Process is a sequential decision-making stochastic process characterized by five elements: decision epochs, states, actions, transition probabilities, and rewards (Puterman, 1994). At each decision epoch, the system occupies a decision-making state. As a result of taking an action in a state, the decision-maker receives a reward (which may be positive or negative) and the system goes to the next state with a certain probability, which is called the transition probability. A decision rule is a function for selecting an action in each state, while a policy is a collection of such decision rules over the state space. Implementation of a policy generates a sequence of rewards. The MDP problem is to choose a policy that optimizes the function of this rewards' sequence.

Markovian formulations are useful in solving a number of real-world problems under uncertainties, such as determining inventory levels for retailers, scheduling maintenance for manufacturers, and scheduling/planning in production management (Giannoccaro and Pontrandolfo, 2002; Yokoyama and Lewis, 2003; Gayon et al., 2009). It is shown that the MDP system results in lower average inventory levels and requires fewer reorders to be placed (Yin et al., 2004). These advantages are more pronounced for products with highly variable demand, for which most other models do not perform well.

Value iteration and policy iteration are two fundamental dynamic programming algorithms for solving MDPs (Howard, 1960), which are sometimes inefficient. Puterman and Shin (1978) proposed a Modified Policy Iteration algorithm, which seeks a trade-off between cheap and effective iterations, and is preferred by some practitioners. In a sense, it is just policy iteration, where the policy evaluation step is carried out via value iteration. This can be shown to produce an approximation and achieve substantial speedups. In our paper, the algorithm is adopted to solve the MDP model. To improve the efficiency of the Modified Policy Iteration algorithm for our case, we add one of the action elimination procedures introduced in Puterman and Shin (1982) at each iteration.

## 4. The delivery and the ordering decisions

Wu Hu port (WHP) is the largest transshipment point of coal along the Yangtze River. There are some other coal suppliers, though their prices are higher than HMGC. In case inventories on hand at the central warehouse are not enough to satisfy subsidiaries' demand, additional quantities can be procured by ordering from other suppliers, with penalty costs.

The specific supply situation in our case study is that the supply of coal from HMGC to WHP, and that from WHP to subsidiaries, are realized by different transportation modes, and by carriers with different contracts and constraints. Therefore, delivery and ordering decisions can be made separately, and the optimal solution of each stage is also the optimal decision for the overall logistics system (Lee et al., 2000), with the assumption that the central warehouse at WHP can obtain some coal from "alternate" sources to meet any shortfalls.

### 4.1. The decision about deliveries to the subsidiaries

In the delivery process, the subsidiaries check their inventory levels periodically, at discrete time point $t$ (in month), $t = 1, 2, \ldots$, and report it to the central warehouse (at WHP). Then, in order to minimize the expected long-run costs for all subsidiaries, including holding, shortage and transportation costs, the central warehouse needs to make an optimal delivery decision for them, based on comprehensive information. It delivers the optimal quantity to each subsidiary, following the JIT principle, to meet the demand in the given period $[t, t + 1)$. The unit inventory of coal is 1000 tons in our case.

To determine the optimal decision, a MDP model is employed. Considering the practical situation that coal carried by one vessel must be shipped to one subsidiary only, due to high docking and unloading costs, the MDP model is formulated and solved for each subsidiary $i$.

Before illustration of the delivery strategy, the notations are defined as follows. All costs are denoted in Chinese currency: Yuan (1US$ $\approx$ 6.8 Yuan in 2009).

- $i$ index of the subsidiary, where 1 is for YA, 2 for YI, 3 for JI and 4 for AN
- $v$ capacity of the vessel
- $c_v$ vessel's unit variable transportation cost
- $C_v$ vessel's fixed transportation cost per trip
- $d_i$ distance from the central warehouse to subsidiary $i$ (in kilometers)
- $h_i$ unit holding cost at subsidiary $i$ per decision period
- $q_i$ unit shortage cost at subsidiary $i$ per decision period
- $\mu_i$ mean of demand occurring at subsidiary $i$ per decision period (in 1000 tons)

Then $P(x_i = r) = e^{-\mu_i}\frac{(\mu_i)^r}{r!}$ is the probability that the demand of subsidiary $i$ is 1,000 $r$ tons, $r = 1, \ldots, \infty$. In our case, demands of the subsidiaries can be regarded as independent of each other, and also time independent.

Private vessels owned by the warehouse act as backups, in case vessels from hired water carriers cannot deliver the cargoes timely; so we assume the quantities delivered by private vessels form only a small part of the total. For the purpose of simplification, we assume that both private and hired vessels have the same costs, $c_v$ and $C_v$, as unit variable and fixed transportation costs, respectively. Instead, since the vessels are bought at the beginning of the planning horizon, the purchasing cost, denoted as $P_v$, is considered in computational analysis.

Considering the unit time is one month in our case study, and the longest round trip to the subsidiaries can be completed within one day, purchase of two vessels of the same size as those owned by the shipping company (with a capacity of 1500 tons each), is considered enough to ensure JIT delivery. Capacity of 1500 tons for the vessels is determined on the basis of prices of vessels and the buoyancy of the river; this decision is essentially beyond the scope of our case study.

For any subsidiary $i$, a delivery policy refers to quantities delivered on the basis of any possible inventory-on-hand in different decision periods. Denote $\pi$ as a stable delivery policy, meaning the decision is stable at different time periods. Our objective is to find the optimal policy $\pi^*$over an infinite time horizon. To determine the optimal inventory-delivery policy, we formulate the above problem as a discrete time Markov Decision Process (MDP) with finite state and action space. It contains the following five components (for the sake of notational convenience, we omit the subscript $i$ in the following expression).

**State space** $S$: $S$ is the set of $X^t$, which is the possible on-hand inventory in the subsidiary at time point $t$ (before the occurrence of the demand). An upper bound $D^{\max}$ and a lower bound $D^{\min}$ for demand can be estimated according to past data, so that the probability of actual demand being out of this range is small (lower than 0.01 in our case). Therefore, we have $X^t \in S = \{0, \ldots, D^{\max} - D^{\min}\}$. We ignore the cases where $X^t < 0$, and take the shortage cost into consideration instead.

**Action space** $A$: For any state $X^t \in S$, action space $A(X^t)$ is the set of delivery quantities. Our objective is to find the optimal policy, which is stable over an infinite time horizon; so the delivery decision is based on the state, not the decision period. For each subsidiary, it is assumed that the space for holding inventory is limited (it is not allowed to keep too much inventory), and hence, the action set is limited by $D^{\max}$. So the delivery quantity as a decision variable is $a \in A(X^t) = \{0, 1, \ldots, D^{\max}\}$.

**Transfer matrix** $G$: Denote $D^t$ as the demand at time period $[t, t + 1)$, so the quantity consumed during this period is min $\{X^t + a, D^t\}$, $a \in A(X^t)$, and the inventory at time point $t + 1$ is $X^{t+1} = \max\{X^t + a - D^t, 0\}$. Denote $P(X^{t+1}|X^t, a)$ as the probability that the inventory state transfers from $X^t$ to $X^{t+1}$, given delivery decision $a \in A(X^t)$. Let $X^t = k$, $X^{t+1} = j$, then

$$P(X^{t+1}|X^t, a) = P(j|k, a) = P_{kj}(a) = \begin{cases} P(D^t \geq k + a), & j = 0 \\ P(D^t = k + a - j), & j \neq 0 \end{cases} \quad (1)$$

Denote $G$ as the state-transfer matrix. $G$ can be represented as a three-dimension matrix $G(k, j, a)$, or a two-dimension matrix $G(k, j)$, or $G(j, a)$, depending on the variables considered.

**Instantaneous reward** $g$: Denote $g(X^t, a)$ as the expected cost of a single decision period t, $[t, t + 1)$, with delivery action $a \in A(X^t)$ under state $X^t$ at the beginning of the period, then

$$g(X^t, a) = n_t(C_v + 2dc_v) + E(I^t) + E(s^t), \quad (2)$$

where $n_t(C_v + 2dc_v)$ is the transportation cost, in which $n_t = \frac{a}{v}$ is the number of vessels required for transporting the order quantity, $C_v$ is the fixed transportation cost for each vessel, and $2dc_v$ is the variable transportation cost of the round trip; $E(I^t)$ and $E(s^t)$ are the expected inventory and expected shortage costs, respectively, and $I^t$, $s^t$ are the corresponding quantities of inventory at the end of period $t$ and lost sales in period $t$, respectively. Denoting $\overline{D} = D^{\max} - D^{\min}$, we have,

$$E(I^t) = \sum_{I^t \in \{0, \overline{D}\}} P(D^t = X^t + a - I^t) I^t h$$

$$E(s^t) = \sum_{s^t \in \{0, \overline{D}\}} P(D^t = X^t + a + s^t) s^t q$$

where $D^t \in [D^{\min}, D^{\max}]$.

**Long-run expected discounted cost** $V$: With an initial state $k$, the objective of the delivery decision is to find a policy $\pi$, such that the expected discounted cost $V(\pi, k)$ over an infinite time horizon is minimal. Denote $\pi = \{\pi(k)|k = 0, 1, 2, \ldots, D^{\max} - D^{\min}\}$, and let $\beta \in [0, 1)$ be the economic discount factor. Then we have

$$V(\pi, k) = \sum_{t=1}^{\infty} \beta^{t-1} \sum_{\substack{j \in S \\ a \in A(k)}} P(X^t = j | X^1 = k, a) g(j, a) = g(k, \pi(k)) + \beta \sum_{j \in S} P(j|k, \pi(k)) V(\pi, j).$$

Here $k \in S$, $\pi \in \prod$, $\prod$ is the set of all Markov policies. Denote $V(\pi^*, k)$ as the optimal expected cost with initial state $k$; $\pi^*$ is the optimal policy; then

$$V(\pi^*, k) = \min_{\pi \in \prod} V(\pi, k) = \min_{\pi \in \prod} \left\{ g(k, \pi(k)) + \beta \sum_{j \in S} P(j|k, \pi(k)) V(\pi, j) \right\}. \quad (3)$$

*4.2. The central warehouse's ordering strategy*

In the proposed system, besides making decisions about deliveries to the subsidiaries to fulfill their demands, the central warehouse needs to make ordering decisions for itself as well. The quantities ordered will be transported by train from HMGC to the warehouse, and will arrive after a certain lead time period because of the long distance between HMGC and the warehouse. For the ordering strategy, we can continue applying a MDP to find the optimal solution.

The notations are listed as follows:

- $c_n$ unit transportation cost from HMGC to the central warehouse
- $C_w$ fixed cost for hiring premises and operating the central warehouse (per decision period), which is supposed to be independent of the quantities of inventory-on-hand
- $C_f$ fixed cost for holding the captively-owned vessels, per decision period
- $q_w$ unit penalty cost for purchasing coal at a higher price from alternate suppliers close to the warehouse instead of HMGC. It happens only when the on-hand inventory is not enough to satisfy the subsidiaries' immediate demands and the warehouse cannot wait for further supplies from HMGC
- $h_w$ unit holding cost per decision period at the central warehouse
- $l$ constant lead time for receipt of ordered quantities at the central warehouse (in months)

Similar to the delivery decision, we aim at finding the optimal ordering policy, which is stable over an infinite time horizon. However, the existence of the lead time and the fact that the lead time is longer than the decision period make the problem complicated. To solve it by the MDP model, we define the decision variable, $L$, at time point $t$ as the summation of the quantities that have been ordered but have not yet arrived, that is, the total quantities ordered from period $(t - l + 1)$ to period $t$.

We can see that the demand faced by the central warehouse during the lead time periods approximately follows a Poisson distribution, with mean $U = \sum_{i=1}^{4} u_i l$ (in our case, lead time $l = 2$). Please refer to Appendix A for detailed explanation. With this property, we can build the transfer matrix efficiently. Now the following MDP elements can be developed.

**State space** $S_w$: $S_w$ is the set of $Y^t$, which is the possible inventory level in the central warehouse at time point $t$ (before realization of the demand).

**Lemma 1.** $Y^t \in \left\{0, \ldots, \sum_{i=1}^4 D_i^{\max} l - \sum_{i=1}^4 D_i^{\min} l\right\}$.

**Proof.** At time point $t$, decision variable $L$ is defined as the sum of quantities ordered but yet to arrive. It can be deduced that quantities on hand plus $L$ are required to meet the demand faced by the central warehouse during the lead time period, which is $L + Y_t \in \left\{\sum_{i=1}^4 D_i^{\min} l, \sum_{i=1}^4 D_i^{\max} l\right\}$. Then $Y^t \in \left\{0, 1, \ldots, \sum_{i=1}^4 D_i^{\max} l - \sum_{i=1}^4 D_i^{\min} l\right\}$. □

As in the case of the subsidiaries, an upper bound $D_w^{\max}$ and a lower bound $D_w^{\min}$ are applied to the demand the central warehouse faces during the lead time period. So,

$$Y^t \in S_w = \{0, 1, \ldots, D_w^{\max} - D_w^{\min}\} \in \left\{0, 1, \ldots, \sum_{i=1}^4 D_i^{\max} l - \sum_{i=1}^4 D_i^{\min} l\right\}.$$

**Action space** $A_w$**:** For any state $Y^t \in S_w$, action space $A_w(Y^t)$ is the set of $L$ defined as the sum of quantities ordered but yet to arrive. It is easily deduced that $L \in A_w(Y^t) = \{0, 1, \ldots, D_w^{\max}\}$.

**Transfer matrix** $G_w$**:** Denote $D_w^t$ as the total demand faced by the central warehouse during lead time period $t, t+1, \ldots, t+1-l$, the quantity consumed (from what is provided by HMGC) during the period is $\min\{L + Y^t, D_w^t\}$, and the inventory at the end of this duration is $Y^{t+l} = \max\{L + Y^t - D_w^t, 0\}$. Denote $F(Y^{t+l}|Y^t, L)$ as the probability of the inventory state transferring from $Y^t$ to $Y^{t+l}$ under decision $L \in A_w(Y^t)$. Let $Y^t = k$, $Y^{t+l} = j$, then

$$F(Y^{t+l}|Y^t, L) = F(j|k, L) = F_{kj}(L) = \begin{cases} P(D_w^t \geq k + L), & j = 0 \\ P(D_w^t = k + L - j), & j \neq 0 \end{cases} \tag{4}$$

where $D_w^t$ follows Poisson distribution with mean $\sum_{i=1}^4 \mu_i l$. State-transfer matrix $G_w$ can be a three-dimensional matrix $G_w(k, j, L)$, or a two-dimensional matrix $G_w(k, j)$, or $G_w(j, L)$, depending on the variables considered.

**Instantaneous reward** $g_w$**:** Denote $g_w(Y^t, L)$ as the single period expected cost with decision $L \in A_w(Y^t)$, then,

$$g_w(Y^t, L) = C_w + C_f + E(R_n^t) + E(I_w^t) + E(s_w^t), \tag{5}$$

where $E(R_n^t)$, $(E(I_w^t)$ and $E(s_w^t)$ are the expected variable transportation costs, expected inventory cost and expected shortage cost, respectively. As $E(R_n^t)$ is related to the quantity ordered at time period $t$, it is determined by $L$, and is difficult to calculate. Here, we estimate $E(R_n^t)$ as

$$E(R_n^t) = \frac{c_n L}{l}.$$

Denote $\overline{U} = \left(\sum_{i=1}^4 U_i^{\max} l - \sum_{i=1}^4 U_i^{\min} l\right)$, $E(I_w^t)$ and $E(s_w^t)$ can be calculated as follows:

$$E(I_w^t) = \sum_{I_w^t \in \{0, \overline{U}\}} F(U^t = Y^t + L - I_w^t) I_w^t h_w,$$

$$E(s_w^t) = \sum_{s_w^t \in \{0, \overline{U}\}} F(U^t = Y^t + L + s_w^t) s_w^t q_w,$$

where $U^t \in [D_w^{\min}, D_w^{\max}]$.

**Long-run expected discounted cost** $V_w$**:** Denote $\pi_w = \{\pi_w(k) | k = 1, \ldots, D_w^{\max} - D_w^{\min}\}$. With an initial state $k$, the objective of the ordering decision is to find a policy $\pi_w$ with minimal expected discounted cost $V_w(\pi_w, k)$ over an infinite time horizon; so we have

$$V_w(\pi_w, k) = \sum_{t=1}^{\infty} \beta^{t-1} \sum_{\substack{j \in S_w \\ L \in A_w(k)}} F(X^{t+l} = j | X^1 = k, L) g_w(j, L) = g_w(k, \pi_w(k)) + \beta \sum_{j \in S_w} F(j | k, \pi_w(k)) V_w(\pi_w, j).$$

Here, $k \in S_w$, $\pi_w \in \prod_w$, $\prod_w$ is the set of all Markov policies. Denote $V_w(\pi_w^*, k)$ as the optimal expected value with initial state $k$, and $\pi_w^*$ is the optimal policy, then

$$V_w(\pi_w^*, k) = \min_{\pi_w \in \prod_w} V_w(\pi_w, k) = \min_{\pi_w \in \prod_w} \left\{g_w(k, \pi_w(k)) + \beta \sum_{j \in S_w} F(j | k, \pi_w(k)) V_w(\pi_w, j)\right\}. \tag{6}$$

## 5. Modified Policy Iteration algorithm

To find the optimal solutions of Eqs. (3) and (6), we apply the Modified Policy Iteration algorithms (MPI) with action elimination procedures, which can obtain the approximate optimal action for each state (Puterman and Shin, 1982). The algorithm is represented by Fig. 3 and the details are presented in Appendix B.

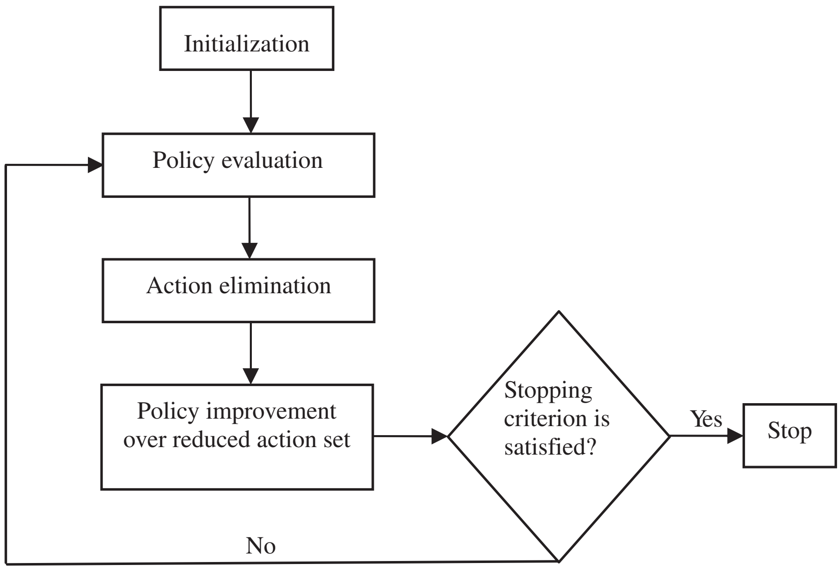


**Fig. 3.** Modified Policy Iteration algorithms with action elimination.

## 6. Comparative analysis

### *6.1. Advantages of the proposed system*

In the proposed system, the ordering decision and the delivery process are integrated and performed by the central warehouse, so the subsidiaries can concentrate on production, without paying much attention to coal ordering, transportation and delivery decisions. In addition, as the Just-in-Time (JIT) mode is introduced, the subsidiaries need not maintain large inventories; the corresponding inventory cost can also be reduced. Some benefits of the proposed system to the subsidiaries seem obvious.

Detailed analysis shows that the advantages of the new plan also include: (1) the order is bigger due to integration of orders of the four subsidiaries, making the supply of coal more stable; (2) all orders and deliveries for the subsidiaries will be integrated, resulting in economies of scale both in transportation and ordering processes; (3) as the unit inventory holding cost at the central warehouse is lower than that at the subsidiaries, and integration can reduce the buffer stock, shifting of the inventory from the subsidiaries to the central warehouse can reduce the inventory cost significantly; and (4) the JIT mode is introduced in the delivery process from WHP to the subsidiaries, effectively lowering the uncertainty faced by the subsidiaries.

However, as cost of establishment of the central warehouse and the ownership of the fleet of vessels is high, quantitative analysis between the current and the proposed systems is needed. In this section, by applying the algorithms given in Section 5 for solving the MDP models, we compare the costs between the two systems.

### *6.2. The costs in the proposed system*

Tables 1 and 2 list estimations of costs and demand parameters with regard to the subsidiaries and the central warehouse, respectively. Because the subsidiaries along the Yangtze River are relatively close, and all are in an area with approximately the same general price level, we assume the same unit holding cost and unit penalty cost for all of them. By using the MDP models and the MPI algorithm, we determine the optimal delivery decisions for each subsidiary, and the optimal ordering decision of the central warehouse, along with the long-run discounted costs. The algorithm is coded in MATLAB software and is run on a Pentium 4 computer with 1400 MHz processor and 256 MB RAM. The parameters for computation are taken as: $m = 5$, $\varepsilon = 0.5$, and $\rho = 0.8$ (See explanation of these parameters in Appendix B). The computational results are listed in Tables 3 and 4. Since the results can be obtained within reasonable computational time, the CPU time is not stated.

In Table 3, optimal delivery quantity for any state, of each subsidiary, is given, along with the long-run discounted cost, when the state is taken as the initial value. It can be seen from the computational results that for each subsidiary, the sum of the inventory level and the delivery quantity varies in a certain range, which is [27, 28] for YA; [20, 22] for YI; [5] for JI and [6, 7] for AN. The sum of the inventory level and the delivery quantity are not a constant, indicating that the constant Base-Stock level policy is not suitable for the problem, because in our case, transportation cost is limited by the capacity of the vessel, and is non-linearly charged.

**Table 1**
Parameters of the subsidiaries.

| $i$ | $h_i$ | $q_i$ | $d_i$ | $D_i^{max}$ | $D_i^{min}$ | $\mu_i$ | $c_v$ | $C_v$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 32,000 | 86,000 | 56 | 35 | 15 | 25 | 90 | 975 |
| 2 | 32,000 | 86,000 | 81 | 28 | 10 | 19 | 90 | 975 |
| 3 | 32,000 | 86,000 | 81 | 9 | 1 | 4 | 90 | 975 |
| 4 | 32,000 | 86,000 | 90 | 11 | 1 | 5 | 90 | 975 |

$i$ = 1 for YA; 2 for YI; 3 for JI; 4 for AN.

**Table 2**
Parameters of the central warehouse.

| $h_w$ | $q_w$ | $l$ | $D_w^{max}$ | $D_w^{min}$ | $\mu_w$ | $c_n$ | $C_w$ | $c_f$ | $P_v$ |
|---|---|---|---|---|---|---|---|---|---|
| 15,000 | 50,000 | 2 | 123 | 89 | 106 | 8000 | 1000,000 | 100,000 | 3600,000 |

**Table 3**
Optimal MDP delivering policies and the long-run discounted costs ($\times 10^6$).

| Inventory level | Delivery amount | | | | Long-run discounted cost | | | |
|---|---|---|---|---|---|---|---|---|
| | YA | YI | JI | AN | YA | YI | JI | AN |
| 0 | 27 | 21 | 5 | 6 | 1.786 | 1.752 | 0.621 | 0.725 |
| 1 | 27 | 21 | 4 | 6 | 1.780 | 1.747 | 0.605 | 0.723 |
| 2 | 25 | 18 | 3 | 4 | 1.775 | 1.734 | 0.510 | 0.708 |
| 3 | 24 | 18 | 2 | 3 | 1.764 | 1.721 | 0.589 | 0.691 |
| 4 | 24 | 18 | 1 | 3 | 1.758 | 1.716 | 0.574 | 0.689 |
| 5 | 22 | 15 | 0 | 1 | 1.753 | 1.703 | 0.559 | 0.674 |
| 6 | 21 | 15 | – | 0 | 1.742 | 1.689 | – | 0.657 |
| 7 | 21 | 15 | – | – | 1.736 | 1.685 | – | – |
| 8 | 19 | 12 | – | – | 1.730 | 1.672 | – | – |
| 9 | 18 | 12 | – | – | 1.719 | 1.659 | – | – |
| 10 | 18 | 12 | – | – | 1.714 | 1.654 | – | – |
| 11 | 16 | 9 | – | – | 1.708 | 1.641 | – | – |
| 12 | 15 | 9 | – | – | 1.697 | 1.626 | – | – |
| 13 | 15 | 9 | – | – | 1.691 | 1.623 | – | – |
| 14 | 13 | 6 | – | – | 1.686 | 1.610 | – | – |
| 15 | 12 | 6 | – | – | 1.675 | 1.596 | – | – |
| 16 | 12 | 6 | – | – | 1.669 | 1.592 | – | – |
| 17 | 10 | 3 | – | – | 1.664 | 1.579 | – | – |
| 18 | 9 | 3 | – | – | 1.653 | 1.565 | – | – |
| 19 | 9 | – | – | – | 1.647 | – | – | – |
| 20 | 7 | – | – | – | 1.642 | – | – | – |

In Table 4, optimal ordering quantity for any state of the central warehouse is given, along with the long-run discounted cost, when the state is taken as the initial value. Different from the computational results in Table 3, the sum of the inventory level and the delivery variable is a constant value, which is 113 here. Since the fixed costs ($C_w + C_f$) happen at each decision period, no matter whether coal is supplied by the HMGC or not, and the variable transportation cost is linearly changed, the optimal decisions follow the constant Base-Stock level strategy.

### 6.3. Comparison of costs of the two systems

Currently, each subsidiary orders coal according to predication and experience. In addition, without the central warehouse and the private vessels, the lead time from the supplier HMGC to the subsidiaries is quite long. Therefore, each subsidiary has to order and hold a larger stock to avoid shortages. Based on the yearly supply contract made between the subsidiaries and HMGC, the estimated cost structure is as given in Table 5. Here, approximation of the long-run discounted cost is computed by summing up the geometric sequence of the period total cost with discount rate $\beta = 0.8$ .

Table 6 lists the comparison of costs under the current policy and the proposed policy. Under the proposed policy, the long-run cost of each subsidiary is the mean value of long-run costs corresponding to every initial inventory level in Table 3. It can be seen from Table 6 that for each subsidiary, cost under the current policy is much higher than that under the proposed policy. In the VMI concept, as the standard deviation of demand faced by the central warehouse is lower than the sum of those of the four subsidiaries, and also the integrated decision made by the central warehouse for all subsidiaries can

**Table 4**
Optimal MDP ordering policy and its long-run discounted cost ($\times 10^6$).

| $k$ | $L$ | $V_w(\pi_w^*, k)$ | $k$ | $L$ | $V_w(\pi_w^*, k)$ | $k$ | $L$ | $V_w(\pi_w^*, k)$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 113 | 8.648 | 24 | 89 | 8.552 | 48 | 65 | 8.456 |
| 1 | 112 | 8.644 | 25 | 88 | 8.548 | 49 | 64 | 8.452 |
| 2 | 111 | 8.640 | 26 | 87 | 8.544 | 50 | 63 | 8.448 |
| 3 | 110 | 8.636 | 27 | 86 | 8.540 | 51 | 62 | 8.444 |
| 4 | 109 | 8.632 | 28 | 85 | 8.536 | 52 | 61 | 8.440 |
| 5 | 108 | 8.628 | 29 | 84 | 8.532 | 53 | 60 | 8.436 |
| 6 | 107 | 8.624 | 30 | 83 | 8.528 | 54 | 59 | 8.432 |
| 7 | 106 | 8.620 | 31 | 82 | 8.524 | 55 | 58 | 8.428 |
| 8 | 105 | 8.616 | 32 | 81 | 8.520 | 56 | 57 | 8.424 |
| 9 | 104 | 8.612 | 33 | 80 | 8.516 | 57 | 56 | 8.420 |
| 10 | 103 | 8.608 | 34 | 79 | 8.512 | 58 | 55 | 8.416 |
| 11 | 102 | 8.604 | 35 | 78 | 8.508 | 59 | 54 | 8.412 |
| 12 | 101 | 8.600 | 36 | 77 | 8.504 | 60 | 53 | 8.408 |
| 13 | 100 | 8.596 | 37 | 76 | 8.500 | 61 | 52 | 8.404 |
| 14 | 99 | 8.592 | 38 | 75 | 8.496 | 62 | 51 | 8.400 |
| 15 | 98 | 8.588 | 39 | 74 | 8.492 | 63 | 50 | 8.396 |
| 16 | 97 | 8.584 | 40 | 73 | 8.488 | 64 | 49 | 8.392 |
| 17 | 96 | 8.580 | 41 | 72 | 8.484 | 65 | 48 | 8.388 |
| 18 | 95 | 8.576 | 42 | 71 | 8.480 | 66 | 47 | 8.384 |
| 19 | 94 | 8.572 | 43 | 70 | 8.476 | 67 | 46 | 8.380 |
| 20 | 93 | 8.568 | 44 | 69 | 8.472 | 68 | 45 | 8.376 |
| 21 | 92 | 8.564 | 45 | 68 | 8.468 | 69 | 44 | 8.372 |
| 22 | 91 | 8.560 | 46 | 67 | 8.464 | 70 | 43 | 8.368 |
| 23 | 90 | 8.556 | 47 | 66 | 8.460 | | | |

**Table 5**
Current periodic cost structure of the subsidiary companies ($\times 10^5$).

| $i$ | Holding cost | Transportation cost | Shortage cost | Total costs | Approximated long-run cost |
|---|---|---|---|---|---|
| 1 | 10.6 | 5.072 | 0 | 15.672 | 78.361 |
| 2 | 8.5 | 4.644 | 2 | 15.144 | 75.722 |
| 3 | 2.1 | 1.102 | 0 | 3.162 | 15.811 |
| 4 | 2.0 | 1.167 | 1 | 4.117 | 20.585 |

**Table 6**
System cost comparison between the two policies ($\times 10^5$).

| Subsidiary | Under the current policy | Under the proposed policy |
|---|---|---|
| YA | 78.361 | 17.137 |
| YI | 75.722 | 16.614 |
| JI | 15.811 | 5.882 |
| AN | 20.590 | 6.917 |
| Central warehouse + vessels | – | 85.050 + 36.0 |
| Total | 190.479 | 167.617 |

achieve economies of scale, both inventory and transportation costs can be reduced substantially. Even after considering the establishment cost of the central warehouse and the purchasing cost of a fleet of private vessels, the proposed system is still preferable.

## 7. Conclusion

This paper addresses a practical logistics problem confronted by SPC, a petroleum and chemical corporation in China, and its subsidiaries along the Yangtze River. We recommend the VMI concept to the parent company, involving establishment of a central warehouse, which can restrain the upstream wave in the supply chain at WHP and also serve as a buffer for stabilizing coal supplies. The central warehouse is responsible for delivery decisions for the subsidiaries, as well as its own ordering decisions. By operating a fleet of private vessels, the JIT mode can be adopted in the delivery process.

To find the optimal delivery and ordering decisions, the MDP models are developed, in which ordering and delivery decisions are integrated concurrently to minimize the long-run expected cost under discount criterion, including transportation cost, expected inventory cost and expected shortage cost. We apply the Modified Policy Iteration algorithm with an action elimination procedure to solve the MDP models and approximately obtain the optimal solutions.

With a set of data, the long-run expected cost of the whole system under the proposed strategy is compared with the current cost. It turns out that the proposed VMI strategy fits well, reducing the cost of the system significantly.

Even though the computational results are obtained under certain assumptions, such as demand characteristics and cost structures, it can be deduced from qualitative and quantitative analysis, as shown in this paper, that VMI concepts and scientific management decisions offer apparent advantages.

Contributions of the paper to the literature are as follows. We provide a general integration framework and solution approach for a set of complicated logistics systems. In such kind of systems, a supply lead time may exist, while customer demands are stochastic, and can be approximated to some discrete distributions. We introduce some advanced supply chain notions as VMI and Just-in-Time to a traditional industry, and also successfully apply a stochastic process MDP model for solving practical real life problems. Different from most extant literature, our models handle the real world case where capacities of different transportation modes, and their fixed and variable costs are different, and other issues like penalty costs and lead times are involved. The proposed method for integration of inventory and transportation decisions is comprehensive and realistic. Therefore, it is concluded that this work provides a good guide not only for SPC, but also for other companies facing similar scenarios.

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## Appendix A. Discussion of distribution of demand faced by the central warehouse

(In this discussion, we limit our scope to the real life case under study, that is, to specify the customer index as $i = 1, \ldots, 4$ and the lead time $l = 2$. However, the rationale and the conclusion are also applicable to general cases where $i$ and $l$ can take any positive integer values).

In the optimal delivery decision, we denote $Z_i^t$, $i = 1, \ldots, 4$ as the sum of quantities received and inventory-on-hand of subsidiary $i$ at time period $t$. If there are no fixed costs, based on Heyman and Sobel (1984), $Z_i = Z_i^t$, $t = 1, \ldots, \infty$.

In the delivery decision process, the transportation cost depends on the capacity of the vessel and the number of trips. So, for achieving economies of scale in transportation, optimal $Z_i^t$ may not be a stable value; it varies in a certain range.

Denoting $a_i^t$ as the quantity delivered to subsidiary $i$ during the time period $[t, t+1)$, according to Heyman and Sobel (1984), we have

$$a_i^t = Z_i^t - Z_i^{t-1} + D_i^t,$$
$$a_i^{t-1} = Z_i^{t-1} - Z_i^{t-2} + D_i^{t-1}.$$

So, demand faced by the warehouse from subsidiary $i$ during lead time $[t-1, t+1)$ is

$$a_i^t + a_i^{t-1} = Z_i^t - Z_i^{t-2} + D_i^t + D_i^{t-1} \tag{A.1}$$

It can be deduced from qualitative analysis that the mean of this random value $(Z_i^t - Z_i^{t-2})$ is 0. The reasons are as follows: (i) in our case study, the final delivery strategy is taken as stable, which means that the quantity delivered depends on the inventory level, not the time period; (ii) $Z_i^t$ relies on the demand faced by subsidiary $i$, which is *i.i.d.* over an infinite time horizon. The second reason also indicates that $Z_i^t$ is correlated with $D_i^t$.

Based on the above analysis, it can be concluded from formulation (A.1) that the distribution of $(a_i^t + a_i^{t-1})$ approximately resembles that of $(D_i^t + D_i^{t-1})$, and the conclusion is suitable to each subsidiary. As $\sum_{i=1}^4 \sum_{t=1}^2 D_i^t$ follows a Poisson distribution with mean $\sum_{i=1}^4 2\mu_i$, it is reasonable to assume that $\sum_{i=1}^4 \sum_{t=1}^2 a_i^t$ also follows a Poisson distribution, with mean $\sum_{i=1}^4 2\mu_i$.

## Appendix B. Modified Policy Iteration algorithm

In the following algorithm, notations are specified for the MDP model of delivery decision. The ordering decision model can be solved by the same algorithm, with notation changes.

**Step 1. Initialization.** Select $\varepsilon > 0$, to be used as a stopping criterion. Denote integers $n$ and $m$ as the iteration stage and the order of the algorithm, respectively. Set $n = 0$ and $m > 0$. Define policy $\pi_k$ and single period reward $g(k, \pi_k) = \min_{a \in A(k)} g(k, a)$, $k \in S$. Define $E_n(k)$ to be the set of actions that have been eliminated in state $k$ at iteration stage $n$ and set $E_n(k)$. Set the long-run expected discounted cost at initial stage $V_0 = \frac{1}{1-\beta}[\max_{k \in S} g_0(k)] \times e$, where $e$ denotes a column vector of ones.

**Step 2. Evaluation phase.** Calculate $V_{n+1}(k) = V_n(k) + \sum_{t=0}^{m} \beta^t P^t(k, \pi_n(k))(TV_n(k) - V_n(k))$, $k \in S$, where $V_n(k)$ represents $V(\pi_n, k)$, which is the total expected value at iteration stage $n$ with initial state $k$ (Here, $\pi_n$ is composed by $a \in A(k) \setminus E_n(k)$, $k \in S$. $TV_n(k)$ and $T_{\pi_n}(k)$ are two operators, defined as:

$$T_{\pi_n}(k) = g(k, \pi_k) + \beta \sum_{j \in S} P_{kj}(\pi_n) V_n(k), \quad k \in S, \ pi_n \in \Pi$$

$$TV_n(k) = \min_{\pi_n \in \Pi} T_{\pi_n} V(k) = \min_{a \in [A(k) - E_n(k)]} \left[ g(k, a) + \beta \sum_{j \in S} P_{kj}(a) V_n(k) \right], \quad k \in S$$

Increment $n$ by one, i.e. $n = n + 1$.

**Step 3. Action elimination.** Suppose that at iteration $n + 1$ and $\forall a \in A(i) \setminus E_n(i)$,

$$g(k, a) + \beta \sum_{j \in S} P_{kj}(a) V_n(k) + \beta \cdot U(DV_{n,1}) - V_{n+1}(k) > \beta \cdot L(PHV_n),$$

then $a \in E_{n+1}(k)$. Here:

$$DV_{n,1}(k) = V_{n+1}(k) - V_n(k),$$
$$U(DV_{n,1}) = \min_{k \in S} DV_{n,1}(k),$$
$$L(PHV_n) = \max_{k \in S} \{(\beta P_n)^m [TV_n(k) - V_n(k)]\},$$

where $P_n$ denotes the state-transfer probability matrix used at iteration $n$, that is $G(k, \pi_n)$.

**Step 4. Improvement phase.** Find policy $\pi_n$, composed of $a \in A(k) \setminus E_{n-1}(k), k \in S$:

$$TV_n = g_{\pi_n} + \beta \cdot P(\pi_n) V_n = \min_{\pi \in \Pi} [g_\pi + \beta \cdot P(\pi) V_n].$$

If $\max(V_n - TV_n) < \varepsilon$, go to Step 5. Otherwise, return to Step 2.

**Step 5. Final extrapolation.** Set $V' = TV_n + \beta(1 - \beta)^{-1} \max_{i \in S} [TV_n(i) - V_n(i)]e$ as approximation of $V_\beta^*$, $\pi_n$ is $\beta(1 - \beta)^{-1}$ optimal.

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