Integration of inventory and transportation decisions in decentralised supply chains

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Abstract: In decentralised supply chains, lack of visibility, long delivery delays and complex transportation networks make it difficult to integrate inventory control with other logistics activities. However, because of the impact of stock turnover on just-in-time operations, inventory control has to be considered in the global optimisation of the supply chain. In the literature, transportation and inventory control decisions are seldom modelled together, because minimising transportation costs and increasing inventory turns are two contradictory objectives. This paper addresses this global optimisation problem. It presents a Decision Support System (DSS) that estimates logistics activities in a decentralised supply chain by integrating inventory control and transportation operations. Delivery frequencies and phases are the decision variables used to study the behaviour of the logistics system.

Keywords: supply chain management; inbound logistics; inventory control; transportation operations; routing; global optimisation; DSS; decision support system.

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1 Introduction

This research addresses the real industrial problem of a company that owns several components and subassembly factories located throughout the USA and an assembly factory in Quebec, Canada. The global supply chain here is large (from the east coast to the west coast, taking into account both external and internal suppliers), and the final assembly line is highly exenterated. Most of the trucks used go out FTL (with a full truckload), but this is usually on the trip from a supplier to the final assembly line. So, frequencies are low and inventories high.

The assembly company in Canada wanted to move from mass production toward mass customisation (most components are brought to the assembly line 'just in time', according to the assembly sequence). Instead of keeping inventories of all items, the line requirements are transmitted to the suppliers, who must meet these requirements in the given order. This means that the sequence has to be frozen at some point during the maximum shipping lead time. The greater the quantity of components transported, the longer the frozen sequence.

That is why the company has been attempting to drastically reduce the lead time by means of a Collaborative Transportation Management (CTM) strategy. Our study concerns a project involving the reengineering of the whole supply chain to allow migration from mass production to mass customisation. All opportunities had to be explored, from the transshipment platform, shipping and/or delivery consolidation shipping and/or delivery routes, etc. Both transportation and inventory costs have to be considered. Finally, the company proposes to integrate production and detailed transportation scheduling. Here, not only the delivery frequencies, but also the phase of the delivery, are considered. Rather than to find an optimal solution, the aim was to set out the possible strategies available, which is why no operational considerations are discussed.

The first part of the research involves a general review of strategies for managing the global supply chain. One of the most important aspects of logistics management is supply chain integration, which is aimed at synchronising every link in the chain, i.e., every trading partner involved. Many enterprises have achieved a high level of integration by applying lean manufacturing principles to optimise their entire chain. Recently developed methods relating to logistics management, like Vendor Managed Inventory (VMI), Collaborative Planning, Forecasting and Replenishment (CPFR) and the CTM strategy introduced above, are all built around the basic idea of lean thinking, i.e., to eliminate waste (Womack and Jones, 2003). However, it appears that these methods are not suitable options for solving the problem studied here. The collaboration between the partners is already very good. The assembly line is already able to transmit the frozen sequence, and even a forecasted one. The aim is to reduce inventories by modifying transportation methods. CTM was the solution, but the global systems to support this strategy had to be designed. That is why the research has been oriented toward the integrated design and control of the supply chain.

This paper is organised as follows. First, the problem is defined and a literature review is presented to provide a brief look at existing models related to global supply chain optimisation. After determining the modelling approach, the Decision Support System (DSS) is presented. The structure of the DSS is explained and the optimisation model is developed to integrate transportation and inventory control decisions. Some experiments are shown to study the behaviour of the logistics system, and the results are analysed. Finally, recommendations are made with respect to global supply chain optimisation.

2 Problem definition

To show where this paper fits in the field of global supply chain research, it is important to define the issues addressed and the problem to be solved, which we do here. The literature review is presented in the following section.

First of all, this paper focuses on the integration of the two most costly logistics operations, which are transportation and inventory control, and develops a model to minimise the total costs related to these activities. As mentioned above, the decentralised supply chain studied in this paper involves multiple suppliers and assembly plants. The transportation network is composed of consolidation and transshipment centres. Thus, we have a multicommodity and multilevel transportation and inventory control problem to solve. Overseas operations are not considered, truck transportation being the only transportation mode used to supply the plants.

In the field of supply chain management research, a great deal of effort has been spent on supply chain design (Verter and Dincer, 1992; Crainic, 2000; Meixell and Gargeya 2005). However, the DSS presented in this paper is aimed at helping logistics managers make decisions related to transportation operations and inventory control. As a result, we focus here on tactical and operational planning, and will not be addressing supply chain design problems like facility location, supplier selection and capacity determination.

3 Literature review

In this literature review, two topics are covered: global supply chain optimisation and transportation operations modelling.

3.1 Global supply chain optimisation

Global supply chain optimisation is called 'global' for two reasons (Vidal and Goetschalckx, 1997). First, it constitutes a study of the integration of multiple logistics activities, like transportation, inventory control, order processing and manufacturing.

Second, it constitutes a study of the integration of the trading partners involved (suppliers, carriers, consolidation and transshipment centres and plants), which implies the development of efficient communication systems. Consequently, global models are, most of the time, very complex.

Many authors have studied transportation and inventory control problems, but only a few have built models integrating both logistics activities. As pointed out by Goetschalckx et al. (2002), much of the research on global supply chain optimisation has failed to recognise inventory control as part of the decision problem. In fact, in a literature review in which they surveyed 18 papers related to this subject, Meixell and Gargeya (2005) observed that only five addressed inventory costs. Moreover, few of them took into account the impact of long transit times. Nevertheless, there are interesting aspects of the existing models that will be of interest for our study here.

Because of the complexity of multicommodity problems, researchers often choose to develop heuristics to solve them. Indeed, Qu et al. (1999) and van Norden and Van de Velde (2005) used a similar approach to simplify problem resolution. They divided their model into two entities: a master problem and a subproblem. For example, the subproblem can be a transportation problem and the master problem an inventory control problem. The subproblem is solved initially, and then the master problem is solved using the results provided by the subproblem. By iteration, it is possible to obtain a good solution, and, even though these models are very theoretical, the modelling approach could be applied to many different problems.

Goetschalckx et al. (2002) developed an interesting but complex model to integrate production, transportation and inventory control operations. The authors solved the multi-commodity network flow problem, for instance including 12 products and 3738 transportation channels. They observed that savings of 2% (on a total cost of $401,000,000 per year) could be realised by including inventory control in the problem, in comparison with cases that did not include it, showing the importance of integrating inventory control into global models. Porras-Musalem and Dekker (2005) also showed the impacts of inventory on transportation decisions.

Some authors have built models in which the decision variables are delivery frequencies. Actually, these models contain many interesting aspects for our research. Bertazzi, Speranza, Favaretto, Pesenti and Ukovich are five authors who studied this type of problem specifically. Their basic idea for minimising total logistics costs was to build a network of the supply chain and to determine the delivery frequency on each arc of the network (Bertazzi and Speranza, 1999; Bertazzi et al., 2000; Favaretto et al., 2001). In the models studied, inventory levels at each node of the network are determined by the delivery frequencies, and the supply chain is modelled with an linear integer program which is NP-hard. Thus, heuristics were developed to generate solutions. These models have two major drawbacks. First, only one supplier is considered, so the transportation networks are extremely simple (no consolidation or transshipment operations). Second, the inventories in transit are not calculated, because instantaneous replenishment is assumed. Chan and Kingsman (2007) proposed a model to synchronise delivery and production cycles, but did not consider transportation decisions.

3.2 Transportation operations modelling

CTM is a transportation management strategy which is becoming increasingly popular. It is a technique that is of special interest for multilocation supply chains, because it allows multiple plants to manage their logistics operations centrally (CTM Sub-committee, 2004). The main advantage of this centralisation is the reduction of transportation costs (Esper and Williams, 2003). Actually, the objective of CTM is

> "to reduce or eliminate inefficiencies in the transportation process (for example, time, inventory, space, errors and distance) through collaboration, in order to bring benefit to all trading partners." (Sutherland, 2003)

The literature related to CTM has not yet provided detailed examples showing how transportation operations are modelled. However, many researchers have already worked on minimum cost flow problems to model transportation operations and other logistics activities. In a survey of optimisation models for long-haul freight transportation, Crainic (2003) presents a minimum cost flow model developed for network design. Even though network design is not studied in this paper, the multicommodity capacity network design formulation proposed by Crainic is interesting because it is path-based, meaning that transportation decisions are based on pre-established paths for each commodity. This type of formulation allows complex transportation networks to be modelled; however, it is not flexible because the paths have to be preestablished. Rieksts and Ventura (2008) present a model to obtain an optimal inventory policy with two modes of freight transportation, namely the 'Less than TruckLoad' (LTL) mode and a package delivery carrier. Chen et al. (2006) present a minimum cost flow formulation to model the procurement of multiple plants by multiple suppliers. The plants are supplied via a consolidation centre, and the inventories there are calculated at each period of the planning horizon. This formulation is NP-hard. These two examples of minimum cost flow problems point out the potential of such models for transportation operations modelling. Another interesting approach, which is a stochastic location model with risk pooling, has been presented by Snyder et al. (2007). Their model uses working inventories and safety stocks which are affected by demand and transportation cost variations. Finally, transshipment strategies have been integrated into the model proposed by Ozdemir et al. (2006), based on a theoretical case.

4 Modelling approach

In a decentralised supply chain, long distances between suppliers and plants increase the complexity of the procurement operations. First, transportation management is complex because multiple methods can be used (direct shipment, shipment with consolidation, transshipment, among others). Second, inventories in transit cannot be neglected for the calculation of total logistics costs, because shipments may be on the road for a few days. Thus, determining the right levels of inventories in the supply chain and the best transportation strategies to minimise the global cost is very difficult.

Because of the complexity of the problem, the choice of modelling approach is important. In the literature, authors identify two main types of approaches: analytical methods (i.e., optimisation) and simulation (Slats et al., 1995). Usually, optimisation is used when the system to be modelled is simple and the objective function can be defined analytically. Simulation is more appropriate for complex systems (Baptiste, 2004). It is important, therefore, to analyse the problem to be modelled before determining the right approach.

As mentioned, very few optimisation models have been developed to optimise transportation operations and inventory control at the same time. Here are the main reasons for this:

- transportation networks are complex
- there are many component, supplier and plant combinations
- inventory levels throughout the supply chain have to be determined
- the integration of logistics operations is difficult.

The integration of transportation operations and inventory control is difficult because these two logistics activities are not computed on the same time horizons. For example, transportation costs are calculated for orders that have to be shipped on a certain day. Thus, the time basis is a period of the planning horizon. By contrast, inventory costs are calculated on average inventory over the entire planning horizon. With two different time bases, it becomes more complicated to develop an optimisation model that minimises total costs. Authors usually bypass this difficulty by calculating the inventories at each period of the planning horizon (one day, for example). However, this method considerably increases the number of decision variables, and does not allow inventories to be computed in transit.

Considering this problem, a combination of optimisation and simulation was chosen to integrate transportation and inventory control operations. This approach constitutes the basic idea of the DSS detailed in the next section.

5 Development of the DSS

This section presents the DSS developed to minimise the transportation and inventory holding costs incurred to ensure the procurement of multiple plants in a decentralised supply chain.

5.1 Structure of the DSS

To study the behaviour of decentralised supply chains, it is necessary to model a supply chain and to design a system that allows several procurement scenarios to be tested in a short space of time. This is the purpose of the DSS presented in this paper.

The structure of the DSS is illustrated in Figure 1. Simulation is used to generate and evaluate different procurement scenarios, and a model is created to optimise the transportation operations corresponding to a particular scenario. For the supply chain studied in this paper (see Section 6), the planning horizon has been fixed to a week (seven days). Assembly plants receive at least one shipment per week from each of their suppliers. Thus, it is possible to simulate a typical week of production to evaluate the relationship between input variables and total logistics costs. The requirements of each plant are known and constant for every scenario tested.

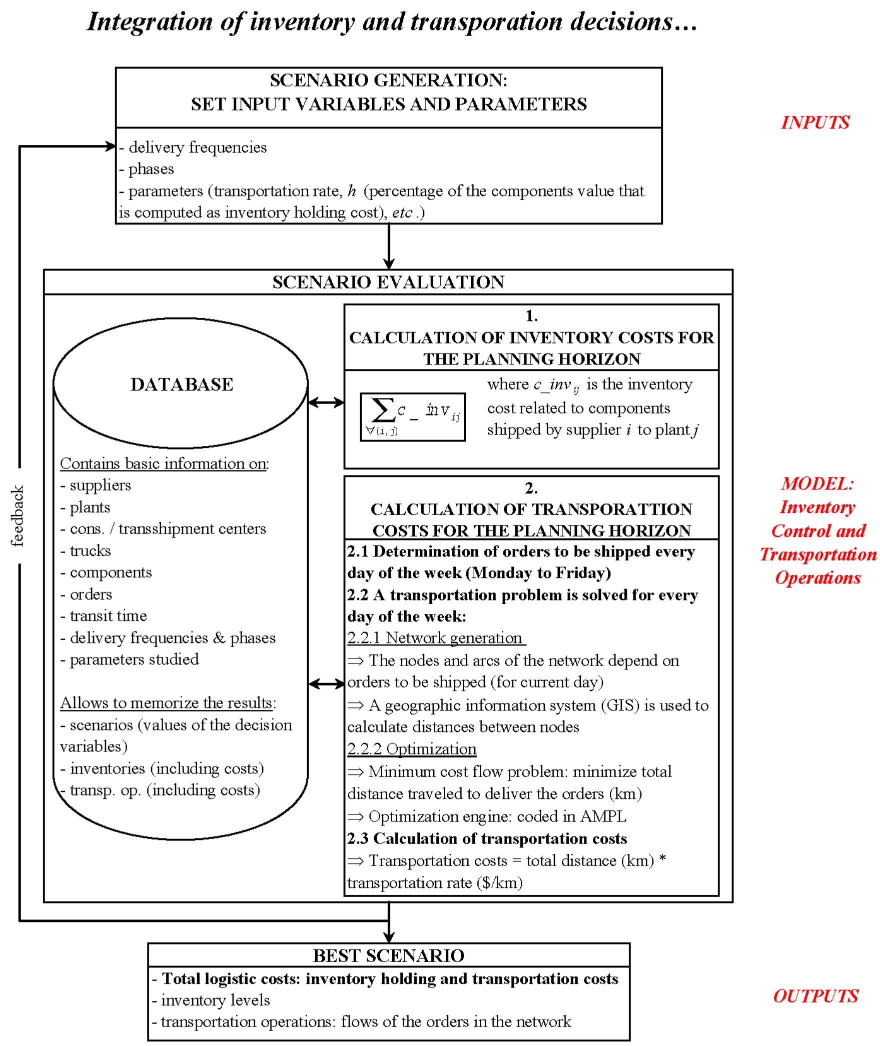
Two types of decision variables are used:

- delivery frequencies
- phases.

A delivery frequency is the number of times a plant receives material from a particular supplier in a fixed planning horizon (for example, twice a week). A phase represents the days on which a plant has to be supplied (for example, Monday and Wednesday). These variables were chosen because they affect both transportation operations and inventory levels. Moreover, the DSS makes it possible to study the impact of the variation of different parameters on logistics costs.

A scenario is generated by setting the decision variables and parameters studied. Delivery frequencies and phases have to be determined for every couple (i, j), where (i, j) refers to the material provided by supplier i to plant j. Once all the inputs of the model have been set, it is possible to evaluate the scenario, i.e., to compute total logistics costs. It is important to note that inventory levels and transportation operations are modelled separately (see Figure 1). The global optimisation is made possible by evaluating multiple scenarios. The two modules of the DSS are detailed in Sections 5.2 and 5.3.

Figure 1 Structure of the DSS (see online version for colours)



It is interesting to note that the DSS can be used for two interrelated purposes. First, by using a neighbourhood search method to generate scenarios (like Tabu Search, for example), the DSS can evaluate multiple scenarios and memorise the best solution. In that case, the DSS works as an optimisation engine to support operational decisions. Second, generating scenarios in accordance with the design of an experiment can help to quantify the impact of different parameters on logistics costs, and thus to support tactical decisions. For example, the quantified effects of the distance separating suppliers and plants can help managers figure out the best transportation strategies to minimise total costs and to ensure an appropriate inventory turnover rate, which makes this type of analysis useful for generating good starting solutions (or scenarios).

Before presenting the model that was integrated in the DSS, it is important to list some assumptions that were made to validate the results obtained with this DSS:

- the demand is considered to be constant during the planning horizon, meaning that the components are consumed by the plants at a constant rate during the week
- the price of the components is constant, i.e., it does not depend on order processing costs
- the components belong to the buyer as soon as they leave the supplier's plant (FOB Origin)
- the suppliers and the plants are open from Monday to Friday.

5.2 Calculation of inventory holding costs

In our paradigm, shipping is a continuous flow. No inventories are kept and the line is fed by a sequence of components arriving 'just in time'. So, the safety stock is a time lag between the predicted arrival time and the due date. The risk is mainly a shipping and transportation risk. The company has proposed considering that this safety stock be proportional to the quantity of the order, and that the risk is greater if the flow is a full truck each week than if it is one-fifth of a truck daily. The choice of a linear model is an approximation.

The model of the DSS is composed of two modules. The first module calculates the inventory holding costs (C_{inv}) by determining the inventory levels in transit and at the plants (on hand). Since delivery frequencies and phases were fixed when the scenario was generated, it is possible to compute, for a couple (i, j), the value of the average inventories in the supply chain during the planning horizon ($\overline{I}_{ij}$):

$$\overline{I}_{ij} = \left[\frac{v_{ij}}{2f_{ij}} + \frac{v_{ij}}{f_{ij}}\alpha_{ij}\right] + \left[\frac{v_{ij}}{f_{ij}}\frac{f_{ij}t_{ij}}{7}\right] = \frac{v_{ij}}{f_{ij}}\left(\frac{1}{2} + \alpha_{ij}\right) + \frac{v_{ij}t_{ij}}{7} \tag{1}$$

where

$\overline{I}_{ij}$: Value of the average inventories for couple (i, j) during the planning horizon ($)

f_{ij}: Delivery frequency of couple (i, j): number of deliveries during the planning horizon (delivery/horizon)

t_{ij}: Transit time for couple (i, j) (days)

v_{ij}: Value of the components provided by supplier i to plant j during the planning horizon ($)

α_{ij}: Coefficient used to consider weekends and the variability of inventory levels (unitless).

In formula (1), $v_{ij}/f_{ij} \times (0.5 + \alpha_{ij})$ is the average value of on-hand inventories (i.e., inventories at the plants, safety stock), whereas $v_{ij} \times t_{ij}/7$ is the average value of in-transit inventories (i.e., inventories moving from the suppliers to the plants, pipeline stock).

The way to determine the value of on-hand inventories is less obvious. For a plant that produces 365 days per year and 24 hours per day, the formula for calculating on-hand inventories is $v_{ij}/f_{ij} \times 0.5$ to model constant demand. However, many companies are closed on weekends and have to hold inventories during that period. Moreover, uncertainty in deliveries also affects inventories on hand. Thus, a coefficient was added to formula (1) to take into account these factors, and corresponds to $v_{ij}/f_{ij} \times \alpha_{ij}$. It is possible to estimate α_{ij} with the following formula:

$$\alpha_{ij} = \alpha_{ij}^1 + \alpha_{ij}^2 \tag{2}$$

where

α_{ij}^1: coefficient used to consider the increase in average on-hand inventory level created by weekends

α_{ij}^2: coefficient used to consider the uncertainty in deliveries

Coefficient α_{ij}^2 can be estimated by analysing historical data (demand variability, punctuality and regularity of deliveries, etc.). In this paper, α_{ij}^2 will be fixed to 0.1 for every couple (i, j) in order to simplify calculations.

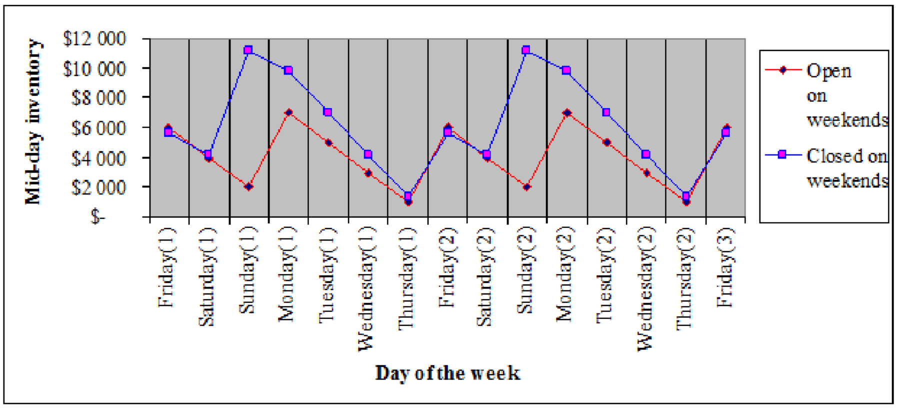
Unlike α_{ij}^2, the coefficient α_{ij}^1 can be determined analytically. To show the effect of weekends on inventory levels, let us continue with the example presented earlier. In that example, $v_{ij} = \$14{,}000$ and $t_{ij} = 2$ days. Adding $f_{ij} = 2$ deliveries/week and phases $\rho_{ij} = \{\text{Tuesday; Friday}\}$, it is possible to determine the average value of on-hand inventories.

Figure 2 shows the variation of on-hand inventory for two weeks of production. The curve *Open on weekends* represents a production system in which the supplier and the plant are open on weekends (average value of on-hand inventories = $v_{ij}/f_{ij} \times 0.5$), whereas the curve *Closed on weekends* reveals a production system in which the supplier can only ship from Monday to Friday and the plant is closed on weekends (average value of on-hand inventories = $v_{ij}/f_{ij} \times (0.5 + \alpha_{ij}^1)$).

The calculation of a midday inventory is the following: midday inventory on day x = midday inventory on day $(x - 1)$ – daily requirements (if applicable) + value of shipment(s) received on day x (it was supposed that the shipments are always received in the morning). For example, the midday inventory on Friday for the curve *Closed on weekends* is: $1400 – $2800 + $7000 = $5600. Actually, the value of the daily requirements is $14000/5 = $2800, and the value of a shipment is $14000/2 = $7000.

As shown in Figure 2, weekends create an increase in the average on-hand inventory. There are two reasons for this. First, inventories left from the weekly production have to be held over the weekend. Second, the fact that suppliers can only ship orders on week days often causes a phase shift, in order that the orders can be received earlier at the plants. In our example, the transit time is two days, and so, because the orders should be received on Tuesday and Friday, the supplier should ship on Sunday and Wednesday. However, Sunday's shipment has to be shifted to Friday because the supplier is closed during the weekend. Thus, the shipment will be received on Sunday instead of Tuesday, which means that the components will be stored for two days, increasing the average inventory.

Figure 2 Variation of on-hand inventory (see online version for colours)



The average inventory is \$3500 for the *Open on weekends* curve and it is \$6200 for the *Closed on weekends* curve, which represents an 80% increase. Because $v_{ij}/f_{ij} \times (0.5 + \alpha_{ij}^{1}) = \$6200$, $\alpha_{ij}^{1} = 0.39$. Thus, α_{ij}^{1} is not negligible.

Once the average inventory is calculated, it is possible to estimate the cost of holding these inventories during the planning horizon. A coefficient h is used to represent the annual inventory holding cost rate ($[h] = \%$ of component value per year of storage). Consequently, the inventory holding cost per week for a couple (i, j) is:

$$c\_\text{inv}_{ij} = \frac{7}{365} h \bar{I}_{ij}. \tag{3}$$

Finally, the total inventory holding cost for a scenario is:

$$C_{\text{inv}} = \sum_{\forall(i,j)} c\_\text{inv}_{ij}. \tag{4}$$

In short, delivery frequencies and phases determine the inventory levels throughout the supply chain. By generating multiple scenarios, it will be possible to analyse the variation in inventory holding costs.

5.3 *Transportation operations modelling*

The second module of the DSS is related to transportation operations (see Figure 1). To calculate transportation costs (C_{transp}), a transportation problem has to be solved for every day of the week. First, the orders to be shipped from Monday to Friday have to be determined.

For a couple (i, j), the shipping days are deduced from the delivery phase (ρ_{ij}) and the transit time (t_{ij}), as shown in the previous section. In addition, the value, volume and weight of the orders are calculated with the delivery frequency (f_{ij}). Actually, because the demand is considered to be constant during the planning horizon, it is supposed that each shipment has the same value. For example, if the weekly requirements of plant j for components produced by supplier i are worth \$10,000, occupy 40 m^3 of space and weigh 5000 kg, a delivery frequency of two per week means that each shipment will be worth \$5000, occupy 20 m^3 of space and weigh 2500 kg.

By tracking the orders and shipping days for every couple (i, j), it becomes possible to group the orders by shipping day and to solve a transportation problem. First, a graph modelling the movements of the orders from the suppliers to the plants is created. Then, a minimum cost flow problem is solved to minimise the total distance travelled by the trucks supplying the various plants. Finally, the transportation cost is calculated by multiplying the distance travelled by the transportation rate.

5.3.1 Minimum cost flow problem

Because of the potential of minimum cost flow formulations to model transportation operations (see Section 3.2 of the literature review), this type of model was incorporated into the optimisation engine of the DSS.

An interesting way to model transportation operations is to build a network in which flows represent the movements of orders being shipped from suppliers to plants, while the nodes symbolise trading partners in the supply chain (suppliers, consolidation/transshipment centres and plants). For each day of the week, transportation operations are modelled with a minimum cost flow problem in which the costs are the distances travelled by the trucks transporting the orders. The objective function is to minimise the total distance travelled to supply the plants. It is worth noting that a Geographic Information System (GIS) is essential for determining the distances between the nodes of the network.

5.3.2 Network generation

In a decentralised supply chain, multiple transportation strategies can be used to optimise transportation operations:

- direct shipment from a supplier to a plant
- shipment via a consolidation or transshipment centre
- consolidation of orders for suppliers located in a particular region
- consolidation of orders intended for different plants.

The model integrated in the DSS allows for a combination of these four transportation methods. To illustrate the problem, it is important to detail the network generation with an example. Let us consider a supply chain with the following characteristics:

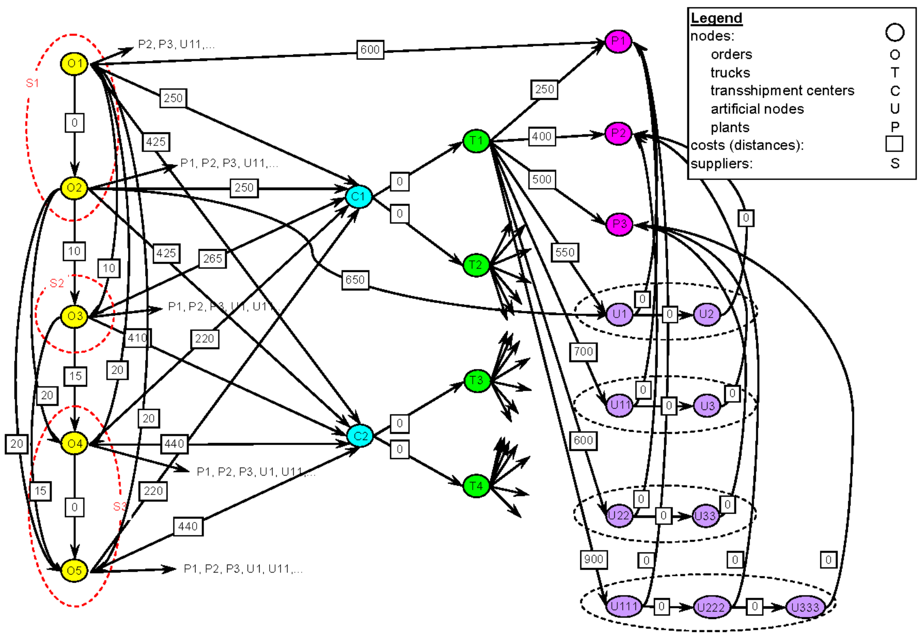
- three plants (*P1*, *P2*, *P3*) have to be supplied
- three suppliers (*S1*, *S2*, *S3*) provide the components
- two transshipment centres (*C1*, *C2*) are available.

Moreover, let us suppose that five orders have to be delivered:

- *O1*: (*S1*, *P1*), meaning that order *O1* was made by plant *P1* and to supplier *S1*
- *O2*: (*S1*, *P2*)
- *O3*: (*S2*, *P1*)
- *O4*: (*S3*, *P2*)
- *O5*: (*S3*, *P3*).

Figure 3 illustrates the network corresponding to this example. It represents every path that an order (or a flow) can borrow to be delivered to the right plant. First, there is a node representing each order (*O1* to *O5*). For each of these nodes, a flow is generated. This flow contains the order information: supplier, plant, value ($), volume ($m^3$) and weight (kg). To model the consolidation of orders between suppliers, arcs have been added between the orders nodes. The costs related to these arcs are the distances separating the corresponding suppliers. For example, a cost of zero means that the corresponding orders are shipped from the same supplier.

Figure 3 Network example (all the items in the legend should be in the singular – ch) (see online version for colours)



Moreover, the flows leaving order nodes (i.e., suppliers) can be dispatched to three other types of nodes:

- *Plant node (P)*. Permits modelling of a direct shipment from the supplier to the plant.
- *Artificial node (U)*. Permits modelling of a shipment containing orders intended for different plants (*milk run*). For example, it may be advantageous to ship two orders on one truck which will visit two different plants, instead of shipping the two orders

separately. *U* nodes are called 'artificial nodes' because they are only used to model milk runs between plants. For example, a shipment entering node *U22* is transported by a truck which will visit both plant *P2* and plant *P3*. The cost related to an arc connecting an order to an artificial node corresponds to the total distance travelled from the supplier to the last plant visited. Thus, artificial nodes are connected to other artificial nodes (*U*) or to plants (*P*) with arcs having a cost equal to zero.

- *Transshipment centre (C)*. Permits shipments from a supplier to a transshipment centre.

Finally, to be able to form shipments with orders passing through a transshipment centre, *C* nodes are connected to trucks (*T*). Orders passing through an arc *C-T* have to respect the capacity of a trailer. The cost related to these arcs is equal to zero because no distance is travelled. Note that the number of trucks associated with a transshipment centre can be determined according to the expected volume of orders that will pass through this transshipment centre.

This example shows that the graph is generated according to the orders to be delivered and the number of trucks needed at each transshipment centre. Thus, this type of graph approach can be used to model very complex transportation networks.

5.3.3 Mathematical formulation

To solve the problem modelled with the graph presented in Figure 3, a mathematical formulation is developed. Here is the linear program corresponding to this minimum cost flow problem:

$\min \sum_{\forall(i,j)\in\beta} L_{ij} y_{ij}$		minimise total distance	(5)
s.t. $\sum_{\forall(i,j)\in\delta} x_{ij}^{i} = 1$,	$\forall I \in \gamma$	flow generation	(6)
$\sum_{\forall(i,j)\in\delta} x_{ij}^{k} = \sum_{\forall(j,l)\in\delta} x_{jl}^{k}$,	$\forall j \in \gamma, \forall k \in \gamma \mid k \neq j$	flow conservation for order nodes (*O*)	(7)
$\sum_{\forall(i,j)\in\delta} x_{ij}^{k} = \sum_{\forall(j,l)\in\delta} x_{jl}^{k}$,	$\forall j \in \lambda, \forall k \in \gamma$	flow conservation for nodes *C*, *T* & *U*	(8)
$\sum_{\forall(i,j)\in\delta} x_{ij}^{k} = 1$,	$\forall j \in \mu, \forall k \in \gamma(j)$	delivering orders to the right plants	(9)
$\sum_{\forall k\in\gamma} r^{k} x_{ij}^{k} \leq R$,	$\forall(i,j) \in \tau, \forall(i,j) \in \omega$	trailer capacity: volume	(10)
$\sum_{\forall k\in\gamma} w^{k} x_{ij}^{k} \leq W$,	$\forall(i,j) \in \tau, \forall(i,j) \in \omega$	trailer capacity: weight	(11)
$\sum_{\forall k\in\gamma} x_{ij}^{k} \leq M y_{ij}$,	$\forall(i,j) \in \beta$	setting variables y_{ij}	(12)
$\sum_{\forall(i,j)\in\delta} y_{ij} = 1$,	$\forall i \in \gamma, \forall i \in \varphi$	prevents shipment division	(13)

$\sum_{\forall(i,j)\in\delta} y_{ij} \le 1,$	$\forall j \in \gamma$	only one shipment can enter an order node	(14)
$x_{ij}^k \in \{0,1\},$	$\forall (i,j) \in \delta,\ \forall k \in \gamma$		(15)
$y_{ij} \in \{0,1\},$	$\forall (i,j) \in \beta$		(16)

where r^k: Volume occupied by order k

w^k: Weight of order k

R: Trailer capacity: volume

W: Trailer capacity: weight

L_{ij}: Distance between nodes i and j

M: Represents a high number (ex.: 100)

γ: Set of orders

μ: Set of plants

$\gamma(j)$: Set of orders made by plant j

φ: Set of trucks

λ: Set of nodes for which flow conservation applies (excluding nodes O): nodes C, T & U

δ: Set containing every arc of the network

β: Set of arcs with cost > 0 (i.e., arcs representing movements of orders)

τ: Set of arcs associated with the orders: *O-O*, *O-C*, *O-U*, *O-P*

ϖ: Set of arcs entering truck nodes (T)

x_{ij}^k : Binary variable equal to one if order k passes through arc (i, j), zero otherwise

y_{ij} : Binary variable equal to one if at least one order passes through arc (i, j), zero otherwise.

As mentioned above, the objective of this linear program is to minimise the total distance travelled to deliver the orders (5). The first series of constraints (6) is necessary to generate the flows. Constraints (7) and (8) are flow conservation constraints. Actually, two series of constraints were necessary because the flows are generated at nodes O. Thus, for these particular nodes, flow conservation does not apply to the flow generated. Moreover, constraint nine was added to ensure that the orders are delivered to the corresponding plant. For example, if plant *U3* made the order *O2*, then there must be one and only one variable x_{iU3}^{O2} equal to one. In addition, constraints (10) and (11) ensure that trailer capacities (maximum volume and weight) are respected. Constraint (12) was added to determine the arcs borrowed by the flows (variables y_{ij}) and, thus, to make it possible to calculate the distance travelled. Constraint (13) was necessary to prevent shipment division. In other words, flows leaving an order or a truck node have to borrow the same arc. Finally, constraint (14) was added to ensure that only one shipment can enter an order node (O). Otherwise, it would be possible to consolidate different shipments at a supplier's plant, which is not a transportation strategy considered in this model.

Naturally, certain assumptions have to be made to validate this transportation model:

- the transit time of couple (i, j) is fixed, i.e., it does not depend on the type of transportation strategy used to deliver the shipments (direct transport, consolidation, etc.)
- the time windows for pickups or deliveries are not considered
- the capacity of a trailer (R) is estimated according to the maximum space that can be occupied in a trailer, taking into account that the pallets can be stacked
- only one type of trailer is available
- in the network, the distance from node i to node j is equal to the distance from node j to node i ($L_{ij} = L_{ji}$)
- a fixed transportation rate is considered (the same rate for TL and LTL)
- the arcs between the order nodes (O) are unidirectional. For example, in the graph illustrated in Figure 3, a flow cannot go from order *O5* to order *O1*.

At first sight, the first assumption may seem illogical. However, there are two reasons for making it. First, in widely distributed supply chains, transit times are calculated in days. Thus, even if, for example, an order from supplier A is consolidated with an order from supplier B located nearby, the transit time associated with a direct shipment from supplier A to plant C compared to the transit time required if the order from supplier A goes to supplier B before arriving at plant C may well be the same. Second, it is possible to determine the transit time by estimating, fairly accurately, the transportation strategy that will be used.

Although these assumptions help to reduce the complexity of the model, the size of the transportation problem to solve is an important issue, as will be discussed in the following section.

5.3.4 Size reduction process

The formulation presented in Section 5.3.3 makes it possible to model complex transportation networks because multiple transportation strategies are considered. However, the size of the problem to solve grows rapidly with the number of orders considered. For example, let us suppose that 70 orders have to be delivered to four different plants and that two transshipment centres are available (at each transshipment centre, ten trucks can be loaded). Table 1 details the size of this problem modelled with the linear program (5)–(16).

In this example, the linear program contains 275,065 variables and 19,555 constraints. Obviously, to be able to find the optimal solution to the problem, its size has to be reduced. Thus, a size reduction process composed of four steps has been applied to the model (see Table 2).

The first step in the size reduction process is aimed at reducing the number of arcs between orders, because, as noted in Table 1, these arcs represent 63% of the network's arcs. Actually, it is possible to reduce the number of arcs between orders by more than

80%, depending on the geographical distribution of the suppliers. Since these arcs model the consolidation of orders between suppliers, arcs linking suppliers which are not located in the same area can be eliminated. In a decentralised supply chain, only the arcs between suppliers that form a cluster will be created.

Table 1 Size of the transportation problem (example)

Problem characteristics	*Quantity*
Orders	70
Nodes	121
Arcs	3875
Variables	275,065
x^k_{ij}	271,250
y_{ij}	3815
Constraints	19,555

Table 2 Steps in the size reduction process

Step	*Strategy*	*Implementation*	*Impact on the experimental error*
1	Elimination of unnecessary *O-O* arcs	Arcs *O-O* between two suppliers that are far from one another will be eliminated	Negligible
2	Elimination of *O-U* and *O-P* arcs that are useless	Arcs *O-U* and *O-P* that do not represent a possible path for the orders will be eliminated	None
3	Elimination of unnecessary *P* nodes	Artificial nodes which involve long milk runs will be eliminated	None
4	Elimination of *T* nodes	Modification of the graph's structure	Depends on the volume of orders passing through the transshipment centres

The second step in the size reduction process is aimed at eliminating *O-U* and *O-P* arcs. Regarding the network detailed in Table 1, 700 arcs are *O-U* arcs and 280 arcs are *O-P* arcs. Actually, some of these arcs are useless. For example, if order *O3* has been made by plant *P1*, the arcs linking *O3* to *P2*, *P3* and *U22* are useless because they do not model a possible path for order *O3* (see Figure 3). Thus, arcs *O3-P2*, *O3-P3* and *O3-U22* can be eliminated.

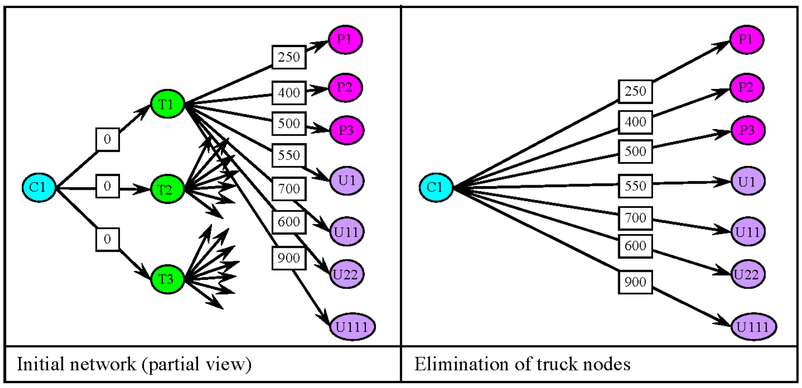
Even if the artificial nodes (*U*) are added to the network to model milk runs between plants, it is not necessary to add every combination of plants. For example, it may not be possible to visit the three plants *P1*, *P2* and *P3*, because the distance between them would result in too big an increase in the transit time. Thus, the third step in the size reduction process consists of eliminating the artificial nodes that do not model realistic milk runs.

Finally, one last step has been added to the size reduction process. This fourth step consists of eliminating the truck nodes. In short, these four steps make it possible to reduce the size of the network considerably. By applying this process, the number of variables for the example detailed in Table 1 could be reduced from 275,065 to around 40,000, depending on the geographical distribution of the suppliers and the plants.

Figure 4 shows how the network has been modified (partial view of Figure 3). As shown in this figure, eliminating truck nodes divides the total number of *C*-*U* and *C*-*P* arcs by the number of trucks. However, with this modification, the truck nodes can no longer be used to form the shipments dispatched from the transshipment centres. Constraints related to trailer capacity (10–11) must then be relaxed. Moreover, the distance travelled by the trucks passing through a *C*-*U* or a *C*-*P* arc will be calculated by multiplying the distance of the arc by the number of trucks (nb_t) needed to ship the orders that have to go through the corresponding arc (nb_t = total volume of orders passing through the arc divided by trailer capacity). As the number of trucks (nb_t) may not be an integer, the elimination of truck nodes introduces an error into the objective function because, in reality, it is impossible to use a fraction of a truck to deliver a shipment. However, this error can either be neglected, if the volume of orders passing through the transshipment centre is high (for example, if 10.1 trucks are needed for an arc *C*-*P*, the error is less than if 1.1 trucks are needed), or compensated for by using a coefficient to increase the distance travelled to consider the fraction of a truck that cannot be included in the objective function. The revised formulation of the linear program considering the elimination of *T* nodes is presented in Appendix A.

In short, these four steps make it possible to reduce the size of the network considerably. By applying this process, the number of variables for the example detailed in Table 1 could be reduced from 275,065 to around 40,000, depending on the geographical distribution of the suppliers and the plants.

Figure 4 Fourth step in the size reduction process: elimination of truck nodes (see online version for colours)



6 Experiments

The DSS detailed in the previous section was developed using various software programs. First, a Microsoft Access database was built to manage the data related to the supply chain studied (see Figure 1), and to generate and evaluate the scenarios. Microsoft MapPoint was used to generate the matrix giving the distances between the nodes (suppliers, transshipment centres and plants). Finally, the linear program modelling the transportation operations was coded with A Mathematical Programming Language (AMPL) and solved with a CPLEX solver for AMPL. This DSS was used to study the impact of delivery frequencies and phases on logistics costs.

6.1 Details on the supply chain modelled

To carry out the experiments, a real supply chain was modelled. The data were gathered from Paccar Inc.'s North American supply chain. Paccar Inc. is a multinational company which assembles trucks. In North America, this company owns six assembly plants and deals with hundreds of suppliers throughout the USA, Canada and Mexico. Obviously, Paccar Inc.'s North American supply chain could not be modelled entirely for the purposes of this research. However, a representative sample of it was studied. This sample has the following characteristics (see appendix B):

- 42 suppliers located in 19 states in the USA and in one province in Canada
- two transshipment centres
- four plants (three American and one Canadian)
- an average distance separating the suppliers and the plants of 1850 km (2570 km for the Canadian plant)
- a transit time that varies from one to seven days
- a database of 255 different components.

Considering this supply chain, a 'typical' week of production was simulated. For every scenario tested, around one million dollars' worth of components had to be supplied to each plant during the week.

6.2 Plan for the experiments

To quantify the impact of delivery frequencies and phases on logistics costs, two experiments were performed. The first experiment (*E1*) consisted of varying the average delivery frequency in the supply chain from 1.5 to 3 deliveries/week. For this experiment, the phases were not optimised. Thus, *E1* shows the impact of delivery frequencies on C_{inv}, C_{transp} and C_{TOT} (total logistics costs).

The second experiment (*E2*) consisted of optimising the phases for a given average delivery frequency. To optimise the phases, two different strategies are tested:

- determining the phases in order to minimise the inventories on hand (Strategy 1)
- determining the phases in order to increase the extent of consolidation operations (Strategy 2).

For a given delivery frequency, it is possible to determine analytically the phase that minimises the average on-hand inventory for a couple (i, j). Thus, scenarios optimised with Strategy 1 were generated according to the phases presented in Appendix C. By contrast, for scenarios optimised with Strategy 2, the phases were determined in such a way that suppliers located in the same region ship their orders on the same day(s).

Results

Table 3 presents the results obtained. This table details, for each scenario, the input variables, the distances travelled and the logistics costs ($US). Seven scenarios were evaluated for experiment *E1* and six for experiment *E2*. It is important to note that a transportation rate of $1.00/km was used and that h = 10%.

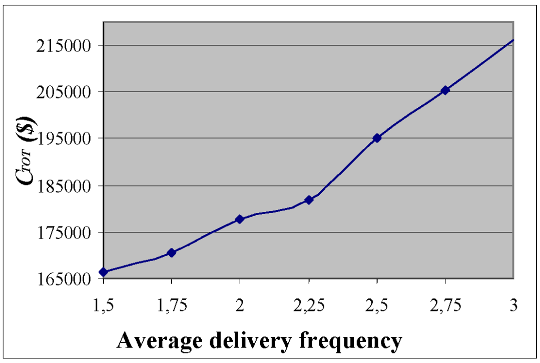
Table 3 Results obtained for the two experiments carried out

Scenario		*Average delivery frequency* (*deliveries/ week*)	*Phase strategy*	C_{inv} (\$)	*Distance travelled* *Monday* (km)	*Tuesday* (km)	*Wednesday* (km)	*Thursday* (km)	*Friday* (km)	C_{transp} (\$)	C_{TOT} (\$)
E1	1	1.5	basic	6464	30083	32533	22292	14553	60325	159786	166250
	2	1.75	basic	5988	31919	28456	21545	14790	67898	164608	170596
	3	2	basic	5674	31910	29194	23813	14692	72320	171929	177603
	4	2.25	basic	5472	31216	30941	24856	14757	74547	176317	181789
	5	2.5	basic	5296	35283	29813	26762	20422	77532	189812	195108
	6	2.75	basic	5160	40621	29864	28548	21218	80034	200285	205445
	7	3	basic	5018	40732	31351	31562	23460	83923	211028	216046
E2	8	2.25	1	5472	30085	30430	32264	28066	68231	189076	194548
	9	2.25	1+	5476	34297	31884	34383	29711	62362	192637	198113
	10	2.25	1++	5401	41365	25187	33647	29810	62142	192151	197552
	11	2.25	2	5400	40626	22262	34634	29508	64043	191073	196473
	12	2.25	2+	5469	40935	16027	24063	13748	71411	166184	171653
	13	2.25	2++	5419	41073	14340	24063	12332	73522	165330	170749

6.2.1 *Experiment E1*

From Scenario 1 to Scenario 7, total logistics costs went from \$166,250 to \$216,046, which represents an increase of 30%. The results show that inventory holding costs represent less than 5% of total logistics costs. Thus, even though the increase in the average delivery frequency permitted a reduction in inventory holding costs by more than 20%, it was not enough to compensate for the increase in transportation costs. Figure 5 illustrates the increase in total costs according to the delivery frequency.

Figure 5 Total logistics costs according to the average delivery frequency (see online version for colours)



This chart shows that, for 1.5–2.25 deliveries/week, total costs only increased by 9%, whereas they increased by 19% for 2.25–3 deliveries/week. Thus, it seems that delivery frequencies can be increased up to a certain point, the point at which transportation costs start to increase faster than inventory costs.

6.2.2 Experiment E2

Scenarios 8–10 show that optimising delivery phases according to Strategy 1 is not advantageous. Actually, inventory holding costs only decreased by 1.3% (compared to scenario 4), whereas total logistics costs increased by 8.7% (an increase of $15,730). By contrast, Strategy 2 generated impressive results. Indeed, for scenarios 10–13, this strategy reduced transportation costs by 14%, while the variation in inventory holding costs can be considered negligible.

In short, compared to scenario 4, phase optimisation made possible a decrease in total logistics costs of 6% (a reduction of $11,040).

6.3 Discussion

The results presented above show that it is possible to increase the inventory turnover rate in a decentralised supply chain without increasing total logistics costs too much. Increasing delivery frequencies accelerates the flow of material throughout the supply chain, and thus the inventory turnover rate. However, delivery frequencies cannot be increased indefinitely. Here are two steps to follow to optimise the delivery frequencies in a decentralised supply chain:

- Determine the ideal average delivery frequency to ensure the desired material flow throughout the supply chain. This average frequency can be determined according to the company's needs and by simulating different scenarios to analyse the variation of total logistics costs.
- Starting with the ideal average frequency, optimise the delivery frequency of each couple (i, j), according to the following rules:
    - for a supplier i located near a plant j or a transshipment centre, increase the delivery frequency if the value or the volume of the orders to be dispatched justifies this decision
    - if there is a cluster of suppliers in the transportation network, increase the delivery frequencies for these suppliers (because operations consolidation will absorb the increase in transportation costs)
    - for a supplier i located far from a plant j, reduce the delivery frequency if necessary.

Generally, these two steps will make it possible to increase the inventory turnover rate and minimise logistics costs. Experiment E2 showed that combining Strategoes one and two reduces total logistics costs, for given delivery frequencies, by more than 6%. Strategy 1 has to be applied first and Strategy 2 second, because Strategy 2 has a direct impact on transportation costs. Even though the majority of companies only focus on delivery frequency, the experiments carried out in this paper show that the delivery phase can reduce logistics costs considerably.

Finally, it is important to note that the largest transportation problems solved in this paper contained around 100 orders. In real-life problems, there may be hundreds of orders to deliver. Thus, the transportation model integrated in the DSS has to be improved to solve larger problems within a short space of time.

7 Conclusion

Building a system that permits optimisation of both inventory control and transportation operations is a difficult task. It is made even more difficult where supply chains are more widely dispersed geographically because of the complexity of the transportation networks involved. In fact, we faced three major challenges. The first was to determine the right decision variables in order to be able to analyse the behaviour of a supply chain. For our DSS, we used delivery frequencies and phases as variables to minimise total logistics costs. The second was to find a way to integrate inventory control and transportation operations. Our system uses a scenario generation and evaluation process to compute inventory and transportation costs. The third was to model a complex network. Our optimisation engine consists of solving a minimum cost flow problem for every period of the planning horizon. Thus, our DSS contains all the elements necessary to globally optimise the logistics operations in a decentralised supply chain.

Further research is needed to improve this DSS, however. More powerful computer tools will be required to solve real-world problems because of their size. For example, to manage a supply chain on a daily basis, specialised mathematical algorithms will have to be used to solve the transportation problems more quickly. Also, the DSS database should be Web-based, so that real-time information can be gathered from suppliers, carriers, transshipment centres and plants. This DSS nonetheless constitutes an interesting advance in the area of global supply chain optimisation.

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Appendix A: Revised Formulation of the Linear Program (truck nodes eliminated)

$$\min \sum_{\forall(i,j)\in\beta^1} L_{ij} y_{ij} + \sum_{\forall(i,j)\in\beta^2} L_{ij} z_{ij} \qquad \text{minimise total distance} \tag{17}$$

s.t. equations (6) to (9), (14)

$$\sum_{\forall k\in\gamma} r^k x_{ij}^k \le R, \qquad \forall(i,j)\in\beta^I \qquad \text{trailer capacity: volume} \tag{18}$$

$$\sum_{\forall k\in\gamma} w^k x_{ij}^k \le W, \qquad \forall(i,j)\in\beta^I \qquad \text{trailer capacity: weight} \tag{19}$$

$$\sum_{\forall k \in \gamma} x_{ij}^{k} \le M y_{ij}, \qquad \forall (i,j) \in \beta^{1} \qquad \text{setting variables } y_{ij} \tag{20}$$

$$\sum_{\forall (i,j) \in \delta} y_{ij} = 1, \qquad \forall i \in \gamma \qquad \text{prevents shipment division} \tag{21}$$

$$z_{ij} = \sum_{\forall k \in \gamma} r^{k} x_{ij}^{k} / R, \qquad \forall (i,j) \in \beta^{2} \qquad \text{number of trucks necessary} \tag{22}$$

$$y_{ij} \in \{0,1\}, \qquad \forall (i,j) \in \beta^{1} \tag{23}$$

$$z_{ij} \in \Re^{+}, \qquad \forall (i,j) \in \beta^{2} \tag{24}$$

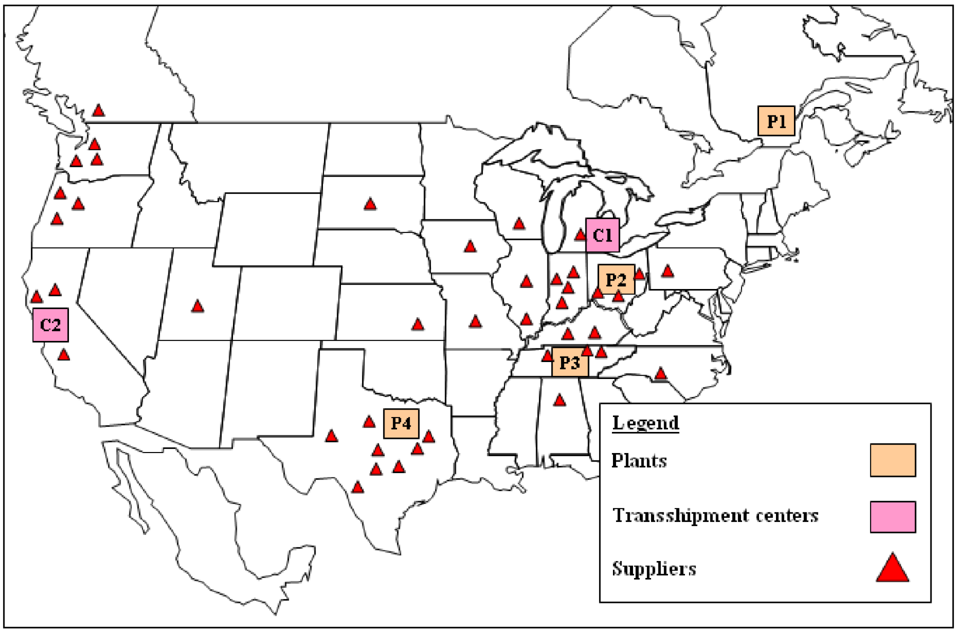
where

β^1 : Set of arcs with cost > 0 (excluding *C-U* and *C-P* arcs)

β^2 : Set of *C-U* et *C-P* arcs

z_{ij} : Variable equal to the number of trucks needed to deliver the orders going through arc (i,j)

Appendix B: Paccar Inc.'s North American supply chain (sample) (see online version for colours)



Appendix C: Phase that minimises on-hand inventory (according to delivery frequency and transit time)

Delivery frequency	*Transit time* (days)	*Optimal phase**	*Average on-hand inventory*** (days)
1	1	Tuesday	2.43
1	2	Wednesday	2.71
1	3	Monday	2.14
1	4	Monday	2.14
1	5	Monday	2.14
1	6	Monday	2.14
1	7	Monday	2.14
2	1	Tuesday and Thursday	1.72
2	2	Monday and Wednesday	1.79
2	3	Monday and Thursday	1.57
2	4	Monday and Tuesday (or Wednesday or Thursday)	1.79
2	5	Monday and Wednesday	1.43
2	6	Monday and Wednesday	1.43
2	7	Monday and Wednesday	1.43
3	1	Tuesday, Wednesday and Friday	1.47
3	2	Monday (or Tuesday), Wednesday and Thursday	1.52
3	3	Monday, Tuesday (or Wednesday) and Thursday	1.43
3	4	Monday, Tuesday (or Wednesday) and Friday	1.62
3	5	Monday, Tuesday, Wednesday (or Thursday or Friday)	1.43
3	6	Monday, Tuesday and Thursday	1.19
3	7	Monday, Tuesday and Thursday	1.19
4	1	Tuesday, Wednesday, Thursday and Friday	1.35
4	2	Monday (or Tuesday), Wednesday, Thursday and Friday	1.46
4	3	Thursday, Friday and two days between Monday, Tuesday and Wednesday	1.39
4	4	Monday, Friday and two days between Tuesday, Wednesday and Thursday	1.32
4	5	Monday, Tuesday and two days between Wednesday, Thursday and Friday	1.25
4	6	Monday, Tuesday, Wednesday and Thursday	1.07
4	7	Monday, Tuesday, Wednesday and Thursday	1.07
5	1	Monday, Tuesday, Wednesday, Thursday and Friday	1
5	2	Monday, Tuesday, Wednesday, Thursday and Friday	1.14
5	3	Monday, Tuesday, Wednesday, Thursday and Friday	1.14
5	4	Monday, Tuesday, Wednesday, Thursday and Friday	1.14
5	5	Monday, Tuesday, Wednesday, Thursday and Friday	1.14
5	6	Monday, Tuesday, Wednesday, Thursday and Friday	0.85
5	7	Monday, Tuesday, Wednesday, Thursday and Friday	0.71

*The optimal phase is the one minimising average on-hand inventory.

**The daily inventory is calculated at the beginning of the day.