Introduction to Mobile Robot Control

Basics and Advances

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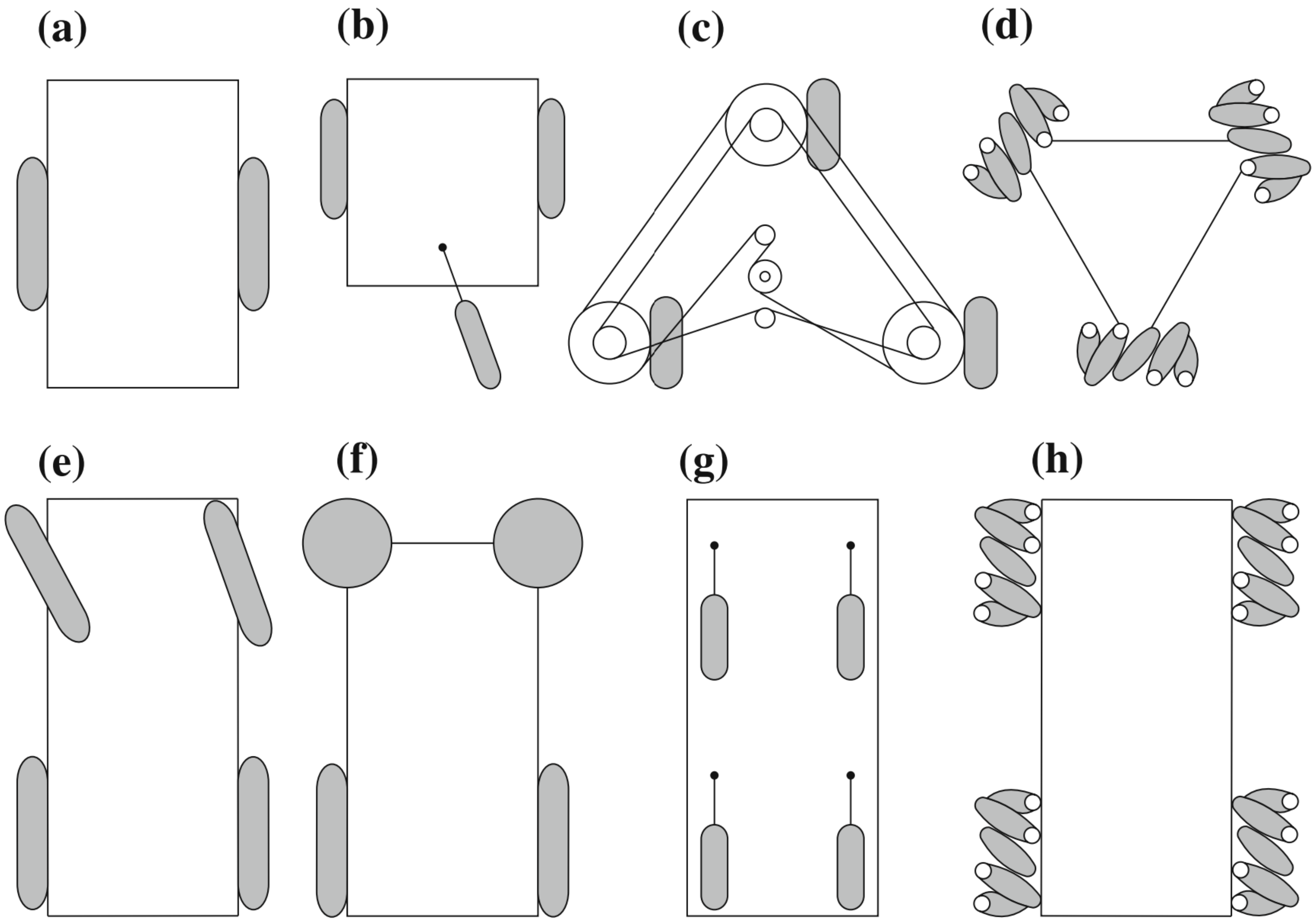
Mobile robot configurations

Mobile Robot Kinematics



I. System model concept

System model is to use to map the input and output

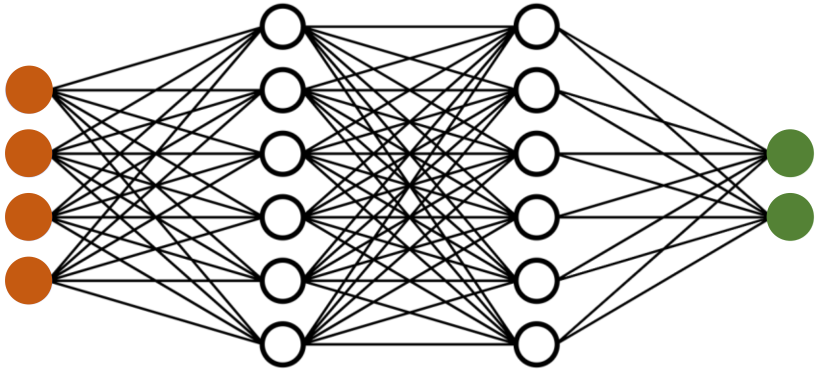
$\boldsymbol{u}(t)$

Input

$\boldsymbol{y}(t)$

Output

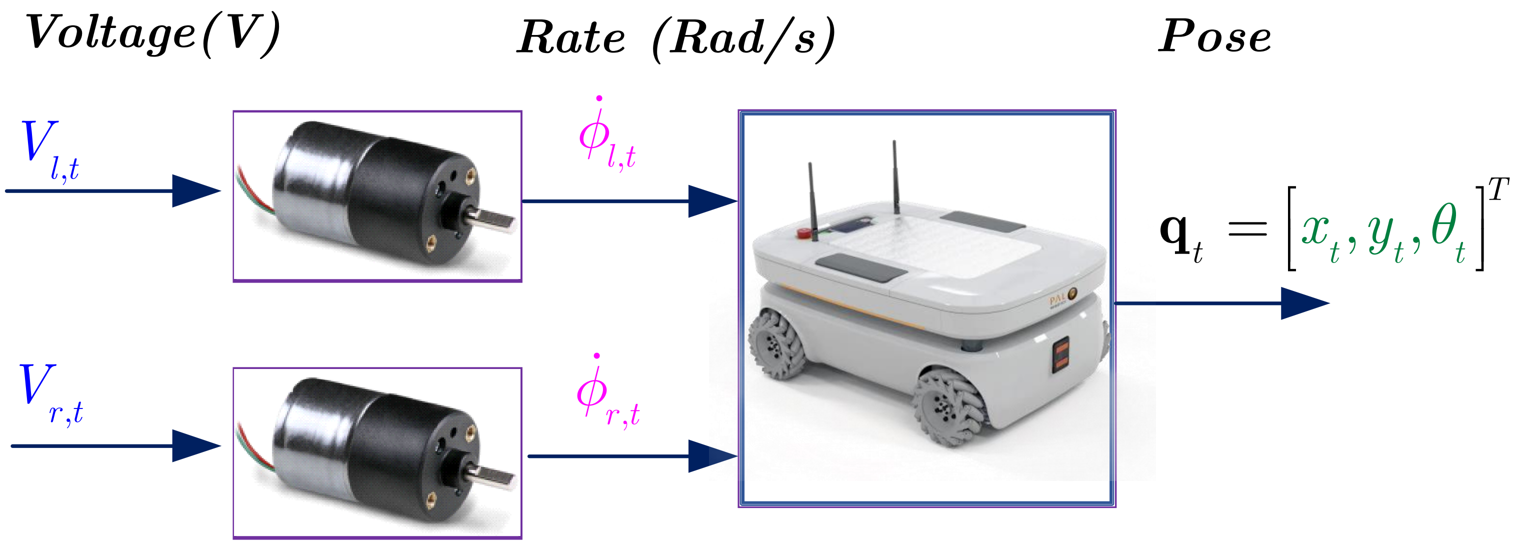
$$
\begin{cases}
\dot{x}(t) = f \quad x(t), \boldsymbol{u}(t) \\
\boldsymbol{y}(t) = g \quad x(t), \boldsymbol{u}(t)
\end{cases}
$$



II. Differential drive WMR

Differential Drive WMR: Motion of each wheel is controlled by a separate DC motor

Model of Differential Drive WMR:



1. **Forward kinematics**:

Given $\dot{\phi}_1, \dot{\phi}_2, \ldots, \dot{\phi}_t$, how will the robot move?

$$\dot{\mathbf{q}}_t = f\begin{pmatrix}\dot{\phi}_r \\ \dot{\phi}_l\end{pmatrix} ???$$

2. **Inverse kinematics**:

Given a desired movement $\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_t$, what commands should we send to the wheels?

$$\begin{pmatrix}\dot{\phi}_r \\ \dot{\phi}_l\end{pmatrix} = f(\dot{\mathbf{q}}_t) ???$$

II. Differential drive WMR

NOTATIONS

Word frame: x^w, y^w

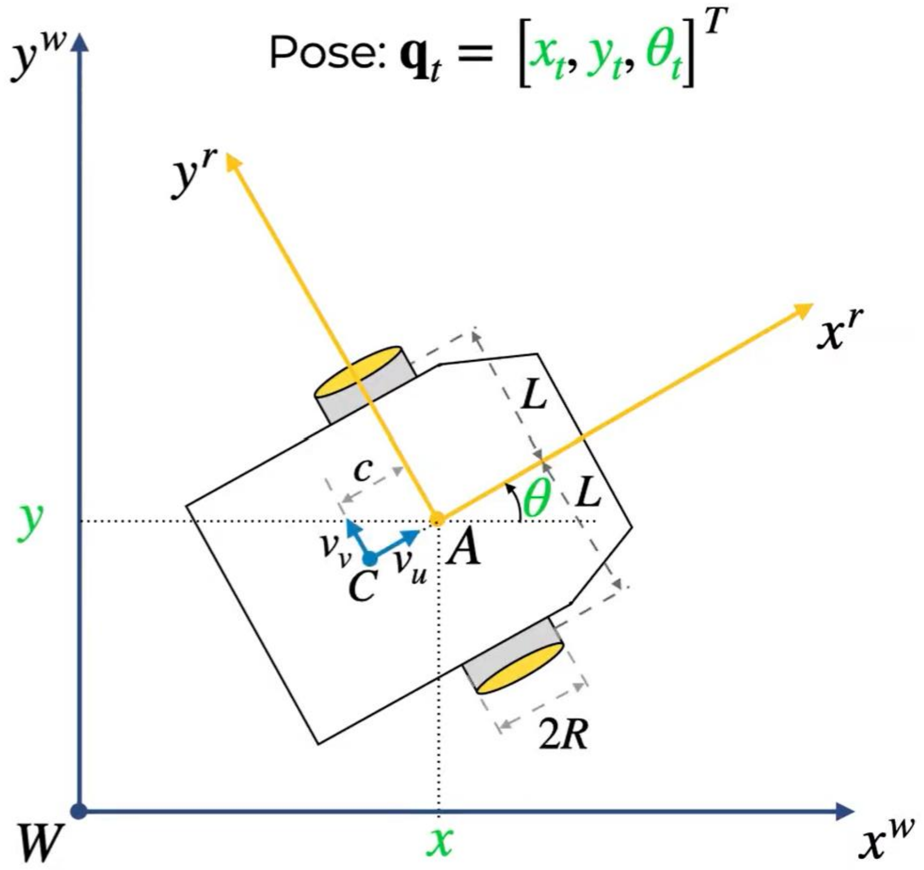
Robot frame: x^r, y^r

⊕ **Assumption 1**: robot is symmetric along longitudinal axis (x^r)

- Equidistant wheels (axle length $= 2L$)
- Identical wheels ($R_l = R_r = R$)
- Center of mass (C) on x^r at distance c from A (A rotation point)

⊕ **Assumption 2**: robot is rigid body

- Distance between any two points of the robot does not change in time
- In particular $\dot{c} = 0$



Forward kinematics, jacobian matrix?

II. Differential drive WMR

NOTATIONS

Rigid body rotation: $\omega = \left[0, 0, \dot{\theta}\right]^T$

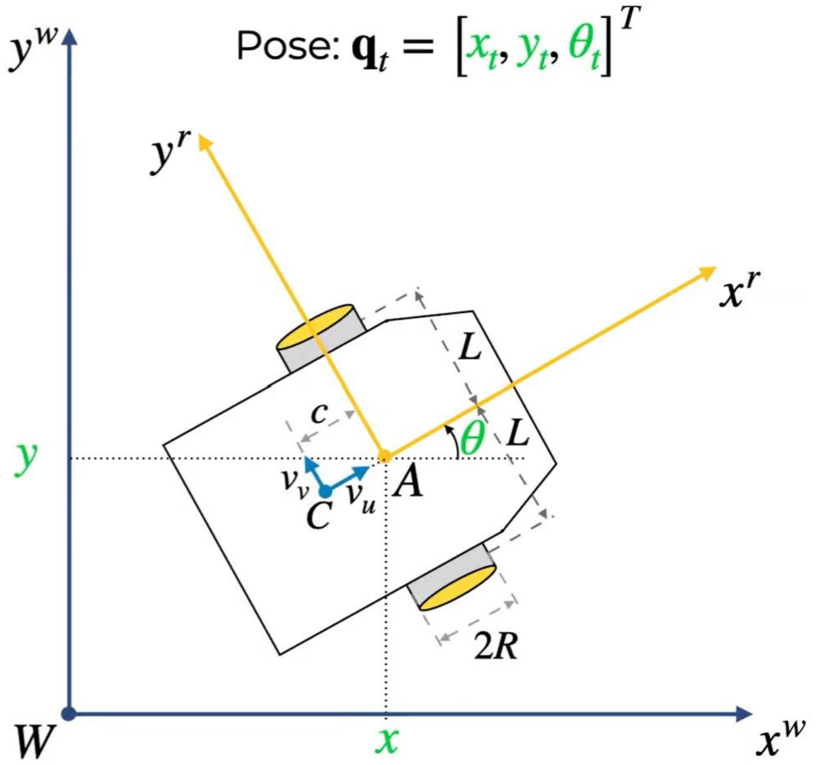
$$\mathbf{v}_A^w = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = {}^w_r R(z, \theta) . \mathbf{v}_A^r$$

$$= \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_u \\ v_v - c\dot{\theta} \\ 0 \end{pmatrix} \qquad (1)$$

$$\Rightarrow \textit{only } x, y \text{ interest} \quad \mathbf{v}_A^w = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} v_u \\ v_v - c\dot{\theta} \end{pmatrix}$$

General kinematic equations : $(\textit{include angular rate})$

$$\begin{cases} \dot{x} = v_u \cos\theta - (v_v - c\dot{\theta}) \sin\theta \\ \dot{y} = v_u \sin\theta + (v_v - c\dot{\theta}) \cos\theta \\ \dot{\theta} = \omega \end{cases} \qquad (2)$$



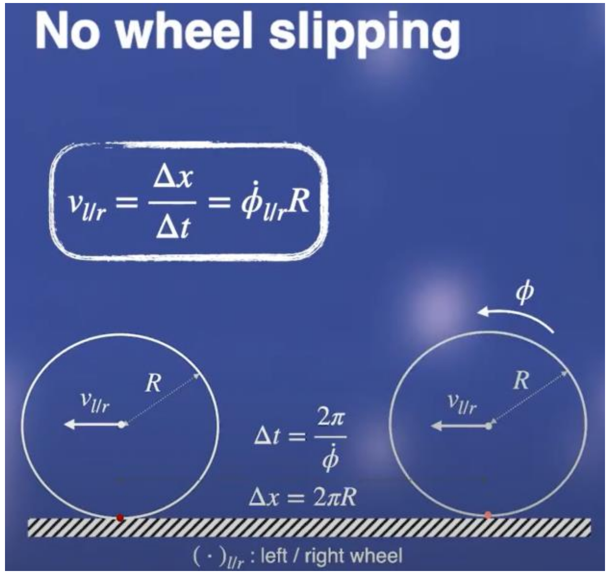
II. Differential drive WMR

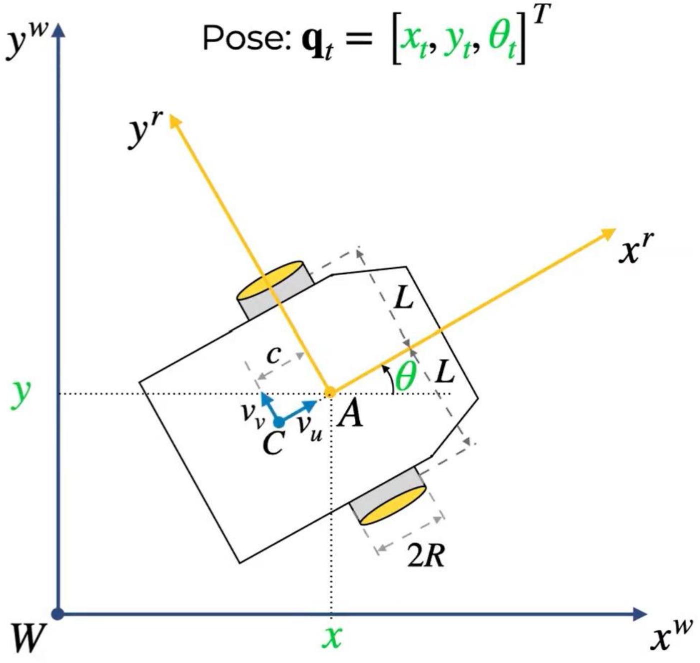
Kinematics constraint: Pure rolling

- Assumption: Pure rolling
  - ✓ (1) No moving sideways (skidding): point $\boldsymbol{A}$ has null lateral component

$$\mathbf{v}_A^r = \begin{bmatrix} v_u \\ \boxed{v_v - c\dot{\theta}} \end{bmatrix} = \begin{bmatrix} v_u \\ \boxed{0} \end{bmatrix}$$

  - ✓ (2) No wheel slipping: every a full revolution ($\Delta\phi = 2\pi$), each wheel travels a distance equal to its circumference ($\Delta x = 2\pi R$)





II. Differential drive WMR

No skidding: $v_v - c\dot{\theta} = 0$

$$\mathit{find} : \dot{\mathbf{q}}_t = \begin{bmatrix} \dot{x}_t \\ \dot{y}_t \\ \dot{\theta}_t \end{bmatrix} = ?$$

$$\xrightarrow[v_A]{\text{Velocities}} \boxed{\begin{cases} \dot{x} = v_u \cos\theta - \cancel{(v_v - c\dot{\theta})} \sin\theta \\ \dot{y} = v_u \sin\theta + \cancel{(v_v - c\dot{\theta})} \cos\theta \\ \dot{\theta} = \omega \end{cases}} \xrightarrow[\dot{\mathbf{q}}_t]{\text{Pose variation}}$$

$$\mathit{let}\ v_A = v_u$$

$$\Leftrightarrow \dot{\mathbf{q}}_t = \begin{bmatrix} \dot{x}_t \\ \dot{y}_t \\ \dot{\theta}_t \end{bmatrix} = \begin{bmatrix} \cos\theta_t & 0 \\ \sin\theta_t & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_A \\ \omega_t \end{bmatrix} \qquad (3)$$

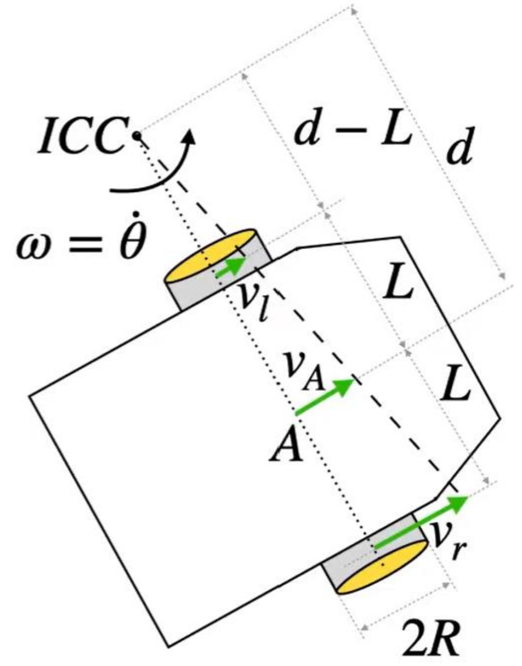
II. Differential drive WMR

Instantaneous Center of Curvature (ICC)

No slipping assumption → All points in a pure rotation field (centered at ICC) have velocity orthogonal to distance to ICC, how to get ***v_l, v_A, v_r ,d***?

$$\begin{cases} v_l = \omega \;\; d - L \\ v_A = \omega d \\ v_r = \omega \;\; d + L \end{cases} \qquad 4$$

$$\Rightarrow d = L \frac{v_r + v_l}{v_r - v_l}$$



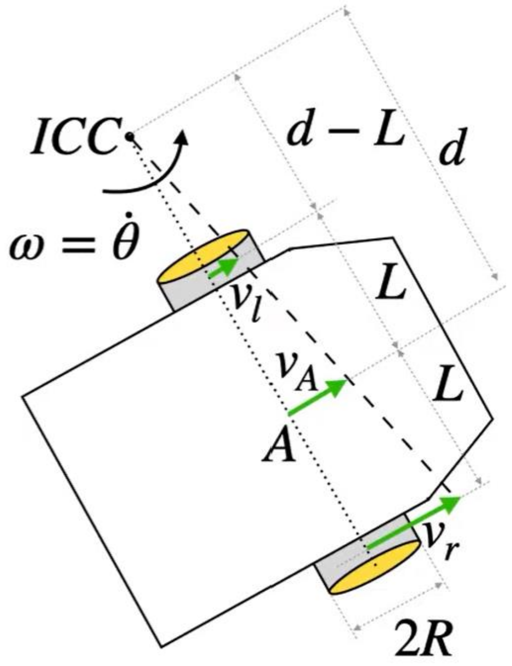
Notes :

1. If $v_r = v_l \Rightarrow$ no turn $\Rightarrow$ ICC is underfined
2. If $v_r = -v_l \Rightarrow$ turn " on itself " $\Rightarrow$ ICC $\equiv$ A
3. If $v_r = 0$ (or $v_l = 0$) $\Rightarrow$ turn " on wheel " $\Rightarrow d = -L$ (or $d = L$)

II. Differential drive WMR

Forward kinematics

$$Find: \begin{bmatrix} v_A \\ \omega \end{bmatrix} = \underset{Jacobian}{J} \begin{bmatrix} \dot{\phi}_r \\ \dot{\phi}_l \end{bmatrix}$$



$$4 \Rightarrow \begin{cases} v_r + v_l = 2\omega d \\ v_r - v_l = 2\omega L \end{cases} \Rightarrow \begin{cases} d = L\dfrac{v_r + v_l}{v_r - v_l} \\ v_A = \omega d = \dfrac{v_r + v_l}{2} \\ \omega = \dfrac{v_r - v_l}{2L} \end{cases}$$

$$v_r = \dot{\phi}_r R,\ v_l = \dot{\phi}_l R$$

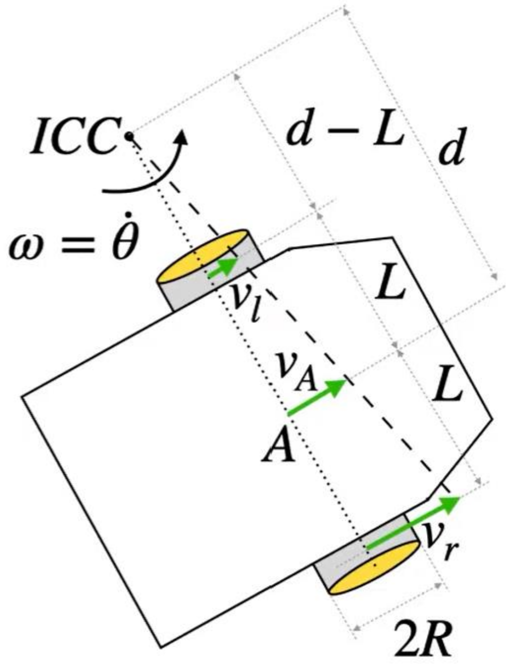
$$\Rightarrow \begin{cases} v_A = \dfrac{R}{2}\ \dot{\phi}_r + \dot{\phi}_l \\ \omega = \dfrac{R}{2L}\ \dot{\phi}_r - \dot{\phi}_l \end{cases} \Leftrightarrow \boxed{\begin{bmatrix} v_A \\ \omega \end{bmatrix} = \underbrace{\frac{R}{2}\begin{bmatrix} 1 & 1 \\ 1/L & -1/L \end{bmatrix}}_{Jacobian}\begin{bmatrix} \dot{\phi}_r \\ \dot{\phi}_l \end{bmatrix}} \qquad (5)$$

II. Differential drive WMR

Forward kinematics

From (3) & (5)

$$\Rightarrow \boxed{\dot{\mathbf{q}}_t = \frac{R}{2}\begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1/L & -1/L \end{bmatrix}\begin{bmatrix} \dot{\phi}_r \\ \dot{\phi}_l \end{bmatrix}}$$

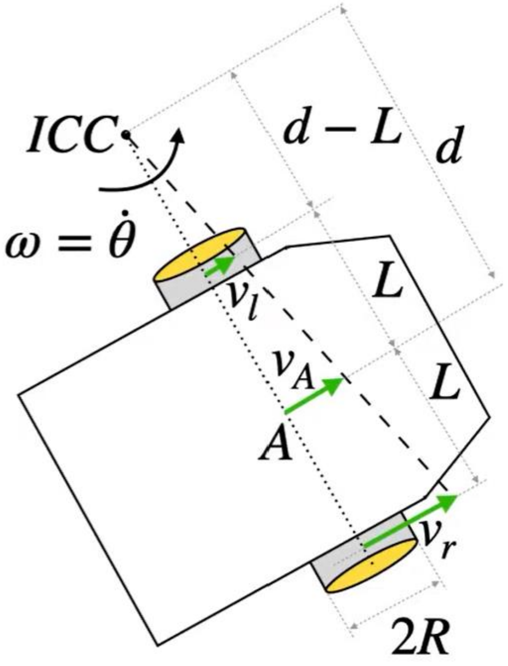


II. Differential drive WMR

Inverse kinematics

From Eq.(5)

$$\Rightarrow \boxed{\begin{bmatrix} \dot{\phi}_r \\ \dot{\phi}_l \end{bmatrix} = \frac{1}{R}\begin{bmatrix} 1 & L \\ 1 & -L \end{bmatrix}\begin{bmatrix} v_A \\ \omega \end{bmatrix}} \qquad (6)$$

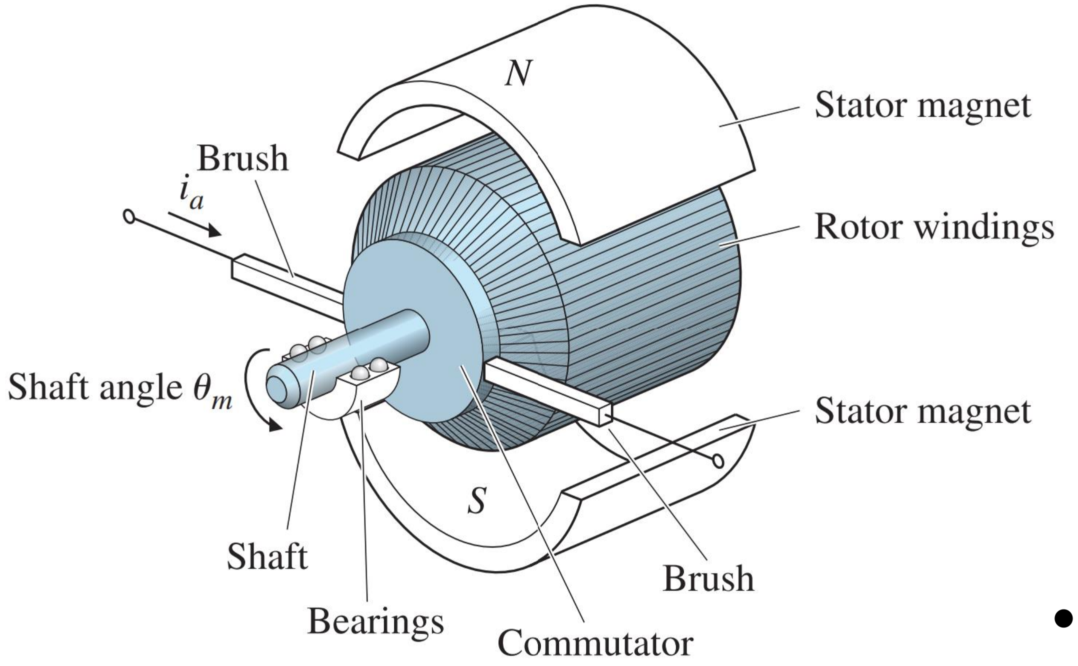


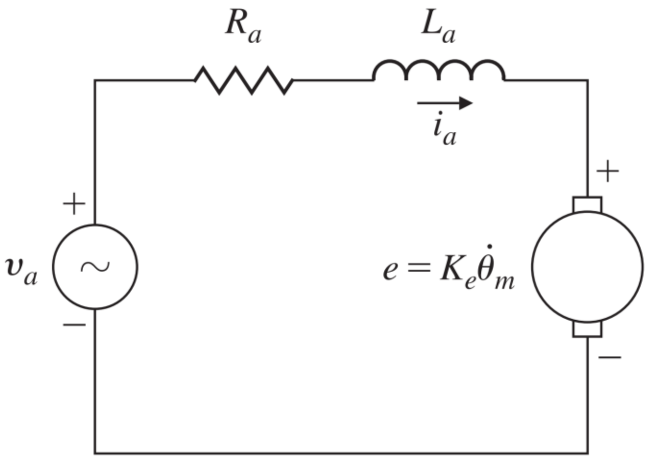
$$where \begin{bmatrix} \dot{x}_t \\ \dot{y}_t \\ \dot{\theta}_t \end{bmatrix} = \begin{bmatrix} \cos\theta_t & 0 \\ \sin\theta_t & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} v_A \\ \omega_t \end{bmatrix}$$

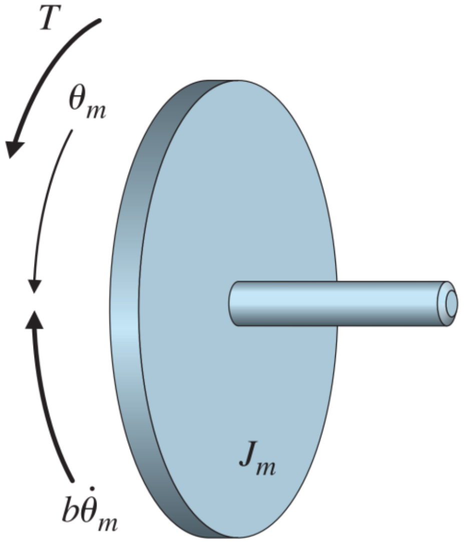
II. Differential drive WMR

DC motor model

Reference: Franklin, "Feedback Control of Dynamic Systems" 8th Edition, Pearson 2020







*The generated electromotive force (emf) works against the applied armature voltage, it is called the **back emf**.*

- The torque T on the rotor in terms of the armature current i_a : $\boxed{T = K_t i_a}$, $K_t\left[\frac{N}{A}\right]$: torque const
- The express of **back emf** voltage e in terms of the shaft's rotational velocity $\dot{\theta}_m$: $\boxed{e = K_e\dot{\theta}_m}$, $K_e\left[\frac{V}{rad/s}\right]$: electric const
- $K_t = K_e$, but unit are different needing to translate to a certain

II. Differential drive WMR

DC motor model

Reference: Franklin, "Feedback Control of Dynamic Systems" 8th Edition, Pearson 2020

- Electrical subsystem (Kirchhoff's law):

$$L_a \frac{di_a}{dt} + R_a i_a = v_a - K_e \dot{\theta}_m \qquad (7)$$

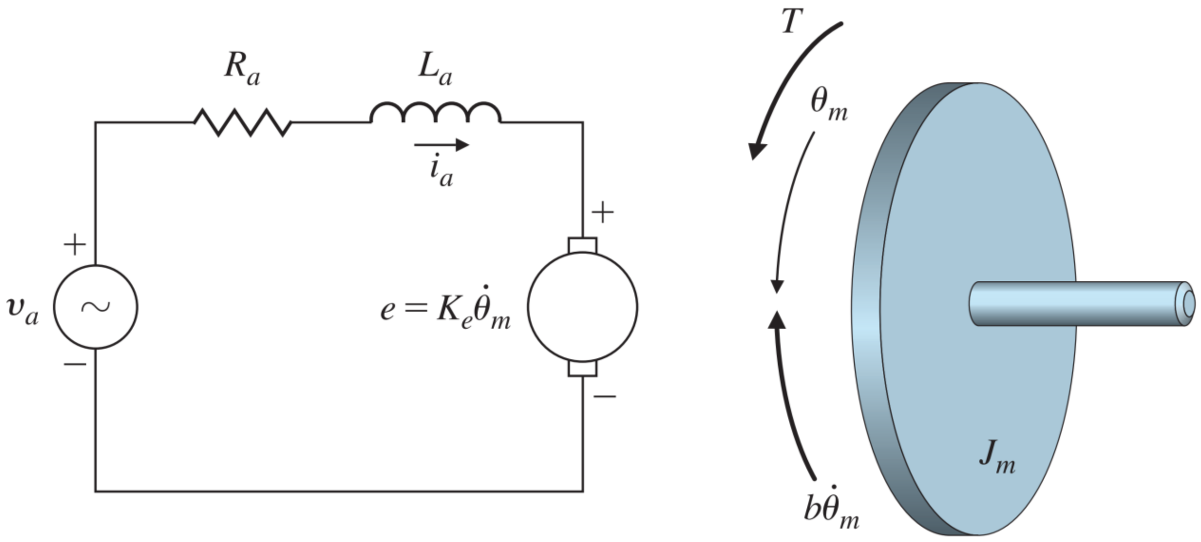
- Mechanical subsystem (Newton's law):

$$J_m \ddot{\theta}_m + b\dot{\theta}_m = K_t i_a \qquad (8)$$

where J_m is inertia, and b viscous friction coefficient

$$(7) \Rightarrow i_a = \frac{v_a - K_e \dot{\theta}_m}{R_a}, \quad \left(L_a : \text{be neglected} \right)$$

In many cases the relative effect of the *inductance* is negligible compared with the mechanical motion



II. Differential drive WMR

DC motor model

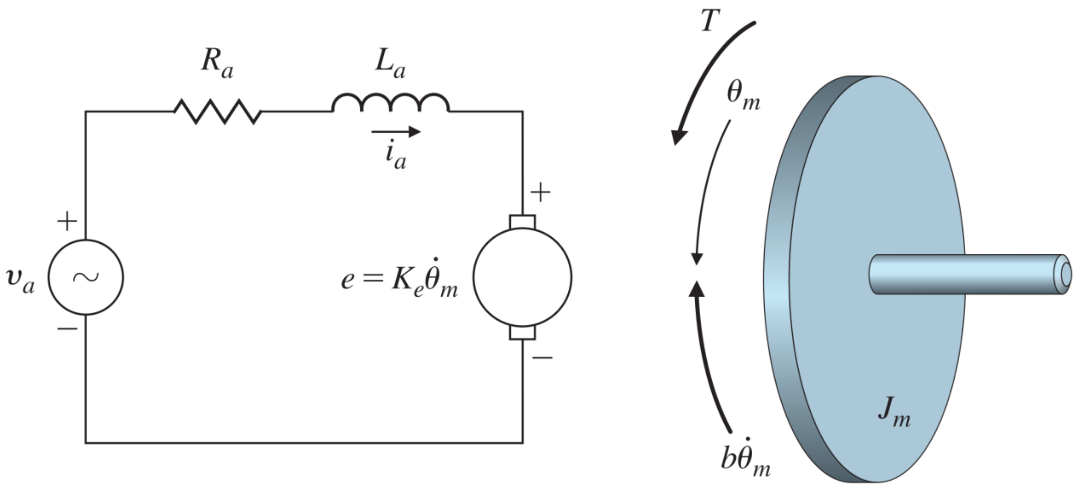
Reference: Franklin, "Feedback Control of Dynamic Systems" 8th Edition, Pearson 2020

$$(8) \Rightarrow \boxed{J_m\ddot{\theta}_m + \left(b + \frac{K_t K_e}{R_a}\right)\dot{\theta}_m = \frac{K_t}{R_a}v_a} \quad (ODE)$$

$$\xrightarrow{Laplace} G(s) = \frac{\Theta_m(s)}{V_a(s)} = \frac{\dfrac{K_t}{R_a}}{J_m s^2 + \left(b + \dfrac{K_t K_e}{R_a}\right)s}$$

$$\Rightarrow \boxed{G(s) = \frac{K}{s\ \ \tau s + 1}} \quad (TF)$$

where $K = \dfrac{K_t}{bR_a + K_t K_e}, \tau = \dfrac{R_a J_m}{bR_a + K_t K_e}$

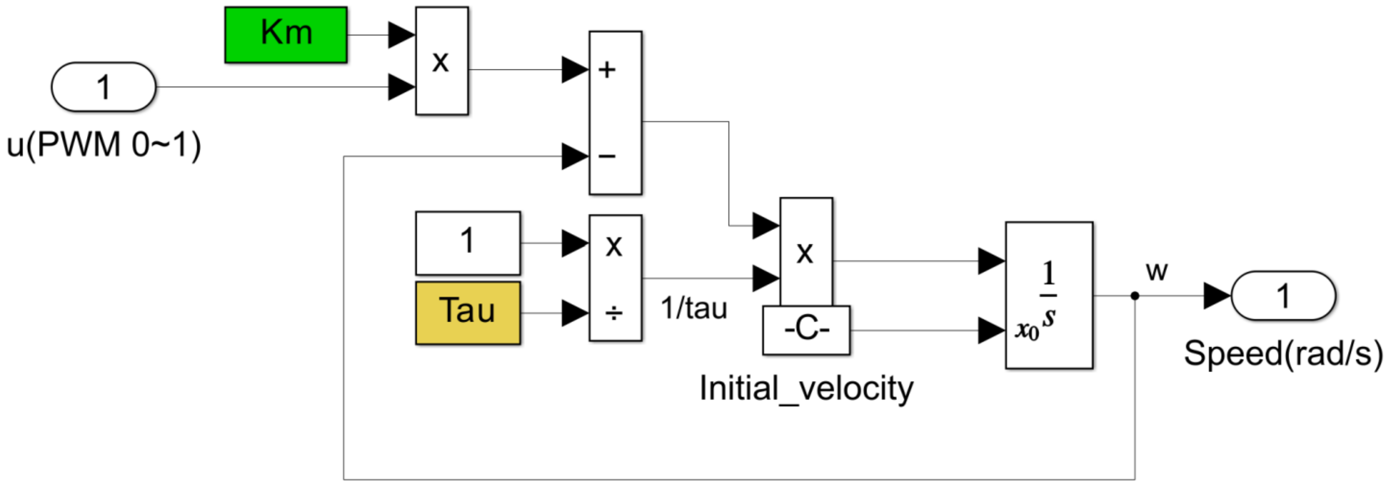


In many cases, a transfer function between the motor input and the output speed ($\omega = \dot{\theta}_m$) is required:

$$\boxed{\frac{\Omega(s)}{V_a(s)} = s\frac{\Theta_m(s)}{V_a(s)} = \frac{K}{\tau s + 1}}$$

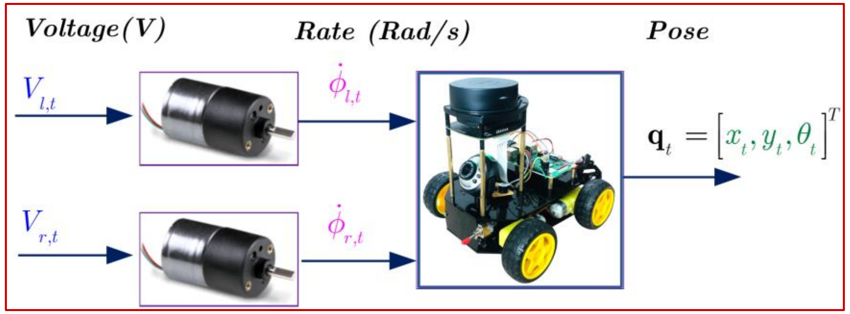
II. Differential drive WMR

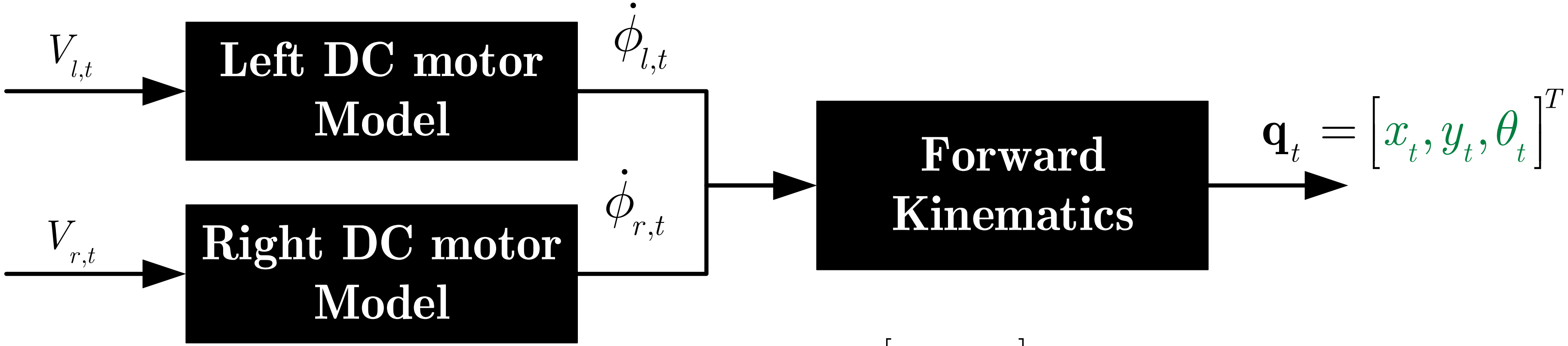
DC motor model



II. Differential drive WMR

DC motor model



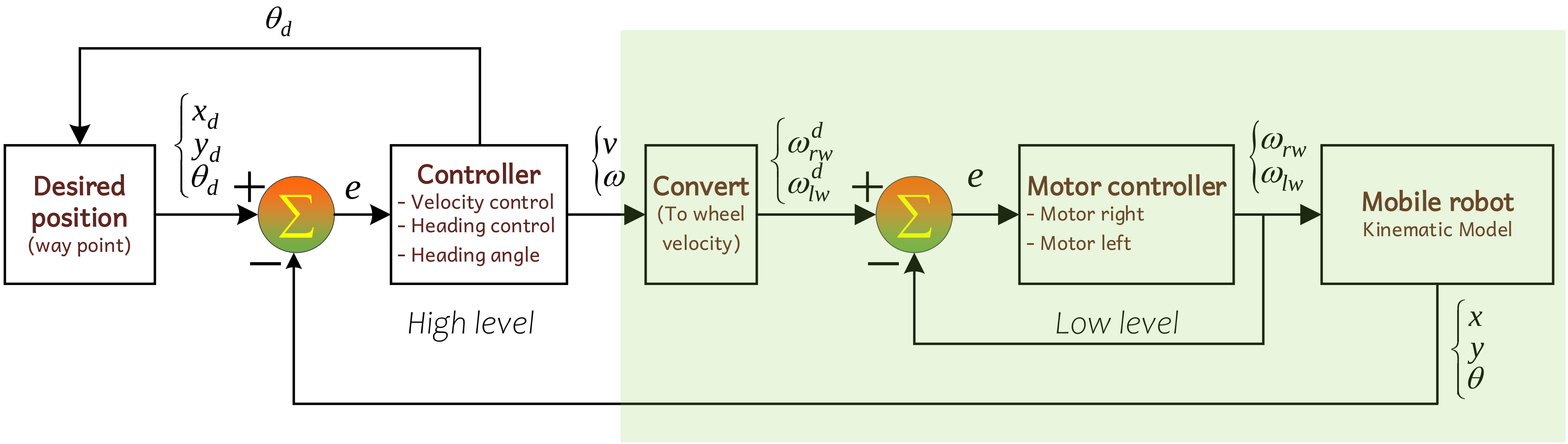


$$\frac{\Omega(s)}{V_a(s)} = \frac{K}{\tau s + 1}$$

$$\dot{\mathbf{q}}_t = \frac{R}{2}\begin{bmatrix}\cos\theta & 0\\ \sin\theta & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 1/L & -1/L\end{bmatrix}\begin{bmatrix}\dot{\phi}_r\\ \dot{\phi}_l\end{bmatrix}$$

Controller design – PID controller (basic)

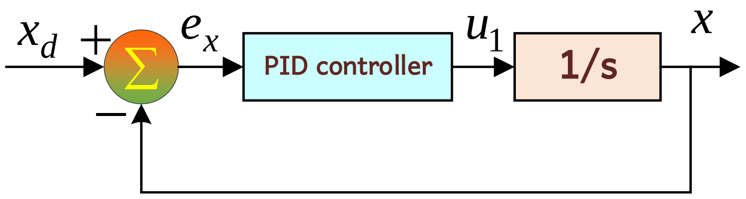
+ Controller: v and ω
+ From controller v and ω, it can be generated to ω_{rw}^d and ω_{lw}^d



Controller design

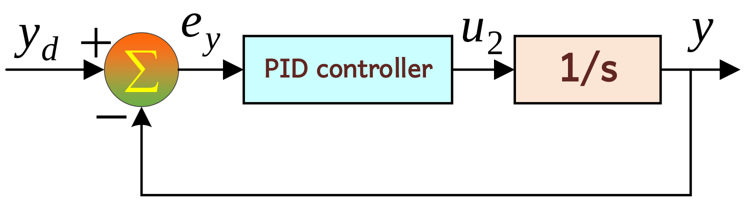
$$From: \begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = \frac{R}{2}\begin{bmatrix}\cos\theta & 0\\ \sin\theta & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 1/L & -1/L\end{bmatrix}\begin{bmatrix}\dot{\phi}_r\\ \dot{\phi}_l\end{bmatrix} \Rightarrow \begin{cases}\dot{x} = \frac{R}{2}\ \omega_{rw} + \omega_{lw}\ \cos\theta\\ \dot{y} = \frac{R}{2}\ \omega_{rw} + \omega_{lw}\ \sin\theta\\ \dot{\theta} = \frac{R}{2}\left(\frac{\omega_{lw} + \omega_{rw}}{L}\right)\end{cases}$$

Controller for x axis: $\dot{x} = u_1$



+ Position (x,y) controller u

Controller for y axis : $\dot{y} = u_2$

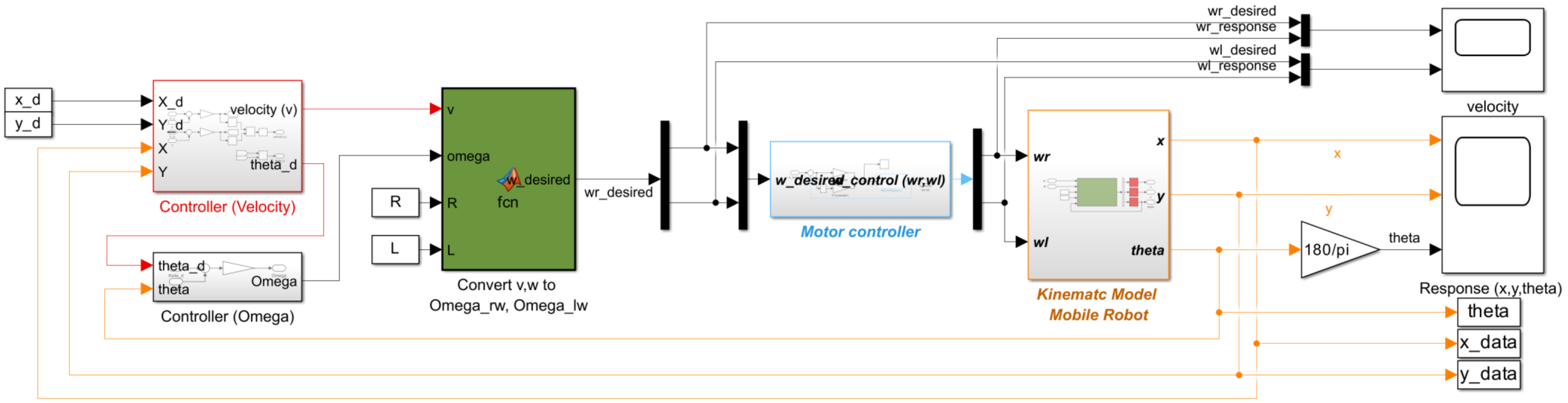


Let give a controller as follows:

$$u = \begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix} = \begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}PID\ \ x_d - x\\ PID\ \ y_d - y\\ PID\ \ \theta_d - \theta\end{bmatrix} = \begin{bmatrix}K_1\ \ x_d - x\\ K_2\ \ y_d - y\\ K_3\ \ \theta_d - \theta\end{bmatrix}$$

Controller design

Simulation:

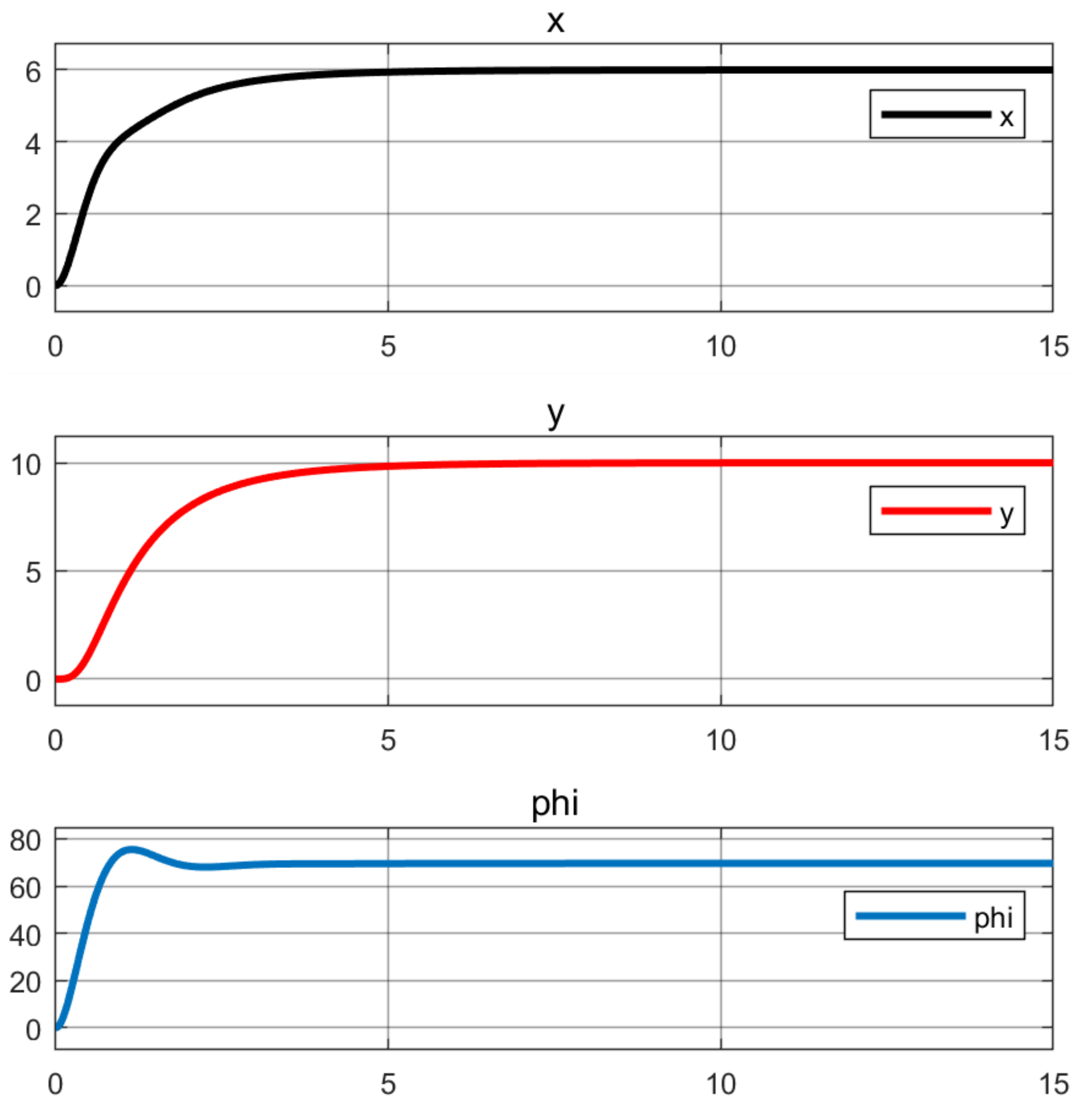


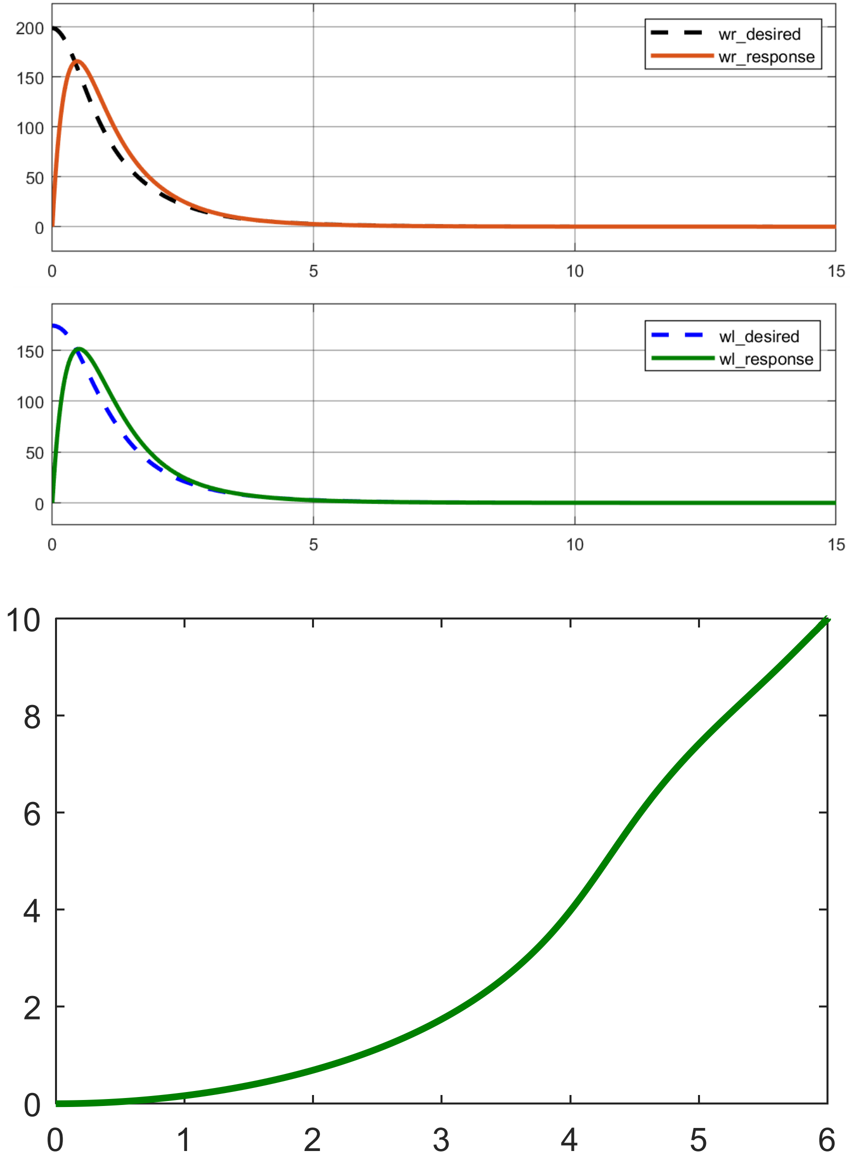

```
clc; clear all; close all;
R=0.05 %5cm
L=0.2 %40cm
x_Init=0
y_Init=0
phi_Init=0*pi/180
%% setpoint
x_d=6
y_d=10
%% Motor Dynamics
Km=500;
Tau = 2;
```

```
%% mobile robot control
kpx=0.8
kpy=0.8
kptheta=3
%% motor control
kp=0.02
ki=0.015
```

Controller design

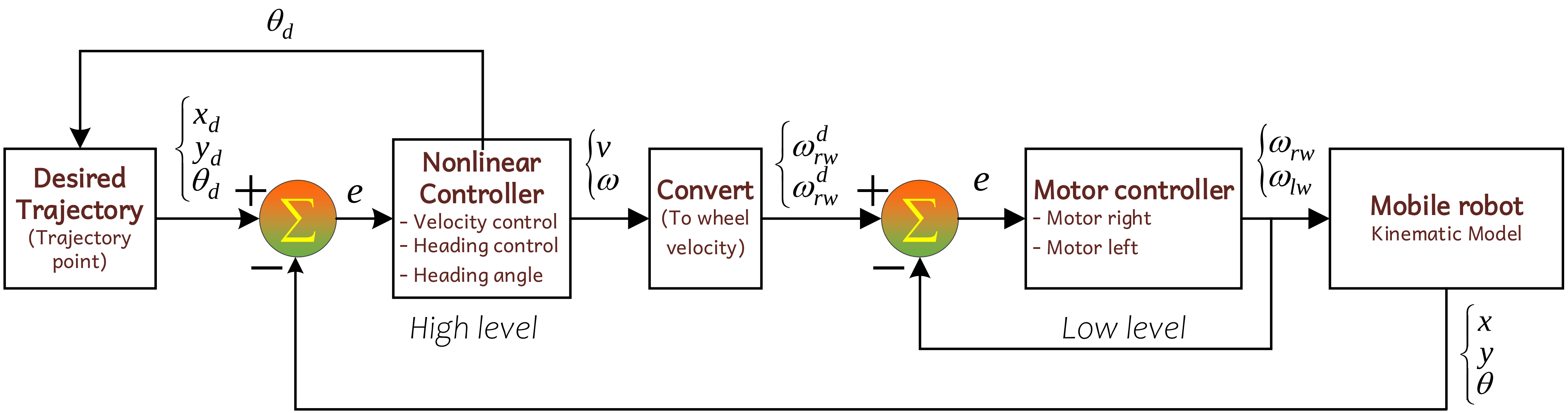
Simulation:



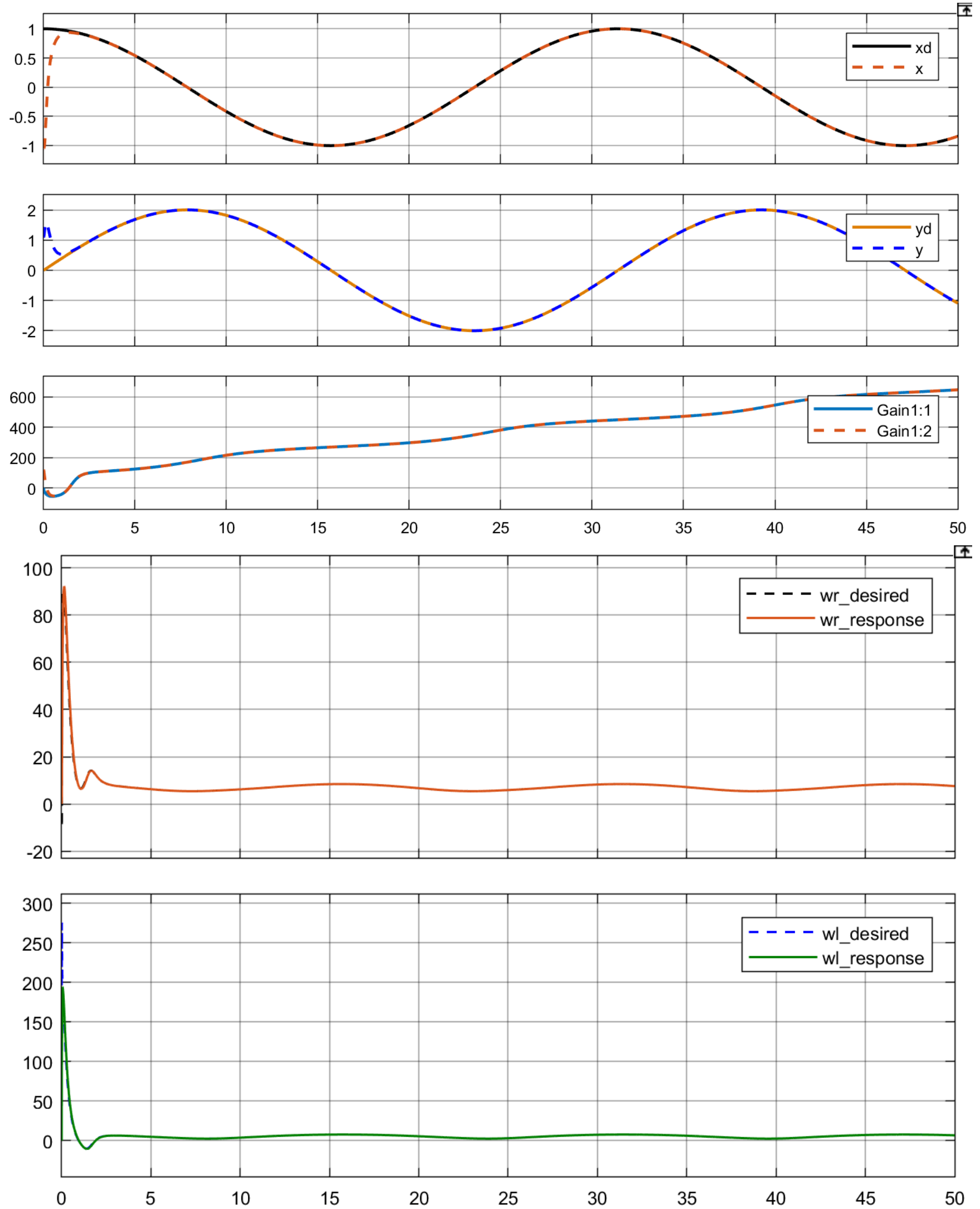


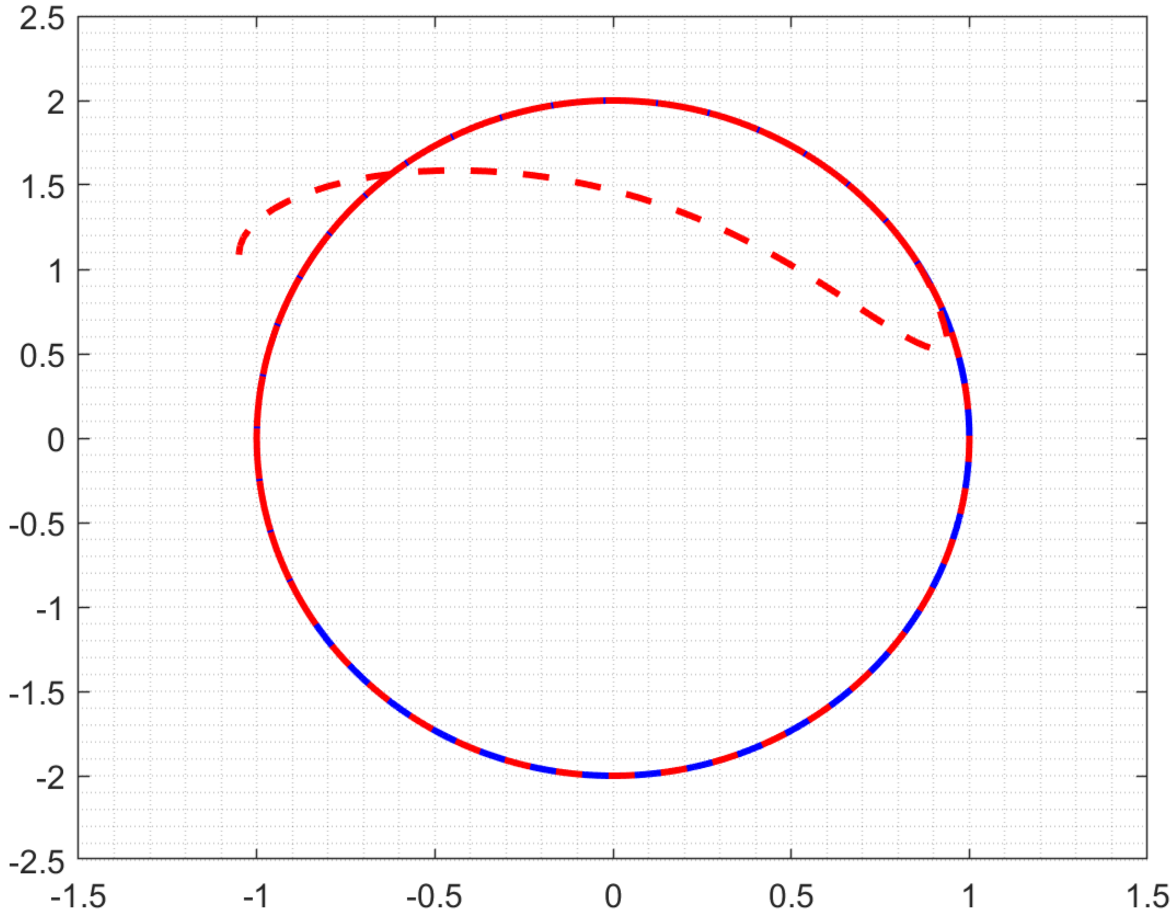
Controller design – Linear, Nonlinear Tracking Control

+ Controller: v and ω
+ From controller v and ω, it can be generated to ω_{rw}^{d} and ω_{lw}^{d}



Controller design – Linear Tracking Control





Controller design – Linear Tracking Control