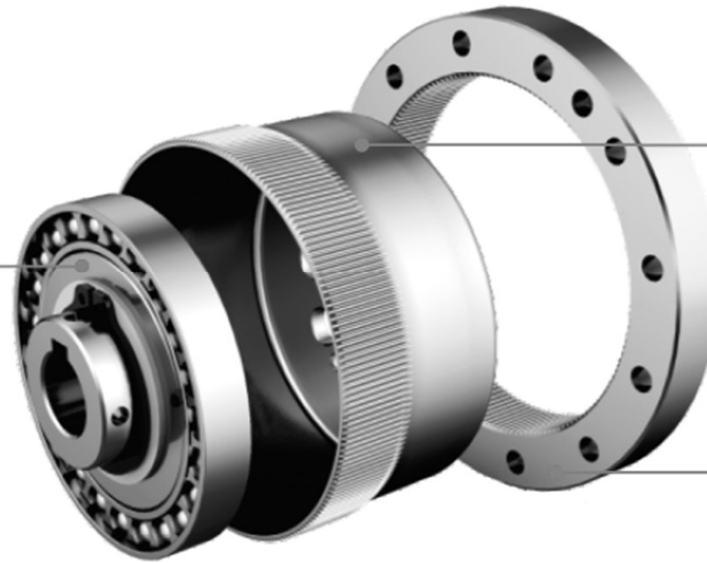


Gear ratio of Strain Wave Gear

Wave generator

Assemble thin - wall ball bearing component in the periphery of elliptic cam. Inner wheel of the bearing can be fixed on the cam and outer wheel can perform elastic deformation through ball. It is usually installed on output shaft.

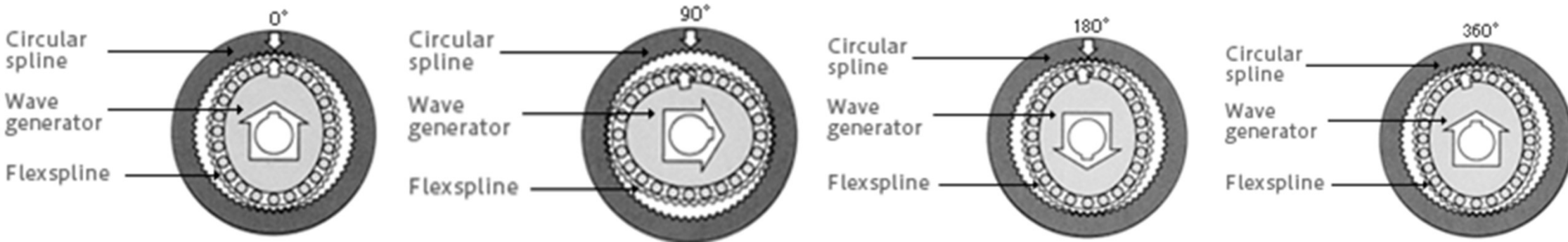


Flexible gear

The flexible gear is a thin-wall cup-shaped metallic elastic component and gears are carved on the periphery of its opening part. Flexible gear bottom (cup-shaped bottom) is called diaphragm part, which is generally installed on the output shaft.

Rigid gear

It is a rigid ring component and gears are carved on the inner ring. The number of the gears is two more than that of flexible gear. It is commonly fixed on the machine shell.



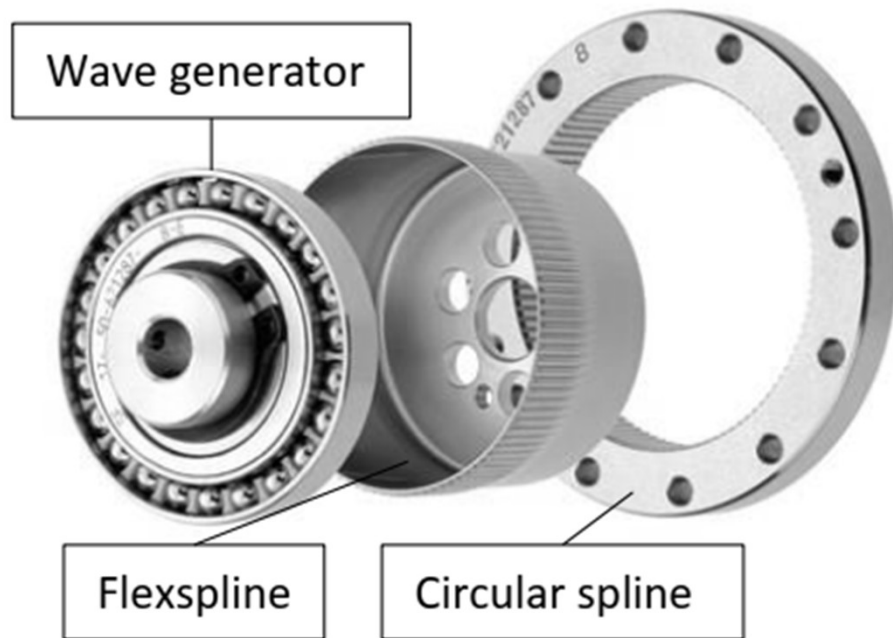
Gear ratio of Strain Wave Gear

Cup style strain wave gear

The wave generator (WG) is a ball bearing with an elliptical cam insert

The flexspline (FS) is a cup shape thin shell

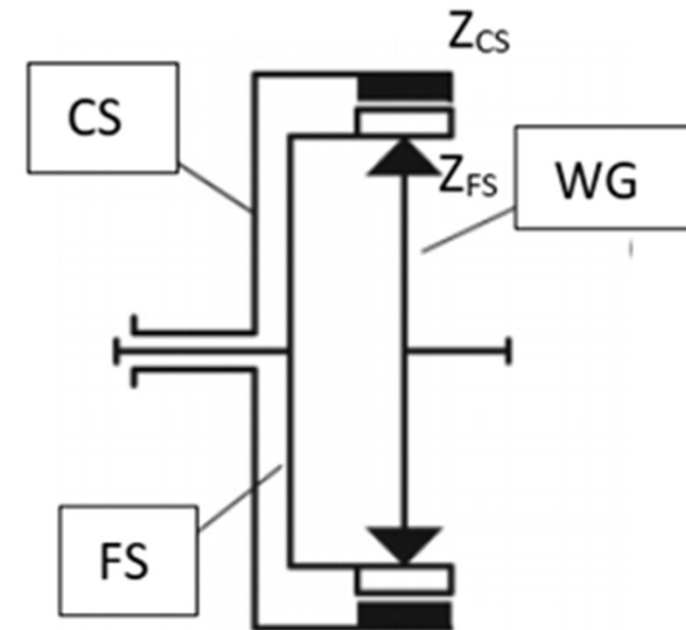
The circular spline (CS) is a rigid ring gear



$$\frac{\omega_{CS} - \omega_{WG}}{\omega_{FS} - \omega_{WG}} = \frac{Z_{FS}}{Z_{CS}}$$

The three components system has two degree of freedom as a planetary gear train

z is tooth number, ω is angular velocity

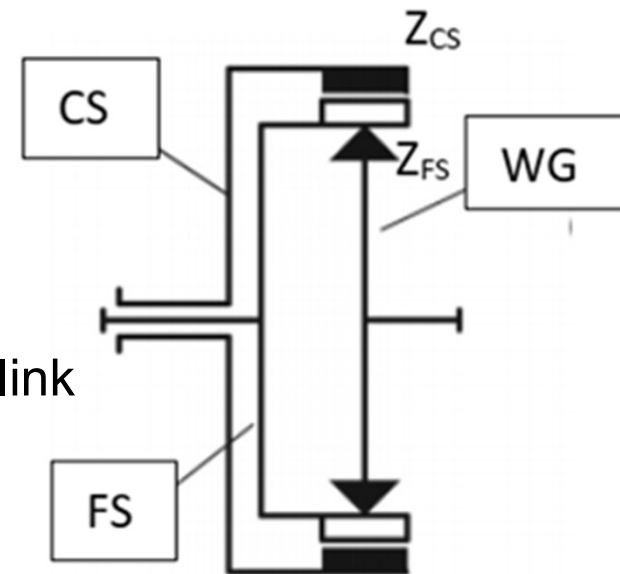


Gear ratio of Strain Wave Gear

Example: if the flexspline has tooth number $Z_{FS} = 160$, circular spline has tooth number $Z_{CS} = 162$. The circular spline is connected to the fixed link of a robotic arm (angular velocity $\omega_{CS} = 0$). The wave generator is driven by servo motor as input ($\omega_{WG} = \omega_{input}$). The flexspline rigid end flange is connected to the robotic arm moving link ($\omega_{FS} = \omega_{output}$)

$$\frac{\omega_{CS} - \omega_{WG}}{\omega_{FS} - \omega_{WG}} = \frac{Z_{FS}}{Z_{CS}} \Rightarrow i = \frac{\omega_{input}}{\omega_{output}} = \frac{\omega_{WG}}{\omega_{FS}} = \frac{Z_{FS}}{-Z_{CS} + Z_{FS}}$$

$$= \frac{160}{-162 + 160} = -80$$



The speed of the servo motor will be reduced 80 times to the moving link

The negative sign means the wave generator and flexspline will rotate in opposite direction

Gear ratio of Strain Wave Gear

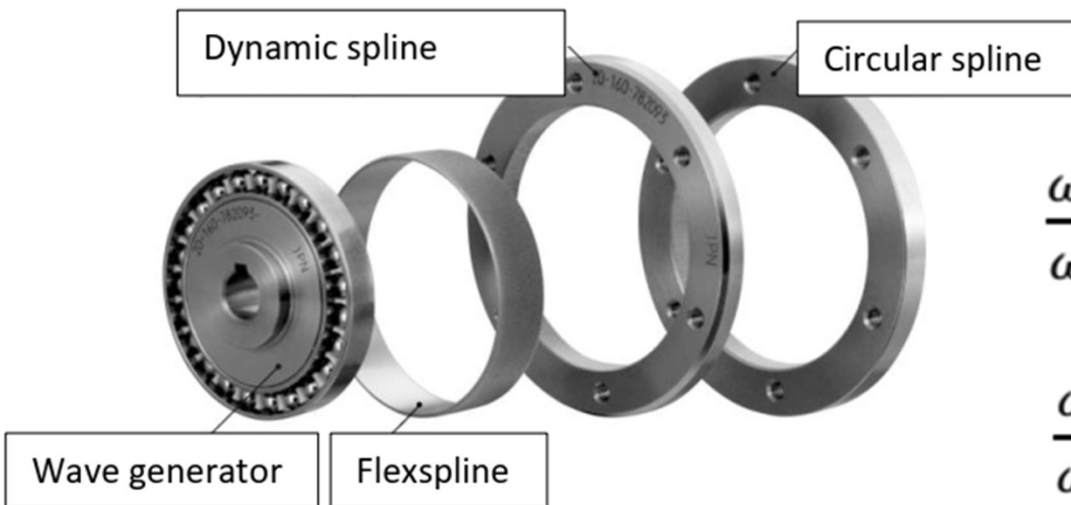
Pancake style strain wave gear

The wave generator (WG)

The flexspline (FS)

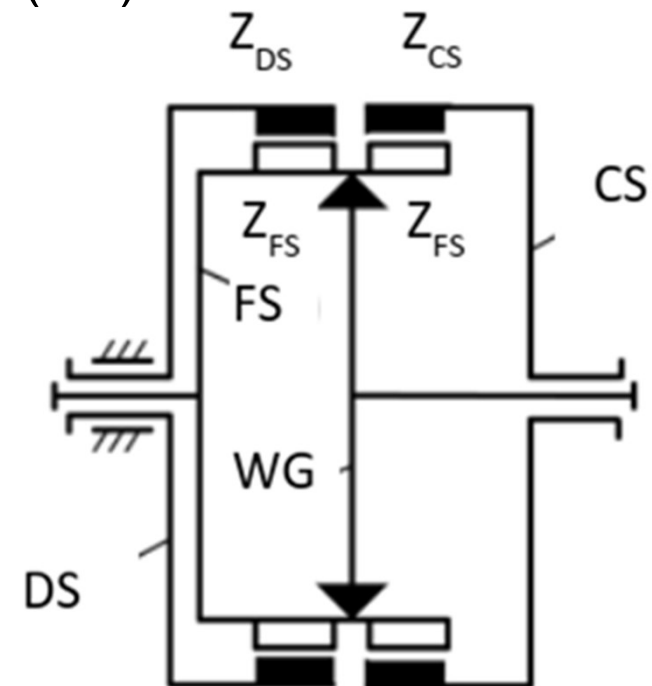
The circular spline (CS)

The dynamic spline (DS)



$$\frac{\omega_{DS} - \omega_{WG}}{\omega_{FS} - \omega_{WG}} = \frac{Z_{FS}}{Z_{DS}}$$

$$\frac{\omega_{CS} - \omega_{WG}}{\omega_{FS} - \omega_{WG}} = \frac{Z_{FS}}{Z_{CS}}$$



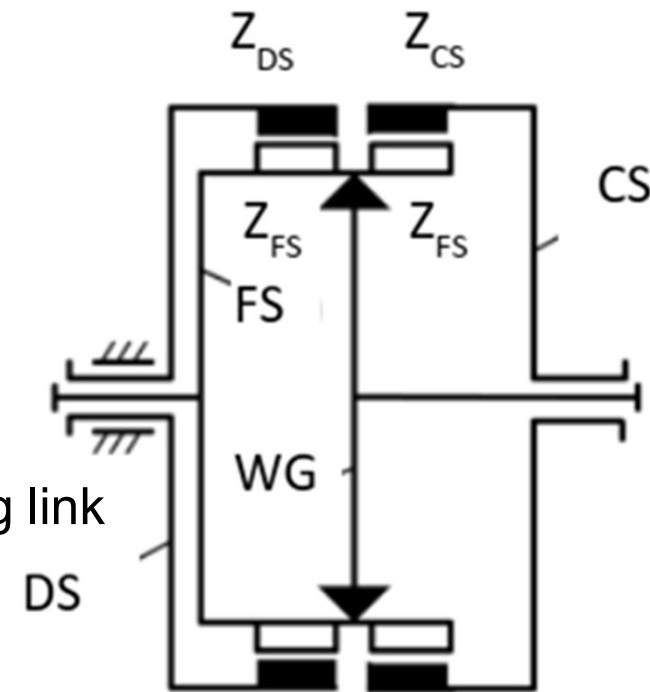
Gear ratio of Strain Wave Gear

Example: if the flexspline has $Z_{FS} = 160$, circular spline has $Z_{CS} = 162$. The circular spline is connected to the fixed link of a robotic arm ($\omega_{CS} = 0$). The dynamic spline has $Z_{DS} = 160$. The wave generator is driven by servo motor as input ($\omega_{WG} = \omega_{input}$). The dynamic spline is connected to the robotic arm moving link ($\omega_{DS} = \omega_{output}$)

The gear ratio can be calculated:

$$\left\{ \begin{array}{l} \frac{\omega_{DS} - \omega_{WG}}{\omega_{FS} - \omega_{WG}} = \frac{Z_{FS}}{Z_{DS}} \\ \frac{\omega_{CS} - \omega_{WG}}{\omega_{FS} - \omega_{WG}} = \frac{Z_{FS}}{Z_{CS}} \end{array} \right.$$

$$i = \frac{\omega_{input}}{\omega_{output}} = \frac{\omega_{WG}}{\omega_{DS}} = \frac{Z_{DS}}{-Z_{CS} + Z_{DS}} = \frac{160}{-162 + 160} = -80$$



The speed of the servo motor will be reduced 80 times to the moving link
 The negative sign means the wave generator and flexspline will rotate in opposite direction

Gear ratio of Strain Wave Gear

Calculate the reduction ratio of Harmonic Gear drive. Given data as below:

Number of teeth of the Flexspline:	Z_f	200
Number of teeth of the Circular Spline:	Z_c	202

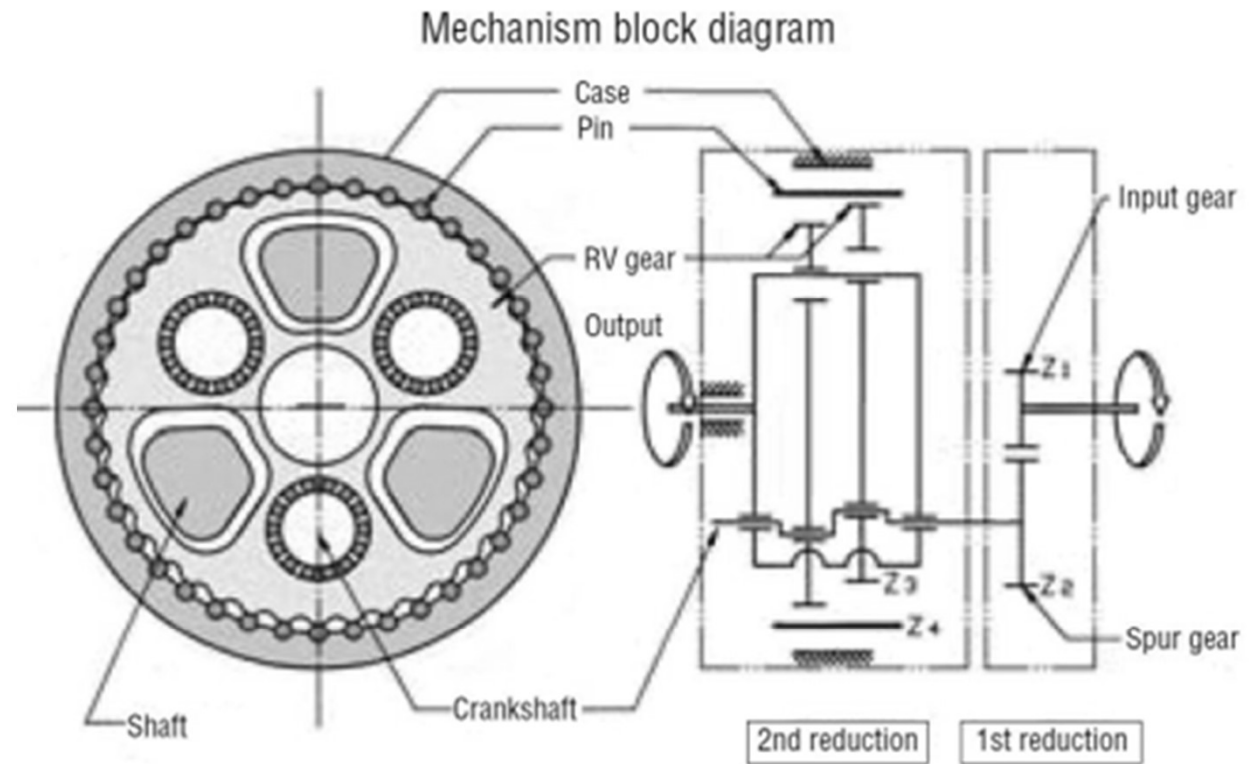
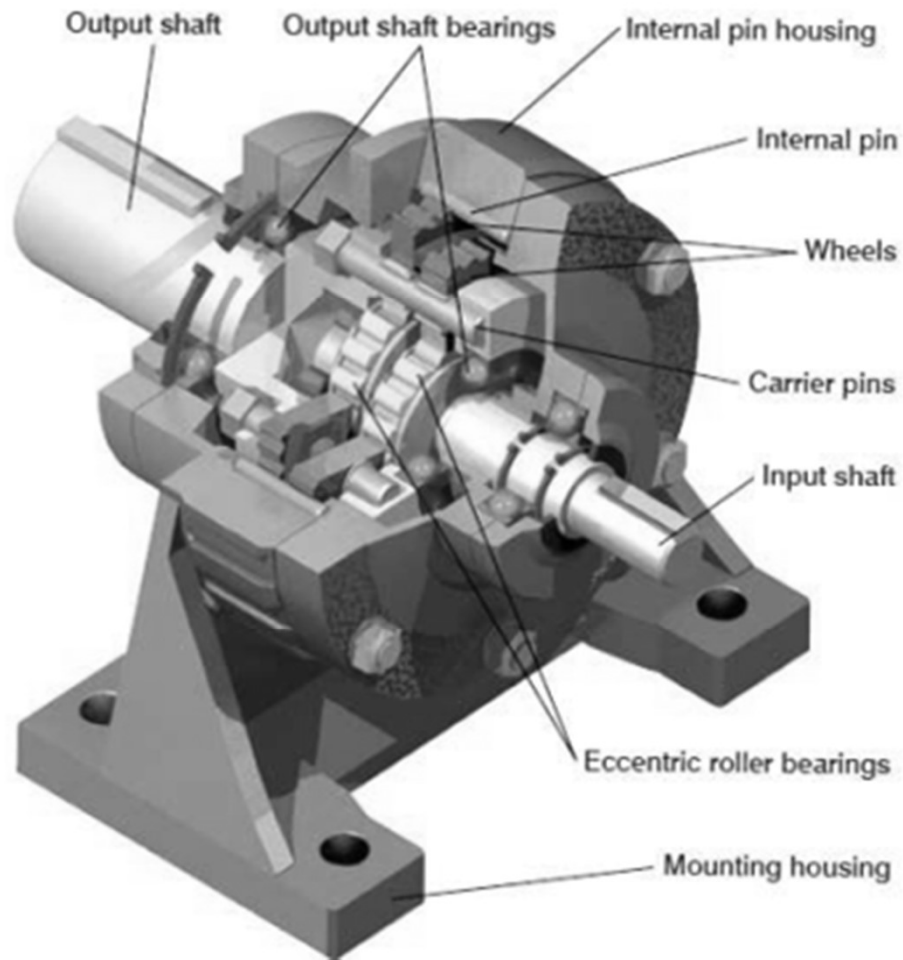
► Input: Wave Generator
Output: Flexspline
Fixed: Circular Spline

$$\Rightarrow i = \frac{w_i}{w_o} \Rightarrow i = \frac{Z_f}{Z_f - Z_c} = -100$$

► Input: Wave Generator
Output: Circular Spline
Fixed: Flexspline

$$\Rightarrow i = \frac{w_i}{w_o} \Rightarrow i = \frac{Z_c}{Z_c - Z_f} = 101$$

Cycloidal Gear Drive



Gear ratio is designated by this expression:

$$i = \frac{z_2 - z_1}{z_1}$$

Where i – reduction rate

z_1 – number of teeth(lobes) on the cycloid disc

z_2 – number of pins in the housing .

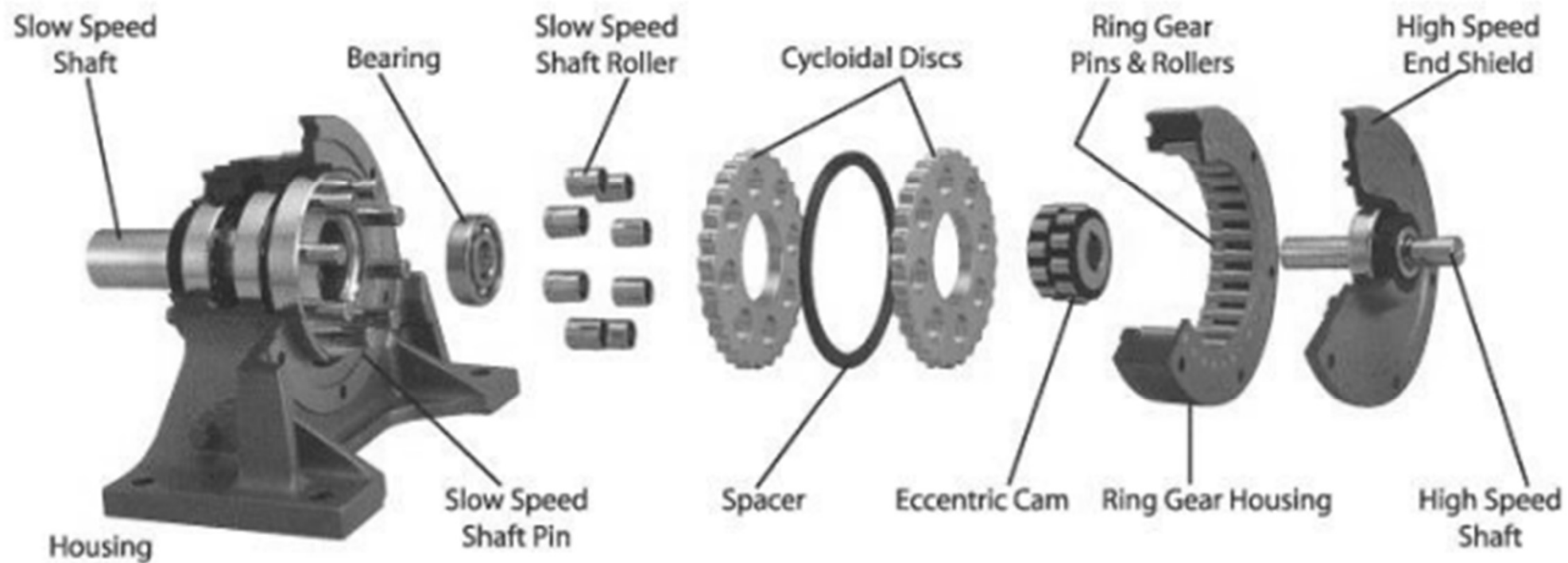
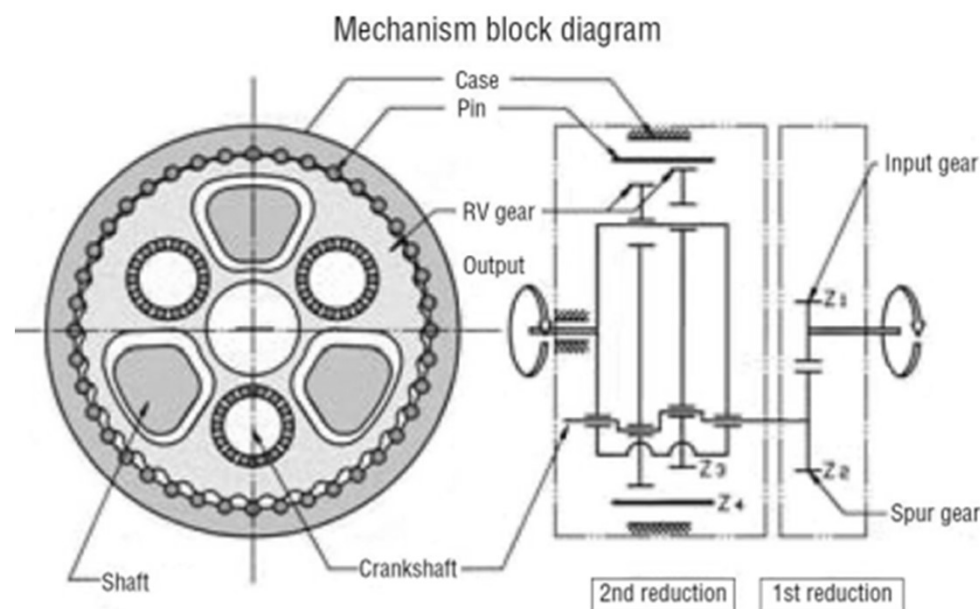


Figure 3. Exploded View of a Cycloid Speed Reducer



The overall reduction ratio of the reducer can be determined from the following equation:

$$i = \frac{1}{1 + \frac{Z_2}{Z_1} Z_4} \quad (\text{Eq 1.1})$$

Where here i - is the reduction ratio, Z_1 - is the number of teeth on the sun gear, Z_2 - number of teeth on the satellite gears, Z_4 - number of pins in case.