

Introduction to Neural Networks: DNN / CNN / RNN

EE807: Recent Advances in Deep Learning

Lecture 1

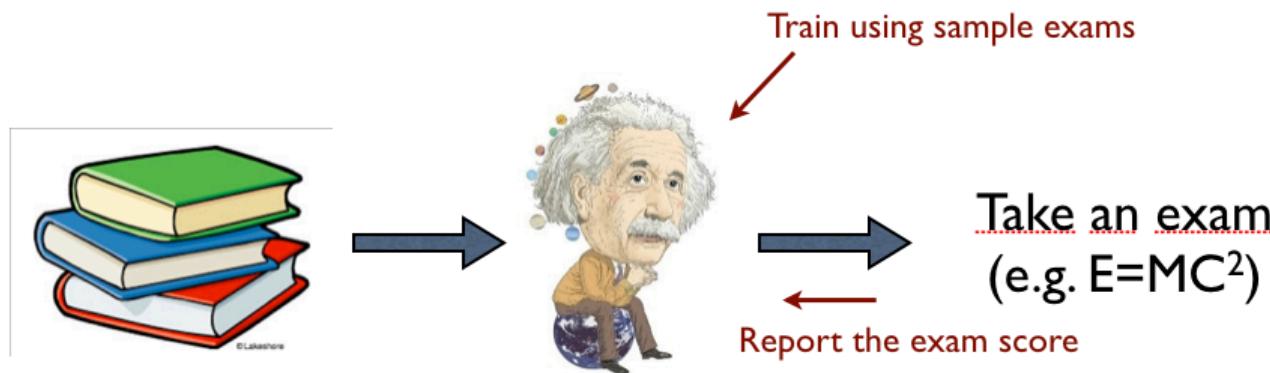
Slide made by

Hyungwon Choi and Yunhun Jang

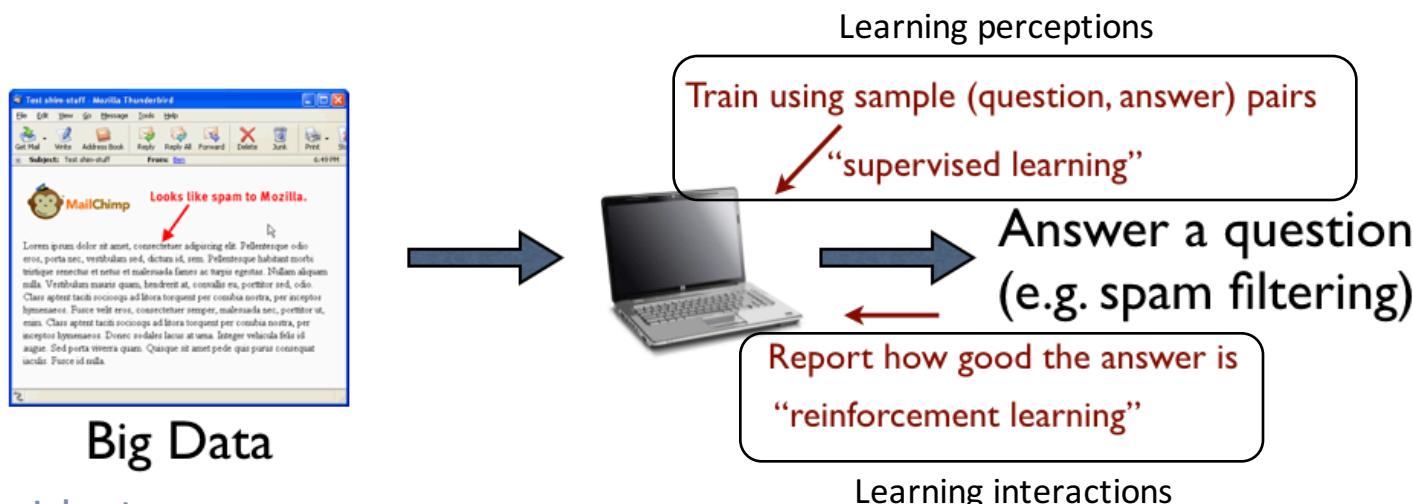
KAIST EE

What is Machine/Deep Learning?

- Human Learning

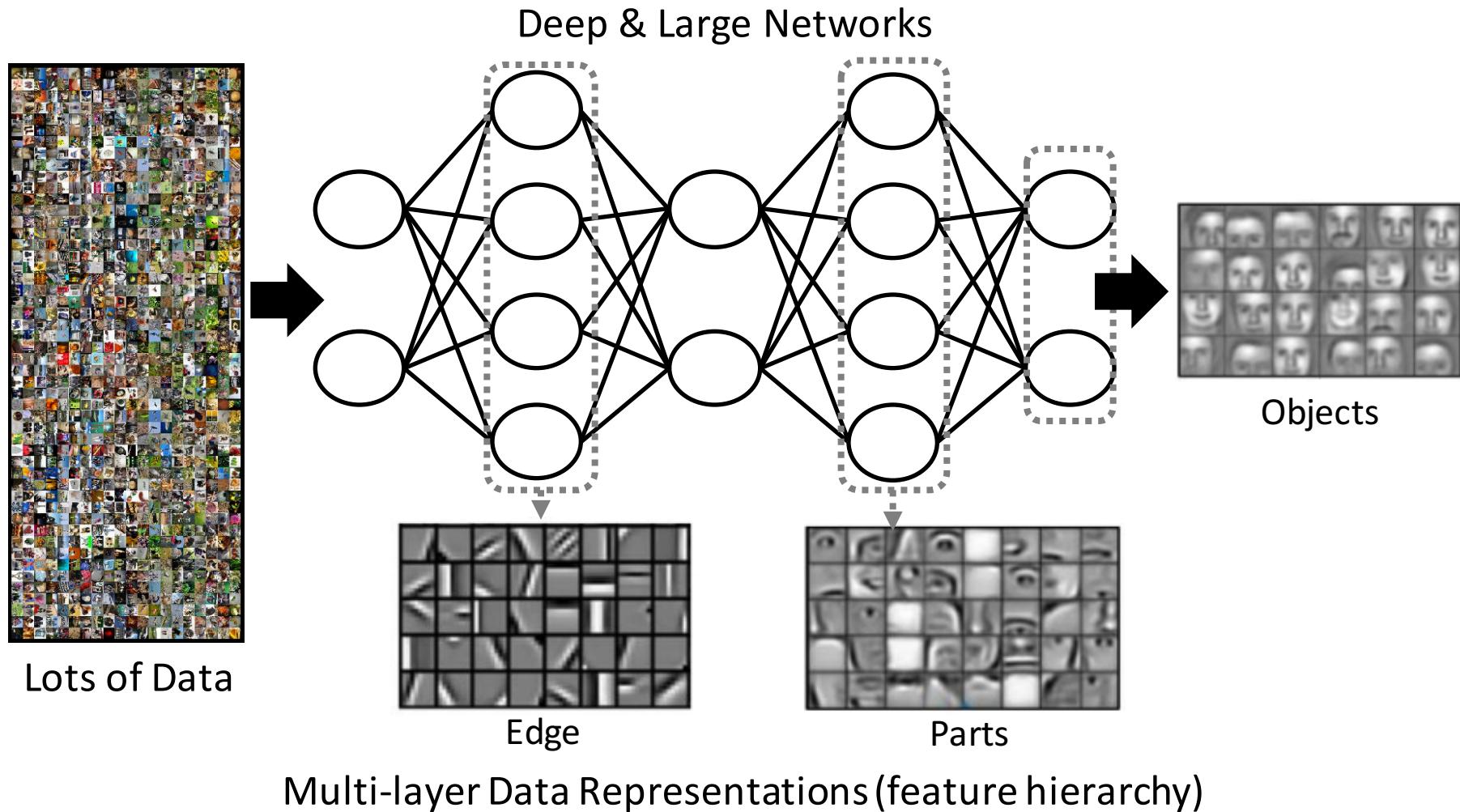


- Machine Learning = Build an algorithm from data
 - Deep learning is a special type of algorithms in machine learning



Definition of Deep Learning

- An algorithm that learns multiple levels of abstractions in data



Deep Learning = Feature Learning

- Why deep learning outperforms other machine learning (ML) approaches for vision, speech, language?

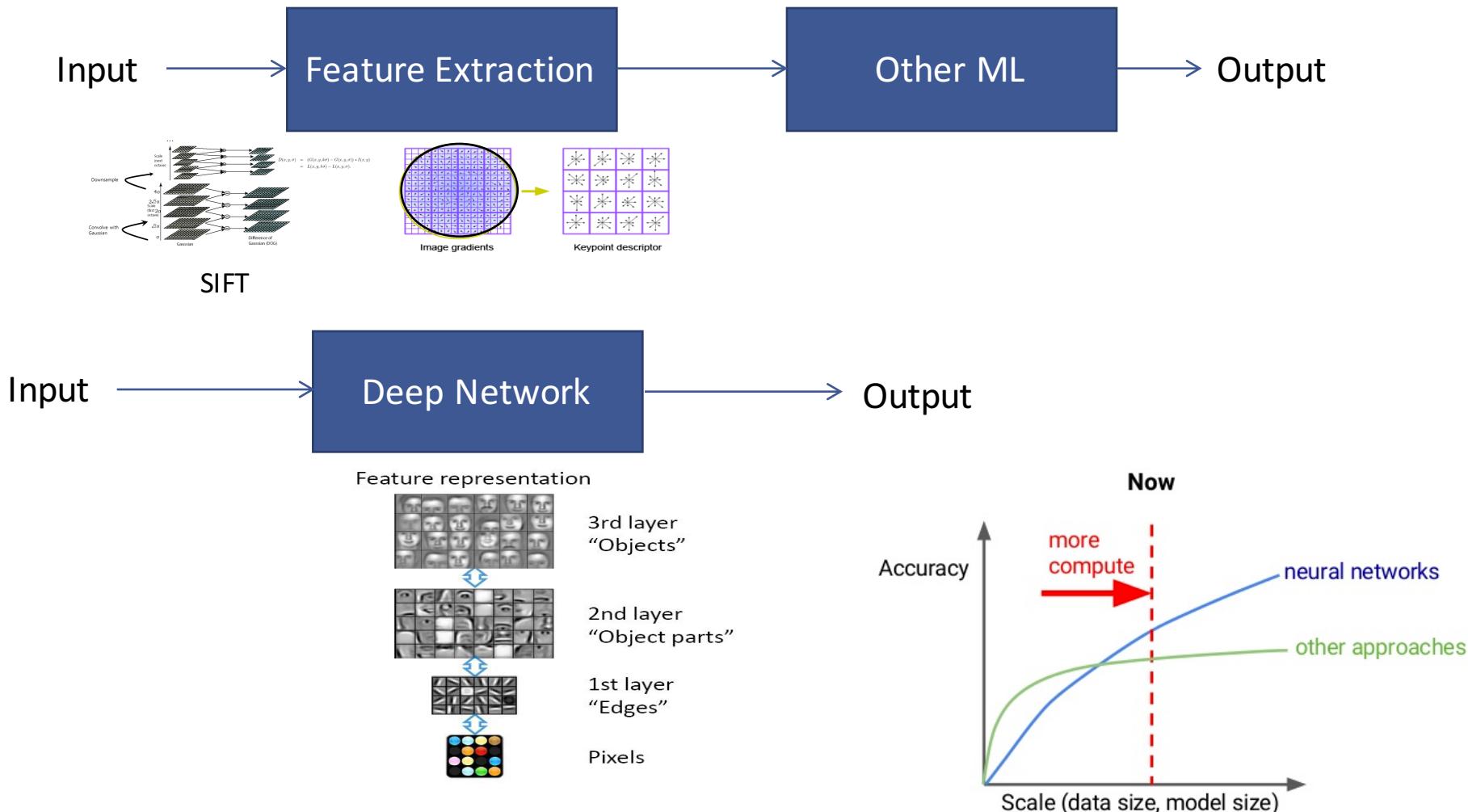


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1. Deep Neural Networks (DNN)

- Basics
- Training : Back propagation

2. Convolutional Neural Networks (CNN)

- Basics
- Convolution and pooling
- Some applications

3. Recurrent Neural Networks (RNN)

- Basics
- Character-level language model (example)

4. Question

- Why is it difficult to train a deep neural network?

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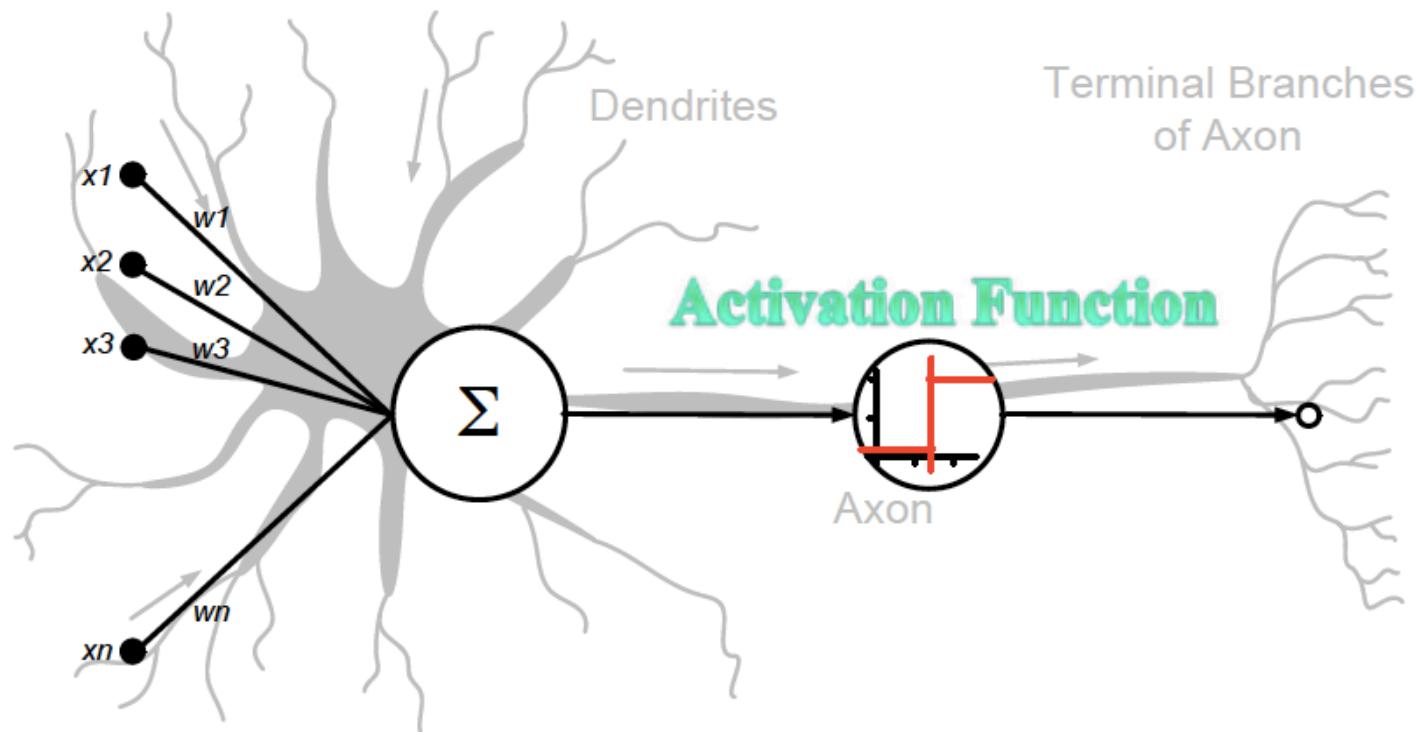
- Basics
- Character-level language model (example)

4. Question

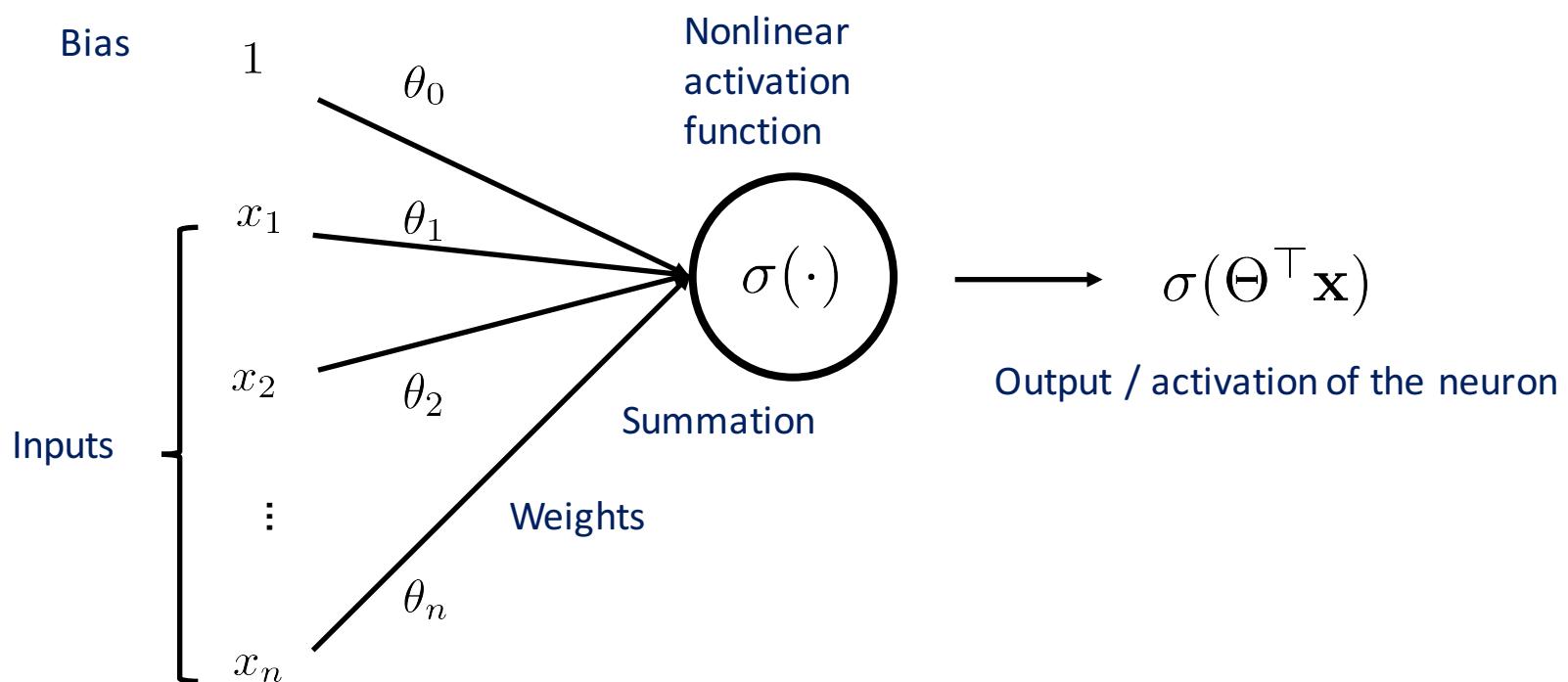
- Why is it difficult to train a deep neural network?

DNN: Neurons in the Brain

- Human brain is made up of 100 billion **neurons**
 - Neurons **receive** electric signals at the dendrites and **send** them to the axon
 - Dendrites can perform complex **non-linear** computations
 - Synapses are not a single weight but a **complex** non-linear dynamical system



- Artificial neural networks
 - A simplified version of biological neural network



DNN: The Brain vs. Artificial Neural Networks

- **Similarities**

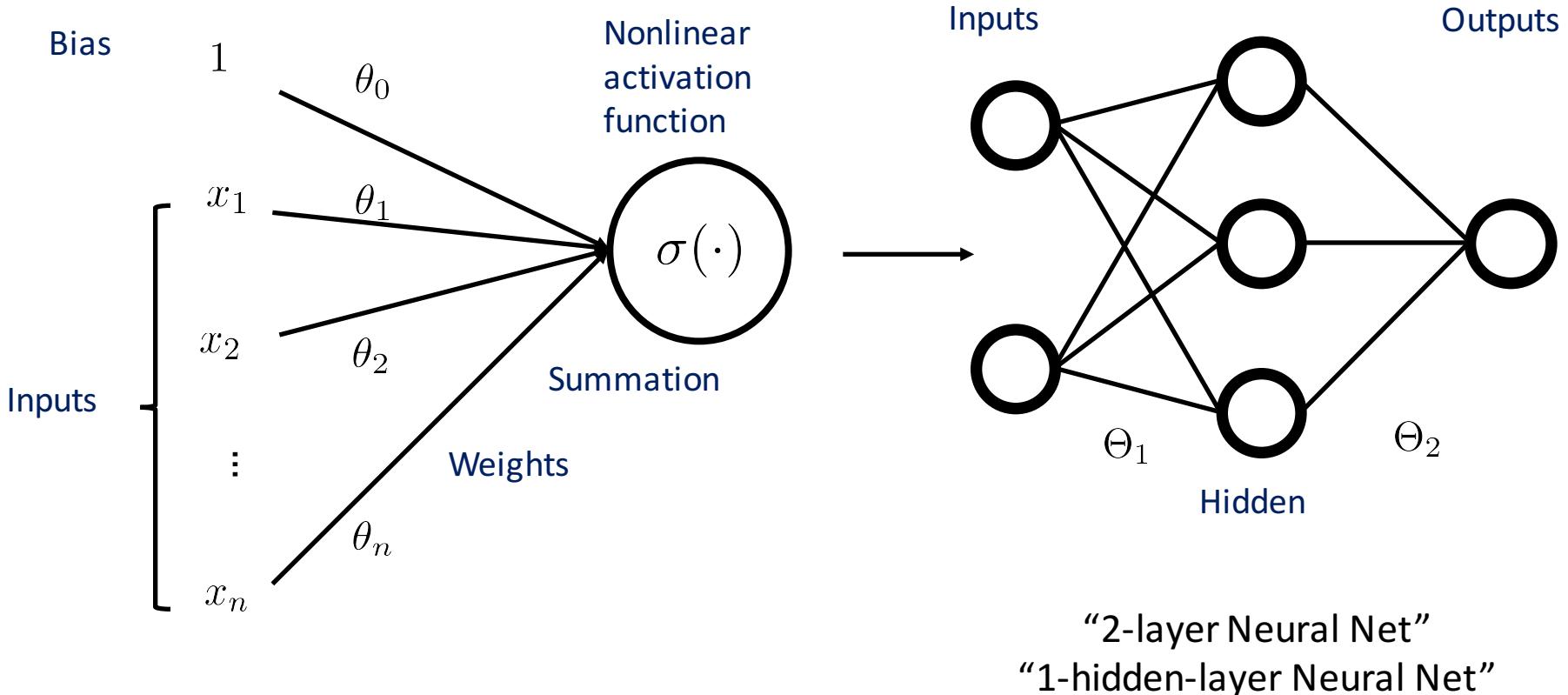
- Consists of neurons & connections between neurons
- Learning process = Update of **connections**
- Massive **parallel** processing

- **Differences**

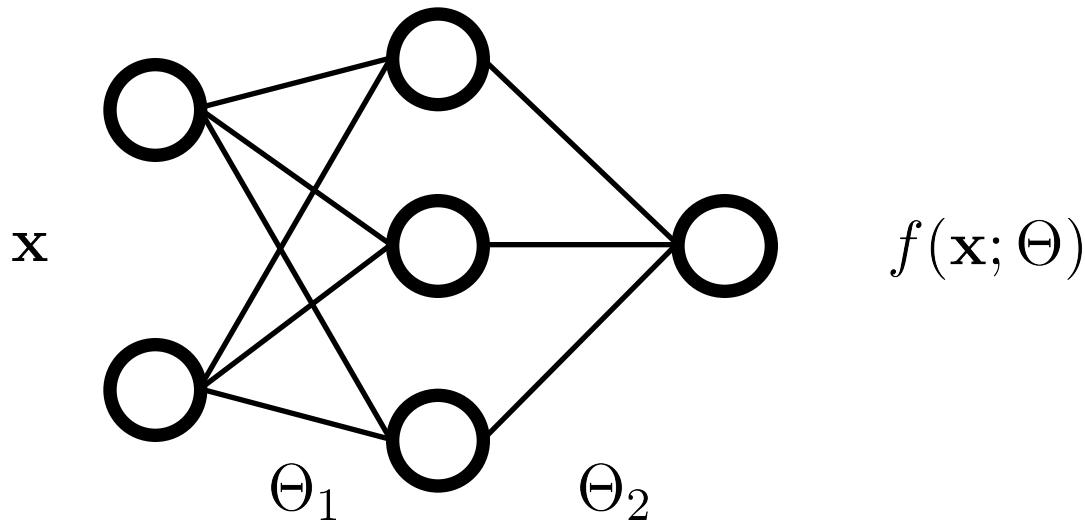
- Computation within neuron vastly **simplified**
- **Discrete** time steps
- Typically some of **supervised** learning with massive number of stimuli



- Deep neural networks
 - Neural network with more than 2 layers
 - Can model more **complex** functions



- Training dataset $\{(\mathbf{x}_1, y_1), \dots, (\mathbf{x}_n, y_n)\}$
 - \mathbf{x}_i : i^{th} input data
 - y_i : i^{th} target data (or label for classification)
- Neural network $f(\mathbf{x}; \Theta) \in \mathbb{R}$ parameterized by Θ



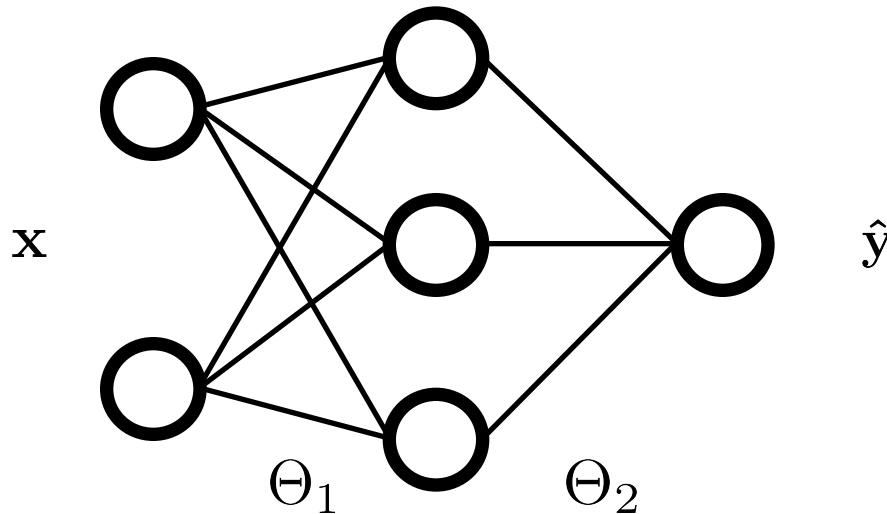
Next, forward propagation

DNN: Forward Propagation

- **Forward propagation:** calculate the output \hat{y} of the neural network

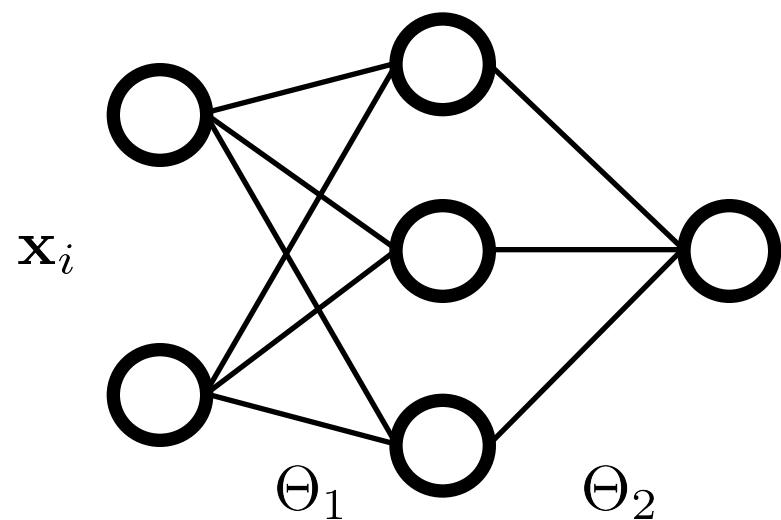
$$\hat{y} = \sigma \left(\Theta_k^\top \sigma \left(\Theta_{k-1}^\top \sigma \left(\cdots \sigma \left(\Theta_1^\top \mathbf{x} \right) \right) \right) \right)$$

where $\sigma(\cdot)$ is activation function (e.g., sigmoid function) and k is number of layers



DNN: Forward Propagation (Example)

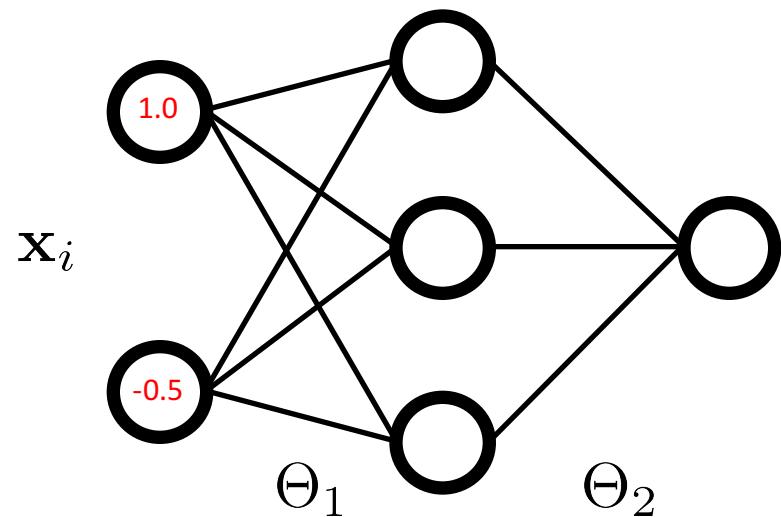
$$\mathbf{x}_i = \begin{pmatrix} 1.0 \\ -0.5 \end{pmatrix} \quad \Theta_1 = \begin{pmatrix} 1.2 & 2.1 & 1.5 \\ -0.3 & -0.7 & 0.3 \end{pmatrix} \quad \Theta_2 = \begin{pmatrix} -0.2 \\ 0.5 \\ 1.3 \end{pmatrix}$$



DNN: Forward Propagation (Example)

$$\mathbf{x}_i = \begin{pmatrix} 1.0 \\ -0.5 \end{pmatrix} \quad \Theta_1 = \begin{pmatrix} 1.2 & 2.1 & 1.5 \\ -0.3 & -0.7 & 0.3 \end{pmatrix} \quad \Theta_2 = \begin{pmatrix} -0.2 \\ 0.5 \\ 1.3 \end{pmatrix}$$

- Input data \mathbf{x}_i



DNN: Forward Propagation (Example)

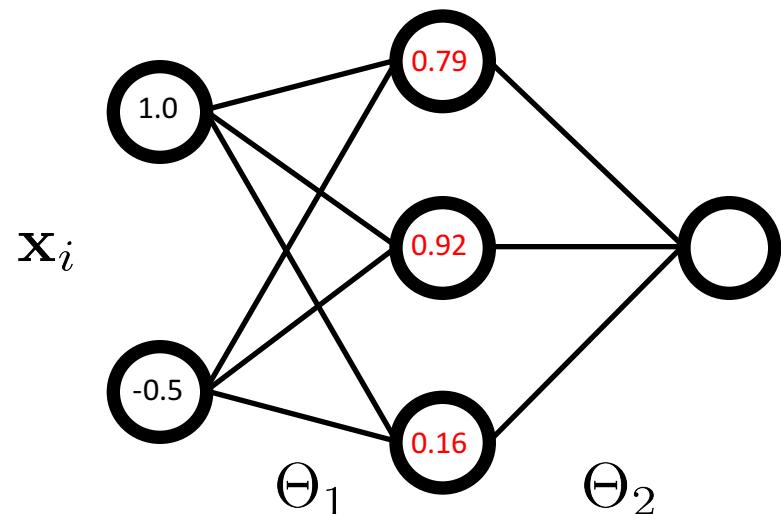
$$\mathbf{x}_i = \begin{pmatrix} 1.0 \\ -0.5 \end{pmatrix} \quad \Theta_1 = \begin{pmatrix} 1.2 & 2.1 & 1.5 \\ -0.3 & -0.7 & 0.3 \end{pmatrix} \quad \Theta_2 = \begin{pmatrix} -0.2 \\ 0.5 \\ 1.3 \end{pmatrix}$$

- Compute hidden units \mathbf{h}_1

$$\Theta_1^\top \mathbf{x}_i = \begin{pmatrix} 1.2 & -0.3 \\ 2.1 & -0.7 \\ -1.5 & 0.3 \end{pmatrix} \begin{pmatrix} 1.0 \\ -0.5 \end{pmatrix} = \begin{pmatrix} 1.35 \\ 2.45 \\ -1.65 \end{pmatrix}$$

$$\mathbf{h}_1 = \sigma(\Theta_1^\top \mathbf{x}_i) = \begin{pmatrix} \sigma(1.35) \\ \sigma(2.45) \\ \sigma(-1.65) \end{pmatrix} = \begin{pmatrix} 0.79 \\ 0.92 \\ 0.16 \end{pmatrix}$$

where $\sigma(x) = \frac{1}{1+e^{-x}}$



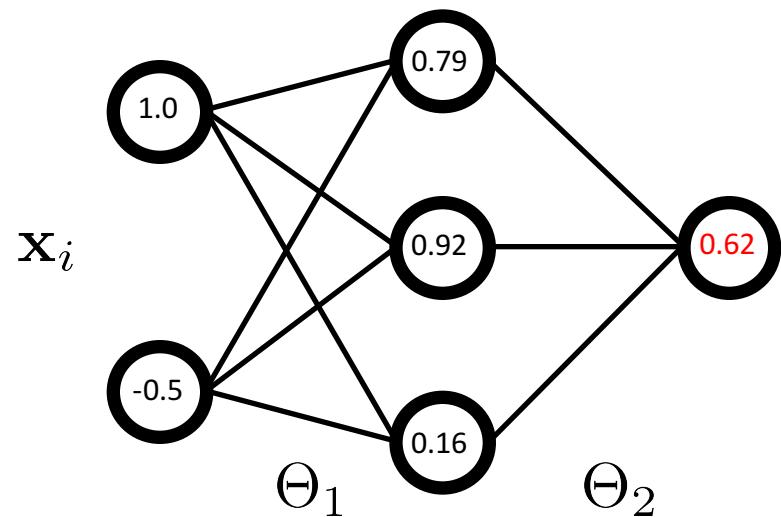
DNN: Forward Propagation (Example)

$$\mathbf{x}_i = \begin{pmatrix} 1.0 \\ -0.5 \end{pmatrix} \quad \Theta_1 = \begin{pmatrix} 1.2 & 2.1 & 1.5 \\ -0.3 & -0.7 & 0.3 \end{pmatrix} \quad \Theta_2 = \begin{pmatrix} -0.2 \\ 0.5 \\ 1.3 \end{pmatrix}$$

- Compute output \hat{y}_i

$$\Theta_2^\top \mathbf{h}_1 = (-0.2 \quad 0.5 \quad 1.3) \begin{pmatrix} 0.79 \\ 0.92 \\ 0.16 \end{pmatrix} = 0.51$$

$$\hat{y}_i = \sigma(\Theta_2^\top \mathbf{h}_1) = \sigma(0.51) = 0.62$$

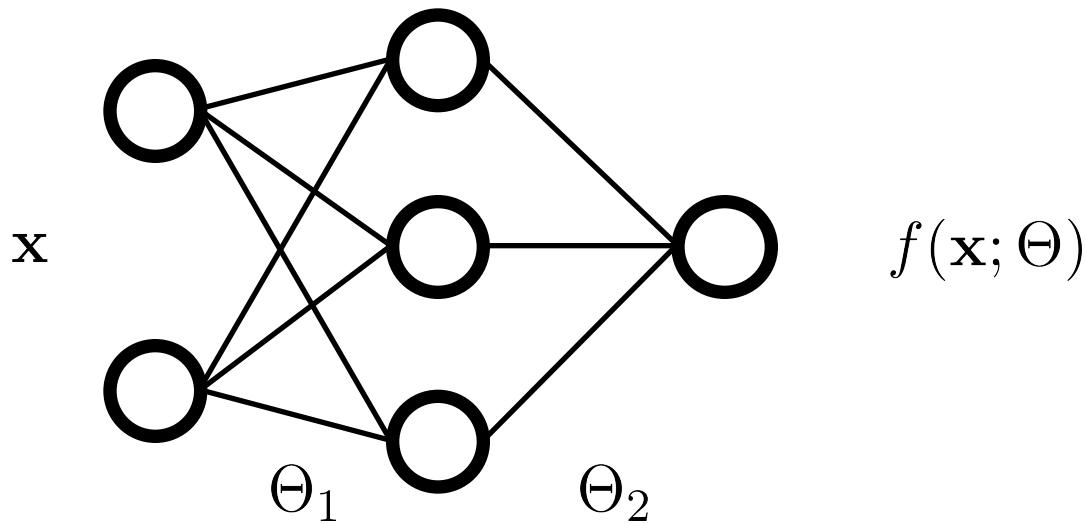


Next, training objective

- **Objective:** Find a parameter that minimizes the error (or empirical risk)

$$\min_{\Theta} \frac{1}{n} \sum_{i=1}^n \ell(f(\mathbf{x}_i; \Theta), y_i) := L(\Theta)$$

where $\ell(\cdot, \cdot)$ is a loss function e.g., MSE(Mean square error) or cross entropy



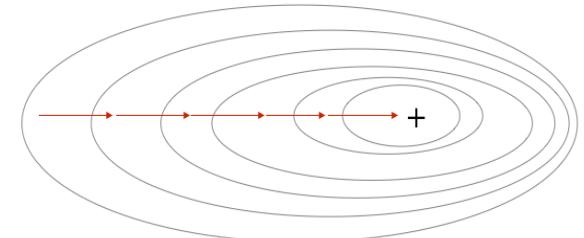
Next, how to optimize $L(\Theta)$?

- **Gradient descent (GD)** updates parameters iteratively to the gradient direction.

$$\Theta^{(t+1)} = \Theta^{(t)} - \gamma \nabla L(\Theta^{(t)})$$

↑ parameters ↑ loss function

learning rate $\nabla L(\Theta^{(t)}) := \frac{1}{n} \sum_{i=1}^n \nabla \ell(\mathbf{x}_i, y_i; \Theta^{(t)})$



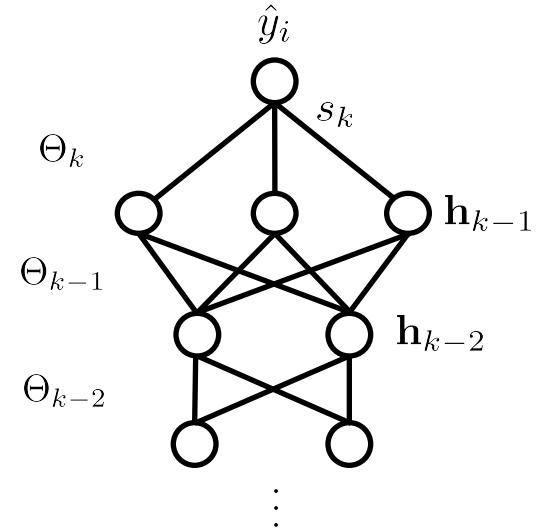
- **Backpropagation**

- First adjust the **last layer** weights Θ_k
- Propagate error back to each previous layers
- Adjust **previous layer** weights $\Theta_{k-1}, \Theta_{k-2}, \dots, \Theta_1$

Next, backpropagation in details

DNN: Backpropagation

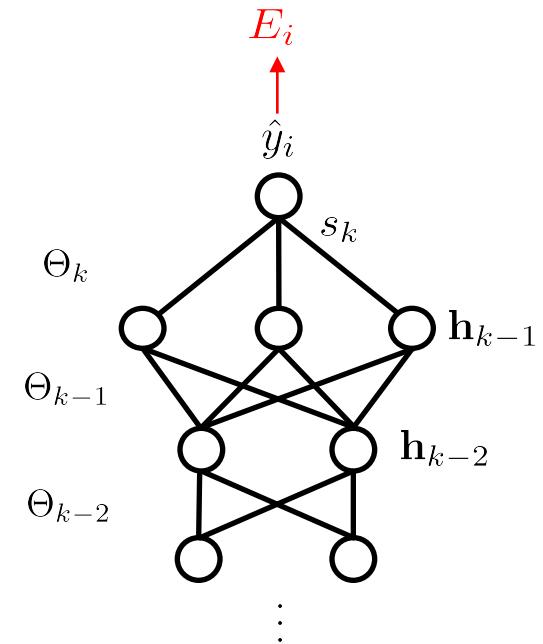
- Consider the input (\mathbf{x}_i, y_i)
- Forward propagation to compute output $\hat{y}_i = f(\mathbf{x}_i; \Theta)$
- i^{th} layer intermediate output $s_i = \Theta_i^\top \mathbf{h}_{i-1}$



DNN: Backpropagation

- Consider the input (\mathbf{x}_i, y_i)
- Forward propagation to compute output $\hat{y}_i = f(\mathbf{x}_i; \Theta)$
- i^{th} layer intermediate output $s_i = \Theta_i^\top \mathbf{h}_{i-1}$
- **Compute error** $\ell(\hat{y}_i, y_i)$ (where $\ell(\cdot, \cdot)$ is MSE loss)

$$\ell(\hat{y}_i, y_i) = \frac{1}{2}(y_i - \hat{y}_i)^2 := E_i$$



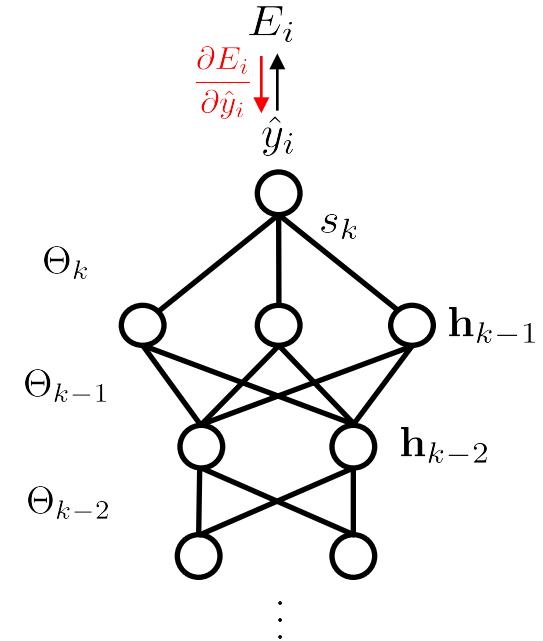
DNN: Backpropagation

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$$\ell(\hat{y}_i, y_i) = \frac{1}{2}(y_i - \hat{y}_i)^2 := E_i$$

- **Compute derivative of E_i with respect to \hat{y}_i**

$$\frac{\partial E_i}{\partial \hat{y}_i} = \frac{\partial}{\partial \hat{y}_i} \frac{1}{2}(y_i - \hat{y}_i)^2 = -(y_i - \hat{y}_i)$$



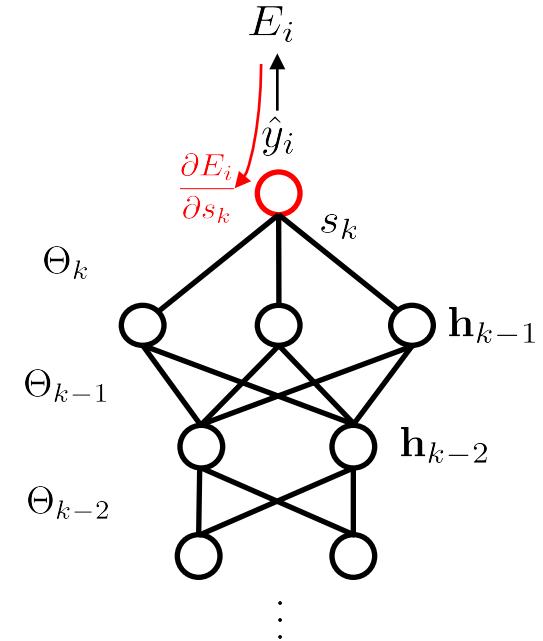
DNN: Backpropagation

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$$\ell(\hat{y}_i, y_i) = \frac{1}{2}(y_i - \hat{y}_i)^2 := E_i$$

- **Compute derivative of E_i with respect to s_k**

$$\frac{\partial E_i}{\partial s_k} = \frac{\partial E_i}{\partial \hat{y}_i} \frac{\partial \hat{y}_i}{\partial s_k} = \frac{\partial E_i}{\partial \hat{y}_i} \frac{\partial}{\partial s_k} \sigma(s_k) = (\hat{y}_i - y_i) \sigma'(s_k)$$



DNN: Backpropagation

- Consider the input (\mathbf{x}_i, y_i)
- Forward propagation to compute output $\hat{y}_i = f(\mathbf{x}_i; \Theta)$
- i^{th} layer intermediate output $s_i = \Theta_i^\top \mathbf{h}_{i-1}$
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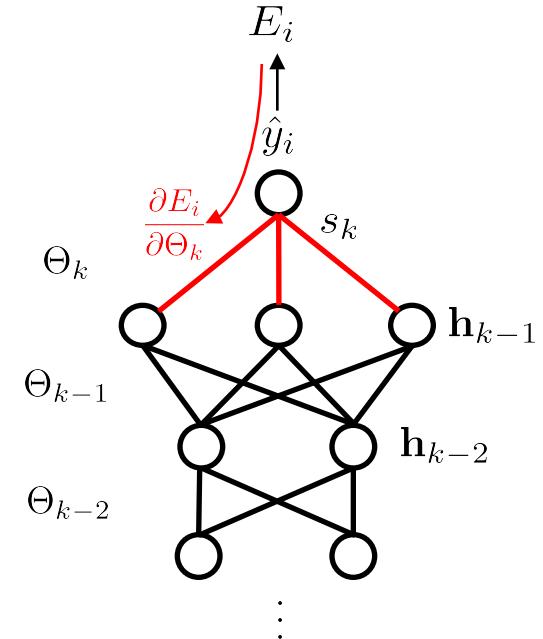
- **Compute derivative of E_i with respect to Θ_k**

$$\frac{\partial E_i}{\partial \Theta_k} = \frac{\partial E_i}{\partial \hat{y}_i} \frac{\partial \hat{y}_i}{\partial s_k} \frac{\partial s_k}{\partial \Theta_k} = \frac{\partial E_i}{\partial \hat{y}_i} \frac{\partial \hat{y}_i}{\partial s_k} \frac{\partial}{\partial \Theta_k} (\Theta_k^\top \mathbf{h}_{k-1}) = (\hat{y}_i - y_i) \sigma'(s_k) \mathbf{h}_{k-1}$$

- Parameter update rule

learning rate

$$\Theta_k \leftarrow \Theta_k - \gamma \frac{\partial E_i}{\partial \Theta_k}$$



DNN: Backpropagation

- Consider the input (\mathbf{x}_i, y_i)
- Forward propagation to compute output $\hat{y}_i = f(\mathbf{x}_i; \Theta)$
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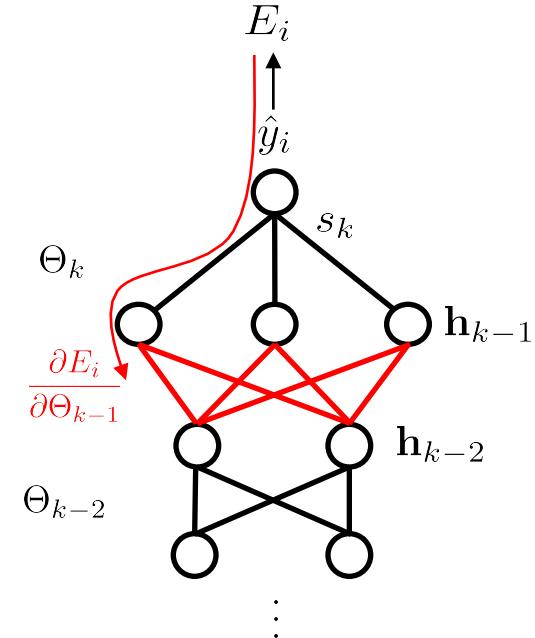
- Compute derivative of E_i with respect to Θ_{k-1}

$$\frac{\partial E_i}{\partial \Theta_{k-1}} = \frac{\partial E_i}{\partial \hat{y}_i} \frac{\partial \hat{y}_i}{\partial s_k} \frac{\partial s_k}{\partial \mathbf{h}_{k-1}} \frac{\partial \mathbf{h}_{k-1}}{\partial \mathbf{s}_{k-1}} \frac{\partial \mathbf{s}_{k-1}}{\partial \Theta_{k-1}} = \frac{\partial E_i}{\partial \hat{y}_i} \frac{\partial \hat{y}_i}{\partial s_k} \frac{\partial s_k}{\partial \mathbf{h}_{k-1}} \frac{\partial \mathbf{h}_{k-1}}{\partial \mathbf{s}_{k-1}} \frac{\partial}{\partial \Theta_{k-1}} (\Theta_{k-1}^\top \mathbf{h}_{k-2})$$

- Parameter update rule

learning rate

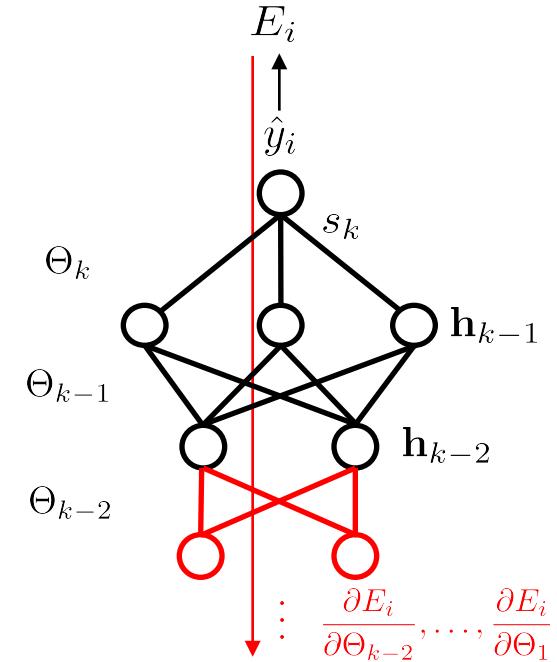
$$\Theta_{k-1} \leftarrow \Theta_{k-1} - \gamma \frac{\partial E_i}{\partial \Theta_{k-1}}$$



DNN: Backpropagation

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$$\ell(\hat{y}_i, y_i) = \frac{1}{2}(y_i - \hat{y}_i)^2 := E_i$$



- Similarly, we can compute gradients with respect to $\Theta_{k-2}, \dots, \Theta_1$
 - And update using the same update rule

DNN: Backpropagation (Example)

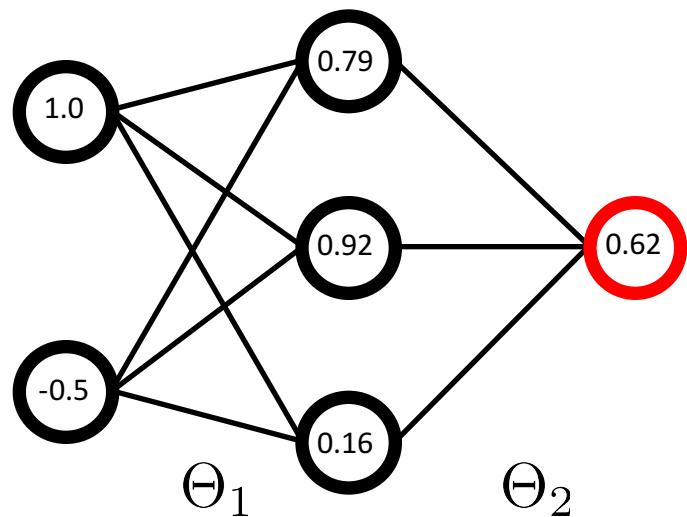
$$\mathbf{x}_i = \begin{pmatrix} 1.0 \\ -0.5 \end{pmatrix} \quad y_i = (1.0) \quad \Theta_1 = \begin{pmatrix} 1.2 & 2.1 & 1.5 \\ -0.3 & -0.7 & 0.3 \end{pmatrix} \quad \Theta_2 = \begin{pmatrix} -0.2 \\ 0.5 \\ 1.3 \end{pmatrix}$$

- Compute the error $\ell(\hat{y}_i, y_i)$

$$\ell(\hat{y}_i, y_i) = \frac{1}{2}(y_i - \hat{y}_i)^2 = 0.072$$

- Compute $\frac{\partial E_i}{\partial \hat{y}_i}$

$$\frac{\partial E_i}{\partial \hat{y}_i} = (\hat{y}_i - y_i) = -0.38$$



DNN: Backpropagation (Example)

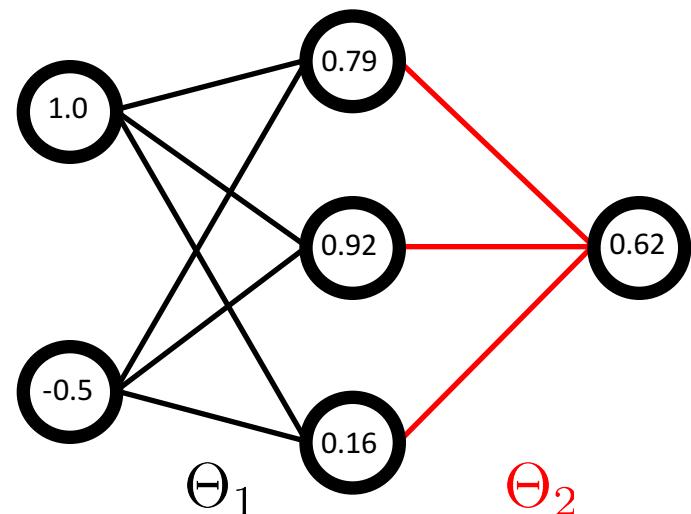
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- Compute $\frac{\partial E_i}{\partial \Theta_2}$

$$\frac{\partial E_i}{\partial \Theta_2} = (\hat{y}_i - y_i)\sigma'(s_2)\mathbf{h}_1 = \begin{pmatrix} 0.02 \\ -0.05 \\ -0.12 \end{pmatrix}$$

- Update Θ_2 with $\gamma = 1$

$$\Theta_2 = \begin{pmatrix} -0.2 \\ 0.5 \\ 1.3 \end{pmatrix} - 1 \begin{pmatrix} 0.02 \\ -0.05 \\ -0.12 \end{pmatrix} = \begin{pmatrix} -0.22 \\ 0.55 \\ 1.42 \end{pmatrix}$$



DNN: Backpropagation (Example)

$$\mathbf{x}_i = \begin{pmatrix} 1.0 \\ -0.5 \end{pmatrix} \quad y_i = (1.0) \quad \Theta_1 = \begin{pmatrix} 1.2 & 2.1 & 1.5 \\ -0.3 & -0.7 & 0.3 \end{pmatrix} \quad \Theta_2 = \begin{pmatrix} -0.2 \\ 0.5 \\ 1.3 \end{pmatrix}$$

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- Similarly, we can update Θ_1

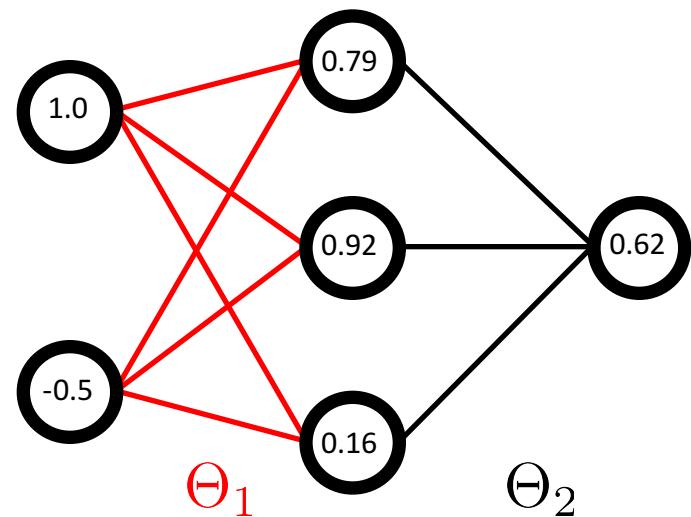


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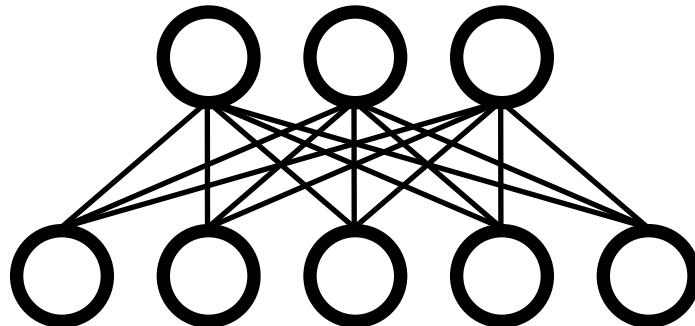
- Basics
- Character-level language model (example)

4. Question

- Why is it difficult to train a deep neural network?

CNN: Drawbacks of Fully-Connected DNN

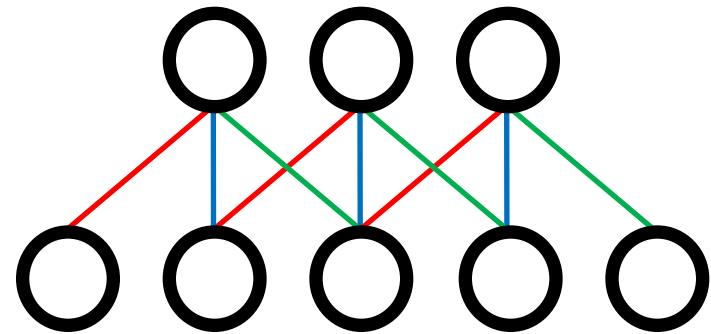
- Previous DNNs use fully-connected layers
 - Connect **all** the neurons between the layers



- Drawbacks
 - **(-)** Large number of parameters
 - Easy to be over-fitted
 - Large memory consumption
 - **(-)** Does not enforce any structure, e.g., *local information*
 - In many applications, local features are important, e.g., images, language, etc.

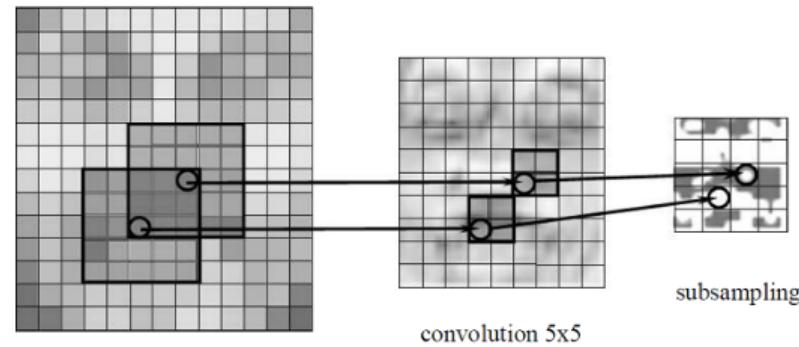
- **Weight sharing and local connectivity** (convolution)

- Use multiple filters convolve over inputs
- (+) Reduce the number of **parameters** (less over-fitting)
- (+) Learn **local** features
- (+) Translation invariance



- **Pooling (or subsampling)**

- Make the **representations smaller**
- (+) Reduce number of parameters and computation



CNN: Weight Sharing and Translation Invariance

- **Weight sharing**

- Apply same weights over the different spatial regions
- One can achieve **translation invariance** (not perfect though)

1 <small>$\times 1$</small>	1 <small>$\times 0$</small>	1 <small>$\times 1$</small>	0	0
0 <small>$\times 0$</small>	1 <small>$\times 1$</small>	1 <small>$\times 0$</small>	1	0
0 <small>$\times 1$</small>	0 <small>$\times 0$</small>	1 <small>$\times 1$</small>	1	1
0	0	1	1	0
0	1	1	0	0

Image

4		

Convolved
Feature

CNN: Weight Sharing and Translation Invariance

- **Weight sharing**

- Apply same weights over the different spatial regions
- One can achieve **translation invariance**

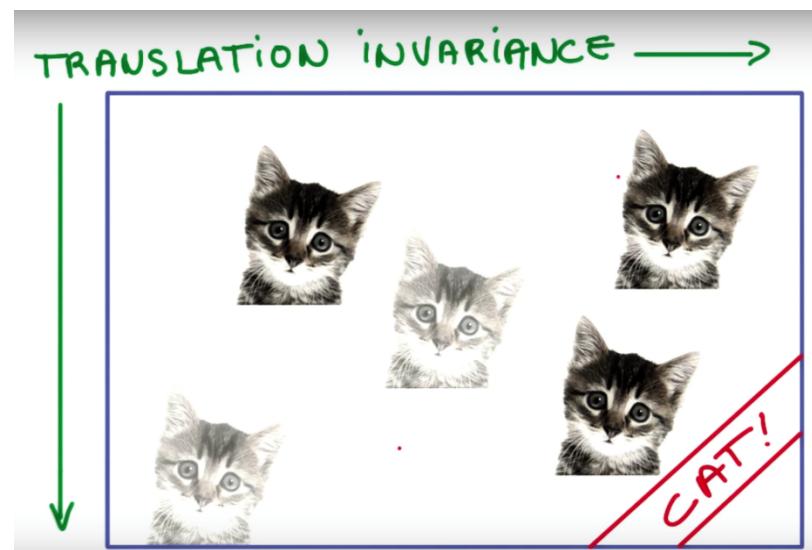
1 _{x1}	1 _{x0}	1 _{x1}	0	0
0 _{x0}	1 _{x1}	1 _{x0}	1	0
0 _{x1}	0 _{x0}	1 _{x1}	1	1
0	0	1	1	0
0	1	1	0	0

Image

Convolved Feature

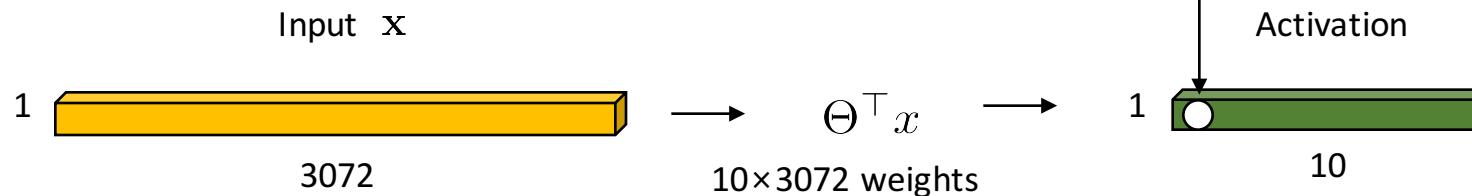
- **Translation invariance**

- When input is changed spatially (translated or shifted), the corresponding output to recognize the object should not be changed
- CNN can produce the same output even though the input image is shifted due to weight sharing



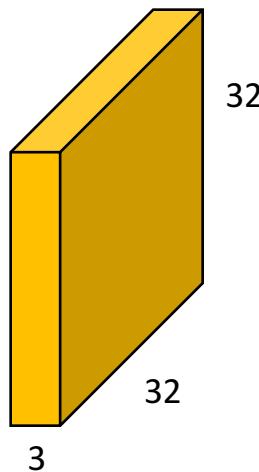
Fully-connected layer

- $32 \times 32 \times 3$ image \rightarrow stretch to 3072×1



Convolution layer

$32 \times 32 \times 3$ image



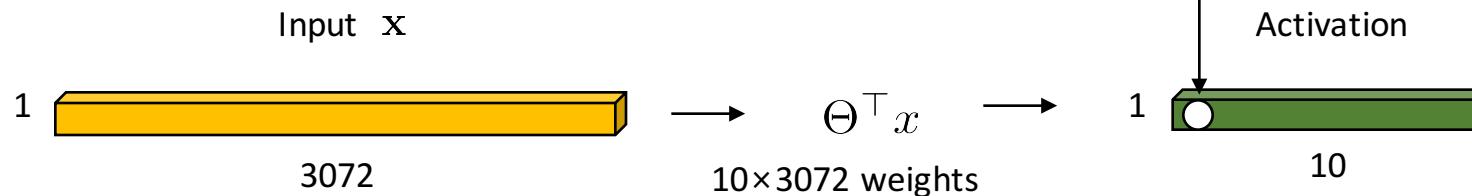
5 \times 5 \times 3 filter
(equivalent to 1 \times 75 weights for FC layer)



Convolve the filter with the image
i.e., “slide over the image spatially,
computing dot products”

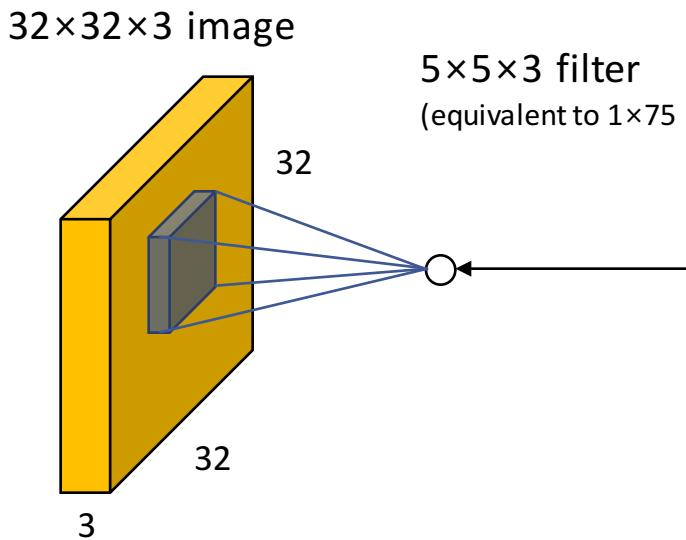
Fully-connected layer

- $32 \times 32 \times 3$ image \rightarrow stretch to 3072×1



The result of taking a dot product between a row of Θ^\top and the input

Convolution layer

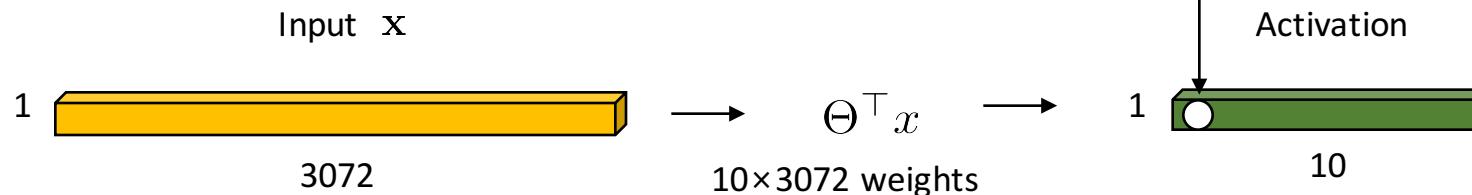


The result of taking a dot product between the filter and a small $5 \times 5 \times 3$ chunk of the image
(i.e., $5 \times 5 \times 3 = 75$ -dimensional dot product + bias)

$$\Theta^\top x + b$$

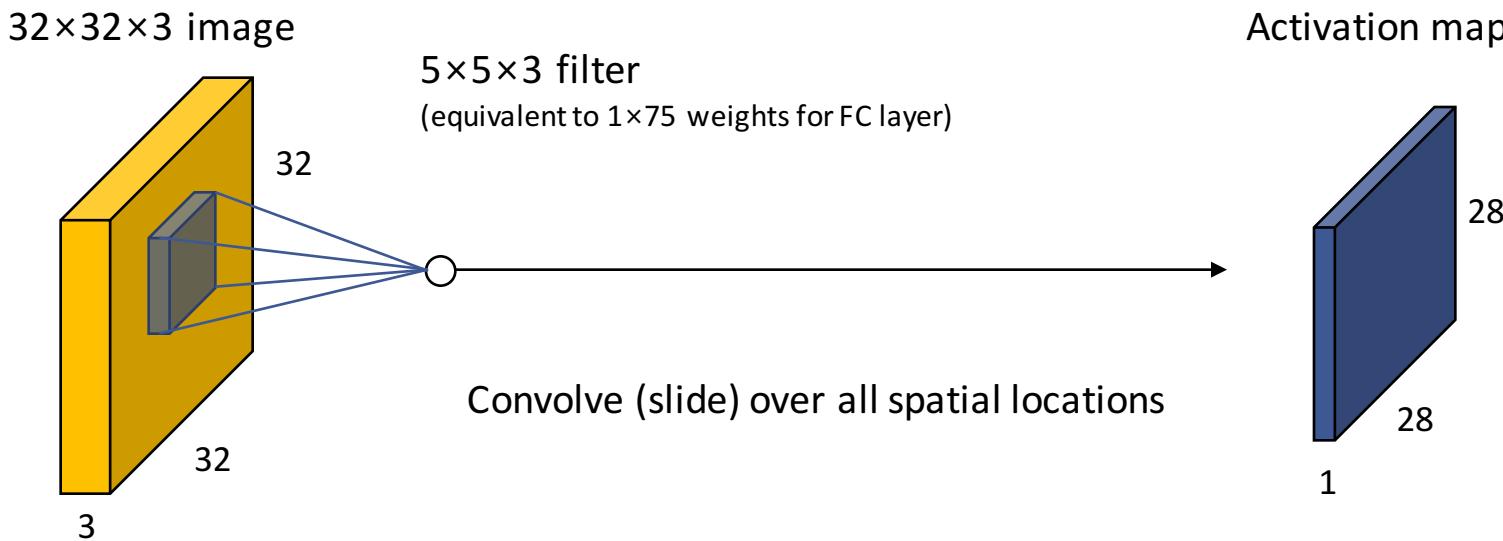
Fully-connected layer

- $32 \times 32 \times 3$ image \rightarrow stretch to 3072×1



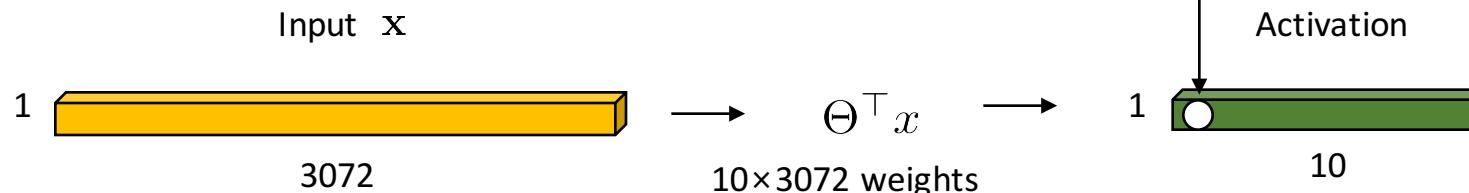
The result of taking a dot product between a row of Θ^\top and the input

Convolution layer



Fully-connected layer

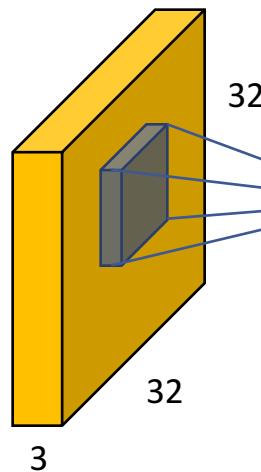
- $32 \times 32 \times 3$ image \rightarrow stretch to 3072×1



The result of taking a dot product between a row of Θ^\top and the input

Convolution layer

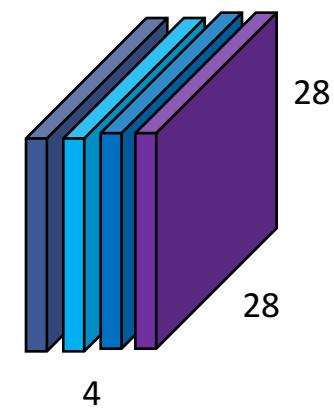
$32 \times 32 \times 3$ image



If there are **four $5 \times 5 \times 3$ filters**

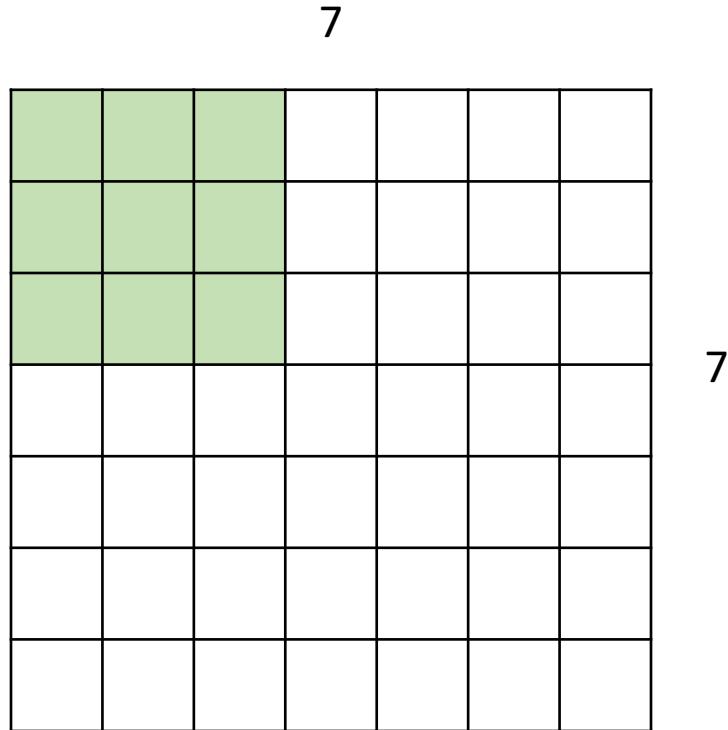
Convolve (slide) over all spatial locations

4 separate activation maps



CNN: An Example

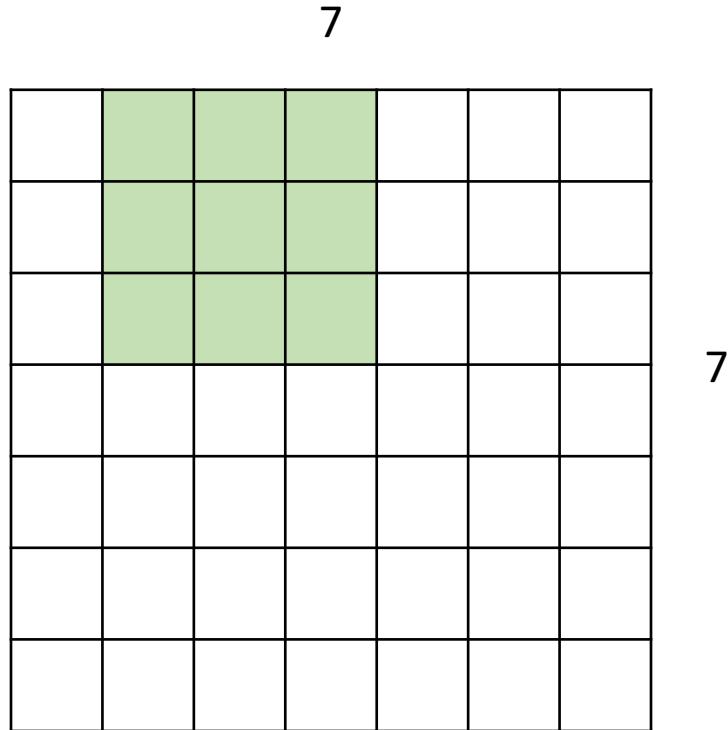
- Closer look at spatial dimensions



7×7 input (spatially)
Assume 3×3 filter
Applied with **stride 1**

CNN: An Example

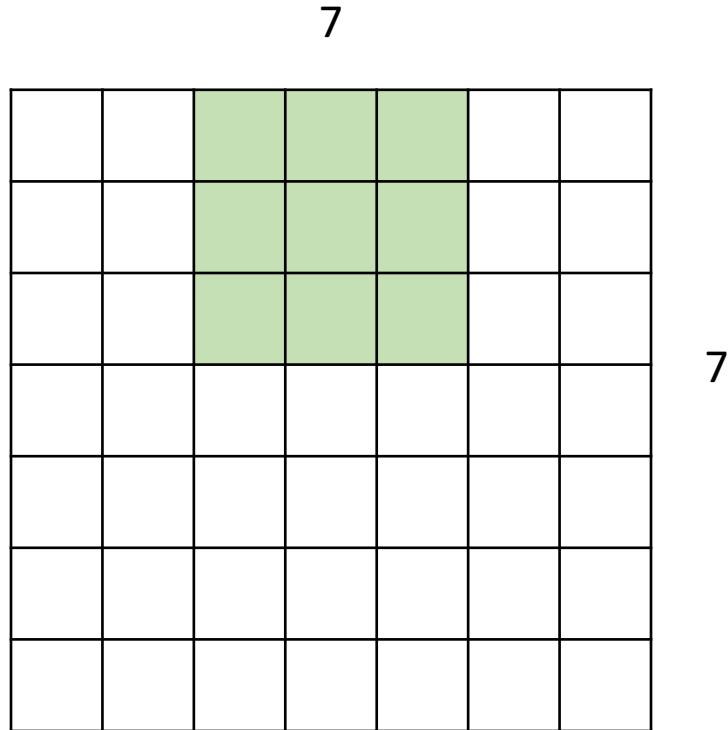
- Closer look at spatial dimensions



7×7 input (spatially)
Assume 3×3 filter
Applied with **stride 1**

CNN: An Example

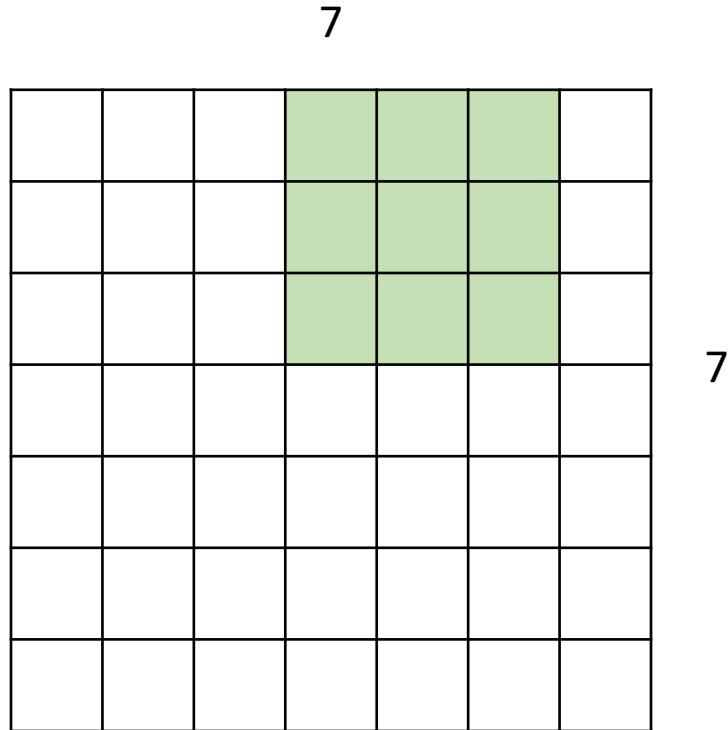
- Closer look at spatial dimensions



7×7 input (spatially)
Assume 3×3 filter
Applied with **stride 1**

CNN: An Example

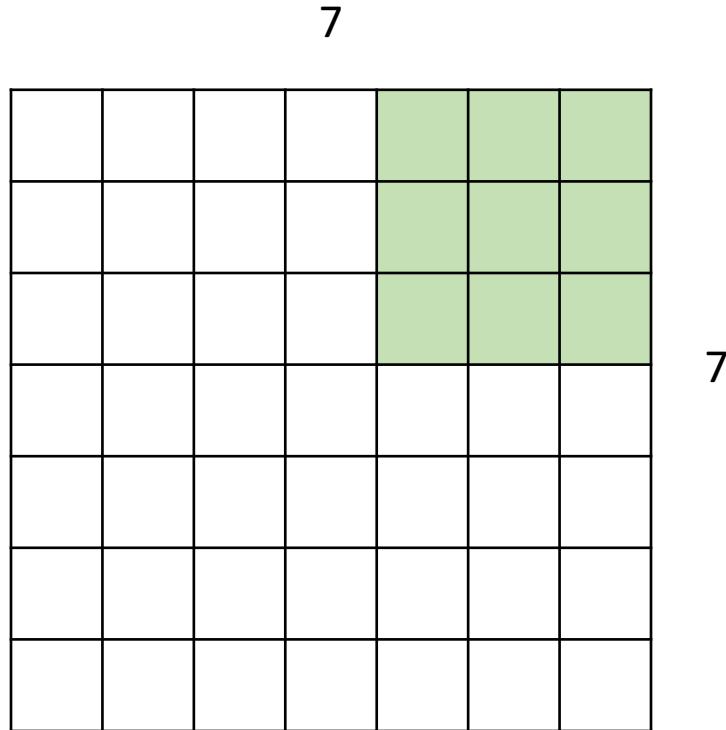
- Closer look at spatial dimensions



7×7 input (spatially)
Assume 3×3 filter
Applied with **stride 1**

CNN: An Example

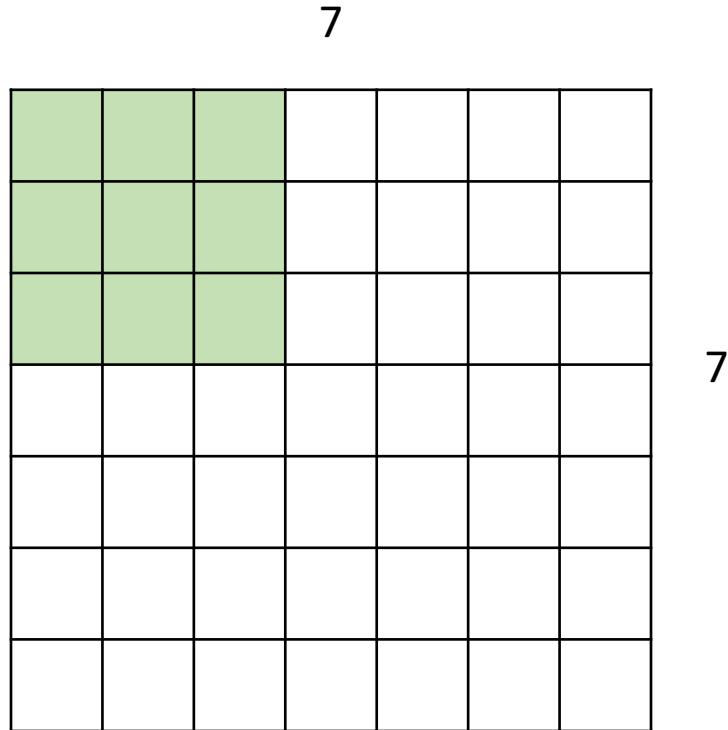
- Closer look at spatial dimensions



7×7 input (spatially)
Assume 3×3 filter
Applied with **stride 1**
→ 5×5 output

CNN: An Example

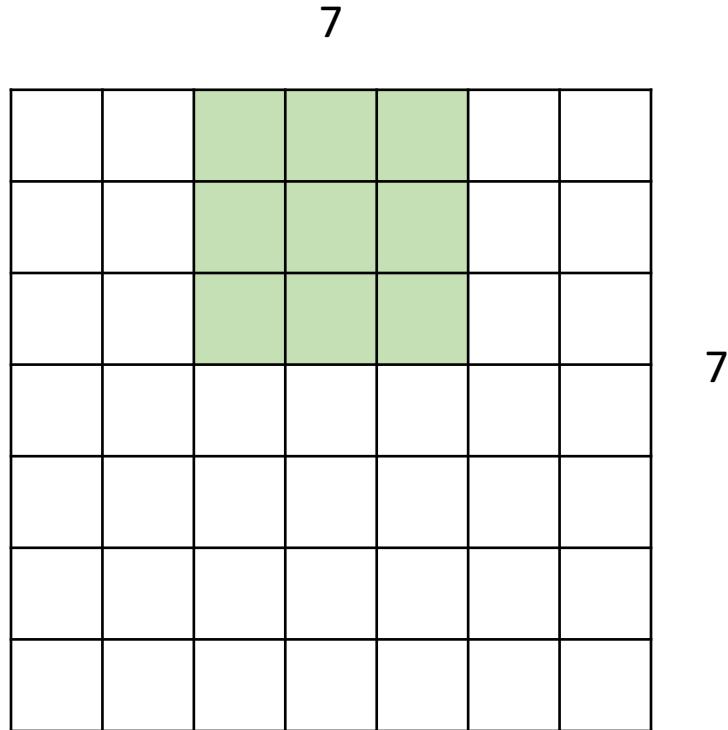
- Closer look at spatial dimensions



7×7 input (spatially)
Assume 3×3 filter
Applied with **stride 2**

CNN: An Example

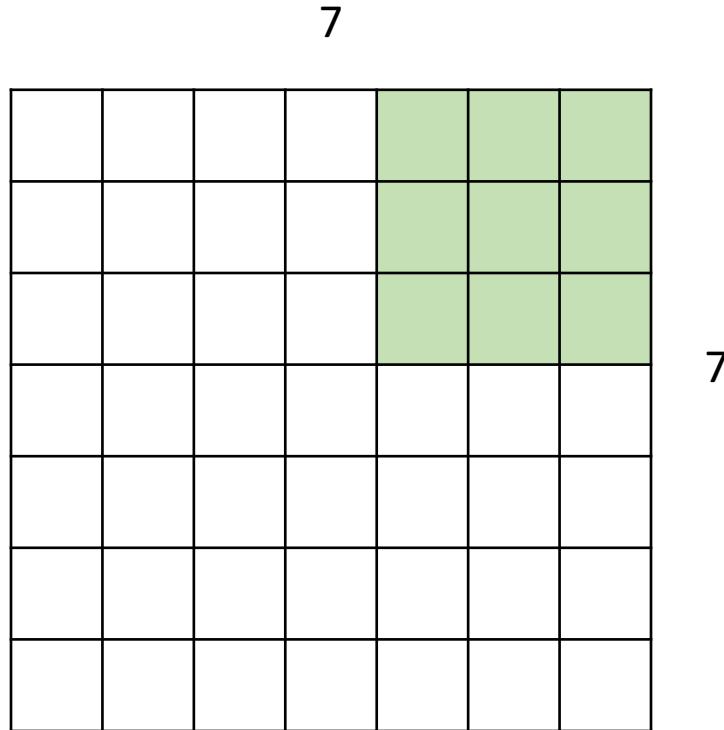
- Closer look at spatial dimensions



7×7 input (spatially)
Assume 3×3 filter
Applied with **stride 2**

CNN: An Example

- Closer look at spatial dimensions



7×7 input (spatially)

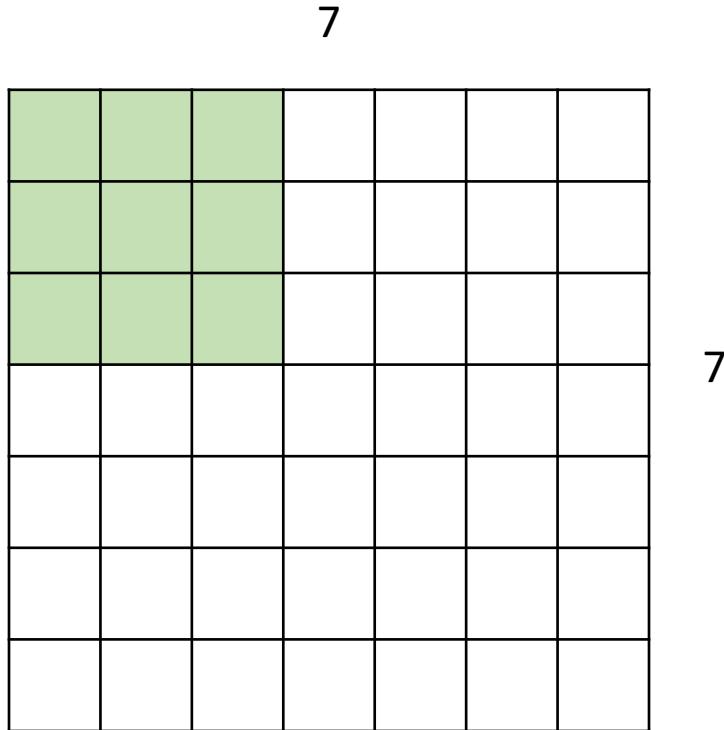
Assume 3×3 filter

Applied with **stride 2**

→ **3×3 output**

CNN: An Example

- Closer look at spatial dimensions



7×7 input (spatially)
Assume 3×3 filter
Applied with **stride 3** ?

Doesn't fit!
Cannot apply 3×3 filter on
 7×7 input with stride 3

CNN: An Example

- In practice: Common to **zero pad** the border
 - Used to control the output filter size

0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0	0	0	0	0	0	0	0	0

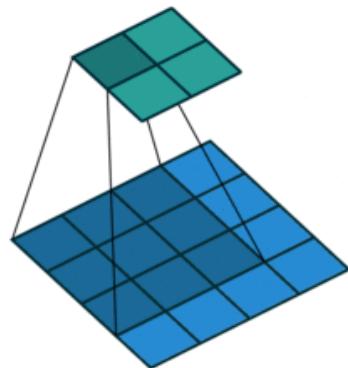
9

7×7 input (spatially)
Zero pad 1 pixel border
Assume 3×3 filter
Applied with **stride 3**

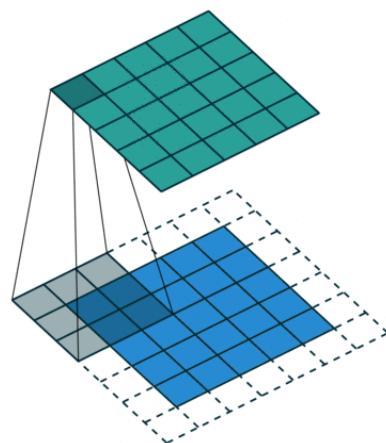
→ 3×3 output

9

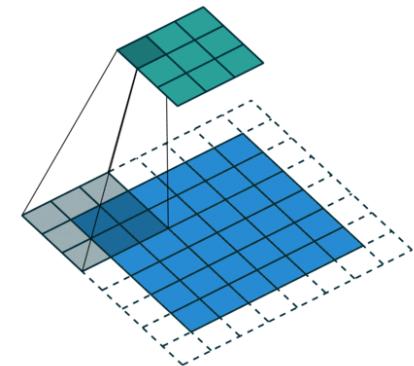
CNN: An Example (Animation)



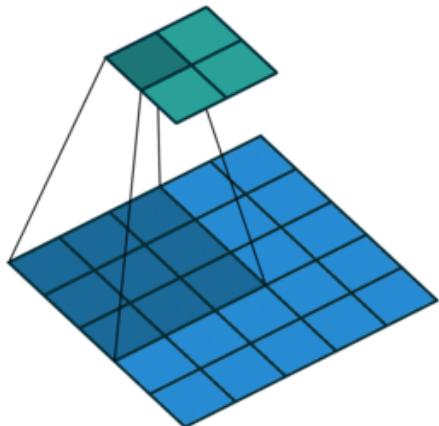
No padding, stride 1



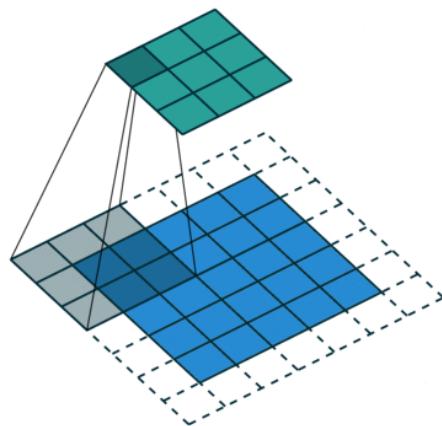
Padding 1, stride 1



Padding 1, stride 2 (odd)



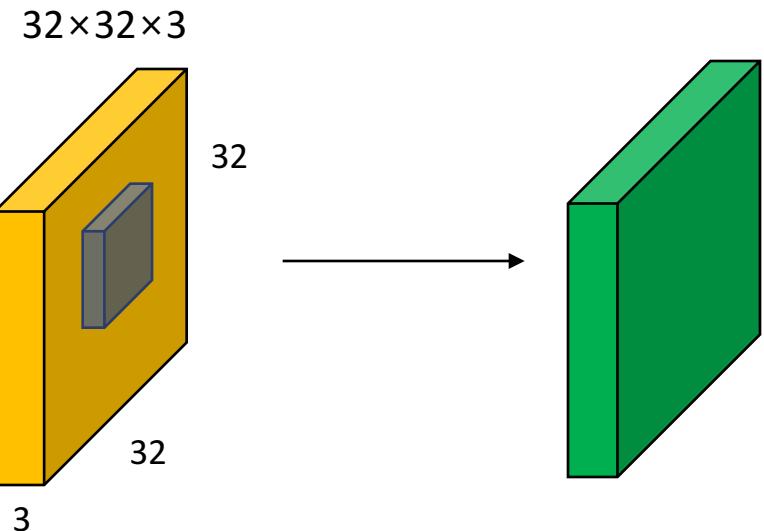
No padding, stride 2



Padding 1, stride 2

CNN: An Example

- Input volume : $32 \times 32 \times 3$
- 10 5×5 filters with stride 1, pad 2



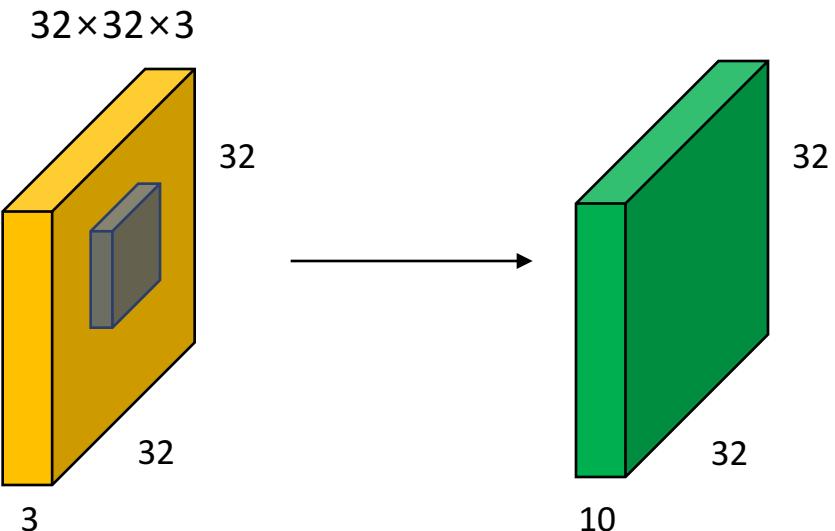
Output volume size = ?

CNN: An Example

- Input volume : $32 \times 32 \times 3$
- 10 5×5 filters with stride 1 , pad 2

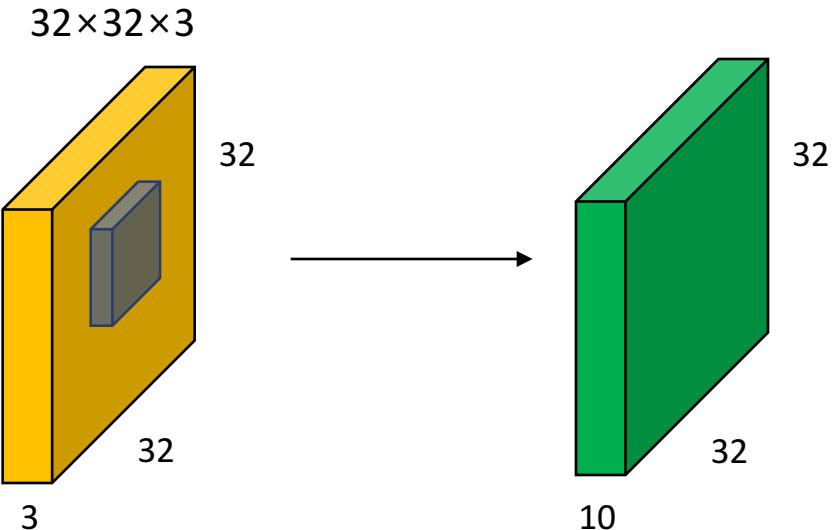
Output volume size = ?

- $(32 + 2 \times 2 - 5)/1 + 1 = 32$ spatially
- $= > 32 \times 32 \times 10$



CNN: An Example

- Input volume : $32 \times 32 \times 3$
- 10 5×5 filters with stride 1, pad 2



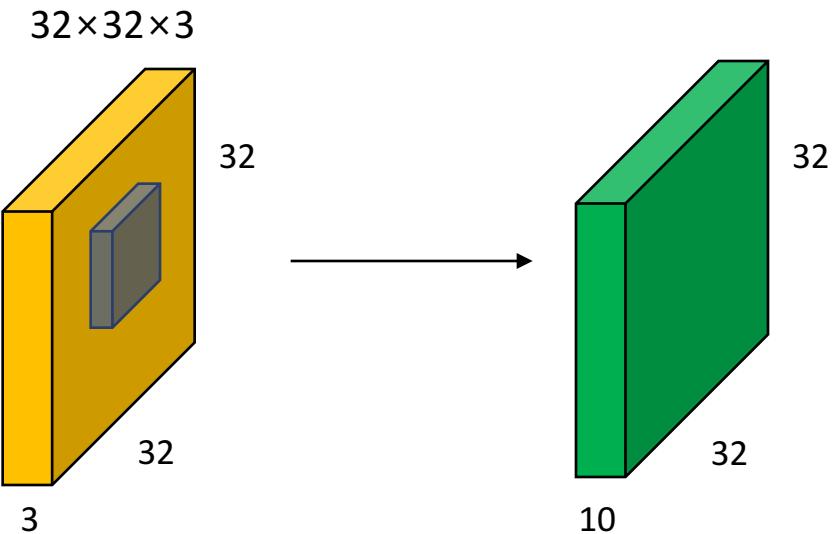
Number of parameters in this layer?

CNN: An Example

- Input volume : $32 \times 32 \times 3$
- 10 5×5 filters with stride 1 , pad 2

Number of parameters in this layer?

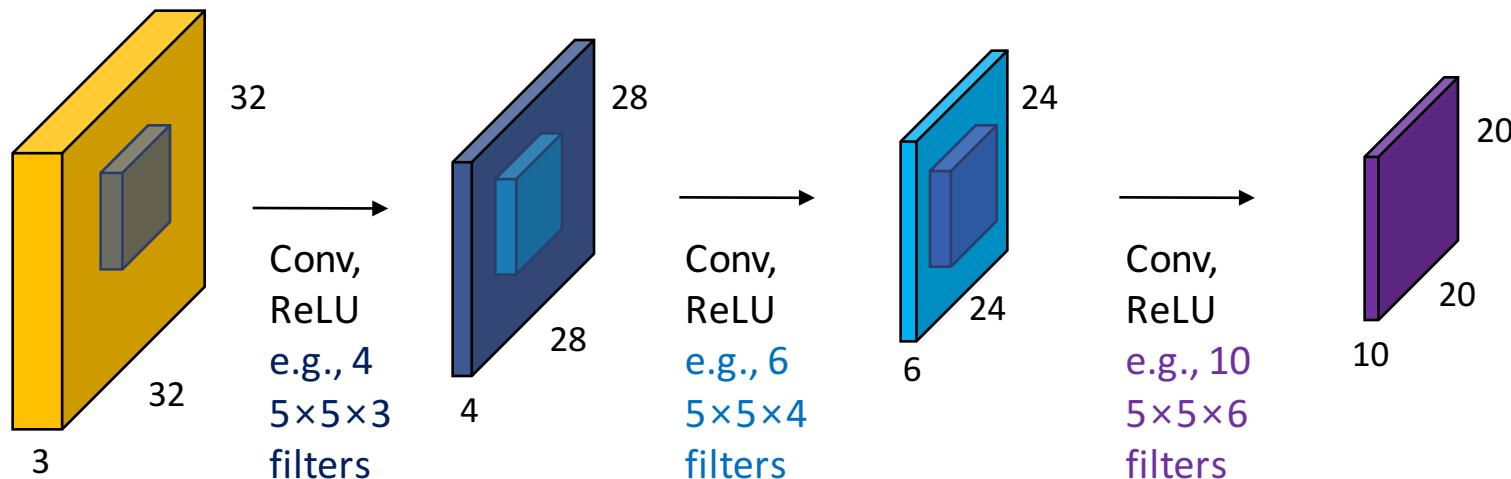
- Each filter has $5 \times 5 \times 3 + 1 = 76$ params (+1 for bias)
- $= > 76 \times 10 = 760$



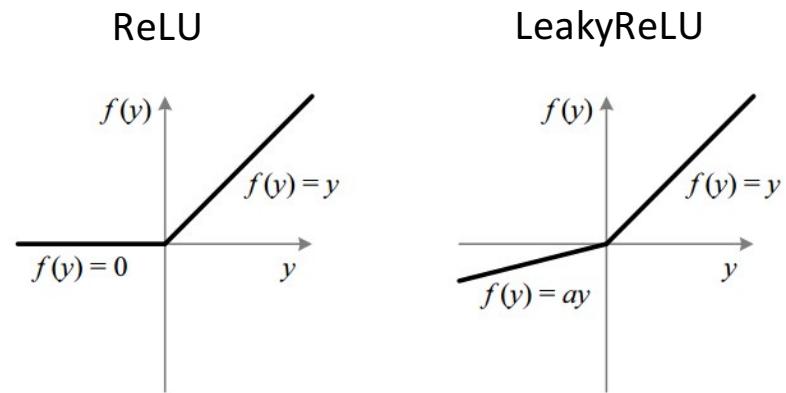
CNN: Convolution

- **ConvNet** is a sequence of Convolutional layers, followed by non-linearity

$32 \times 32 \times 3$ image



- Choices of other **non-linearity**
 - Tanh/Sigmoid
 - ReLU [Nair et al., 2010]
 - Leaky ReLU [Maas et. al., 2013]

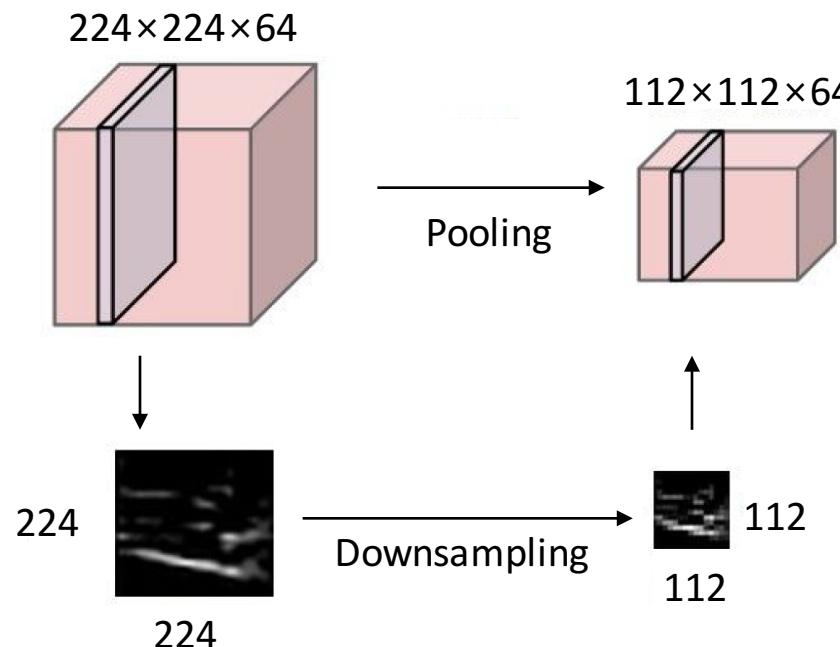


*reference: <http://cs231n.stanford.edu/2017/>

*Image source: <https://towardsdatascience.com/activation-functions-neural-networks-1cbd9f8d91d6> 53

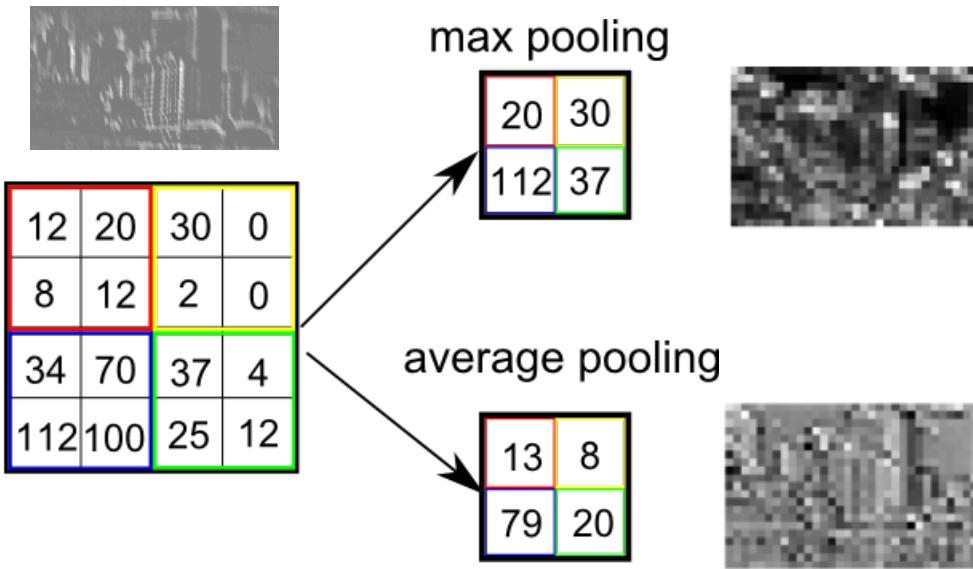
- **Pooling layer**

- Makes the **representations smaller** and more **manageable**
- Operates over each activation map independently
- Enhance **translation invariance** (invariance to small transformation)
- Larger receptive fields (see more of input)
- Regularization effect



CNN: Pooling

- Max pooling and average pooling
 - With 2×2 filters and stride 2



input								
0.88	0.44	0.14	0.16	0.37	0.77	0.96	0.27	
0.19	0.45	0.57	0.16	0.63	0.29	0.71	0.70	
0.66	0.26	0.82	0.64	0.54	0.73	0.59	0.26	
0.85	0.34	0.76	0.84	0.29	0.75	0.62	0.25	
0.32	0.74	0.21	0.39	0.34	0.03	0.33	0.48	
0.20	0.14	0.16	0.13	0.73	0.65	0.96	0.32	
0.19	0.69	0.09	0.86	0.88	0.07	0.01	0.48	
0.83	0.24	0.97	0.04	0.24	0.35	0.50	0.91	

ROI pooling

- Another kind of pooling layers are also used
 - e.g. stochastic pooling, ROI pooling

*source:

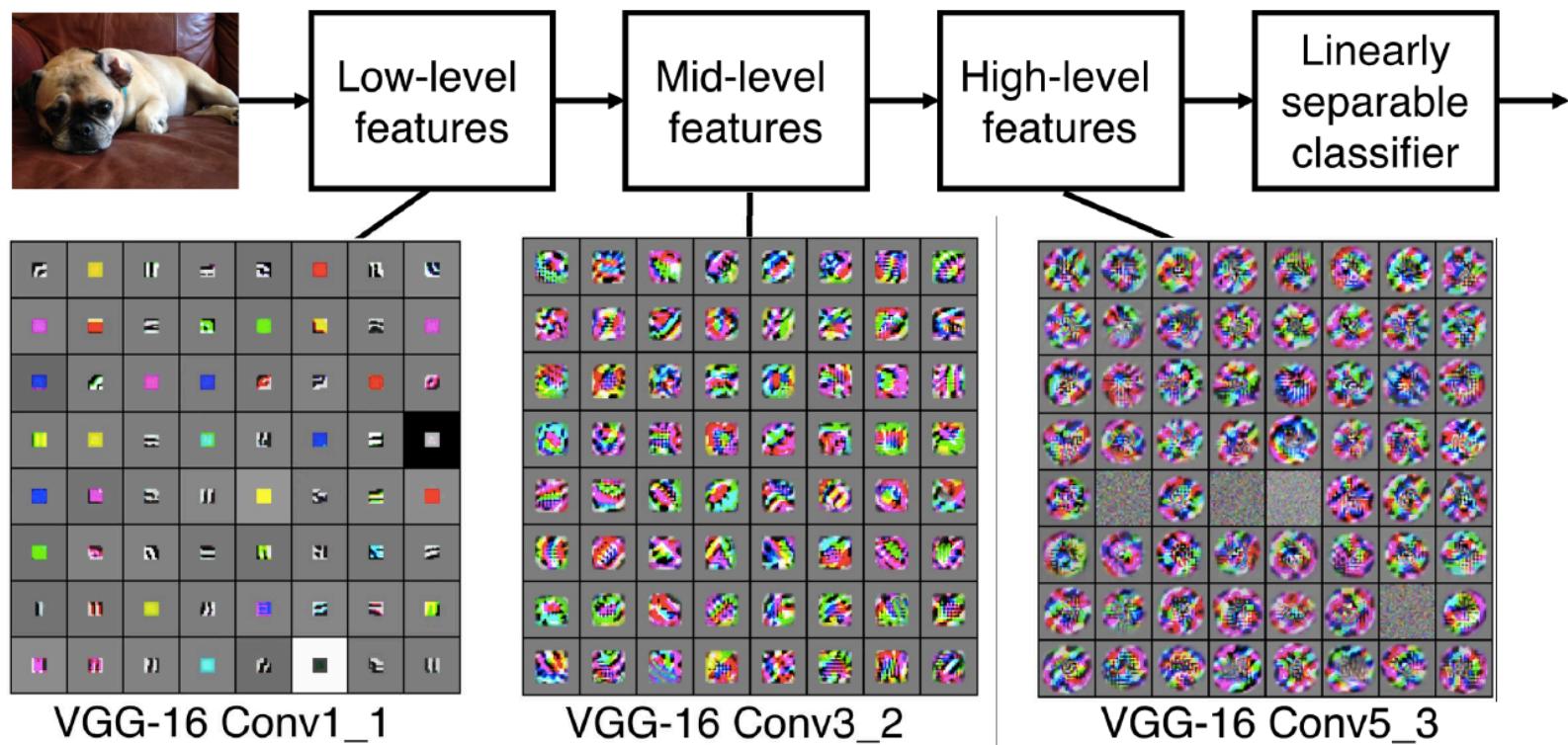
<https://deepsense.ai/region-of-interest-pooling-explained/>

http://mlss.tuebingen.mpg.de/2015/slides/fergus/Fergus_1.pdf

<https://vaaaaaanquish.hatenablog.com/entry/2015/01/26/060622> 55

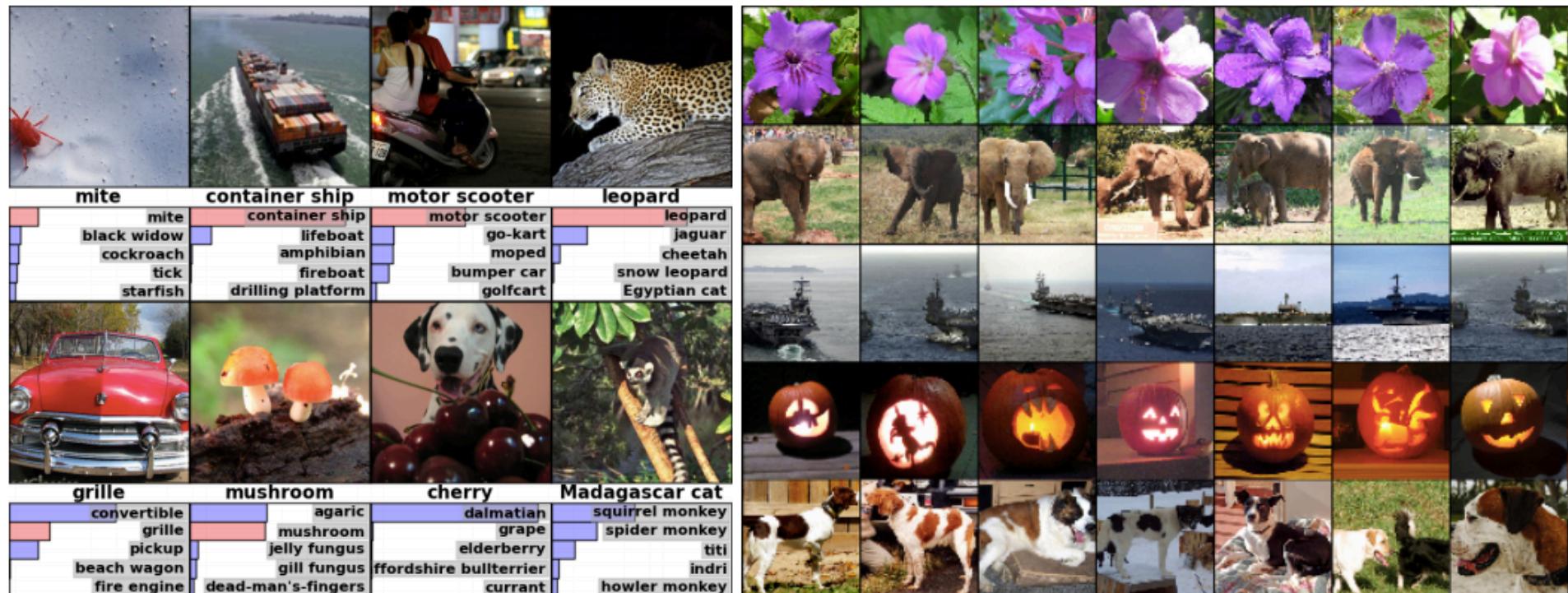
CNN: Visualization

- Visualization of CNN feature representations [Zeiler et al., 2014]
 - VGG-16 [Simonyan et al., 2015]



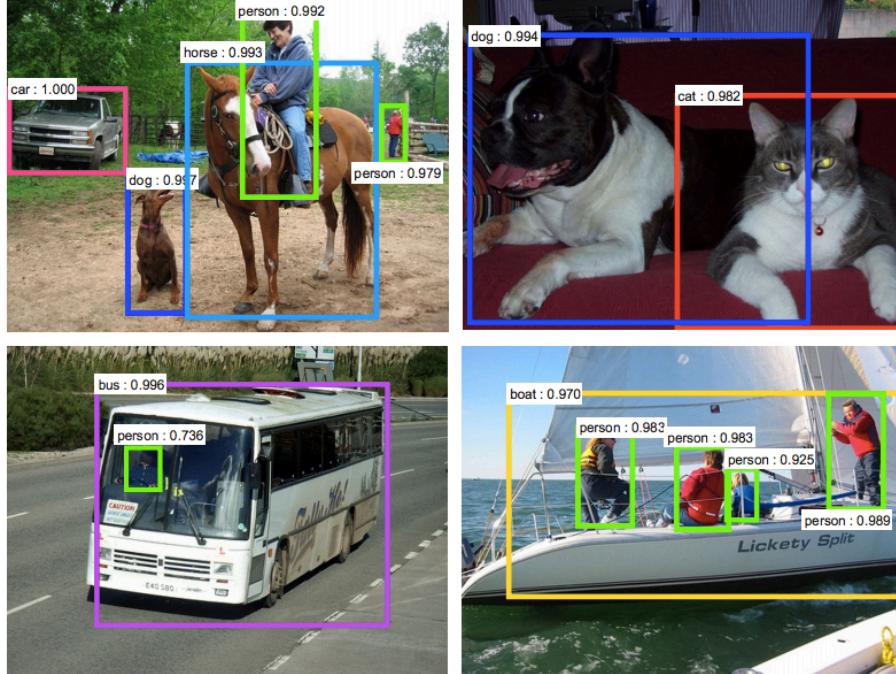
CNN in Computer Vision: Everywhere

Classification and retrieval [Krizhevsky et al., 2012]



CNN in Computer Vision: Everywhere

Detection [Ren et al., 2015]

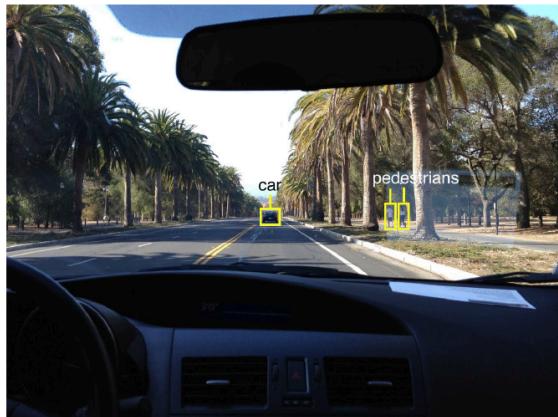


Segmentation [Farabet et al., 2013]



CNN in Computer Vision: Everywhere

Self-driving cars



Human pose estimation [Cae et al., 2017]

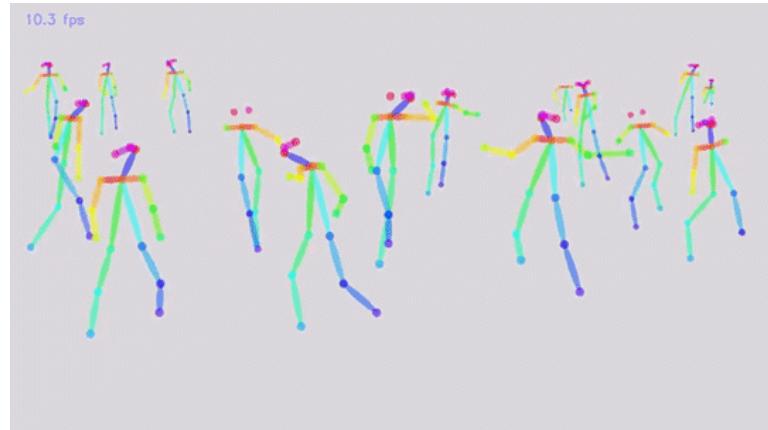


Image captioning [Vinyals et al., 2015][Karpathy et al., 2015]

No errors



A white teddy bear sitting in the grass

Minor errors



A man in a baseball uniform throwing a ball

Somewhat related



A woman is holding a cat in her hand

Table of Contents

1. Deep Neural Networks (DNN)

- Basics
- Training : Back propagation

2. Convolutional Neural Networks (CNN)

- Basics
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- Some applications

3. Recurrent Neural Networks (RNN)

- Basics
- Character-level language model (example)

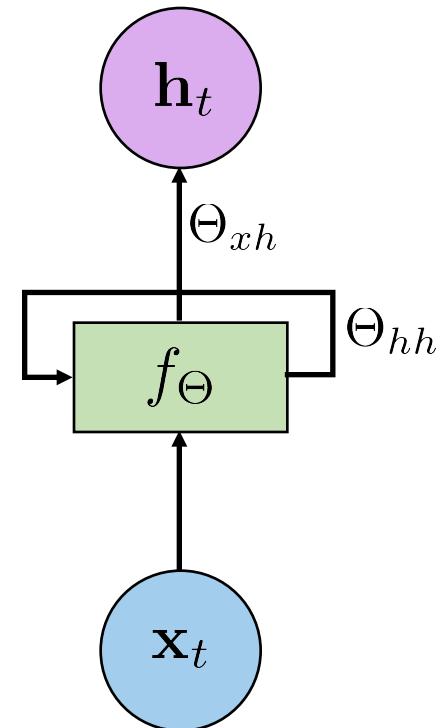
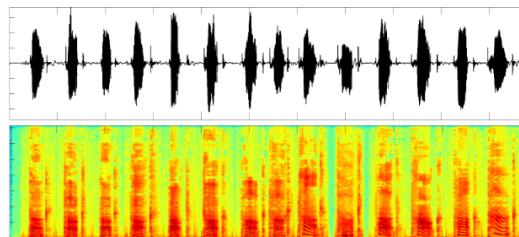
4. Question

- Why is it difficult to train a deep neural network ?

- CNN models spatial invariance information
- **Recurrent Neural Network (RNN)**
 - Models **temporal** information
 - Hidden states as a function of inputs and **previous** time step information

$$\mathbf{h}_t = f(\mathbf{h}_{t-1}, \mathbf{x}_t; \Theta)$$

- Temporal information is important in many applications
 - Language
 - Speech
 - Video

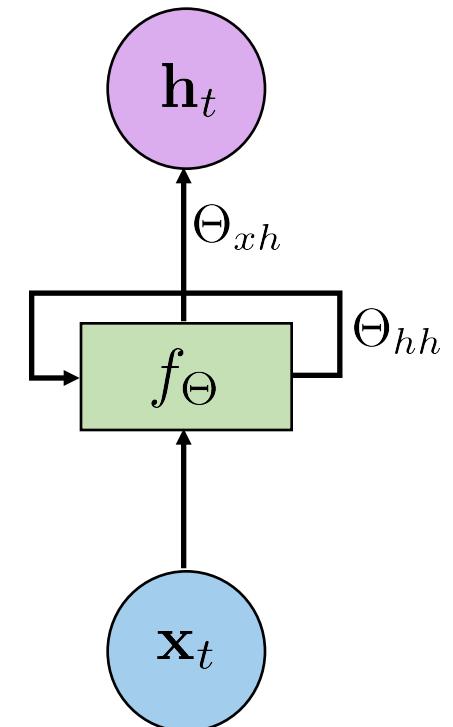


- Process a sequence of vectors by applying **recurrence formula** at **every time step** :

$$\mathbf{h}_t = f(\mathbf{h}_{t-1}, \mathbf{x}_t; \Theta)$$

New state | Old state Input
 vector at
 time step t

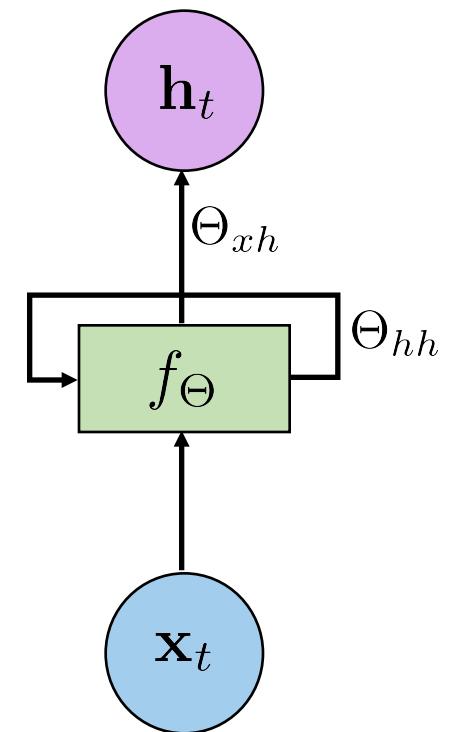
Function parameterized by Θ e.g, DNN, CNN



- Process a sequence of vectors by applying **recurrence formula** at **every time step** :

$$\mathbf{h}_t = f(\mathbf{h}_{t-1}, \mathbf{x}_t; \Theta)$$

- **Same function** and the **same set of parameters** f_Θ are used at every time step



RNN: Vanilla RNN

- Simple RNN

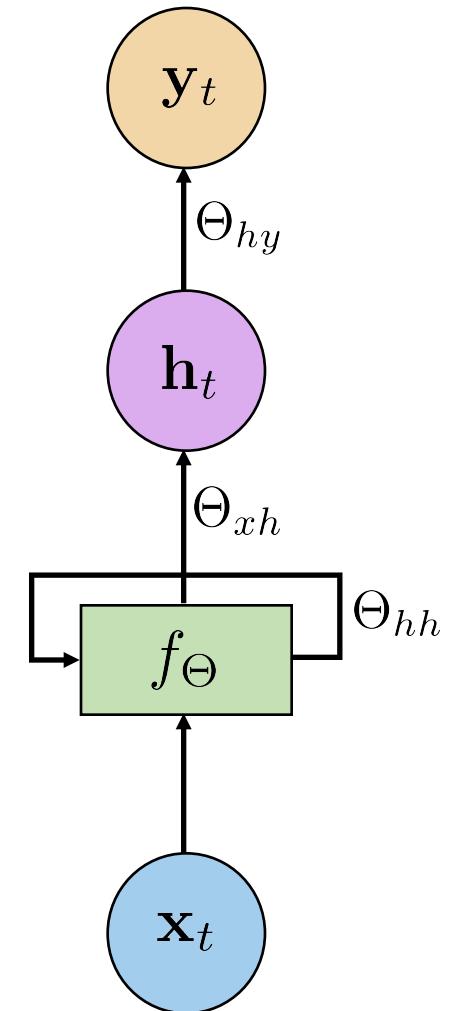
- The state consists of a single “hidden” vector \mathbf{h}_t
- Vanilla RNN (or sometimes called Elman RNN)

$$\mathbf{h}_t = f(\mathbf{h}_{t-1}, \mathbf{x}_t; \Theta)$$



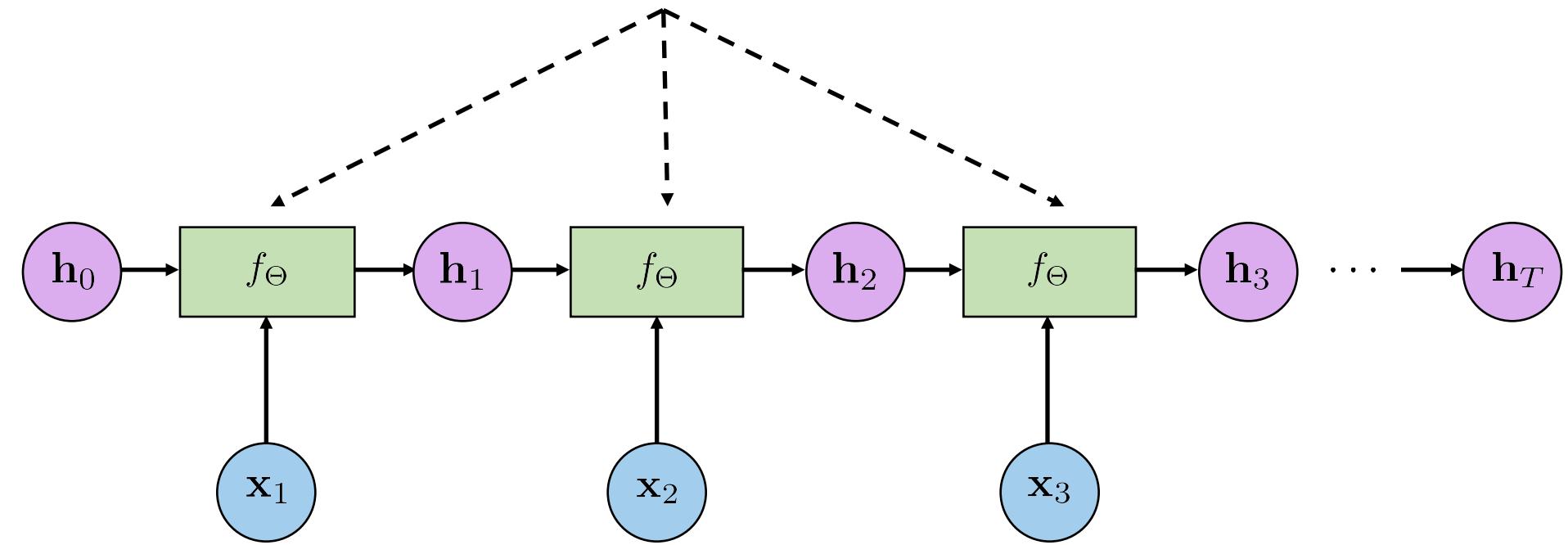
$$\mathbf{h}_t = \tanh(\Theta_{hh}\mathbf{h}_{t-1} + \Theta_{xh}\mathbf{x}_t)$$

$$\mathbf{y}_t = \Theta_{hy}\mathbf{h}_t$$

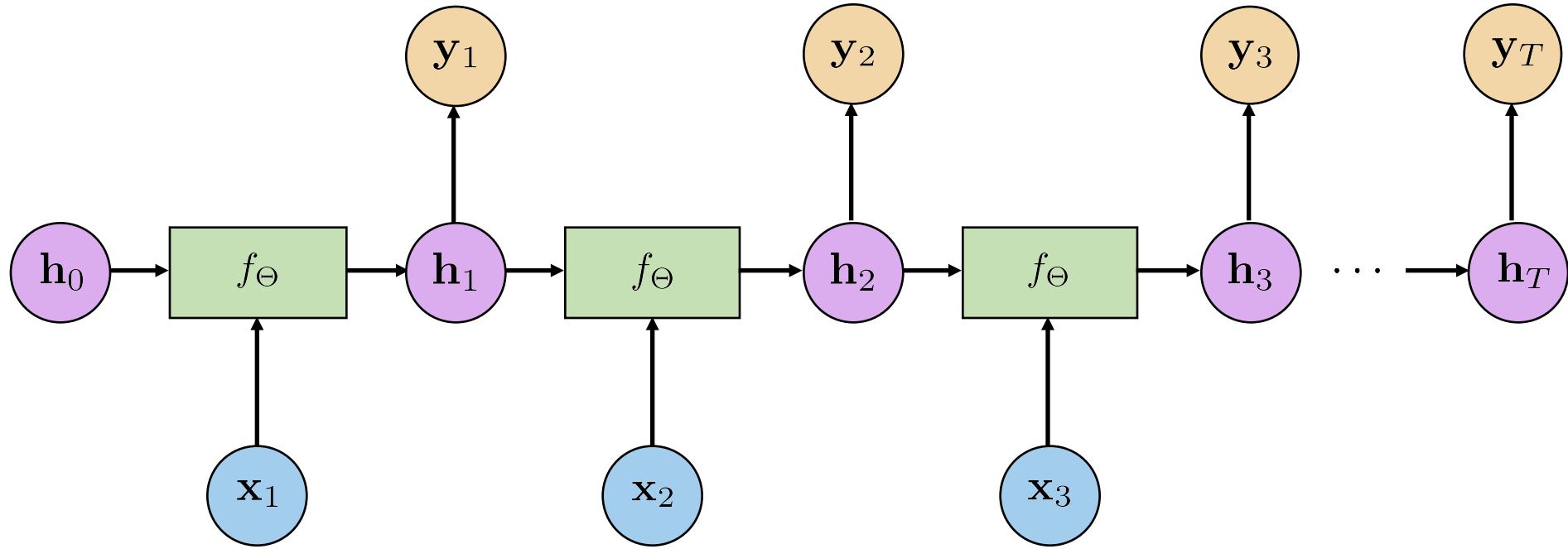


RNN: Computation Graph

Re-use the **same** weight matrix Θ at every time step



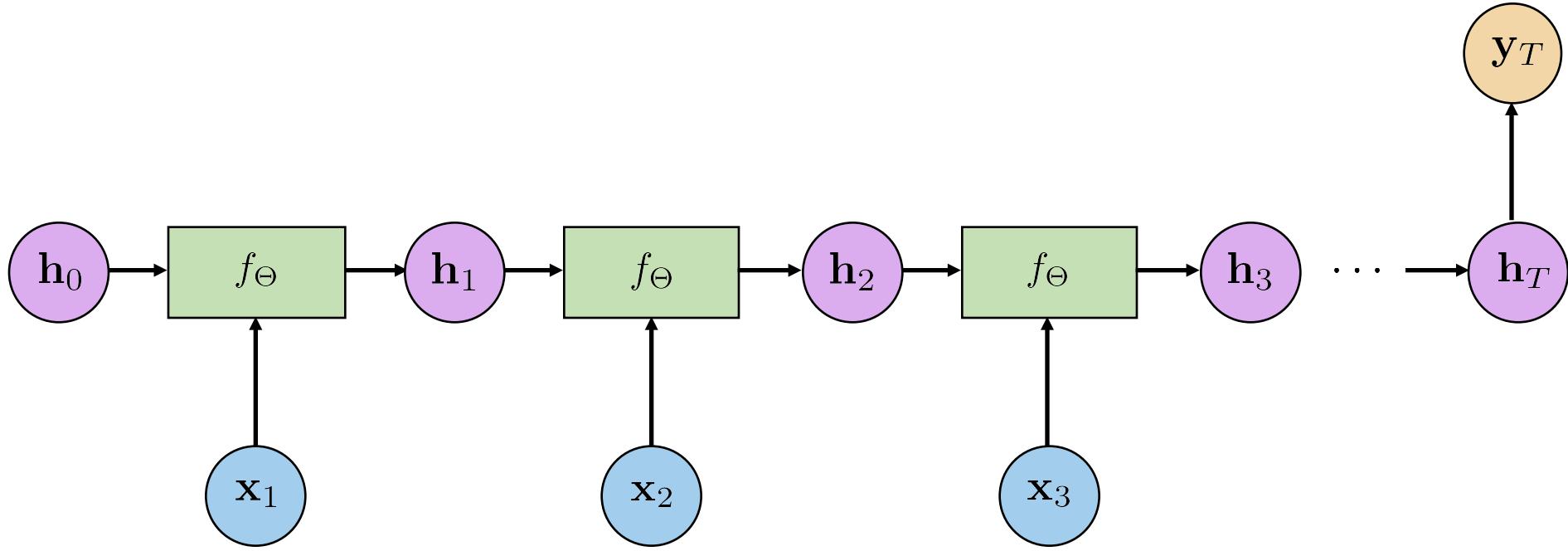
RNN: Computation Graph (Many to Many)



e.g., Machine Translation
(Sequence of words → Sequence of words)

Input sentence:	Translation (PBMT):	Translation (GNMT):	Translation (human):
李克強此行將啟動中加總理年度對話機制，與加拿大總理杜魯多舉行兩國總理首次年度對話。	Li Keqiang premier added this line to start the annual dialogue mechanism with the Canadian Prime Minister Trudeau two prime ministers held its first annual session.	Li Keqiang will start the annual dialogue mechanism with Prime Minister Trudeau of Canada and hold the first annual dialogue between the two premiers.	Li Keqiang will initiate the annual dialogue mechanism between premiers of China and Canada during this visit, and hold the first annual dialogue with Premier Trudeau of Canada.

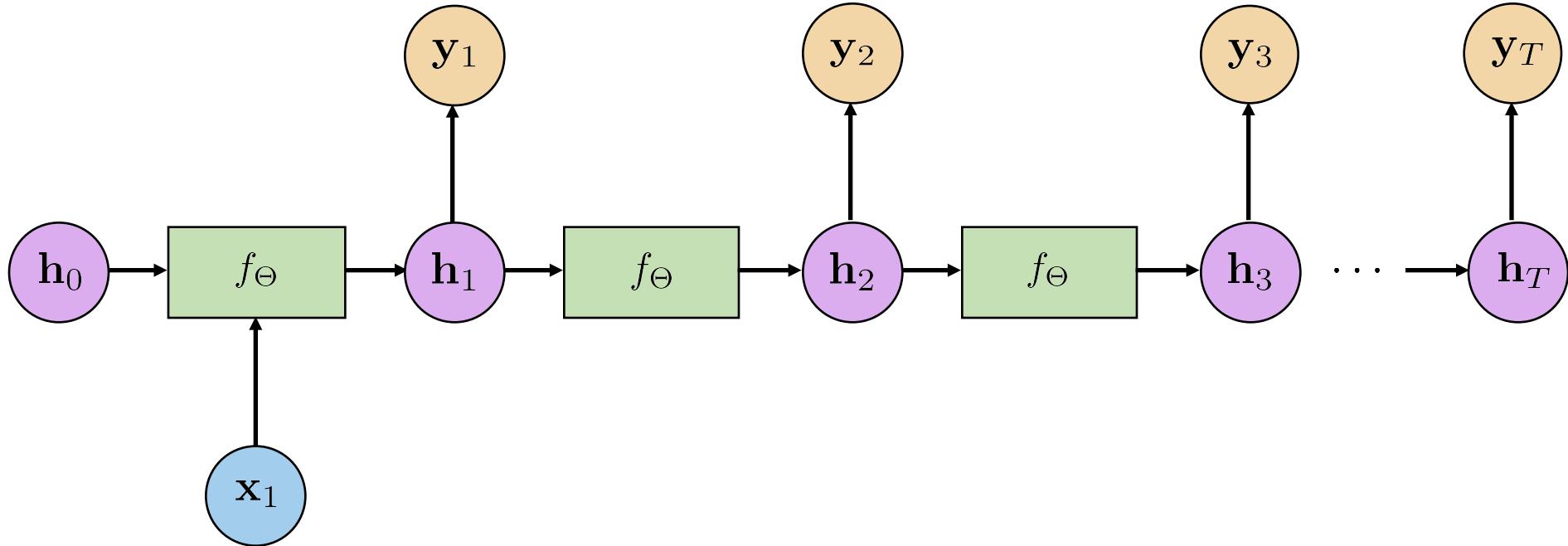
RNN: Computation Graph (Many to One)



e.g., **Sentiment Classification**
(Sequence of words → sentiment)

→ Good paper or not?

RNN: Computation Graph (One to Many)



e.g., **Image Captioning**
(Image → sequence of words)



A white teddy bear sitting in the grass



A man in a baseball uniform throwing a ball



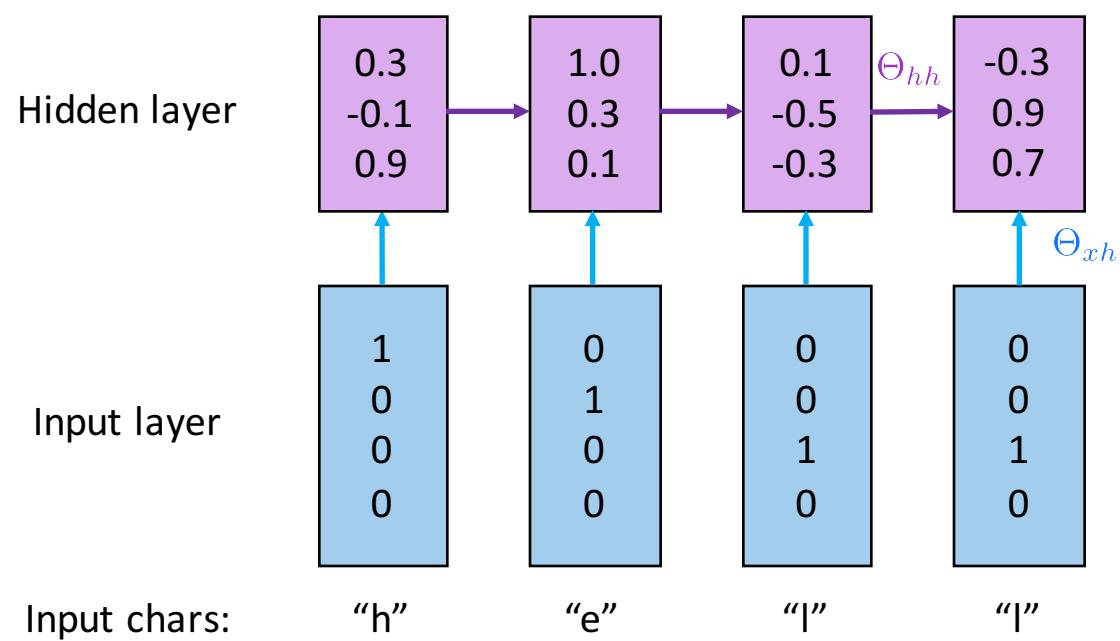
A woman is holding a cat in her hand

RNN: An Example

- Character-level language model
- Vocabulary : [h,e,l,o]

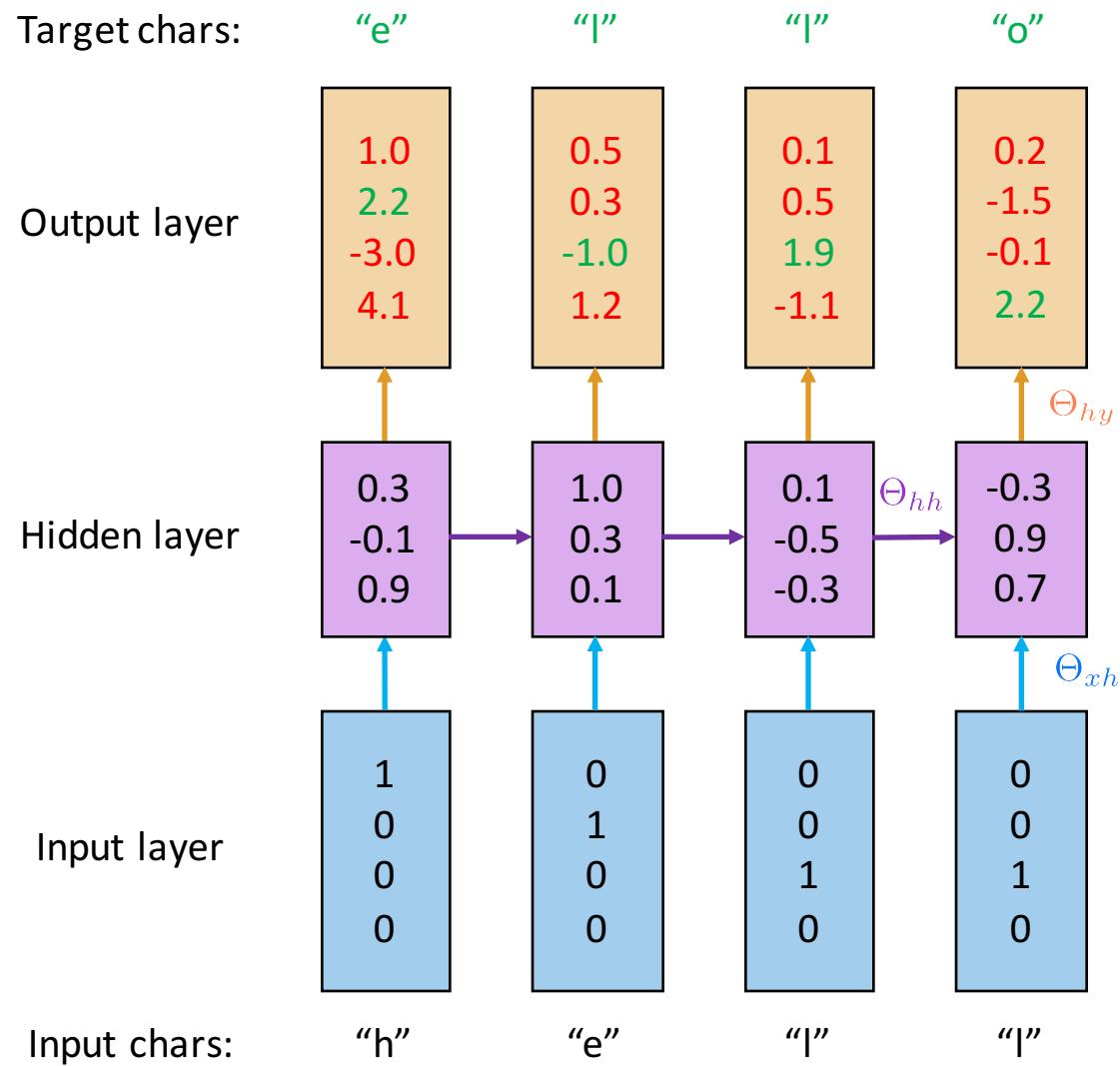
- Example training sequence : “hello”

$$\mathbf{h}_t = \tanh(\Theta_{hh}\mathbf{h}_{t-1} + \Theta_{xh}\mathbf{x}_t)$$



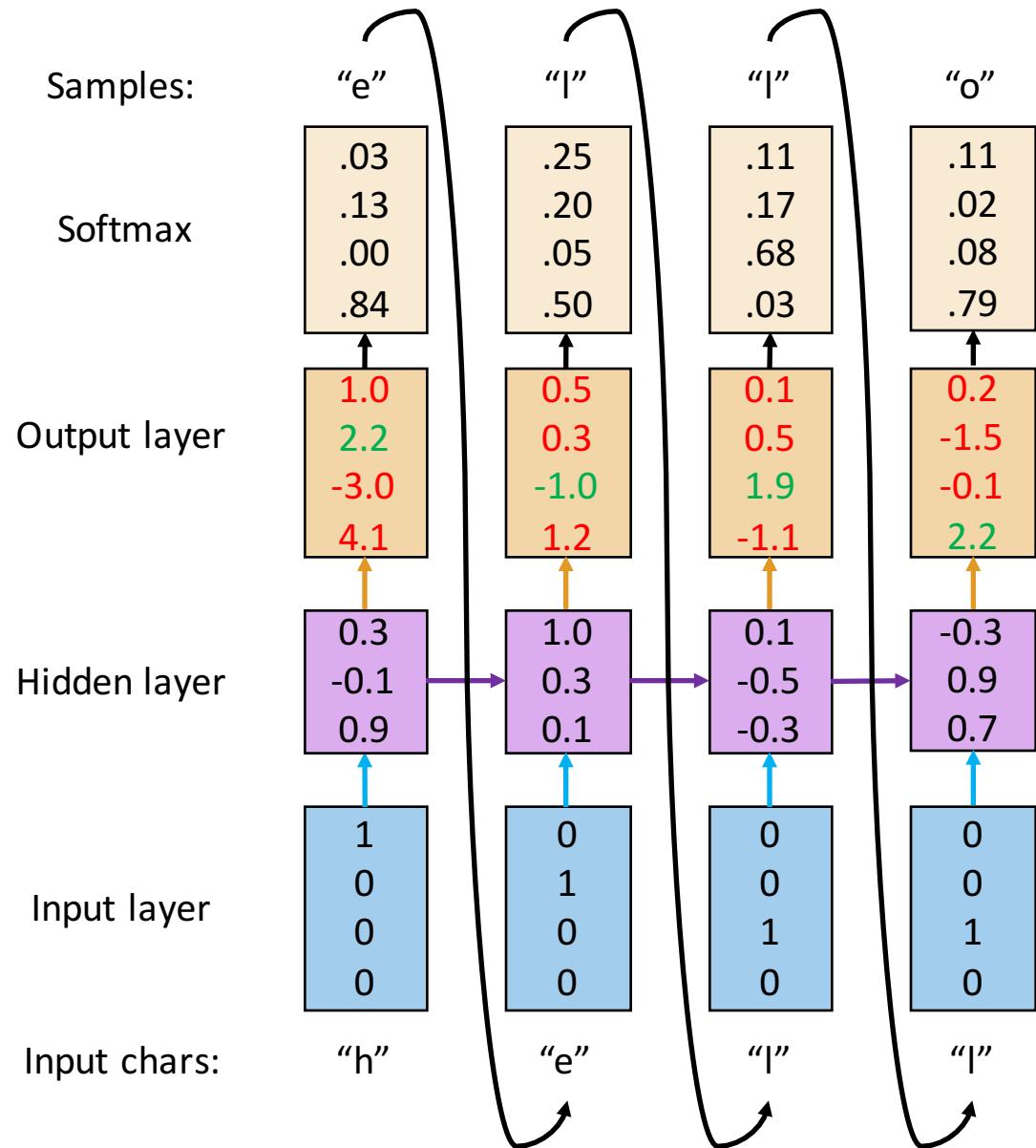
RNN: An Example

- Character-level language model
- Vocabulary : [h,e,l,o]
- Example training sequence : “hello”



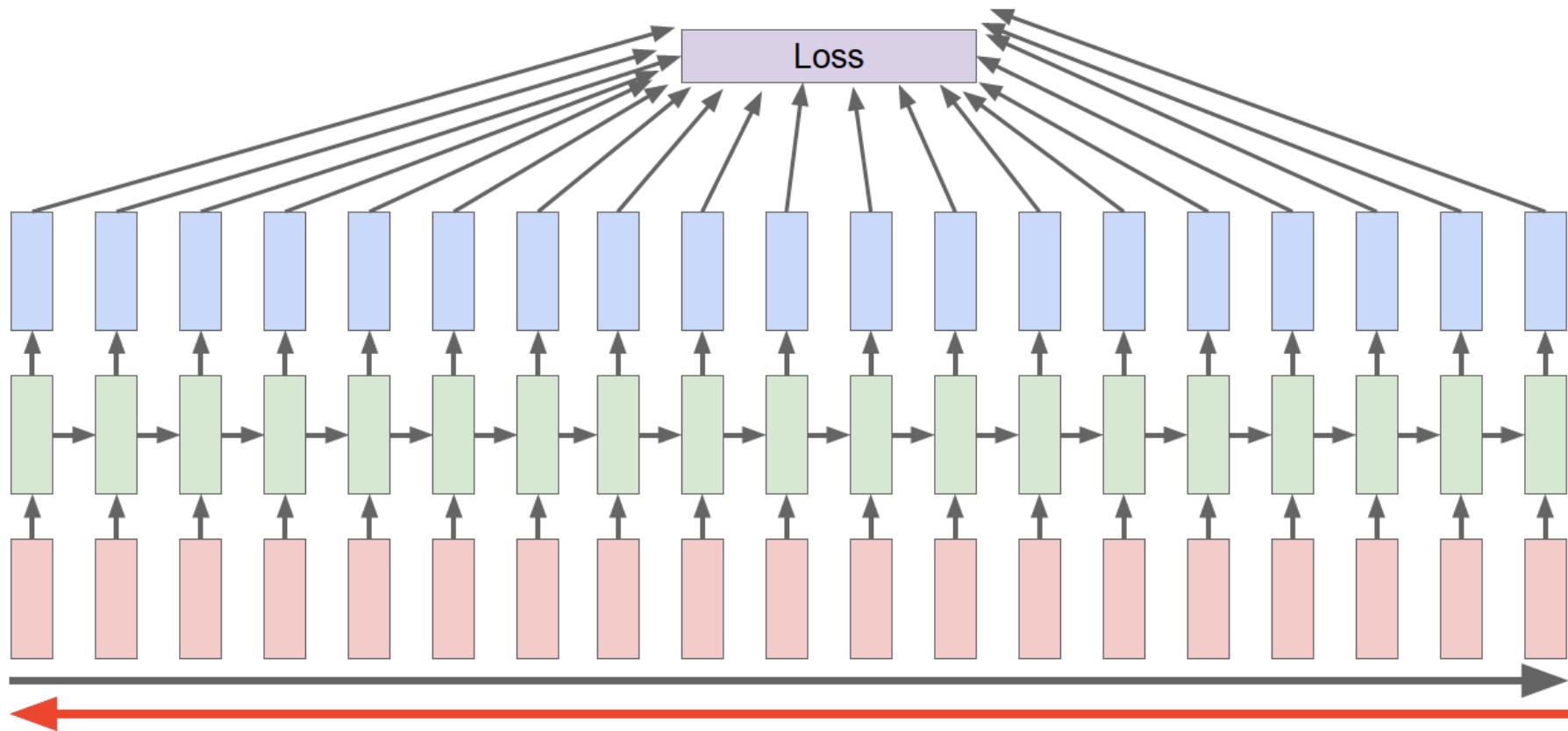
RNN: An Example

- Character-level language model
- Vocabulary : [h,e,l,o]
- At **test** time, sample character one at a time and feedback to model



RNN: Backpropagation Through Time (BPTT)

- Backpropagation through time (BPTT)
- Forward through entire sequence to compute loss, then backward through entire sequence to compute gradient



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4. Question

- Why is it difficult to train a deep neural network?

Question

- Why is it difficult to train a deep neural network?
- Can we just simply stack multiple layers and train them all?
 - Unfortunately, it does not work well
 - Even if we have infinite amount of computational resource

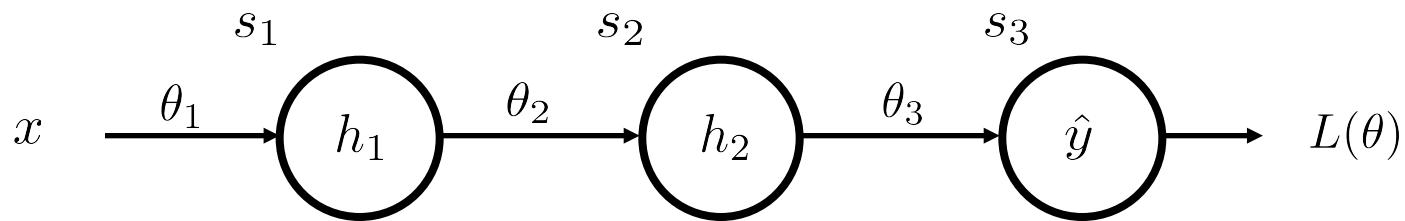
Vanishing gradient problem :

- The magnitude of the gradients **shrink exponentially** as we backpropagate through **many layers**
- Since typical activation functions such as sigmoid or tanh are **bounded**
- The phenomenon is called ***vanishing gradient*** problem

Vanishing Gradient Problem

- Why do gradients vanish?
- Think of a simplified 3-layer neural network

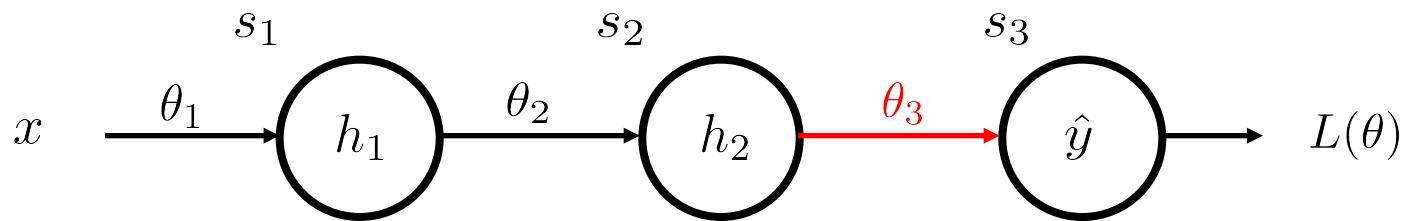
$$\hat{y} = \sigma (\theta_3 \sigma (\theta_2 \sigma (\theta_1 x)))$$



Vanishing Gradient Problem

- Why do gradients vanish?
- Think of a simplified 3-layer neural network

$$\hat{y} = \sigma (\theta_3 \sigma (\theta_2 \sigma (\theta_1 x)))$$



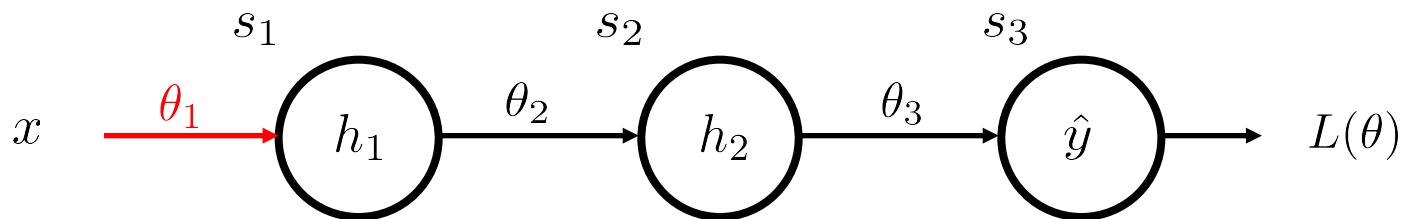
- First, let's update θ_3
 - Calculate the gradient of the loss with respect to θ_3

$$\frac{\partial L}{\partial \theta_3} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial s_3} \frac{\partial s_3}{\partial \theta_3} = \frac{\partial L}{\partial \hat{y}} \sigma'(s_3) h_2$$

Vanishing Gradient Problem

- Why do gradients vanish?
- Think of a simplified 3-layer neural network

$$\hat{y} = \sigma (\theta_3 \sigma (\theta_2 \sigma (\theta_1 x)))$$



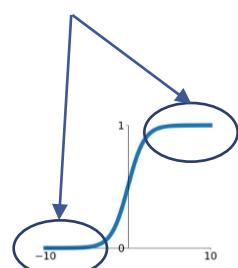
- How about θ_1 ?
 - Calculate the gradient of the loss with respect to θ_1

$$\frac{\partial L}{\partial \theta_1} = \frac{\partial L}{\partial \hat{y}} \boxed{\sigma'(s_3)} h_2 \boxed{\sigma'(s_2)} h_1 \boxed{\sigma'(s_1)} x$$

Gradients < 1

Sigmoid

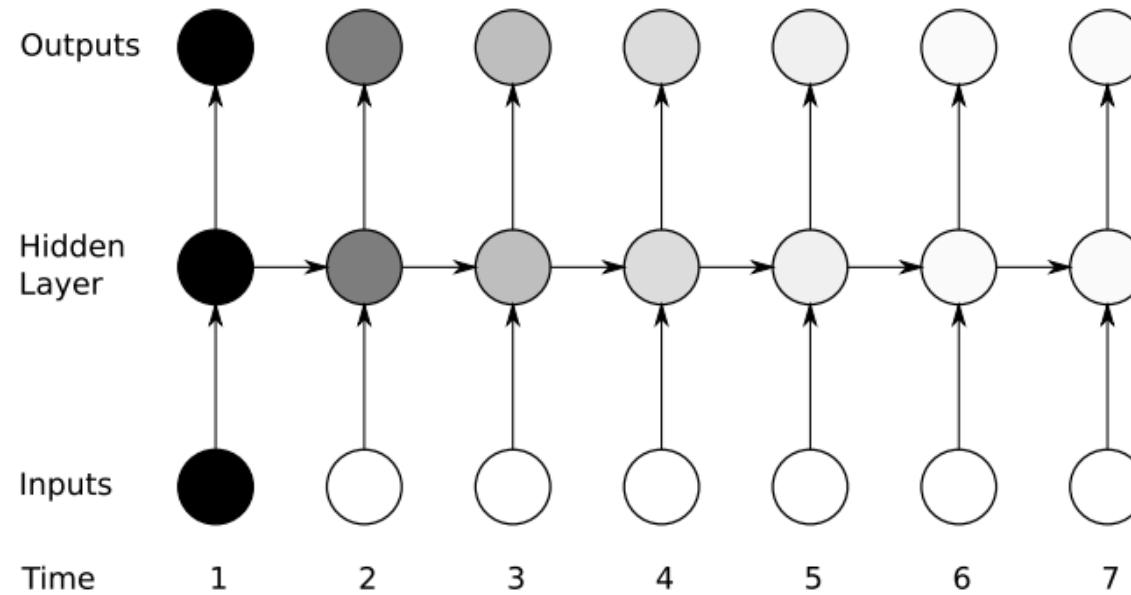
$$\sigma(x) = \frac{1}{1+e^{-x}}$$



Keep multiplying values < 1 will decrease the amount exponentially

Vanishing Gradient Over Time

- This is more problematic in vanilla RNN (with tanh/sigmoid activation)
 - When trying to handle long temporal dependency
 - Similar to previous example, the **gradient vanishes** over time



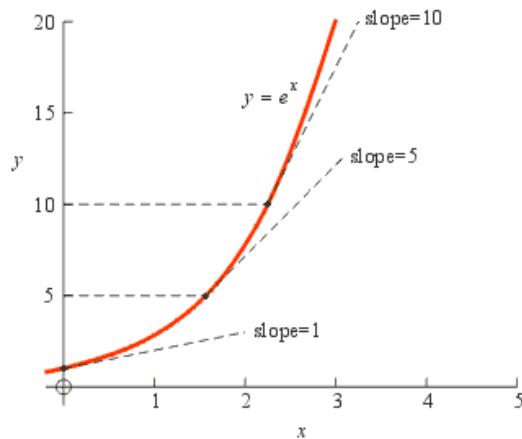
Quiz

- Vanishing gradient problem is critical in training neural network
- Q: Can we just use activation function that has gradients > 1 ?



Answer for Quiz

- Not really. It will cause another problem so called **exploding gradients**.
- Let's consider if we use exponential activation function:
 - The magnitude of gradient is always larger than 1 when input > 0
 - If output of the networks are positive, then the gradients to update θ_1 will explode



Gradients > 1

$$\frac{\partial L}{\partial \theta_1} = \frac{\partial L}{\partial \hat{y}} \sigma'(s_3) h_2 \sigma'(s_2) h_1 \sigma'(s_1) x$$

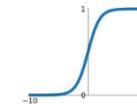
- This will cause the training very **unstable**
 - The weights will be updated in very large amount, resulting in NaN values
 - Very critical problem in training neural networks

How Can We Overcome Vanishing Gradient Problems?

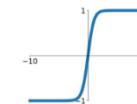
- Possible solutions
 - Activation functions
 - CNN: Residual networks [He et al., 2016]
 - RNN: LSTM (Long Short-Term Memory)

Activation Functions

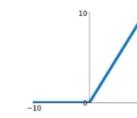
Sigmoid
 $\sigma(x) = \frac{1}{1+e^{-x}}$



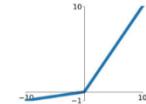
tanh
 $\tanh(x)$



ReLU
 $\max(0, x)$

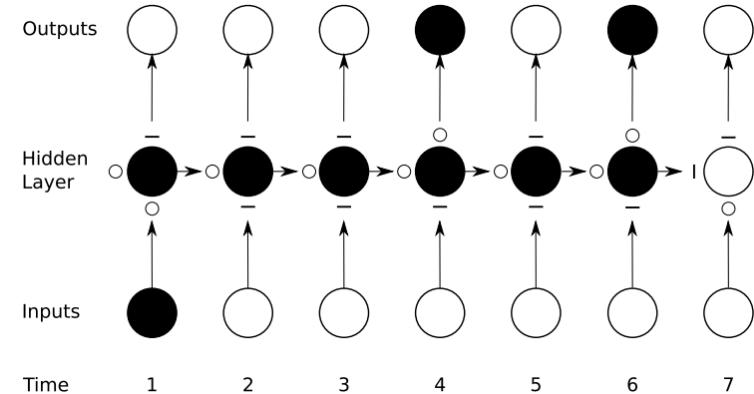
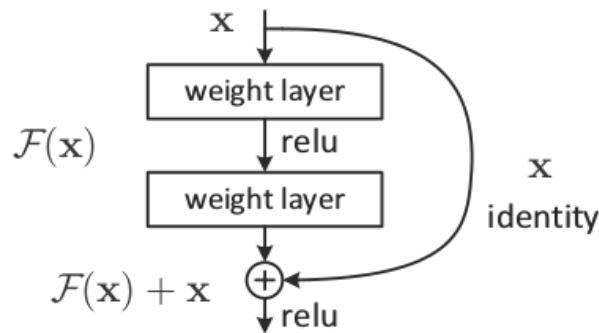
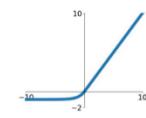


Leaky ReLU
 $\max(0.1x, x)$



Maxout
 $\max(w_1^T x + b_1, w_2^T x + b_2)$

ELU
$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



LSTM (Long Short-Term Memory)

*source

<https://mediatum.ub.tum.de/doc/673554/file.pdf>

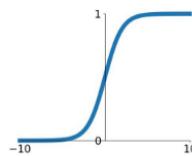
<https://medium.com/@shrutijadon10104776/survey-on-activation-functions-for-deep-learning-9689331ba092> 81

Solving Vanishing Gradient: Activation Functions

- Use different activation functions that are not bounded:
 - Recent works largely use **ReLU** or their variants
 - No saturation, easy to optimize

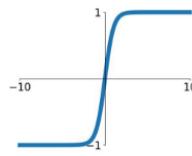
Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



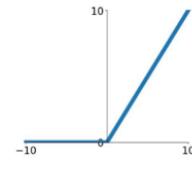
tanh

$$\tanh(x)$$



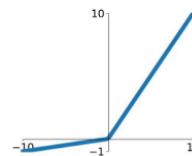
ReLU

$$\max(0, x)$$



Leaky ReLU

$$\max(0.1x, x)$$

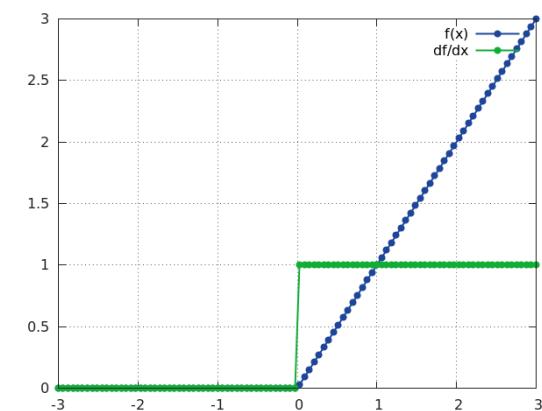
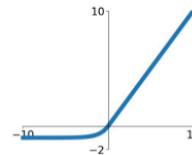


Maxout

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

ELU

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



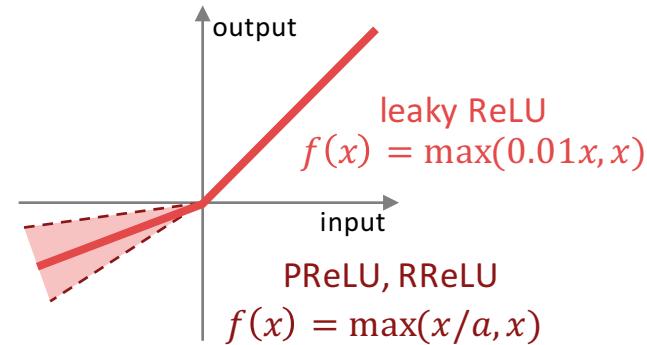
*source: <https://medium.com/@shrutijadon10104776/survey-on-activation-functions-for-deep-learning-9689331ba092>

Solving Vanishing Gradient: Activation Functions

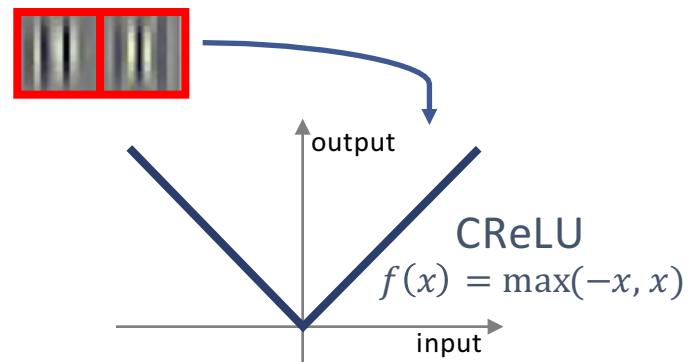
- Several generalizations of ReLU
 - Leaky ReLU [Maas et. al., 2013]: Introducing non-zero gradient for ‘dying’ ReLUs’
 - Parameteric ReLU (PReLU) [He et al., 2015]: Additional learnable parameter a on leaky ReLUs.
 - Randomized ReLU (RReLU) [Xu et al., 2015]: Samples parameter a from uniform distribution.

Activation	Training Error	Test Error
ReLU	0.00318	0.1245
Leaky ReLU, $a = 100$	0.0031	0.1266
Leaky ReLU, $a = 5.5$	0.00362	0.1120
PReLU	0.00178	0.1179
RReLU ($y_{ji} = x_{ji}/\frac{l+u}{2}$)	0.00550	0.1119

Table 3. Error rate of CIFAR-10 Network in Network with different activation function

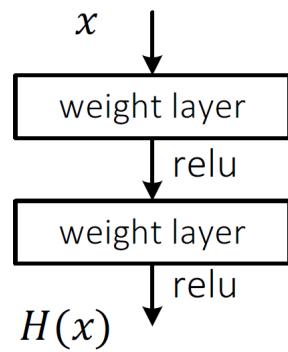


- Concatenated ReLU (CReLU) [Shang et al., 2016]
 - ‘Opposite pairs’ of filters found in CNN
 - Needs to learn twice the information
 - Two-sided ReLU

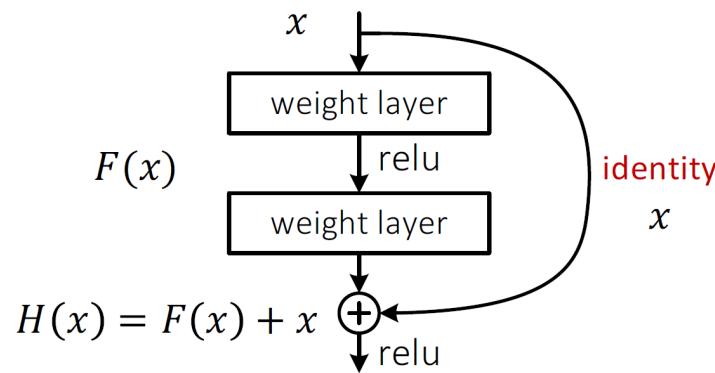


Solving Vanishing Gradient: Residual Networks

- Residual networks (ResNet [He et al., 2016])
 - Feed-forward NN with “**shortcut connections**”
 - Can preserve gradient flow throughout the entire depth of the network
 - Possible to train **more than 100 layers** by simply stacking residual blocks



Plain network



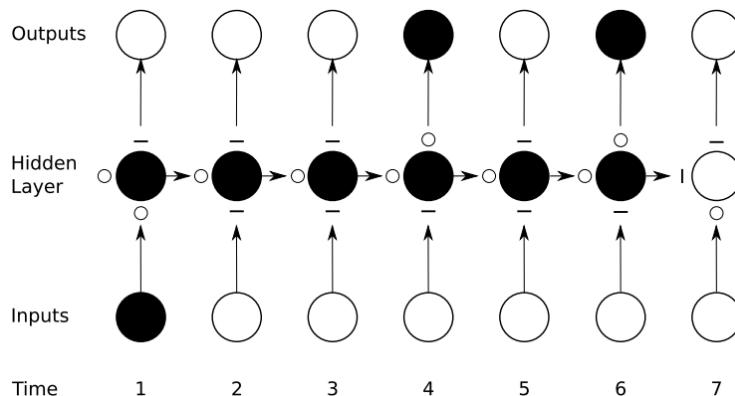
Residual network

Solving Vanishing Gradient: LSTM and GRU

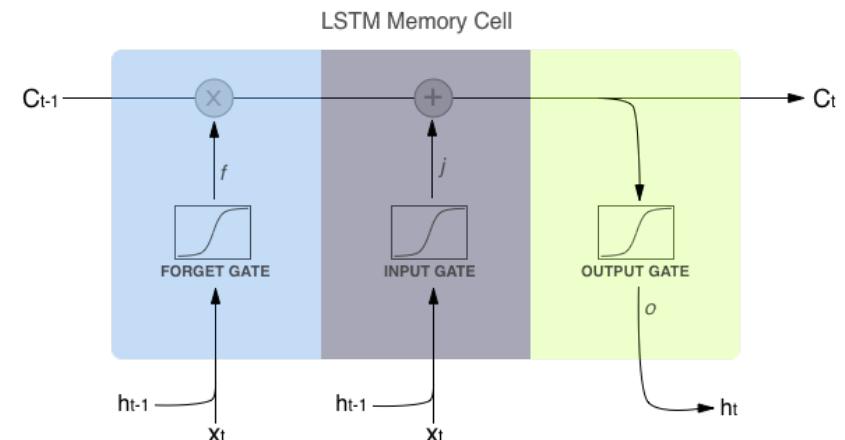
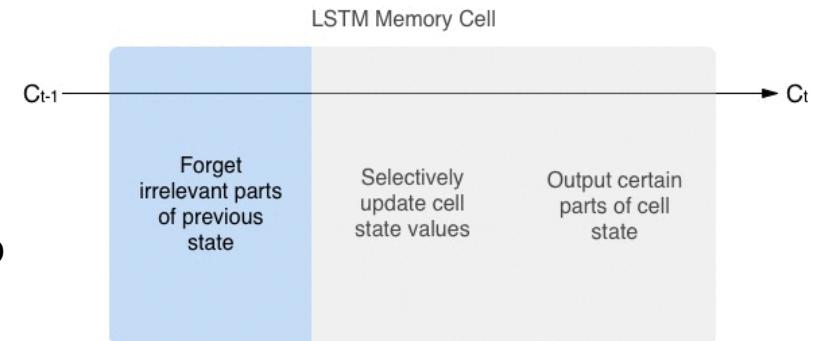
- LSTM (Long Short-Term Memory) and GRU (Gated Recurrent Units)
 - Specially designed RNN which can **remember** information for much **longer period**

3 main steps:

- **Forget irrelevant parts** of previous state
- **Selectively update** the cell state based on the new input
- **Selectively decide** what part of the cell state to output as the new hidden state



Preservation of gradient information in LSTM



*source :
<http://harinisuresh.com/2016/10/09/lstms/>
<https://mediatum.ub.tum.de/doc/673554/file.pdf> 85

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