

# How to break EW symmetry naturally?

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C. Csaki, T. Ma, J. Shu., Phys.Rev.Lett. 119 (2017) no13, 131803

C. Csaki, T. Ma, J. Shu., Phys.Rev.Lett. 121 (2018) no23, 231801

Many many other  
papers.....

C. Csaki, T. Ma, J. Shu, J-H. Yu, arxiv:1810.07704

C. Csaki, F. Freitas, L. Huang, T. Ma, J. Shu, M. Perelstein, arxiv:1811.01961

# Outline



- Basic introduction
- Brief background knowledge on Maximally Symmetric Composite Higgs
- How to realize maximal symmetry, even from warped extra dimensions (emergence!!!)
- Naturalness sum rule, how to test?
- Trigonometric Parity for the Composite Higgs
- Outlook on HEP and other fields

# High energy physics

TeV

GeV

MeV

eV

t

b  
c  
τ  
μ  
s  
u, d

w, z h

2012

p, n

nucleus

12 orders of  
magnitude

e

v<sub>3</sub>  
v<sub>2</sub>  
v<sub>1</sub>

atoms

1895

Open the door of  
sub-atom physics

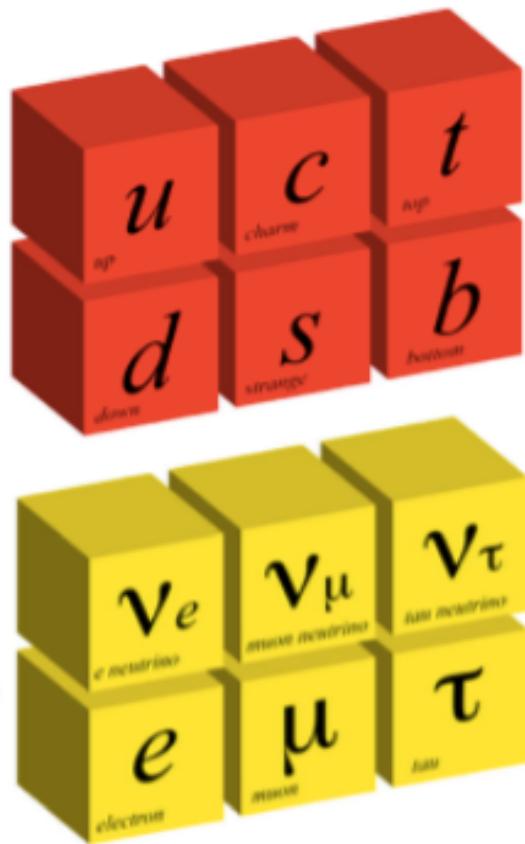


Higher and  
higher energy

Last 122 years

# “Old” physics up to date

Quarks  
Leptons

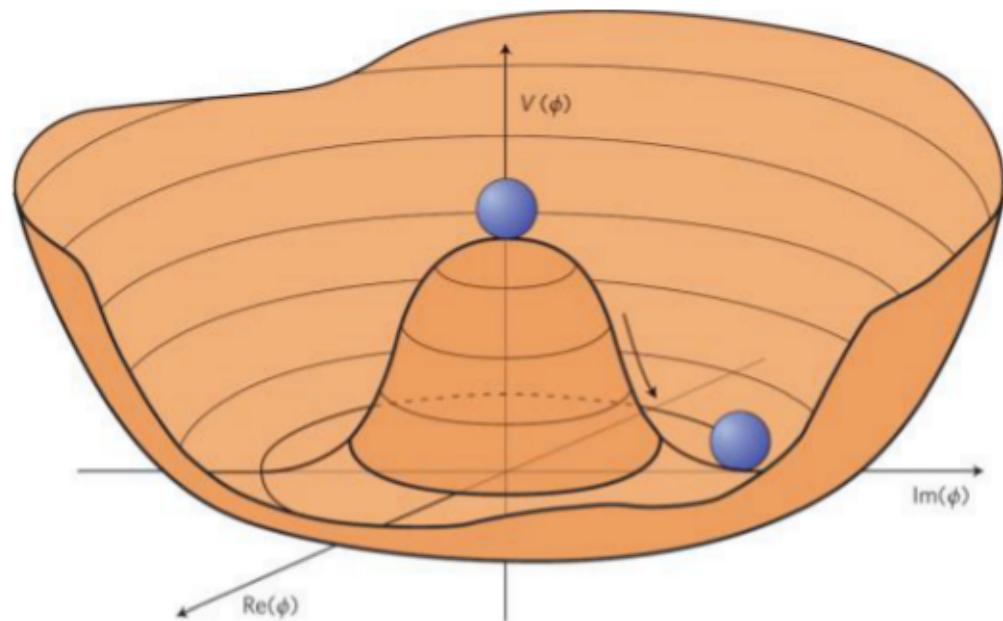


## The Weinberg-Salam Model

$$\begin{aligned}\mathcal{L} = & \bar{E}_L(i\partial)E_L + \bar{e}_R(i\partial)e_R + \bar{Q}_L(i\partial)Q_L + \bar{u}_R(i\partial)u_R + \bar{d}_R(i\partial)d_R \\ & + g(W_\mu^+ J_W^{\mu+} + W_\mu^- J_W^{\mu-} + Z_\mu^0 J_Z^\mu) + e A_\mu J_{EM}^\mu,\end{aligned}$$
$$J_W^{\mu+} = \frac{1}{\sqrt{2}}(\bar{\nu}_L \gamma^\mu e_L + \bar{u}_L \gamma^\mu d_L);$$
$$J_W^{\mu-} = \frac{1}{\sqrt{2}}(\bar{e}_L \gamma^\mu \nu_L + \bar{d}_L \gamma^\mu u_L); \quad e = \frac{gg'}{\sqrt{g^2 + g'^2}},$$
$$\begin{aligned}J_Z^\mu = & \frac{1}{\cos \theta_w} \left[ \bar{\nu}_L \gamma^\mu \left( \frac{1}{2} \right) \nu_L + \bar{e}_L \gamma^\mu \left( -\frac{1}{2} + \sin^2 \theta_w \right) e_L + \bar{e}_R \gamma^\mu \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) e_R \right. \\ & \left. + \bar{u}_L \gamma^\mu \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) u_L + \bar{u}_R \gamma^\mu \left( -\frac{2}{3} \sin^2 \theta_w \right) u_R \right. \\ & \left. + \bar{d}_L \gamma^\mu \left( -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_w \right) d_L + \bar{d}_R \gamma^\mu \left( \frac{1}{3} \sin^2 \theta_w \right) d_R \right],\end{aligned}$$
$$J_{EM}^\mu = \bar{e} \gamma^\mu (-1) e + \bar{u} \gamma^\mu (+\frac{2}{3}) u + \bar{d} \gamma^\mu (-\frac{1}{3}) d.$$

The chosen one!

# Why God's particle?



Higgs potential

$$V(h) = \frac{1}{2}\mu^2 h^2 + \frac{\lambda}{4}h^4$$

EWSB  
(Higgs mechanism)

$$\langle h \rangle \equiv v \neq 0 \rightarrow m_W = g_W \frac{v}{2}$$

Gives all particles mass

The origin of the mass

# Unknown in “old” physics



Higgs potential

$$V(h) = \frac{1}{2}\mu^2 h^2 + \frac{\lambda}{4}h^4$$

Laudau-Ginzberg potential (Superconductivity)

$$m_h^2(h^\dagger h) + \frac{1}{2}\lambda(h^\dagger h)^2 + \frac{1}{3!}\Lambda^2(h^\dagger h)^3.$$

negative, why?

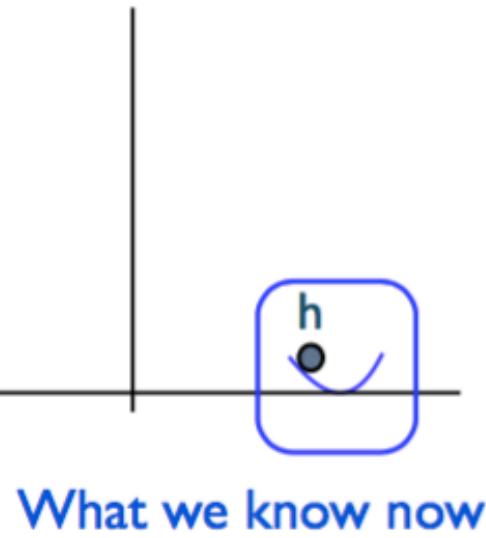
$$\frac{1}{2}\lambda(h^\dagger h)^2 \log \left[ \frac{(h^\dagger h)}{m^2} \right]$$

$$V(h) \simeq -\gamma s_h^2 + \beta s_h^4.$$

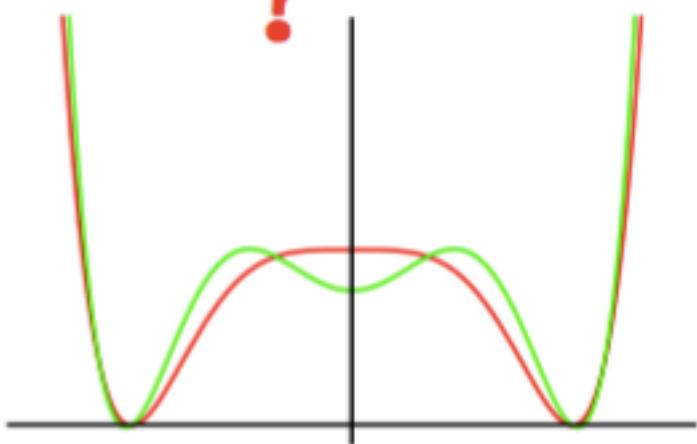
We actually never know the Higgs potential and why EWSB?

CORE question in particle physics

# Unknown in “old” physics



?



Taylor expand on the quantum fluctuation of higgs potential

$$h^3 \quad h^4 \quad h^5 \dots \dots \dots h^9$$

Future Collider Not known how to probe

$$V(h) = \frac{1}{2}\mu^2 h^2 + \frac{\lambda}{4}h^4$$

Same collider signal, different potential

$$V(h) = \frac{1}{2}\mu^2 h^2 - \frac{\lambda}{4}h^4 + \frac{1}{\Lambda^2}h^6$$

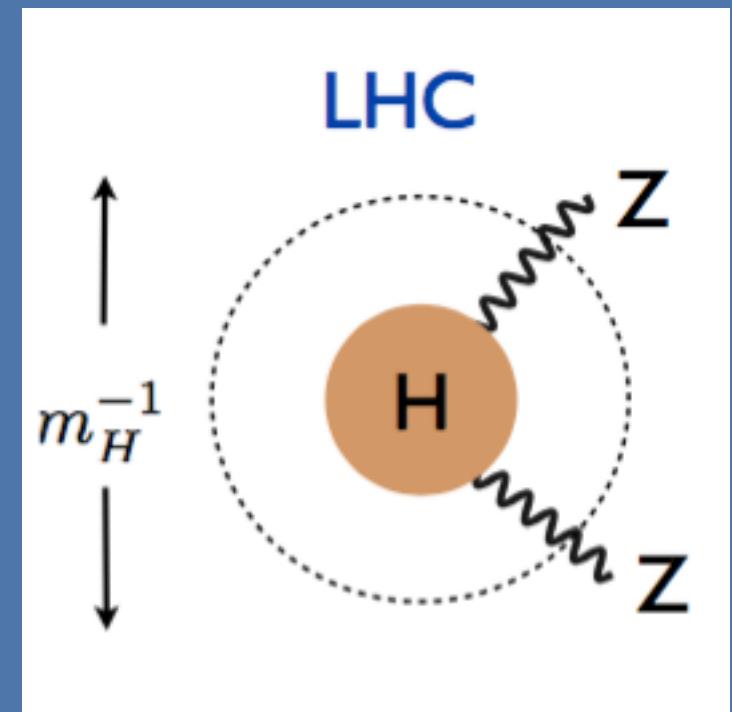
# Substructure of Higgs?



Suppose in NP scale, we see  
substructure of Higgs (like  
QCD Pi form factor deviations)

Possible NP deviation

$$\delta = c \frac{m_W^2}{M_{\text{NP}}^2}, \quad c = \mathcal{O}(1)$$

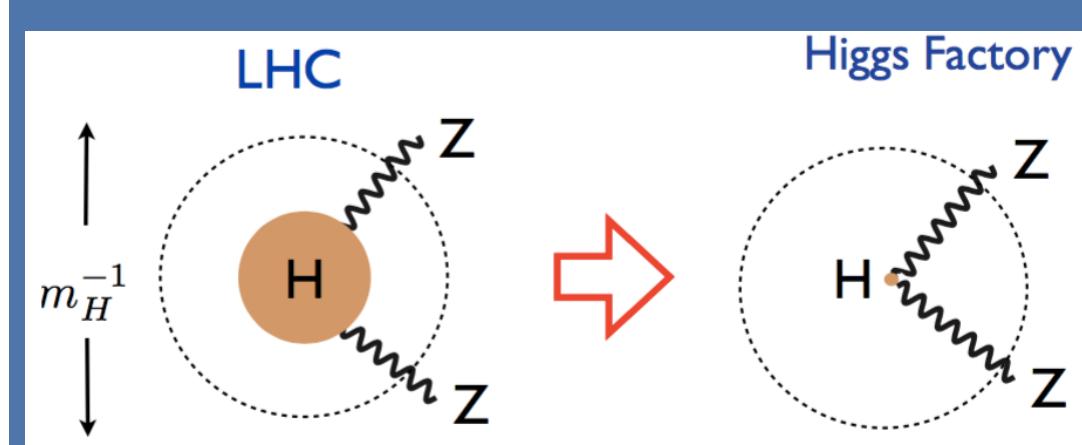
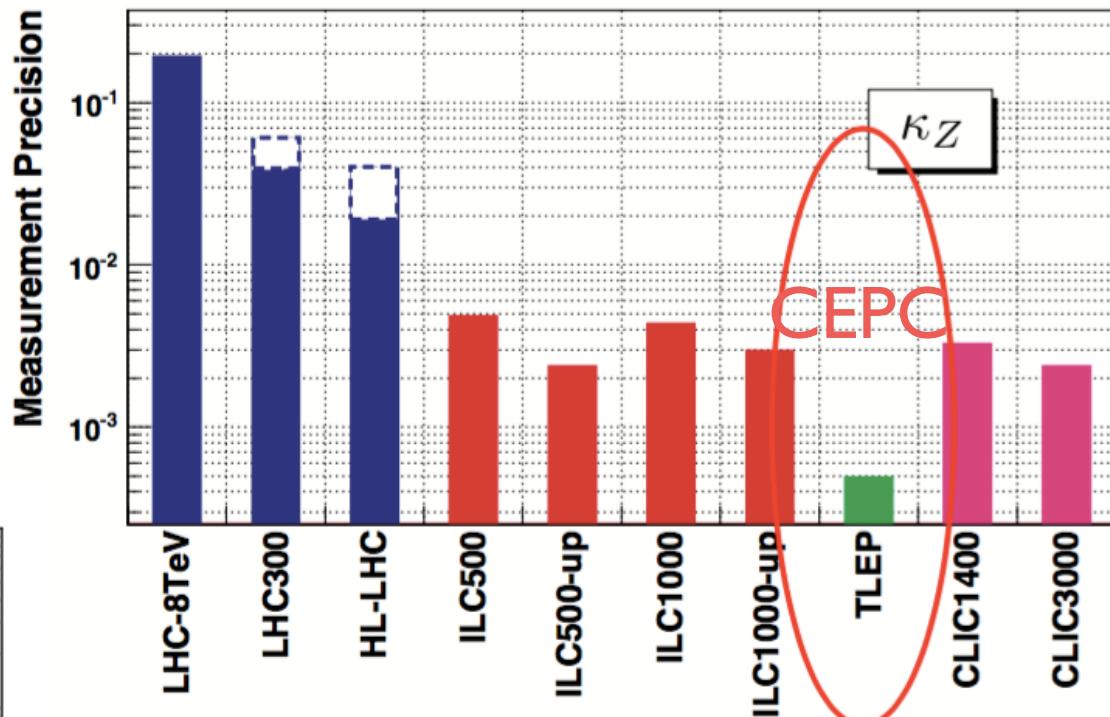
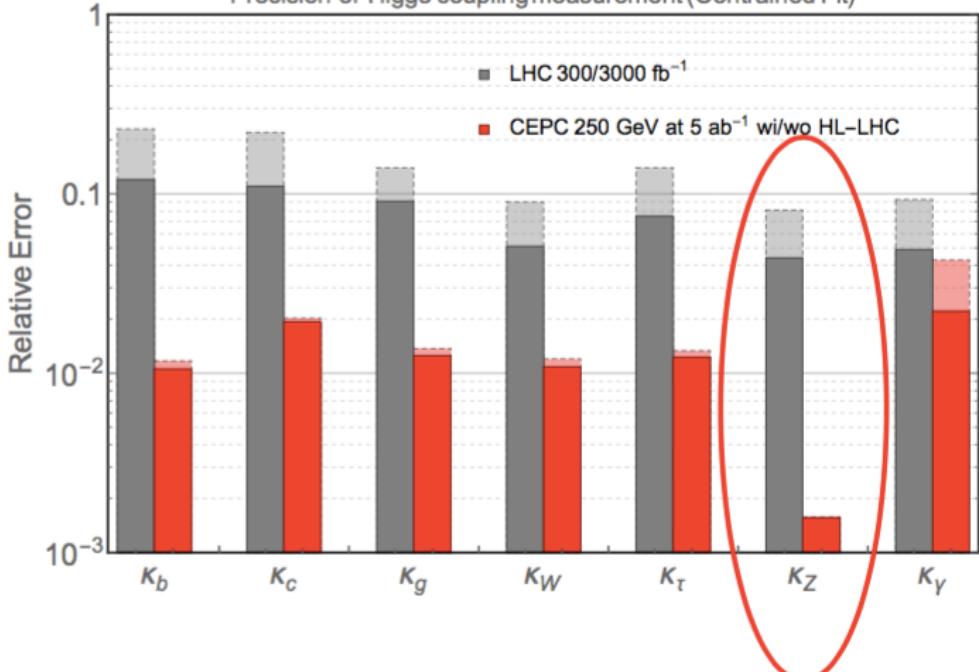


# Precision Higgs measure



$$\kappa_Z = \frac{g_{hZ}(\text{Measured})}{g_{hZ}(\text{SM})}$$

Precision of Higgs coupling measurement (Constrained Fit)



# Higgs compositeness for Higgs factory

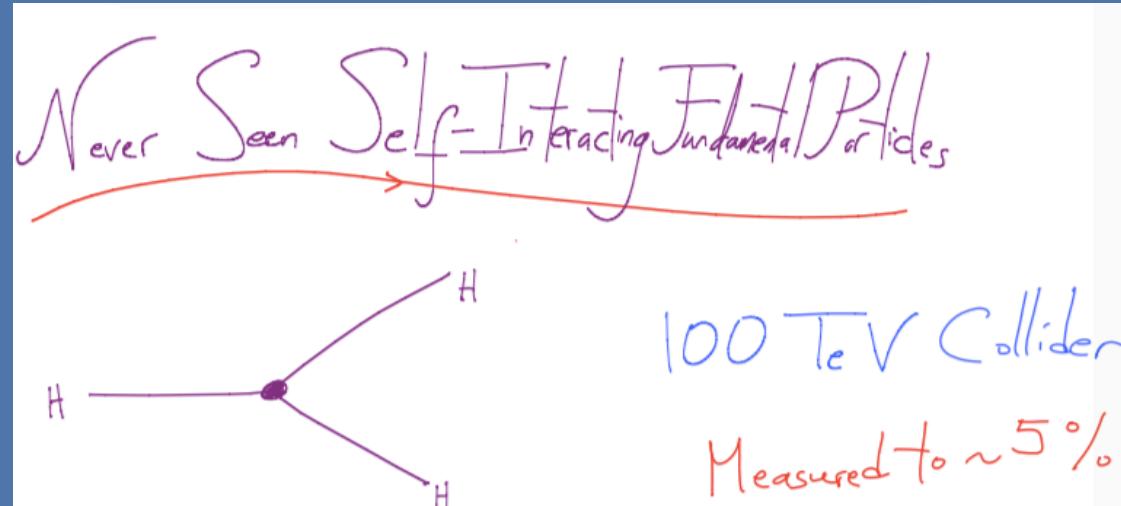


Our chairman Mao says all particles  
are composite from his philosophy!  
A great eye-catching and physics motivation  
for Higgs factory and great collider  
Nima's talk in IHEP 2018

Higgs precision



Self-interaction



100 TeV Collider

Measured to ~5%

# Higgs as a pNGB



## Why Higgs as a pNGB?

Kaplan, H. Georgi, Phys.Lett.B 136 (1984) 183

Kaplan, H. Georgi, Phys.Lett.B 145 (1984) 216

- Higgs mass small compared to confinement scale ( $1 \sim 10 \text{ TeV}$ ) Highly constrained by LEP
- The radiatively generated Higgs potential
- universal prediction on Higgs couplings (Like pion soft theorem)

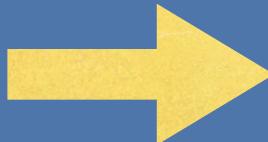
If the **strong dynamics** triggers the breaking G/H,  
pNGB is a **composite** particle.

QCD chiral symmetry, pion

# The origin of Higgs potential

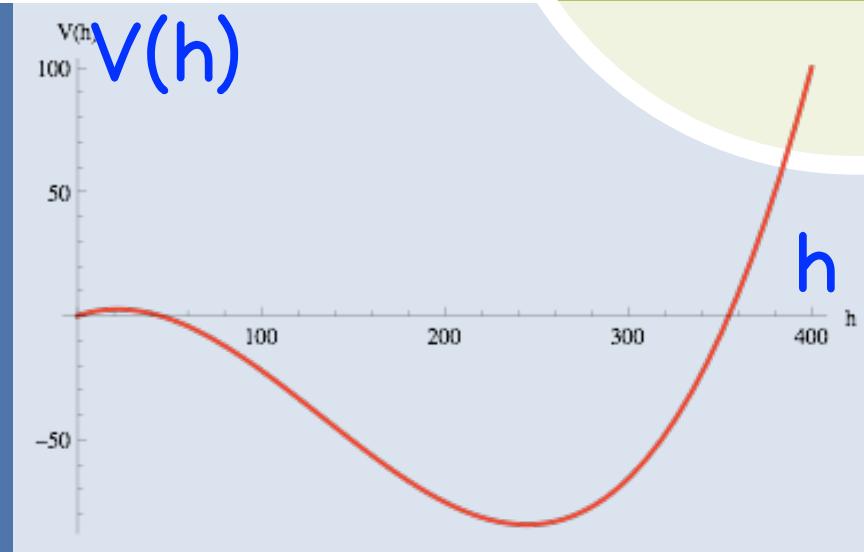


But pions has no vev



only a positive mass

The origin of the Higgs potential

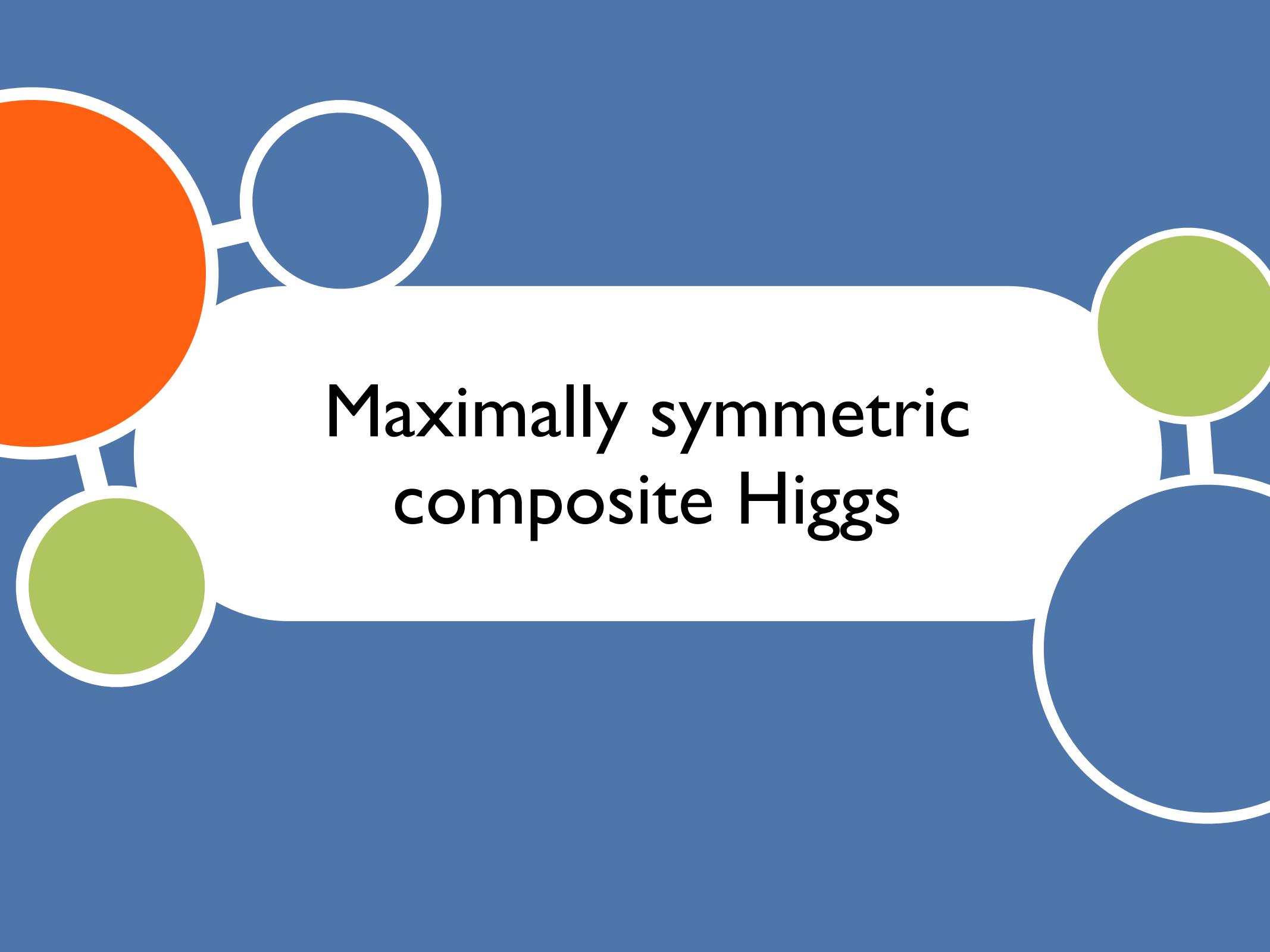


- The “parton” mass for the composite Higgs

L. Da, T. Ma, J. Shu, in preparation

- Quantum corrections from the SM particles (Mostly top)?

More focused top sector



# Maximally symmetric composite Higgs

# Why another CHM?

○○○ Can get the correct EWSB.

- Can easily get the 125 GeV light Higgs mass?
- No UV dependence of Higgs potential.  
gauge hierarchy problem
- Can we have minimal technical tuning
- A general methods based on symmetric coset space  
to describe the EWSB (Higgs as a pNGB) in an unified  
manner.  
Simplest Structure
- Find the new symmetry breaking pattern (Maximal  
symmetry) automatically solves all problems above

# Symmetric space



For any global symmetry  $G$  breaks into  $H$

$T^{\hat{a}}(T^a)$  is the (un)broken generator

$$[T^a, T^a] \sim T^a, [T^a, T^{\hat{a}}] \sim T^{\hat{a}}$$

$$[T^{\hat{a}}, T^{\hat{a}}] \sim T^a$$

$H$  is a closed group

symmetric coset space

$G$  always has an automorphism

$$V T^a V^\dagger = T^a$$

$$V T^{\hat{a}} V^\dagger = -T^{\hat{a}}$$

Higgs is the NGB in the symmetric coset space

$V$  is also the Higgs parity operator, like pion parity in QCD

# Examples:



Examples of Symmetric Coset Space:

$$SU(M+N)/SU(M) \times SU(N) \times U(1)$$

$$SO(M+N)/SO(M) \times SO(N)$$

$$SO(5)/SO(4)$$

$$SU(2N)/Sp(2N)$$

$$SU(N)/SO(N)$$

$$G \times G/G_V$$

$$V = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & -1 & -1 \end{pmatrix}$$

$$V = \begin{pmatrix} \mathbf{1}_{4 \times 4} & 0 \\ 0 & -1 \end{pmatrix}$$

Actually  
cover  
almost all  
useful  
cosets

$$\Phi = \mathbf{1}_{N \times N} \times (i\sigma_2)$$

$$\Phi = \begin{pmatrix} 0 & 1_{\frac{N}{2} \times \frac{N}{2}} \\ 1_{\frac{N}{2} \times \frac{N}{2}} & 0 \end{pmatrix} \quad N = 2l$$

$$\Phi = \begin{pmatrix} 0 & 1_{\frac{N-1}{2} \times \frac{N-1}{2}} \\ 0 & 1 & 0 \\ 1_{\frac{N-1}{2} \times \frac{N-1}{2}} & & 0 \end{pmatrix} \quad N = 2l + 1$$

# Goldstone in symmetric space



For any global symmetry  $G$   
spontaneously breaks into  $H$

If decouple the “Higgs”

$$U = \exp\left(\frac{ih^{\hat{a}}T^{\hat{a}}}{f}\right)$$

The CCWZ transformation

$$U \rightarrow gUh(h^{\hat{a}}, g)^\dagger$$

For any symmetric coset space

Key  
construction

$$\Sigma' = U^2V$$

Information of  $G/H$   
is included in  $V$

Simple linear transformation

$$\tilde{U} = VUV^\dagger = U^\dagger.$$

$$\Sigma' \rightarrow g\Sigma'g^\dagger .$$

Goldstone matrix transform linearly!

# G/H CW potential from top



Consider the MCHM5  $\text{SO}(5)/\text{SO}(4)$

SM Fermion

Embed the SM fields into fund rep of G (spurionic)

$$\Psi_{q_L} = \frac{1}{\sqrt{2}} \begin{pmatrix} b_L \\ -ib_L \\ t_L \\ it_L \\ 0 \end{pmatrix} \quad \Psi_{t_R} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ t_R \end{pmatrix}$$

$$\Sigma' \rightarrow g\Sigma'g^\dagger .$$

$$\Lambda^L = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 1 & -i & 0 \\ 1 & i & 0 & 0 & 0 \end{pmatrix} \text{ spurion vev } \Lambda^R = (0 \ 0 \ 0 \ 0 \ 1)$$

$$\Psi_{Q_L} = \Lambda_L^\alpha Q_L^\alpha$$

$$\Psi_{t_R} = \Lambda_R t_R$$

$$Q_L^\alpha t_R \text{ SU(2) multiplet}$$

$$\Sigma' = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & c_{2h} & -s_{2h} \\ 0 & 0 & 0 & -s_{2h} & -c_{2h} \end{pmatrix}$$

# G/H CW potential from top



Master formular

$$\begin{aligned}\mathcal{L}_{\text{eff}} = & \bar{\Psi}_{Q_L} \not{p} (\Pi_0^q(p) + \Pi_1^q(p)\Sigma') \Psi_{Q_L} \\ & + \bar{\Psi}_{t_R} \not{p} (\Pi_0^t(p) + \Pi_1^t(p)\Sigma') \Psi_{t_R} \\ & + \bar{\Psi}_{Q_L} M_1^t(p)\Sigma' \Psi_{t_R} + h.c.\end{aligned}$$

Converting back  
to the SM fields by  
using spurions.

$$P_l^{\alpha\beta} = (\Lambda_L^\beta)^\dagger \Lambda_L^\alpha, \quad P_r = (\Lambda_R)^\dagger \Lambda_R \quad P_{lr}^\alpha = (\Lambda_R)^\dagger \Lambda_L^\alpha.$$

Master formular

Most general Lagrangian

$$\Sigma'^2 = 1$$

$$\Sigma' \rightarrow g\Sigma'g^\dagger.$$

$$\Psi \rightarrow g\Psi \quad \text{fund rep}$$

Derivatives of GB does not contribute to Higgs potential

$$\begin{aligned}\mathcal{L}_{\text{eff}} = & \bar{Q}_L^\alpha \not{p} \text{Tr}[(\Pi_0^q + \Pi_1^q \Sigma') P_l^{\alpha\beta}] Q_L^\beta \\ & + \bar{t}_R \not{p} \text{Tr}[(\Pi_0^t + \Pi_1^t \Sigma') P_r] t_R \\ & + M_1^t \bar{Q}_L^\alpha t_R \text{Tr}[\Sigma'. P_{lr}^\alpha],\end{aligned}$$

Based on G/H, one can write whatever Lag  
contribute to Higgs potential

# Enlarged Global Symmetry

$$\Pi_1^{q,t} = 0$$

LH & RH top each has  $G_L \times G_R$  symmetry

$$G_L \quad \Psi_{Q_L} \rightarrow g_L \Psi_{Q_L}$$

$$G_L \quad \Psi_{t_R} \rightarrow g_R \Psi_{t_R}$$

global

Only acting on fermions, not  
Goldstones

Only the mass term breaks them into  $G_{V'}$

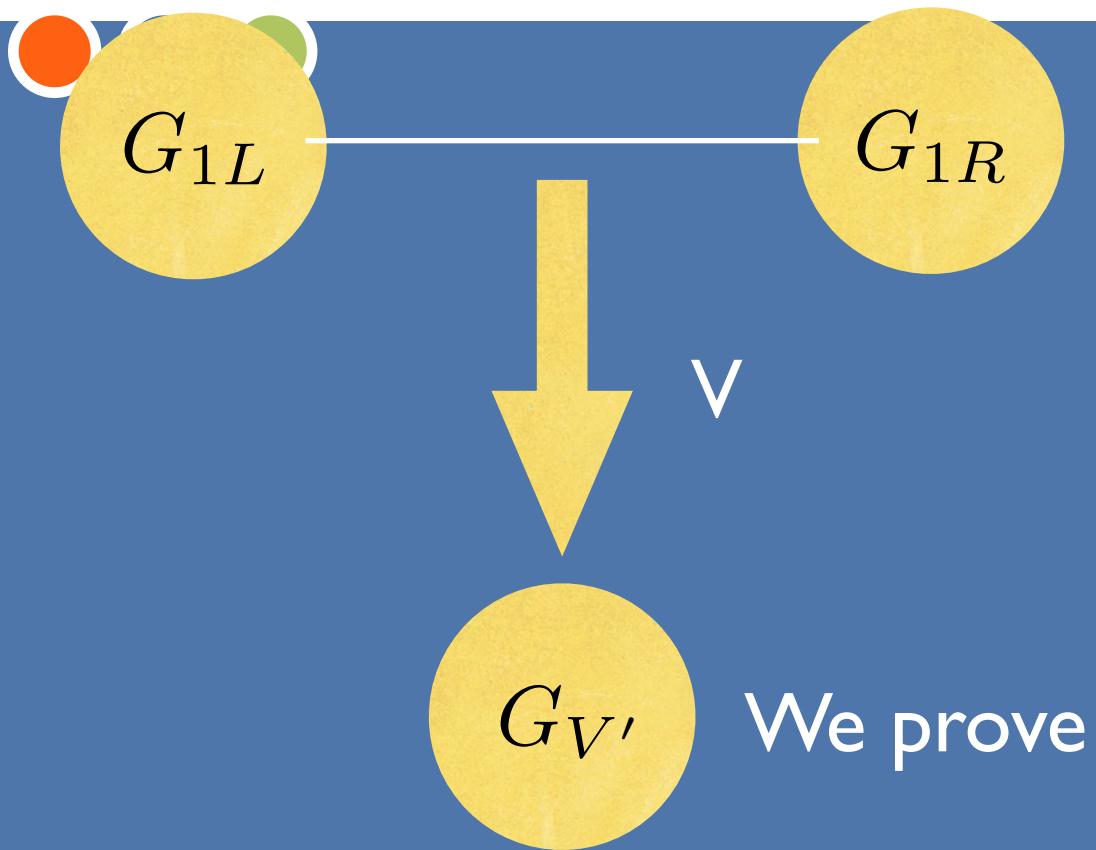
$g_L \Sigma' g_R^\dagger = \Sigma'$ . Maximal subgroup leaves the GB invariant

$$g'_{L,R} = U^\dagger g_{L,R} U$$

Global Symmetry from composites

$$g'_L V {g'_R}^\dagger = V$$

# What is Maximal Symmetry?



Can be used as a new UV  
completion of maximal symmetry  
just like moose model for LH,

$$SO(5)/SO(4)$$

$$V = \begin{pmatrix} 1_{4 \times 4} & 0 \\ 0 & -1 \end{pmatrix}$$

$$H_V$$

$$(G/H)_A$$

Mathematical structure

C. Csaki, T. Ma, J. Shu., Phys.Rev.Lett.  
119 (2017) no13, 131803

Appendix A

# The Higgs potential

$$V(h) = -2N_c \int \frac{d^4 p}{2\pi^4} \log \left[ 1 + \frac{(M_1^t)^2 |\text{Tr}[\Sigma' \cdot P_{lr}^1]|^2}{p^2 \text{Tr}[\Pi_0^q P_l^{11}] \text{Tr}[\Pi_0^t P_r]} \right]$$

$\sin(2h/f)$

Higgs potential is just the top mass square up to some factors after integration on momentum.

Log(1+x) expansion

CW  
potential

top mass from both LH & RH top  
mixture with top partners

No UV Divergence

$$M_1^t \sim \lambda_L \lambda_R f^2 (M_Q - M_S)/p^2$$

$$V(h) \sim \lambda_L^2 \lambda_R^2 f^4 (M_Q - M_S)^2 / \Lambda^2$$

$$m_t \sim \sin_2 h$$

$$\text{Higgs potential } (\sin_2 h)^2 = s_h^2 - s_h^4.$$

# Understand deconstruction



Collective Symmetry Breaking, “Little Higgs”

$$\sim \int d^4 p \Pi_1^{q,t}$$

G/H 

G/H

G/H One need 3 G/H

G/H

structures

$$\Pi_1^{q,t} \propto p^{-2N} \text{ at UV}$$

N=3 for finite potential

Higgs potential tends to have large  $v/f > 1$  (double tuning)

Even for large N, still have very troublesome finite piece

Higgs mass tends to be large than 300GeV

Fine-tuning

# Realization from MCHM5



Fermion mass from linear mixing

$$\mathcal{L}_{mix} = \lambda \bar{q}_i \mathcal{O}_i$$

$$\mathcal{O}_i \sim U \Psi_i$$

partial compositeness

$$\mathcal{L} = \lambda_L \bar{q}_L^\alpha \Lambda_{\alpha I}^L \mathcal{O}_R^I + \lambda_R \bar{t}_R \Lambda_{\alpha I}^R \mathcal{O}_L^I + h.c.$$

5=4+ | Composite top partners

$$\Psi_Q = \frac{1}{\sqrt{2}} \begin{pmatrix} iB - iX_{5/3} \\ B + X_{5/3} \\ iT + iX_{2/3} \\ -T + X_{2/3} \\ 0 \end{pmatrix} \quad \Psi_S = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ T_1 \end{pmatrix}$$

$$U \rightarrow gUh(h^{\hat{a}}, g)^\dagger$$

Fermionic Lagrangian:

Top and top partner masses:

$$m_t = \frac{\epsilon_{qQ}\epsilon_{tS}f^2}{2M_T M_{T_1}} \left| \frac{\epsilon_{qS}}{\epsilon_{qQ}} M_Q - \frac{\epsilon_{tQ}}{\epsilon_{tS}} M_S \right| \sin \frac{\langle h \rangle}{f}$$

$$\begin{aligned} \mathcal{L}_f &= \bar{\Psi}_Q (i\not{\!D} - M_Q) \Psi_Q + \bar{\Psi}_S (i\not{\!D} - M_S) \Psi_S \\ &+ \frac{\lambda_R f}{\sqrt{2}} \bar{\Psi}_{t_R} P_L (\epsilon_{tS} U \Psi_S + \epsilon_{tQ} U \Psi_Q) \\ &+ \lambda_L f \bar{\Psi}_{q_L} P_R (\epsilon_{qS} U \Psi_S + \epsilon_{qQ} U \Psi_Q) + h.c., \end{aligned}$$

$$M_T = \sqrt{\epsilon_{qQ}^2 f^2 + M_Q^2}, \quad M_{T_1} = \sqrt{\frac{\epsilon_{tS}^2}{2} f^2 + M_S^2}.$$

# The use of V



Rewrite the top partners into a full rep of G

$$\Psi_+ = V\Psi_- \quad V = \begin{pmatrix} \mathbf{1}_{4 \times 4} & 0 \\ 0 & -1 \end{pmatrix}$$

$$\Psi_+ = \frac{1}{\sqrt{2}}(\Psi_2 + \Psi_1) \quad \Psi_- = \frac{1}{\sqrt{2}}(\Psi_2 - \Psi_1)$$

$$c_{\pm R} = \frac{\epsilon_{tQ} \pm \epsilon_{tS}}{2}, \quad c_{\pm L} = \frac{\epsilon_{qQ} \pm \epsilon_{qS}}{\sqrt{2}}.$$

The Lagrangian is G invariant except for V

Elementary-composite  
mixing is G invariant

$$c_{-L} = c_{-R} = 0.$$

$$\begin{aligned} \mathcal{L}_f = & \bar{\Psi}_+ i\not{\partial} \Psi_+ + \bar{\Psi}_- i\not{\partial} \Psi_- + \lambda_R f c_{+R} \bar{\Psi}_{t_R} P_L U \Psi_+ \\ & + \lambda_L f c_{+L} \bar{\Psi}_{q_L} P_R U \Psi_+ - (M_Q + M_S) \bar{\Psi}_{+L} \Psi_{+R} \\ & - (M_Q - M_S) \bar{\Psi}_{+L} V \Psi_{+R} + h.c. \end{aligned} \quad (22)$$

Enlarged global sym:

$$SO(5)_L \times SO(5)_R$$

# Symmetries in CS



Two vector mass: twisted and untwisted:

$$SO(5)_L \quad \Psi_{+L} \rightarrow g'_L \Psi_{+L}$$

$$SO(5)_R \quad \Psi_{+L} \rightarrow g'_L \Psi_{+L}$$

The mass term **explicitly** break the global symmetry

**Maximal Symmetry: Only the Twisted Mass**

$$M_Q - M_S = 0 \Rightarrow SO(5)_L \times SO(5)_R / SO(5)_V$$

$$M_Q + M_S = 0 \Rightarrow SO(5)_L \times SO(5)_R / SO(5)_{V'} \quad g'_L V g'_R^\dagger = V$$

$$|M_Q| \neq |M_S| \Rightarrow SO(5)_L \times SO(5)_R / SO(4)_V \quad (23)$$

# The form factors

Integrating out the top partners, we have the form factors in the EFT

$$\begin{aligned} \frac{\Pi_0^{q,t}}{\lambda_{L,R}^2 f^2} &= 1 + \frac{(c_{-L,R}^2 + c_{+L,R}^2)(M_Q^2 + M_S^2 - 2p^2)}{2(p^2 - M_S^2)(M_Q^2 - p^2)} \\ &\quad + \frac{c_{-L,R}c_{+L,R}(M_S + M_Q)(M_S - M_Q)}{(p^2 - M_S^2)(M_Q^2 - p^2)} \\ \frac{\Pi_1^{q,t}}{\lambda_{L,R}^2 f^2} &= \frac{c_{+L,R}c_{-L,R}(M_Q^2 + M_S^2 - 2p^2)}{(p^2 - M_S^2)(M_Q^2 - p^2)} \\ &\quad + \frac{(c_{+L,R}^2 + c_{-L,R}^2)(M_S - M_Q)(M_S + M_Q)}{2(p^2 - M_S^2)(M_Q^2 - p^2)} \\ \frac{M_1^t}{\lambda_L \lambda_R f^2} &= \frac{M_Q^2 M_S (c_{-L} - c_{+L})(c_{-R} - c_{+R})}{2(p^2 - M_Q^2)(p^2 - M_S^2)} \\ &\quad - \frac{M_S^2 M_Q (c_{-L} + c_{+L})(c_{-R} + c_{+R})}{2(p^2 - M_Q^2)(p^2 - M_S^2)} \\ &\quad + \frac{M_Q (c_{-L} + c_{+L})(c_{-R} + c_{+R}) p^2}{2(p^2 - M_Q^2)(p^2 - M_S^2)} \\ &\quad - \frac{M_S (c_{-L} - c_{+L})(c_{-R} - c_{+R}) p^2}{2(p^2 - M_Q^2)(p^2 - M_S^2)}, \end{aligned} \tag{54}$$

# Vh structure from symmetry



$$SO(5)_{V'}$$

UV finite

$c_{+L}$  is turned off, Higgs shift symmetry  $h^{\hat{a}} \rightarrow h^{\hat{a}} + \alpha^{\hat{a}}$

$$\Psi_{+L} \rightarrow g'_L \Psi_{+L}$$

subgroup of  
transformation on the left

$$\Psi_{+R} \rightarrow V g'_L \Psi_{+R}$$

$$(g'_L = \exp(i\alpha^{\hat{a}} T^{\hat{a}}))$$

$$|\lambda_L \lambda_R|^2 c_{+L}^2 c_{+R}^2 f^4 (M_1 - M_2)^2 / \Lambda^2. \quad (26)$$

Top mass square!

$$m_t = c_{+L} c_{+R} (M_Q - M_S) f^2 / (2 M_T M_{T_1})$$

$$SO(4)_V$$

Log divergent

$$V_{L\xi} \sim |\lambda_L|^2 c_{+L}^2 f^2 (M_1 + M_2)(M_1 - M_2) \log \Lambda^2 \quad (24)$$

# Higgs potential tuning

$$V_f(h) \simeq -\gamma_f s_h^2 + \beta_f s_h^4,$$

$$\xi = \frac{\gamma_f}{2\beta_f}$$

To obtain  $\xi \ll 1$

$\gamma_{\text{div}}$  much smaller than  $\beta_{\text{div}}$

$$\begin{aligned} V_{\text{div}} &= \frac{N_c M_f^4}{16\pi^2 g_f^2} \left[ \left( \frac{c_L}{2} \epsilon_L^2 - c_R \epsilon_R^2 + c_{LL} \frac{\epsilon_L^4}{g_f^2} + c_{RR} \frac{\epsilon_R^4}{g_f^2} \right) s_h^2 \right. \\ &\quad \left. + \left( c'_{LL} \frac{\epsilon_L^4}{g_f^2} + c'_{RR} \frac{\epsilon_R^4}{g_f^2} \right) s_h^4 \right] \\ &\equiv -\gamma_{\text{div}} s_h^2 + \beta_{\text{div}} s_h^4 \end{aligned} \quad (34)$$

Large finite piece from \PI\_I form factor (expansion over s\_h or c\_h) tends to make \gamma >> \beta

log divergent

$\mathcal{O}(\epsilon_L^4)$  and  $\mathcal{O}(\epsilon_R^4)$ .

quadratic divergent parts

$\mathcal{O}(\epsilon_L^2)$  and  $\mathcal{O}(\epsilon_R^2)$

$c_L \sim c_R \sim \Lambda^2$

If UV divs cancels but finite remains

$c_{LL} \sim c'_{LL} \sim c_{RR} \sim c'_{RR} \sim \log \Lambda$

$$\Delta^{5+5} \simeq \frac{1}{\xi} \frac{g_f^2}{\epsilon^2}$$

Double tuning

# Tunnings in EWSB

$$\begin{aligned}
 V_h &= c_{LR} \frac{N_c M_f^4}{16\pi^2} \left( \frac{\epsilon_L^2 \epsilon_R^2}{g_f^4} \right) [-s_h^2 + s_h^4] + \mathcal{O}\left(\frac{\epsilon_L^4 \epsilon_R^4}{g_f^8}\right) \\
 &\simeq c_{LR} \frac{N_c M_f^4}{16\pi^2} \left( \frac{y_t}{g_f} \right)^2 [-s_h^2 + s_h^4] + \mathcal{O}\left(\frac{y_t^4}{g_f^4}\right) \\
 &\equiv -\gamma_f s_h^2 + \beta_f s_h^4
 \end{aligned} \tag{39}$$

Maximally  
symmetric case

$$\xi = \frac{\gamma_f}{2\beta_f} = 0.5$$

Cancellation from the gauge sector

$$\gamma_g = -\frac{9f^2 g^2 m_\rho^2 \log 2}{64\pi^2}$$

$\xi \ll 1$ , we require  $\gamma_f \simeq -\gamma_g$ .

Assuming 1st & 2nd Weinberg sum rule, UV finite

$$\Delta^{(5+5)} = \frac{\max(|\gamma_f|, |\gamma_g|)}{|\gamma_f + \gamma_g|} \simeq \max\left(\frac{1}{2\xi}, \frac{1}{2\xi} - 1\right) = \frac{1}{2\xi} \quad (44)$$

20% tuning

# How to get 125 GeV Higgs?



$$m_t \sim \sin \theta_L \sin \theta_R |M_Q - M_S| s_h$$

Usually top is too heavy, difficult to get a light Higgs

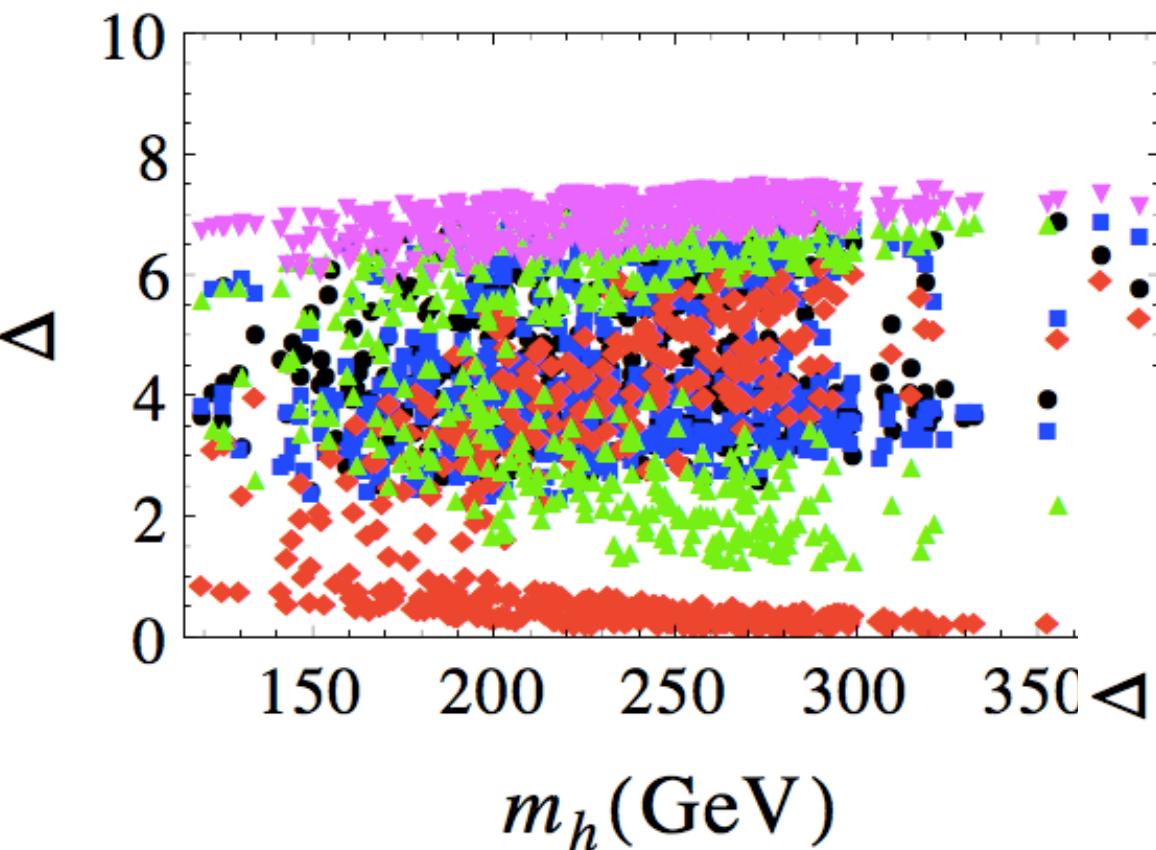
$\theta_L$  and  $\theta_R$  minimal

$$M_Q = -M_S$$

$$\min\{M_T, M_{T_1}\} = \min\left\{\frac{M_S}{\cos \theta_L}, \frac{M_Q}{\cos \theta_R}\right\} \text{ minimal}$$

$$m_H \propto \min\{M_T, M_{T_1}\} m_t / f$$

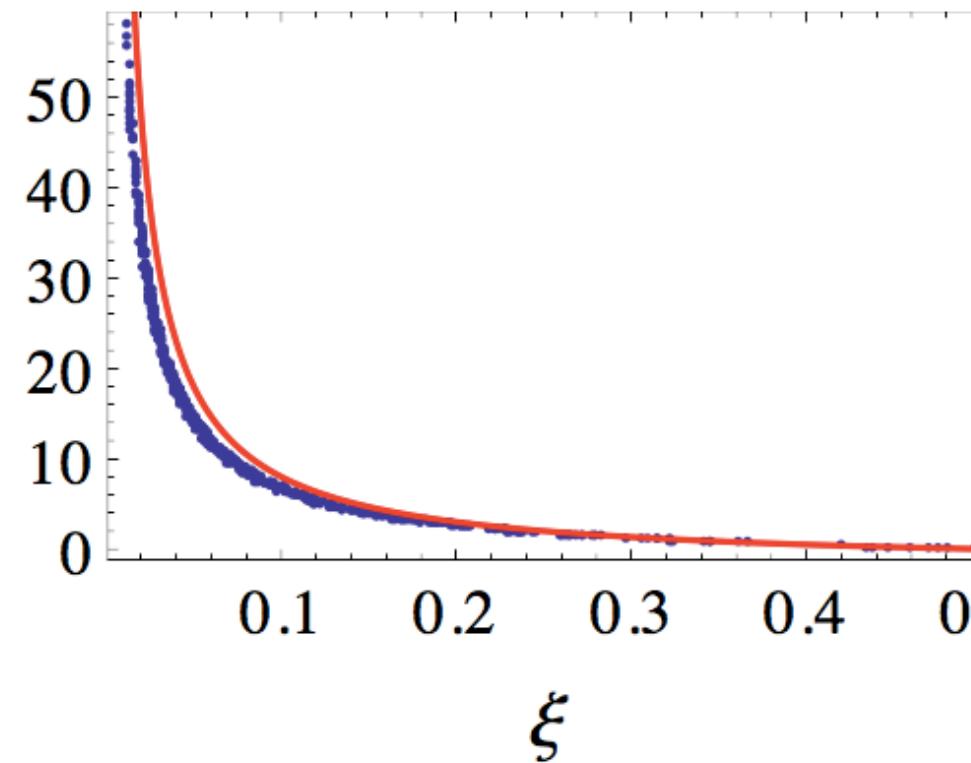
# Numerical tuning



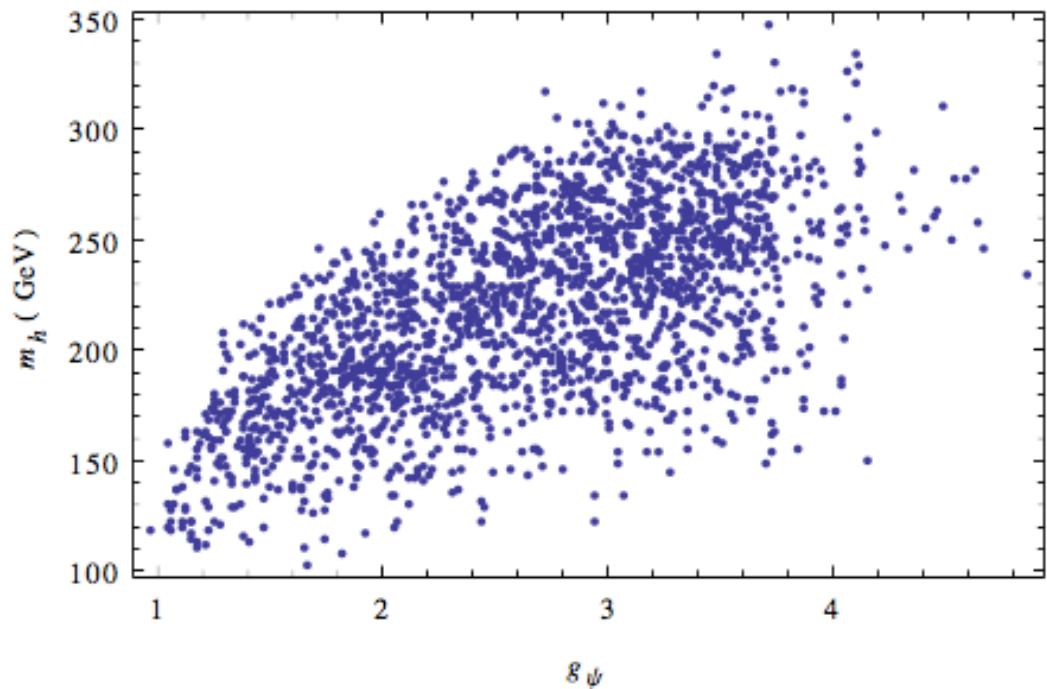
$$\Delta = \max \left| \frac{2x_i}{s_h} \frac{c_h^2}{f^2 m_h^2} \frac{\partial^2 V}{\partial x_i \partial s_h} \right|$$

$\Delta_m \simeq 2\gamma_g / |\gamma_f + \gamma_g| \simeq \frac{1}{\xi} - 2$

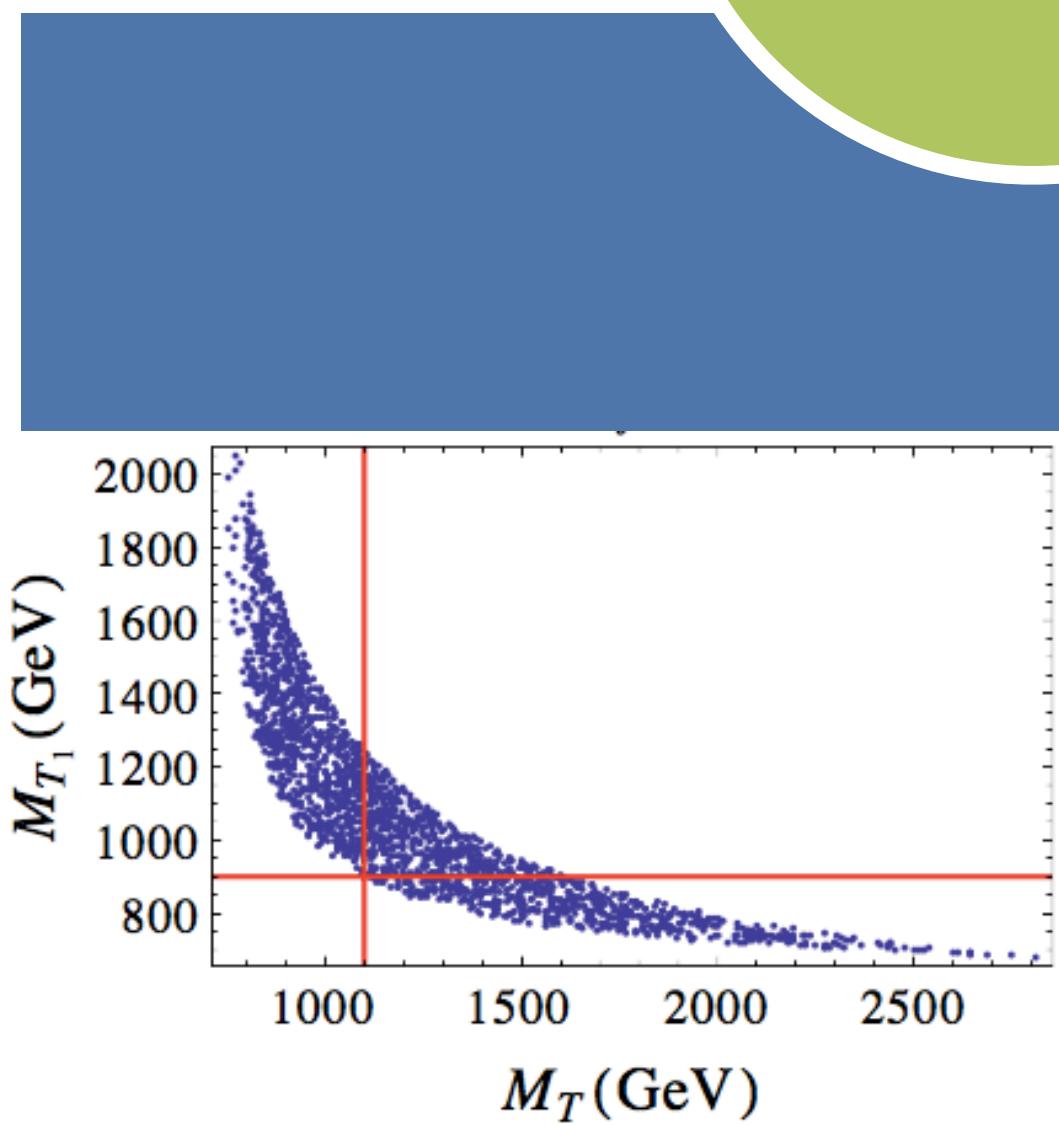
One free parameter except f

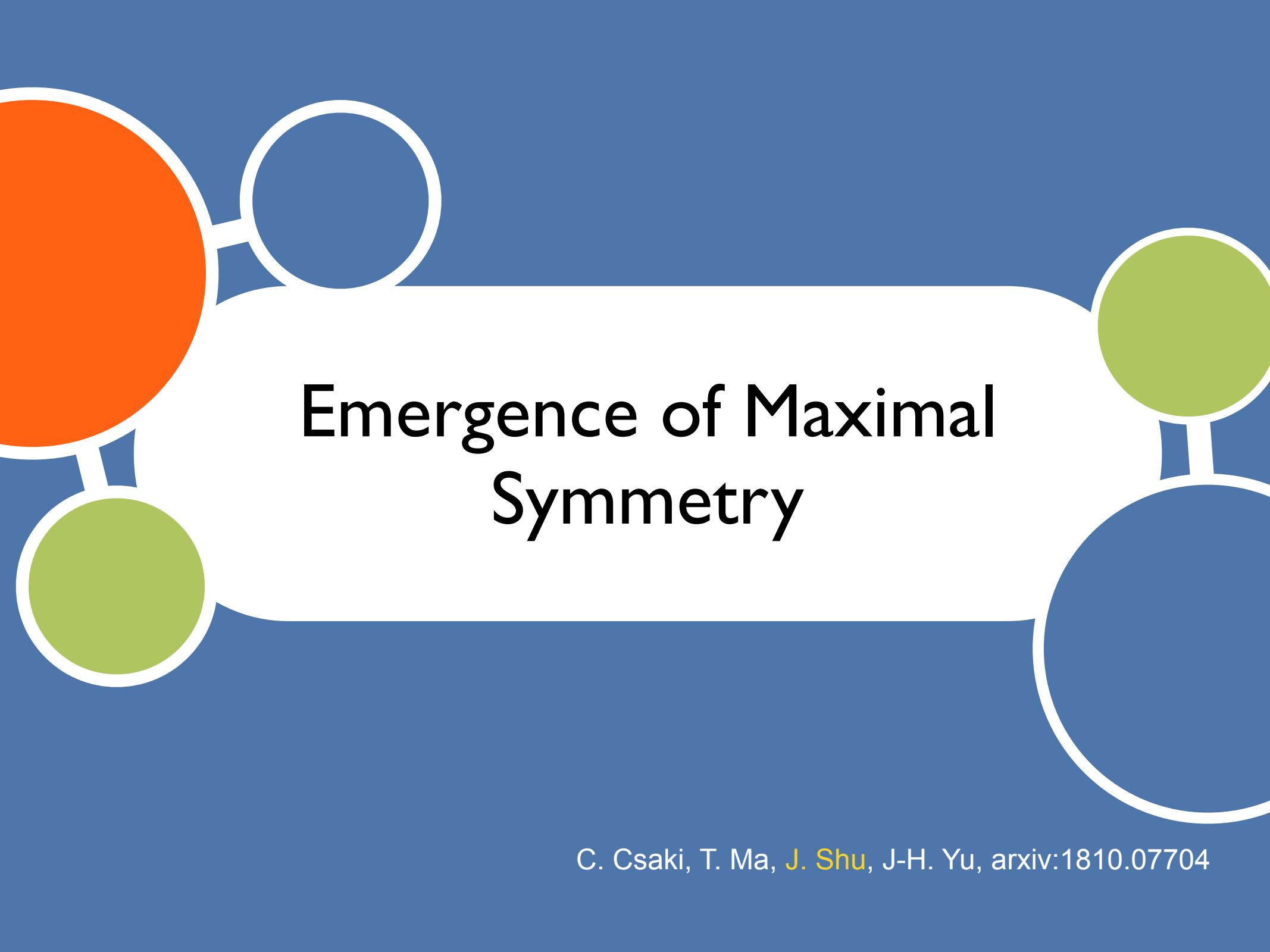


# Scan



One free parameter  
except for  $f$

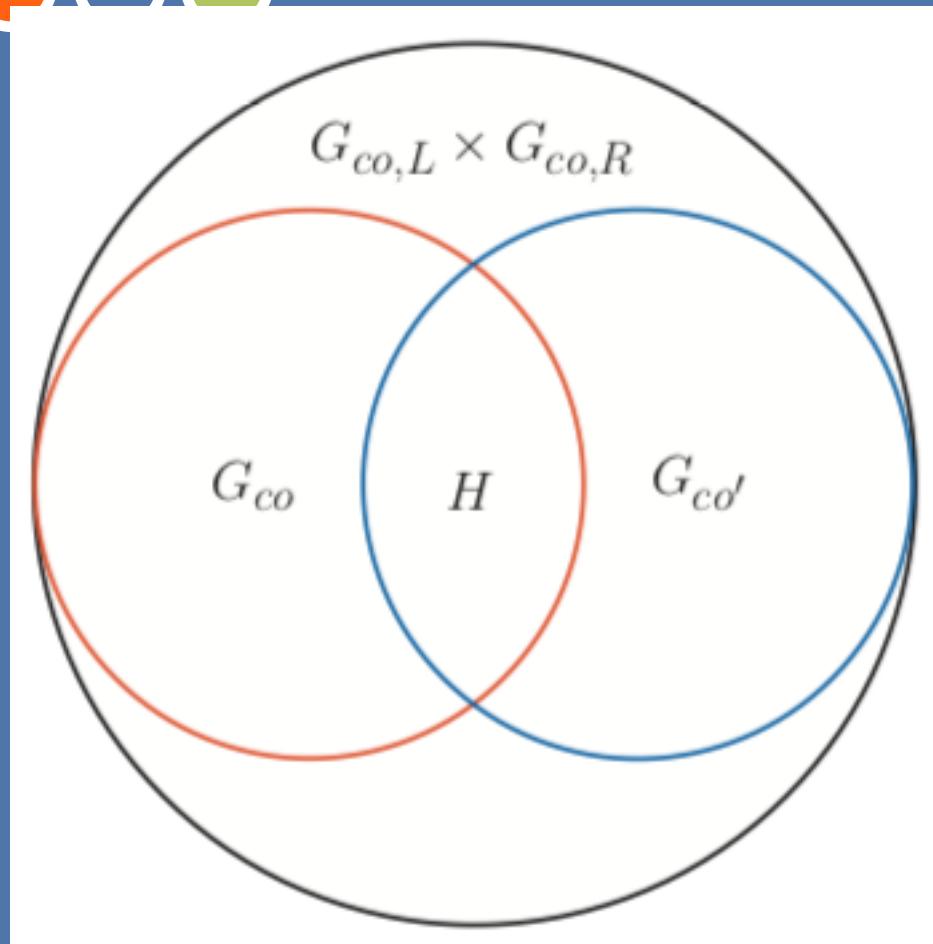




# Emergence of Maximal Symmetry

C. Csaki, T. Ma, J. Shu, J-H. Yu, arxiv:1810.07704

# Why a maximal symmetry?



sounds like symmetry inflow

Ordinary case global symmetry breaks into  $H$  at the boundary

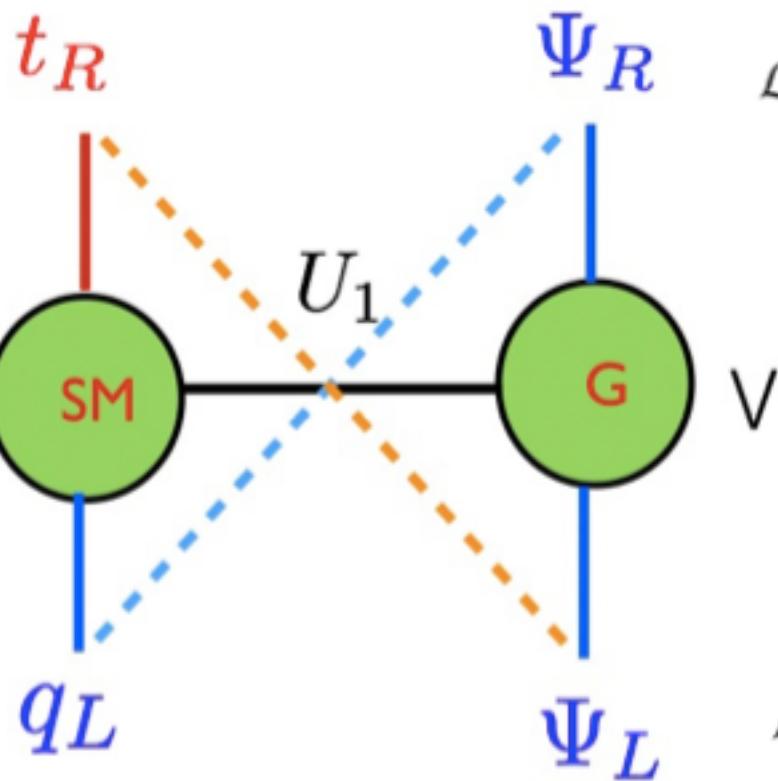
Maximal symmetry: global symmetry breaks into  $G_{\{V\}}$  at the boundary

Integrating out the bulk from this boundary (composite) preserve this global symmetry and transmitted it to the other boundary (SM elementary)

# How to realize a maximal symmetry?



Bulk fermion:



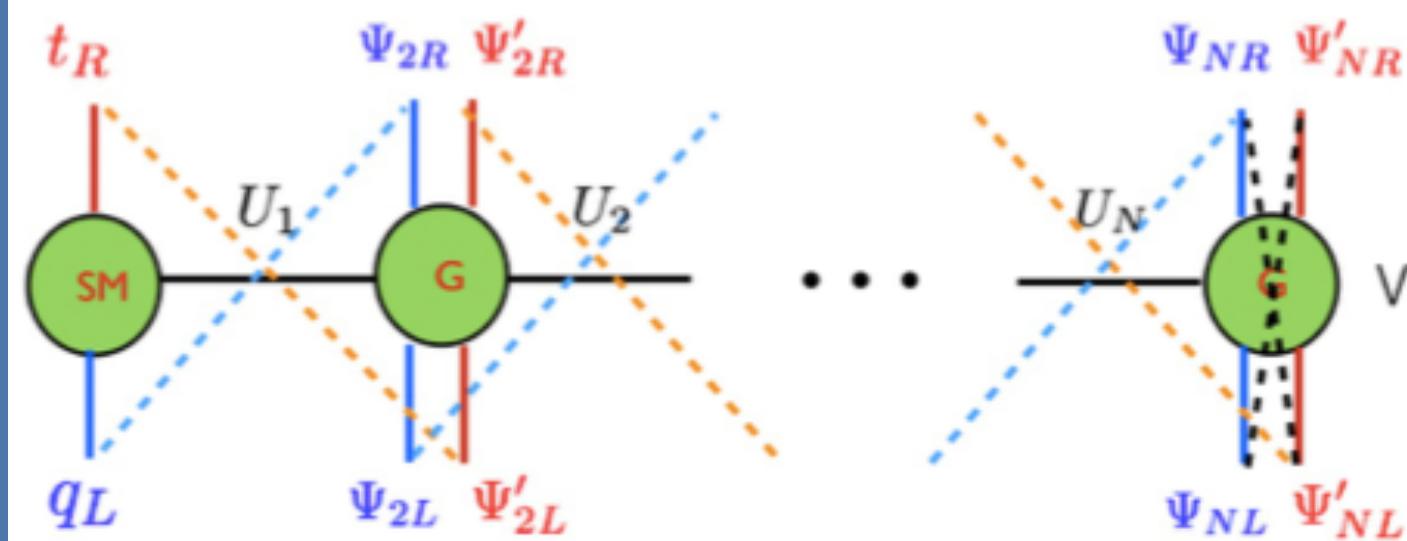
$$\mathcal{L}_{\text{eff}} = \bar{\Psi}_{q_L} \not{p} (\Pi_0^L(p) + \Pi_1^L(p)\Sigma') \Psi_{q_L} - \bar{\Psi}_{q_L} M_1^t(p)\Sigma' \Psi_{t_R} \\ + \bar{\Psi}_{t_R} \not{p} (\Pi_0^R(p) + \Pi_1^R(p)\Sigma') \Psi_{t_R} + h.c. , \quad (2)$$

Both LH & RH fields are in the fundamental representation

UV completion of two site moose

$$\mathcal{L}_f = \bar{q}_L i\not{D} q_L + \bar{\Psi} i\not{D} \Psi + \bar{t}_R i\not{D} t_R \\ - \epsilon_L \bar{\Psi}_{q_L} U_1 \Psi_R - M \bar{\Psi}_L \Sigma \Psi_R - \epsilon_R \bar{\Psi}_L U_1^\dagger \Psi_{t_R} + h.c. \quad (5)$$

# How to realize a maximal symmetry?

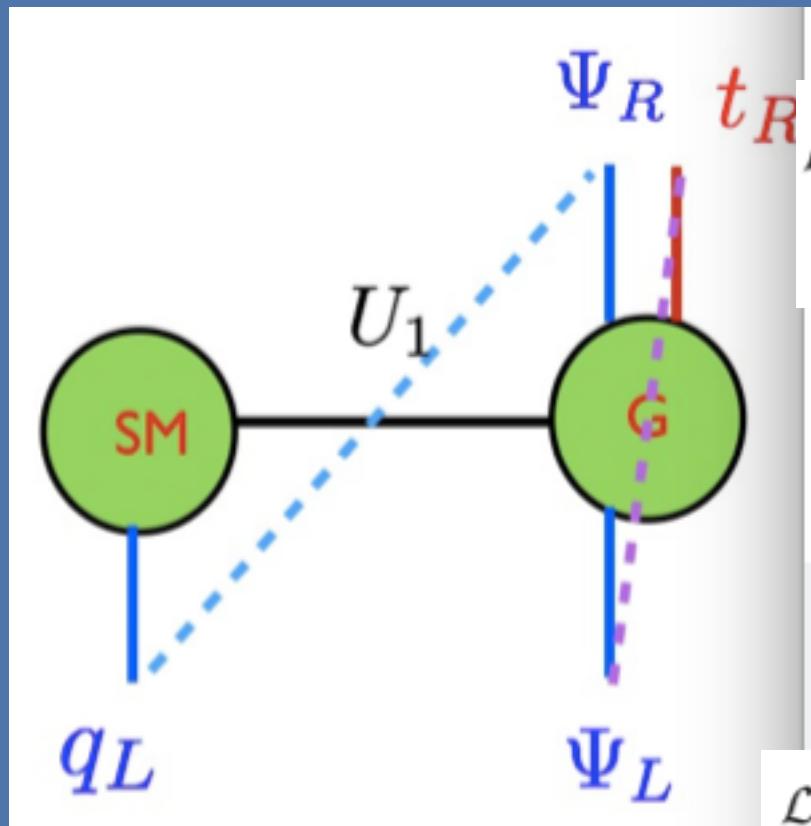


$$\begin{aligned}
 \mathcal{L}_f = & \bar{q}_L i \not{D} q_L + \bar{t}_R i \not{D} t_R + \sum_{i=2}^N (\bar{\Psi}_i i \not{D}_i \Psi_i) \\
 & - \epsilon_1 \bar{\Psi}_{q_L} U_1 \Psi_{2R} - \sum_{j=2}^{N-1} \epsilon_j \bar{\Psi}_{jL} U_j \Psi_{j+1R} - M_i \sum_{i=2}^N \bar{\Psi}_{iL} \Psi_{iR} \\
 & - \epsilon'_1 \bar{\Psi}_{t_R} U_1 \Psi'_{2L} - \sum_{j=2}^{N-1} \epsilon'_j \bar{\Psi}'_{jL} U_j \Psi'_{j+1R} - M'_i \sum_{i=2}^N \bar{\Psi}'_{iL} \Psi'_{iR} \\
 & - M (\bar{\Psi}_{NL} \Sigma \Psi'_{NR} + \bar{\Psi}'_{NL} \Sigma \Psi_{NR}) + h.c. \tag{11}
 \end{aligned}$$

# Why a maximal symmetry?



Boundary fermion:



$$\begin{aligned}\mathcal{L}_{\text{eff}} = & \bar{\Psi}_{q_L} \not{p} (\Pi_0^L(p) + \Pi_1^L(p)\Sigma') \Psi_{q_L} + \bar{\Psi}_{t_R} \not{p} \Pi_0^R(p) \Psi_{t_R} \\ & + \bar{\Psi}_{q_L} M_1^t(p) U \Psi_{t_R} + h.c.\end{aligned}\quad (3)$$

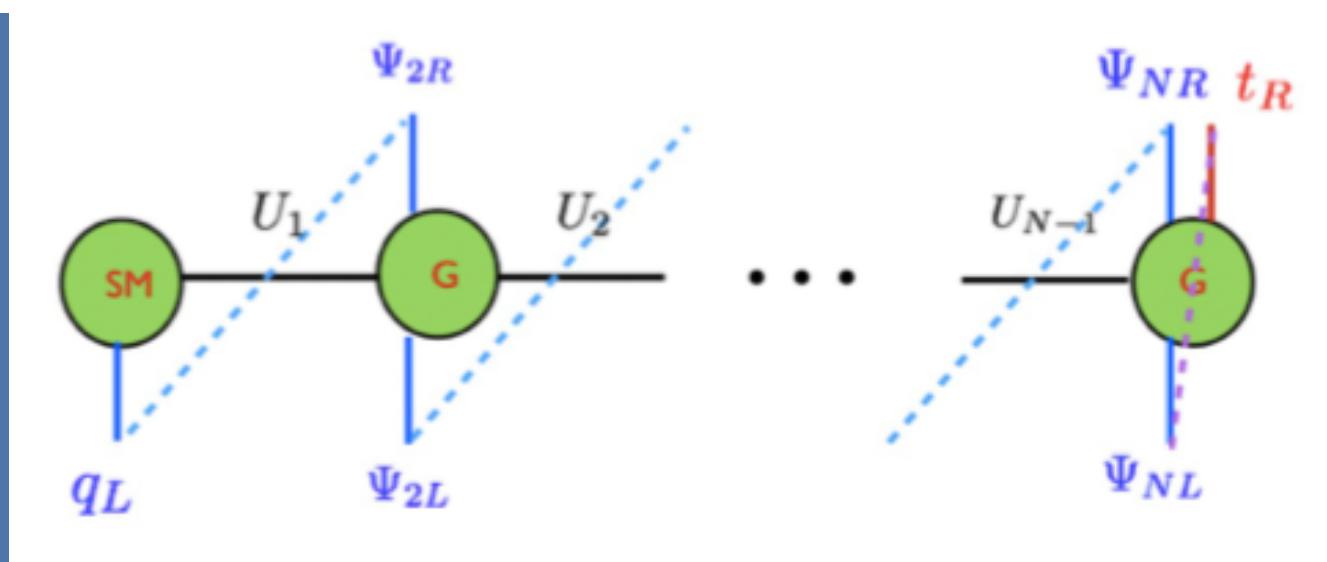
RH SM fermions are **singlets**

UV completion of two site moose

$$\mathcal{H} = U^\dagger \mathcal{V} \text{ with } \mathcal{V} = (0, 0, 0, 0, 1).$$

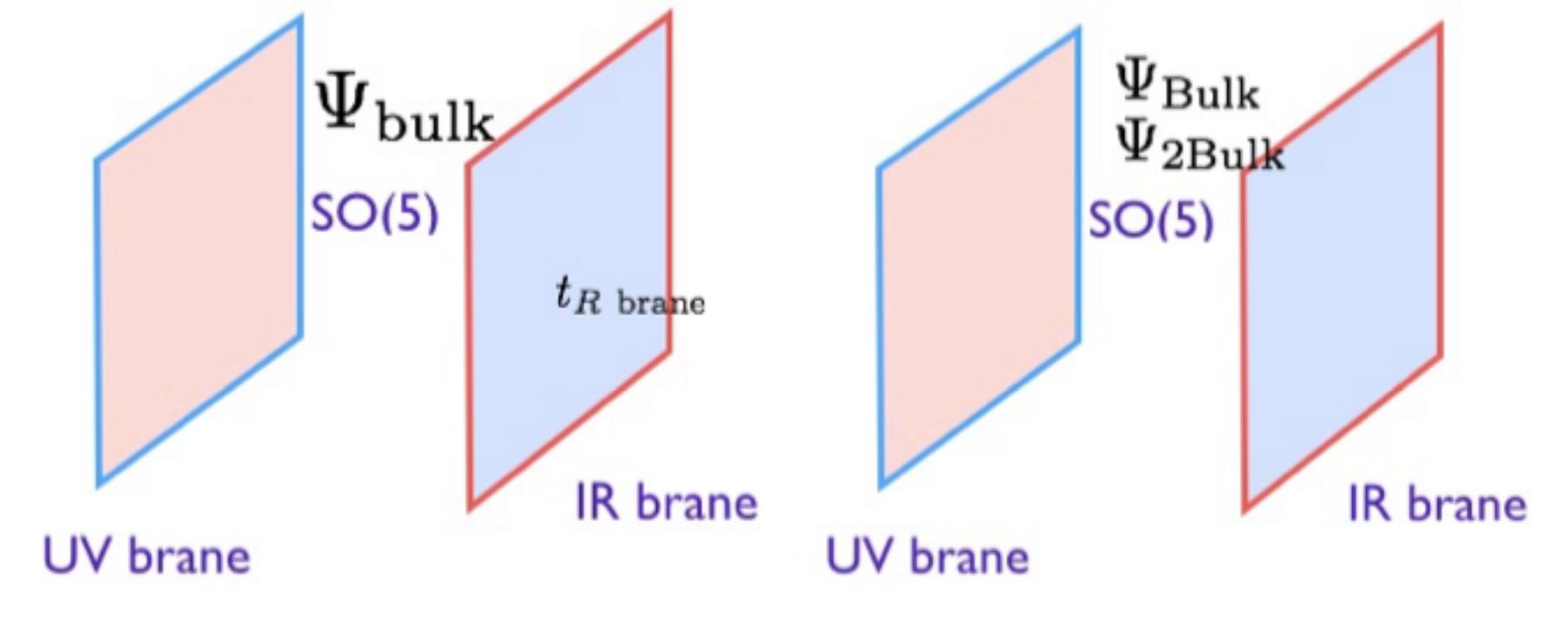
$$\begin{aligned}\mathcal{L}_f = & \bar{q}_L i \not{D} q_L + \bar{\Psi} i \not{D} \Psi + \bar{t}_R i \not{D} t_R \\ & - \epsilon_L \bar{\Psi}_{q_L} U_1 \Psi_R - M \bar{\Psi}_L \Psi_R - \epsilon_R \bar{\Psi}_L \mathcal{H}' t_R + h.c.\end{aligned}\quad (6)$$

# Why a maximal symmetry?



$$\begin{aligned}\mathcal{L}_f = & \bar{q}_L iD\!\!\!/ q_L + \sum_{i=2}^N \bar{\Psi}_i iD\!\!\!/ \Psi_i + \bar{t}_R iD\!\!\!/ t_R \\ & - \epsilon_1 \bar{\Psi}_{q_L} U_1 \Psi_{2R} - \sum_{j=2}^{N-1} \epsilon_j \bar{\Psi}_{jL} U_j \Psi_{j+1R} \\ & - \sum_{i=2}^N M_i \bar{\Psi}_{iL} \Psi_{iR} - \epsilon_N \bar{\Psi}_{NL} \mathcal{H}' t_R + h.c. \quad (7)\end{aligned}$$

# Extra dimension case:



$$U(R, R') = \text{Exp}\left(i \frac{-\sqrt{2}\pi^{\hat{a}} T^{\hat{a}}}{f}\right),$$

After integrating out the bulk

$$\begin{aligned} \mathcal{L}_{eff} = & \bar{\chi}'_L \not{p} \Pi_L(p) \chi'_L + \bar{t}_R \not{p} t_R \\ & + (M(p) \bar{\chi}'_L \mathcal{H} t_R + h.c.) \end{aligned}$$

$$\begin{aligned} \mathcal{L}_H = & \bar{\chi}_L \not{p} \Pi^L(\tilde{m}) \chi_L - \bar{\psi}_R \not{p} \Pi^R(\tilde{m}) \psi_R \\ & + M^{LR} (\bar{\chi}_L V \psi_R + \bar{\psi}_R V \chi_L), \end{aligned}$$

# Comment



What I feel interesting or critical is that:

The **boundary symmetry** completely controls the **bulk pNGB properties**, in particular, the UV sensitivity of the pNGB Coleman-Weinberg potential

I wonder if there is a application  
in condense matter physics?

MS in the lattice can also be applied to low-dim  
condense matter system (Bilayer Quantum Hall System?)

# Naturalness Sum rules

C. Csaki, F. Freitas, L. Huang, T. Ma, J. Shu, M. Perelstein, arxiv:1811.01961

# Test and predictions



Top kinetic terms: no corrections from Higgs

$$M_t(h) \sim \sin\left(\frac{2h}{f}\right) \left(1 + \frac{1}{2} \sin^2(h/f) (\Pi_1^q(0) - \Pi_1^t(0))\right)$$

C. Csaki, T. Ma, J. Shu., 1702.00405  
D. Liu, I. Low, C. Wagner, 1703.07791

However, the ggh coupling only scale with  
the derivative of the first part.

$$c_g = c_t$$

Maximal Symmetry limit

100 TeV perhaps tth 1%

# Test and predictions



Find the top partner resonance (charge 2/3), sum rule of  
diagonal Higgs Yukawa & mass  
**Mass eigenstates**

$$\text{Tr}[Y_m M_D] = 0 + \mathcal{O}(v^2)$$

$$\text{Tr}[Y_m M_D^3] = 0 + \mathcal{O}(v^2/M_f^2)$$

No quadratic div

No log div

C. Csaki, T. Ma, J. Shu., Phys.Rev.Lett. 119 (2017) no13, 131803

$$M_Q + M_S = 0$$

Lightest exotic charge (5/3)

# Gauge Sum rules



Quadratic divergence

$$\text{Tr}[g_{VV}h] = 0 + \mathcal{O}(\tilde{v}^2/f^2).$$

Log divergence

$$\text{Tr}[g_{VV}h M_V^2] = 0 + \mathcal{O}(\tilde{v}^2/f^2),$$

# SUSY Case

$$\text{Tr}[g_{SSH}] - 2\text{Tr}[Y_M M_D^\dagger + M_D Y_M^\dagger] + 3\text{Tr}[g_{VVH}] = 0,$$

$$\text{Tr}[g_{SSH}] - 4\text{Tr}[Y_M M_D] + 3\text{Tr}[g_{VVH}] = 0.$$

Quadratic divergence

Top sector/stop sector

Gauge/gaugino/Higgs/Higgsino sector

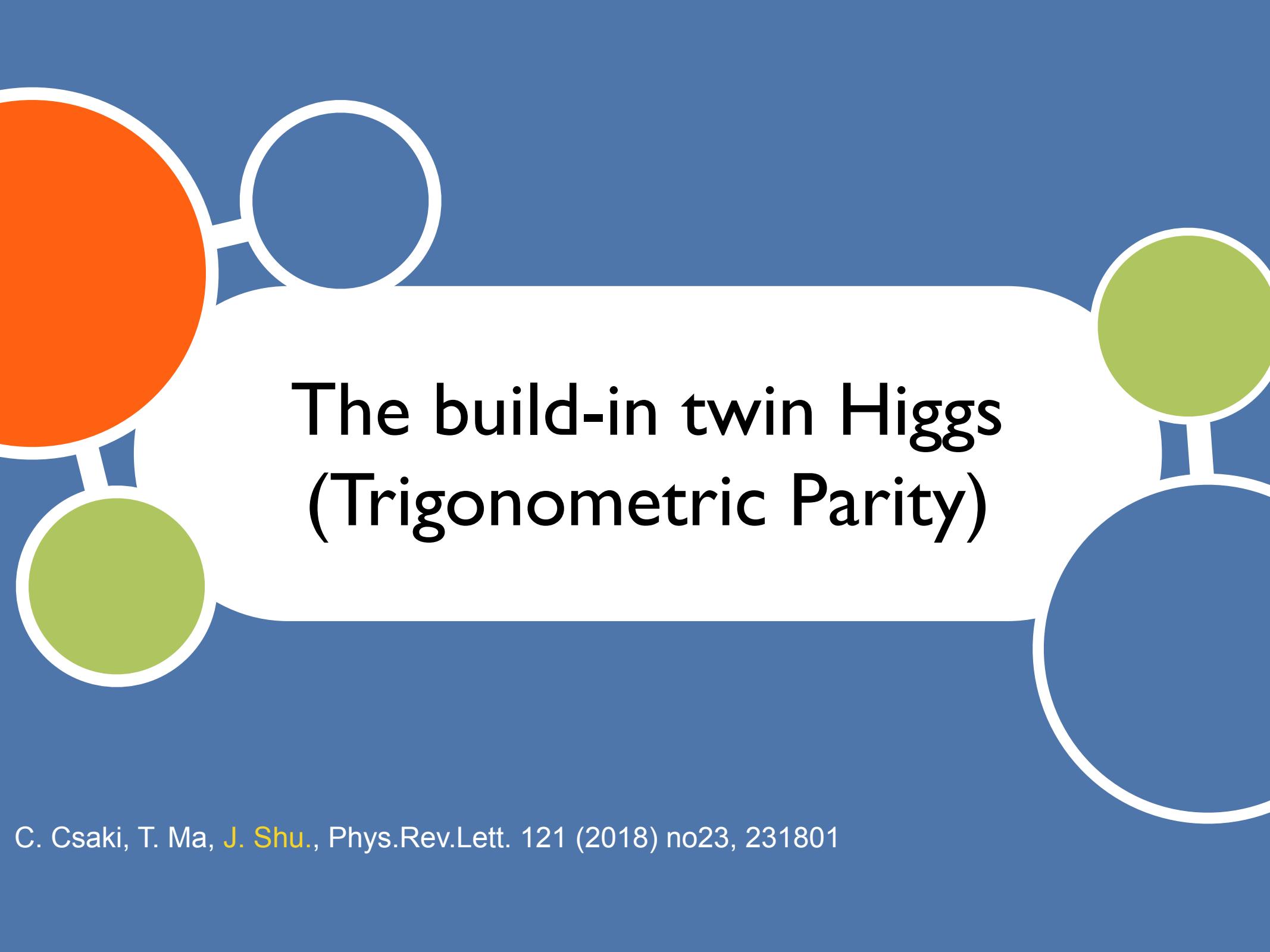
$$\sum_i g_{\tilde{t}_i \tilde{t}_i h} - 4y_t m_t = 0,$$

$$4 \sum_i (y_{C_i^+ C_i^- h} m_{C_i} + y_{N_i N_i h} m_{N_i}) - 3(g_{W^+ W^- h} + g_{ZZ h}) \\ - \sum_i (g_{H_i^0 H_i^0 h} + g_{H_i^+ H_i^- h}) - g_{hh h} = 0$$

# Non-SUSY Case: Collider



Non-susy case done with signs! See the talk tomorrow!



# The build-in twin Higgs (Trigonometric Parity)

C. Csaki, T. Ma, J. Shu., Phys.Rev.Lett. 121 (2018) no23, 231801

# Why twin Higgs?

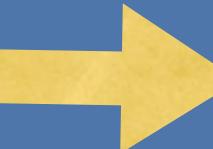


Z. Chacko, H.-S. Goh, R. Harnik, Phys.Rev.Lett. 96 (2006) 231802

The key reason is that we still do not see the colored top partner yet!

Colored top partners are the **most sensitive probe** of composite Higgs models

Proved upper limit of lightest top partners for given symmetry breaking scale  $f$

Light Higgs  Light Top Partners

D. Marzocca, M. Serone, J. Shu., JHEP 1208, 013 (2012)

O.Matsedonskyi, G. Panico, A. Wulzer, JHEP 1301, 164 (2013)

EW charged twin top almost have **zero LHC bounds**

**Neutral Naturalness**

See for instance

N. Craig, A.Katz, M.Strassler, R. Sundrum, JHEP 1507, 105 (2015)

# Why composite twin Higgs?



Funny trigonometric parity

$$s_h \leftrightarrow c_h.$$

M. Geller, O. Telem, PRL 114, 191801 (2015)

R. Barbieri, D. Greco, R. Rattazzi, A. Wulzer, JHEP 1508, 161 (2015)

- Why Twin Higgs? M. Low, A. Tesi, L.T. Wang, PRD 91, 095012 (2015)  
**Highly constrained by LEP**

- The radiatively generated Higgs potential
- universal prediction on Higgs couplings (Like pion soft theorem)

If the **strong dynamics** triggers the breaking G/H,  
pNGB is a **composite** particle.

QCD chiral symmetry, pion

# Trigonometric Parity as the build-in Twin Parity

However, the goldstone itself does have the spontaneous broken symmetry!

The symmetry of the G/H coset space manifold!

Inside any coset space manifold, there is a trigonometric parity

Physical higgs has a shift symmetry in the corresponding unbroken direction

C. Csaki, T. Ma, J. Shu., Phys.Rev.Lett. 121 (2018) no23, 231801

$$\pi^i/f \rightarrow \pi^i/f + \epsilon^i.$$

$$\frac{\pi^i}{f} \rightarrow -\frac{\pi^i}{f} + \frac{\pi}{2}$$

Higgs parity:

$$\pi^i \rightarrow -\pi^i.$$

$$U(1) \sim SO(2)$$

$SO(N+1)/SO(N)$   $S^N$

$$U = \begin{pmatrix} \mathbf{1}_3 & & & \\ & \cos \frac{h}{f} & \sin \frac{h}{f} & \\ & -\sin \frac{h}{f} & \cos \frac{h}{f} & \\ & & & 1 \end{pmatrix}.$$

Exchange of the 4th and 6th row

# Adding matter fields



The matter fields have to conserve such a build-in trigonometric parity

$$\Psi_{Q_L} = \frac{1}{\sqrt{2}} \begin{pmatrix} b_L \\ -ib_L \\ t_L \\ it_L \\ 0 \\ 0 \end{pmatrix}.$$

$$\Psi_{\tilde{t}_L} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \tilde{t}_L \\ i\tilde{t}_L \end{pmatrix},$$

Exchange the coordinates in the 3rd and 5th, 4th and 6th row

$$P = P_0 P_1^h = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \\ & 1 & & \\ & & 1 & \end{pmatrix}$$

$$P_1^h = \begin{pmatrix} \mathbb{1}_3 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}.$$

$$y_t = \tilde{y}_t$$

$$\Psi_{Q_L} \leftrightarrow P\Psi_{\tilde{t}_L}, \quad t_R \leftrightarrow \tilde{t}_R, \quad \Sigma \rightarrow P\Sigma$$

# Fermion Lag

The top and bottom sector

$$\begin{aligned}\mathcal{L}_{eff}^t &= \bar{b}_L \not{p} \Pi_0^q(p) b_L + \bar{t}_L \not{p} (\Pi_0^q(p) + \Pi_1^q(p) c_h^2) t_L \\ &+ \bar{t}_R \not{p} \Pi_0^t(p) t_R + \bar{\tilde{t}}_L \not{p} (\Pi_0^q(p) + \Pi_1^q(p) s_h^2) \tilde{t}_L \\ &+ \bar{\tilde{t}}_R \not{p} \Pi_0^t(p) \tilde{t}_R - \frac{i M_1^t(p)}{\sqrt{2}} (\bar{t}_L t_R s_h + \bar{\tilde{t}}_L \tilde{t}_R c_h) + h.c.\end{aligned}$$

# A UV Completion

$$SO(6)/SO(5) \simeq SU(4)/Sp(4)$$

The latter has the fermion condensation

	$Sp(2N)$	$SU(2)_L$	$U(1)_Y$	$SU(3)_c$	$U(1)_\eta$
$(\psi_1, \psi_2)$	□	□	0	1	1
$\psi_3$	□	1	$-\frac{1}{2}$	1	-1
$\psi_4$	□	1	$\frac{1}{2}$	1	-1

Gauge sector automatically satisfy the Weinberg sum rule  
Lowest chiral breaking operators at UV: 4-fermions dim 6.

# SU(4)/Sp(4) matter content

$$\Psi_{Q_L} = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{0} & Q \\ -Q^T & \mathbf{0} \end{pmatrix} \text{ and } \Psi_{\tilde{t}_L} = \frac{1}{\sqrt{2}} \begin{pmatrix} i\tilde{t}_L \sigma^2 & 0 \\ 0 & \mathbf{0} \end{pmatrix}$$

$$U = \begin{pmatrix} c' \mathbb{1}_2 & i\sigma^2 h s' \\ i\sigma^2 h s' & c' \mathbb{1}_2 \end{pmatrix} \quad \eta : i(\psi_1 \psi_2 + \psi_3 \psi_4 - \psi_1^c \psi_2^c - \psi_3^c \psi_4^c)$$

$$\begin{aligned} \mathcal{L}_{eff}^t &= \bar{b}_L \not{p} \Pi_0^q(p) b_L + \bar{t}_L \not{p} (\Pi_0^q(p) - 2\Pi_1^q(p)s_h^2) t_L \\ &+ \bar{t}_R \not{p} \Pi_0^t(p) t_R + \bar{\tilde{t}}_R \not{p} \Pi_0^t(p) \tilde{t}_R \\ &+ \bar{\tilde{t}}_L \not{p} (\Pi_0^q(p) - 2\Pi_1^q(p)c_h^2) \tilde{t}_L \\ &- \sqrt{2} M_1^t(p) \left( \bar{t}_L t_R s_h + \bar{\tilde{t}}_L \tilde{t}_R c_h \right) + h.c. \quad (36) \end{aligned}$$

# Extension for Composite Top

	$Sp(2N)$	$SU(2)_L$	$U(1)_Y$	$SU(3)_c$	$SU(3)'_c$
$\chi_L$	$\begin{smallmatrix} & 1 \\ 1 & \end{smallmatrix}$	1	$\frac{2}{3}$	$\square$	1
$\chi_R^c$	$\begin{smallmatrix} & 1 \\ 1 & \end{smallmatrix}$	1	$-\frac{2}{3}$	$\bar{\square}$	1
$\tilde{\chi}_L$	$\begin{smallmatrix} & 1 \\ 1 & \end{smallmatrix}$	1	1	1	$\square$
$\tilde{\chi}_R^c$	$\begin{smallmatrix} & 1 \\ 1 & \end{smallmatrix}$	1	1	1	$\bar{\square}$

	$SU(4) \times SU(12)$	$Sp(4) \times SO(12)$
$\chi(\psi\psi)$	(6, 12)	(5, 12), (1, 12)
$\chi(\psi^c\psi^c)$	(6, 12)	(5, 12), (1, 12)
$\psi(\chi\psi)$	(10, 12)	(10, 12)
$\psi(\chi^c\psi^c)$	(1, $\overline{12}$ )	(1, 12)
$\psi(\chi^c\psi^c)$	(15, $\overline{12}$ )	(15, 12)

# Extension for Composite Top

$$\begin{aligned}\mathcal{L} = & f\bar{\Psi}_L U(\epsilon_{5L}\Psi_{5R} + \epsilon_{1L}\Psi_{1R}) + f\epsilon_R \bar{\Psi}_R \Psi_{1L} \\ & + M_5 \bar{\Psi}_{5L} \Psi_{5R} + M_1 \bar{\Psi}_{1L} \Psi_{1R} + h.c,\end{aligned}$$

$$\Psi_Q = \begin{pmatrix} iB - iX_{5/3} \\ B + X_{5/3} \\ T + X_{2/3} \\ -T + X_{2/3} \\ iT'_+ - iT'_- \\ 0 \end{pmatrix} \quad \Psi_S = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ T'_+ + T'_- \end{pmatrix}$$

$$\tilde{\Psi}_Q = \begin{pmatrix} i\tilde{B}_{-1} - i\tilde{X}_1 \\ \tilde{B}_{-1} + \tilde{X}_1 \\ \tilde{T}_0 + \tilde{X}_0 \\ -\tilde{T}_0 + \tilde{X}_0 \\ i\tilde{T}'_+ - i\tilde{T}'_- \\ 0 \end{pmatrix} \quad \tilde{\Psi}_S = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \tilde{T}'_+ + \tilde{T}'_- \end{pmatrix}$$

# Higgs potential

$$V_g = \gamma_g s_h^2 \quad V_f = \gamma_f (-s_h^2 + s_h^4),$$

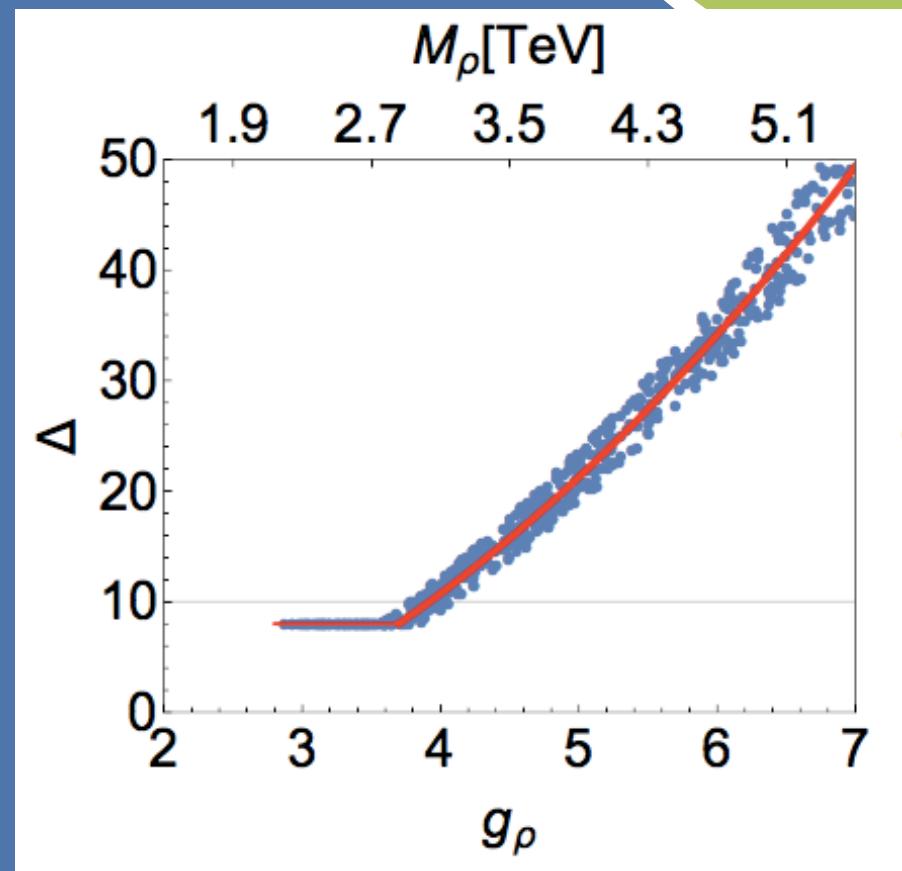
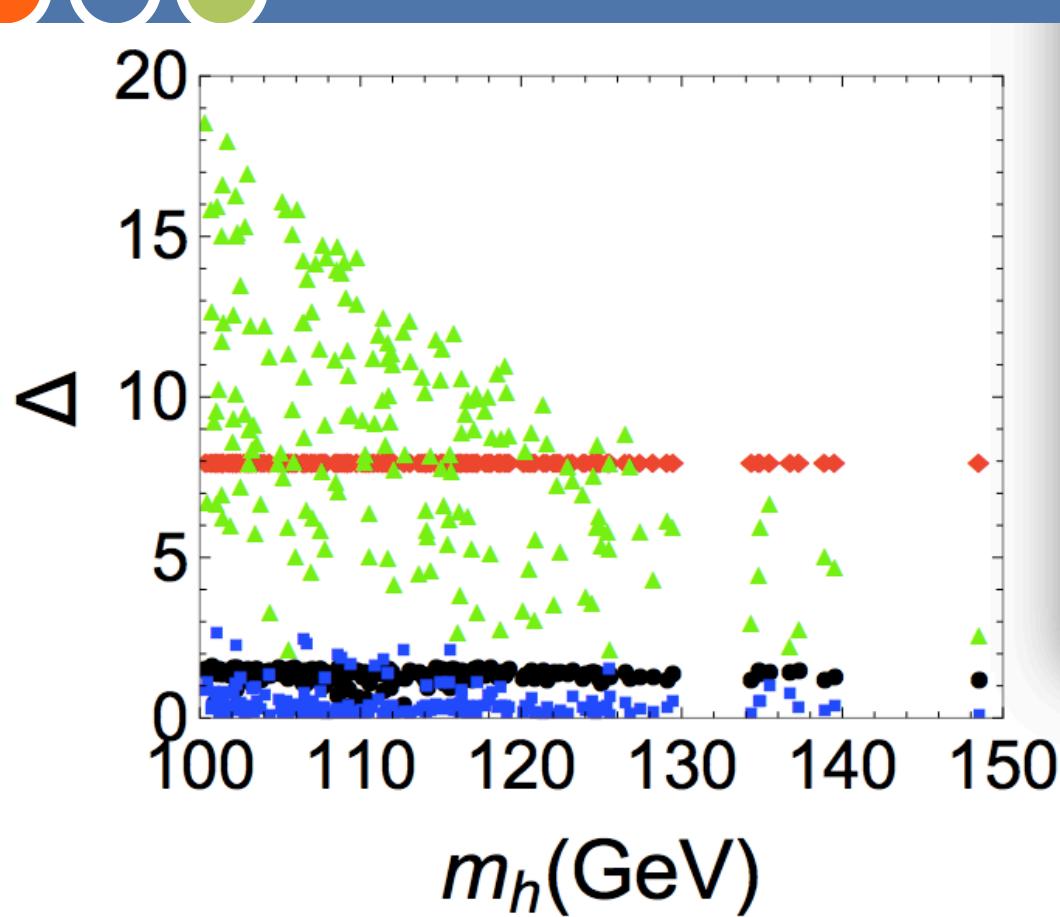
$$V = V_g + V_f = -\gamma s_h^2 + \beta s_h^4,$$

$$\gamma = \gamma_f - \gamma_g \text{ and } \beta = \gamma_f$$

$$\begin{aligned} V_f &\simeq c' \frac{N_c M_f^4}{16\pi^2} \left(\frac{y_t}{g_f}\right)^4 [-s_h^2 + s_h^4] \\ &\simeq c' \frac{N_c f^4}{16\pi^2} y_t^4 [-s_h^2 + s_h^4], \end{aligned}$$

Notice top  
Yukawa 4th power

# Higgs potential



# Novel six top signals



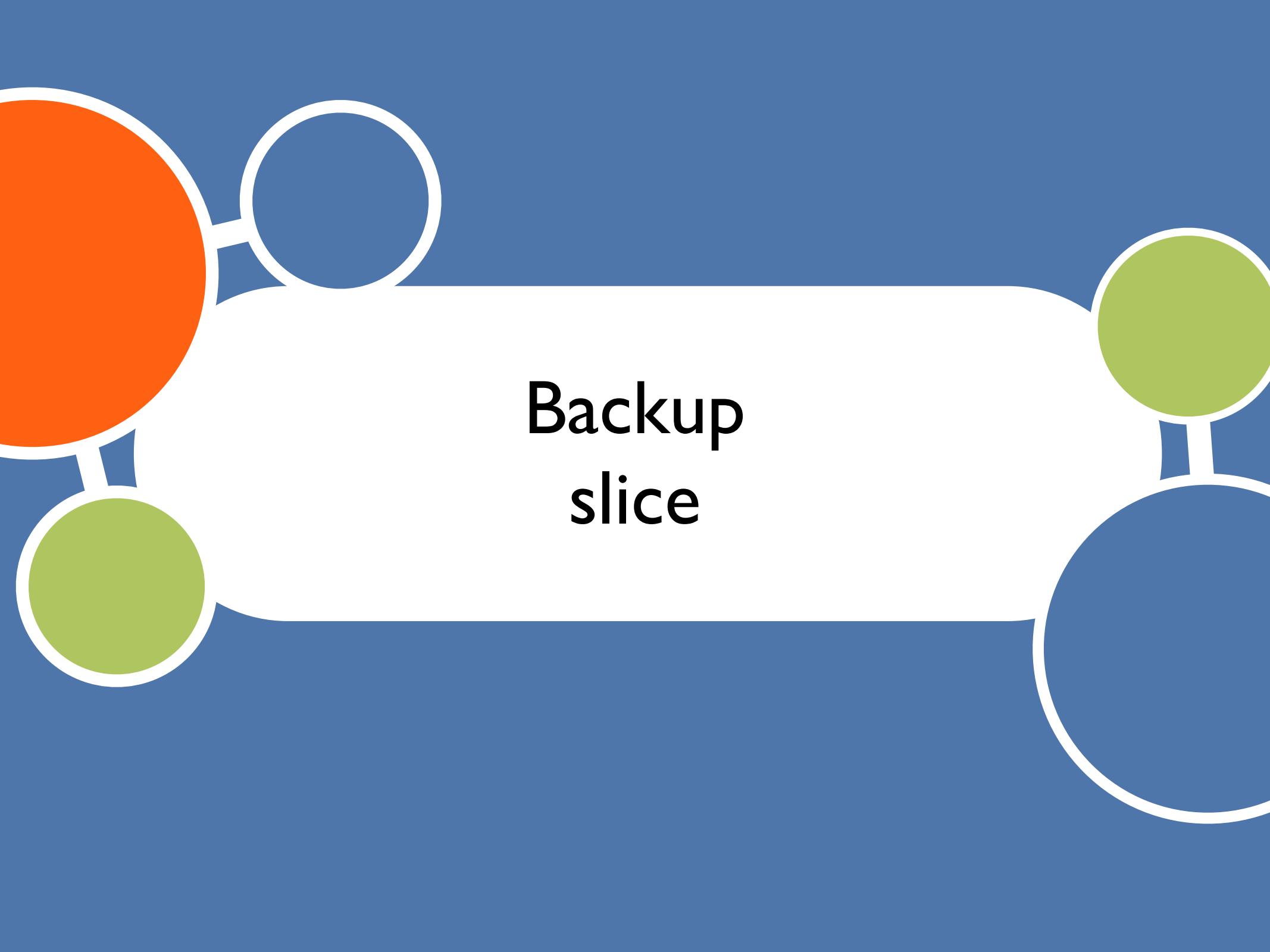
$$t' \rightarrow B'_\mu \ t \rightarrow t \bar{t} t.$$

Completely new and novel channels

# Future Prospects



- Understanding models of EWSB (real progress after 2000)
- EFT approach to EWSB, connect collider physics with true natural of EWSB
- Theoretical Framework can be applied to many other aspects? (Inflation, axion, condensed matter?)



Backup  
slice

# Discrete Parities

Hidden additional  $Z_2$  forbids the tuning term: (like composite twin Higgs)

M. Geller, O. Telem, PRL 114, 191801 (2015)

R. Barbieri, D. Greco, R. Rattazzi, A. Wulzer, JHEP 1508, 161 (2015)

M. Low, A. Tesi, L.T. Wang, PRD 91, 095012 (2015)

$s_h \Leftrightarrow -c_h$  in the Higgs potential

Can be realized under the following transformation

$$\begin{aligned} \Psi_{+L} &\rightarrow P_1 \Psi_{+L}, \quad \Psi_{+R} \rightarrow V P_1 V \Psi_{+R}, \quad U \rightarrow V U V P_1 V, \\ \Psi_{q_L} &\rightarrow V \Psi_{q_L} = \Psi_{q_L}, \quad \Psi_{t_R} \rightarrow P_2 \Psi_{t_R} = \Psi_{t_R} \end{aligned} \quad (28)$$

$$P_1 = \text{diag}(1_{3 \times 3}, \sigma_1), \quad P_2 = \text{diag}(1_{3 \times 3}, -\sigma_3).$$

# Vector bosons



SO(5)/SO(4)

Consider one vector meson and one axi-vector meson

$$\rho_\mu \equiv \mathbf{6}$$

$$a_\mu \equiv \mathbf{4}$$

$$\rho_\mu \rightarrow h\rho_\mu h^\dagger + \frac{i}{g_\rho} h\partial_\mu h^\dagger$$

$$a_\mu \rightarrow ha_\mu h^\dagger$$

The Lag based on HLS

$$\mathcal{L}^v = -\frac{1}{4} \text{Tr}[\rho_{\mu\nu}\rho^{\mu\nu}] + \frac{f_\rho^2}{2} \text{Tr}[(g_\rho\rho_\mu - E_\mu)^2]$$

$$\mathcal{L}^a = -\frac{1}{4} \text{Tr}[a_{\mu\nu}a^{\mu\nu}] + \frac{f_a^2}{2\Delta^2} \text{Tr}[(g_a a_\mu - \Delta d_\mu)^2]$$

$$\mathcal{L}_{\text{kin}} = \frac{f^2}{4} \text{Tr}[d_\mu d^\mu] \quad (3)$$

$$\begin{aligned} \rho_{\mu\nu} &= \partial_\mu\rho_\nu - \partial_\nu\rho_\mu - ig_\rho[\rho_\mu, \rho_\nu], \\ a_{\mu\nu} &= \Delta_\mu a_\nu - \Delta_\nu a_\mu, \quad \Delta = \partial - iE. \end{aligned}$$

$$m_\rho^2 = g_\rho^2 f_\rho^2 \quad m_a^2 = \frac{g_a^2 f_a^2}{\Delta^2}$$

$\Delta$  is a free parameter

# Vector bosons

Further simplified as

$$\begin{aligned}\mathcal{L} = & \frac{f^2 + 2f_a^2}{4} \text{Tr}[d_\mu d^\mu] - m_a f_a \text{Tr}[a_\mu(E_\mu + d_\mu)] + \frac{g_a^2 f_a^2}{2\Delta^2} \text{Tr}[a_\mu a^\mu] \\ & + \frac{f_\rho^2}{2} \text{Tr}[E_\mu E^\mu] - m_\rho f_\rho \text{Tr}[\rho_\mu(E_\mu + d_\mu)] + \frac{g_\rho^2 f_\rho^2}{2} \text{Tr}[\rho_\mu \rho^\mu]\end{aligned}\quad (7)$$

For symmetric coset space, G-invariant building blocks

$$\rho_\mu \pm a_\mu$$

$$E_\mu \pm d_\mu$$

$$\begin{aligned}\mathcal{L} = & f_+^2 \text{Tr}[(d_\mu + E_\mu)^2] + f_-^2 \text{Tr}[V(E_\mu + d_\mu)V(E^\mu + d_\mu)] \\ & - m_+^2 \text{Tr}[(\rho_\mu + a_\mu)(d_\mu + E_\mu)] - m_-^2 \text{Tr}[V(\rho_\mu + a_\mu)V(d_\mu + E_\mu)] \\ & + \frac{m_\rho^2 + m_a^2}{4} \text{Tr}[(\rho_\mu + a_\mu)(\rho_\mu + a_\mu)] \\ & + \frac{m_\rho^2 - m_a^2}{4} \text{Tr}[V(\rho_\mu + a_\mu)V(\rho_\mu + a_\mu)]\end{aligned}\quad (8)$$

$$f_+^2 = \frac{f^2 + 2f_a^2 + 2f_\rho^2}{8}, \quad m_+^2 = \frac{m_\rho f_\rho + m_a f_a}{2}, \quad m_-^2 = \frac{m_\rho f_\rho - m_a f_a}{2}$$

$$f_-^2 = \frac{f^2 + 2f_a^2 - 2f_\rho^2}{8}$$

# Vector bosons



Again, theory is made of **one** G-invariant adjoints for **one** G and also V

$$SO(5)_1 \quad U^\dagger D_\mu U \rightarrow \Omega_1 U^\dagger D_\mu U \Omega_1^\dagger \quad \text{Higgs shift sym lies in } [SO(5)/SO(4)]_1$$

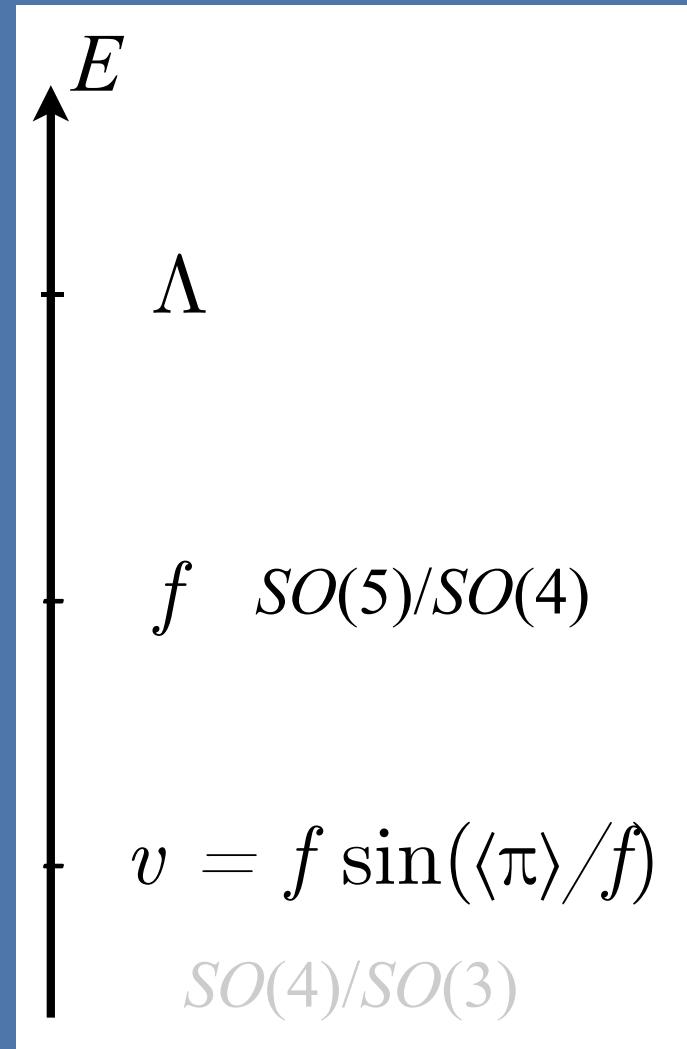
$$SO(5)_2 \quad \rho_\mu + a_\mu \rightarrow \Omega_2(\rho_\mu + a_\mu)\Omega_2^\dagger \quad \text{Automatically get the Weinberg sum rules}$$

$$\text{CYB in the 1st line} \quad V_g \sim g_0^2 f_-^2 \Lambda^2 \quad f_- = 0 \text{ 1st WS}$$

$$\text{CYB in the 2nd line} \quad V_g \sim g_0^2 m_+^2 m_-^2 \log \Lambda^2 \quad m_- = 0 \text{ 2nd WS}$$

$$\text{CYB in the 3rd line} \quad V_g \sim g_0^2 m_+^4 (m_\rho^2 - m_a^2) / \Lambda^2 \quad m_\rho \approx m_a$$

# Higgs as pNGB



Consider the minimal group  $G/H$

$$SO(5) \times U(1)_X \rightarrow SO(4) \times U(1)_X$$

K. Agashe, R. Contino, A. Pomarol, NPB 719 (2005) 165

at the scale  $f > v$

$$\xi \equiv \frac{v^2}{f^2}$$

There are four NGBs:  $\pi^{\hat{a}}$ , with  $\hat{a} = 1, 2, 3, 4$ .

They transform as a **4 of  $SO(4)$**

**(2,2) of  $SU(2) \times SU(2) \sim SO(4)$ .**

$$Y = T_{3R} + X$$

$$SU(2)_L \times U(1)_Y \subset SU(2)_L \times SU(2)_R \times U(1)_X \sim \text{ **$SO(4)' \times U(1)_X$** }$$

See other holographic models based on  $SU(3)/SU(2)$

R. Contino, Y. Nomura, A. Pomarol, NPB 671 (2003) 148

# CCWZ of GCHM



pNGB matrix:

$$U = \exp \left( i \frac{\sqrt{2}}{f} h^{\hat{a}} T^{\hat{a}} \right).$$

$$\hat{a} = 1, 2, 3, 4.$$

The CCWZ transformation

C.G. Callan, S.R. Coleman, J.Wess, B. Zumino, PR 177 (1969) 2247  
,

$$U \rightarrow g U h(h^{\hat{a}}, g)^{\dagger}$$

$$i U^{\dagger} D_{\mu} U = \hat{d}_{\mu}^{\hat{a}} T^{\hat{a}} + \hat{E}_{\mu} T^a$$

SM gauge fields

Leading order chiral Lag

$$\mathcal{L}_{\sigma_g} = -\frac{1}{4} W_{\mu\nu}^{aL} W^{aL\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{f^2}{4} \text{Tr} \left( \hat{d}_{\mu} \hat{d}^{\mu} \right)$$

# CCWZ of GCHM



$$iU^\dagger D_\mu U = \hat{d}_\mu^{\hat{a}} T^{\hat{a}} + \hat{E}_\mu T^a$$

SM gauged

$$\hat{d}_\mu = -\frac{\sqrt{2}}{f} (D_\mu h) + \dots$$

$$\hat{E}_\mu = g_0 A_\mu + \frac{i}{f^2} (h \leftrightarrow D_\mu h) + \dots$$

Transform like a gauge field

$$m_W = \frac{gf}{2} \sin \frac{\langle h \rangle}{f} \equiv \frac{gv}{2}$$

$$s_h = \sin \frac{\langle h \rangle}{f}, \quad \xi \equiv s_h^2$$

# Higgs physics



$$\begin{aligned} f^2 \sin^2 \frac{h}{f} &= f^2 \left[ \sin^2 \frac{\langle h \rangle}{f} + 2 \sin \frac{\langle h \rangle}{f} \cos \frac{\langle h \rangle}{f} \left( \frac{h}{f} \right) \right. \\ &\quad \left. + \left( 1 - 2 \sin^2 \frac{\langle h \rangle}{f} \right) \left( \frac{h}{f} \right)^2 + \dots \right] \\ &= v^2 + 2v\sqrt{1-\xi}h + (1-2\xi)h^2 + \dots \end{aligned}$$

W boson mass

modification of  $hVV$   
coupling

$$a = \sqrt{1 - \xi} \qquad b = 1 - 2\xi$$

Similarly for fermions.

$$m_f(h) \propto \sin \left( \frac{2h}{f} \right) \qquad c = \frac{1 - 2\xi}{\sqrt{1 - \xi}} \qquad 5, 10$$

$$m_f(h) \propto \sin \left( \frac{h}{f} \right) \qquad c = \sqrt{1 - \xi} \qquad \text{Spinorial 4}$$

# 电弱对称破缺机制



Higgs 势能: 辐射修正

$$V_f(h) \simeq -\gamma_f s_h^2 + \beta_f s_h^4,$$

$$\sin^2 \langle H \rangle / f = \xi \ll 1$$

$$m_H^2 = 8 \xi (1 - \xi) \beta.$$

$$\begin{aligned}\gamma_g &= -\frac{3}{8(4\pi)^2} \int_0^\infty dp_E^2 p_E^2 \left( \frac{3}{\Pi_0} + \frac{c_X^2}{\Pi_B} \right) \Pi_1, \\ \beta_g &= -\frac{3}{64(4\pi)^2} \int_{\mu_g^2}^\infty dp_E^2 p_E^2 \left( \frac{2}{\Pi_0^2} + \left( \frac{1}{\Pi_0} + \frac{c_X^2}{\Pi_B} \right)^2 \right) \Pi_1^2.\end{aligned}$$

规范波色子贡献

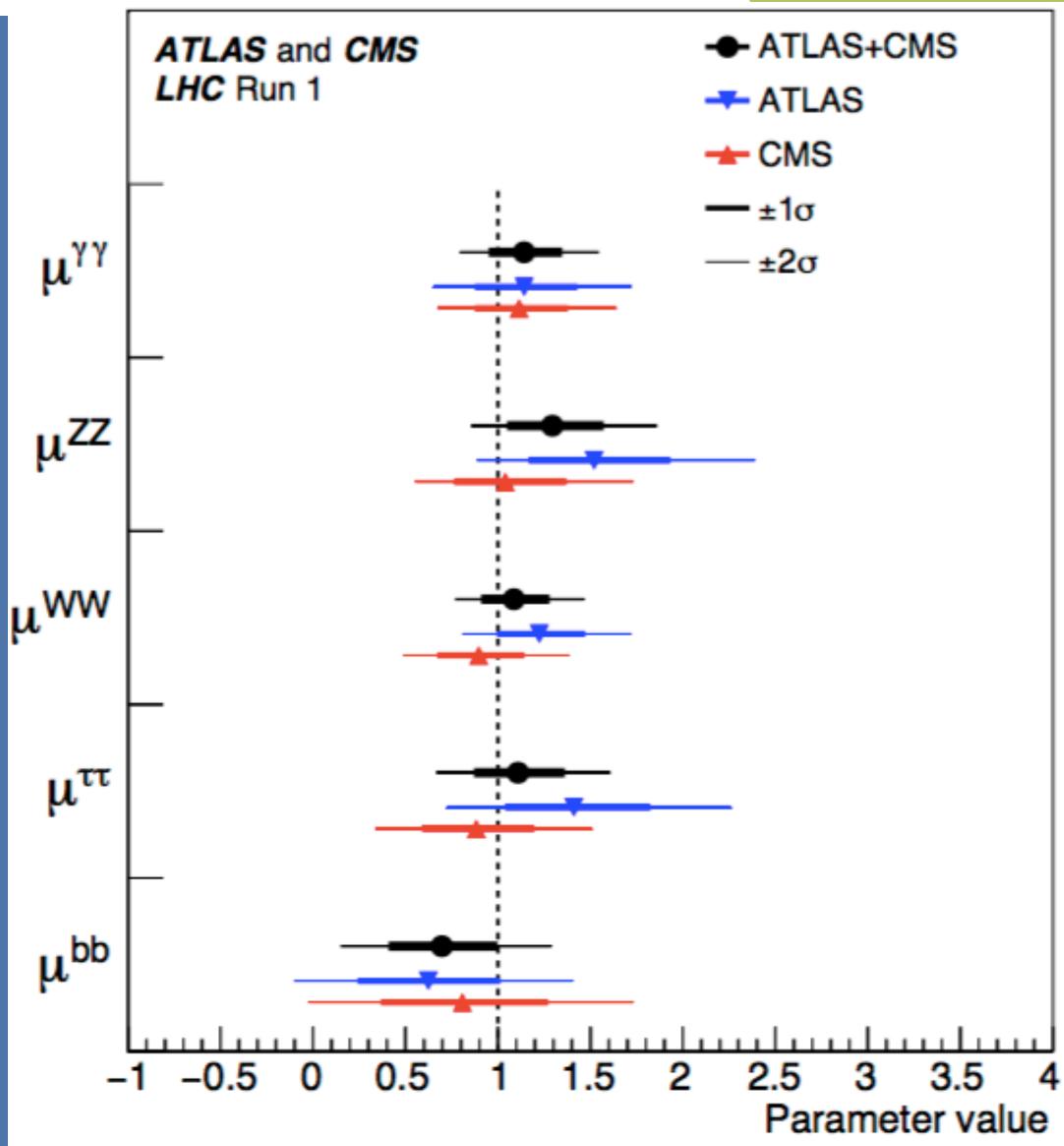
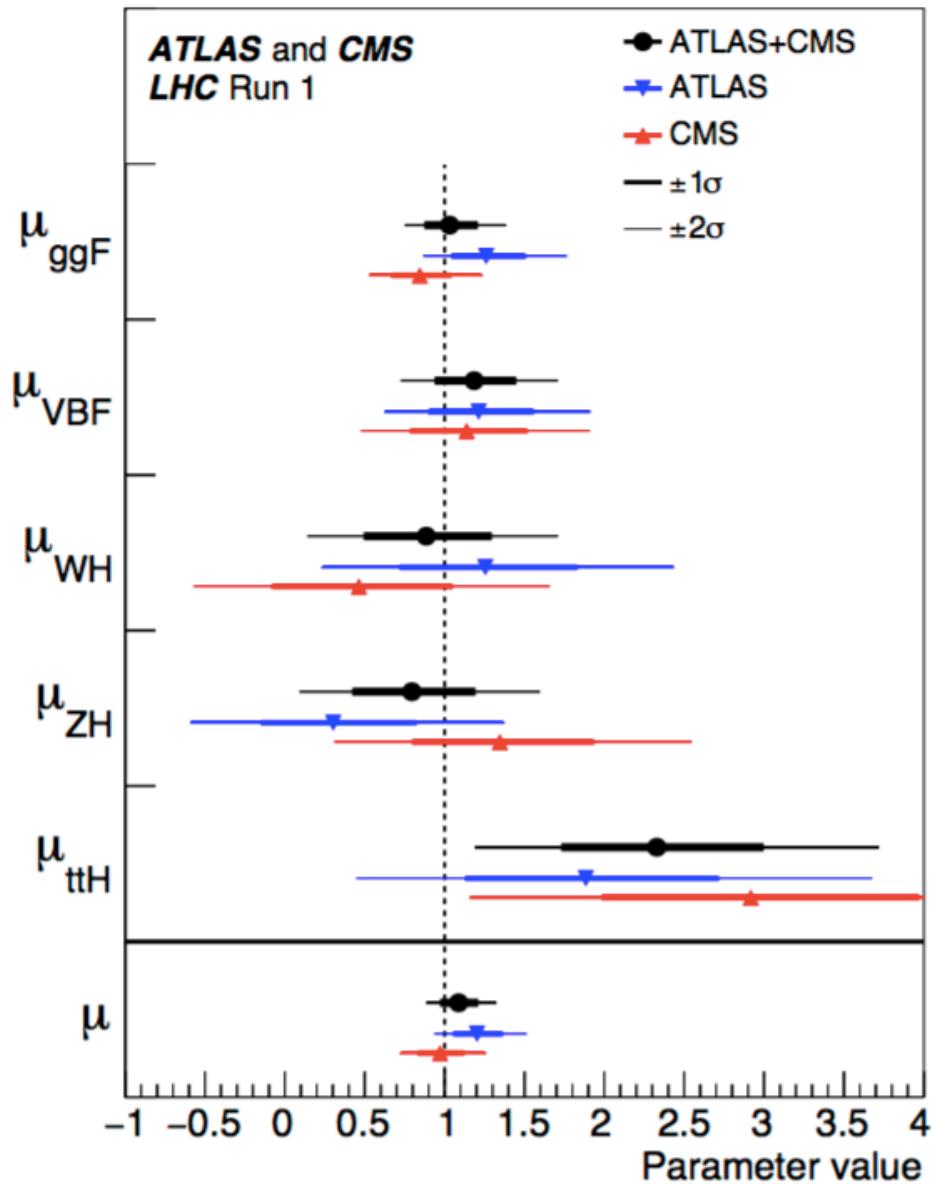
$$\begin{aligned}\gamma_f &= \frac{2N_c}{(4\pi)^2} \int_0^\infty dp_E^2 p_E^2 \left( \frac{\Pi_{1Q}}{\Pi_Q} + \frac{\Pi_{1S}}{\Pi_S} + \frac{\Pi_{QS}^2}{p_E^2 \Pi_Q \Pi_S} \right), \\ \beta_f &= \frac{N_c}{(4\pi)^2} \int_{\mu_f^2}^\infty dp_E^2 p_E^2 \left( \left( \frac{\Pi_{QS}^2}{p_E^2 \Pi_Q \Pi_S} + \frac{\Pi_{1Q}}{\Pi_Q} + \frac{\Pi_{1S}}{\Pi_S} \right)^2 - \frac{2(p_E^2 \Pi_{1Q} \Pi_{1S} - \Pi_{QS}^2)}{p_E^2 \Pi_Q \Pi_S} \right).\end{aligned}$$

费米子贡献

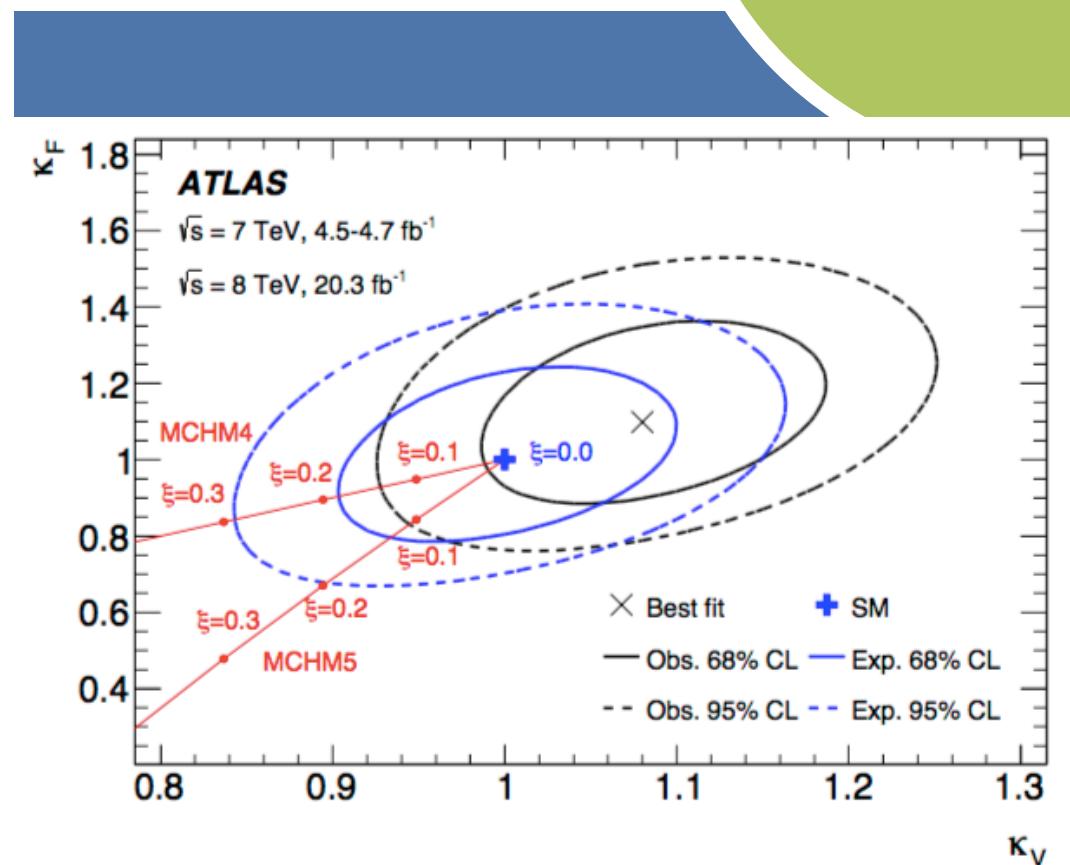
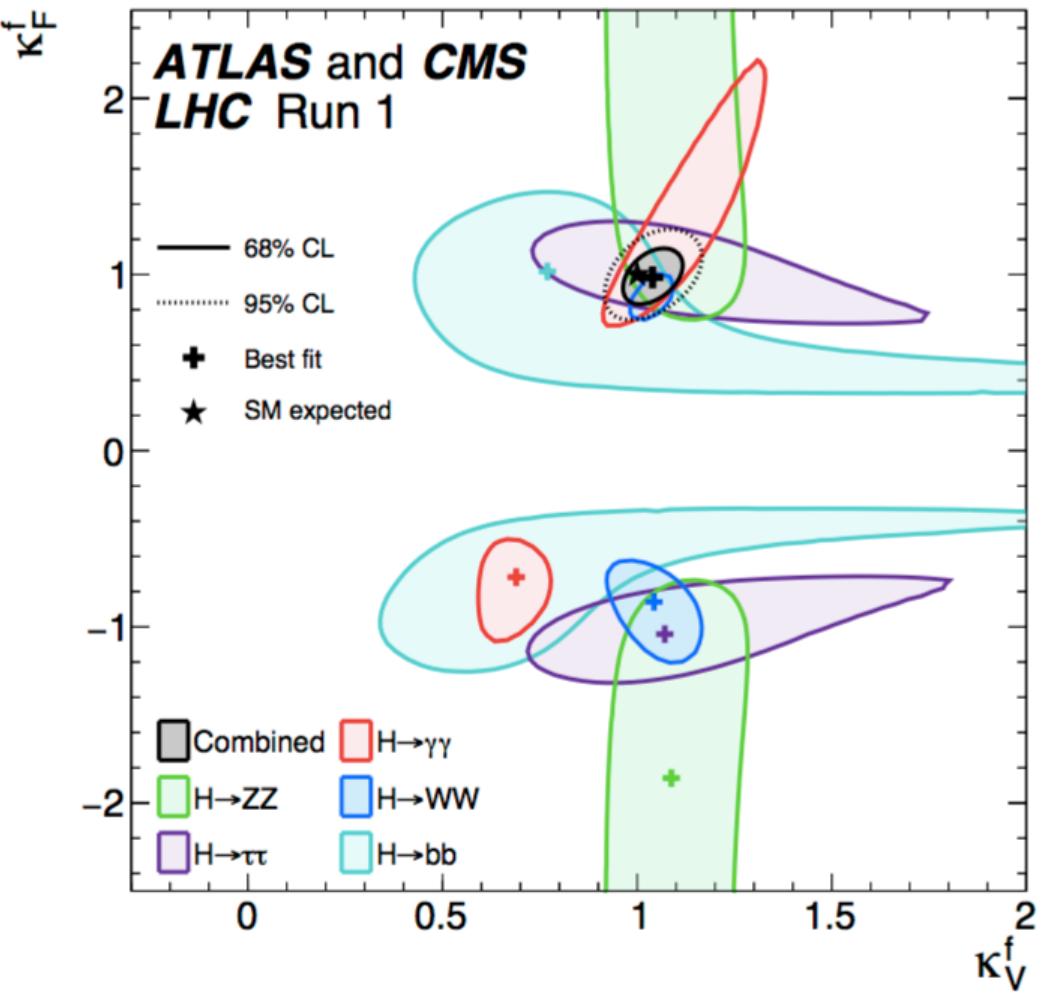
$$M_t^2(q^2, \langle h \rangle) = \frac{|\Pi_{t_L t_R}(q^2, \langle h \rangle)|}{\sqrt{\Pi_{t_L}(q^2, \langle h \rangle) \Pi_{t_R}(q^2, \langle h \rangle)}}.$$

Top 夸克质量

# Higgs产生和衰变



# Higgs物理



Top耦合为负的情况不再存在

Higgs 拟合  $\xi < 0.1$