CP Symmetry and Its Violation

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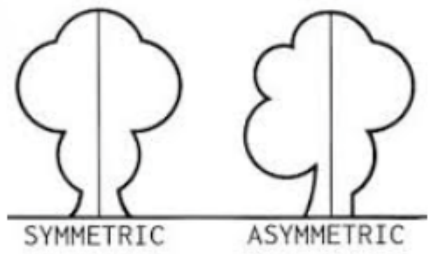
Lecture One

1. A Brief History of P and CP symmetries

Symmetry (from Greek συμμετρία *symmetria* "agreement" In dimensions, due proportion, arrangement", in Chinese "对称性") in everyday language refers to a sense of harmonious and beautiful proportion and balance.



Symmetry in physics has been generalized to mean invariance—that is, lack of change—under certain kind of transformation.

This concept has become one of the most powerful tools of theoretical physics, as it has become evident that practically all laws of nature originate in symmetries

Example: A circle is unchanged under a rotation around its center.

An interaction is invariant under rotation transformation, the angular momentum is conserved in this system.

In 1918, Emmy Noether showed that each symmetry of a physical system corresponds to a conservation law.

Some important symmetries in Physics

Continuous space time symmetries: translation, rotation and Lorentz boost.

Discrete space-time symmetries: space inversion (P-parity), time reversal (T-parity) and particle-antiparticle conjugation (C-parity).

Permutation symmetries: Bose-Einstein and Fermi-Dirac symmetries.

Global continuous symmetries: baryon and lepton numbers.

Local gauge symmetries: electromagnetic $U(1)_{em}$, strong and electroweak $SU(3)_C \times SU(2)_L \times U(1)_Y$.

Classify and simplify analysis of problems, lead to conservation laws, and also determine dynamics…

Symmetry -> Exact symmetry,

Broken symmetry (small or maximal)

All important in Nature!

P, T, and C symmetries are broken symmetries, our emphasis of this lecture!

Discrete space-time symmetries

Spatial reversal P (P-Parity): change space coordinate **x** -> -**x**

Time reversal T (T-parity): change time coordinate t -> -t

Charge Conjugation C (C-parity): change a particle to its anti-particle

All are discrete transformations. Systems do not change under these transformations, exhibits these symmetries and have corresponding conservation laws.

Why C is also grouped into space-time discrete transformation?

Intuitive view: An anti-particle is a particle traveling backwords in time.

Negative energy represents an anti-particle in Dirac and Klein-Gorden equations.

Plane wave: $e^{-i E t} = e^{-i((-E)(-t))}$

For many years, P, C and T symmetries are thought to be exact ones in interactions for fundamental particles.

The believe that P-parity is a good symmetry was brought to an end by T-D Lee and C-N. Yang in 1956. Nobel Prize in 1957.

Weak interaction violates P and was V-A form, played an important role in building Standard electroweak interaction Model in 1961 by S. Glashow, in 1968 by S. Weinberg and 1969 by A. Salam. Nobel Prize in 1979.

The believe that CP symmetry is a good symmetry was brought to an end by experiment led by J. Cronin and V. Fitch in 1964. Nobel Prize in 1980.

In 1972, Standard Model for CP violation was proposed by M. Kobayashi and T. Maskawa. Nobel Prize in 2008.

Many experiments provided information about CP violation even now.

Big puzzles still exist! What is the Origin of CP violation?

Why in our universe matter dominates over anit-matter?

The downfall of P symmetry

PHYSICAL REVIEW VOLUME 104, NUMBER 1 OCTOBER 1, 1956

Question of Parity Conservation in Weak Interactions*

T. D. LEE, *Columbia University, New York, New York*

AND

C. N. YANG,† *Brookhaven National Laboratory, Upton, New York*

(Received June 22, 1956)

The question of parity conservation in β decays and in hyperon and meson decays is examined. Possible experiments are suggested which might test parity conservation in these interactions.

To be more specific, let us consider the allowed β transition of any oriented nucleus, say Co^{60}. The angular distribution of the β radiation is of the form (see Appendix):

$$I(\theta)d\theta = (\text{constant})(1+\alpha\cos\theta)\sin\theta d\theta, \qquad (2)$$

where α is proportional to the interference term CC'. If $\alpha \neq 0$, one would then have a positive proof of parity nonconservation in β decay. The quantity α can be obtained by measuring the fractional asymmetry between $\theta < 90°$ and $\theta > 90°$; i.e.,

$$\alpha = 2\left[\int_0^{\pi/2} I(\theta)d\theta - \int_{\pi/2}^{\pi} I(\theta)d\theta\right] \Big/ \int_0^{\pi} I(\theta)d\theta.$$

In the decay processes

$$\pi \to \mu + \nu, \qquad (5)$$

$$\mu \to e + \nu + \nu, \qquad (6)$$

starting from a π meson at rest, one could study the distribution of the angle θ between the μ-meson momentum and the electron momentum, the latter being in the center-of-mass system of the μ meson. If parity is conserved in neither (5) nor (6), the distribution will not in general be identical for θ and $\pi - \theta$. To understand

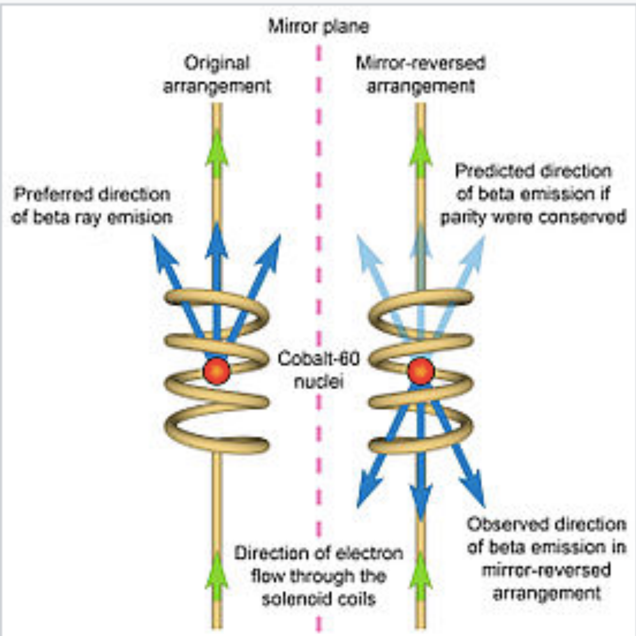
Experimental Test of Parity Conservation in Beta Decay*

C. S. Wu, *Columbia University, New York, New York*

AND

E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson,
National Bureau of Standards, Washington, D. C.

(Received January 15, 1957)



Principle of the Wu experiment 1956 to detect parity violation in nuclear beta decay

Observations of the Failure of Conservation of Parity and Charge Conjugation in Meson Decays: the Magnetic Moment of the Free Muon*

Richard L. Garwin,† Leon M. Lederman, and Marcel Weinrich

Physics Department, Nevis Cyclotron Laboratories, Columbia University, Irvington-on-Hudson, New York

(Received January 15, 1957)

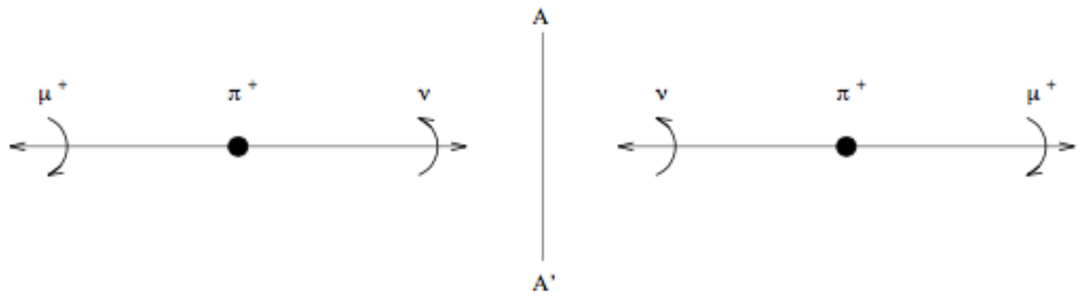


Figure 1: Mirror processes.

Mirror process is not observed, neutrino is left-handed.

The Nobel Prize in Physics 1957

Chen Ning Yang
Prize share: 1/2

Tsung-Dao (T.D.) Lee
Prize share: 1/2

The Nobel Prize in Physics 1957 was awarded jointly to Chen Ning Yang and Tsung-Dao (T.D.) Lee *"for their penetrating investigation of the so-called parity laws which has led to important discoveries regarding the elementary particles"*

Discovery of V-A current of weak interaction

PHYSICAL REVIEW VOLUME 109, NUMBER 1 JANUARY 1, 1958

Theory of the Fermi Interaction

R. P. FEYNMAN AND M. GELL-MANN
California Institute of Technology, Pasadena, California
(Received September 16, 1957)

The representation of Fermi particles by two-component Pauli spinors satisfying a second order differential equation and the suggestion that in β decay these spinors act without gradient couplings leads to an essentially unique weak four-fermion coupling. It is equivalent to equal amounts of vector and axial vector coupling with two-component neutrinos and conservation of leptons. (The relative sign is not determined theoretically.) It is taken to be "universal"; the lifetime of the μ agrees to within the experimental errors of 2%. The vector part of the coupling is, by analogy with electric charge, assumed to be not renormalized by virtual mesons. This requires, for example, that pions are also "charged" in the sense that there is a direct interaction in which, say, a π^0 goes to π^- and an electron goes to a neutrino. The weak decays of strange particles will result qualitatively if the universality is extended to include a coupling involving a Λ or Σ fermion. Parity is then not conserved even for those decays like $K\rightarrow 2\pi$ or 3π which involve no neutrinos. The theory is at variance with the measured angular correlation of electron and neutrino in He^6, and with the fact that fewer than 10^{-4} pion decay into electron and neutrino.

ACKNOWLEDGMENTS

The authors have profited by conversations with F. Boehm, A. H. Wapstra, and B. Stech. One of us (M. G. M.) would like to thank R. E. Marshak and E. C. G. Sudarshan for valuable discussions.

Chirality Invariance and the Universal Fermi Interaction*

E. C. G. SUDARSHAN, *Harvard University, Cambridge, Massachusetts*
AND
R. E. MARSHAK, *University of Rochester, Rochester, New York*
(Received January 10, 1958)

WE have shown[1] that the imposition of the requirement of chirality invariance[2] on each covariant in the four-fermion interaction Hamiltonian leads to the essentially unique expression[3]:

$$G\{[\bar{A}\gamma_\mu(1+\gamma_5)B]^\dagger[\bar{C}\gamma_\mu(1+\gamma_5)D]+\text{H.c.}\} \qquad \text{(I)}$$

where G is the coupling constant and A, B, C, D are four Dirac *particle* fields. In the standard terminology of parity-conserving interactions, (I) represents the combination $V-A$ (V is vector, A is axial vector). We

Phys. Rev. 109, 1860(1958)

Birth of the Standard Model

8.B

Nuclear Physics **22** (1961) 579—588;

PARTIAL-SYMMETRIES OF WEAK INTERACTIONS

SHELDON L. GLASHOW †

Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark

Received 9 September 1960

Abstract: Weak and electromagnetic interactions of the leptons are examined under the hypothesis that the weak interactions are mediated by vector bosons. With only an isotopic triplet of leptons coupled to a triplet of vector bosons (two charged decay-intermediaries and the photon) the theory possesses no partial-symmetries. Such symmetries may be established if additional vector bosons or additional leptons are introduced. Since the latter possibility yields a theory disagreeing with experiment, the simplest partially-symmetric model reproducing the observed electromagnetic and weak interactions of leptons requires the existence of at least four vector-boson fields (including the photon). Corresponding partially-conserved quantities suggest leptonic analogues to the conserved quantities associated with strong interactions: strangeness and isobaric spin.

A Model of Leptons

Steven Weinberg (MIT, LNS). Nov 1967. 3 pp.
Published in **Phys.Rev.Lett. 19 (1967) 1264-1266**
DOI: 10.1103/PhysRevLett.19.1264

Weak and Electromagnetic Interactions

Abdus Salam (Imperial Coll., London & ICTP, Trieste). May 1968. 11 pp.
Published in **Conf.Proc. C680519 (1968) 367-377**
Conference: C68-05-19

The Nobel Prize in Physics 1979

Sheldon Lee Glashow
Prize share: 1/3

Abdus Salam
Prize share: 1/3

Steven Weinberg
Prize share: 1/3

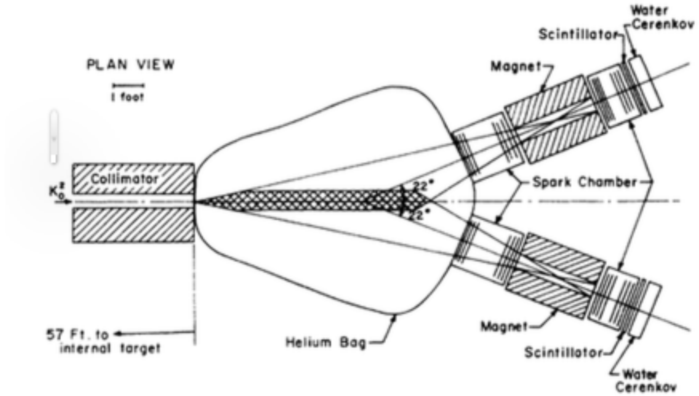
The Nobel Prize in Physics 1979 was awarded jointly to Sheldon Lee Glashow, Abdus Salam and Steven Weinberg *"for their contributions to the theory of the unified weak and electromagnetic interaction between elementary particles, including, inter alia, the prediction of the weak neutral current".*

The downfall of CP symmetry

PHYSICAL REVIEW LETTERS

EVIDENCE FOR THE 2π DECAY OF THE K_2^0 MESON*†

J. H. Christenson, J. W. Cronin,‡ V. L. Fitch,‡ and R. Turlay§
Princeton University, Princeton, New Jersey
(Received 10 July 1964)



There are two neutral Kaons, K_1 and K_2 with $K_1 \to \pi\pi$ (shorter lifetime) and $K_2 \to \pi\pi\pi$ (longer lifetime).

At far enough distance, initial K_1 has decayed and should only see $K_2 \to \pi\pi\pi$.

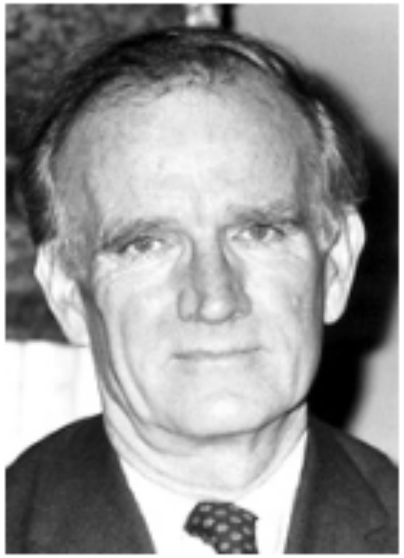
But still observed $\pi\pi$ final state, looks like K_2 also decays into $\pi\pi$.

K_2 is a mixed state of two states, each only decay into $\pi\pi$ or $\pi\pi\pi$, respectively.

The mixing ε between these two states has a value $|\varepsilon| = 2.3 \times 10^{-3}$.

The Nobel Prize in Physics 1980

James Watson Cronin
Prize share: 1/2



Val Logsdon Fitch
Prize share: 1/2

The Nobel Prize in Physics 1980 was awarded jointly to James Watson Cronin and Val Logsdon Fitch *"for the discovery of violations of fundamental symmetry principles in the decay of neutral K-mesons"*

Birth of the SM of CP violation

Progress of Theoretical Physics, Vol. 49, No. 2, February 1973

CP-Violation in the Renormalizable Theory of Weak Interaction

Makoto KOBAYASHI and Toshihide MASKAWA

Department of Physics, Kyoto University, Kyoto

(Received September 1, 1972)

In a framework of the renormalizable theory of weak interaction, problems of CP-violation are studied. It is concluded that no realistic models of CP-violation exist in the quartet scheme without introducing any other new fields. Some possible models of CP-violation are also discussed.

$$\begin{pmatrix} \cos\theta_1 & -\sin\theta_1\cos\theta_3 & -\sin\theta_1\sin\theta_3 \\ \sin\theta_1\cos\theta_2 & \cos\theta_1\cos\theta_2\cos\theta_3-\sin\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\cos\theta_2\sin\theta_3+\sin\theta_2\cos\theta_3 e^{i\delta} \\ \sin\theta_1\sin\theta_2 & \cos\theta_1\sin\theta_2\cos\theta_3+\cos\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\sin\theta_2\sin\theta_3-\cos\theta_2\sin\theta_3 e^{i\delta} \end{pmatrix}.$$

| a typo !

$\cos\theta_3$

The Nobel Prize in Physics 2008

Photo: University of Chicago
Yoichiro Nambu
Prize share: 1/2

© The Nobel Foundation
Photo: U. Montan
Makoto Kobayashi
Prize share: 1/4

© The Nobel Foundation
Photo: U. Montan
Toshihide Maskawa
Prize share: 1/4

The Nobel Prize in Physics 2008 was divided, one half awarded to Yoichiro Nambu *"for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"*, the other half jointly to Makoto Kobayashi and Toshihide Maskawa *"for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature"*.

Importance of CP violation

Played important roles in understanding fundamental laws of Nature!

One of the most crucial elements why we exist in the Universe.

Sakharov (1967): Violation of CP invariance, C asymmetry and baryon asymmetry of the universe.

Andrei Sakharov on *Soviet Nobel Peace Prize winners*, the USSR stamp issued on 14 May 1991

A matter dominating anti-Matter universe resulted from a symmetric one in the Big-Bang cosmology

- Baryon number B violation.
- C-symmetry and CP-symmetry violation.
- Interactions out of thermal equilibrium.

2. P, C, T and CP violation, and CPT theorem

P symmetry and violation

The parity operation P, a spatial inversion through the origin: $P\vec{x} = -\vec{x}$

The Schrödinger equation

$$i\hbar \frac{d\psi}{dt} = [-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{x})]\psi \ ,$$

P operates on the wave function $\psi(\vec{x})$ of a state $|N, \vec{p}, \vec{s}>$,
N internal quantum numbers: electric charge, baryon number and etc.
$\vec{p}$ and $\vec{s}$ are the momentum and spin,

$$P\psi(\vec{x}) \to \psi(-\vec{x}) \ , \ \ P|N, \ \vec{p}, \ \vec{s}>= \eta_P|N, \ -\vec{p}, \ \vec{s}> \ ,$$

η_P the intrinsic parity of the particle (system).

Parity symmetry (invariance) of the interactions, implies: $V(\vec{x}) = V(-\vec{x})$
Then: $\psi(-\vec{x})\psi^*(-\vec{x})$ is equal to $\psi(\vec{x})\psi^*(\vec{x})$,

The probability of the transition $i \to f$ is the same as that for $Pi \to Pf$.

Observe left process, but not right one,
P is violated!

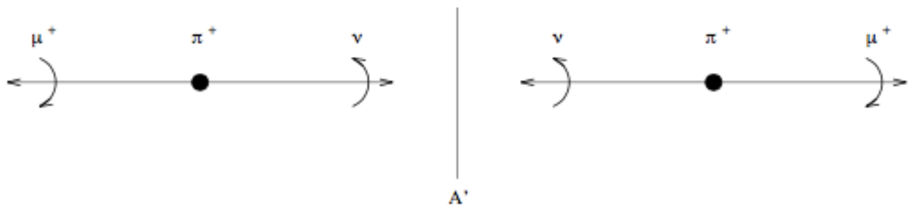


Figure 1: Mirror processes.

T symmetry and violation

In classic mechanics

Classically time reversal operation T: $t \rightarrow -t$.
Reverse momenta $\vec{p} \rightarrow = \vec{p}$, spins $\vec{s} \rightarrow -\vec{s}$
interchanging the initial state $|i>$ with final $|f>$ state.

A physics example: a damped pendulum
The equation of motion for the damped pendulum

$$m\frac{d^2x}{d^2t} + r\frac{dx}{dt} + kx = 0$$

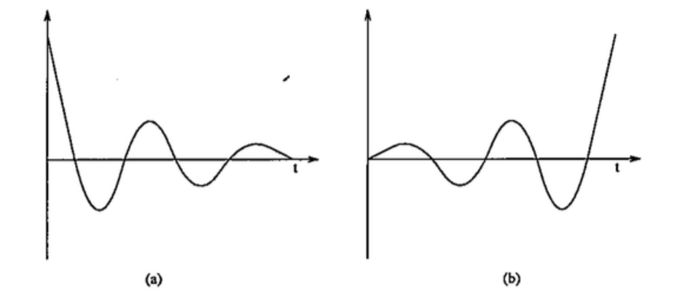


Figure 2: (a) The pendulum swing amplitude vs. time. (b) The time reversed processe.

gives the T transformed equation

$$m\frac{d^2x}{d^2(-t)} + r\frac{dx}{d(-t)} + kx = 0 \rightarrow m\frac{d^2x}{d^2t} - r\frac{dx}{dt} + kx = 0$$

Clearly the first order derivative in t is the reason why it is not T invariant.
$r = 0$, no damping, T invariant!

In quantum mechanics, the situation is more complicated.
The Schrödinger equation has a first order derivative in t

$$i\hbar\frac{d\psi}{dt} = [-\frac{\hbar^2}{2m}\nabla^2 + V(t)]\psi$$

T reversal operation : $t \to -t$

$$i\hbar\frac{d\psi(-t)}{d(-t)} = [-\frac{\hbar^2}{2m}\nabla^2 + V(-t)]\psi(-t)$$

$$\to \quad -i\hbar\frac{d\psi(-t)}{dt} = [-\frac{\hbar^2}{2m}\nabla^2 + V(-t)]\psi(-t)\ .$$

Even $V(t) = V(-t)$, not possible to go back as before T transformation!

Contradiction with the observation for the damped pendulum
$V(t) = kx = V(-t)$, $r = 0$ is possible to define T invariance!

How to make T transformation still possible to be invariant?
This puzzle was solved by Wigner in 1932

T transformation: *Change t to $-t$ and take the complex conjugate.*
The T transformed version is

$$(-i)^*\hbar\frac{d\psi(-t)^*}{dt} = [-\frac{\hbar^2}{2m}\nabla^2 + V^*(-t)]\psi(-t)^*$$
$$\rightarrow\ i\hbar\frac{d\psi(-t)^*}{dt} = [-\frac{\hbar^2}{2m}\nabla^2 + V^*(-t)]\psi(-t)^*$$

As quantum observables are expectation values involving only $\psi^*\psi$,
If the interaction V is real: $V(-t)^* = V(t)$),
then $\psi(t)^*\psi(t) = \psi(-t)^*\psi(-t)$. Same physics!

Time reversal invariance imposes reality conditions on the interaction.
To break T symmetry, one needs to introduce complex valued interactions.

C symmetry and violation

Dirac equation for spin-1/2 electron particle contains negative energy states. According to Dirac hole theory, they corresponds to anti-particle with opposite charges. These Are interpreted as anti-particles!

The Nobel Prize in Physics 1936

Victor Franz Hess
Prize share: 1/2

Carl David Anderson
Prize share: 1/2

The Nobel Prize in Physics 1936 was divided equally between Victor Franz Hess *"for his discovery of cosmic radiation"* and Carl David Anderson *"for his discovery of the positron"*.

Positron was discovered in 1932 by C. Anderson

The Nobel Prize in Physics 1933

Erwin Schrödinger
Prize share: 1/2

Paul Adrien Maurice Dirac
Prize share: 1/2

The Nobel Prize in Physics 1933 was awarded jointly to Erwin Schrödinger and Paul Adrien Maurice Dirac *"for the discovery of new productive forms of atomic theory"*

Hole theory is completely symmetric Between negative and positive charges.

Dirac theory is symmetric under particle and anti-particle transformation.

Particle and anti-particle have opposite additive quantum numbers,
C parity changes the signs of all additive quantum numbers,

$$C|N,\, \vec{p},\, \vec{s} >= \eta_C| - N,\, \vec{p},\, \vec{s} >$$

η_C is a phase factor.
Only if $N = 0$, a particle or a particle system can be eigen-state of C.

Example: π^0 which satisfies:

$$C|\pi^0 >= +|\pi^0 > \text{ self-conjugate}$$

Charge conjugation symmetry also plays an important role in particle physics.
Apply C transformation on (a) the reactio becomes to (b)

$$(a)\ \pi^+ \to \mu^+\nu\ ,\quad (b)\ \pi^- \to \mu^-\bar{\nu}$$

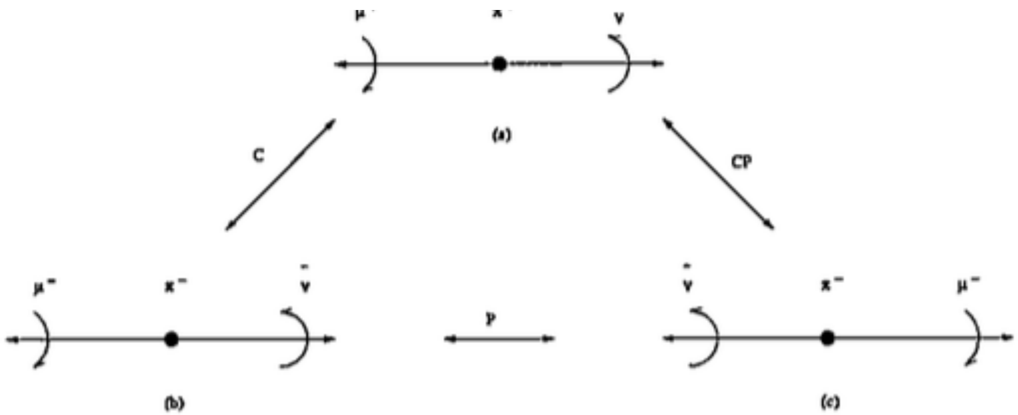


Figure 3: C, P and CP transformed processes.

(a) Process observed

(b) Process not observed.
C symmetry violated.

(c) process observed.
CP symmetry is respected.

Is CP symmetry always true?

CP symmetry and violation

CP was still considered to be exact untill experiment led by Fitch and Cronin found mixing between K_1 (K_S) and $K_2(K_L)$ in 1964!

Experimental data show: K_L mainly decays into $\pi\pi\pi$, but about a few per thousand times decays into $\pi\pi$.

Why the above fact implies violation of CP symmetry?

Kaons and pions are pseudoscalar, under P transformation

$$\pi \to -\pi \, , \quad K \to -K \, ,$$

Under a C transformation one has

$$\pi^+(u\bar{d}) \to \pi^-(d\bar{u}) \, , \quad \pi^0((u\bar{u} - d\bar{d})/\sqrt{2}) \to \pi^0 \, , \quad K^0(d\bar{s}) \to \bar{K}^0(s\bar{d}) \, .$$

Two neutral kaon CP eigenstates be constructed from K^0 and $\bar{K}^0$,

$$K_1^0 = \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0)\ , \quad CP \text{ even}\ ; \quad K_2^0 = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0)\ , \quad CP \text{ odd}\ .$$

The pion systems $\pi\pi$ and $\pi\pi\pi$ are all in S-wave states.
$(\pi^+\pi^-\ , \pi^0\pi^0)$ in CP even states. $(\pi^+\pi^-\pi^0\ , \pi^0\pi^0\pi^0)$ in CP odd states.

$$K_1^0 \to \pi^+\pi^-\ ,\ \pi^0\pi^0\ , \quad K_2^0 \to \pi^+\pi^-\pi^0\ ,\ \pi^0\pi^0\pi^0\ .$$

Phase space considerations, K_2^0 decays slower than the K_1^0 decay.
Experiment show the lifetimes are 10^{-7}s and 10^{-10}s, respectively.
K_L state mainly decay into $\pi\pi\pi$, it should be identified as K_2^0

At far enough distance (all original K_1^0 should have all decayed),
Still see $\pi\pi$ final state, K_L and K_S are admixture of K_1^0 and K_2^0.

$$K_L = \frac{K_2^0 + \epsilon_1 K_1^0}{\sqrt{1+|\epsilon_1|^2}}\ , \quad K_S = \frac{K_1^0 + \epsilon_2 K_2^0}{\sqrt{1+|\epsilon_2|^2}}\ .$$

$K^0 - \bar{K}^0$ mixing leads to CP violation!

CP symmetry is violated!

What about CPT symmetry?

No experimental evidence which shows violation of CPT symmetry.
There are more fundamental reasons for CPT symmetry to be exact.

In the 1950s, it was shown that CPT symmetry holds
if the Lagrangian of a local quantum field theory is

Lorentz invariance, Hermitician and the fields obey usual Spin-Statistics
(Bose-Einstein statistics for bosons, and Fermi-Dirac statistics for fermions).

Schwinger (1951); Lüders, (1954); Pauli(1955); Streater and Wightman(1964).

This is the so called CPT theorem.

There are many implications of the CPT theorem.

The masses, and life-times for particles and their anti-particles are all equal.

If CP is violated, T is violated in a way that CPT is still conserved!

These properties provide practical ways to test the CPT theorem.
The best limit on CPT symmetry is from

the mass difference between the masses of K^0 and $\bar{K}^0$,

one has $|m_{K^0} - m_{\bar{K}^0}|/m_{K^0} < 6 \times 10^{-19}$.

CPLEAR, observed direct T violation in $K^0 - \bar{K}^0$ system in 1998
as expected from CP violation in this system (PLB444, 43(1998)).

In most of the discussion later, CPT symmetry is assumed to hold.

Phenomenology of neutral Kaon mixing

The weak interaction mixes K^0 and $\bar{K}^0$

Work in the basis $\Phi(t) = (K^0(t), \bar{K}^0(t))^T$, $\; i\frac{d}{dt}\Phi = H\Phi(t)$,

H can written as the sum of two Hermitian 2×2 matrices M and Γ,

$$H = M - i\frac{\Gamma}{2} = \begin{pmatrix} M_{11} - i\Gamma_{11}/2 & M_{12} - i\Gamma_{12}/2 \\ M_{12}^* - i\Gamma_{12}^*/2 & M_{22} - i\Gamma_{22}/2 \end{pmatrix},$$

M is related to the masses of the particles, Γ is related to the life-times
Both separately must be Hermitian

The appearance i in front Γ, naive T violation because particles decay.

CPT symmetry is exact, $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$.
The off diagonal ones M_{12} and Γ_{12} mix K^0 and $\bar{K}^0$.

CP violation requires M_{12} and/or Γ_{12} be complex!

$\frac{d}{dt}(\Phi^\dagger\Phi) = \frac{d\Phi^\dagger}{dt}\Phi + \Phi^\dagger\frac{d\Phi}{dt}\Phi = \Phi^\dagger(iM^\dagger - \Gamma^\dagger/2)\Phi - \Phi^\dagger(iM - \Gamma/2)\Phi = -\Phi^\dagger\Gamma\Phi$

Γ must be positively defined! $\Gamma_{11} = \Gamma_{22} > 0$ and $Det(\Gamma) > 0$.

Diagonalize the mixing Hamiltonian H

One obtains the mass and life-time eigenvalues for K_S and K_L

$$(m - i\frac{\Gamma}{2})_S = M_{11} - i\frac{\Gamma_{11}}{2} - E\,, \quad (m - i\frac{\Gamma}{2})_L = M_{11} - i\frac{\Gamma_{11}}{2} + E\,,$$
$$E = \sqrt{(M_{12} - i\Gamma_{12}/2)(M_{12}^* - i\Gamma_{12}^*/2)}\,.$$

One also finds, $\epsilon_1 = \epsilon_2$ which will be denoted by ϵ. One obtains:
$\Delta m_{L-S} = m_L - m_S$ and $\Delta\Gamma_{S-L} = \Gamma_S - \Gamma_L$.

$$K_L = \frac{K_2^0 + \epsilon K_1^0}{\sqrt{1+|\epsilon|^2}} = \frac{(1+\epsilon)K^0 + (1-\epsilon)\bar{K}^0}{\sqrt{1+|\epsilon|^2}}\,, \quad \left(\frac{1+\epsilon}{1-\epsilon}\right)^2 = \frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}$$
$$K_S = \frac{K_1^0 + \epsilon K_2^0}{\sqrt{1+|\epsilon|^2}} = \frac{(1+\epsilon)K^0 - (1-\epsilon)\bar{K}^0}{\sqrt{1+|\epsilon|^2}}\,, \quad \epsilon \approx \frac{iIm(M_{12}) + Im(\Gamma_{12}/2)}{\Delta m_{L-S} + i\Delta\Gamma_{S-L}/2}$$

Data: $\Delta m_{L-S} \approx \Delta\Gamma_{S-L}/2 = (3.484 \pm 0.006) \times 10^{-12}$ MeV,

$\epsilon = (2.228 \pm 0.011) \times 10^{-3} exp(i\phi_\epsilon)$ with $(\phi_\epsilon = 43.52 \pm 0.05)^o$.

Assuming $Im(\Gamma_{12})$ is much smaller than $Im(M_{12})$
Theoretical estimate OK

One finally obtains

$$\epsilon \approx \frac{Im(M_{12})}{\sqrt{2}\Delta m_{L-S}} e^{i\phi_\epsilon} \ ,$$

To understand CP violation, one must understand

How $Im(M_{12})$ is generated and what is the origin of it.

Homework

Diagonalize the Hamiltonian $H = M - i\,\Gamma/2$

Find the eigenvalues E_1, E_2, and the matrix V diagonalizing H

$V\,H\,V^{-1} = \mathrm{diag}(E_1, E_2)$

Note that H is not Himitian, V is not Unitary.

Lecture Two

3. Some basics of QFT for CP violation

How fields and Lagrangian transform under C, P and T?

Take QED with fermion ψ and scalar ϕ fields as example.

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma_\mu - m_\psi)D^\mu\psi(D^\mu\phi)^\dagger(D_\mu\phi) - m_\phi^2\phi^\dagger\phi - V(\phi^\dagger\phi) \ ,$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ , \ \ D_\mu = \partial_\mu + ieQA_\mu \ ,$$

ϕ spin-0, A^μ spin-1 (communting), ψ spin-1/2 (anti-communting)
$V(\phi^\dagger\phi)$ - potential of ϕ and is invariant under Lorentz Transformation.

The theory is invariant under the following gauge transformation,
$A_\mu \to A_\mu - \partial_\mu\alpha(x)$, $\psi(x) \to e^{ieQ\alpha(x)}\psi(x)$ and $\phi(x) \to e^{ieQ\alpha}\phi(x)$.

The Dirac γ-matrices are

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \ , \ \ g^{\mu\nu} = \mathrm{Diag}(1, -1, -1, -1) \ ,$$

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \ , \ \ \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \ .$$

Under C transformation, x^μ and ∂^μ do not transform,

$\phi \to \phi_C = \eta_C \phi^\dagger \; (\eta_C^2 = 1), \;\; A^\mu \to A_C^\mu = -A^\mu \;, \;\; \psi \to \phi_C = C\bar{\psi}^T = i\gamma^2\psi^*$

$C = i\gamma^2\gamma^0 \;, \;\; C^{-1} = C^\dagger = -C \;, \;\; C\gamma_\mu^T C^\dagger = -\gamma^\mu .$

Under P transformation, $x^\mu = (x^0, x^i) \to x_\mu = (x^0, -x_i)$ and $\partial^\mu \to \partial_\mu$,

$\phi(\vec{x}) \to \phi_P = \pm\phi^\dagger(-\vec{x}) : +scalar, -pesudoscalar, \; A^\mu(\vec{x}) \to A_P^\mu = A_\mu(-\vec{x}),$

$\psi(\vec{x}) \to \psi_P = \eta_P \gamma^0 \psi(-\vec{x}) \; (\eta_P \; phase \; factor), \;\; \gamma^0\gamma^\mu\gamma^0 = \gamma_\mu$

Under T transformation, $x^\mu \to -x_\mu$ and $\partial^\mu \to -\partial_\mu$,

$\phi(t) \to \phi_P = \eta_T^\phi \phi(-t) \;, \;\; A^\mu(t) \to A_T^\mu = A_\mu(-t) \;,$

$\psi(t) \to \psi_T = \eta_T^\psi T\psi(-t) \; (\eta_T^\psi \; phase \; factor),$

$T = i\gamma^1\gamma^3, \, T^\dagger = T^{-1}, \, T^\dagger \gamma_\mu^* T = \gamma^\mu$

$axial \; vector \; a^\mu \to a_C^\mu = a^\mu, \; a_P^\mu = -a_\mu(-\vec{x}), \; a_T^\mu = a_\mu(-t)$

Define the following bi-spiner products

$$S(x) = \bar{\psi}(x)\psi(x)\ , \quad A^\mu(x) = \bar{\psi}(x)\gamma^\mu\psi(x)\ , \quad T^{\mu\nu}(x) = \bar{\psi}(x)\sigma^{\mu\nu}\psi(x)\ ,$$

$$a^\mu(x) = \bar{\psi}(x)\gamma_5\gamma^\mu\psi(x)\ , \quad P(x) = i\bar{\psi}(x)\gamma_5\psi(x)\ , \quad \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]\ .$$

Transformation properties under C, P, T and CPT

$S(x)$ as a scalar, $P(x)$ as a pesudoscalar, $A^\mu(x)$ as a vector,

$a^\mu(x)$ as an axial vector, and $T^{\mu\nu}(x)$ as tensor.

	$S(x)$	$A^\mu(x)$	$T^{\mu\nu}(x)$	$a^\mu(x)$	$P(x)$
C	$S(x)$	$-A^\mu(x)$	$-T^{\mu\nu}(x)$	$a^\mu(x)$	$P(x)$
P	$S(x')$	$A_\mu(x')$	$T_{\mu\nu}(x')$	$-a_\mu(x')$	$-P(x')$
T	$S(-x')$	$A_\mu(-x')$	$-T_{\mu\nu}(-x')$	$a_\mu(-x')$	$-P(-x')$
CPT	$S(-x)$	$-A^\mu(-x)$	$T^{\mu\nu}(-x)$	$-a^\mu(-x)$	$P(-x)$

P transformed Lagrangian $L^P(x) = L^P(x, \partial^\mu, \psi_P(x), \phi_P(x), A^\mu_P(x))$

Using transformation table $L^P(x) = L(x')$ $(x = (x^0, x^i)$ and $x' = (x^0, -x^i))$

Example:
$F^{\mu\nu}_P(x) F_{P,\mu\nu}(x) = (\partial^\mu A^\nu_P(x) - \partial^\nu A^\mu_P(x))(\partial_\mu A_{P,\nu}(x) - \partial_\nu A_{P,\mu}(x))$

$= (\partial^\mu A_\nu(x') - \partial^\nu A_\mu(x'))(\partial_\mu A^\nu(x') - \partial_\nu A^\mu(x'))$

$= (\partial'_\mu A_\nu(x') - \partial'_\nu A_\mu(x'))(\partial'^\mu A^\nu(x') - \partial'^\nu A^\mu(x')) = F^{\mu\nu}(x') F_{\mu\nu}(x')$

$\bar{\psi}_P(x) \psi_P(x) = \bar{\psi}(x') \psi(x')$, ...

$S = \int_{-\infty}^{\infty} dx^4 L^P(x) = \int_{-\infty}^{\infty} dx^4 L(x' = (x^0, -x^i))$

$= \int_{-\infty}^{\infty} dx^4 L(x = (x^0, x^i)) = \int_{-\infty}^{\infty} dx^4 L(x)$

The action does not change!

Similarly $S = \int_{-\infty}^{\infty} L^C d^4x = \int_{-\infty}^{\infty} L^T d^4x = \int_{-\infty}^{\infty} L^{CP} d^4x = \int_{-\infty}^{\infty} L^{CPT} d^4x$

Similarly $S = \int_{-\infty}^{\infty} L^C d^4x = \int_{-\infty}^{\infty} L^T d^4x = \int_{-\infty}^{\infty} L^{CP} d^4x = \int_{-\infty}^{\infty} L^{CPT} d^4x$

Be careful when when making T transformed L^T,
a complex conjugate action should be taken

$L^T = L^*(\phi_T, \psi_T, A_T^\mu)$

Any constant c in L is transformed to c^*.

Example: $\bar{\psi}(x) i\gamma^\mu \partial_\mu \psi(x) \ \rightarrow \ \bar{\psi}_T(x)(-i\partial^\mu)\gamma_\mu^* \psi_T(x)$

$= \bar{\psi}^\dagger(x')(-i\gamma^{3\dagger}\gamma^{1\dagger})\gamma^0(-i\partial^\mu)\gamma_\mu^*)(i\gamma^1\gamma^3)\psi(x') = \bar{\psi}(x')(-i\partial^\mu)\gamma^\mu\psi(x') = \bar{\psi}(x')i\partial'_\mu\gamma^\mu\partial(x')$

An outline for CTP theorem proof

For a more general L, there may be tensors with arbitrary numbers of Lorentz

indices $\Gamma^{\mu_1\mu_2\cdots\mu_N}$ results from combinations of ∂^μ, $\phi(S,P)$, $T^{\mu\nu}$, A^μ and a^μ

The $\epsilon^{\mu\nu\alpha\beta}$ tensor

Under P and T it changes to $-\epsilon_{\mu\nu\alpha\beta}$. It does not change under C.

Using transformation properties of S, P, $T^{\mu\nu}$, A^μ, a^μ and $\epsilon^{\mu\nu\alpha\beta}$
and under CPT a constant c is transformed into c^*

One obtains: $(\Gamma^{\mu_1\cdots\mu_N})^{CPT} = (-1)^N(\Gamma^{\mu_1\cdots\mu_N})^\dagger$

Since L is Lorentz scalar, the Lorentz indices must be contracted
Then $L^{CPT}(x) = (-1)^{2N}L^\dagger(-x)$

If the theory is Hermmitian $L^\dagger = L \to L^{CPT}(x) = L(-x)$!

Where spin-statistics matters?

When prove the transformation table for C transformation.

A sample calculation: $\bar{\psi}'_C(x)\psi_C(x)$

$\psi_C = C\gamma^0\psi^*$, $\bar{\psi}_C = \psi^T C^\dagger \gamma^0$

$\bar{\psi}'_C\psi_C = \psi'^T C^\dagger \gamma^0 C \psi^* = -\psi'^T \gamma^{0T} \psi^*$

Switching ψ and ψ' a minus sign is generated because they are fermion fields

$\bar{\psi}'_C\psi_C = -\psi'^T \gamma^{0T} \psi^* = \psi^\dagger \gamma^0 \psi' = \bar{\psi}\psi'$

Particle and anti-Particle masses and lifetimes

a) The existence of an anti-particle for each particle ($|P>_m$, $m - spin$)

$\Theta|P>_m = \eta^{CPT}|\bar{P}>_{-m}$. If $\Theta = CPT$ is good, $|\bar{P}>_{-m}$ exists.

b) $m_P = m_{\bar{P}}$

$$m_P = <P|H|P>_m = <P|\Theta^{-1}\Theta H \Theta^{-1}\Theta|P>_m$$

$$= <\bar{P}|H^*|\bar{P}>_{-m} = <\bar{P}|H|\bar{P}>_{-m} = m_{\bar{P}}.$$

c) $\tau_P = \tau_{\bar{P}}$

$$\tau_P^{-1} = 2\pi \sum_i \delta(E_i - E_P)| < i(\infty)|U(\infty, 0)H_{int}|P >_m |^2$$
$$\tau_{\bar{P}}^{-1} = 2\pi \sum_{\bar{i}} \delta(E_{\bar{i}} - E_{\bar{P}})| < \bar{i}(\infty)|U(\infty, 0)H_{int}|\bar{P} >_m |^2$$

$$\tau_P^{-1} = 2\pi \sum_i \delta(E_i - E_P)| < i(\infty)|\Theta^{-1}\Theta U(\infty, 0)H_{int}\Theta\Theta^{-1}|P >_m |^2$$
$$= 2\pi \sum_i \delta(E_i - E_P)| < \bar{i}(-\infty)|U(-\infty, 0)H_{int}|\bar{P} >_{-m} |^2$$

$E_P = E_{\bar{P}}$ and $E_i = E_{\bar{i}}$

$$\rightarrow 2\pi \sum_{\bar{i}} \delta(E_{\bar{i}} - E_{\bar{P}})| \sum_{\bar{j}} < \bar{i}(-\infty)|S\dagger|\bar{j}(\infty) >< \bar{j}(\infty)|U(\infty, 0)H_{int}|\bar{P} >_{-m} |^2$$
$$= 2\pi \sum_{\bar{i},\bar{j},\bar{j}'} < \bar{i}(-\infty)|S\dagger|\bar{j}(\infty) >< \bar{j}(\infty)|U(\infty, 0)H_{int}|\bar{P} >_{-m}$$
$$(< \bar{i}(-\infty)|S\dagger|\bar{j}'(\infty) >)^*(< \bar{j}'(\infty)|U(\infty, 0)H_{int}|\bar{P} >_{-m})^*$$

Summ over $\bar{i}$,

$$\rightarrow 2\pi \sum_{\bar{j},\bar{j}'} \delta_{jj'} < \bar{j}(\infty)|U(\infty, 0)H_{int}|\bar{P} >_{-m} (< \bar{j}'(\infty)|U(\infty, 0)H_{int}|\bar{P} >_{-m})^*$$

$$= 2\pi \sum_{\bar{j}} \delta(E_{\bar{j}} - E_{\bar{P}})| < \bar{j}(\infty)|U(\infty, 0)H_{int}|\bar{P} >_{-m} |^2$$

There is no difference in lifetime with different third spin component: $\tau_P = \tau_{\bar{P}}$!

Homework

Using the C, P and T transformation properties for spinors (when exchange two spinors, be careful about the sign changes) and γ^μ matrices obtain S, A^μ, $T^{\mu\nu}$, a^μ, and P transformation table.

4. Standard Mode for CP violation

How to have P, CP violation, but CTP conservation?

A toy model interaction violating P, C, CP, but conserving CTP

$$L = \bar{\psi}'\kappa\gamma^\mu(1-\gamma_5)\psi A_\mu + \bar{\psi}\kappa^\dagger\gamma^\mu(1-\gamma_5)\psi' A_\mu^\dagger$$

The P transformed Lagrangian, $x \to x^\mu$ goes to $x' = x_\mu$.

$$L^P(x) = \bar{\psi}'_P(x)\kappa\gamma^\mu(1-\gamma_5)\psi_P(x)A_{P,\mu}(x) + \bar{\psi}_P(x)\kappa^\dagger\gamma^\mu(1-\gamma_5)\psi'(x)A^\dagger_{P,\mu}(x)$$

Using the fact: $\psi_P(x) = \gamma^0\psi(x')$, $A_{P,\mu}(x) = -A^\mu(x')$

$$\bar{\psi}'_P(x)\gamma^\mu\psi_P(x) = \bar{\psi}'(x')\gamma_\mu\psi(x'),\ \bar{\psi}'_P(x)\gamma^\mu\gamma_5\psi_P(x) = -\bar{\psi}'(x')\gamma_\mu\gamma_5\psi(x')$$

$$L^P(x) = \bar{\psi}'(x')\kappa\gamma^\mu(1+\gamma_5)\psi(x')A_\mu(x') + \bar{\psi}(x')\kappa^\dagger\gamma^\mu(1+\gamma_5)\psi'(x')A^\dagger_\mu(x')$$

$L^P(x)$ does not goes to $L(x')$, mixture of $\psi'\gamma^\mu\psi$ and $\psi'\gamma^\mu\gamma_5\psi$!

Under C transformation, $\psi_C(x) = C\bar{\psi}^*$ and $A_C(x) = -A^\dagger(x)$

$$L^C(x) = \bar{\psi}'_C(x)\kappa\gamma^\mu(1-\gamma_5)\psi_C(x)A_{C,\mu}(x) + \bar{\psi}_C(x)\kappa^\dagger\gamma^\mu(1-\gamma_5)\psi'(x)A^\dagger_{C,\mu}(x)$$

$$L^C(x) = \bar{\psi}(x)\kappa^T\gamma^\mu(1+\gamma_5)\psi'(x)A^\dagger_\mu(x) + \bar{\psi}'(x)\kappa^*\gamma^\mu(1+\gamma_5)\psi(x)A_\mu(x)$$

$L^C(x)$ **does not goes to** $L(x)$**,**

mixture of $\psi'\gamma^\mu\psi$ **and** $\psi'\gamma^\mu\gamma_5\psi$**,** κ **is complex!**

Under CP transformation

$$L^{CP}(x) = \bar{\psi}(x')\kappa^T\gamma^\mu(1-\gamma_5)\psi'(x')A^\dagger_\mu(x') + \bar{\psi}'(x')\kappa*\gamma^\mu(1-\gamma_5)\psi(x')A_\mu(x')$$

$L^{CP}(x)$ **will goes to** $L(x')$ **is** κ **is real! CP is then conserved!!**

Under CPT transformation, because the T will change constant κ to κ^*,

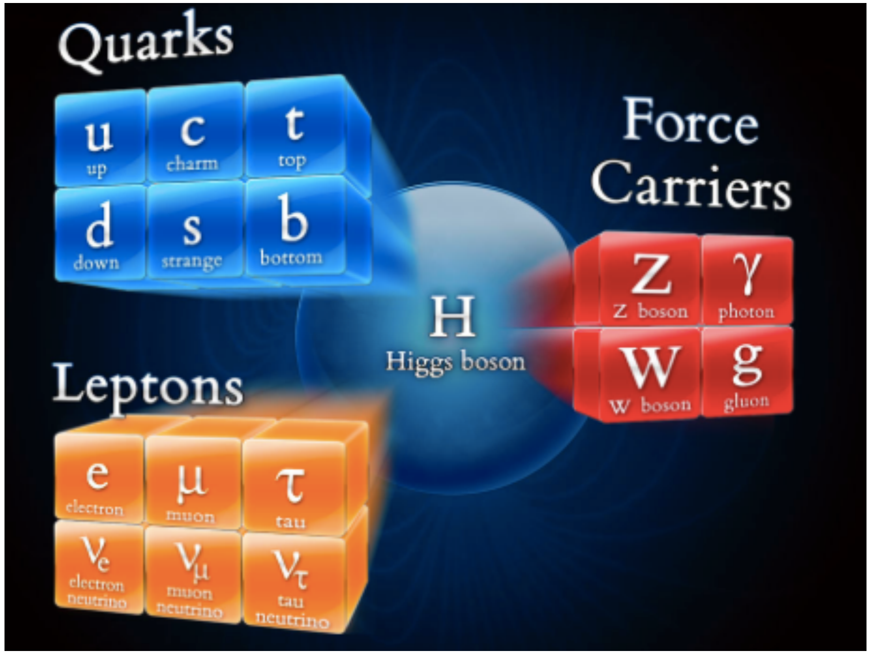
$$L^{CPT}(x) = \bar{\psi}(x')\kappa^\dagger\gamma^\mu(1-\gamma_5)\psi'(x')A^\dagger_\mu(x') + \bar{\psi}'(x')\kappa\gamma^\mu(1-\gamma_5)\psi(x')A_\mu(x')$$

$L^{CPT}(x) = L(-x)$, **CPT is conserved!!**

Standard Model and CP Violation

Standard Model is based on $SU(3)_C x SU(2)_L x U(1)_Y$ gauge interaction.

In SM mis-match of weak and mass eigen-bases, leads to flavor mixing and CP violation.



When going beyond SM,
more possibilities!

Standard Model: CKM and Strong CP Violation

The standard model of strong and electroweak interaction has gauge group

$SU(3)_C \times SU(2)_L \times U(1)_Y$ with gauge bosons

$8\ SU(3)_C$ Gluons : $G^\mu = \frac{\lambda^a}{2} G^\mu_a, \quad Tr(\frac{\lambda^a}{2}\frac{\lambda^b}{2}) = \frac{\delta^{ab}}{2}$.

$3\ SU(2)_L$ W-bosons : $W^\mu = \frac{\sigma^i}{2} W^\mu_i, \ Tr(\frac{\sigma^i}{2}\frac{\sigma^j}{2}) = \frac{\delta^{ij}}{2}$.

$1\ U(1)_Y$ B boson : B^μ

The building blocks of fermions are chiral fields $f_{L,R} = \frac{1 \mp \gamma_5}{2} f$

The SM fermions are leptons L_L, E_R and quarks Q_L, U_R and D_R

$L_L = (\nu_L, e_L : (1,2)(-1/2)^T\ , \ \ e_R : (1,1)(-1)\ ,$

$Q_L = (u_L, d_L)^T : (3,2)(1/6)\ , \ \ u_R : (3,1)(2/3)\ , \ \ d_R : (3,1)(-1/3)\ .$

Also a Higgs boublet $H = (h^+, (v + h + iI)/\sqrt{2})^T : (1, 2)(1/2)$

The non-zero vev v to break electroweak symmetries

and give masses to all particles.

$h^\pm$ and I are Goldstone bosons "eaten" by $W^\pm$ and Z bosons.

h is a neutral boson, the famous $Higgs$ boson discovered in 2012 at LHC.

Renormalizable SM Lagrangian

$$
\begin{aligned}
L &= -\frac{1}{2}Tr(G_{\mu\nu}G^{\mu\nu}) - \frac{1}{2}Tr(W_{\mu\nu}W^{\mu\nu}) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \theta\frac{g_3^2}{16\pi}Tr(\tilde{G}^{\mu\nu}G_{\mu\nu}) \\
&+ \bar{Q}_L i\gamma^\mu D_\mu Q_L + \bar{u}_R i\gamma^\mu D_\mu u_R + \bar{d}_R i\gamma^\mu D_\mu d_R + \bar{L}_L i\gamma^\mu D_\mu L_L + \bar{e}_R i\gamma^\mu D_\mu e_R \\
&- \bar{Q}_L Y_u \tilde{H} u_R + \bar{Q}_L Y_d H d_R + \bar{L}_L Y_e H e_R + H.C. + (D_\mu H)^\dagger (D^\mu H) - V(H)
\end{aligned}
$$

$\tilde{G}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}G_{\alpha\beta}$

$V(H) = \mu^2 H^\dagger H + \lambda(H^\dagger H)^2$ is the Higgs potential.
Positivity of potential for large h, $\lambda > 0$.

$D_\mu = \partial_\mu - ig_3 G_\mu - ig_2 W_\mu - ig_1 Y B_\mu$

If $D_f^\mu = \partial^\mu - igf^\mu$, then $f^{\mu\nu} = \partial^\mu f^\nu - \partial^\nu f^\mu - ig[f^\mu, f^\nu]$

At the vacuum $< H >= v/\sqrt{2}$, minimize $V(v)$ results in $\mu^2 = -\lambda v^2$

$V(h) = -\lambda\frac{v^4}{4} + \lambda v^2 h^2 + \lambda v h^3 + \lambda\frac{h^4}{4}, \;\; m_h^2 = 2\lambda v^2$

$(D_\mu H)^\dagger(D^\mu H) \to m_W^2 = g_2^2 v^2/4$ and $m_Z^2 = (g_2^2 + g_1^2)v^2/4 = \frac{g_2^2}{4c_W^2}v^2$

$A_\mu = c_w B_\mu + s_W W_\mu^3,\; Z_\mu = -s_W B_\mu + c_W W_\mu^3,\; W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2)$

$J_g^{a,\mu} G_\mu^a = -g_3 \bar{f} \frac{\lambda^a}{2} \gamma^\mu f G_\mu^a$. P, C, T symmetric.

$J_{em}^\mu A_\mu = -eQ_f \bar{f} \gamma^\mu f A_\mu$. P, C, T symmetric.

$J_Z^\mu Z_\mu = -\frac{g_2}{2c_W} \bar{f} \gamma^\mu (g_V^f - g_A^f \gamma_5) f Z_\mu$, $g_V^f = T_f^3 - 2Q_f s_W^2$ and $g_A^f = T_f^3$

T_f^3 weak isospin, up type (u, ν) $T_f^3 = 1/2$, down type (d, e) $T_f^3 = -1/2$

Violates P, C symmetry, but CPT, CP, T symmetric.

$J_W^\mu W_\mu^+ = -\frac{g_2}{\sqrt{2}} \bar{u}_L \gamma^\mu d_L W_\mu^+ = -\frac{g_2}{2\sqrt{2}} \bar{u} \gamma^\mu (1 - \gamma_5) d W_\mu^+$.

Violates P, C symmetry. CPT symmetric. What about CP, T?

Needs to work with the charged current and also the Yukawa interactions.

$L_m = -(\bar{Q}_L Y_u \tilde{H} u_R + \bar{Q}_L Y_d H d_R + \bar{L}_L Y_e H e_R + H.C.)$,

$\bar{f} M_f (1 + \frac{h}{v}) f$ with $M_f = Y_f v/\sqrt{2}$ which is usually not diagonalized.

Bi-unitary diagonalization:

$M_f = V^\dagger_{fL} \hat{M}_f V_{fR}$, $\hat{M}_f = diag(m_f^1, m_f^2 ... m_f^N)$, $V_{L,R}$ unitary matrices.

Mass eigenstate basis: $f_L^m = V_{fL} f_L$ and $f_R^m = V_{fR} f_R$

$L_m = -\bar{f}^m \hat{M}_f (1 + \frac{h}{v}) f^m$ is very simple. **P, C, T symmetric!**

$J^\mu_{g,em,Z}$ just replace f by f^m, form no change. **CP properties no change!**

But the charged current will be modified to

$$J_W^\mu W_\mu^+ = -\frac{g_2}{\sqrt{2}} \bar{u}_L^m \gamma^\mu V_{uL} V_{dL}^\dagger d_L^m W_\mu^+, \ V_{CKM} = V_{uL} V_{dL}^\dagger$$

$$L = -\frac{g_2}{\sqrt{2}} [\bar{u}_L^m \gamma^\mu V_{KM} d_L^m W_\mu^+ + \bar{d}_L^m \gamma^\mu V_{KM}^\dagger u_L^m W_\mu^-]$$

$$= -\frac{g_2}{2\sqrt{2}} [\bar{u}_i^m \gamma^\mu (1-\gamma_5) V_{KM}^{ij} d_j^m W_\mu^+ + \bar{d}_j^m \gamma^\mu (1-\gamma_5)(V_{KM}^{ij})^* u_i^m W_\mu^-]$$

$$L^{CP}(x) = -\frac{g_2}{2\sqrt{2}} [\bar{d}_j^m \gamma^\mu (1-\gamma_5) V_{KM}^{ij} u_i^m W_\mu^- + \bar{u}_i^m \gamma^\mu (1-\gamma_5)(V_{KM}^{ij})^* u_j^m W_\mu^+](x')$$

If V_{KM} is real, then $L^{CP}(x) = L(x')$, CP is conserved!

Is V_{KM} real? Or can V_{KM} be complex?

Conditions for complex V_{KM}

$V_{KM} = (V^{ij})$, $N \times N$ unitary matrix. Naively $2N^2$ parameters, V^{ij} complex.

N^2 constraining equations: $\sum_i V^{ij} V^{ik*} = \delta^{jk}$ and $\sum_i V^{ji} V^{ki*} = \delta^{jk}$

So a unitary matrix contains N^2 parameters for N generations.

$2N - 1$ parameters, absorbed into quarks $q_i \to e^{i\alpha_i} q_i$, not physical.
Why not 2N but 2N-1?

Needs $N(N-1)/2$ parameters describe rotation angles (Euler angles)

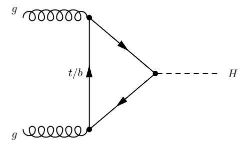
Finally $(N-1)(N-2)/2$ non-removable phases, physical, $\to V_{KM}$ complex.

The physical phases are the sources for KM model of CP violation!

Number of SM generations

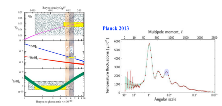
In the SM, only 3 generations of quarks and leptons are allowed.

gg -> Higgs ~ (number of heavy quarks)^2, if fourth generation exist, their mass should be large, 9 times bigger production of Higgs. LHC data ruled out more than 3 generations of quarks.

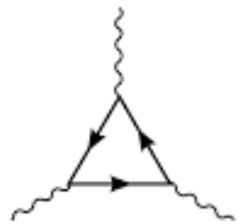


LEP already ruled out more than 3 neutrinos with mass less than $m_Z/2$.

Cosmology and astrophysics, number of light neutrinos also less than 4.



SM, triangle anomaly cancellation: equal number of quarks and leptons!



There are only three generations of sequential quarks and leptons!

Why 3 generations? How do they mix with each other?

Beyond SM, conclusions may change, X-G He and G. Valencia, PPLB707 (2012)

Idea of quark mixing and source for CP violation

The Cabbibo angle

UNITARY SYMMETRY AND LEPTONIC DECAYS

Nicola Cabibbo
CERN, Geneva, Switzerland
(Received 29 April 1963)

To determine θ, let us compare the rates for $K^+ \to \mu^+ + \nu$ and $\pi^+ \to \mu^+ + \nu$; we find

$$\Gamma(K^+ \to \mu\nu)/\Gamma(\pi^+ \to \mu\nu)$$
$$= \tan^2\theta M_K (1 - M_\mu{}^2/M_K{}^2)^2 / M_\pi (1 - M_\mu{}^2/M_\pi{}^2)^2. \quad (3)$$

From the experimental data, we then get[5,6]

$$\theta = 0.257. \quad (4)$$

Application of KM model for ε

CP violation in the six-quark model*

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(Received 29 September 1975)

We construct a Weinberg-Salam-type gauge theory of a weak interaction with CP violation based on the six-quark model. Under the assumption of the validity of the Zweig-Iizuka rule and (quark mass/W-meson mass)$^2 \ll 1$ this leads to the superweak theory of CP violation for both uncharmed and charmed hadrons. We also propose a new assignment for the J and other ψ particles, which predicts the existence of a 3.5-GeV 0^- meson using the 2.85-GeV 0^- state as input.

CP VIOLATION IN PURELY LEFTHANDED WEAK INTERACTIONS

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The Kobayashi-Maskawa Model in 1973!

Received 3 November 1975
Revised manuscript received 18 February 1976

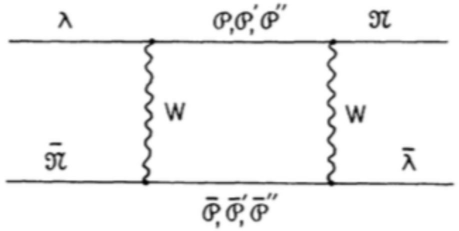
In a model with six quarks and pure V-A weak interactions CP violation can be introduced in weak currents by spontaneous breaking. The resulting milliweak model is shown to lead uniquely, with good approximation, to the results of the superweak theory both for K decays and for the neutron electric dipole moment.

The GIM mechanism

Weak Interactions with Lepton-Hadron Symmetry*

S. L. Glashow, J. Iliopoulos, and L. Maiani†
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02139
(Received 5 March 1970)

We propose a model of weak interactions in which the currents are constructed out of four basic quark fields and interact with a charged massive vector boson. We show, to all orders in perturbation theory, that the leading divergences do not violate any strong-interaction symmetry and the next to the leading divergences respect all observed weak-interaction selection rules. The model features a remarkable symmetry between leptons and quarks. The extension of our model to a complete Yang-Milis theory is discussed.



(c)

(c) Second-order diagram for ϵ.

Birth of the SM of CP violation

Progress of Theoretical Physics, Vol. 49, No. 2, February 1973

CP-Violation in the Renormalizable Theory of Weak Interaction

Makoto KOBAYASHI and Toshihide MASKAWA

Department of Physics, Kyoto University, Kyoto

(Received September 1, 1972)

In a framework of the renormalizable theory of weak interaction, problems of CP-violation are studied. It is concluded that no realistic models of CP-violation exist in the quartet scheme without introducing any other new fields. Some possible models of CP-violation are also discussed.

$$\begin{pmatrix} \cos\theta_1 & -\sin\theta_1\cos\theta_3 & -\sin\theta_1\sin\theta_3 \\ \sin\theta_1\cos\theta_2 & \cos\theta_1\cos\theta_2\cos\theta_3-\sin\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\cos\theta_2\sin\theta_3+\sin\theta_2\cos\theta_3 e^{i\delta} \\ \sin\theta_1\sin\theta_2 & \cos\theta_1\sin\theta_2\cos\theta_3+\cos\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\sin\theta_2\sin\theta_3-\cos\theta_2\sin\theta_3 e^{i\delta} \end{pmatrix}.$$

typo !
$\cos\theta_3$

Mechanism for CP violation in SM. Predicted the existence of the third generation!

Yet, another CP violation source possible: under P and CP, $\varepsilon^{\mu\nu\alpha\beta}$ -> - $\varepsilon_{\mu\nu\alpha\beta}$,

$$-\theta\frac{g_3^2}{16\pi}Tr(\tilde{G}^{\mu\nu}G_{\mu\nu})$$ – term violates P and CP

This term θ-term gives too large neutron EDM and cause problem,
Strong CP problem. Later

KM matrix parametrizations

More commonly used PDG parametrization of V_{KM}

$$V_{KM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

L.Maiani, 1976; L.L. Chau and W. Y. Keung, 1984

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$s_i = sin\theta_i$ and $c_i = cos\theta_i$ with θ_i

A non-zero value for δ violates CP.
The Wolfenstein parameterization

$$V_{KM} \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}.$$

When discussing CP violation

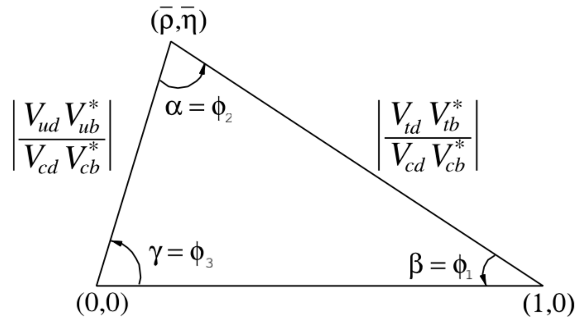
should add $-A^2\lambda^5(\rho+i\eta)$ and $-A\lambda^4(\rho+i\eta)$ to V_{cd} and V_{ts}, respectively.

Quark and Lepton mixing patterns

The mis-match of weak and mass eigen-state bases lead quark and lepton mix within generations.

Quark mixing the Cabibbo -Kobayashi-Maskawa (CKM) matrix $V_{\rm CKM}$,

lepton mixing the Pontecorvo -Maki-Nakawaga-Sakata (PMNS) matrix $U_{\rm PMNS}$



$$L = -\frac{g}{\sqrt{2}}\overline{U}_L\gamma^\mu V_{\rm CKM} D_L W_\mu^+ - \frac{g}{\sqrt{2}}\overline{E}_L\gamma^\mu U_{\rm PMNS} N_L W_\mu^- + H.C.\,,$$

$U_L = (u_L, c_L, t_L, ...)^T$, $D_L = (d_L, s_L, b_L, ...)^T$, $E_L = (e_L, \mu_L, \tau_L, ...)^T$, and $N_L = (\nu_1, \nu_2, \nu_3, ...)^T$

For n-generations, $V = V_{\rm CKM}$ or $U_{\rm PMNS}$ is an $n \times n$ unitary matrix.

A commonly used form of mixing matrix for three generations of fermions is given by

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$ are the mixing angles and δ is the CP violating phase.

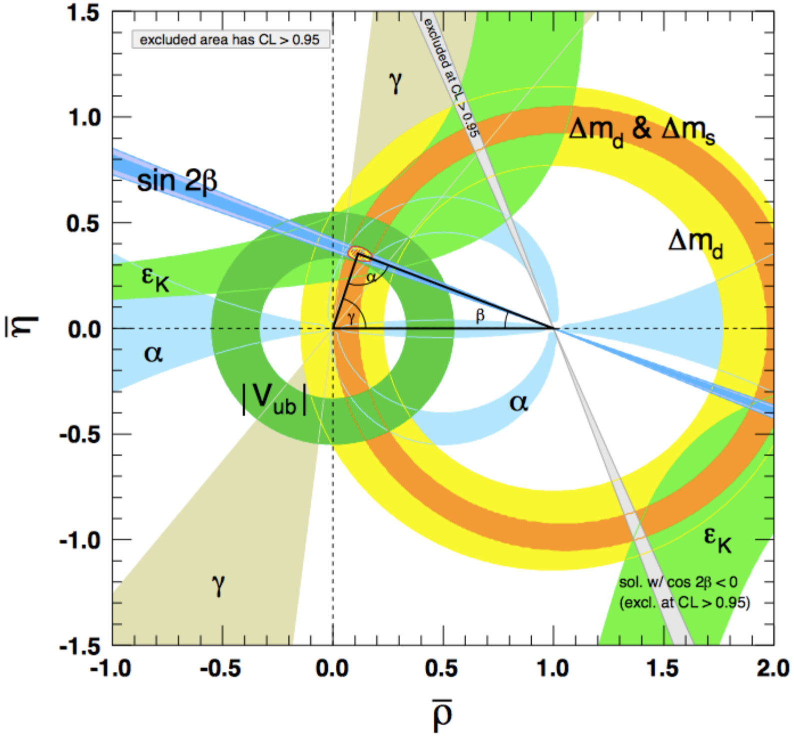
If neutrinos are of Majorana type, for the PMNS matrix one should include an additional diagonal

matrix with two Majorana phases $\mathrm{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)$ multiplied to the matrix from right in the above.

Status of Quark and Lepton

Quark Mixing | PDG | Neutrino Mixing

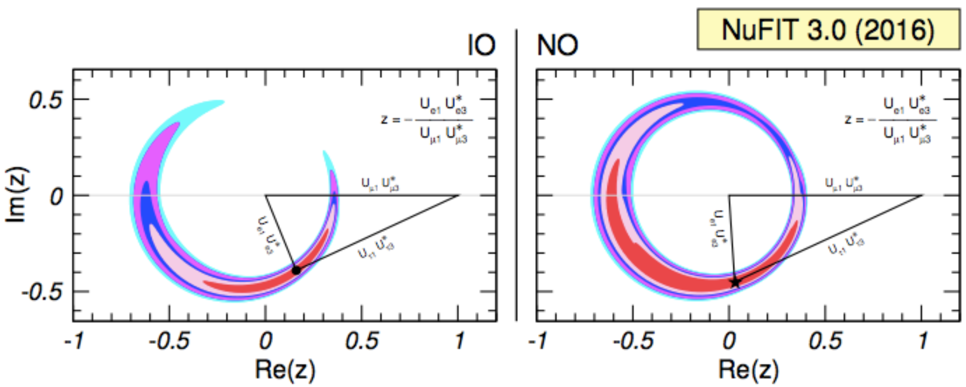
$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$



$$\lambda = 0.22537 \pm 0.00061\,, \qquad A = 0.814^{+0.023}_{-0.024}\,,$$
$$\bar\rho = 0.117 \pm 0.021\,, \qquad \bar\eta = 0.353 \pm 0.013\,.$$

$\Delta m^2 = m_3^2 - (m_2^2 + m_1^2)/2$. Thus, $\Delta m^2 = \Delta m_{31}^2 - \Delta m_{21}^2/2 > 0$, if $m_1 < m_2 < m_3$ and $\Delta m^2 = \Delta m_{32}^2 + \Delta m_{21}^2/2 < 0$ for $m_3 < m_1 < m_2$.

Parameter	best-fit	3σ
Δm^2_{21} [10^{-5} eV ^{2}]	7.37	6.93 – 7.97
$\lvert\Delta m^2\rvert$ [10^{-3} eV ^{2}]	2.50 (2.46)	2.37 – 2.63 (2.33 – 2.60)
$\sin^2\theta_{12}$	0.297	0.250 – 0.354
$\sin^2\theta_{23}$, $\Delta m^2 > 0$	0.437	0.379 – 0.616
$\sin^2\theta_{23}$, $\Delta m^2 < 0$	0.569	0.383 – 0.637
$\sin^2\theta_{13}$, $\Delta m^2 > 0$	0.0214	0.0185 – 0.0246
$\sin^2\theta_{13}$, $\Delta m^2 < 0$	0.0218	0.0186 – 0.0248
δ/π	1.35 (1.32)	(0.92 – 1.99)
		((0.83 – 1.99))

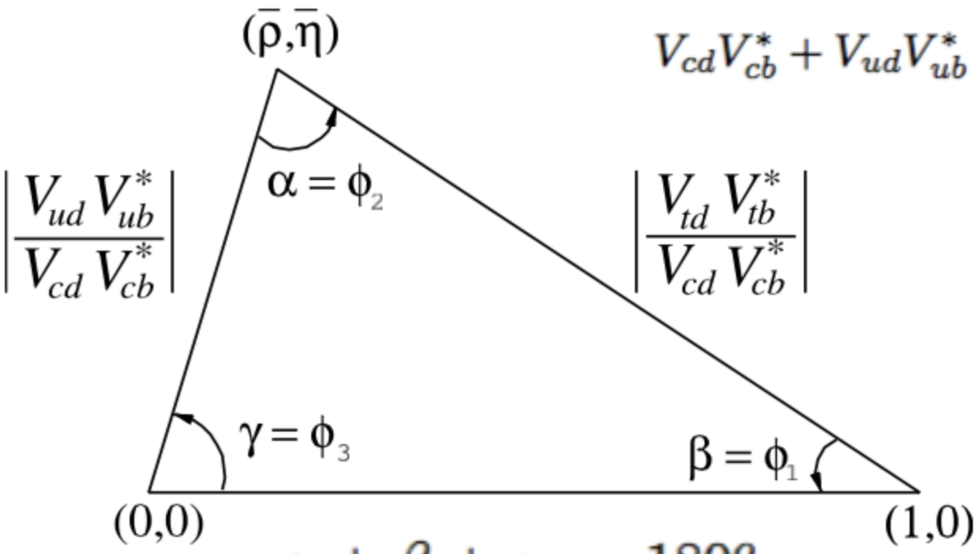


Why they mix the pattern shown above? Some understanding.

The Unitarity Triangle

$$\sum_i V_{ij}V_{ik}^* = \delta_{jk}\ , \quad \sum_i V_{ji}V_{ki}^* = \delta_{jk}\ ,$$

$$V_{cd}V_{cb}^* + V_{ud}V_{ub}^* + V_{td}V_{tb}^* = 0 \ \rightarrow\ 1 + \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0\ .$$



$$\alpha + \beta + \gamma = 180^o$$

$$\sin 2\beta = 0.691 \pm 0.017$$

$$\alpha = (87.6^{+3.5}_{-3.3})^\circ \qquad \gamma = (73.2^{+6.3}_{-7.0})^\circ.$$

The Jarlskog parameter J (1985)

$$\mathrm{Im}\left[V_{ij}V_{kl}V_{il}^*V_{kj}^*\right] = J\sum_{m,n}\varepsilon_{ikm}\varepsilon_{jln}$$

$J = s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^2\sin\delta$

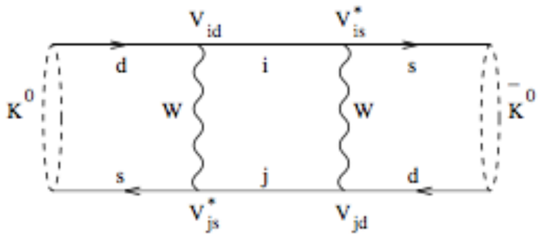
$= (3.04^{+0.21}_{-0.20})\times 10^{-5}$

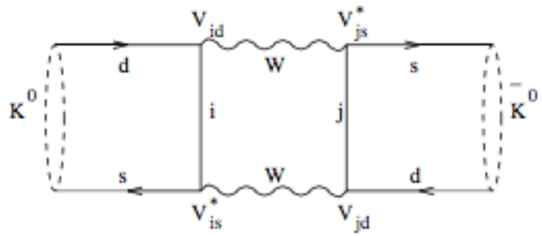
The area of the triangle = J/2

CPV in SM is always proportional to J

$$\alpha = \mathrm{Arg}(-V_{td}V_{tb}^*/V_{ub}^*V_{ud}),\ \beta = \mathrm{Arg}(-V_{cd}V_{cb}^*/V_{tb}^*V_{td}),\ \text{and}\ \gamma = \mathrm{Arg}(-V_{ud}V_{ub}^*/V_{cb}^*V_{cd})$$

Calculation of Im(M_{12})





$$H_{eff} = -\frac{2}{3}\frac{G_F^2 m_W^2}{\pi^2}\sum_{i,j}(V_{id}V_{is}^*)(V_{jd}V_{js}^*)B(\alpha_i,\alpha_j)\bar{s}\gamma_\mu L d\bar{s}\gamma^\mu L d\ ,$$

$$B(x,y) = (1+\frac{xy}{4})(\frac{1}{(1-x)(1-y)} + \frac{1}{x-y}[\frac{x^2 lnx}{(1-x)^2} - \frac{y^2 lny}{(1-y)^2}])$$
$$-2xy(\frac{1}{(1-x)(1-y)} + \frac{1}{x-y}[\frac{x lnx}{(1-x)^2} - \frac{y lny}{(1-y)^2}])\ ,$$

$$M_{12} \quad = \quad <\bar{K}^0|H_{eff}|K^0> = -\frac{1}{8}\frac{G_F^2 m_W^2}{\pi^2}\sum_{i,j}(V_{id}V_{is}^*)(V_{jd}V_{js}^*)B(\alpha_i,\alpha_j)C\ ,$$

$$C = <\bar{K}^0|\bar{s}\gamma_\mu L d\bar{s}\gamma^\mu L d|K^0>$$

Vacuum saturation approximation $C =< \bar{K}^0|\bar{s}\gamma_\mu L d\bar{s}\gamma^\mu L d|K^0 >$

$=< \bar{K}^0|(\bar{s}^\alpha\gamma_\mu L d_\alpha \bar{s}^\beta\gamma^\mu L d_\beta) + (\bar{s}^\alpha\gamma_\mu L d_\beta \bar{s}^\beta\gamma^\mu L d_\alpha)K^0 >$

$= 2 < \bar{K}^0|(\bar{s}^\alpha\gamma_\mu L d_\alpha|0 >< 0|\bar{s}^\beta\gamma^\mu L d_\beta) + (\bar{s}^\alpha\gamma_\mu L d_\beta|0 >< 0|\bar{s}^\beta\gamma^\mu L d_\alpha)K^0 >$

$= 2(1 + 1/3)(1/4)f_K^2 m_K^2/(2m_k) = -2/3 f_K^2 m_K.$

$\alpha_i = m_i^2/m_W^2$, $f_K = 160$ MeV is the kaon decay constant

$< 0|\bar{s}\gamma^\mu\gamma_5 d|K^0 >=< \bar{K}^0|\bar{s}\gamma^\mu\gamma_5 d|0 >= i f_K p_k^\mu$

None factorizeble effects introduce bag factor B_K, $C = -2/3 f_K^2 m_K B_K$

Vacuum saturation, $B_K = 1$. Lattice calculation gives $B_K = 0.766 \pm 0.010$

With QCD corrections, the matrix element M_{12} is given by

$$
\begin{aligned}
M_{12} &= \frac{f_K^2 m_K G_F^2 m_W^2}{12\pi^2} B_K [\eta_1 \tilde{B}_1 (V_{cd} V_{cs}^*)^2 + \eta_2 \tilde{B}_2 (V_{td} V_{ts}^*)^2 \\
&+ 2\eta_3 \tilde{B}_3 (V_{cd} V_{cs}^* V_{td} V_{ts}^*)] , \\
\tilde{B}_1 &= B(\alpha_c, \alpha_c) - B(\alpha_u, \alpha_c) - B(\alpha_c, \alpha_u) + B(\alpha_u, \alpha_u) , \\
\tilde{B}_2 &= B(\alpha_t, \alpha_t) - B(\alpha_u, \alpha_t) - B(\alpha_t, \alpha_u) + B(\alpha_u, \alpha_u) , \\
\tilde{B}_3 &= B(\alpha_u, \alpha_u) - B(\alpha_c, \alpha_u) - B(\alpha_t, \alpha_u) + B(\alpha_t, \alpha_c) ,
\end{aligned}
$$

η_i QCD correction factors η_i,next-to-leading order and are given by:

$\eta_1 = 1.38$, $\eta_2 = 0.574$, and $\eta_3 = 0.47$

The parameter ϵ is given by

$$|\epsilon| = 4.39 A^2 B_K \eta [\eta_3 \tilde{B}_3 - \eta_1 \tilde{B}_1 + \eta_2 A^2 \lambda^4 (1-\rho) \tilde{B}_2] .$$

Successful explain CP violation in Neutral Kaon Mixing!

CP violation for ε and consistency with a heavy top

To produce the right e, a large top is need and have many implications.

AN ULTRA-HEAVY TOP QUARK?

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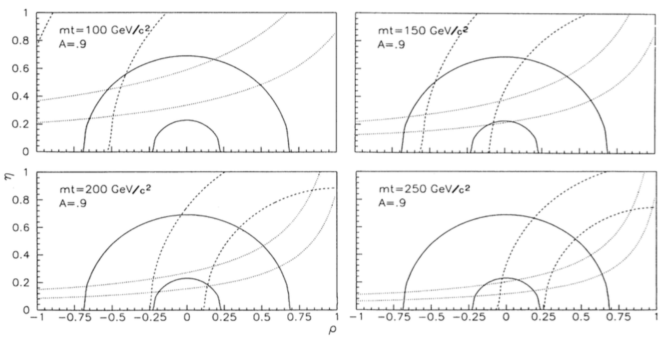
Received 11 May 1987

We consider an ansatz for the mass of the t-quark yielding a value as high as it is allowed to be viz. about 200 GeV. We check the consistency of this hypothesis with all known low energy physics and delineate the consequences for ϵ'/ϵ, mixing in B_d^0–$\bar{B}_d^0$ and B_s^0–$\bar{B}_s^0$ systems and rare decay modes of K and B. An interesting feature is the existence of two classes of solutions for the Kobayashi–Maskawa angles; one of which corresponds to small ϵ'/ϵ and large B_d^0–$\bar{B}_d^0$ mixing and the other to large ϵ'/ϵ and small B_d^0–$\bar{B}_d^0$ mixing.



New range of mixing parameters and rare K decays

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(Received 24 August 1990)

The constraints on the elements of the quark mixing matrix are updated and are used to study rare kaon decays. The top-quark-mass-dependent limit on the mixing matrix from the measurement of the branching ratio $K_L \to \mu\bar{\mu}$ is also included, and the effects on other rare decays are presented.

Homework

Show that for V_{KM}, the identify is true

$$\mathrm{Im}\left[V_{ij}V_{kl}V_{il}^{*}V_{kj}^{*}\right] = J\sum_{m,n}\varepsilon_{ikm}\varepsilon_{jln}.$$

Calculate $\Delta m_{S\text{-}L}$ using the result from Box diagram and compare with data.

Lecture Three

5. Tests for Standard Model of CV Violation

SM can explain CPV in neutral Kaon mixing. Only doing that job is not enough to become part of a SM and being awarded Nobel prize.

Predictions made and confirmed.

Many predictions been confirmed!

Observables: ε', time dependent A_{CP} and independent rate asymmetry S_f and C_f, unitarity triangle, electric dipole moment d of fundamental particle, …

5.1 CP violating observable

ε'/ε in K -> ππ

The ϵ' in $K_{L,S} \to \pi\pi$, a measurement of direct CPV

$$\epsilon' = \frac{\eta_{+-} - \eta_{00}}{3} \; , \quad \eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \; , \quad \eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} \; .$$

What ϵ' is measuring?

$$A(K_L \to \pi\pi) = \frac{1+\epsilon}{\sqrt{1+|\epsilon|^2}} A(K^0 \to \pi\pi) + \frac{1-\epsilon}{\sqrt{1+|\epsilon|^2}} A(\bar{K}^0 \to \pi\pi)$$

$$A(K_S \to \pi\pi) = \frac{1+\epsilon}{\sqrt{1+|\epsilon|^2}} A(K^0 \to \pi\pi) - \frac{1-\epsilon}{\sqrt{1+|\epsilon|^2}} A(\bar{K}^0 \to \pi\pi)$$

Isospin decay decomposition for $K^0(\bar{K}^0) \to \pi\pi$ decay amplitudes

Isospins I of π is 1, isospin components $(\pi^+, \pi^0, \pi^-) \to (1, 0, -1)$,

Isospin I of K is 1/2, isospin components $(K^0, \bar{K}^0) \to (-1/2, 1/2)$

$$|\pi^+\pi^- >= \sqrt{1/3}|2,0> + \sqrt{2/3}|0,0>, \ \ |\pi^0\pi^0>= \sqrt{2/3}|2,0> - \sqrt{1/3}|0,0>$$

$$< K^0| \ |\pi^+\pi^- >= |\frac{1}{2},\frac{1}{2}> |\pi^+\pi^- >= \sqrt{\frac{1}{5}}|\frac{5}{2},\frac{1}{2}> + \sqrt{\frac{2}{15}}|\frac{3}{2},\frac{1}{2}> + \sqrt{\frac{2}{9}}|\frac{1}{2},\frac{1}{2}>$$

$$< K^0| \ |\pi^0\pi^0 >= |\frac{1}{2},\frac{1}{2}> |\pi^0\pi^0 >= -\sqrt{\frac{2}{5}}|\frac{5}{2},\frac{1}{2}> + \sqrt{\frac{4}{15}}|\frac{3}{2},\frac{1}{2}> + \sqrt{\frac{2}{9}}|\frac{1}{2},\frac{1}{2}>$$

To induce the decay to happen, the Hamiltonian needs carry isospin I=5/2, I=3/2 and I=1/2 inducing $A_{5/2}$, $A_{3/2}$ and $A_{1/2}$ amplitudes

In the SM, 5/2 isospin amplitude is very small (more than four quark operators to generate). Neglect them!

Renaming $-(2/\sqrt{5})A_{3/2} = A_2 e^{i\delta_2}$ and $A_{1/2} = -\sqrt{2}A_0 e^{i\delta_0}$

$$\begin{aligned} A(K^0 \to \pi^+\pi^-) &= \sqrt{\frac{2}{3}}A_0 e^{i\delta_0} + \sqrt{\frac{1}{3}}A_2 e^{i\delta_2} , \\ A(K^0 \to \pi^0\pi^0) &= \sqrt{\frac{1}{3}}A_0 e^{i\delta_0} - \sqrt{\frac{2}{3}}A_2 e^{i\delta_2} , \end{aligned}$$

δ_i are the strong final state rescattering phases (strong phase)
A_0 and A_2 are complex in general due to weak CP violating phases.
The corresponding anti-particle decay amplitudes are

$$\begin{aligned} A(\bar{K}^0 \to \pi^+\pi^-) &= -\sqrt{\frac{2}{3}}A_0^* e^{i\delta_0} - \sqrt{\frac{1}{3}}A_2^* e^{i\delta_2} , \\ A(\bar{K}^0 \to \pi^0\pi^0) &= -\sqrt{\frac{1}{3}}A_0^* e^{i\delta_0} + \sqrt{\frac{2}{3}}A_2^* e^{i\delta_2} . \end{aligned}$$

$$\eta_{+-} = \epsilon + i\frac{ImA_0}{ReA_0} + e^{i(\pi/2+\delta_2-\delta_0)}\frac{ReA_2}{\sqrt{2}ReA_2}\left(\frac{ImA_2}{ReA_2} - \frac{ImA_0}{ReA_0}\right),$$

$$\eta_{00} = \epsilon + i\frac{ImA_0}{ReA_0} - 2e^{i(\pi/2+\delta_2-\delta_0)}\frac{ReA_2}{\sqrt{2}ReA_0}\left(\frac{ImA_2}{ReA_2} - \frac{ImA_0}{A_0}\right),$$

$$\epsilon' = \frac{\eta_{+-} - \eta_{00}}{3} = \frac{ReA_2}{\sqrt{2}ReA_0}\left(\frac{ImA_2}{ReA_2} - \frac{ImA_0}{ReA_0}\right)e^{i(\pi/2+\delta_2-\delta_0)}.$$

δ_i are determined from phase shift analyses in $\pi - \pi$ scattering, and $\pi/2 + \delta_2 - \delta_0$ is found to be close to $\pi/4$.

CPT symmetry implies that this phase is equal to the phase ϕ_ϵ for ϵ.

In the literature the quantity ϵ'/ϵ is usually used.

Experimental measurement of ϵ'/ϵ

1993 NA31 at CERN, $\epsilon'/\epsilon = (2.3 \pm 0.7) \times 10^{-3}$

1993 E731 at Fermilab, $\epsilon'/\epsilon = 0.74 \pm 0.59) \times 10^{-3}$.

1999 KTeV at Fermlab, $\epsilon'/\epsilon = (2.8 \pm 0.41) \times 10^{-3}$

1999 NA48 at CERN, $Re(\epsilon'/\epsilon) = (1.85 \pm 0.45 \pm 9,58) \times 10^{-3}$

Current value: $Re(\epsilon'/\epsilon) = (1.65 \pm 0.26) \times 10^{-3}$

SM calculation for ε'/ε

Tree and penguin contributions

$$\mathcal{H}_{\rm eff}(\Delta S=1)=\frac{G_F}{\sqrt{2}}\,V_{us}^*V_{ud}\sum_{i=1}^{10}\left[z_i(\mu)+\tau\, y_i(\mu)\right]Q_i(\mu),$$

$$\tau=-\frac{V_{ts}^*V_{td}}{V_{us}^*V_{ud}}.$$

$$Q_1=(\bar{s}_iu_j)_{V-A}(\bar{u}_jd_i)_{V-A},$$

$$Q_2=(\bar{s}u)_{V-A}(\bar{u}d)_{V-A},$$

$$Q_3=(\bar{s}d)_{V-A}\sum_q(\bar{q}q)_{V-A},$$

$$Q_4=(\bar{s}_id_j)_{V-A}\sum_q(\bar{q}_jq_i)_{V-A},$$

$$Q_5=(\bar{s}d)_{V-A}\sum_q(\bar{q}q)_{V+A},$$

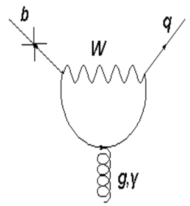
$$Q_6=(\bar{s}_id_j)_{V-A}\sum_q(\bar{q}_jq_i)_{V+A}.$$

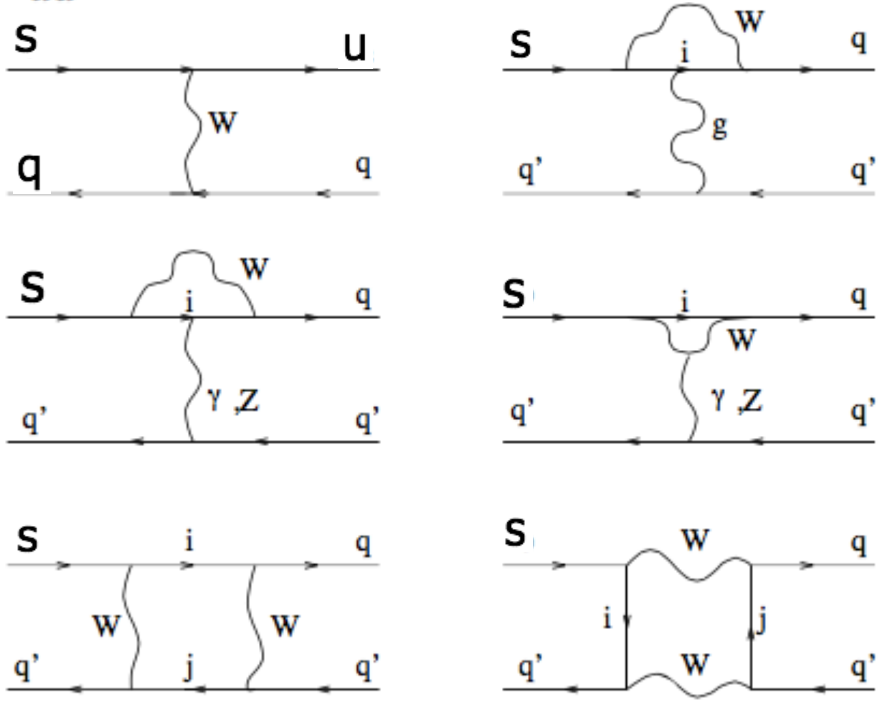
$$Q_7=\frac{3}{2}(\bar{s}d)_{V-A}\sum_q e_q(\bar{q}q)_{V+A},$$

$$Q_8=\frac{3}{2}(\bar{s}_id_j)_{V-A}\sum_q e_q(\bar{q}_jq_i)_{V+A},$$

$$Q_9=\frac{3}{2}(\bar{s}d)_{V-A}\sum_q e_q(\bar{q}q)_{V-A},$$

$$Q_{10}=\frac{3}{2}(\bar{s}_id_j)_{V-A}\sum_q e_q(\bar{q}_jq_i)_{V-A}.$$





ΔS=1 Wilson coefficients at μ=1 GeV for m_t=170 GeV. $y_1 = y_2 \equiv 0$.

	$\Lambda_{\overline{MS}}^{(4)}$=215 MeV			$\Lambda_{\overline{MS}}^{(4)}$=325 MeV			$\Lambda_{\overline{MS}}^{(4)}$=435 MeV		
Scheme	LO	NDR	HV	LO	NDR	HV	LO	NDR	HV
z_1	−0.607	−0.409	−0.494	−0.748	−0.509	−0.640	−0.907	−0.625	−0.841
z_2	1.333	1.212	1.267	1.433	1.278	1.371	1.552	1.361	1.525
z_3	0.003	0.008	0.004	0.004	0.013	0.007	0.006	0.023	0.015
z_4	−0.008	−0.022	−0.010	−0.012	−0.035	−0.017	−0.017	−0.058	−0.029
z_5	0.003	0.006	0.003	0.004	0.008	0.004	0.005	0.009	0.005
z_6	−0.009	−0.022	−0.009	−0.013	−0.035	−0.014	−0.018	−0.059	−0.025
z_7/α	0.004	0.003	−0.003	0.008	0.011	−0.002	0.011	0.021	−0.001
z_8/α	0	0.008	0.006	0.001	0.014	0.010	0.001	0.027	0.017
z_9/α	0.005	0.007	0	0.008	0.018	0.005	0.012	0.034	0.011
z_{10}/α	0	−0.005	−0.006	−0.001	−0.008	−0.010	−0.001	−0.014	−0.017
y_3	0.030	0.025	0.028	0.038	0.032	0.037	0.047	0.042	0.050
y_4	−0.052	−0.048	−0.050	−0.061	−0.058	−0.061	−0.071	−0.068	−0.074
y_5	0.012	0.005	0.013	0.013	−0.001	0.016	0.014	−0.013	0.021
y_6	−0.085	−0.078	−0.071	−0.113	−0.111	−0.097	−0.148	−0.169	−0.139
y_7/α	0.027	−0.033	−0.032	0.036	−0.032	−0.030	0.043	−0.031	−0.027
y_8/α	0.114	0.121	0.133	0.158	0.173	0.188	0.216	0.254	0.275
y_9/α	−1.491	−1.479	−1.480	−1.585	−1.576	−1.577	−1.700	−1.718	−1.722
y_{10}/α	0.650	0.540	0.547	0.800	0.690	0.699	0.968	0.892	0.906

Buchalla et al., Rev. Mod. Phys. 68, 1125(1996)

$$\frac{\varepsilon'}{\varepsilon} = 10^{-4}\left[\frac{\mathrm{Im}\lambda_{\rm t}}{1.4\cdot 10^{-4}}\right]\left[-6.5(3.2) + 25.3\, B_6^{(1/2)} + 1.2(8) - 10.2\, B_8^{(3/2)}\right]$$

$$(\varepsilon'/\varepsilon)_{\rm SM} = (11.1 \pm 3.2) \times 10^{-4}, \qquad (B_6^{(1/2)} = 1.0, \quad B_8^{(3/2)} = 0.76)$$

(Buras e t al., arXiv:1507.06345)

Lattice calculation $(\varepsilon'/\varepsilon)_{\rm SM} = (1.4 \pm 7.0) \times 10^{-4}$

(Z. Bai et al., arXiv:1505.07863)

Needs further confirmed!

Room for new Physics beyond SM!?

EMMANUEL A. PASCHOS and Y.L. WU, *Mod. Phys. Lett. A* **06**, 93 (1991).
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CORRELATIONS BETWEEN ε′/ε AND HEAVY TOP

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The paper reviews new theoretical developments for the CP-parameter ε′/ε and its intimate connection with the mass of a heavy top quark. It presents an extensive list of results and their implications for experiments.

Time dependent and independent Rate asymmetry

The mass eigenstates are

$$|M_L>= p|M> +q|\bar{M}> \,, \quad M_H>= p|M> -q|\bar{M}> \,,$$
$$|M>= \frac{1}{2p}(|M_L> +|M_H>) \,, \ |\bar{M}>= \frac{1}{2q}(|M_L> -|M_H>) \,.$$

$$|M(t)>= \frac{1}{2p}\left(e^{-im_Lt-\Gamma_Lt/2}|M_L> +e^{-im_Ht-\Gamma_Ht/2}|M_H>\right) \,,$$
$$= \frac{1}{2}e^{-im_Ht-\Gamma_Ht/2}\left((1+e^{i\Delta m+\Delta\Gamma t/2})|M> -\frac{q}{p}(1-e^{i\Delta mt+\Delta\Gamma t/2})|\bar{M}>\right) \,,$$
$$|\bar{M}(t)>= \frac{1}{2q}\left(e^{-im_Lt-\Gamma_Lt/2}|M_L> -e^{-im_Ht-\Gamma_Ht/2}|M_H>\right) \,,$$
$$= \frac{1}{2}e^{-im_Ht-\Gamma_Ht/2}\left(-\frac{p}{q}(1-e^{i\Delta m+\Delta\Gamma t/2})|M> +(1+e^{i\Delta mt+\Delta\Gamma t/2})|\bar{M}>\right) \,,$$

$\Delta m = m_H - m_L$, $\Delta\Gamma = \Gamma_H - \Gamma_L$, $\Gamma = (\Gamma_H + \Gamma_L)/2$, $(\frac{p}{q})^2 = \frac{M^*_{12} - i\Gamma^*_{12}/2}{M_{12} - i\Gamma_{12}/2}$

Define decay amplitudes

$M \to f$: $A_f =< f|H|M >$, $M_{\to}\bar{f}$: $A_{\bar{f}} =< \bar{f}|H|M >$,

$\bar{M} \to \bar{f}$: $\bar{A}_{\bar{f}} =< \bar{f}|H|\bar{M} >$, $\bar{M} \to f$: $\bar{A}_f =< f|H\bar{M} >$

$$< f|H|M(t) >= \frac{1}{2}e^{-im_H t-\Gamma_H/2}(1+e^{i\Delta mt+\Delta\Gamma t/2})A_f \ ,$$
$$< \bar{f}|H|M(t) >= -\frac{1}{2}e^{-im_H t-\Gamma_H/2}\frac{q}{p}(1-e^{i\Delta mt+\Delta\Gamma t/2})A_{\bar{f}} \ ,$$
$$< \bar{f}|H|\bar{M}(t) >= \frac{1}{2}e^{-im_H t-\Gamma_H/2}(1+e^{i\Delta mt+\Delta\Gamma t/2})\bar{A}_{\bar{f}} \ ,$$
$$< f|H|\bar{M}(t) >= -\frac{1}{2}e^{-im_H t-\Gamma_H/2}\frac{p}{q}(1-e^{i\Delta mt+\Delta\Gamma t/2})\bar{A}_f \ ,$$

Uncorrelated M and $\bar{M}$ production

Flavor specific case, $f \neq \bar{f}$**,** $A_{\bar{f}} = \bar{A}_f = 0$

Time dependent CP asymmetry,

Such as $B_d^0 \to \pi^+ K^-$ and $B_s^0 \to \pi^- K^-$

$$A(t)_{CP} = \frac{\Gamma(\bar{M}(t) \to \bar{f}) - \Gamma(M(t) \to f)}{\Gamma(M(t) \to f) + \Gamma(\bar{M}(t) \to \bar{f})} = \frac{|\bar{A}_{\bar{f}}|^2 - |A_f|^2}{|A_f|^2 + |A_{\bar{f}}|^2} \, .$$

Actually no time dependence!

How to measure this experimentally?

Usually M and $\bar{M}$ are produced in pairs.

If produced uncorrelated, like production at hadron colliders

Make sure each decay is originated from M for $M(t) \to f$

by having good tracking measurement

One trace at origin, whether the particle is M or $\bar{M}$.

Time integrated CP asymmetry,

$$A_{CP} = \frac{\int_0^\infty \Gamma(\bar{M}(t) \to \bar{f}) - \int_0^\infty \Gamma(M(t) \to f)}{\int_0^\infty \Gamma(M(t) \to f) + \int_0^\infty \Gamma(\bar{M}(t) \to \bar{f})} = \frac{|\bar{A}_{\bar{f}}|^2 - |A_f|^2}{|A_f|^2 + |A_{\bar{f}}|^2} \, .$$

$A(t)_{CP} = A_{CP}$. Direct CP violation.

For charged M no need of tagging because there is no M and $\bar{M}$ oscillation.

There is no mixing between M and $\bar{M}$, such as $B^+ \to K^+\pi$...

Conditions for CP asymmetry: $|A_f| \neq |\bar{A}_{\bar{f}}|$

Parametrized

$$\begin{aligned} A_f &= A_1 e^{i(\delta_1^s+\delta_1^w)} + A_2 e^{i(\delta_2^s+\delta_2^w)} \, , \\ \bar{A}_{\bar{f}} &= \eta^{CP}(A_1 e^{i(\delta_1^s-\delta_1^w)} + A_2 e^{i(\delta_2^s-\delta_2^w)}) \, , \end{aligned}$$

δ_i^s are the strong phases and δ_i^w are the CP violating weak phases. $|\eta^{CP}| = 1$

$$A_{CP} = \frac{-2A_1A_2 sin(\delta_1^w - \delta_2^w) sin(\delta_1^s - \delta_2^s)}{A_1^2 + A_2^2 + 2A_1A_2 cos(\delta_1^w - \delta_2^w) cos(\delta_1^w - \delta_w^2)} \, .$$

There must more than one amplitudes with different strong and weak phases!

$A_{CP}(B_s^0 \to p^+K^-) = 0.26 \pm 0.04$ and $A_{CP}(B^0 \to \pi^-K^+) = -0.082 \pm 0.006$!

Purely mixing induced CP

If measuring $M(t) \to \bar{f}$ and $\bar{M}(t) \to f$

$$A(t)_{CP} = \frac{\Gamma(M(t) \to \bar{f}) - \Gamma(\bar{M}(t) \to f)}{\Gamma(M(t) \to \bar{f}) + \Gamma(\bar{M}(t) \to f)} = \frac{\left|\frac{q}{p}\right|^2 |\bar{A}_{\bar{f}}|^2 - \left|\frac{p}{q}\right|^2 |A_f|^2}{\left|\frac{q}{p}\right|^2 |\bar{A}_{\bar{f}}|^2 + \left|\frac{p}{q}\right|^2 |A_f|^2} .$$

Information about mixing can be extracted!

In the case $|A_f| = |\bar{A}_{\bar{f}}|$,

$$A^{mix}(t)_{CP} = A^{mix}_{CP} = \frac{\left|\frac{q}{p}\right|^2 - \left|\frac{p}{q}\right|^2}{\left|\frac{q}{p}\right|^2 + \left|\frac{p}{q}\right|^2} .$$

Example: $K^0 \to \mu^+\nu$ and $\bar{K}^0 \to \mu^-\bar{\nu}$. $A_f = \bar{A}_{\bar{f}}$

$p = (1+\epsilon)/\sqrt{1+|\epsilon|^2}$ and $q = (1-\epsilon)/\sqrt{1+|\epsilon|^2}$

$p/q = \sqrt{\frac{M_{12}^* - i\Gamma^*/2}{M_{12} - i\Gamma_{12}/1}}$

$A_{CP}^{mix} = (|1-\epsilon|^4 - |1+\epsilon|^4)/(|1+\epsilon|^4 + |1-\epsilon|^4) \approx -2Re(\epsilon) \approx 2\frac{Im(M_{12}\Gamma^*)}{|M_{12}|^2}$

If one can identify K_L first, then

$$\delta_L = \frac{\Gamma(K_L - > l^+\nu_l\pi^-) - \Gamma(K_L \to l^-\bar{\nu}_l\pi^+)}{\Gamma(K_L - > l^+\nu_l\pi^-) + \Gamma(K_L \to l^-\bar{\nu}_l\pi^+)}$$

$\delta_L \approx 2Re(\epsilon) = (3.32 \pm 0.06) \times 10^{-3}$. Agree with data!

For $B_s^0 \to l^- X$, $A_{SL}^s = A_{CP}^{mix} = (-7.5 \pm 4.1) \times 10^{-3}$.

Compared with SM $A_{SL}^s = (1.9 \pm 3.0) \times 10^{-5}$.

M **and** $\bar{M}$ **decay into CP eigenstate** $f = \bar{f} = f_{CP}$

$$
\begin{aligned}
\Gamma(M(t) \to f_{CP}) \quad &\sim \quad \frac{1}{2} e^{\Gamma_H t} \Bigg(\frac{1 + e^{\Delta\Gamma t}}{2} (|A_{CP}|^2 + |\frac{q}{p}\bar{A}_{CP}|^2) + e^{\Delta\Gamma t} cos(\Delta m t)(|A_{CP}|^2 - |\frac{q}{p}\bar{A}_{CP}|^2) \\
&- \quad |A_{CP}|^2 (1 - e^{\Delta\Gamma t}) Re(\frac{q}{p}\frac{\bar{A}_{CP}}{A_{CP}}) - |A_f|^2 sin(\Delta m t) e^{\Delta\Gamma t/2} Im(\frac{q}{p}\frac{\bar{A}_f}{A_f}) \Bigg) \ , \\
\Gamma(\bar{M}(t) \to f_{CP}) \quad &\sim \quad \frac{1}{2} e^{\Gamma_H t} \Bigg(\frac{1 + e^{\Delta\Gamma t}}{2} (|\bar{A}_{CP}|^2 + |\frac{p}{q} A_{CP}|^2) + e^{\Delta\Gamma t} cos(\Delta m t)(|\bar{A}_{CP}|^2 - |\frac{p}{q} A_{CP}|^2) \\
&- \quad |\bar{A}_{CP}|^2 (1 - e^{\Delta\Gamma t}) Re(\frac{p}{q}\frac{A_{CP}}{\bar{A}_{CP}}) - |\bar{A}_{CP}|^2 sin(\Delta m t) e^{\Delta\Gamma t} Im(\frac{p}{q}\frac{A_{CP}}{\bar{A}_{CP}}) \Bigg) \ ,
\end{aligned}
$$

Time dependent asymmetry

$$
A(t)_{CP} = \frac{\Gamma(\bar{M}(t) \to f_{CP}) - \Gamma(M(t) \to f_{CP})}{\Gamma(\bar{M}(t) \to f_{CP}) + \Gamma(M(t) \to f_{CP})} \ .
$$

Time dependent asymmetry

$$A(t)_{CP} = \frac{\Gamma(\bar{M}(t) \to f_{CP}) - \Gamma(M(t) \to f_{CP})}{\Gamma(\bar{M}(t) \to f_{CP}) + \Gamma(M(t) \to f_{CP})}\,.$$

In the limit $|q/p| = 1$, one obtains

$$A(t)_{CP} = \frac{-C_f cos(\Delta m t) + S_f sin(\Delta m t)}{cosh(\Delta\Gamma t/2) + A_f^{\Delta\Gamma} sinh(\Delta\Gamma t/2)}\,,$$

$$C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}\,,\quad S_f = \frac{2\Im(\lambda_f)}{1+|\lambda_f|^2}\,,\quad A_f^{\Delta\Gamma|} = \frac{2\Re(\lambda_f)}{1+|\lambda_f|^2}\,,\quad \lambda_f = \frac{q}{p}\frac{\bar{A}_{CP}}{A_{CP}}\,.$$

CPT sum rule: $|C_f|^2 + |S_f|^2 + |A_f^{\Delta\Gamma}|^2 = 1.$

In the SM, for $B_s^0 - \bar{B}_s^0$ system, good approximation $q/p = V_{tb}^* V_{ts}/V_{tb} V_{ts}^*$,
For $B_0 - \bar{B}^0$ system, $q/p = V_{tb}^* V_{td}/V_{tb} V_{td}^*$. $|q/p| = 1$.

Measurements of S_f and C_f in B decays played an important role in verifying the standard model for CP violation.

Example: $Im(\lambda_{B^0 \to J/\psi K_s} = sin(2\beta) = 0.691 \pm 0.017$

More later

Without Tagging, one can also obtain important information

Let f to be a positively charged particle and $\bar{f}$ then has negative charge.

The time integrated event number N^+ is proportional to

$$N^+(M) \sim \int_0^\infty |<f|H|M(t)|>^2\, dt = \frac{|A_f|^2}{2}\frac{\Gamma}{\Gamma_H\Gamma_L}(1+\frac{\Gamma_H\Gamma_L}{\Delta m^2+\Gamma^2})$$

$$N^+(\bar{M}) \sim \int_0^\infty |<f|H|M(t)>|^2 dt = \frac{|A_f|^2}{2}\frac{\Gamma}{\Gamma_H\Gamma_L}(1-\frac{\Gamma_H\Gamma_L}{\Delta m^2+\Gamma^2})|p/q|^2$$

$$N^-(\bar{M}) \sim \int_0^\infty |<\bar{f}|H|\bar{M}(t)>|^2 dt = \frac{|\bar{A}_{\bar{f}}|^2}{2}\frac{\Gamma}{\Gamma_H\Gamma_L}(1+\frac{\Gamma_H\Gamma_L}{\Delta m^2+\Gamma^2})$$

$$N^-(M) \sim \int_0^\infty |<\bar{f}|H|M(t)>|^2 dt = \frac{|\bar{A}_{\bar{f}}|^2}{2}\frac{\Gamma}{\Gamma_H\Gamma_L}(1-\frac{\Gamma_H\Gamma_L}{\Delta m^2+\Gamma^2})|q/p|^2$$

The event numbers N^{+-}, N^{++} and N^{--} for observing $f\bar{f}$, ff and $\bar{f}\bar{f}$ are

$$N^{+-} = N^{+}(M)N^{-}(\bar{M}) + N^{+}(\bar{M})N^{-}(M)$$
$$\sim (\tfrac{\Gamma}{2\Gamma_H\Gamma_L})^2|A_f|^2|\bar{A}_{\bar{f}}|^2((1+\tfrac{\Gamma_H\Gamma_L}{\Delta m^2+\Gamma^2})^2+(1-\tfrac{\Gamma_H\Gamma_L}{\Delta m^2+\Gamma^2})^2)$$

$$N^{++} = N^{+}(M)N^{+}(\bar{M}) \sim (\tfrac{\Gamma}{2\Gamma_H\Gamma_L})^2|A_f|^2|A_f|^2(1-(\tfrac{\Gamma_H\Gamma_L}{\Delta m^2+\Gamma^2})^2)|p/q|^2$$

$$N^{--} = N^{-}(M)N^{-}(\bar{M}) \sim (\tfrac{\Gamma}{2\Gamma_H\Gamma_L})^2|\bar{A}_{\bar{f}}|^2|\bar{A}_{\bar{f}}|^2(1-(\tfrac{\Gamma_H\Gamma_L}{\Delta m^2+\Gamma^2})^2)|p/q|^2$$

Defining $\Delta = \frac{\Gamma_H\Gamma_L}{\Delta m^2+\Gamma^2} = \frac{\Gamma^2-\Delta\Gamma^2/4}{\Delta m^2+\Gamma^2}$

$$r = \frac{N^{++}+N^{--}}{N^{+-}} = \frac{|q/p|^2|\bar{A}_{\bar{f}}|^4+|p/q|^2|A_f|^4}{2|A_f|^2|\bar{A}_{\bar{f}}|^2}\frac{1-\Delta^2}{1+\Delta^2}\,,$$
$$a = \frac{N^{--}-N^{++}}{N^{--}+N^{++}} = \frac{|q/p|^2|\bar{A}_{\bar{f}}|^4-|p/q|^2|A_f|^4}{|q/p|^2|\bar{A}_{\bar{f}}|^4+|p/q|^2|A_f|^4}$$

In the limit $|A_f| = \bar{A}_{\bar{f}}|$, $\Delta\Gamma = 0$ and $p/q| = 1$
such as $B^0 \to l^+X$ and $\bar{B}^0 \to l^-\bar{X}$

$$r = \frac{(1+\Delta m^2/\Gamma^2)^2-1}{(1+\Delta m^2/\Gamma^2)^2+1}\,,\quad \Delta m_{B^0}/\Gamma_{B^2} = 0.77 \pm 0.004\,.$$

The need of asymmetric e⁺e⁻ collider for B factories

The need of asymmetric e^+e^- collider for B factories

Produce $B^0\bar{B}^0$ pair at $\Upsilon(4S)$
B are almost at rest and decay at production point and $\Delta\Gamma = 0$.

Aim to measure $Im(\lambda_f)$

Coherent production $M\bar{M}$ at resonance in $e + e^-$ collider at $t = 0$,

Wave function $\Psi(t_1, r_1; t_2, r_2)$ system at for M or $\bar{M}$ at t_1, r_1 and t_2, r_2 is

$$\Psi(t_1, r_1; t_2, r_2) = \frac{1}{\sqrt{2}}(|M(t_1, r_1)\bar{M}(t_2, r_2) > +(-1)^l|M(t_2, r_2)\bar{M}(t_1, r_1) >)$$

For $M(t_1)\bar{M}(t_2)$ decay to $f_{CP}f$ and $f_{CP}\bar{f}$, the decay amplitudes are

$$< f_{CP}(t_1)f(t_2)|\Psi(t_1;t_2>= \frac{\bar{A}_{CP}A_f}{4}e^{-im_H(t_1+t_2)-\Gamma_H(t_1+t_2)/2}$$
$$\times\left\{[1-\bar{\lambda}_f-(1+\bar{\lambda}_f)e^{i\Delta mt_1+\Delta\Gamma t_1/2}](1-e^{i\Delta mt_2+\Delta\Gamma t_2/2})\right.$$
$$\left.+(-1)^l[1-\bar{\lambda}_f+(1+\bar{\lambda}_f)e^{i\Delta mt_1+\Delta\Gamma t_1/2}](1+e^{i\Delta mt_2+\Delta\Gamma t_2/2})\right\}$$

$$< f_{CP}(t_1)\bar{f}(t_2)|\Psi(t_1;t_2>= \frac{A_{CP}\bar{A}_{\bar{f}}}{4}e^{-im_H(t_1+t_2)-\Gamma_H(t_1+t_2)/2}$$
$$\times(-1)^l\left\{(-1)^l[1-\lambda_f+(1+\lambda_f)e^{i\Delta mt_1+\Delta\Gamma t_1/2}](1+e^{i\Delta mt_2+\Delta\Gamma t_2/2})\right.$$
$$\left.+[1-\lambda_f-(1+\lambda_f)e^{i\Delta mt_1+\Delta\Gamma t_1/2}](1+e^{i\Delta mt_2-\Delta\Gamma t_2/2})\right\}$$

Consider $B^0\bar{B}^0$ system, $\Delta\Gamma = 0$

$$\bar{\Gamma}(t_1,t_2) \sim \bar{R}(t_1,t_2) = |< f_{CP}(t_1)\bar{f}(t_2)|\Psi(t_1;t_2) > |^2 \frac{|A_{CP}\bar{A}_{\bar{f}}|^2}{2} e^{-\Gamma(t_1+t_2)}$$

$$\times \{1 + |\lambda_f|^2 + [(1 - |\lambda_f|^2) cos(\Delta m t_1) - 2 Im\lambda_f sin(\Delta m t_1)] cos(\Delta m t_2)$$

$$-(-1)^l\ [(1 - |\lambda_f|^2 sin(\Delta m t_1) + 2 Im\lambda_f cos(\Delta m t_1)] sin(\Delta m t_2)\}$$

$$\Gamma(t_1,t_2) \sim \bar{R}(t_1,t_2) = |< f_{CP}(t_1) f(t_2)|\Psi(t_1;t_2) > |^2 \frac{|\bar{A}_{CP}A_{\bar{f}}|^2}{2} e^{-\Gamma(t_1+t_2)}$$

$$\times \{1 + |\bar{\lambda}_f|^2 + [(1 - |\bar{\lambda}_f|^2) cos(\Delta m t_1) - 2 Im\bar{\lambda}_f sin(\Delta m t_1)] cos(\Delta m t_2)$$

$$-(-1)^l\ [(1 - |\bar{\lambda}_f|^2 sin(\Delta m t_1) + 2 Im\bar{\lambda}_f cos(\Delta m t_1)] sin(\Delta m t_2)\}$$

If one does not need to know when f or $\bar{f}$, integrate t_2,

$$\bar{R}(t_1) = \int_0^\infty \bar{R}(t_1, t_2)dt_2 = \frac{|A_{CP}\bar{A}_{\bar{f}}|^2}{2} e^{-\Gamma t_1}$$
$$\times [\frac{1+|\bar{\lambda}_f|^2}{\Gamma} + ((1-|\lambda_f|^2)cos(\Delta m t_1) - 2Im\bar{\lambda}_f sin(\Delta m t_1))\frac{\Gamma}{\Delta m^2 + \Gamma^2}$$
$$-(-1)^l((1-|\lambda_f|^2)sin(\Delta m t_1) + 2Im\bar{\lambda}_f cos(\Delta m t_1))\frac{\Delta m}{\Delta m^2+\Gamma^2}] \; ,$$

Further if one does not needs to know where f_{CP} was borne, integrate t_1

$$\bar{R} = \int_0^\infty \bar{R}(t_1)dt_1 = \frac{|A_{CP}\bar{A}_{\bar{f}}}{2}\left(\frac{1+|\lambda_f|^2}{\Gamma^2} + (1-|\lambda_f|^2)\frac{\Gamma^2-(-1)^l\Delta m^2}{(\Delta m^2+\Gamma^2)^2} - 2Im\lambda_f\frac{\Delta m\Gamma}{(\Gamma^2+\Delta m^2)^2}(1+(-1)^l)\right)$$

Similarly one obtains

$$R = \int_0^\infty R(t_1)dt_1 = \frac{|\bar{A}_{CP}A_f}{2}\left(\frac{1+|\bar{\lambda}_f|^2}{\Gamma^2} + (1-|\bar{\lambda}_f|^2)\frac{\Gamma^2-(-1)^l\Delta m^2}{(\Delta m^2+\Gamma^2)^2} - 2Im\bar{\lambda}_f\frac{\Delta m\Gamma}{(\Gamma^2+\Delta m^2)^2}(1+(-1)^l)\right)$$

For B factories, resonant production of $e^+e^- \to \Upsilon(4S) \to B^0\bar{B}^0$, $l = 1$

$$\bar{R} = \frac{|A_{CP}\bar{A}_{\bar{f}}|}{2}\left(\frac{1+|\lambda_f|^2}{\Gamma^2} + \frac{1-|\lambda_f|^2}{\Delta m^2 + \Gamma^2}\right)$$
$$R = \frac{|\bar{A}_{CP}A_f|}{2}\left(\frac{1+|\bar{\lambda}_f|^2}{\Gamma^2} + \frac{1-|\bar{\lambda}_f|^2}{\Delta m^2 + \Gamma^2}\right)$$

Contains no information about $Im\lambda_f$ and $\bar{\lambda}_f$ at all.

One should keep time dependent information without integrating

Not only that, one should boost B^0 and $\bar{B}^0$ to move after been produced

Asymmetric e^+ energy E_+ and e^- energy E_- are needed

For example, with $E_+ > E_-$, B^0 and $\bar{B}^0$ would be boosted in the e^+ direction

With $l = 1$

$$\bar{R}(t_1, t_2) = \frac{|A_{CP}\bar{A}_{\bar{f}}|}{2} e^{-\Gamma(t_1+t_2)}$$

$$\times \left(2(1 + |\lambda_f|^2) + (1 - |\lambda_f|^2) cos(\Delta m(t_1 - t_2) + 2Im\lambda_f sin(\Delta(t_1 - t_2))\right) \ ,$$

$$R(t_1, t_2) = \frac{|\bar{A}_{CP}A_f|}{2} e^{-\Gamma(t_1+t_2)}$$

$$\times \left(2(1 + |\bar{\lambda}_f|^2) + (1 - |\bar{\lambda}_f|^2) cos(\Delta m(t_1 - t_2) + 2Im\bar{\lambda}_f sin(\Delta(t_1 - t_2))\right) \ .$$

By measuring where f_{CP}, f and $\bar{f}$ are produced

Information on $Im\lambda_f$ and $\bar{\lambda}_f$ can be extracted!

This is the principle for Belle and Babar to measure CP violation

$$Im(\lambda_{B^0 \to J/\psi K_s} = sin(2\beta) = 0.691 \pm 0.017$$

Homework

For coherently produced M and $\bar{M}$, calculate

$A^{+-} =< f_1 \bar{f}_2|H|\Psi(t_1;t_2>$, $N^{+-}(+1,-2) \sim \int_0^\infty |A^{+-}|^2 dt_1 dt_2$
and $N^{+-}(+2,-1)$ (+i, -i indicate that t_i correspond to f and $\bar{f}$ respectively.)

obtain $N^{+-} = N^{+-}(+1,-2) + N^{+-}(+2,-1)$

$A^{++} =< f_1 f_2|H|\Psi(t_1;t_2>$, $N^{++}(+1,+2) \sim \int_0^\infty |A^{++}|^2 dt_1 dt_2$

$A^{--} =< \bar{f}_1 \bar{f}_2|H|\Psi(t_1;t_2>$, $N^{--}(-1,-2) \sim \int_0^\infty |A^{--}|^2 dt_1 dt_2$

Obtain r and a

The EDM of a fundamental particle

Classically a EDM $\vec{D} = \int d^3x\vec{x}\rho(\vec{x})$ interacts with an electric field $\vec{E}$

The interaction energy is given by $H = \vec{D}\cdot\vec{E}$, allowed by P and T symmetries.

Under P, $\vec{D} \to -\vec{D}$ and $\vec{E} \to -\vec{E}$, H conserves both P and T.

A fundamental particle, $\vec{D}$ is equal to $d\vec{S}$, $H_{edm} = d\vec{S}\cdot\vec{E}$.

Since under P, $\vec{S} \to \vec{S}$ and under T, $\vec{S} \to -\vec{S}$

Magnetic Dipole conserves P and T

$H_{mdm} = d_m\vec{S}\cdot\vec{B}$,

Under P: $\vec{B} \to \vec{B}$ and under T: $\vec{B} \to -\vec{B}$

Relativistic expression: $d_m\bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu}$.

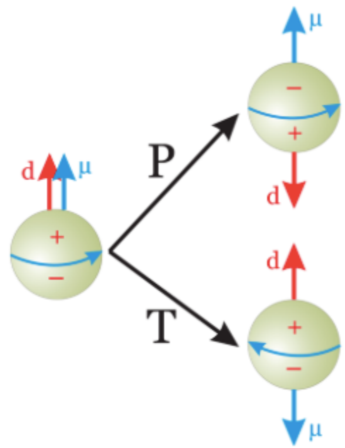
H_{edm} violates both P and T, CPT is conserved, CP is also violated!

Quantum field theory, $H_{edm} = -i\frac{1}{2}d\bar{\psi}\sigma^{\mu\nu}\psi\tilde{F}_{mu\nu} = -i\frac{1}{2}d\bar{\psi}\sigma^{\mu\nu}\gamma_5\psi F_{\mu\nu}$

In non-relativestic limit H_{edm} reduce to $d\frac{\vec{\sigma}}{2}\cdot\vec{E} = d\vec{S}\cdot\vec{E}$.

One easily sees that H_{edm} violates P and T, violates CP, but conserve CPT.

A non-zero fundamental particle EDM, violates P, T and CP!



First fundamental particle EMD measurement: neutron EDM in 1950 by Purcell and Ramsey.

Landau first pointed out that EDM violates P and T symmetry.

No measurement of a fundamental particle EDM, yet!

Current 90% C.L. limits on EDM:

Neutron $|D_n| < 3 \times 10^{-27}$ edm, electron $|D_e| < 0.87 \times 10^{-28}$ edm

EDM of neutron and electron in KM model

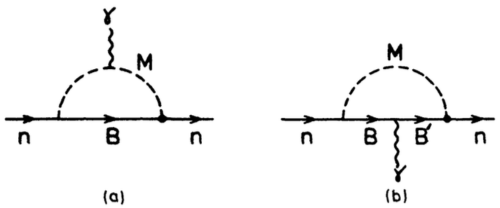
Quark EDM D_q and neutron EDM D_n, $D_n = (4D_d - D_u)/3$

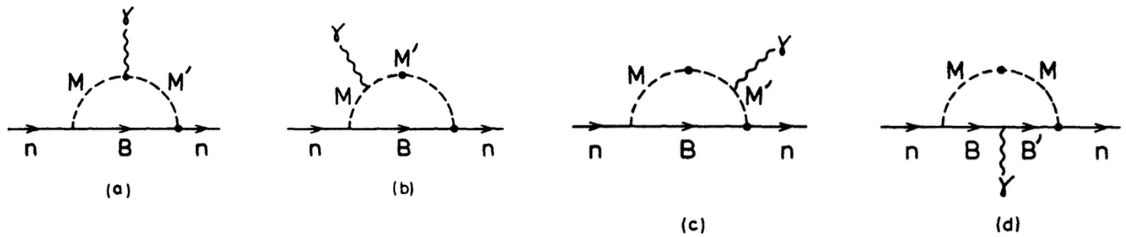
In KM model, quark EDM only generated at two electroweak and one strong loop level (3 loop effects)l, very small ~ 10^{-33} e.cm. (Shabalin, 1978, 1980)

In fact with two weak and one strong interaction vertices, EDM can also be generated!

(He, McKellar and Pakvasa, PLB197, 556(1987), J. Mod. Phys. A4, 5011(1989)

$$1.6 \times 10^{-31}\ \text{e.cm} \geq |D_n| \geq 1.4 \times 10^{-33}\ \text{e.cm}$$





Electron EDM is even smaller, generated at fourth loop level, $D_e < 10^{-38}$e.cm

The strong CP problem and neutron EDM

The CP violating term

$$\delta L_{QCD} = -\theta \frac{g^2}{16\pi^2} Tr(\tilde{G}^{\mu\nu} G_{\mu\nu}) = -\theta \frac{g^2}{32\pi^2} \tilde{G}^{\mu\nu}_a G^a_{\mu\nu} \, .$$

can be written as surface integral for the action

$$\delta S = -\theta \frac{g^2}{32\pi^2} \int d^4x \tilde{G}^{\mu\nu}_a G^a_{\mu\nu} = -\theta \frac{g^2}{32\pi^2} \int d^4x \partial_\mu K^\mu = -\theta \frac{g^2}{32\pi^2} \int d\sigma_\mu \partial_\mu K^\mu$$

with $K^\mu = (\epsilon^{\mu\alpha\beta\gamma} G^a_\alpha (G^a_{\beta\gamma} - (g_3/3) f^{abc} G^b_\beta G^c_\gamma)$

The surface integral is usually dropped.

However, there are configuration the above integral is non-zero, θ-vacuum.

The integral is actually corresponding to the topological winding number ν

$$\nu = \frac{g^2}{32\pi^2} \int d\sigma_\mu \partial_\mu K^\mu$$

One cannot simply disregard it!

Some useful relations

The chiral anomaly relation, $\partial^\mu(\bar{\psi}\gamma_\mu\gamma_5\psi) = \frac{g_3^2}{16\pi^2}Tr(\tilde{G}G)$,

leads to a chiral rotation $\psi \to e^{i\alpha\gamma_5/2}\psi$ generates in the Lagrangian

$$\delta L_\alpha = -\alpha\frac{g_3^2}{16\pi^2}Tr(\tilde{G}G) \ .$$

An imaginary matter term $\delta L = -\bar{\psi}m(cos\alpha + isin\alpha)\psi$

can be transformed away by define $\psi' = e^{-\alpha\gamma_5/2}\psi$ and to

$$\delta L = -\bar{\psi}'m\psi' + \alpha\frac{g_3^2}{16\pi^2}Tr(\tilde{G}G) \ .$$

If one write with more than one ψ the mass matrices as $\psi_R M \psi_L$

In general M is complex. Then

$$\begin{aligned}\delta L &= -(\bar{\psi}_R M\psi_L + H.C.) - \theta\frac{g_3^2}{16\pi^2}Tr(\tilde{G}G) \\ &= -\bar{\psi}\hat{M}\psi - (\theta - Arg(Det(M))\frac{g_3^2}{16\pi^2}Tr(\tilde{G}G) \ .\end{aligned}$$

$\hat{M} = diag(m_1, m_2, ...)$, with $m_i > 0$

Calculation of EDM from θ-term

Making on each of the light quarks (u, d, s).

With appropriate chiral transformation, assuming small θ

$$L = -m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s - \theta \frac{g_3^2}{16\pi^2} Tr(\tilde{G}G)$$

$$\rightarrow \quad -m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s - i\theta \frac{m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s} \bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s$$

Using current algerbra, turn the above into nucleon-pion interactions

$$< P^a B_f \bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s | n > = -i(\sqrt{2}/f_p i) < B_f | \bar{q}\lambda^a q | n >$$

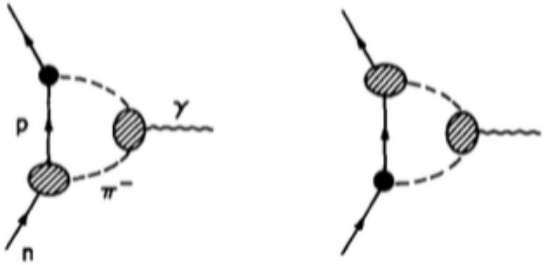
$$L_{\pi^i B_f B} = -\sqrt{2}\bar{N}_f \sigma^i (i\gamma_5 g_{\pi NN} + f_{\pi NN}|N>$$

$g_{\pi NN} \approx 14$ is CP conserving, and $f_{\pi NN}$ is CP violating coupling with

Crewther, Di Vecchia, Veneziano and Witten, PLB88, 13,(1979)

$$f_{\pi NN} = -2\frac{(m_\Xi - m_\Sigma)}{f_\pi}\frac{m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s},$$

$D_n \sim -3.8 \times 10^{-16}\theta$



Including all SU(3) octet contributions:

$2.5 \times 10^{-16}\theta e.cm < |D_n| < 4.6 \times \theta e.cm$

He, McKellar and Pakvasa, IJMP A4, 5011 (1989))

Using data $|D_n| < 3 \times 10^{-27} e.cm$, $|\theta| < 10^{-11}$!

Why θ is small is the strong CP problem.

Solutions to the strong CP problems

1. One of the quark mass is zero, since D_n is proportional to $m_u m_d m_s$. But all quarks have non-zero masses!

2. Spontaneous CP violation, making θ equal to zero first. Need to check whether after symmetry breaking, θ is not generated.

3. Dynamic solution driving θ small by imposing an additional chiral symmetry, the Peccei-Quinn symmetry. This solution leads to Axion which has not been discovered.

Not yet discovered!

Strong CP problem and Axion

The most attractive solution is provided by Peccei-Quinn symmetry

Lagrangian symmetric under a $U(1)_A$ global chiral transformation

Chiral transformation for SM Q_L, u_R, d_R, L_L and e_R are independent

How f_L and f_R, if f transforms under $e^{i\alpha\gamma_5/2}f$?

$f_R = \frac{1}{2}(1+\gamma_5)f \rightarrow \frac{1}{2}(1+\gamma_5)e^{i\alpha\gamma_5/2}f$

$$\frac{1}{2}(1+\gamma_5)e^{i\alpha\gamma_5/2}f = \frac{1}{2}(1+\gamma_5)(cos\alpha + i\gamma_5 sin\alpha)f = \frac{1}{2}(cos\alpha + isin\alpha)(1+\gamma_5)f = e^{i\alpha}f_R \ ,$$

$$\frac{1}{2}(1-\gamma_5)e^{i\alpha\gamma_5/2}f = \frac{1}{2}(1-\gamma_5)(cos\alpha + i\gamma_5 sin\alpha)f = \frac{1}{2}(cos\alpha - isin\alpha)(1-\gamma_5)f = e^{-i\alpha}f_L \ .$$

$U(1)_A$ chiral model of QP symmetry for strong CP problem

$L = L_{SM} + \delta L_\theta$, $\delta L_\theta = -\theta(g_3^2/16\pi^2)Tr(\tilde{G}G)$

$u_R \to e^{i\alpha}u_R$, $d_R \to e^{i\alpha}$, $Q_L \to Q_L$, $L_L \to L_l$ and $e_R \to e^{i\alpha}e_R$

$\bar{\theta} = \theta \to \theta - 2\alpha$,

If L_{SM} is symmetric under $U(1)_A$, $L \to L_{SM} + \delta L_{\bar{\theta}=\theta-2\alpha}$

For L_{SM}, α is arbitrary, choose one such that $\bar{\theta} = \theta - 2\alpha = 0$.

No strong CP term!

One then needs to show that the corresponding potentials are minimal to have a stable solution.

How to make L_{SM} symmetric under $U(1)_A$?

With just one Higgs, H, $L_Y = -\bar{Q}_L Y_u \tilde{H} u_R - \bar{Q}_L Y_d H d_R$

If require the first term be PQ invariant, $\tilde{H} \to e^{-i\alpha} \tilde{H}$

Since $\tilde{H} = i\sigma_2 H^*$, then $H \to e^{i\alpha} H$,

Second term not allowed, d-quarks do not get masses for vev of H

Not possible to make the second term PQ invariant.

Minimal SM does not work!

Extend the Higgs secotor to have two Higgs doublets H_1 and H_2

$H_1 \to e^{i\alpha} H_1$ and $H_2 \to e^{-i\alpha} H_2$,

Then $L_Y = -\bar{Q}_L Y_u \tilde{H}_1 u_R - \bar{Q}_L Y_d H_2 d_R$

Should make the potential $V(H_1, H_2)$ invariant.

Both H_1 and H_2 should have non-zero vev, v_1 and v_2

The PQ symmetry in $V(H_1, H_2)$ is spontaneously broken by v_i,

There is a massless GOLDSTONE boson, Axion.

Finding the Goldstone boson

Two Higgs doublet model: (v_1, v_2) breaks $U(1)_Y$ symmetry

The imaginary fields (I_1, I_2) carries Y charges is proportional to $(1, 1)$

The Z needs to "eats" a neutral pesudo-Goldstone boson to become massive

The one "eaten" is just proportional to $\sum_i v_i Y_i I_i$.

If there is another $U(1)_A$ is broken by the same vev,

If no Z to "eats" anything, the Goldstone boson is proportional to: $\sum_i v_i A_i I_i$.

If there is a combination Z_I "eaten" by a gauge boson Z, find it first

The one orthogonal to Z_I is the other Goldstone boson!

In PQ model, there are two neutral imaginary fields I_1 and I_2

$Z_I = \frac{v_1 I_1 + v_2 I_2}{\sqrt{v_1^2+v_2^2}}$ "eaten" by Z

The orthoganol combination is massless Axion: $a = \frac{-v_2 I_1 + v_1 I_2}{\sqrt{v_1^2+v_2^2}}$.

The PQ symmetry is anomalous, there is a mass of order hundrads of KeV.

Couplings to quarks: $\sim i[-\bar{u}\frac{\hat{M}_u}{v}\frac{v_2}{v}u + \bar{d}\frac{\hat{M}}{v}\frac{v_1}{v}d]a$

Too large couplings. Ruled out!

Invisible Axion. Introduce an additional singlet $S : (1,1)(1)_{PQ}$ with vev v_s

Axion; $a = \frac{-2v_1 v_2^2 I_1 + 2v_2 V_1^2 I_2 + v^2 v_s I_s}{\sqrt{4v_1^2 v_2^2 (v_1^2 + v_2^2) + v^4 v_s^2}}$

Couplings to quarks: $\sim i \frac{-\bar{u}(\hat{M}_u/v)(2v_1 v_2^2)u + \bar{d}(\hat{M}/v)(2v_1^2 v_2)d}{\sqrt{4v_1^2 v_2^2 (v_1^2 + v_2^2) + v^4 v_s^2}} a$

If $v_s >> v_1$, the couplings are: $\sim i[-\bar{u}\frac{\hat{M}_u}{v}\frac{2v_1 v_2^2}{v^2 v_s}u + \bar{d}\frac{\hat{M}}{v}\frac{2v_1^2 v_2}{v^2 v_s}d]a$

Axion mass of order $m_\pi^2 \frac{v^2}{v_s^2}$.

Invisible Axion. Still alive!

A lot of interestgin physics related to Axion: find the Axion, applicatin to astrophysics, cosmology and etc.!!!

CP violation with polarization measurement

a. spin-1/2 -> spin-0 + spin-1/2

$$\mathcal{A} = \bar{\mathcal{F}}(A_v + iA_c\gamma_5)\mathcal{B} = \mathcal{S} + \mathcal{P}\sigma \cdot \vec{p}_c \qquad |\vec{p}_c| = \sqrt{E_{\mathcal{F}}^2 - m_{\mathcal{F}}^2}$$

$$\mathcal{S} = A_v\sqrt{\frac{(m_{\mathcal{B}} + m_{\mathcal{F}})^2 - m_{\mathcal{M}}^2}{16\pi m_{\mathcal{B}}^2}}\,, \quad \mathcal{P} = A_c\sqrt{\frac{(m_{\mathcal{B}} - m_{\mathcal{F}})^2 - m_{\mathcal{M}}^2}{16\pi m_{\mathcal{B}}^2}}\,.$$

$$\bar{\mathcal{A}} = -\bar{\mathcal{S}} + \bar{\mathcal{P}}\sigma \cdot \vec{p}_c\,.$$

$$\frac{4\pi}{\Gamma}\frac{d\Gamma}{d\Omega} = 1 + \alpha\vec{s}_{\mathcal{B}} \cdot \vec{n} + \vec{s}_{\mathcal{F}} \cdot [(\alpha + \vec{s}_{\mathcal{B}} \cdot \vec{n})\vec{n} + \beta\vec{s}_{\mathcal{B}} \times \vec{n} + \gamma(\vec{n} \times (\vec{s}_{\mathcal{B}} \times \vec{n}))]$$

$$\Delta = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}\,, \quad A_\alpha = \frac{\Gamma\alpha + \bar{\Gamma}\bar{\alpha}}{\Gamma\alpha - \bar{\Gamma}\bar{\alpha}}\,, \quad B_\beta = \frac{\Gamma\beta + \bar{\Gamma}\bar{\beta}}{\Gamma\beta - \bar{\Gamma}\bar{\beta}}\,.$$

$$\vec{n} = \vec{p_c}/|p_c|$$

$$\beta = (1 - \alpha^2)^{1/2} sin\phi$$

CP violation in Hyperons

TABLE I. Experimental measurements of α and ϕ.

	α	ϕ
$\Lambda^0 \to n\pi^0$	0.642 ± 0.013	$-6.5° \pm 3.5°$
$\Lambda^0 \to p\pi^-$	0.642 ± 0.013	$-6.5° \pm 3.5°$
$\Xi^0 \to \Lambda^0\pi^0$	-0.413 ± 0.022	$20.7° \pm 11.7°$
$\Xi^- \to \Lambda^0\pi^-$	-0.434 ± 0.015	$2.0° \pm 5.7°$
$\Sigma^- \to n\pi^-$	-0.0681 ± 0.0077	$10.3° \pm 4.6°$
$\Sigma^+ \to p\pi^0$	-0.979 ± 0.016	$35.8° \pm 33.7°$
$\Sigma^+ \to n\pi^+$	0.068 ± 0.013	$167.3° \pm 20.1°$

Hyperon decays and *CP* nonconservation

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(Received 7 March 1986)

We study all modes of hyperon nonleptonic decay and consider the *CP*-odd observables which result. Explicit calculations are provided in the Kobayashi-Maskawa, Weinberg-Higgs, and left-right-symmetric models of *CP* nonconservation.

Signals of {CP} Nonconservation in Hyperon Decay
John F. Donoghue (Massachusetts U., Amherst), Sandip Pakvasa (Hawaii U.).
Published in **Phys.Rev.Lett. 55 (1985) 162**

	Δ	A	B
$\Lambda^0 \to p\pi^-$	-5.4×10^{-7}	-0.5×10^{-4}	3.0×10^{-3}
$\Xi^- \to \Lambda^0\pi^-$	0	-0.7×10^{-4}	8.4×10^{-4}
$\Sigma^- \to n\pi^-$	0	1.6×10^{-4}	-1.2×10^{-2}
$\Sigma^+ \to p\pi^0$	-6.2×10^{-7}	-3.2×10^{-7}	-4.2×10^{-4}
$\Sigma^+ \to n\pi^+$	6.0×10^{-7}	-1.6×10^{-4}	-8.4×10^{-7}

$A_{\Xi\Lambda} = A_\Xi + A_\Lambda$ HyperCP (Femilab E871):

$$A_{\Xi\Lambda} = [-6.0 \pm 2.1(\text{stat}) \pm 2.0(\text{syst})] \times 10^{-4},$$

CP violation in Higgs h decays into $\tau^+\tau^-$

(Hayreter, He, Valencia, arXiv:1603.06326, arXiv:1606.00951)

(He, Ma, McKellar, Mod. Phys Lett. A9, 205(1994);
Berge, Bereuther, Kirchner, PRD92,096012(2015))

General Higgs to fermion coupling: $L = -\bar{f}(r_f + i\tilde{r}_f\gamma_5) f h$

Define the density matrix R with polarization $\vec{n}_f(\vec{n}_{\bar{f}})$ for $f(\bar{f})$

$$R = N_f \beta_f [Im(r_f \tilde{r}_f^*)\hat{p}_f \cdot (\vec{n}_{\bar{f}} - \vec{n}_f) - Re(r_f \tilde{r}_f)\hat{p}_f \cdot (\vec{n}_f \times \vec{n}_{\bar{f}})]$$

N_f - normalization constant, $\hat{p}$ - three moment of f, $\beta_f = \sqrt{1 - 4m_f^2/m_h^2}$

Application to $h \to \tau^+\tau^-$

Using $\tau \to \pi^- \nu_\tau$ to measure $\vec{n}_f$, $\frac{4\pi}{\Gamma}\frac{d\Gamma}{d\Omega} = (1 + \alpha_\tau \vec{n}_\tau \cdot \hat{p}_\tau)$, $\alpha_\tau = 1$.

$$\hat{p}_\tau \cdot (\vec{n}_f \times \vec{n}_{\bar{f}}) \to \hat{p}_\tau \cdot (\hat{p}_{\pi^-} \times \hat{p}_{\pi^+})$$

One construct CP violating observable

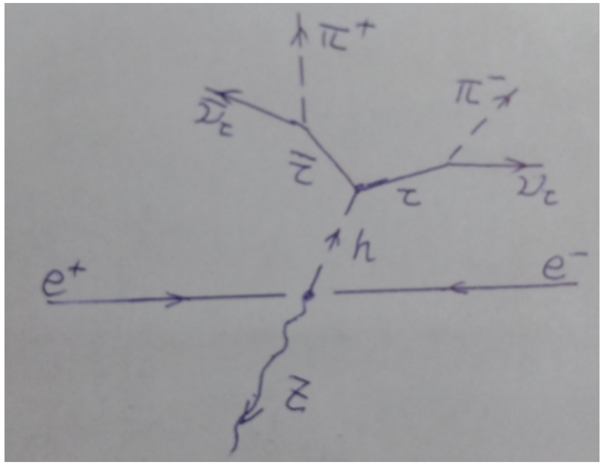
$$A_\tau = \frac{N(O_\pi > 0) - N(O_\pi < 0)}{N(O_\pi > 0) + N(O_\pi < 0)} \,, \quad O_\pi = \hat{p}_\tau \cdot (\hat{p}_{\pi^+} \times \hat{p}_{\pi^-}) \,.$$

Theoretically

$$A_\tau = \frac{N(O_\pi > 0) - N(O_\pi < 0)}{N(O_\pi > 0) + N(O_\pi < 0)} = \frac{\pi}{4}\beta_\tau \alpha_\tau \alpha_{\mp} \frac{r_\tau \tilde{r}_\tau}{\beta_\tau^2 r_\tau^2 + \tilde{r}_\tau^2} \,.$$

Data still allow A to be as large as $\pi/8$. Experiments should look such CPV.

In the SM $A\tau = 0$



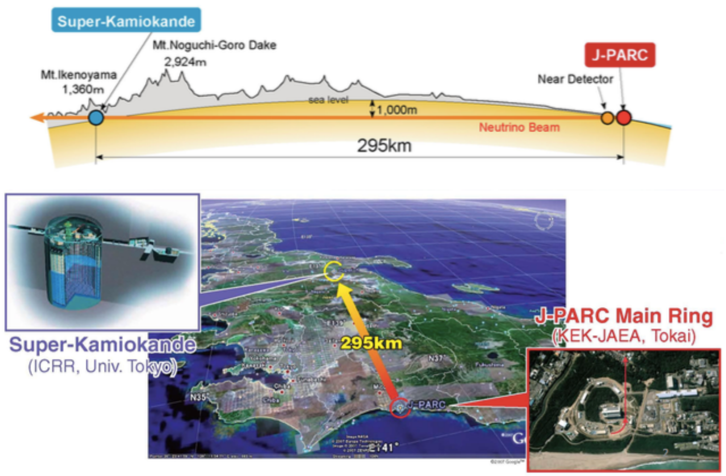
$Br(h \to \tau\tau) \sim 5\times10^{-2}$,

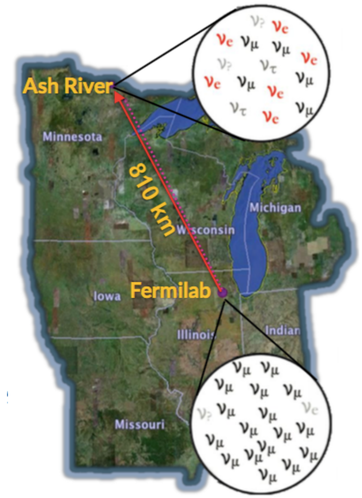
$Br(\tau \to \pi\,\nu) \sim 0.1$

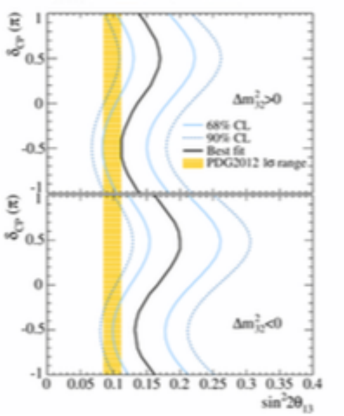
10^6 Higgs bosons, sensitivity to A_π can be 10% at CEPC.

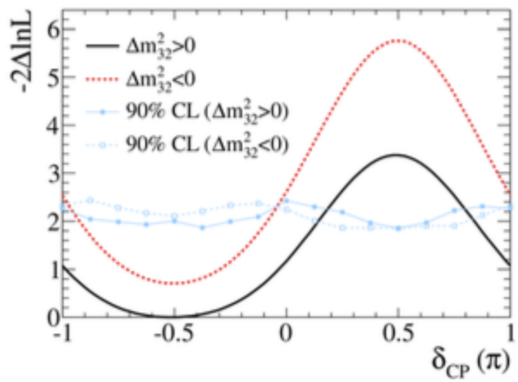
CP violation in neutrino oscillation

T2K Experiment



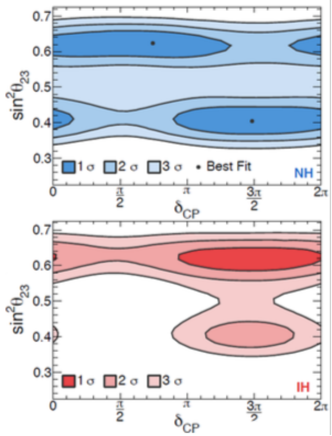






ν_e appearance results

- 2 degenerate best fit points:
  - NH, $\delta_{CP} = 1.48\pi$ $\sin^2\theta_{23} = 0.404$
  - NH, $\delta_{CP} = 0.74\pi$ $\sin^2\theta_{23} = 0.623$
- Inverted hierarchy slightly disfavored - $\Delta\chi^2 = 0.47$
- Lower octant in the IH is disfavored at 93% CL
- arXiv:1703.03328



Neutrino oscillation and anon-zero neutrino masses

$$\begin{pmatrix} \nu_1(0) \\ \nu_2(0) \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_{m1} \\ \nu_{m2} \end{pmatrix}$$

At t=0, a neutrino $\nu_1(0)$ produced

$|\nu_1(0) >= \cos\theta|\nu_{m1} > -\sin\theta|\nu_{m2} >.$

At time t and travelled a distance L, the state becomes

$|\nu_1(t) > \cos\theta e^{-i(Et-p_1L)}|\nu_{m1} > -\sin\theta e^{-(Et-p_2L)}|\nu_{m2} >.$

Using $t \approx L$ and $E - P_i = E - \sqrt{E^2 - m_i^2} \approx m_1^2/2E$,

the probability amplitude of find $\nu_2(0)$ at time t is given by

$$< \nu_2(0)|\nu_1(t) >= \cos\theta \sin\theta (e^{-im_1^2L/2E} - e^{-im_2^2L/2E})$$

which leads to the probability of finding $\nu_2(0)$ is

$$P(\nu_1 \to \nu_2) = | < \nu_1(0)|\nu_2(t) > |^2 = \sin^2(2\theta)\sin^2(\Delta m_{21}^2 L/4E),$$

$$\Delta m_{21}^2 = m_2^2 - m_1^2.$$

B. Pontecorvo (1957. *Zh. Eksp. Teor. Fiz.* 33: 549–551. *Zh. Eksp. Teor. Fiz.* 53: 1717

Z. Maki, M. Nakagawa, and S. Sakata (1962). *Progress of Theoretical Physics* 28 (5): 870.

S. Sakata Z. Maki M. Nakagawa

Compare neutrino and anti-neutrino oscillation information on CP violation can be extracted

With three generations of neutrinos

$$P(\nu_\alpha \to \nu_\beta) = |\sum_i V^*_{\beta i} V_{\alpha i} e^{-i m_i^2 L/2E}|^2$$

$$= \delta_{\alpha\beta} - 4 \sum_{i>j} Re(V^*_{\alpha i} V_{\beta i} V_{\alpha j} V^*_{\beta j}) \sin^2(\Delta m^2_{ij} L/4E)$$

$$- 2 \sum_{i>j} Im(V^*_{\alpha i} V_{\beta i} V_{\alpha j} V^*_{\beta j}) \sin(\Delta m^2_{ij} L/2E) \ ,$$

$$P(\nu_\alpha - \nu_\beta - P(\nu_{\bar{\alpha}} - \nu_{\bar{\beta}} = 4J \textstyle\sum_{m,n} \epsilon_{\alpha\beta m} \epsilon_{ijn} sin^2(\Delta m^2_{ij} L/4E). \ (i > j)$$

T2K already starting to see differences.

Be careful: on earth, neutrino interacts with matter, not anti-matter. Needs to properly normalize the data.

	Expected number of events (MC, Normal Hierarchy)				Observed data
	$\delta_{CP}=-\pi/2$	$\delta_{CP}=0$	$\delta_{CP}=+\pi/2$	$\delta_{CP}=\pi$	T2K
ν_e	28.7	24.2	19.6	24.2	32
$\bar{\nu}_e$	6.0	6.9	7.7	6.8	4

5.2 Test SM in B Decays

SM for CPV has many interesting predictions:

small EDM, Zero $A\tau$, CPV in Hyperon decay of order $A \sim 10^{-4}$ ….

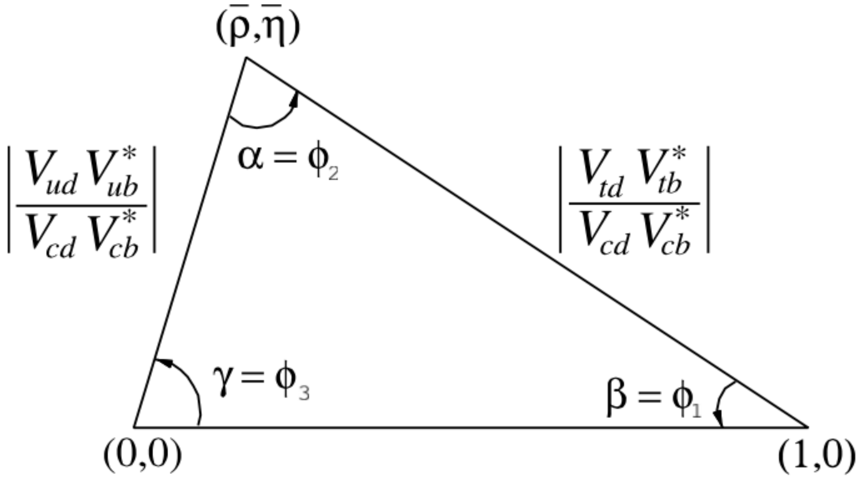
Anything bigger a sign of new physics…

It would nice to have some positive ones to veryfiy SM CPV!

One of the most prominent feature is that CP violation comes from the KM matrix. The unitary conditions: $\Sigma_i V_{ij} V_{ik}^* = \delta_{jk}$; $\Sigma_i V_{ji} V_{ki}^* = \delta_{jk}$.
can be represented by 6 unitarity triangles. The most experimentally accessible one is by the following

If the angles α, β and γ
can be independently measured,
whether $\alpha+\beta+\gamma = 180^o$ can test the model.

This relation indeed holds!
Have been tested from B decays.

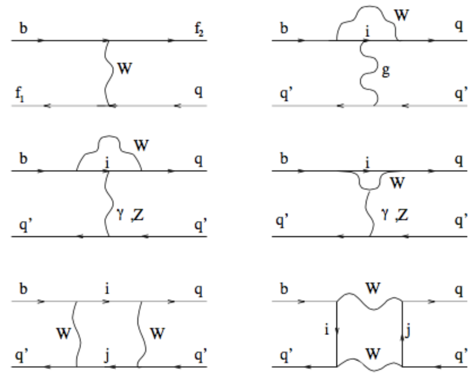


Hadronic B decays – The effective Hamiltonian

For hadronic B decays, the effective Hamiltonian is given by

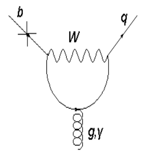
$$H_{\Delta B=1}(q) = \frac{4G_F}{\sqrt{2}}[V_{ub}V_{cq}^*(c_1O_1^{uc}(q)+c_2O_2^{uc}(q))+V_{cb}V_{uq}^*(c_1O_1^{cu}(q)+c_2O_2^{cu}(q))+V_{ub}V_{uq}^*(c_1O_1^{u}(q)$$

$$+ \quad c_2O_2^{u}(q))+V_{cb}V_{cq}^*(c_1O_1^{c}(q)+c_2O_2^{c}(q)) - \sum_{j=u,c,t} V_{jb}V_{jq}^* \sum_{i=3}^{10} c_i^j O_i(q)] + H.C.,$$



O_i's are defined as

$$O_1^{f_1f_2}(q) = \bar{q}_\alpha\gamma_\mu L f_{1\beta}\bar{f}_{2\beta}\gamma^\mu L b_\alpha, \quad O_2^{f_1f_2}(q) = \bar{q}\gamma_\mu L f_1\bar{f}_2\gamma^\mu L b,$$
$$O_1^{f}(q) = \bar{q}_\alpha\gamma_\mu L f_\beta\bar{f}_\beta\gamma^\mu L b_\alpha, \quad O_2^{f}(q) = \bar{q}\gamma_\mu L f\bar{f}\gamma^\mu L b,$$
$$O_{3(5)}(q) = \bar{q}\gamma_\mu L b\bar{q}'\gamma^\mu L(R)q', \quad O_{4(6)}(q) = \bar{q}_\alpha\gamma_\mu L b_\beta\bar{q}'_\beta\gamma^\mu L(R)q'_\alpha,$$
$$O_{7(9)}(q) = \frac{3}{2}\bar{q}\gamma_\mu L b e_{q'}\bar{q}'\gamma^\mu R(L)q', \quad O_{8(10)}(q) = \frac{3}{2}\bar{q}_\alpha\gamma_\mu L b_\beta e_{q'}\bar{q}'_\beta\gamma^\mu R(L)q'_\alpha,$$



f can be u or c quark, q can be d or s quark,

q' is summed over u, d, s, and c quarks.

The leading QCD corrected Wilson Coefficients c_i at

$\alpha_s(m_Z) = 0.118$, $\alpha_{em}(m_Z) = 1/128$, $m_t = 176$ GeV and $\mu \approx m_b = 5$ GeV,

$$
\begin{aligned}
c_1 &= -0.3125, \quad c_2 = 1.1502, \quad c_3^t = 0.0174, \quad c_4^t = -0.0373, \\
c_5^t &= 0.0104, \quad c_6^t = -0.0459, \quad c_7^t = -1.050 \times 10^{-5}, \\
c_8^t &= 3.839 \times 10^{-4}, \quad c_9^t = -0.0101, \quad c_{10}^t = 1.959 \times 10^{-3}\,, \\
c_{3,5}^{u,c} &= -c_{4,6}^{u,c}/N_c = P_s^{u,c}/N_c\,, c_{7,9}^{u,c} = P_{em}^{u,c}\,, \quad c_{8,10}^{u,c} = 0\,,
\end{aligned}
$$

N_c is the number of color, $P_s^i = (\alpha_s/8\pi)c_2[10/9 + G(m_i, \mu, q^2)]$,

$P_{em}^i = \alpha_{em}/9\pi)(N_c c_1 + c_2)[10/9 + G(m_i, \mu, q^2)]$.

$G(m, \mu, q^2) = 4\int_0^1 x(1-x) ln[(m^2 - x(1-x)q^2)/\mu^2]dx$.

Determination of α

The phase angle α can be determined from $B \to \pi\pi$ decays.

The decay amplitude can be parametrized as

$$\bar{A}_{\pi^+\pi^-} = V_{ub}V_{ud}^* T_{\pi^+\pi^-} + V_{tb}V_{td}^* P_{\pi^+\pi^-} \ ,$$

The decay $\bar{B}^0 \to \pi^+\pi^-$ is induced by $H_{\Delta B=1}(d)$, and can be written as

$$\begin{aligned} T_{\pi^+\pi^-} &= \frac{4G_F}{\sqrt{2}} < \pi^+\pi^-|[c_1 O_1^u(d) + c_2 O_2^u(d)) + \sum_{i=3}^{10}(c_i^t - c_i^u)O_i(d)]|\bar{B}^0 > \\ P_{\pi^+\pi^-} &= \frac{4G_F}{\sqrt{2}} \sum_{i=3}^{12} < \pi^+\pi^-|(c_i^t - c_i^c)O_i(d)|\bar{B}^0 > \ . \end{aligned}$$

If the penguin amplitude $P_{\pi^+\pi^-}$ can be neglected,

$$Im\lambda_{\pi^+\pi^-} = Im\left(\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\frac{V_{ub}V_{ud}^*}{V_{ub}^*V_{ud}}\right) = \sin(2\alpha) \ .$$

The angle α can therefore be determined.

However, if penguin effects are significant, the above method fails.

The KM factors for Tree $V_{ub}V_{ud}^*$ and penguin $V_{cb}V_{cd}^*$ is the same order

The error is of order 12°.

It is necessary to find ways to isolate the penguin contributions.

When penguin effects are included,

$$Im\lambda_{\pi^+\pi^-} = \frac{|\bar{A}|}{|A|}\sin(2\alpha+\theta)\ .$$

Gronau and London, PRL65, 3381(1990)
Snyder and Quinn, PRD48, 2139(1993)

To determine θ, Gronau and London[40] proposed to use isospin relation

$$\sqrt{2}\bar{A}(\bar{B}^0 \to \pi^0\pi^0) + \sqrt{2}\bar{A}(B^- \to \pi^-\pi^0) = \bar{A}(\bar{B}^0 \to \pi^+\pi^-)\ ,$$

Similar relation for the corresponding the anti-particle decays.

If all amplitudes are measured, the angle θ can be determined.

Including $B \to \pi\rho, \rho\rho$, $\alpha = (87.6^{+3.5}_{-3.3})^o$.

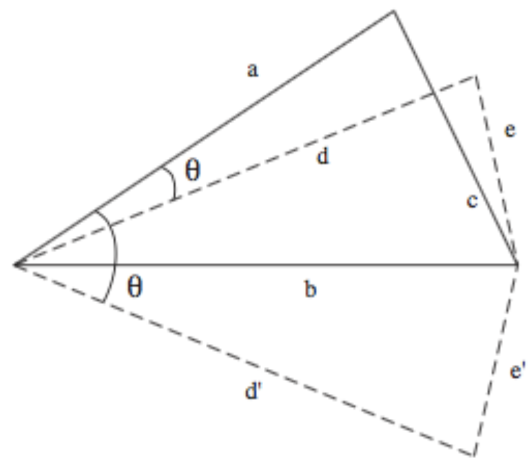


Figure 6: Isospin triangles in the complex plane. Lines a, b, and c denote the amplitudes $\bar{A}(B^0 \to \pi^+\pi^-)$, $\sqrt{2}\bar{A}(B^- \to \pi^-\pi^0) = \sqrt{2}A(B^+ \to \pi^+\pi^0)$, and $\sqrt{2}\bar{A}(B^0 \to \pi^0\pi^0)$,respectively. The dashed lines d and e (or d' and e') denote the amplitudes $A(B^0 \to \pi^+\pi^-)$ and $\sqrt{2}A(B^0 \to \pi^0\pi^0)$, respectively.

Determination of β

The best way to determine β is to measure $Im\lambda_{\psi K_S}$ for $\bar{B}^0(B^0) \to J/\psi K_S$.

The decay amplitude can be parameterized as

$A(\bar{B}^0 \to J/\psi K_S) = \langle K_S J/\psi | H_{eff} | \bar{B}^0 \rangle = V_{cb} V_{cs}^* T_{\psi K} + V_{ub} V_{us}^* P_{\psi K}$.

The WC's involved indicate that $|T_{\psi K}|$ is much larger than $|P_{\psi K}|$,

Also $|V_{cb} V_{cs}^*|$ is about 50 times larger than $|V_{ub} V_{us}^*|$ from experimental data

The $P_{\psi K_S}$ term can be ignored, then $\frac{\bar{A}}{A} = \frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}^*}$.

To a very good approximation,

$$Im\lambda_{\psi K_s} = Im\left(\left(\frac{q}{p}\right)_{B_d} \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \left(\frac{q}{p}\right)_K\right) = -\sin(2\beta) \,.$$

The Gold-plated place for CP violation (Cater, Sanda, and Bigi, 1980, 1981)

Data: $Im(\lambda_{B^0 \to J/\psi K_s} = sin(2\beta) = 0.691 \pm 0.017$

Determination of γ

Gronau and Wyler, PLB256, 172(1991)
Atwood et al., PRL 78, 3257(1997)

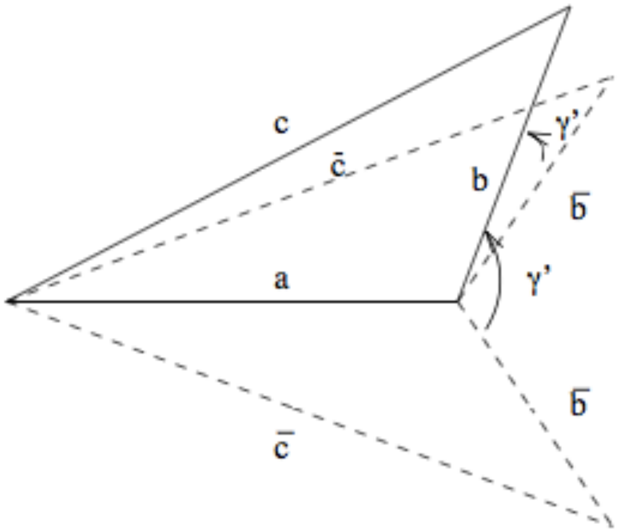
Determination of the phase angle γ using: $B^- \to (D^0,\ \bar{D}^0,\ D_{CP})K^-$.

Here $D_{CP} = (D^0 - \bar{D}^0)/\sqrt{2}$ is the CP even state.

The decay amplitudes can be parameterised as

$$\bar{A}(\bar{D}^0K^-) = V_{ub}V_{cs}^*T_{\bar{D}K}\ ,\quad \bar{A}(D^0K^-) = V_{cb}V_{us}^*T_{D^0K}\ ,$$
$$\bar{A}(D_{CP}K^-) = \frac{1}{\sqrt{2}}(\bar{A}(D^0K^-) - \bar{A}(\bar{D}^0K^-))\ .$$

The angle γ can be measured as shown in the figure



D_{CP} identified is through processes induced by

$c \to ud\bar{d}$ and $\bar{c} \to \bar{u}\bar{d}d$.

The angle γ' in the figure is given by the absolute value of

$\mathrm{Arg}[(V_{ub}V_{cs}^*/V_{cb}V_{us}^*)(V_{cd}V_{ud}^*/V_{cd}^*V_{ud})] = -2(\gamma - \sigma')$.

In the SM σ' is very small, so γ' is equal to 2γ to a very good approximation.

Including $B^- \to DK^{*-}, D^*K^{*-}$ and other similar decays: $(73.2^{+6.3}_{-7.0})^o$.

Figure 7: The measurement of γ through $B^-(B^+) \to DK^-(K^+)$ decays with $a = |\bar{A}(B^- \to D^0K^-)| = |A(B^+ \to \bar{D}^0K^+)|$, $b = \bar{A}(B^- \to \bar{D}^0)$, $\bar{b} = A(B^+ \to D^0K^+)$, $c = \sqrt{2}\bar{A}(B^- \to D_{CP}K^-)$, and $\bar{c} = \sqrt{2}A(B^+ \to D_{CP}K^+)$.

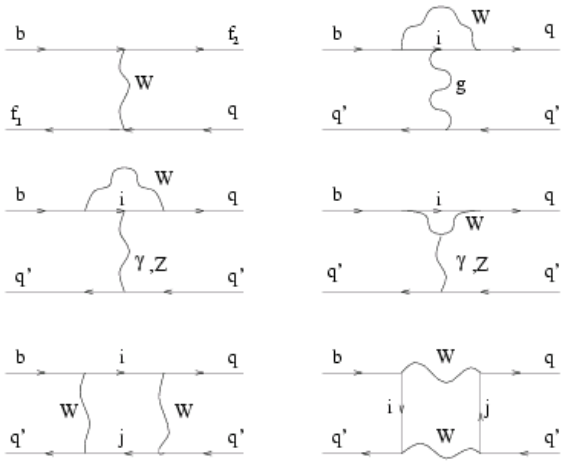
Unitarity triangle confirmed!

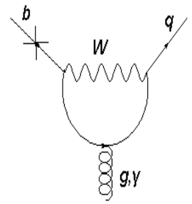
$\alpha+\beta+\gamma = 182.6^{o} \; +\text{-} \; 8^{o}$

U-spin CP violating relations in B -> PP decays

Effective Hamiltonian for B to PP decays in the SM

$$H^q_{eff} = \frac{4G_F}{\sqrt{2}}[V_{ub}V^*_{uq}(c_1O_1 + c_2O_2) - \sum_{i=3}^{12}(V_{ub}V^*_{uq}c_i^{uc} + V_{tb}V^*_{tq}c_i^{tc})O_i].$$





$$O_1 = (\bar{q}_i u_j)_{V-A}(\bar{u}_i b_j)_{V-A}\ , \qquad O_2 = (\bar{q}u)_{V-A}(\bar{u}b)_{V-A}\ ,$$

$$O_{3,5} = (\bar{q}b)_{V-A}\sum\nolimits_{q'}(\bar{q}'q')_{V\mp A}\ , \qquad O_{4,6} = (\bar{q}_i b_j)_{V-A}\sum\nolimits_{q'}(\bar{q}'_j q'_i)_{V\mp A}\ ,$$

$$O_{7,9} = \tfrac{3}{2}(\bar{q}b)_{V-A}\sum\nolimits_{q'} e_{q'}(\bar{q}'q')_{V\pm A}\ , \qquad O_{8,10} = \tfrac{3}{2}(\bar{q}_i b_j)_{V-A}\sum\nolimits_{q'} e_{q'}(\bar{q}'_j q'_i)_V$$

$$O_{11} = \tfrac{g_s}{16\pi^2}\bar{q}\sigma_{\mu\nu}G^{\mu\nu}(1+\gamma_5)b\ , \qquad O_{12} = \tfrac{Q_b e}{16\pi^2}\bar{q}\sigma_{\mu\nu}F^{\mu\nu}(1+\gamma_5)b.$$

(Wise&Gillman; Haglin;Buchalla et al; Ciuchini et al…..)

Theoretical calculations of B -> PP

$$A(B \to PP) = \langle PP|H_{eff}|B\rangle$$

Not completely understood QCD at low energies and therefore not possible to have a first principle QCD calculation, Lattice QCD always gives promises.

On the market, several main approaches of model calculations

Naïve Factorization calculations: factorize four quark into products of two quark operators and decay constants, form factors to fix decay amplitudes. (Bauer,Stech&Wirbel; Ali,Kramer&C-D Lu; H-Y Cheng et al.; Deshpande&Trampetic....)

QCD improved calculations. (Beneke,Buchalla,Neubert& Sachrajda....)
pQCD calculations. (Keum,Li&Sanda; Lu; Yang&Yang;....)
SCET calculations. (Bauer,Fleming,Pirjol&Stewart;...)

Have some understanding of the decays.
But Non-factorizeable contributions.... Still a lot of parameters, wave functions, form factors ...

Flavor SU(3) Symmetry

QCD poses an approximate global flavor SU(3) symmetry because the u, d, s quarks are light compared with QCD scale.

Under this SU(3), (u, d, s) transform as fundamental representation.

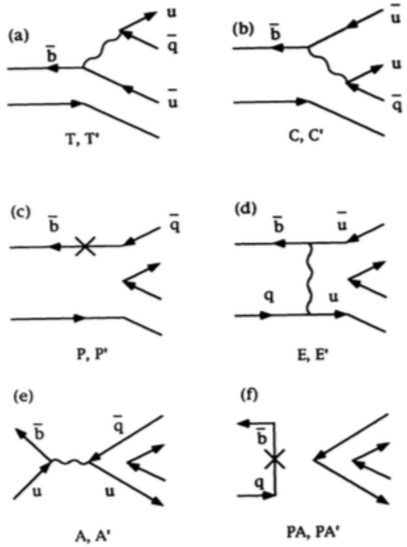
It worked well for light mesons and baryons, the quark model (Gell-Mann; Ne'eman…)

Application to B decays: Also works well!

Algebraic Way: Zepenfeld; Savage&Wise;Deshapnde&He;Gronau,Pirojl&T.M.Yuan…

Diagramatic Way: Chau&Cheng; Gronau,Lodon,Hernadez&Rosner;Cheng&Chiang…

Diagram approach relate them by SU(3) symmetry



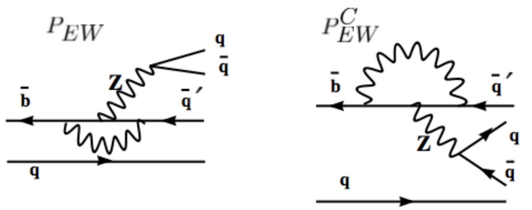
	Final state	T	C	P	E	A	PA
$B^+ \to$	$\pi^+\pi^0$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	0	0	0	0
	$K^+\bar{K}^0$	0	0	1	0	1	0
	$\pi^+\eta_8$	$-1/\sqrt{6}$	$-1/\sqrt{6}$	$-2/\sqrt{6}$	0	$-2/\sqrt{6}$	0
$B^0 \to$	$\pi^+\pi^-$	-1	0	-1	-1	0	-1
	$\pi^0\pi^0$	0	$-1/\sqrt{2}$	$1/\sqrt{2}$	$1/\sqrt{2}$	0	$1/\sqrt{2}$
	K^+K^-	0	0	0	-1	0	-1
	$K^0\bar{K}^0$	0	0	1	0	0	1
	$\pi^0\eta_8$	0	0	$-1/\sqrt{3}$	$1/\sqrt{3}$	0	0
	$\eta_8\eta_8$	0	$1/3\sqrt{2}$	$1/3\sqrt{2}$	$1/3\sqrt{2}$	0	$1/\sqrt{2}$
$B_s \to$	π^+K^-	-1	0	-1	0	0	0
	$\pi^0\bar{K}^0$	0	$-1/\sqrt{2}$	$1/\sqrt{2}$	0	0	0
	$\eta_8\bar{K}^0$	0	$-1/\sqrt{6}$	$1/\sqrt{6}$	0	0	0

	Final state	T'	C'	P'	E'	A'	PA'
$B^+ \to$	π^+K^0	0	0	1	0	1	0
	π^0K^+	$-1/\sqrt{2}$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	0	$-1/\sqrt{2}$	0
	η_8K^+	$-1/\sqrt{6}$	$-1/\sqrt{6}$	$1/\sqrt{6}$	0	$1/\sqrt{6}$	0
$B^0 \to$	π^-K^+	-1	0	-1	0	0	0
	π^0K^0	0	$-1/\sqrt{2}$	$1/\sqrt{2}$	0	0	0
	η_8K^0	0	$-1/\sqrt{6}$	$1/\sqrt{6}$	0	0	0
$B_s \to$	$\pi^+\pi^-$	0	0	0	-1	0	-1
	$\pi^0\pi^0$	0	0	0	$1/\sqrt{2}$	0	$1/\sqrt{2}$
	K^+K^-	-1	0	-1	-1	0	-1
	$K^0\bar{K}^0$	0	0	1	0	0	1
	$\pi^0\eta_8$	0	$-1/\sqrt{3}$	0	$1/\sqrt{3}$	0	0
	$\eta_8\eta_8$	0	$-\sqrt{2}/3$	$2\sqrt{2}/3$	$1/3\sqrt{2}$	0	$1/\sqrt{2}$

Gronau et al.

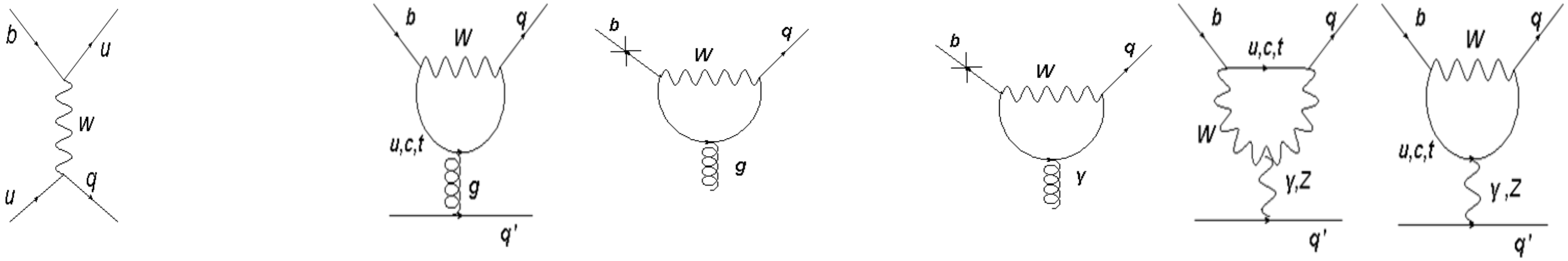
Electroweak penguin important

Deshpande&He; Fleischer;…1994



	Final state	T,C,P contributions	Electroweak Penguins
$B^+ \to$	$\pi^+\pi^0$	$-(T+C)/\sqrt{2}$	$-[(c_u-c_d)P_{EW}+(c_u-c_d)P^C_{EW}]/\sqrt{2}$
	$K^+\bar{K}^0$	$P+A$	$c_dP^C_{EW}$
$B^0 \to$	$\pi^+\pi^-$	$-(T+P)$	$-c_uP^C_{EW}$
	$\pi^0\pi^0$	$-(C-P-E)/\sqrt{2}$	$-[(c_u-c_d)P_{EW}+c_dP^C_{EW}]/\sqrt{2}$
	$K^0\bar{K}^0$	P	$c_dP^C_{EW}$
$B_s \to$	π^+K^-	$-(T+P)$	$-c_uP^C_{EW}$
	$\pi^0\bar{K}^0$	$-(C-P)/\sqrt{2}$	$-[(c_u-c_d)P_{EW}-c_dP^C_{EW}]/\sqrt{2}$
$B^+ \to$	π^+K^0	P'	$c_dP'^C_{EW}$
	π^0K^+	$-(P'+T'+C')/\sqrt{2}$	$-[(c_u-c_d)P'_{EW}+c_uP'^C_{EW}]/\sqrt{2}$
$B^0 \to$	π^-K^+	$-(P'+T')$	$-c_uP'^C_{EW}$
	π^0K^0	$-(P'-C')/\sqrt{2}$	$-[(c_u-c_d)P'_{EW}-c_dP'^C_{EW}]/\sqrt{2}$
$B_s \to$	K^+K^-	$-(P'+T')$	$-c_uP'^C_{EW}$
	$K^0\bar{K}^0$	P'	$c_dP'^C_{EW}$

A complete SU(3) analysis



$\bar{3}\otimes 3\otimes\bar{3} = \overline{15}\oplus 6\oplus\bar{3}\oplus\bar{3}$ $\bar{3}\otimes 1=\bar{3}$ $\bar{3}\otimes 8=\overline{15}\oplus 6\oplus\bar{3}$

Under $SU(3)$ flavor symmetry, while the Lorentz-Dirac structure and color index are both omitted, $O_{1,2}$, $O_{3-6,11}$, and O_{7-10} transform as $\bar{3}+\bar{3}'+6+\overline{15}$, $\bar{3}$, and $\bar{3}+\bar{3}'+6+\overline{15}$, respectively. As a result, H^q_{eff} can be decomposed as the matrices of $H(\bar{3})$, $H(6)$, and $H(\overline{15})$ with their non-zero entries to be, for $q = d$,

$$H(\bar{3})^2 = 1\ ,\ H(6)^{12}_1 = H(6)^{23}_3 = 1\ ,\ H(6)^{21}_1 = H(6)^{32}_3 = -1\ ,$$
$$H(\overline{15})^{12}_1 = H(\overline{15})^{21}_1 = 3,\ H(\overline{15})^{22}_2 = -2\ ,\ H(\overline{15})^{32}_3 = H(\overline{15})^{23}_3 = -1\ ,$$

and for $q = s$

$$H(\bar{3})^3 = 1\ ,\ H(6)^{13}_1 = H(6)^{32}_2 = 1\ ,\ H(6)^{31}_1 = H(6)^{23}_2 = -1\ ,$$
$$H(\overline{15})^{13}_1 = H(\overline{15})^{31}_1 = 3\ ,\ H(\overline{15})^{33}_3 = -2\ ,\ H(\overline{15})^{32}_2 = H(\overline{15})^{23}_2 = -1\ .$$

$$A =< final\ state|H_{eff}^q|\bar{B} >= V_{ub}V_{uq}^* T(q) + V_{tb}V_{tq}^* P(q)$$

$$\begin{aligned} T(q) &= A_{\bar{3}}^T \bar{B}_i H(\bar{3})^i (M_l^k M_k^l) + C_{\bar{3}}^T \bar{B}_i M_k^i M_j^k H(\bar{3})^j \\ &+ A_6^T \bar{B}_i H(6)_k^{ij} M_j^l M_l^k + C_6^T \bar{B}_i M_j^i H(6)_l^{jk} M_k^l \\ &+ A_{\overline{15}}^T \bar{B}_i H(\overline{15})_k^{ij} M_j^l M_l^k + C_{\overline{15}}^T \bar{B}_i M_j^i H(\overline{15})_l^{jk} M_k^l \end{aligned}$$

$B_i = (B^+, B^0, B_s^0)$ is an $SU(3)$ triplet.

M_i^j is the $SU(3)$ pseudoscalar octet, $M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}$

Ai-annihilation amplitude, small.
C6-A6 appear together, just use C6.
Smilar structure for penguin amplitude,

indicate the corresponding amplitudes by A_i^P and C_i^P.

SU(3) decay amplitudes for B → PP

$\Delta S = 0$

$$T^{B_u}_{\pi^-\pi^0}(d) = \frac{8}{\sqrt{2}} C^T_{\overline{15}},$$
$$T^{B_u}_{\pi^-\eta_8}(d) = \frac{2}{\sqrt{6}}(C^T_{\bar 3} - C^T_6 + 3A^T_{\overline{15}} + 3C_{\overline{15}}),$$
$$T^{B_u}_{K^-K^0}(d) = C^T_{\bar 3} - C^T_6 + 3A^T_{\overline{15}} - C^T_{\overline{15}},$$
$$T^{B_d}_{\pi^+\pi^-}(d) = 2A^T_{\bar 3} + C^T_{\bar 3} + C^T_6 + A^T_{\overline{15}} + 3C^T_{\overline{15}},$$
$$T^{B_d}_{\pi^0\pi^0}(d) = \frac{1}{\sqrt{2}}(2A^T_{\bar 3} + C^T_{\bar 3} + C^T_6 + A^T_{\overline{15}} - 5C^T_{\overline{15}}),$$
$$T^{B_d}_{K^-K^+}(d) = 2(A^T_{\bar 3} + A^T_{\overline{15}}),$$
$$T^{B_d}_{\bar K^0K^0}(d) = 2A_{\bar 3} + C^T_{\bar 3} - C^T_6 - 3A^T_{\overline{15}} - C_{\overline{15}},$$
$$T^{B_d}_{\pi^0\eta_8}(d) = \frac{1}{\sqrt{3}}(-C^T_{\bar 3} + C^T_6 + 5A^T_{\overline{15}} + C_{\overline{15}}),$$
$$T^{B_d}_{\eta_8\eta_8}(d) = \frac{1}{\sqrt{2}}(2A_{\bar 3} + \frac{1}{3}C^T_{\bar 3} - C^T_6 - A^T_{\overline{15}} + C_{\overline{15}}),$$
$$T^{B_s}_{K^+\pi^-}(d) = C^T_{\bar 3} + C^T_6 - A^T_{\overline{15}} + 3C_{\overline{15}},$$
$$T^{B_s}_{K^0\pi^0}(d) = -\frac{1}{\sqrt{2}}(C^T_{\bar 3} + C^T_6 - A^T_{\overline{15}} - 5C_{\overline{15}}),$$
$$T^{B_s}_{K^0\eta_8}(d) = -\frac{1}{\sqrt{6}}(C^T_{\bar 3} + C^T_6 - A^T_{\overline{15}} - 5C_{\overline{15}}),$$

$\Delta S = -1$

$$T^{B_u}_{\pi^-\bar K^0}(s) = C^T_{\bar 3} - C^T_6 + 3A^T_{\overline{15}} - C^T_{\overline{15}},$$
$$T^{B_u}_{\pi^0K^-}(s) = \frac{1}{\sqrt{2}}(C^T_{\bar 3} - C^T_6 + 3A^T_{\overline{15}} + 7C^T_{\overline{15}})\ ,$$
$$T^{B_u}_{\eta_8K^-}(s) = \frac{1}{\sqrt{6}}(-C^T_{\bar 3} + C^T_6 - 3A^T_{\overline{15}} + 9C^T_{\overline{15}}),$$
$$T^{B_d}_{\pi^+K^-}(s) = C^T_{\bar 3} + C^T_6 - A^T_{\overline{15}} + 3C^T_{\overline{15}},$$
$$T^{B_d}_{\pi^0\bar K^0}(s) = -\frac{1}{\sqrt{2}}(C^T_{\bar 3} + C^T_6 - A^T_{\overline{15}} - 5C^T_{\overline{15}}),$$
$$T^{B_d}_{\eta_8\bar K^0}(s) = -\frac{1}{\sqrt{6}}(C^T_{\bar 3} + C^T_6 - A^T_{\overline{15}} - 5C^T_{\overline{15}}),$$
$$T^{B_s}_{\pi^+\pi^-}(s) = 2(A^T_{\bar 3} + A^T_{\overline{15}}),$$
$$T^{B_s}_{\pi^0\pi^0}(s) = \sqrt{2}(A^T_{\bar 3} + A^T_{\overline{15}}),$$
$$T^{B_s}_{K^+K^-}(s) = 2A^T_{\bar 3} + C^T_{\bar 3} + C^T_6 + A^T_{\overline{15}} + 3C^T_{\overline{15}},$$
$$T^{B_s}_{K^0\bar K^0}(s) = 2A^T_{\bar 3} + C^T_{\bar 3} - C^T_6 - 3A^T_{\overline{15}} - C^T_{\overline{15}},$$
$$T^{B_s}_{\pi^0\eta_8}(s) = \frac{2}{\sqrt{3}}(C^T_6 + 2A^T_{\overline{15}} - 2C^T_{\overline{15}}),$$
$$T^{B_s}_{\eta_8\eta_8}(s) = \sqrt{2}(A^T_{\bar 3} + \frac{2}{3}C^T_{\bar 3} - A^T_{\overline{15}} - 2C^T_{\overline{15}}).$$

U-spin symmetry d <-> s channels, $T_d=T_s=T$, $P_d=P_s=P$

Example Deshpande&X-G He(1995), X-G He(1999), Gronau&Rosner (2000)…

He, Li, Ren and Yuan, arXiv:1704.05788

$$A(\bar{B}^0 \to K^-\pi^+) = V_{ub}V_{us}^*T + V_{tb}V_{ts}^*P\,, \quad A(B^0 \to K^+\pi^-) = V_{ub}^*V_{us}T + V_{tb}^*V_{ts}P$$

$$A(\bar{B}_s^0 \to K^+\pi^-) = V_{ub}V_{ud}^*T + V_{tb}V_{td}^*P\,, \quad A(B_s^0 \to K^-\pi^+) = V_{ub}^*V_{ud}T + V_{tb}^*V_{td}P$$

$$T = C_{\bar{3}}^T + C_6^T - A_{\overline{15}}^T + 3C_{\overline{15}}^T\,, \quad P = C_{\bar{3}}^P + C_6^P - A_{\overline{15}}^P + 3C_{\overline{15}}^P.$$

$$\Delta(B \to PP) = \Gamma(\bar{B} \to \bar{P}\,\bar{P}) - \Gamma(B \to P\,P) = \frac{\lambda_{ab}}{8\pi m_B}(|A(\bar{B} \to \bar{P}\,\bar{P})|^2 - |A(B \to P\,P)|^2)\,,$$

$$Im(V_{ub}V_{ud}^*V_{tb}^*V_{td}) = -Im(V_{ub}V_{us}^*V_{tb}^*V_{ts}), \qquad \frac{\mathcal{A}_{\rm CP}(B_s^0 \to K^+K^-)}{\mathcal{A}_{\rm CP}(B^0 \to \pi^+\pi^-)} = -r_c\frac{\mathcal{B}(B^0 \to \pi^+\pi^-)\tau_{B_d^0}}{\mathcal{B}(B_s^0 \to K^+K^-)\tau_{B^0}}\,,$$

$$\Delta(B^0 \to K^+\pi^-) = -\Delta(B_s^0 \to K^-\pi^+) \qquad r_c = \frac{\mathrm{Im}(T_sP_s^*)}{\mathrm{Im}(T_dP_d^*)} = \frac{|T_s||P_s|\sin(\phi_{T_s} - \phi_{P_s})}{|T_d||P_d|\sin(\phi_{T_d} - \phi_{P_d})}\,.$$

SU(3)/U symmetric, $r_c = 1$

Test for SU(3) flavor symmetry, and also SM with 3 generations!

Test U relations with several other B decays

X-G He, S-F Li, H-H Li, JHEP, 2013; He, Li, Ren and Yuan, arXiv:1704.05788

	mode	$\overline{\mathcal{B}}\,[10^{-6}]$	$\mathcal{A}_{CP}\,[10^{-2}]$	mode	$\overline{\mathcal{B}}\,[10^{-6}]$	$\mathcal{A}_{CP}\,[10^{-2}]$
P1)	$B_s^0 \to K^-\pi^+$	5.5 ± 0.5	26 ± 4	$B^0 \to K^+\pi^-$	$19.57^{+0.53}_{-0.52}$	-8.2 ± 0.6
P2)	$B^0 \to \overline{K}^0 K^0$	1.21 ± 0.16	-0 ± 40	$B_s^0 \to K^0\overline{K}^0$	$19.6^{+9.7}_{-9.3}$	—
P3)	$B_s^0 \to \overline{K}^0\pi^0$	—	—	$B^0 \to K^0\pi^0$	9.93 ± 0.49	-0 ± 13
P4)	$B^+ \to K^+\overline{K}^0$	1.32 ± 0.14	-8.7 ± 10.0	$B^+ \to K^0\pi^+$	23.79 ± 0.75	-1.7 ± 1.6
P5)	$B^0 \to \pi^+\pi^-$	5.10 ± 0.19	$24 \pm 7 \pm 1$ [1]	$B_s^0 \to K^+K^-$	24.8 ± 1.7	$-24 \pm 6 \pm 2$ [1]
			27 ± 4			
P6)	$B^0 \to K^+K^-$	0.111 ± 0.0565	—	$B_s^0 \to \pi^+\pi^-$	0.671 ± 0.083	—
				$B_s^0 \to \pi^0\pi^0$	—	—

For $P_1 \sim P_6$, theoretical calculations $1 \sim 2$

r_c for P_1 experimental data: 1.11 +- 0.22 (expected with SU(3) breaking)

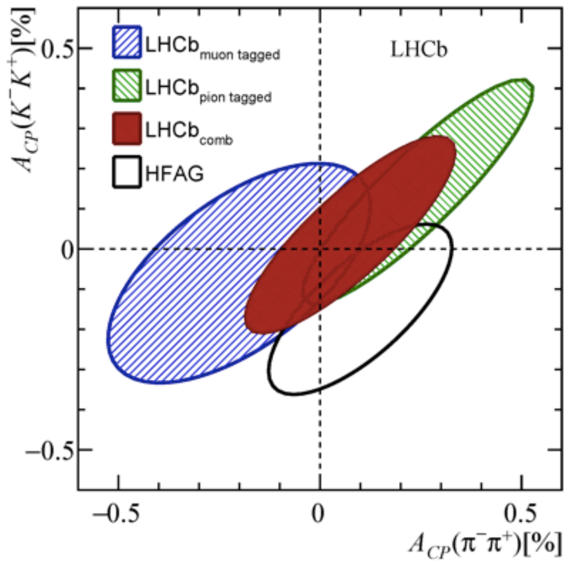
Why for P_5, r_c = 4.67+- 1.88, so much different?

Violation of U relation

Measurement of time-dependent *CP*-violating asymmetries in $B^0 \to \pi^+\pi^-$ and $B_s^0 \to K^+K^-$ decays at LHCb

LHCb-CONF-2016-018

$$\begin{aligned} C_{\pi^+\pi^-} &= -0.243 \pm 0.069, \\ S_{\pi^+\pi^-} &= -0.681 \pm 0.060, \\ C_{K^+K^-} &= 0.236 \pm 0.062, \\ S_{K^+K^-} &= 0.216 \pm 0.062, \\ A^{\Delta\Gamma}_{K^+K^-} &= -0.751 \pm 0.075. \end{aligned}$$



$$\frac{\mathcal{A}_{\rm CP}(B_s^0 \to K^+K^-)}{\mathcal{A}_{\rm CP}(B^0 \to \pi^+\pi^-)} = -r_c \frac{\overline{\mathcal{B}}(B^0 \to \pi^+\pi^-)\tau_{B_d^0}}{\overline{\mathcal{B}}(B_s^0 \to K^+K^-)\tau_{B^0}},$$

SU(3) or U-spin symmetric: $r_c = 1$

LHCb data: $r_c = 4.67 \pm 1.88$

SU(3)/U-spin relation violation!

Large FSI phases?!

For P_1, both $B^0 \to K^+\pi^-$ and $B_s^0 \to \pi^+K^-$, the final states are $K^\pm \pi^\pm$ and are CP conjugate of each other. Their final state phase spaces are the same and also FSI should be similar.

But for $B^0 \to \pi^+\pi^-$ and $B_s^0 \to K^+K^-$ decays, data

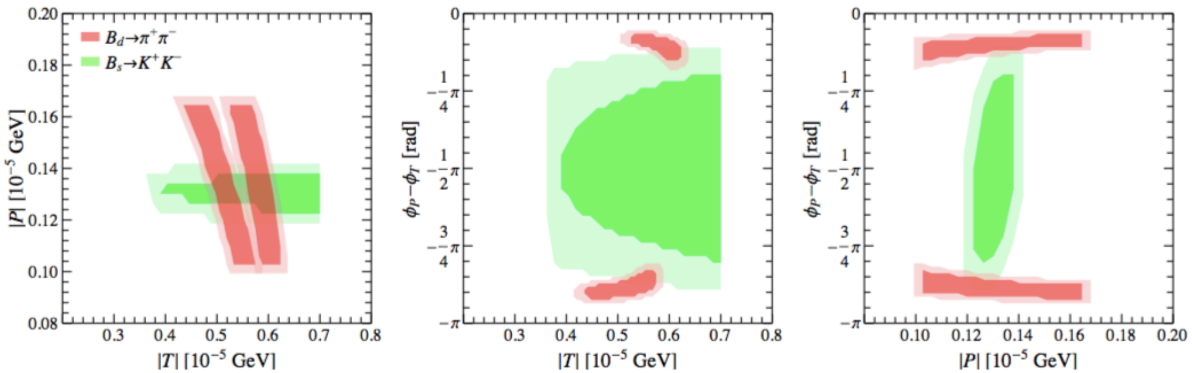
Support from D -> KK, Kπ, ππ

$$\tilde{r}_c(K^+\pi^-, K^-\pi^+) = \frac{\bar{\mathcal{B}}(D^0 \to K^+\pi^-)}{\bar{\mathcal{B}}(D^0 \to K^-\pi^+)} \cdot \frac{|V_{cs}^* V_{ud}|^2}{|V_{cd}^* V_{us}|^2} = 1.3,$$

$$\tilde{r}_c(K^+K^-, \pi^+\pi^-) = \frac{\bar{\mathcal{B}}(D^0 \to K^+K^-)}{\bar{\mathcal{B}}(D^0 \to \pi^+\pi^-)} \cdot \frac{|V_{cd}^* V_{ud}|^2}{|V_{cs}^* V_{us}|^2} = 2.8.$$

the final state $\pi^+\pi^-$ are very much different than the final state K^+K^-.

The phases of T and P caused the difference!



current data prefer $\Delta\phi_s \sim -\pi/2$ and $\Delta\phi_d \sim 0$ or $-\pi$

Further tests: pattern of U relation breaking

P1) $B_s^0 \to \pi^+ K^-$ and $B^0 \to K^+ \pi^-$,
$r_c = 1.11 \pm 0.22$ violation SU(3)/U relation is less than 20% level.

P2) $B^0 \to \bar{K}^0 K^0$ and $B_s^0 \to K^0 \bar{K}^0$ and P3) $B_s^0 \to \bar{K}^0 \pi^0$ and $B^0 \to K^0 \pi^0$
expected to have 1.11 or so.

P5) $B^0 \to \pi^+ \pi^-$ and $B_s^0 \to K^+ K^-$, $r_c = 3.5 \pm 1.06$.
P6) $B^0 \to K^+ K^-$ and $B_s^0 \to \pi^+ \pi^-, \pi^0 \pi^0$, r_c large.

P4) $B^+ \to K^+ \bar{K}^0$ and $B^+ \to K^0 \pi^+$, r_c between P1 and P5.

6. Beyond SM CP violation

What is the origin of CPV? Is there other ways CPV can present in a model?

There are many ways CPV can show up when going beyond SM:

Superweak model, CP is only violated in $\Delta S=2$ current-current interactions. Too small ε'/ε. Ruled out by data.

In left-right $SU(3)_C xSU(2)_L xSU(2)_R xU(1)_{B-L}$ symmetric model, there are similar mixng matrix V^R_{KM} charged current for right-handed fermions by exchanging W_R. More phases, only two generations can have CPV.

Seesaw Model, there are new phases in Right-handed neutrino mass matrix.

…

CP violated by vacuum? Not explicitly violated as that in SM, T-D Lee, spontaneous CP violation.

…

Spontaenous CP violation

A toy model for spontaneous CP violation, basic idea illustrated by T.D. Lee

Consider the following Lagrangian

$$L = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{i}{2}(\bar{\psi}\gamma_\mu\partial^\mu\psi - \partial^\mu\bar{\psi}\gamma_\mu\psi) + m\bar{\psi}\psi - ig\bar{\psi}\gamma_5\psi\phi \ ,$$

ϕ: a pseudoscalar field, ψ is a spinor,

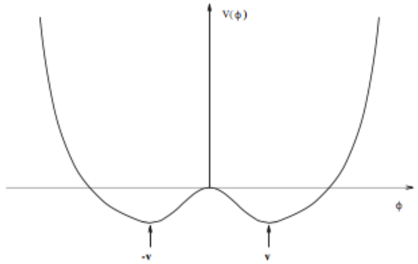
$V(\phi)$ is the potential for ϕ field given by $V(\phi) = \frac{1}{8}k^2(\phi^2 - v^2)^2$.
The transformation properties of ϕ under P, C and T are:

$$P\phi = -\phi \ , \ \ C\phi = \phi \ , \ \ T\phi = -\phi \ .$$

The spinor has the usual transformation properties.

If the ϕ field does not develop any VEV, that is, $<\phi>= 0$,
the above model is invariant under CP transformation.

However, the zero VEV for ϕ is not the minimal of the potential.
The minimal occurs at $\langle\phi\rangle = \pm v$ (v must be real) as shown in the figure



In the broken phase ($\langle\phi\rangle = v$), the Lagrangian is given by,

$$L = -\frac{1}{2}\partial_\mu H \partial^\mu H - V(H)$$
$$-\frac{i}{2}(\bar{\psi}\gamma_\mu\partial^\mu\psi - \partial^\mu\bar{\psi}\gamma_\mu\psi) + m\bar{\psi}\psi - igv\bar{\psi}\gamma_5\psi - ig\bar{\psi}\gamma_5\psi H \ ,$$
$$V(H) = \frac{1}{8}k^2(H^2 + 2vH)^2 \ .$$

H, from $\phi = v + H$, has the same C, P and T transformation properties as ϕ.

The VEV v is a constant, does not transform under P, C, under T $v \to v^*$.

Under CP transformation becomes:

$V^{CP}(H) = (1/8)k^2(H^2 - 2vH)^2$, and the term $igv\bar{\psi}\gamma_5\psi \to -igv\bar{\psi}\gamma_5\psi$

CP is broken by a non-zero v, spontaneous CP violation!

Making a chiral rotation on the fermion, $\psi' = exp(i\alpha\gamma_5/2)\psi$, $tan(\alpha) = gv/m$

In this basis, the kinetic energy term has the same form as in the ψ basis,

but the mass and fermion-scalar interaction terms change

$$L(\psi') = \sqrt{m^2 + g^2v^2}\bar{\psi}'\psi' - g\bar{\psi}'(i\gamma_5 cos\alpha - sin\alpha)\psi' H + \dots .$$

The H field has both scalar and pseudoscalar couplings to the fermion ψ'.

Exchange of H between fermions violates CP.

Spontaneous CP violation in two Higgs doublet model

In the SM it is not possible to have spontaneous CP violation.

It requires at least two Higgs doublets to have a realistic model,

H_1 and H_2 transforming as $(1, 2, 1/2)$

$$\begin{aligned} V(H_1, H_2) &= \mu_{ij}^2 H_i^\dagger H_j + \lambda_i (H_i^\dagger H_i)^2 + \lambda'_{ij} (H_i^\dagger H_j)(H_j^\dagger H_i) \\ &+ [\delta (H_1^\dagger H_2)(H_1^\dagger H_2) + \delta_1 (H_1^\dagger H_2)(H_1^\dagger H_1) \\ &+ \delta_2 (H_1^\dagger H_2)(H_2^\dagger H_2) + H.C.] \,. \end{aligned}$$

The vevs minimizes the potentais are: $\langle H_i \rangle \to v_i exp(i\theta_i)/\sqrt{2}$,

The Higgs fields can be written as: $H_i = (h_i^+, \frac{1}{\sqrt{2}}(v_i e^{i\theta_i} + h_i^0 + iI_i^0))$

The minmimaztion condition for the phase is

$$(\mu_{12}^2 + \mu_{21}^2)v_1 v_2 \sin\theta + \delta v_1^2 v_2^2 \cos(2\theta) + (\delta_1 v^3 v_2 + \delta_2 v_1 v_2^3)\sin\theta = 0\ ,$$

$$\theta = 0\ ,\ \text{or}\ ,\ \theta = -arccos\left(\frac{2(\mu_{12}^2 + \mu_{21}^2) + \delta_2 v_1^2 + \delta_3 v_2^2}{4\delta v_1 v_2}\right)$$

Here $\theta = \theta_1 - \theta_2$.

$\sin\theta = 0$ solution is no CP violating, $\cos\theta \neq \pm 1$, violates CP.

The most general Yukawa interactions with quarks are given by

$$L_Y = -\bar{Q}_L(\lambda_1^U \tilde{H}_1 + \lambda_2^U \tilde{H}_2)U_R - \bar{Q}_L(\lambda_1^D H_1 + \lambda_2^D H_2)D_R + H.C.$$

All $\lambda_i^{U,D}$ are real (spontaneous CPV)

$$\begin{aligned}
L_Y &= -\bar{D}_L[(\lambda_1^U)(-(h_1^+)^*) + (\lambda_2^U)(-(h_2^+)^*))D_L - \bar{U}_L(\lambda_1^D h_1^+ + \lambda_2^D h_2^+]D_R \\
&- \bar{U}_L[\lambda_1^U \frac{1}{\sqrt{2}}(v_1 e^{-i\theta_1} + (h_1^0 + iI_1^0)^*) + \lambda_2^U \frac{1}{\sqrt{2}}(v_2 e^{-i\theta_2} + (h_2^0 + iI_2^0)^*)]U_R \\
&- \bar{D}_L[\lambda_1^D \frac{1}{\sqrt{2}}(v_1 e^{i\theta_1} + h_1^0 + iI_1^0) + \lambda_2^D \frac{1}{\sqrt{2}}(v_2 e^{i\theta_2} + h_2^0 + iI_2^0)]D_R \\
&+ H.C.
\end{aligned}$$

Remove Goldstone bosons Z_I and h_W^+ "eaten" by Z and W^+

$$Z_I = \frac{v_1 e^{-i\theta_1} I_1^0 + v_2 e^{-i\theta_2} I_2^0}{\sqrt{v_1^2 + v_2^2}}\,, \quad h_W^+ = \frac{v_1 e^{-i\theta_1} h_1^+ + v_2 e^{-i\theta_2} h_2^+}{\sqrt{v_1^2 + v_2^2}}\,,$$

The orthogonal components are

$$A = \frac{-v_2 e^{i\theta_2} I_1^0 + v_1 e^{i\theta_1} I_2^0}{\sqrt{v_1^2 + v_2^2}}\,, \quad H^+ = \frac{-v_2 e^{i\theta_2} h_1^+ + v_1 e^{i\theta_1} h_2^+}{\sqrt{v_1^2 + v_2^2}}\,,$$

Normalize in a same way for real scalars

$$h = \frac{v_1 e^{-i\theta_1} h_1^0 + v_2 e^{-i\theta_2} h_2^0}{\sqrt{v_1^2 + v_2^2}}\,, \quad H = \frac{-v_2 e^{i\theta_2} h_1^0 + v_1 e^{i\theta_1} h_2^0}{\sqrt{v_1^2 + v_2^2}}\,,$$

Quark mass matrices

$$\bar{U}_L M^u U_R = \bar{U}_L \frac{1}{\sqrt{2}}(\lambda_1^U v_1 e^{-i\theta_1} + \lambda_2^U v_2 e^{-i\theta_2}) U_R$$
$$\bar{D}_L M^d D_R = \bar{D}_L \frac{1}{\sqrt{2}}(\lambda_1^D v_1 e^{i\theta_1} + \lambda_2^D v_2 e^{i\theta_2}) D_R$$

$$\begin{aligned}
L_Y &= -\bar{D}_L(\lambda_1^U v_2 e^{-i\theta_2} - \lambda_2^U v_1 e^{-i\theta_1}) U_R \frac{h^-}{v} + \bar{U}_L(\lambda_1^D v_2 e^{i\theta_2} - \lambda_2^D v_1 e^{i\theta_1}) D_R \frac{h^+}{v} \\
&- \bar{U}_L[M^u(1+\frac{h}{v}) - (\lambda_1^U v_2 e^{-i\theta_2} - \lambda_2^U v_1 e^{-i\theta_1}) \frac{H - iA}{v}] U_R \\
&- \bar{D}_L[M^d(1+\frac{h}{v}) - (\lambda_1^D v_2 e^{i\theta_2} - \lambda_2^D v_1 e^{i\theta_1}) \frac{H + iA}{v}] D_R \\
&+ H.C.
\end{aligned}$$

Making making chiral rotations: $U_R \to e^{-i\theta_1\gamma_5} U_R$ and $D_R \to e^{i\theta_1\gamma_5} D_R$

Note that the net contribution to strong θ is zero: $\theta_u + \theta_d = \theta_1 - \theta_1 = 0$

Finally, we have

$$
\begin{aligned}
L_Y &= -\bar{D}_L(\lambda_1^U v_2 e^{-i\theta} - \lambda_2^U v_1)U_R\frac{h^-}{v} + \bar{U}_L(\lambda_1^D v_2 e^{i\theta} - \lambda_2^D v_1)D_R\frac{h^+}{v} \\
&- \bar{U}_L[M^u(1+\frac{h}{v}) - (\lambda_1^U v_2 e^{-i\theta} - \lambda_2^U v_1)\frac{H-iA}{v}]U_R \\
&- \bar{D}_L[M^d(1+\frac{h}{v}) - (\lambda_1^D v_2 e^{i\theta} - \lambda_2^D v_1)\frac{H+iA}{v}]D_R \\
&+ H.C.
\end{aligned}
$$

In this basis,

$$
\bar{U}_L M^u U_R = \bar{U}_L\frac{1}{\sqrt{2}}(\lambda_1^U v_1 + \lambda_2^U v_2 e^{-i\theta})U_R
$$

$$
\bar{D}_L M^d D_R = \bar{D}_L\frac{1}{\sqrt{2}}(\lambda_1^D v_1 + \lambda_2^D v_2 e^{i\theta})D_R
$$

Where are CP violation?

1. The $M^{u,d}$ are complex, rotating into real quark eigen-mass basis,

generate a $\theta - term = Arg(\Delta(M^u M^d))$.

2. Diagonalize quark mass matrices, generating complex V_{KM}.

3. h^+, H and A have complex couplings, new source of CP violation!

4. h, H and A are not mass eigen-state yet.

Mixing of h and H with A, new source of CP violation.

Similar analysis can be carried out.

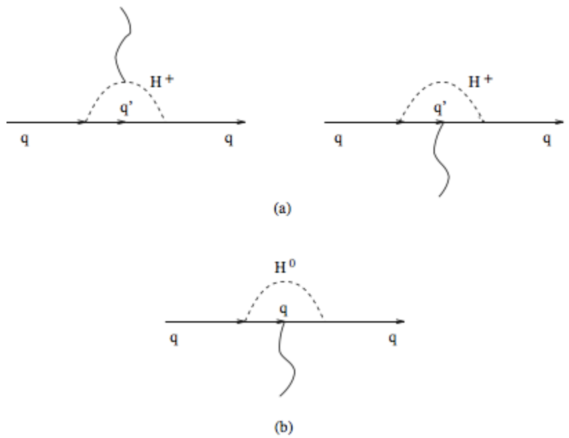
Consequence？ large fermion EDMs

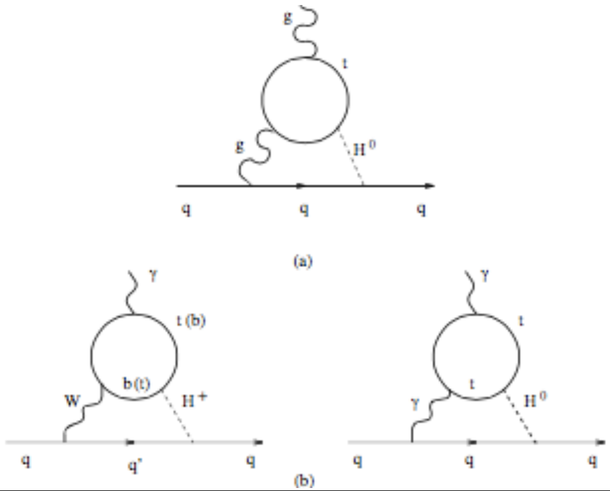
One loop
Review, He et al.
IJMPA, A4, 5011(1989)

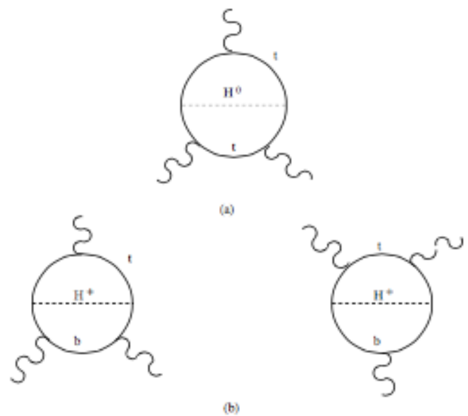
Weinberg operator
PRL63, 2333, (1989)
Braaten, C-S Li, T-C Yuan
PRL 64, 1709(1990)
Correct CD running

Barr-Zee, Gunion-Wyler
BZ, PRL 65, 21(1990)
GW, PLB 248, 170(1990)

Neutron and electron EDMs can be as large as experimental bounds.







Homework

Work out the masses (mass matrices) for h^+, h, H and A for the two Higgs doublet potential given in the lecture.

Theoretical Models for Neutrinos

In the minimal SM: Gauge group: $SU(3)_C \times SU(2)_L \times U(1)_Y$

$$
\begin{aligned}
&G(8,1)\ (0), W(1,3)\ (0), B\ (1,1)(0)\ ,\\
&Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix}\ (3,2)(1/6)\ ,\ \ U_R\ (3,1)(2/3)\ ,\ \ D_R\ (3,1)(-1/3)\ ,\\
&L_l = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}\ (1,2)(-1/2)\ ,\ \ E_R\ (1,1)(-1)\ ,\\
&H = \begin{pmatrix} h^+ \\ (v+h^0)/\sqrt{2} \end{pmatrix}\ (1,2,1/2)\ ,\ \ \text{v - vev of Higgs}\ .
\end{aligned}
$$

Quark and charged lepton masses are from the following Yukawa coupolings

$$\bar{Q}_L \tilde{H} U_R\ ,\ \ \bar{Q}_L H D_R\ ,\ \ \bar{L}_L H E_R\ .$$

Nothing to pair up with $L_L(\nu_L)$. In minimal SM, neutrinos are massless!

Extensions needed: Give neutrino masses and small ones!

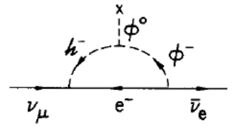
To have Dirac mass, need to introduce right handed neutrinos ν_R: (1,1)(0)

Dirac neutrino mass term

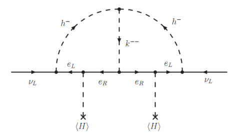
$$L = -\bar{L}_L Y_\nu \tilde{H} \nu_R + H.C\ , \to\ \ -\bar{\nu}_L m_\nu \nu_R\ , \to m_\nu = \frac{v}{\sqrt{2}} Y_\nu$$

$$m_{\nu_e} < 0.3\ \text{eV},\ \to\ \ Y_{\nu_e}/Y_e < 10^{-5},\ \text{very much fine tuned!}$$

Some theoretical models for neutrino masses



Loop generated neutrino masses:



The Zee Model(1980); Zee-Babu Model (zee 1980; Babu, 1988)
Other loop models: Babu&He; E. Ma; Mohapatra et al; Geng et al

Seesaw Models:

$$M_\nu = \begin{pmatrix} 0 & Y_\nu v/\sqrt{2} \\ Y_\nu^T v/\sqrt{2} & M_R \end{pmatrix}$$

$$m_\nu \approx Y_\nu^2 v^2/M_R \,, \qquad M_N \approx M_R$$

Type I Introduce singlet neutrinos

(Minkowski (1977); Gell-Mann, Ramond, and Slansky (1979); Yanagida (1979); Glashow (1980); Mohapatra and Senjanovic(1980))

$$L = \bar{\nu}_L (Y_\nu v/\sqrt{2}) \nu_R + \bar{\nu}_R^c M_R \nu_R/2$$

Type II: Introduce triplet Higgs representation $\Delta : (1,3,1)$, (W. Konetschny, and W. Kummer, 1977; L.-F. Li and T.-P. Cheng, 1980; Gelmini and Roncadelli, 1981)

Type-III: Introduce triplet lepton representations Σ: (1,3,0)) (Foot, Lew, He and Joshi, 1989).

Type I Seesaw model for neutrino mass

In Type I Seesaw model, there are three light and N heavy neutrinos,

The general mass term for neutrinos can be written as

$$L_M = -\bar{L}_L Y_e \tilde{H} E_R - \bar{L}_L Y_\nu H \nu_R - \frac{1}{2}(\bar{\nu}_L, \bar{\nu}_R'^c) M^\nu \begin{pmatrix} \nu_L^c \\ \nu_R' \end{pmatrix} + H.C.$$

The neutrino mass matrix is of the following form in the (ν_L^c, ν_R) basis

$$M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}, \quad M_D = \frac{Y_\nu v}{\sqrt{2}}$$

Approximate solution in the basis M_R is diagonalized $\hat{M}_N$

$$L \approx \bar{\nu}_L m_\nu \nu_L^c + \bar{\nu}_R^c \hat{M}_N \nu_R\,, \quad m_\nu = -M^D \frac{1}{\hat{M}_N} M_D^T\,,$$

$$m_\nu = V_{PMNS} \hat{m} V_{PMNS}^T\,, \;\; M_D = i V_{PMNS} \hat{m}^{1/2} O \hat{M}_N^{1/2}\,.$$

$$V_{PMNS} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix}$$

$$= \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \begin{pmatrix} e^{\imath\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

α_i are Majorana phases.

A non-zero δ phase, cause CPV in neutrino oscillation.

Also O is a matrix $3 \times N$ satisfying: $OO^T = I$.

It can be complex, new source for CPV!

Work in the basis that charged leptons are in mass eigen-states,

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} W^{\mu}[\bar{\ell}^m \gamma_\mu P_L U^{L*}_{3\times 3}\nu^m + \bar{\ell}^m \gamma_\mu P_L U^{L*}_{3\times N} N^m - \frac{g}{\sqrt{2}} W^{\mu} \bar{U}^m_i \gamma_\mu P_L V_{KM} D^m + H.C.$$

In the heavy M_N limit, $U^{L*}_{3\times 3}$ is almost unitary and it is the V_{PMNS} matrix

$$V_{PMNS} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix}$$

$$= \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \begin{pmatrix} e^{\imath\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

α_i are Majorana phases.

New CP violating phases are generated in the lepton sector.

Solve the mass matrix more carefully

M_ν is a symmetric matrix which can be diagonalized

$$U^T M_\nu U = \hat{M}_\nu,$$

U a unitariy matrix.

$\hat{M}_\nu = diag(m_1, m_2, m_3, M_4, \cdots, M_{3+N}),$

m_i and M_i are the light ν^m and heavy N^m mass eigenvalues respectively.

U is a $(3+N)\times(3+N)$ matrix, write in two block $3\times(3+N)$ matrices

$$U = \begin{pmatrix} U^L \\ U^R \end{pmatrix},$$

N should be equal or larger than 2 to fit neutrino data, $\Delta m^2_{21,32} \neq 0$.

$$U^L = \begin{pmatrix} U^L_{e1} & U^L_{e2} & U^L_{e3} & U^L_{e4} & \cdots & U^L_{e3+N} \\ U^L_{\mu1} & U^L_{\mu2} & U^L_{\mu3} & U^L_{\mu4} & \cdots & U^L_{\mu3+N} \\ U^L_{\tau1} & U^L_{\tau2} & U^L_{\tau3} & U^L_{\tau4} & \cdots & U^L_{\tau3+N} \end{pmatrix} = \begin{pmatrix} U^L_{3\times3} & U^L_{3\times N} \end{pmatrix} ,$$

$$U^R = \begin{pmatrix} U^R_{e1} & U^R_{e2} & U^R_{e3} & U^R_{e4} & \cdots & U^R_{e3+N} \\ U^R_{\mu1} & U^R_{\mu2} & U^R_{\mu3} & U^R_{\mu4} & \cdots & U^R_{\mu3+N} \\ U^L_{\tau1} & U^R_{\tau2} & U^R_{\tau3} & U^R_{\tau4} & \cdots & U^R_{\tau3+N} \end{pmatrix} = \begin{pmatrix} U^R_{N\times3} & U^R_{N\times N} \end{pmatrix} .$$

Work in the basis that charged leptons are in mass eigen-states,

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} W^\mu [\bar{\ell}^m \gamma_\mu P_L U^{L*}_{3\times3} \nu^m + \bar{\ell}^m \gamma_\mu P_L U^{L*}_{3\times N} N^m - \frac{g}{\sqrt{2}} W^\mu \bar{U}^m_i \gamma_\mu P_L V_{KM} D^m + H.C.$$

In the heavy M_N limit, $U^{L*}_{3\times3}$ is almost unitary and it is the V_{PMNS} matrix

New CP violating phases are generated in the lepton sector from U.

Also W and l interact with heavy neutrino are related to $U^L_{3\times N}$.

New CPV source for $N^m \to Wl$ decay can happen.

Will U^R show up some where? Yes.

$$L_\nu = -\bar{\nu} M_D \nu_R (1 + \frac{h}{v}) \to (\bar{\nu}^m, \bar{N}^m)\tilde{M} \begin{pmatrix} \nu^m \\ N^m \end{pmatrix} (1 + \frac{h}{v})$$

$$\tilde{M} = \begin{pmatrix} (U^L)^T_{3\times 3} U^{L*}_{3\times 3} & (U^L)^T_{3\times 3} U^{L*}_{3\times N} \\ (U^L)^T_{3\times N} U^{L*}_{3\times 3} & (U^L)^T_{3\times N} U^{L*}_{3\times N} \end{pmatrix} \begin{pmatrix} \hat{m} & 0 \\ 0 & \hat{M} \end{pmatrix} \begin{pmatrix} (U^R)^\dagger_{3\times 3} U^R_{3\times 3} & (U^R)^\dagger_{3\times 3} U^R_{N\times N} \\ (U^R)^\dagger_{3\times N} U^R_{3\times 3} & (U^R)^\dagger_{N\times N} U^R_{N\times N} \end{pmatrix}$$

New CPV source for $N^m \to \nu^m h$.

Seesaw model, originally, proposed to explain why neutrino masses are so much smaller than their charged partners, e, μ, τ. Later pepole found that this model also provide a natural solution to the baryon asymmetry of our universe.

7. Baryon Asymmetry of Our Universe

Thermal History of Our Universe

In early universe, all energy forms existed in form of elementary particles, or ….
Temperature is high and were in thermal equilibrium

Criteria for thermal equilibrium: particle interaction length $1/\Gamma$ (Γ interaction rate) is smaller than Hubble length $1/H_0$

?beginning?Planck mass $T \sim 10^{19}$GeV

Inflation

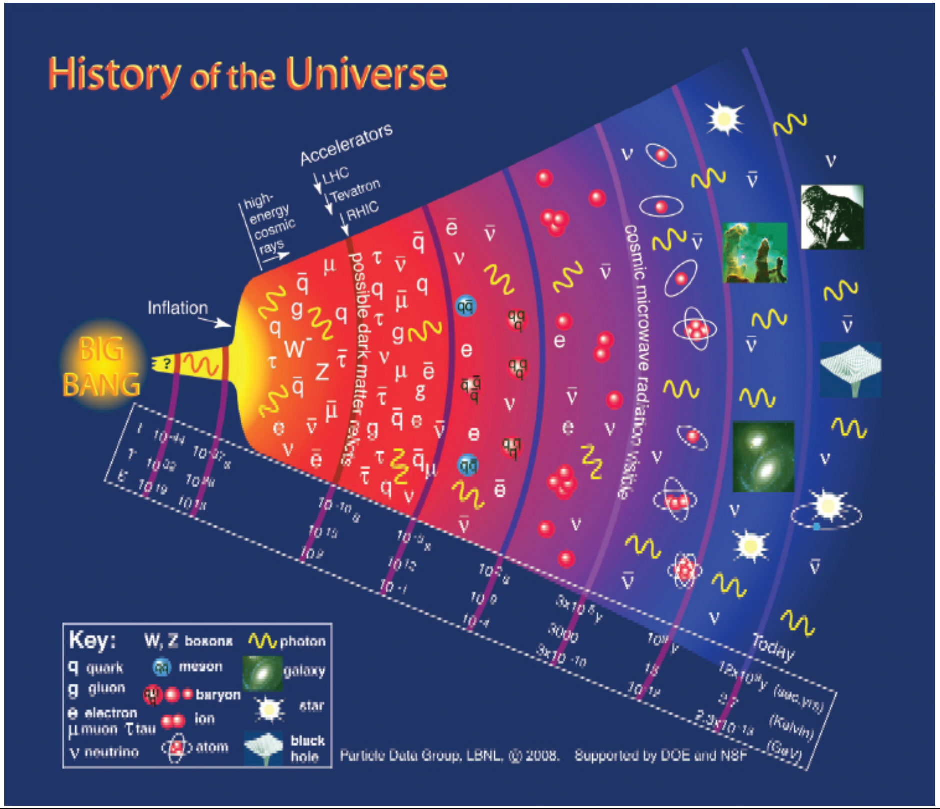
Big Bang ~ T> 10^{16} GeV(Not in thermal Equilibrium by SM for particle physics)

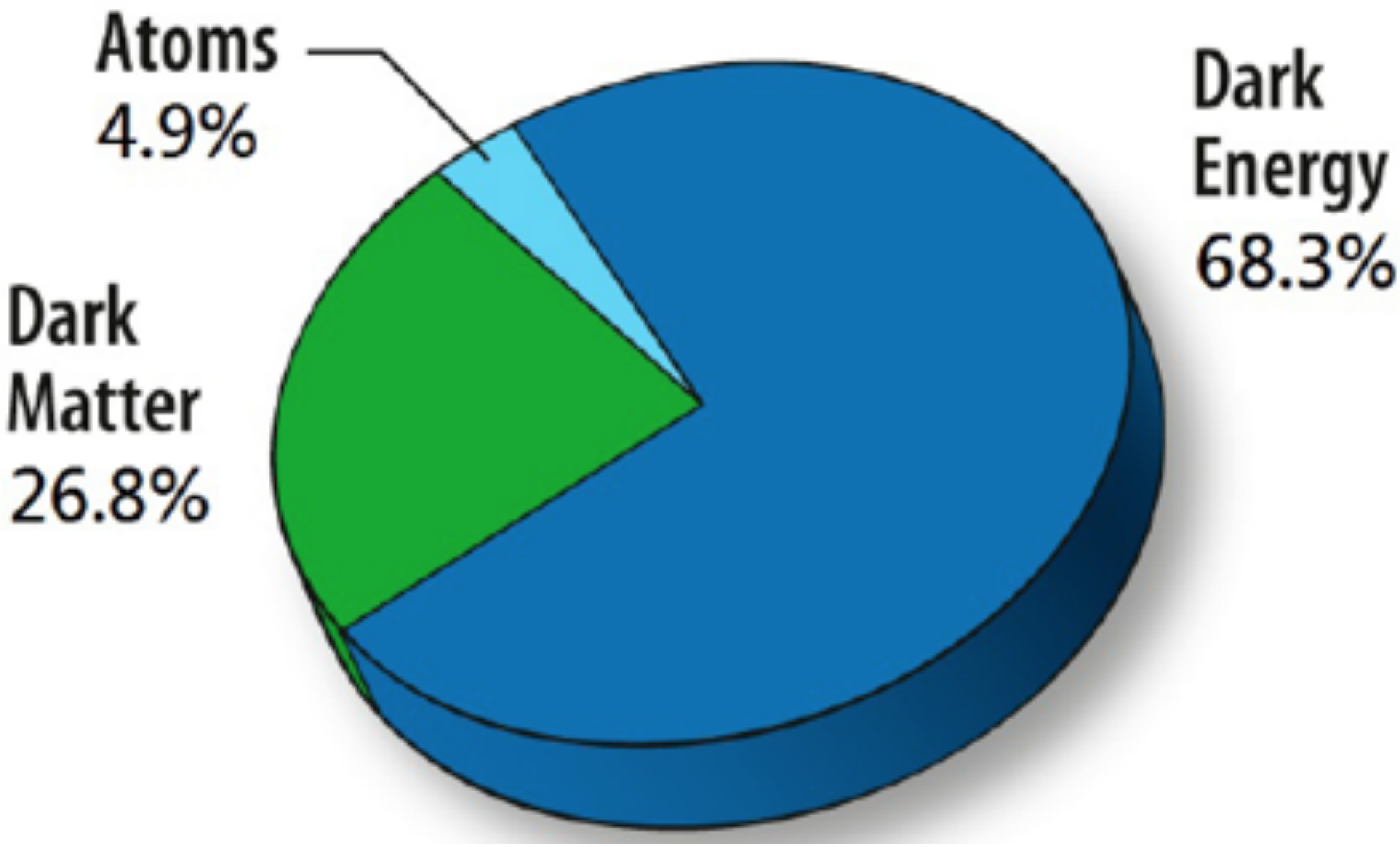
Grand Unification ~ 10^{16} GeV
EW symmetry breaking 300GeV

Color confinement ~300 MeV ->Decoupling of baryons

BBN ~ 1 MeV -> CMB ~0.3 eV

Large structure formation -> Today ~2.7K




Atoms
4.9%
Dark
Energy
68.3%
Dark
Matter
26.8%

7. Baryon Asymmetry of Our Universe

In our Universe, matter dominates over anti-matter

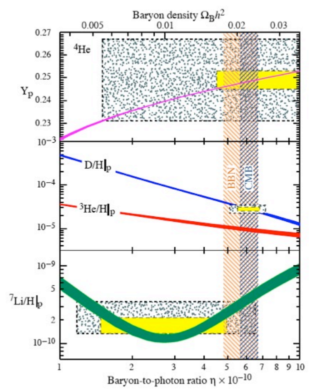
- Why this is so is the problem of Baryon Asymmetry of our Universe (BAU)

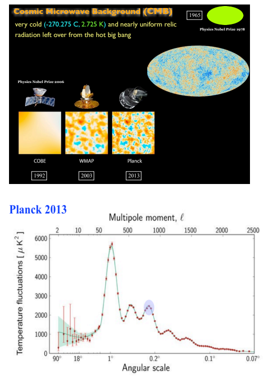
In cosmological terms, the problem is as follows

If initially, the universe is matter and anti-matter symmetric

$n_B/n_\gamma = n_{\bar{B}})/n_\gamma \sim 10^{-20}$

$n_B(n_{\bar{B}})$ - baryon (anti-baryon) number density, n_γ - photon number density





However observation, BBN and CMB, show that

$\eta = (n_B - n_{\bar{B}})/n_\gamma \sim 6 \times 10^{-10}$

There is a 10^{10} order of magnitude difference.

Initially, there is a baryon asymmetry?

But inflation will dilute any asymmetry to zero.

Possible to generate a η which fits observtion

from an, initially, matter anti-matter symmetry universe?

Possible, but certain conditions need to be satisfied.

The Sakharov Conditions for Baryogenesis (1967)

Baryon Number B Violation
C and CP Violation
Interactions Out Of Thermal Equilibrium

The need of baryon number violation is obvious, without this, B will not change. If start with total baryon number = 0, it will remain.

C changes Γ(X -> q…) to Γ(anit-X -> anti-q…), if C is conserved, X and anti-X decay will counterbalanced. C needs to be violated.

CP changes Γ(X -> $q_{(L,R)}$…) to Γ(anit-X -> anti-$q_{(L,R)}$…), if CP conserved, again no net baryon generated.

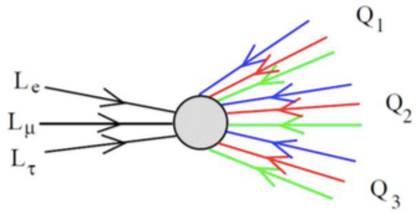
$$n(T, m_B) = \frac{g}{(2\pi)^3} \int_0^\infty f(T, p, E = \sqrt{p^2 + m_B^2}) d^3p\,, \quad \bar{n}(\bar{T}, m_{\bar{B}}) = \frac{g}{(2\pi)^3} \int_0^\infty f\bar{T}, p, E = \sqrt{(}p^2 + m_{\bar{B}}^2) d^3p$$

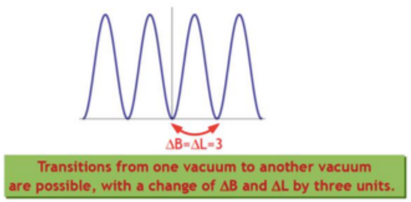
$$f(E, T) = \frac{1}{e^{E/T} \pm 1} \quad \begin{cases} + & \text{fermion} \\ - & \text{boson} \end{cases}$$

CPT theorem: $m_B = m_{\bar{B}} = m$, if in thermal equallibrium $T = \bar{T} \to n(T, m) = \bar{n}(T, m)$!

Standard Model has allingradients

Baryon number violation: Sphelaron effects-tunneling effects from different vacuum states with non-zero baryon number differences. Violated B+L, but conserves B-L.





C and CP violation: Electroweak interaction violates C, and phase in Kobayashi-Maskawa mixing matrix violates CP.

Out of thermal equilibrium: Electroweak symmetry breaking

But, CP violation rate too small, out of thermal equilibrium too weak. Not enough to generate a large enough Baryon Asymmetry.

If Higgs mass is less than 70 GeV, second order phase transition at electroweak symmetry breaking, too weak.

$\eta \sim 10-20$ Too small. Needs to go beyond SM!

Electroweak baryogenesis, Leptogenesis, Gut baryogenesis….

Leptogenesis

Fukugita and Yanagida, PLB174, 45(1986)

Translate lepton number asymmetry generated in the early universe to baryon number asymmetry!

Requires lepton asymmetry generated before Sphelaron effects to be in effective (T ~ 10^{12} – a few TeV). Initial $a_L(i)=a$, $a_B(i)=0$.

Sphelaron effect: Conserve B-L, but violates B+L

After: $a_L(i)+ a_B(i) = a_L(f)+a_B(f) = 0$, $a_L(i) - a_B(i) = a_L(f) - a_B(f)$

$$a_L(f) = a/2; \quad a_B(f) = -a/2$$

half of initial lepton asymmetry will be translated into baryon asymmetry if complete.

SM Sphelaron effect: $a_B = -(28/79)a_L$

Seesaw model plays the right role

$$L_M = -\bar{L}_L Y_e \tilde{H} E_R - \bar{L}_L Y_\nu H \nu_R - \frac{1}{2}(\bar{\nu}_L, \bar{\nu}_R^{\prime c}) M^\nu \begin{pmatrix} \nu_L^{c} \\ \nu_R^{\prime} \end{pmatrix} + H.C.$$

The last term violates lepton number L by two units!

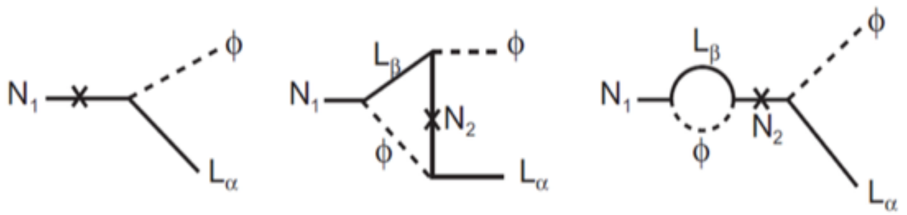
Out of thermal equilibrium decay, new CP violation in N -> L h(ϕ)

N decays into L and anti-L differently

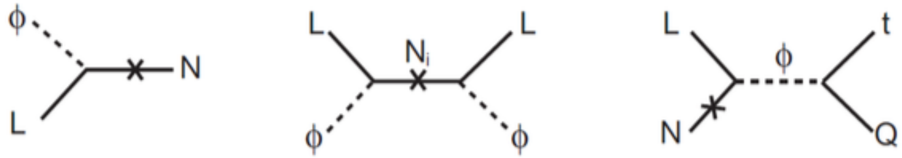
$$a_i = \frac{\Gamma(N_i \to L\phi) - \bar{\Gamma}(N_i \to \bar{L}\phi)}{\Gamma(N_i \to L\phi) + \bar{\Gamma}(N_i \to \bar{L}\phi)}$$

$$a_i \approx -\frac{1}{8\pi} \frac{1}{[\hat{Y}_\nu \hat{Y}_\nu^\dagger]_{ii}} \sum_j Im\{[\hat{Y}_\nu \hat{Y}_\nu^\dagger]_{ij}^2\} f\left(\frac{M_j^2}{M_i^2}\right)$$

$$f(x) = \sqrt{x}\left(\frac{2}{x-1} + \ln\frac{1+x}{x}\right).$$



Needs to consider washout effects due to L conserving interactions in a expanding universe



Washout processes: inverse decays, $\Delta L = 2$ scattering, and $\Delta L = 1$ scattering.

$$\eta_B = \frac{s}{n_\gamma}\frac{28}{79}a_l\frac{\kappa_l}{g_{*l}}\ ,$$ g_{*l} effective degrees, in Type I Seesaw, ~ 100

$$s = \frac{2\pi^2}{45}g_{*l}|_o T^3|_o\ ,\quad n_\gamma = 2\pi^2\xi(3)T^3|_o\ .$$

$g_{*l}|_o = 43/11$ is the present effective degrees of freedom.

Nielsen and Takanishi, PLB507, 241(2001)

$\kappa_l \approx 0.3/K_l(lnK_l)^3/5$ with
$K_l = \Gamma_l(\text{interaction rate})/H_l(\text{Hubber constant})1 \sim 10^6$

M_N and mν masses are correlated to obtain the right number for η, mν of order 0.0.5 eV, M_N ~ 1012 – 1015 GeV.

Seesaw model is a viable model for Baryon Asymmetry of our Universe

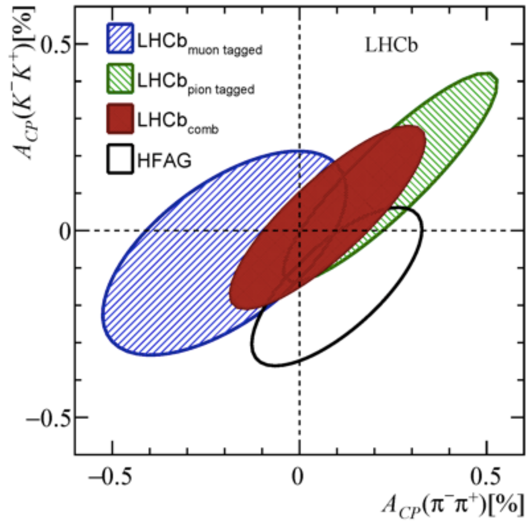
8. A Final comment: CPT violation?

CPT sum rule in time dependent B decays

Measurement of time-dependent CP-violating asymmetries in $B^0 \to \pi^+\pi^-$ and $B_s^0 \to K^+K^-$ decays at LHCb

LHCb-CONF-2016-018

$$
\begin{aligned}
C_{\pi^+\pi^-} &= -0.243 \pm 0.069, \\
S_{\pi^+\pi^-} &= -0.681 \pm 0.060, \\
C_{K^+K^-} &= 0.236 \pm 0.062, \\
S_{K^+K^-} &= 0.216 \pm 0.062, \\
A^{\Delta\Gamma}_{K^+K^-} &= -0.751 \pm 0.075.
\end{aligned}
$$



CPT sum rule $$|\mathcal{C}_f|^2 + |\mathcal{S}_f|^2 + |\mathcal{A}_f^{\Delta\Gamma}|^2 = 1$$

LHCb data $$|\mathcal{C}_{K^+K^-}|^2 + |\mathcal{S}_{K^+K^-}|^2 + |\mathcal{A}^{\Delta\Gamma}_{K^+K^-}|^2 = 0.67 \pm 0.20$$

Sum rule violated!

Further tests of CPT sum rule

CPT symmetry tested to great precision.
Not attempt to build a theoretical model to explain violation of CPT rum rule.

If there is a mixing with some other sector (or sectors) with the correct quantum numbers, the sum rule may change.
Do not have a good candidate to choose from because the mass of the candidate system should have a mass very close to B_s^L and B_s^H.

Further tests time-dependent CP violation $B_s^0 \to K^0 \bar{K}^0$, $B_s^0 \to \pi^+\pi^-$, $B_s^0 \to \pi^0\pi^0$

Model independent SU(3) global fitting on the other hand give

$$\mathcal{C}_{\pi^+\pi^-}(B_s) = \left(16.1^{+1.9}_{-1.6}\right)\%,\ \mathcal{S}_{\pi^+\pi^-}(B_s) = \left(-2.3^{+2.5}_{-2.4}\right)\%,\ \mathcal{A}^{\Delta\Gamma}_{\pi^+\pi^-}(B_s) = \left(-98.6^{+0.4}_{-0.2}\right)\%$$

$$\mathcal{C}_{K^0\bar{K}^0}(B_s) = \left(-0.9^{+0.5}_{-0.5}\right)\%,\ \mathcal{S}_{K^0\bar{K}^0}(B_s) = \left(-3.5^{+0.5}_{-0.5}\right)\%,\ \mathcal{A}^{\Delta\Gamma}_{K^0\bar{K}^0}(B_s) = \left(-99.9^{+0.0}_{-0.0}\right)\%$$

$B_s^0 \to \pi^0\pi^0$ similar to $B_s^0 \to \pi^+\pi^-$, may be harder to measure.

Thank you all for listen to my lectures