MODULE- 3
DC Machines & Transformers

DC GENERATORS

Any electrical machine that converts mechanical energy to electrical energy is called as Generators. The electrical machine that converts electrical energy to mechanical energy is called as motor. They work on the principle of electromechanical energy conversion. However, the construction of both motors and generators are same, they differ by the principle of operation.

The first DC electrical machine was invented in 1839 in Edinburgh but took 4 decades to be commercialized.

Construction of DC machine

The construction of the motor parts can be broadly classified into two: stator and rotor. Stator is the stationary part and rotor is rotating. The stator parts include: Base Plate, yoke, field system, lifting eye and terminal box. The rotor parts include: armature. Commutator and brushes is shown in Fig 4.1.

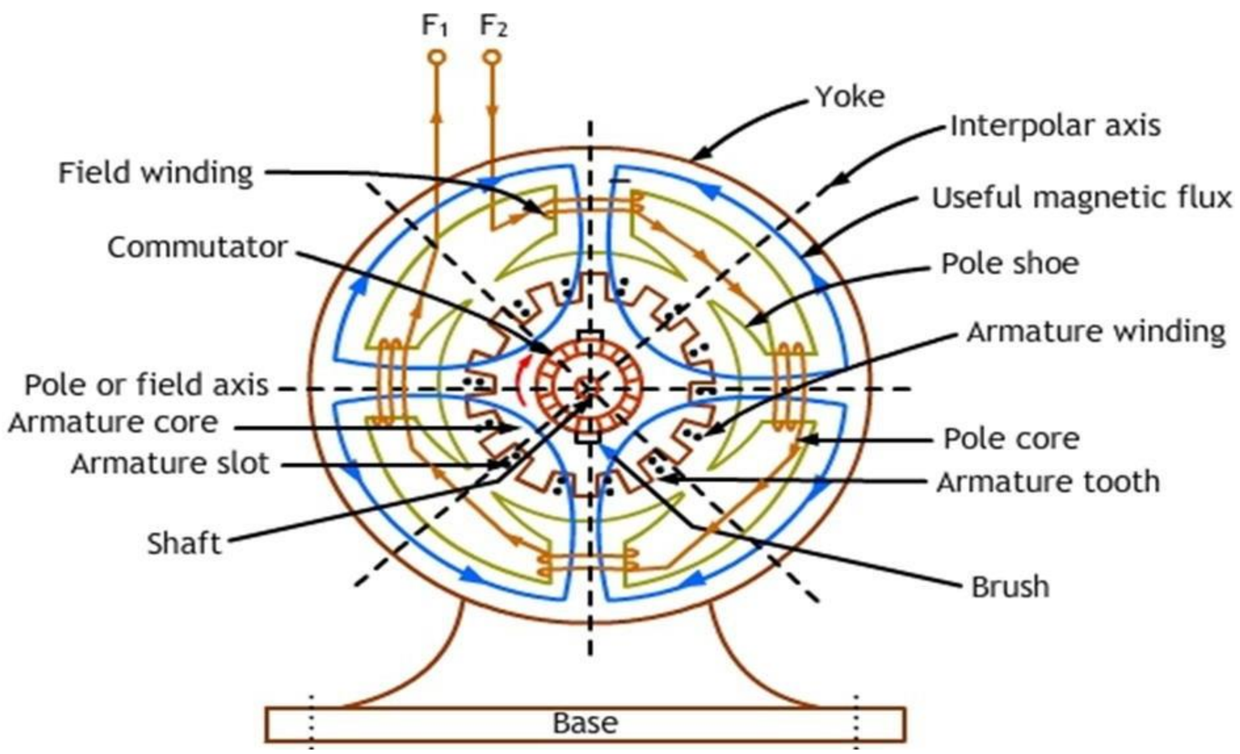


Fig: 4.1. Parts of DC Machine

Yoke: yoke is the outermost part of the machine which is made of either cast iron or cast steel depending on the application. For instance, for small applications such as toys, fans etc, it is made of cast iron. For bigger application such as industrial and generating stations the yoke is made up of cast steel. The outer frame or yoke serves double purpose: (i) It provides mechanical support for the poles and acts as a protecting cover for the whole machine. (ii) It carries the magnetic flux produced by the poles. Base plate is to give the support for the entire system to stand.

Pole Cores and Pole Shoes: The field system consists of pole shoe and pole field. The poles are made up of alloy steel that has high permeability and less hysteresis loss. The coil is wound on the pole which is excited to provide the magnetic field. The poles are tightly reverted to the yoke using nuts and bolts. The pole shoes serve two purposes: (i) They spread out the flux in the air gap and also, being of larger cross-section, reduce the reluctance of the magnetic path. (ii) They support the exciting coils (or field coils) as shown below. There will be small ducts provided along the pole shoe that keeps the air circulation intact.

Pole Coils: The field coils or pole coils, which consist of copper wire or strip, are former-wound for the correct dimension. Then, the former is removed and wound coil is put into place over the core. When current is passed through these coils, they electro magnetize the poles which produce the necessary flux that is cut by revolving armature conductors.

Armature Core: It houses the armature conductors or coils and causes them to rotate and hence cut the magnetic flux of the field magnets. In addition to this, its most important function is to provide a path of very low reluctance to the flux through the armature from a N-pole to a S-pole. It is cylindrical or drum-shaped and is built up of usually circular sheet steel discs or laminations approximately 0.5 mm thick. The slots are either die-cut or punched on the outer periphery of the disc and the keyway is located on the inner diameter as shown. In small machines, the armature stampings are keyed directly to the shaft. Usually, these laminations are perforated for air ducts which permit axial flow of air through the armature for cooling purposes. The purpose of using laminations is to reduce the loss due to eddy currents. Thinner the laminations, greater is the resistance offered to the induced emf, smaller the current and hence lesser the $I^2 R$ loss in the core.

Armature Windings: The armature windings are usually former-wound. These are first wound in the form of flat rectangular coils and are then pulled into their proper shape in a coil puller. Various conductors of the coils are insulated from each other. The conductors are placed in the armature slots which are lined with tough insulating material. This slot insulation is folded over above the armature conductors placed in the slot and is secured in place by special hard wooden or fiber wedges.

Commutator: The functions of the commutator are to facilitate collection of current from the armature conductors, and to convert the alternating current induced in the armature conductors into unidirectional current in the external load circuit. It is of cylindrical structure and is built up of wedge-shaped segments of high-conductivity hard-drawn or drop forged copper. These segments are insulated from each other by thin layers of mica. The number of segments is equal to the number of armature coils. Each commutator segment is connected to the armature conductor by means of a copper lug or riser. To prevent them from flying out under the action of centrifugal forces, the segments have V-grooves, these grooves being insulated by conical micanite rings.

Brushes and Bearings: The brushes, whose function is to collect current from commutator, are usually made of carbon or graphite and are in the shape of a rectangular block. These brushes are housed in brush-holders, the brush-holder is mounted on a spindle and the brushes can slide in the rectangular box open at both ends. The brushes are made to bear down on the commutator by a spring. A flexible copper pigtail mounted at the top of the brush conveys current from the brushes to the holder. The number of brushes per spindle depends on the magnitude of the current to be collected from the commutator.

Because of their reliability, ball-bearings are frequently employed, though for heavy duties, roller bearings are preferable. The ball and rollers are generally packed in hard oil for quieter operation and for reduced bearing wear, sleeve bearings are used which are lubricated by ring oilers fed from oil reservoir in the bearing bracket.

Basic principle of operation of D.C machine as a generator

Generator works on the principle of Faraday's laws of electromagnetic induction and the type of emf is dynamically induced emf. When a conductor cuts the magnetic flux lines of flux an emf is induced in the conductor. The magnitude of the emf induced in the conductor is given by:

$$E = Blv\sin\theta$$

Where B: magnetic flux density in wb/m^2

l : length of the portion of the conductor in the magnetic field in m

v : velocity of the conductor in m/s

θ : Angle between direction of movement of the conductor in the magnetic field and the direction of magnetic flux

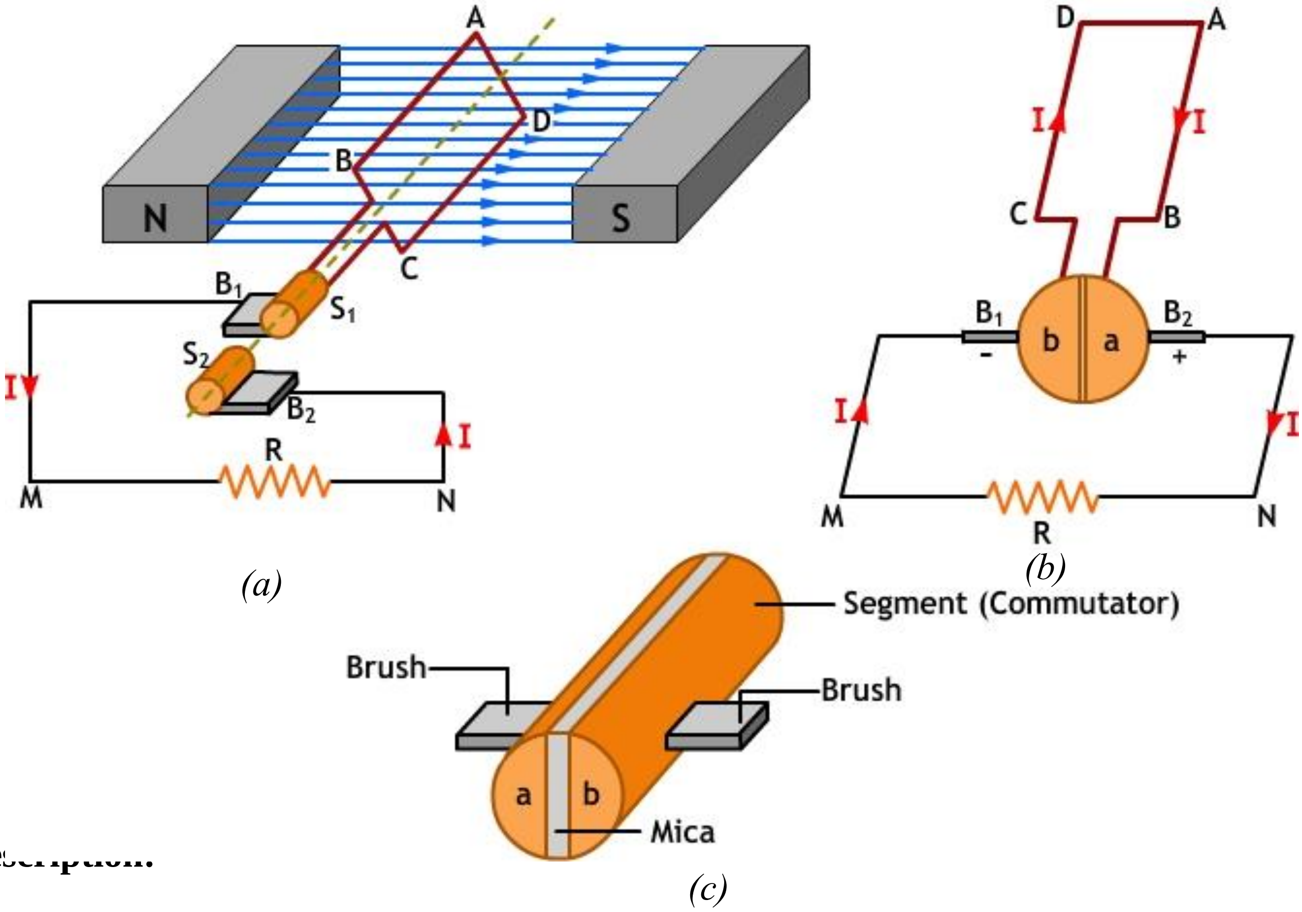


Fig 4.2: Generator working principle

Description:

The working principle of a generator can be better understood by simple concept of faradays laws. Consider two conductors AB and CD which are tied together in the back end and to the separate slip rings S1 and S2 in the front end as shown in the Fig 4.2(a). the conductors ABCD as a whole forms one coil. Let the coil be rotated in the counterclockwise direction such that AB rotates under the influence of north pole and CD under south pole and the current flows from M to N through the resistor as shown. Similarly, after rotation of 180^0 the conductor Cd will be under the influence of north pole and AB under the influence of south pole thus from 180^0 to 360^0 rotation the emf induced in the conductor gets reversed and the current flows from N to M through resistor R as shown in Fig 4.2(b). The type of emf generated will be alternating.

In view of obtaining DC voltage, split ring can be replaced with slip rings as depicted in Fig 4.2(c). In the split rings there are two segments a and b which are separated by a insulting medium and the brushes are placed on these segments. When AB is under the influence of south pole and CD under north pole the brushes just slide through the split rings to maintain the direction of current flow from M to N only.

Fleming's right-hand rule (for generators): - shows the direction of induced emf (current) when a conductor moves in a magnetic field.

The right hand is held with the thumb, first finger and second finger mutually perpendicular to each other (at right angles)

- The Thumb represents the direction of Motion of the conductor
- First finger represents the direction of the Field or Flux. (north to south)
- The Second finger represents the direction of the induced or generated Current (the direction of the induced current or emf will be the direction of conventional current; from positive to negative).

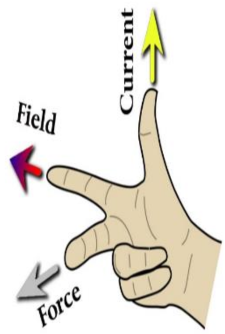


Fig 4.3: Flemings Right hand rule

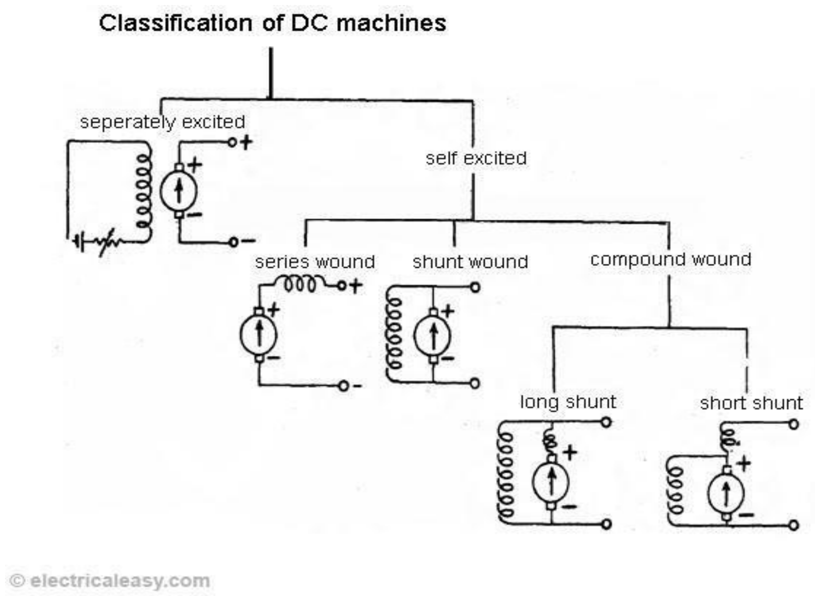
Classification of Generators: -

Generators are usually classified according to the way in which their fields are excited. The field windings provide the excitation necessary to set up the magnetic fields in the machine. There are various types of field windings that can be used in the generator or motor circuit.

In addition to the following field winding types, permanent magnet fields are used on some smaller DC products. Generators may be divided in to

(a) Separately-excited generators and

(b) Self-excited generators



(a) **Separately-excited generators** are those whose field magnets are energized from an independent external source of DC current

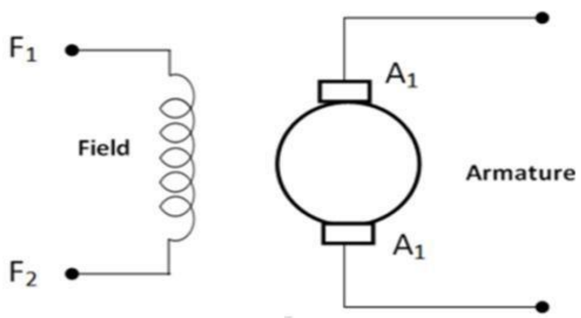


Fig 4.4: Separately-excited generator

Armature current $I_a = I_L$\

Terminal voltage $V = E_g - I_a R_a$ volts

Power developed $P = E_g I_a$ watts

Power delivered to the load $=E_gI_a-I_a^2R_a=I_a(E_g-I_aR_a)=VI_a$ watt

(b) **Self-excited generators** are those whose field magnets are energized by the current produced by the generators themselves. Due to residual magnetism, there is always present some flux in the poles. When the armature is rotated, some emf and hence some induced current is produced which is partly or fully passed through the field coils thereby strengthening the residual pole flux.

Self-excited generators are classed according to the type of field connection they use.

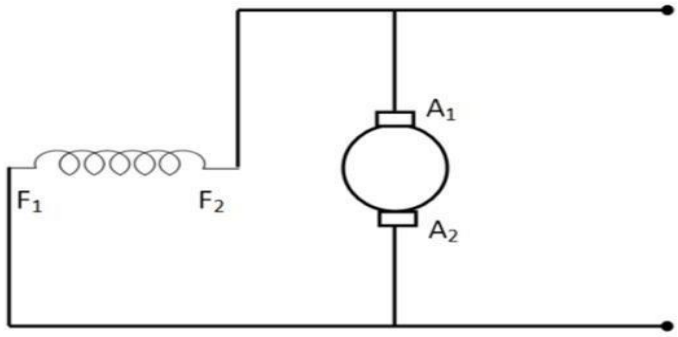


Fig 4.5: Self-excited generators

There are three general types of field connections:

(a) Series-Wound,

(b) Shunt- Wound (parallel),

(c) Compound-wound

Compound-wound generators are further classified as cumulative-compound and differential-compound.

Series-wound generator: - In the series-wound generator, shown in Fig 4.6, the field windings are connected in series with the armature. Current that flows in the armature flows through the external circuit and through the field windings. The external circuit connected to the generator is called load circuit.

A series-wound generator uses very low resistance field coils, which consist of a few turns of large diameter wire.

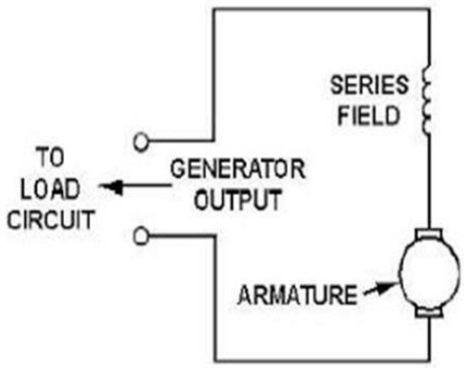


Fig 4.6: Series-wound generator

The voltage output increases as the load circuit starts drawing more current. Under low-load current conditions, the current that flows in the load and through the generator is small. Since small current means that a small magnetic field is set up by the field poles, only a small voltage is induced in the armature. If the resistance of the load decreases, the load current increases. Under this condition, more current flows through the field. This increases the magnetic field and increases the output voltage. A series-wound dc generator has the characteristic that the output voltage varies with load current. This is undesirable in most applications. For this reason, this type of generator is rarely used in everyday practice.

Armature current $Ia=Ise=I_L=I$

Terminal voltage $V=Eg-I(Ra+Rse)$

Power developed $P=EgIa$

Power delivered to the load= $EgIa-I^2(Ra+Rse)=I[Eg-I(Ra+Rse)]=VI$

Shunt wound: - In this field winding is connected in parallel with the armature conductors and have the full voltage of the generator applied across them. The field coils consist of many turns of small wire. They are connected in parallel with the load. In other words, they are connected across the output voltage of the armature.

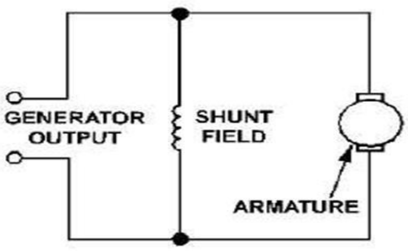


Fig 4.7: Shunt wound

Current in the field windings of a shunt-wound generator is independent of the load current (currents in parallel branches are independent of each other). Since field current, and therefore field strength, is not affected by load current, the output voltage remains more nearly constant than does the output voltage of the series-wound generator.

In actual use, the output voltage in a dc shunt-wound generator varies inversely as load current varies. The output voltage decreases as load current increases because the voltage drop across the armature resistance increases (E = IR).

In a series-wound generator, output voltage varies directly with load current. In the shunt-wound generator, output voltage varies inversely with load current. A combination of the two types can overcome the disadvantages of both. This combination of windings is called the compound-wound dc generator.

- Armature current $I_a=I_L+I_{sh}$
- Shunt field current $I_{sh}=(V/R_{sh})$
- Terminal voltage $V=I_{sh}\ R_{sh}$
- Power delivered $P=E_g\ I_a$
- Power given to the load $=VI_L$

Compound-wound generator: -

Compound-wound generators have a series-field winding in addition to a shunt-field winding, as shown in Fig 4.8. The shunt and series windings are wound on the same pole pieces. They can be either short-shunt or long shunt as shown in Fig 4.8. In a compound generator, the shunt field is stronger than the series field. When series field aids the shunt field, generator is said to be commutatively-compounded. On the other hand if series field opposes the shunt field, the generator is said to be differentially compounded.

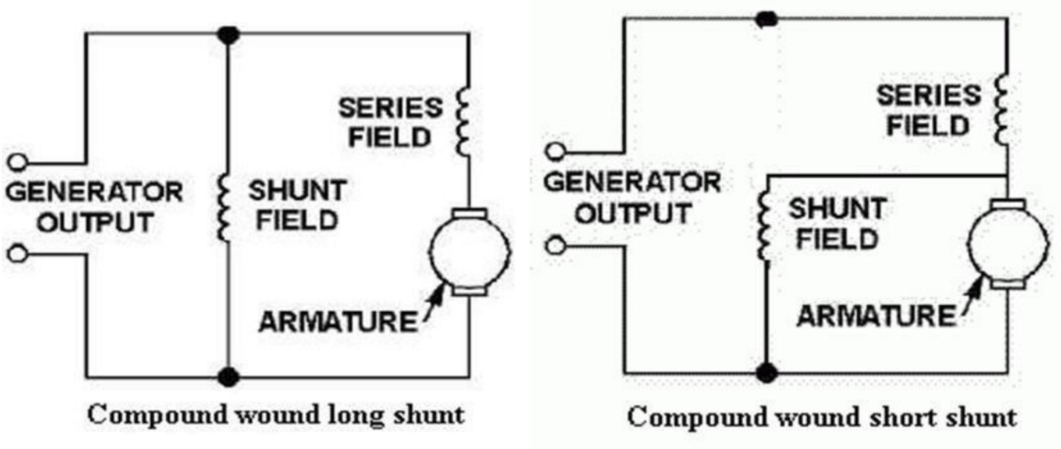


Fig 4.8: Differential and compound generator

In the compound-wound generator when load current increases, the armature voltage decreases just as in the shunt-wound generator. This causes the voltage applied to the shunt-field winding to decrease, which results in a decrease in the magnetic field. This same increase in load current, since it flows through the series winding, causes an increase in the magnetic field produced by that winding.

By proportioning the two fields so that the decrease in the shunt field is just compensated by the increase in the series field, the output voltage remains constant. This is shown in Fig, which shows the voltage characteristics of the series-, shunt-,and compound-wound generators. As you can see, by proportioning the effects of the two fields (series and shunt), a compound-wound generator provides a constant output voltage under varying load conditions. Actual curves are seldom, if ever, as perfect as shown.

Short shunt compound wound generator: -

- Series field current $Ise=I_L$
- Shunt field current $I_{sh}=(V+I_{se}\ R_{se})/R_{sh}$
- Terminal voltage $V=E_g-I_a\ Ra-I_{se}\ R_{se}$
- Power delivered $P=E_g\ I_g$
- Power given to the load $=VI_L$

Long shunt compound wound generator:-

- Series field current $I_{se}=I_L=I_a=I_{sh}+I_L$
- Shunt field current $I_{sh}=(V/R_{sh})$

- Terminal voltage $V=E_g-I_a(R_a+R_{se})$
- Power delivered $P=E_g\ I_g$
- Power given to the load $=VI_L$

E.M.F Equation of DC Generator:

Let, f = Flux / pole in webers Change in flux d ϕ = P ϕ webers

Z = Total number armature conductors

= Number of slots x Number of conductors per slot

P = Number of poles

A = Number of parallel paths in the armature.

N = Rotational speed of armature in revolutions per minute (r.p.m)

Time taken to complete one revolution = 60/N sec.

E = e.m.f induced / parallel path in armature.

By Faradays law

$$\text{E.M.F generated per conductor} = \frac{d\phi}{dt} = \frac{\phi PN}{60 \text{ volts}}$$

$$\text{Number of armature conductors per parallel path} = \frac{Z}{A}$$

$$E_g = \text{e.m.f generated per conductor} \times \text{Number of conductors ineach parallel path}$$

$$E_g = \left(\frac{\phi PN}{60}\right) \times \frac{Z}{A} \text{ volts} \qquad \ldots\ldots\ldots(i)$$

For a Simplex Wave-Wound Generator

Number of parallel paths A=2

$$E_g = \frac{\phi PN.\left(\frac{Z}{2}\right)}{60} = \frac{\phi ZPN}{120} \text{volts}$$

For Simplex Lap-Wound Generator:

Number of parallel paths, A = P

Equation (i) becomes

$$E_g = \frac{\phi PN.\left(\frac{Z}{P}\right)}{60} = \frac{\phi ZN}{60} \text{volts}$$

DC MOTOR

Whenever a current carrying conductor is placed in a magnetic field it experiences a force and the force is given by:

$$F = BIl$$

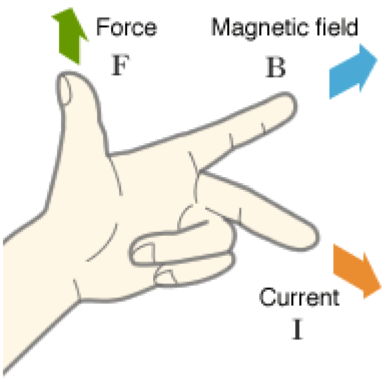
Where F: Force experienced in Newtons

B: Flux Density of magnetic field in Wb/m^2

I: current flowing through the conductor in amperes

l: length of the conductors in meters

By Fleming's left-hand rule: -It states that "when the thumb, fore finger and middle finger are held mutually perpendicular to each other, with the fore finger in the direction of magnetic field, middle finger in the direction of the current, then the direction of thumb indicates the direction of force experienced by the conductor".



Types of DC Motors: -Separately Excited DC Motor: - As the name suggests, in case of a separately excited DC motor the supply is given separately to the field and armature windings. The main distinguishing fact in these types of dc motor is that, the armature current does not flow through the field windings, the field winding is energized from a separate external source.

From the torque equation of dc motor we know Tg = Ka φ Ia So the torque in this case can be varied by varying field flux φ, independent of the armature current Ia.

Self-Excited DC Motor:-In case of self-excited dc motor, the field winding is connected either in series or in parallel or partly in series, partly in parallel to the armature winding, and on this basis its further classified as

(1) DC Shunt Motor

(2) DC Series Motor

(3) DC Compound Motor

(i) Cumulative Compound Motor

(a) Long shunt

(b) Short shunt

(ii) Differential Compound Motor

(a) Long shunt

(b) Short shunt

DC Shunt Motor: -In this type of motor, the field winding is connected in parallel with armature as shown in Fig4.9 (a). There are as many number of field coils as there are poles. When connected to supply, constant voltage appears across the field windings (as they are connected in parallel with armature). The field current is therefore constant and is independent of the load current.Shunt field winding usually are designed to have large number of turns of fine wire. Its resistance, therefore, is high enough to limit the shunt field current to about 1 to 4 percent of the rated motor current

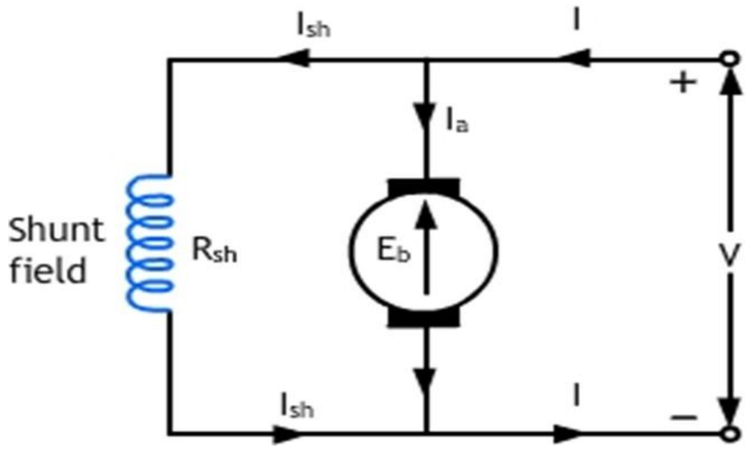


Fig 4.9 DC Shunt Motor

$I_{sh} = V/R_{sh}$ and $I_a = I - I_{sh}$.

where I is the line current

$E_b = V - I_a R_a - B.C.D - A.R.D$

where B.C.D is brush contact drop (1 V/brush, A.R.D is the armature reaction drop

DC Series Motor: -A series motor receives its excitation from a winding which is connected in series with the armature and carries load current Fig 4.9.1 dc series motor. As the series field has to carry high load current, it is made of a thick wire and a few turns. As the resistance is low, the voltage drop across the series winding is small.This motor has excellent starting and over-load torque characteristics. The disadvantages are that the motor attains dangerously high speed at no-load. Speed adjustment of the motor is somewhat difficult.

$I_a = I = I_{se}$

$E_b = V – I_a (R_a + R_{se}) – B.C.D – A.R.D.$

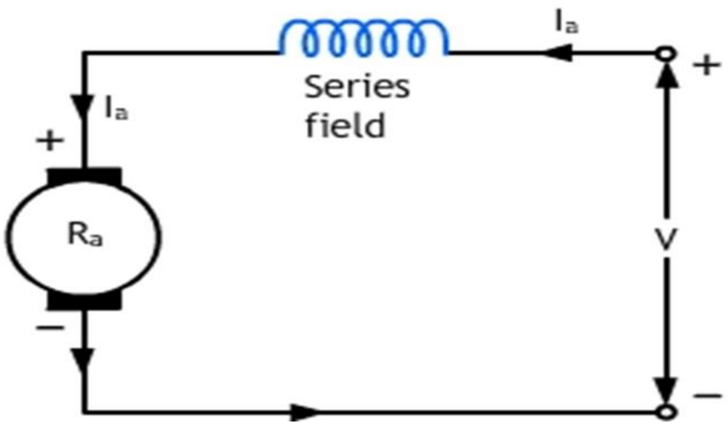


Fig 4.9.1 dc series motor

DC Compound Motor: -In compound motors excitation results from combined action of both shunt field winding and series field winding. In the short-shunt connection, which is sometimes used, the shunt field is directly connected in parallel with the armature, in which case, the series field current is the same as the line current. Excitation of a compound motor is a combination of series and shunt excitation. The motor, therefore, has mixed characteristic between that of a series motor and a shunt motor. These motors behaves somewhat better than a shunt motor from the point of view of starting and overload torque; and has definite stable no-load speed like a shunt motor. Speed of this motor is adjustable as easily as that of a shunt motor. It's speed, however, tends to change as much as 25 percent between full-load and no-load due to the effect of series winding.

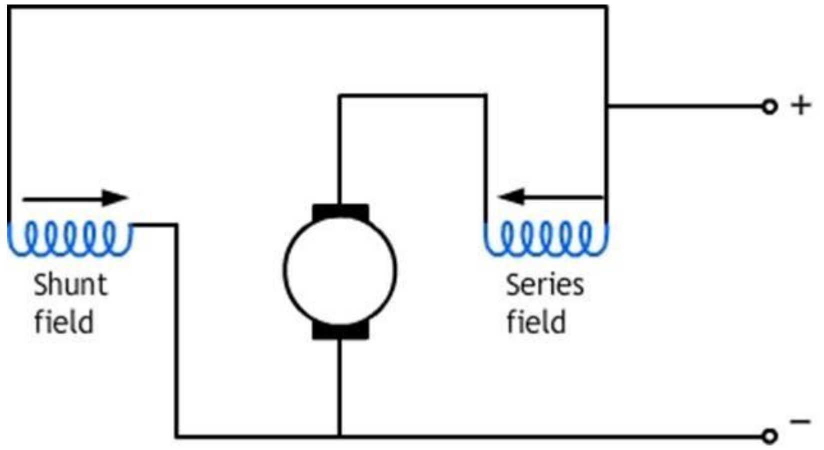


Fig 4.10 Differential compound motor

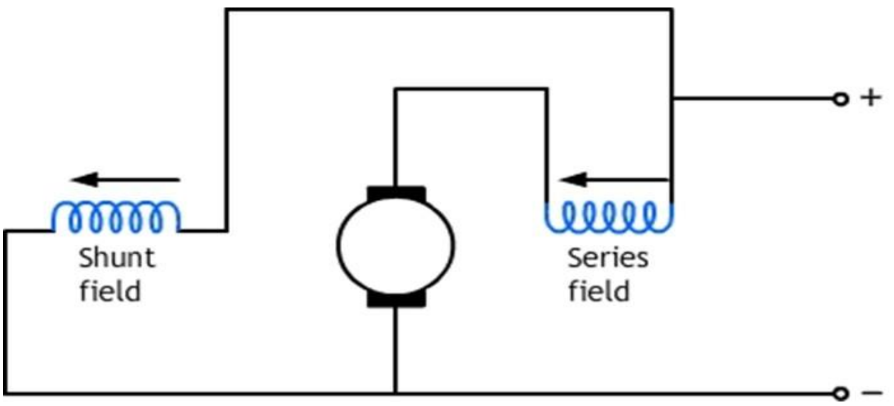


Fig 4.11 Cumulative compound motor

Back EMF (E_b): When the voltage V is applied to the motor, current I_a will flow through the armature and I_{sh} will flow through the field of the motor which will set the flux causing EMF. The EMF developed in the armature opposes the applied voltage and hence it is called the back e.m.f (E_b).The applied voltage V has to drive current through the armature conductors against the opposition of the back E.M.F and hence work has to be done. It is in the form of mechanical power developed by the armature. The armature current Ia is given by eq (1)

$$I_a = \frac{V - E_b}{R_a}$$

Significance of Back EMF: Back EMF is a must in a motor which helps to regulate the armature current and also the real cause for the production of torque.

$$V = E_b + I_a R_a$$

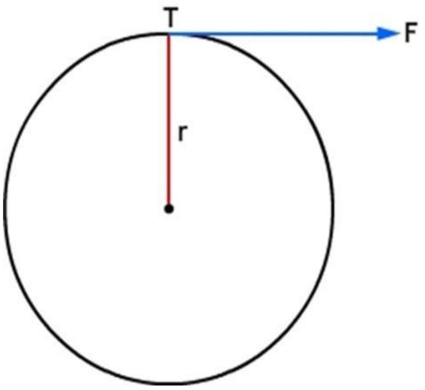
Expression for the back Emf is given by **E=V-IaRa,**

$$\text{and } E_b = \frac{\phi Z N . \left(\frac{P}{A}\right)}{60} \text{volts}$$

Where E is the back emf, V is the applied emf, Ia is the armature current and Ra is the armature circuit resistance. And also, **E= PZNΦ/60A** volts, from the machine parameters.

Torque Equation: -

Torque is the turning moment about its axis. It is also equal to Force x Distance



Consider the armature of the DC Motor of radius r and let F be the force acting tangential to its surface as shown in Fig .

Therefore, Torque = T_a = F x r in Newton meter-------------------------(1)

The work done by this force F in one revolution

W = F x distance covered in revolution

W = F x 2πr watt second.

The power developed by the armature = work done in one second.

= F x r x 2π N / 60 where N = No of revolutions / minute

= (2πN / 60) x T_a watts

But power developed in the armature =E_b I_a

$$\text{Therefore } E_b I_a = \left(\frac{2\pi N}{60}\right) \times T_a$$

$$\left(\frac{\phi ZN}{60}\right)\left(\frac{P}{A}\right) \times I_a = \left(\frac{2\pi N}{60}\right) \times T_a \left(\because E_b = \frac{\phi ZN}{60}\frac{P}{A}\right)$$

$$\text{Therefore, } T_a = \left(\frac{1}{2\pi}\right) \phi Z I_a \cdot \frac{P}{A} \text{ Newton meter}$$

$$= 0.159\, \phi Z I_a \cdot \frac{P}{A} \text{ Newton meter}$$

The actual torque or shaft torque (torque available at the shaft) or Useful torque = T_{sh} = T_a –

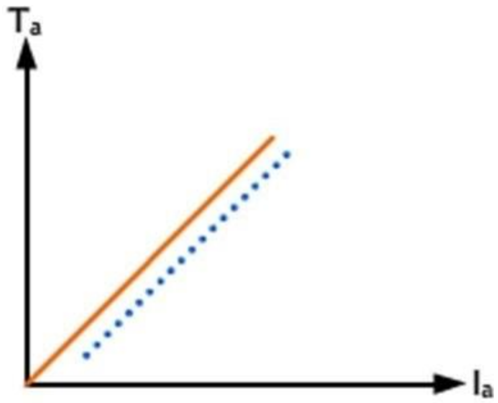
T_L whereT_{sh} = shaft torque

T_a = armature torque
T_L = lost torque due to iron losses and mechanical losses

Output = $2\pi N T_{sh}/60$

T_{sh} = output x 60 / $2\pi N$
If output is in Horse Power,
T_{sh} = output in H.P x 735.5/($2\pi N$/60) N-M

Characteristics of DC Shunt Motor:

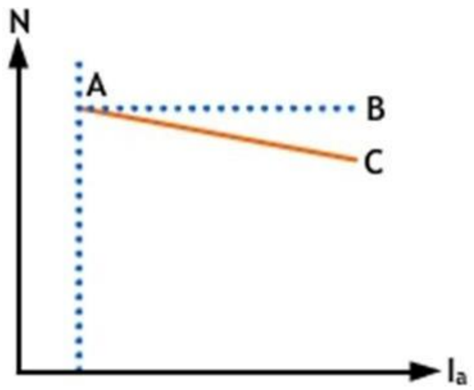


(a) Ta/Ia characteristics (electrical characteristics):- As assumed that flux ϕ is constant in the shunt machine

$Ta \propto Ia$

This implies that the characteristic is a straight line. Larger armature current is required to start a heavy load. Therefore, a shunt motor should not be started on heavy load.

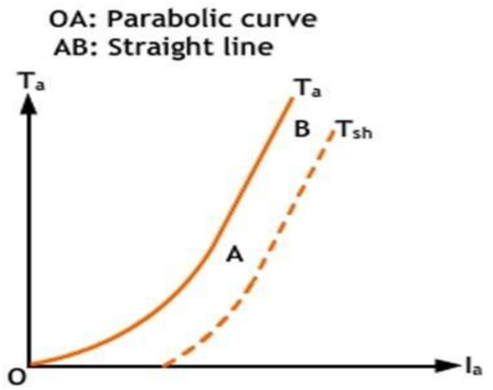
(b) N/Ia characteristics: -



$N \propto (E_b/\phi)$, As ϕ is assumed to be constant, $N \propto E_b$. As E_b is also practically constant, the speed is constant.

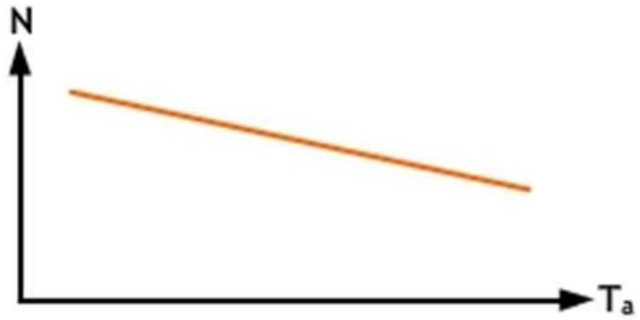
However, to be accurate both E_b and ϕ decrease with increasing load. But E_b decreases somewhat more than ϕ so that there is some decrease in speed, the drop ranging from 5 to 15 % of full load, depending on certain other conditions. The actual speed curve will be somewhat dropping as shown by line AC.

The characteristic does not have a point of zero armature current, because a small current is necessary to maintain the rotation of motor at no-load.



As there is no change in the speed of shunt motor, during the transition from no load to full load, it may be connected to loads which can be suddenly disconnected without fear of excessive speeding.

(c) N/Ta characteristics or mechanical characteristics:-



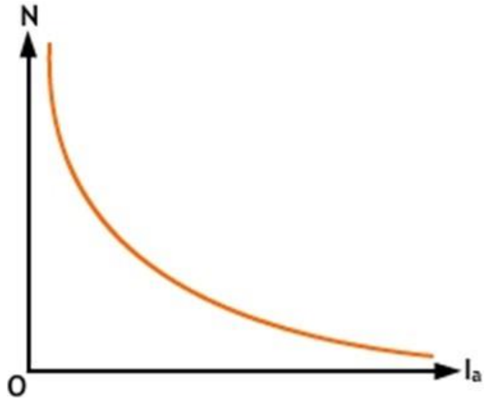
The values of N and T_a for various armature currents I_a is shown. The speed falls as the load torque increases.The N/T_a characteristic is of great importance in determining which type of motor is best suited to drive a given load.

Characteristics of series motor

(a) Torque vs. armature current characteristic

Since T α $I_a{}^2$ in the linear zone and Tα I_a in the saturation zone, the T vs. I_a characteristic is as shown in Fig. At light loads, I_a and hence ϕ is small, but as I_a increases, T_a increases as the square of the current in a parabolic manner till the point of saturation A is reached. After saturation ϕ is practically independent of I_a, hence T_a α I_a and so that the characteristic becomes straight line.

(b) Speed vs. armature current

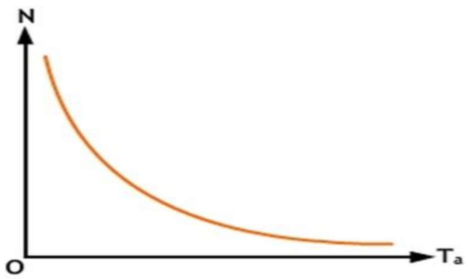


The changes in speed can be determined from the formula

$$N \alpha \, (E_b/\phi)$$

variation of E_b for different load currents is negligible that E_b may be treated as a constant. If I_a is increased, flux ϕ too increases. So speed is inversely proportional to the armature current.When there is heavy load I_a is large. But when the load and consequently I_a decreases to a low value, the speed becomes dangerously high. Hence, a series motor should invariably be started with some mechanical load on it, to prevent excessive speed and damage due to heavy centrifugal forces produced.

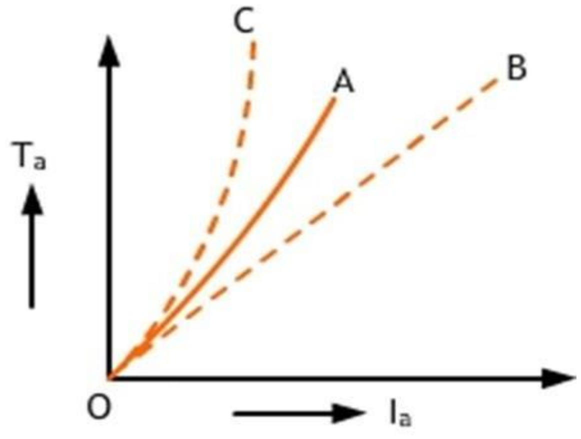
(c) Speed vs. Torque characteristic



The speed vs torque characteristic of a series motor is shown. From the curve, it is apparent that the series motor develops a high torque at low speed and vice versa. This is because an increase in torque requires an increase in armature current, which is also the field current. The result is that the flux is strengthened and hence speed drops. Similarly, at low torque, the motor speed is high.

Characteristics of DC Compound Motor:

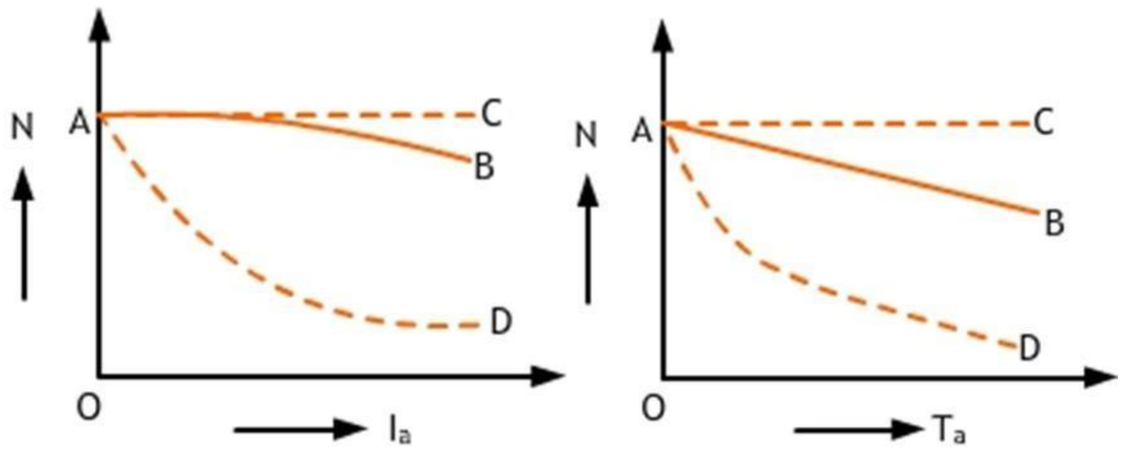
In the cumulative compound motor as Ia increases, flux Φse increases but the shunt field current Ish and ϕsh remain constant and total flux increases



Ta∝ϕIa

Fig(A)

As the armature current is increased, the series flux increases, thus increasing the total flux of the motor. As a result of this, the torque is increased. The increase of torque T_a with armature current is shown by a T_a/I_a characteristic curve OA. This increase of Ta with I_a is greater than what it is in the case of shunt motor (dotted curve OB) less than what it is in the case of series motor develops a high torque with sudden increase in load.



We have just discussed that, with the increase of I_a, the series flux and hence total flux increases. This leads to decrease in motor speed, starting from a particular value given by the point A at no- load. The variation of N with I_a is given by the characteristic AB in Fig (B). Again, the decrease in speed is greater than what it would be in the case of a shunt motor (given by the dotted curve AC), but less than what it would be in the case of a series motor.As series excitation assists shunt excitation, the N/T_a characteristic curve AB will lie between that of a shunt motor(dotted line AC) and of a series motor(dotted line AD)

Applications of DC Motors:

(1) DC Shunt Motor: When constant speed is required DC shunt motors are used.

Example: Lathes, Centrifugal pumps, fans, drilling machines. etc.

(2) DC Series Motor: For high starting torque we prefer DC series motor. Example: Electric traction, electric locomotive, cranes, hoists, conveyors etc.

(3) DC Compound Motor: When we require constant speed and high starting torque Cumulative compound motors are preferred. Example: shears, punches, coal cutting machine, elevators, conveyors, printing presses etc. Differential compound motors have no practical applications (being unstable).

Problems

1) A 6 pole lap wound dc generator has 51 slots, each slot has 18 conductors. The useful flux per pole is 35 mwb. Find the generated emf in the armature, if it is driven at a speed of 750 rpm.

Given: P = 6

A = P (lap wound) Number of slots = 51

Conductors/slot = 18

Total No. of conductors = 51* 18 = Z

ϕ= 35mwb; N = 750 rpm,

emf generated

$$= \frac{\phi ZN}{60}\left(\frac{P}{A}\right) \text{volts} = \frac{(35\times10^{-3})\times(51\times18)\times750\times6}{60\times6}$$

= 401.6 volts.

2) An 8 pole d.c. generator has 650 armature conductors. The flux per pole is 20 mWb. Find the value of emf generated when the armature is wave wound and is rotating at a speed of 1200 rpm. What must be speed at which the armature is to be driven to generate the same emf, if the armature is lap wound.

generate the same emf, if the armature is lap wound. Given: P = 8;

A = 2 (wave wound)

No. of conductors = 650

ϕ = 20 mWb; N = 1200 rpm,

emf generated

$$= \frac{\phi ZNP}{60A} \text{ volts} = \frac{(20\times10^{-3})\times(650)\times1200\times8}{60\times2} = 1040 \text{ volts}$$

To find the speed of armature, when it is lap wound,

$$N = \frac{E_g \times 60A}{\phi ZP} = \frac{(1040)\times60\times8}{(20\times10^{-3})\times(650)\times8} = 4800 \text{ rpm}$$

3) A d.c series motor is running with a speed 800 rpm while taking a current of 20 A from the supply. If the load is changed such that the current drawn by the motor is increased to 50A, calculate the speed of the motor on new load. The armature and series field winding resistances are 0.2 ohm and 0.3 ohm respectively. Assume that the flux produced is proportional to the current. Assume the supply voltage as 250 V.

For load 1, N_1 = 800 rpm, $I_1 = I_{a1}$ = 20 A

For load 2, $I_1 = I_{a2}$ = 50 A

E_{b1} = 240 V .... ($E_{b1} = V - I_{a1}(R_a + R_{se})$)

E_{b2} = 225 V .... ($E_{b2} = V - I_{a2}(R_a + R_{se})$)

$$\frac{N_2}{N_1} = \left(\frac{E_{b2}}{E_{b1}}\right)\times\left(\frac{I_{a1}}{I_{a2}}\right)$$

N_2 = 300 rpm

4) The armature current of a series motor is 60 A when on full load. If the load is adjusted so that this current decreases to 40 A, find the new torque expressed as a percentage of full load torque. The flux for a current of 40 A is 70% of that when the current is 60 A.

$T \propto \phi I_a$,

i) Full load torque = $T_{fl} = \phi \times 60$

ii) $T_{40} = 0.7\, \phi \times 40$

$T_{fl}/ T_{40} = 60\ \phi/ (0.7\ \phi x\ 40)$

$T_{40} = 0.4667\ T_{fl}$

Torque at 40 A is 46.67% of full load torque

5) A 4 pole 250 V d.c. shunt motor has a back emf of 240.8 V and takes a current of 20 A. Calculate the power developed. Take the resistance of the field winding as 250 ohms.

P = 4
V = 250 V
E_b= 240.8 V
I_L=20A
R_{sh}= 250 ohms
R_{sh}= 250 ohms
Power developed = E_bI_a
Ia = $I_L – I_{sh}$ and
$I_{sh} = V/R_{sh}$
Ish = 250/250 =1 A
Ia = 20 – 1 = 19 A
Power developed =240.8 x 19 = 4572.8 W.

6) A 230 V dc series motor takes 12 A and runs at 800 rpm. At what speed will it run, when 10 ohm resistance is connected in series with the armature the motor taking the same current at the same supply voltage. Take Ra and Rse of the motor as 0.5 ohm each.

V = 230 V
IL = Ia = Ise = 12 A
Ra = Rse = 0.5 ohms
$E_{b1} = V – I_{se}R_{se}$- IaRa.
= 230 - 12 x 0.5 - 12 x 0.5 = 218.
When 10-ohm resistance is connected in series with the armature, then
E_{b2} = V – Ise (Rse + Ra + 10)
= 230 - 12(0.5 + 10 + 0.5) = 98 V.
Let N_2 be the corresponding speed then
E_{b2}/E_{b1}=(N2/N1) x (f1/f2)
Since $f_1 = f_2$
$N_2 = (E_{b2}/E_{b1}) xN_1$
N_2 = 98 x 800/218 =359.6 rpm.

Exercise problems

A 4 pole DC shunt motor takes 22A from 220V supply. The armature and the field resistances are 0.5 Ω and 100Ω respectively. The armature is lap connected with 300 conductors. If the flux per pole is 20mWb, calculate the speed and gross torque.

1. A 20kW, 200V dc shunt motor has armature and field resistances of 0.05 ohm and 100 ohm respectively. Calculate the total power developed by the armature when it delivers full output power

2. A DC series motor connected to a connected to a 440V supply runs at 600rpm when taking a current of 50A. Calculate the value of resistor which when inserted in series with the motor will reduce the speed to 400rpm, the gross torque being then half its previous value. Resistance of motor is 0.2Ω. Assume the flux to be proportional to the field current.

3. . A D.C series motor running with a speed of 1000rpm, while taking a current of 22amp from the supply. If the load is changed such that the current drawn by the motor is increased to 55amp, calculate the speed of the motor on new load. The armature and series winding resistances are 0.3ohm and 0.4ohm respectively. Assume supply voltage as 250V.

4. A 200V, 4 pole, lap wound dc shunt motor has 600 conductors on its armature. The resistance of the armature winding is 0.5Ω and shunt field winding is 200Ω. The motor takes a current of 21A, the flux per pole is 30mWb, find the speed and gross torque developed in the motor

SINGLE PHASE TRANSFORMERS

TRANSFORMER is a static device which transfer electric energy from one electric circuit to another at any desired voltage without any change in frequency.

PRINCIPLE:- A transformer works on the principle of mutual induction. "Whenever a change in current takes place in a coil there will be an induced emf in the other coil wound over the same magnetic core". This is the principle of mutual induction by which the two coils are said to be coupled with each other.

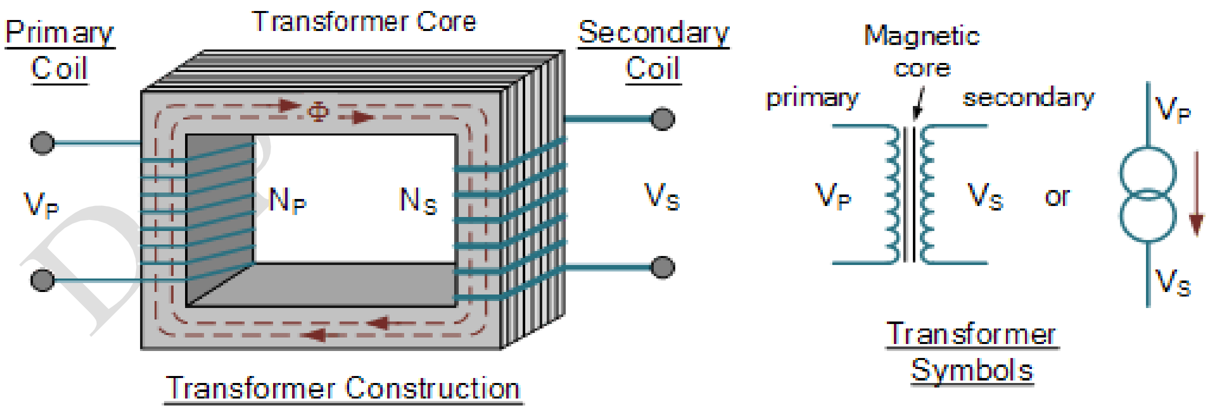


Fig 3.1 Principle of transformer

The Fig 3.1 shows the general arrangement of a transformer. C is the iron core made of laminated sheets of about 0.35mm thick insulated from one another by varnish or thin paper. The purpose of laminating the core is to reduce the power loss due to eddy currents induced by the alternating magnetic flux. The vertical portions of the core are called limbs and the top and bottom portions are called the yokes. Coils P and S are wound on the limbs. Coil P is connected to the supply and therefore called as the primary, coil S is connected to the load and is called as the secondary. An alternating voltage applied to P drives an alternating current though P and this current produces an alternating flux in the iron core, the mean path of the flux is represented by the dotted line D. This flux links with the coil S and thereby induces an emf in S.

Construction of Transformer:

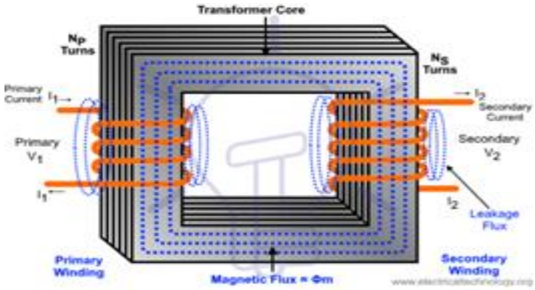


Fig 3.2 Construction of Transformers

For the simple construction of a transformer, you must need two coils having mutual inductance and a laminated steel core. The two coils are insulated from each other and from the steel core. The device will also need some suitable container for the assembled core and windings, a medium with which the core and its windings from its container can be insulated. In order to insulate and to bring out the terminals of the winding from the tank, apt bushings that are made from either porcelain or capacitor type must be used. In all transformers that are used commercially, the core is made out of transformer sheet steel laminations assembled to provide a continuous magnetic path with minimum of air-gap included. The steel should have high permeability and low hysteresis loss. For this to happen, the steel should be made of high silicon content and must also be heat treated. By effectively laminating the core, the eddy-current losses can be reduced. The lamination can be done with the help of a light coat of core plate varnish or lay an oxide layer on the surface. For a frequency of 50 Hertz, the thickness of the lamination varies from 0.35mm to 0.5mm for a frequency of 25 Hertz.

Types of transformers:

1. Core- Type Transformer

In core-type transformer, windings are positioned on two limbs of the core and there is ONLY one flux path and windings are circumventing the core as shown in Fig 3.3 The coils used for this transformer are form-wound and are of cylindrical type. Such a type of transformer can be applicable for small sized and large sized transformers. In the small sized type, the core will be rectangular in shape and the coils used are cylindrical. However, this type of **transformer construction** where the two windings are wound on separate limbs is not very efficient since the primary and secondary windings are well separated from each other. This results in a low magnetic coupling between the two windings as well as large amounts of magnetic flux leakage from the transformer itself. The efficiency of a simple transformer construction can be improved by bringing the two windings within close contact with each other thereby improving the magnetic coupling as shown in Fig 3.4. In this case half of high voltage (HV) winding is placed on one limb while the other half of HV on the other limb. Similarly half of low voltage (LV) winding is placed over HV winding on one limb while the other half of LV is wound over HV on the other limb.

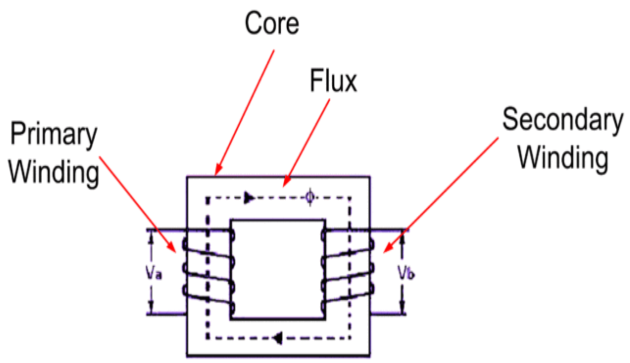


Fig 3.3 Core type transformer

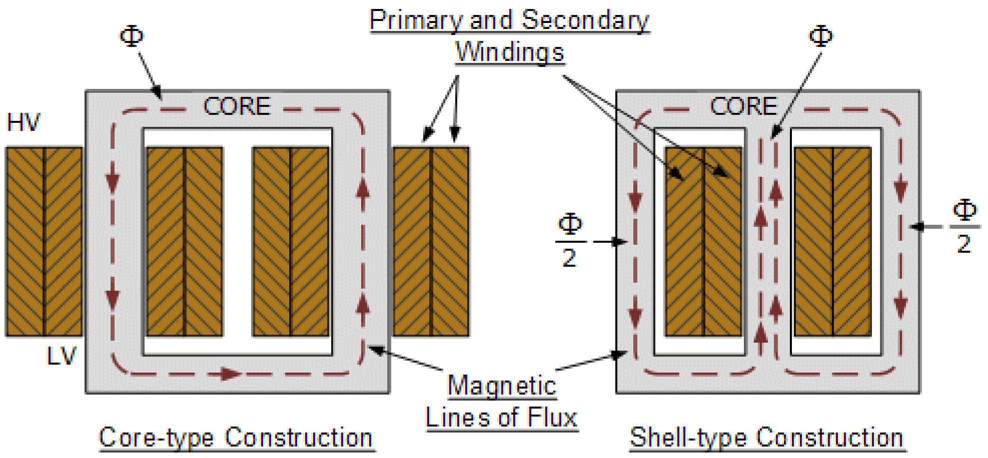


Fig 3.4 Construction of core type and shell type transformers

2. Shell-Type Transformer

In shell-type transformers, windings are positioned on the middle limb of the core while other limbs are utilized as the mechanical support as shown in Fig 3.5. In this case the core surrounds a considerable portion of the windings.

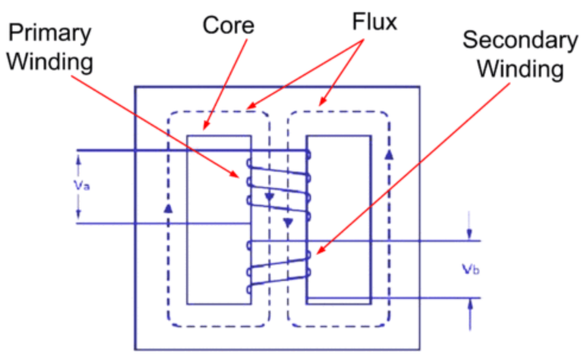


Fig 3.5 Shell type transformer

Shell type transformer core's overcome the leakage flux as both the primary and secondary windings are wound on the same centre leg or limb which has twice the cross-sectional area of the two outer limbs. The advantage here is that the magnetic flux has two closed magnetic paths to flow around external to the coils on both left and right-hand sides before returning back to the central coils. This means that the magnetic flux circulating around the outer limbs of this type of transformer construction is equal to $\Phi/2$. As the magnetic flux has a closed path around the coils, this has the advantage of decreasing core losses and increasing overall efficiency. To further reduce the leakage flux the HV winding is placed over the LV winding as shown in Fig. 3.4.

Transformer Laminations

For the purpose of winding the primary and secondary windings around the laminated iron or steel cores for this type of transformer constructions, the coils are firstly wound on a former which has a cylindrical, rectangular or oval type cross section to suit the construction of the laminated core. In both the shell and core type transformer constructions, in order to mount the coil windings, the individual laminations are stamped or punched out from larger steel sheets and formed into strips of thin steel resembling the letters "E"s, "L"s, "U"s and "I"s as shown in Fig 3.6 below.

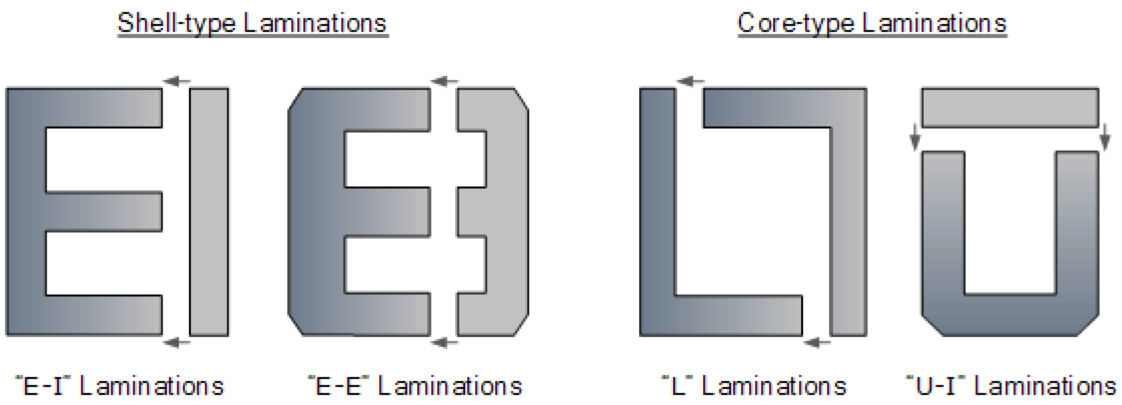


Fig 3.6 Shapes of transformer laminations

These lamination stampings when connected together form the required **core** shape. For example, two "E" stampings plus two end closing "I" stampings to give an E-I core forming one element of a standard shell-type transformer core. These individual laminations are tightly butted together during it's construction to reduce the reluctance of the air gap at the joints producing a highly saturated magnetic flux density.

Transformer core laminations are usually stacked alternately to each other to produce an overlapping joint with more lamination pairs being added to make up the correct core thickness. This alternate stacking of the laminations also gives the transformer the advantage of reduced flux leakage and iron losses. E-I core laminated transformer construction is mostly used in isolation transformer's, step-up and step-down transformer's as well as the auto transformer.

The representation of single phase and three phase core type and shell type transformers are shown in Fig3.7 for better understanding.

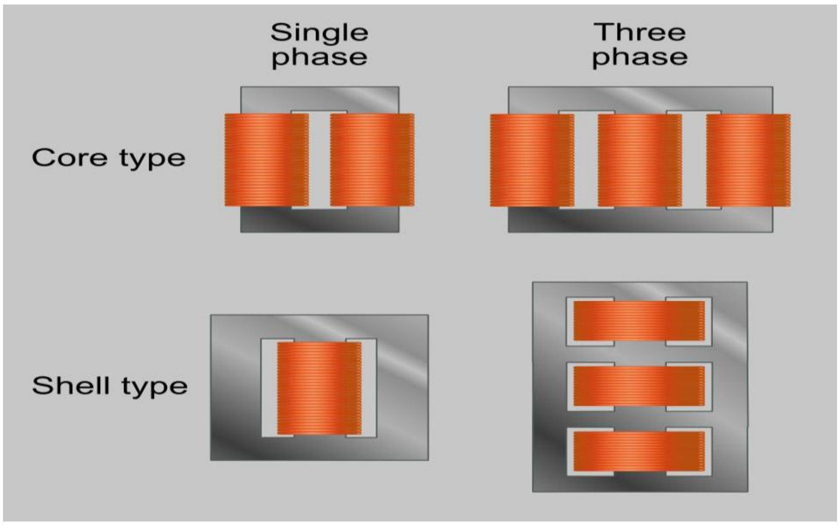
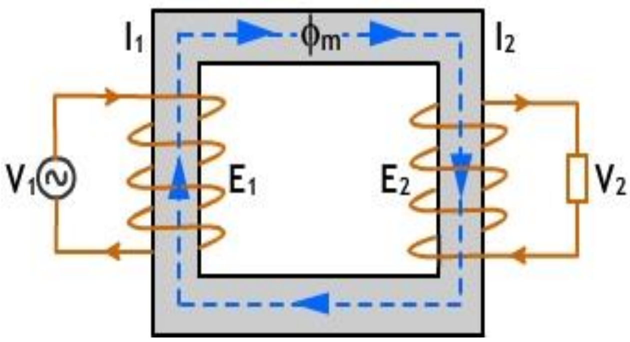
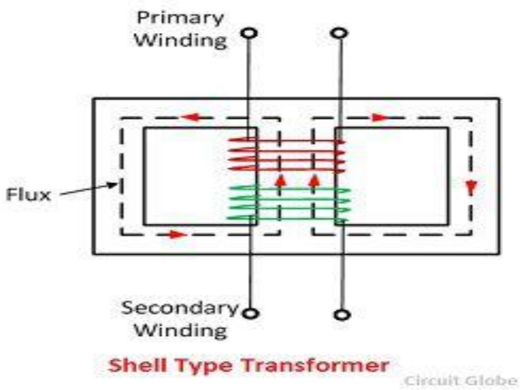


Fig 3.7 Core and Shell Type Transformer Representation

COMPARISON BETWEEN CORE AND SHELL TYPE TRANSFORMER

Table 1: Comparison Between Core and Shell Type Transformer

CORE TYPE	SHELL TYPE
	
The winding encircles the core	**The core encircles most part of the windings**
As the windings are distributed the natural cooling is more effective	**As windings are surrounded by the core the natural cooling doesn't exists**
The cylindrical type of coils are used	**Generally multilayer disc type or sandwich coils are used**
The coils can be easily removed from maintenance point of view	**For removing any winding for maintenance large number of laminations are required to be removed.**

The construction is preferred for low voltage transformers	**The construction is used for very high voltage transformers**
It has single magnetic circuit	**It has double magnetic circuit**
In a single phase type the core has 2 limbs	**In a single phase type the core has 3 limbs**

Transformer – Working Principle

The main principle of operation of a transformer is mutual inductance between two circuits which is linked by a common magnetic flux. A basic transformer consists of two coils that are electrically separate and inductive but are magnetically linked through a path of reluctance. The working principle of the transformer can be understood from the Fig 3.8 below.

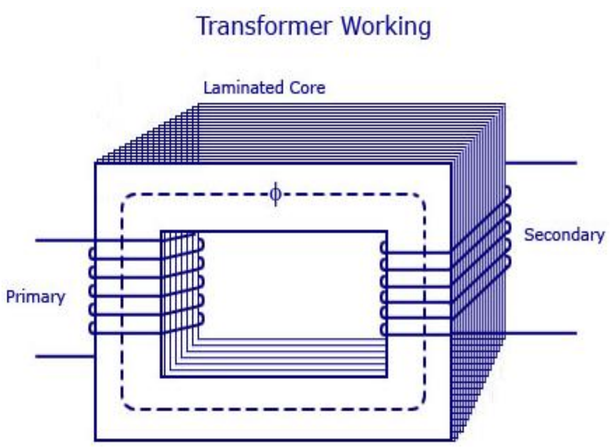


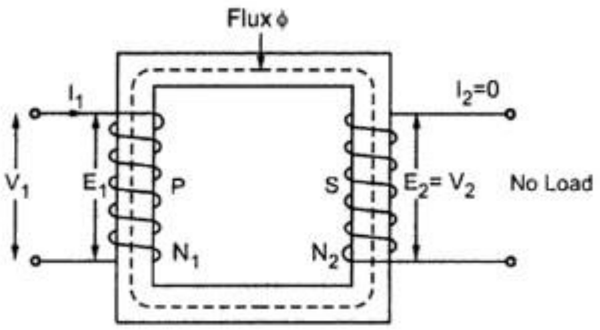
Fig 3.8 Working Principle of Transformers

As shown above the electrical transformer has primary and secondary windings. The core laminations are joined in the form of strips in between the strips you can see that there are some narrow gaps right through the cross-section of the core. These staggered joints are said to be 'imbricated'. Both the coils have high mutual inductance. A mutual electro-motive force is induced in the transformer from the alternating flux that is set up in the laminated core, due to the coil that is connected to a source of alternating voltage. Most of the alternating flux developed by this coil is linked with the other coil and thus produces the mutual induced electro-motive force. The so produced electro-motive force can be explained with the help of Faraday's laws of Electromagnetic Induction as $e=M*dI/dt$

If the second coil circuit is closed, a current flows in it and thus electrical energy is transferred magnetically from the first to the second coil.The alternating current supply is given to the first coil and hence it can be called as the primary winding. The energy is drawn out from the second coil and thus can be called as the secondary winding.In short, a transformer carries the operations shown below:

1. Transfer of electric power from one circuit to another.
2. Transfer of electric power without any change in frequency.
3. Transfer with the principle of electromagnetic induction.
4. The two electrical circuits are linked by mutual induction.

Concept of Ideal transformer on no-load:



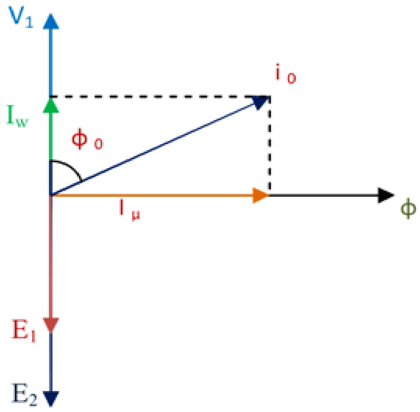


Fig 3.9 Concept of Ideal transformer on no-load

When the transformer is operating at no load, the secondary winding is open-circuited as shown in Fig 3.9, which means there is no load on the secondary side of the transformer and, therefore, current in the secondary will be zero. While primary winding carries a small current I_0 called no-load current which is **2 to 10% of the rated current**. This current is responsible for supplying the iron losses (hysteresis and eddy current losses) in the core and a very small amount of copper losses in the primary winding. The angle of lag depends upon the losses in the transformer. The power factor is very low and varies from **0.1 to 0.15**. The equivalent circuit representation of ideal transformer on no load is as shown in Fig 3.10.

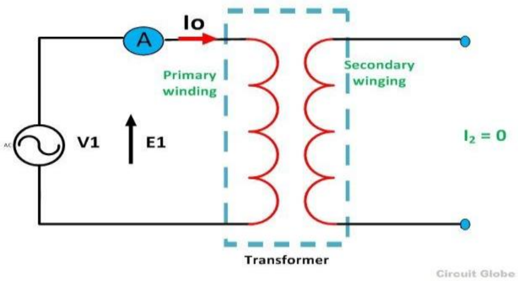


Fig 3.10 Concept of Ideal transformer on no-load

The no-load current consists of two components:

- **Reactive or magnetizing component I_m**
  (It is in quadrature with the applied voltage V_1. It produces flux in the core and does not consume any power).
- **Active or power component I_w**, also know as a working component
  (It is in phase with the applied voltage V_1. It supplies the iron losses and a small amount of primary copper loss).

The following steps are given below to draw the phasor diagram (refer Fig 3.11):

1. The function of the magnetizing component is to produce the magnetizing flux, and thus, it will be in phase with the flux.
2. Induced emf in the primary and the secondary winding lags the flux ϕ by 90 degrees.
3. The primary copper loss is neglected, and secondary current losses are zero as
   $I_2 = 0$.
   Therefore, the current I_0 lags behind the voltage vector V_1 by an angle ϕ_0 called the no-load power factor angle and is shown in the phasor diagram above.
4. The applied voltage V_1 is drawn equal and opposite to the induced emf E_1 because the difference between the two, at no load, is negligible.
5. Active component I_w is drawn in phase with the applied voltage V_1.
6. The phasor sum of magnetizing current I_m and the working current I_w gives the no-load current I_0.

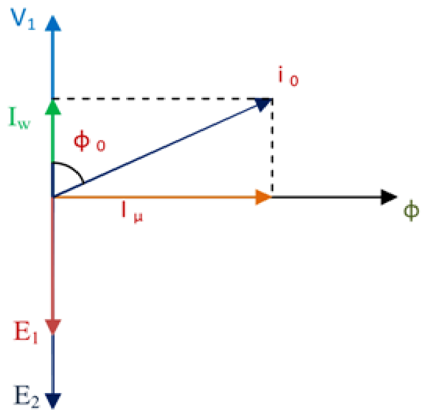


Fig 3.11 Phasor diagram concept of Ideal transformer on no-load

Working component $I_w = I_0 Cos\varphi_0$

No load current $I_0 = \sqrt{I_w^2 + I_m^2}$

Magnetizing component $I_m = I_0 \, Sin\varphi_0$

Power factor $Cos\ \varphi_0 = \frac{I_w}{I_0}$

No load power input $P_0 = V_1 I_0 Cos\varphi_0$

Concept of Ideal transformer on load:

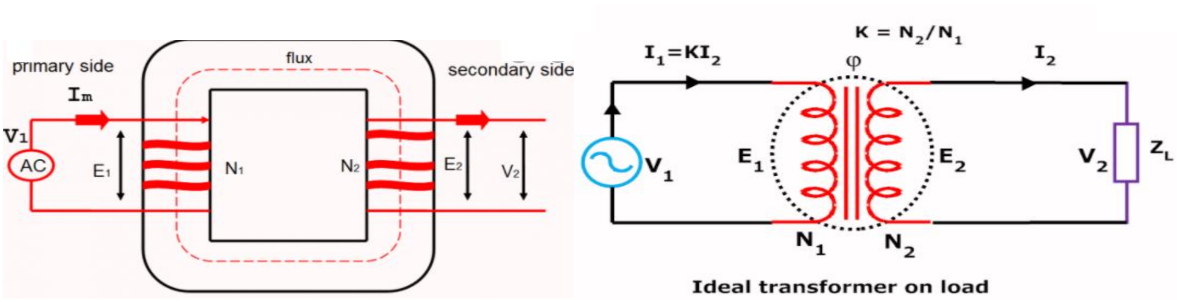


Fig 3.12 Concept of Ideal transformer on load

When the ideal transformer is on the loaded condition, the secondary of the transformer is connected to load as shown in Fig 3.12. The load can be resistive, inductive or capacitive. The current I2 flows through the secondary winding of the transformer. The magnitude of the secondary current depends on the terminal voltage V2 and the load impedance. The phase angle between the secondary current and voltage depends on the nature of the load.

The Operation of the Transformer on Load Condition is explained below:

- When the secondary of the transformer is kept open, it draws the no-load current from the main supply. The no-load current induces the magnetomotive force N_0I_0 and this force set up the flux Φ in the core of the transformer.
- When the load is connected to the secondary of the transformer, I_2 current flows through their secondary winding. The secondary current induces the magnetomotive force N_2I_2 on the secondary winding of the transformer. This force set up the flux φ_2 in the transformer core. The flux φ_2 opposes the flux φ, according to **Lenz's law**.
- As the flux φ_2 opposes the flux φ, the resultant flux of the transformer decreases and this flux reduces the induced EMF E_1. Thus, the strength of the V_1 is more than E_1 and an additional primary current I'_1 drawn from the main supply. The additional current is used for restoring the original value of the

flux in the core of the transformer so that $V_1 = E_1$. The primary current I'_1 is in phase opposition with the secondary current I_2. Thus, it is called the **primary counter-balancing current**.

- The additional current I'_1 induces the magnetomotive force $N_1I'_1$. And this force set up the flux φ'_1. The direction of the flux is the same as that of the φ and it cancels the flux φ_2 which induces because of the MMF N_2I_2

Now, $N_1I_1' = N_2I_2$

Therefore, $$I_1' = \left(\frac{N_2}{N_1}\right) I_2 = KI_2$$

The phase difference between V_1 and I_1 gives the power factor angle ϕ_1 of the primary side of the transformer.

- The power factor of the secondary side depends upon the type of load connected to the transformer.
- If the load is inductive as shown in the above phasor diagram, the power factor will be lagging, and if the load is capacitive, the power factor will be leading. The total primary current I_1 is the vector sum of the currents I_0 and I_1'. i.e

$$\overline{I_1} = \overline{I_0} + \overline{I_1'}$$

- The working flux φ remains same irrespective of the load current and hence the iron losses are constant and do not vary with load current.

EMF Equation:

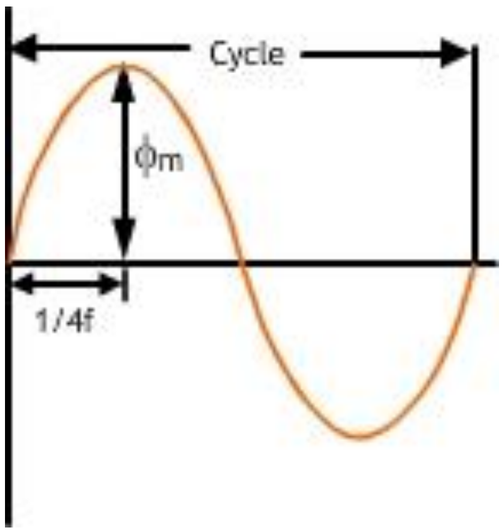


Fig 3.13 Wave form of flux of a transformer

Consider the 1/4th cycle of the flux as shown in Fig.3.13. One complete cycle gets completed in 1/f seconds. In $1/4^{th}$ time period, the change in flux is from 0 to ϕ_m.

$$\therefore \quad \frac{d\phi}{dt} = \frac{\phi_m - 0}{\left(\frac{1}{4f}\right)} \qquad \text{as dt for } 1/4^{th} \text{ time period is } 1/4f \text{ seconds}$$

$$= 4 f \phi_m \text{ Wb/sec}$$

$\therefore$ **average e.m.f. per turn** = $4 f \phi_m$ volts

As ϕ is sinusoidal, the induced e.m.f. in each turn of both the windings is also sinusoidal in nature. For sinusoidal quantity,

$$\text{Form Factor} = \frac{\text{R.M.S. value}}{\text{Average value}} = 1.11$$

$$\therefore \quad \text{R.M.S. value} = 1.11 \times \text{Average value}$$

$\therefore$ R.M.S. value of induced e.m.f. per turn

$$= 1.11 \times 4 f \phi_m$$

$$= 4.44 f \phi_m$$

There are N_1 number of primary turns hence the R.M.S value of induced e.m.f. of primary denoted as E_1 is,

$$E_1 = N_1 \times 4.44 f \phi_m \quad \text{volts}$$

While as there are N_2 number of secondary turns the R.M.S value of induced e.m.f. of secondary denoted E_2 is,

$$E_2 = N_2 \times 4.44 f \phi_m \quad \text{volts}$$

The expressions of E_1 and E_2 are called e.m.f. equations of a transformer.

Thus e.m.f. equations are,

$$\therefore \quad \boxed{\begin{aligned} E_1 &= 4.44 f \phi_m N_1 \quad \text{volts} \\ E_2 &= 4.44 f \phi_m N_2 \quad \text{volts} \end{aligned}} \qquad \begin{aligned} &\ldots(1) \\ &\ldots(2) \end{aligned}$$

Transformation ratio:

It is defined as the ratio of the secondary induced emf to the primary induced emf.

Therefore, $\mathbf{E_1 / E_2 = N_1 / N_2 = K}$

For an ideal (loss free) transformer, the input power is equal to the output power.

Therefore $\mathbf{E_1 I_1 = E_2 I_2}$, from which, $\mathbf{E_2/E_1 = I_1/I_2}$

Also the induced emf per turn is same for both the primary and secondary turns.

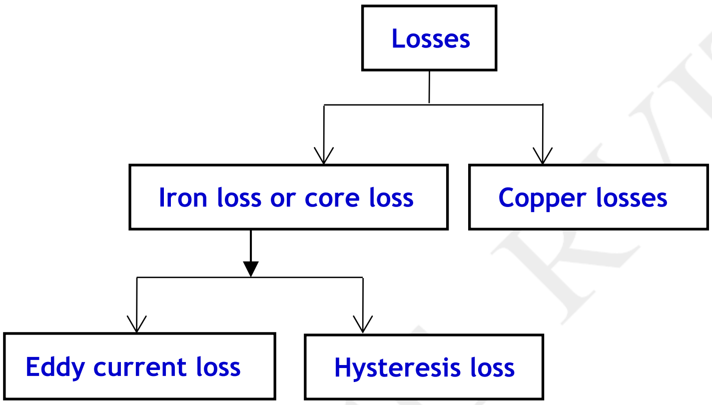
If the value of the transformation ratio **K>1,** then it is called **step up transformer**.

If the value of the transformation ratio **K<1**, then it is called **step down transformer**.

If the value of the transformation ratio **K=1**, then it is a **one:one** or **isolation** transformer.

Losses and Efficiency:

There a two types of power losses occur in a transformer as indicated below.



1) **Iron Loss:** This is the power loss that occurs in the iron part. This loss is due to the alternating frequency of the emf. Iron loss in further classified into two other losses.

Due to alternating flux set up in the magnetic core of the transformer, it undergoes a cycle of magnetisation and demagnetisation. Due to hysteresis effect there is loss of energy in this process which is called hysteresis loss.

It is given by, $\boxed{\text{hysteresis loss} = K_h B_m^{1.67} f v \text{ watts}}$

where
K_h = hysteresis constant depends on material
B_m = maximum flux density.
f = frequency.
v = volume of the core

$$\boxed{\text{eddy current loss} = K_e B_m^2 f^2 t^2 \text{ watts/unit volume}}$$

where
K_e = eddy current constant
t = thickness of the core

a) Eddy current loss **b) Hysteresis loss**

a) **Eddy current loss:** This power loss is due to the alternating flux linking the core, which will induced an emf in the core called the eddy emf, due to which a current called the eddy current is being circulated in the core and is shown in Fig 3.14. As there is some resistance in the core with this eddy current circulation converts into heat called the eddy current power loss. Eddy current loss is proportional to the square of the supply frequency.

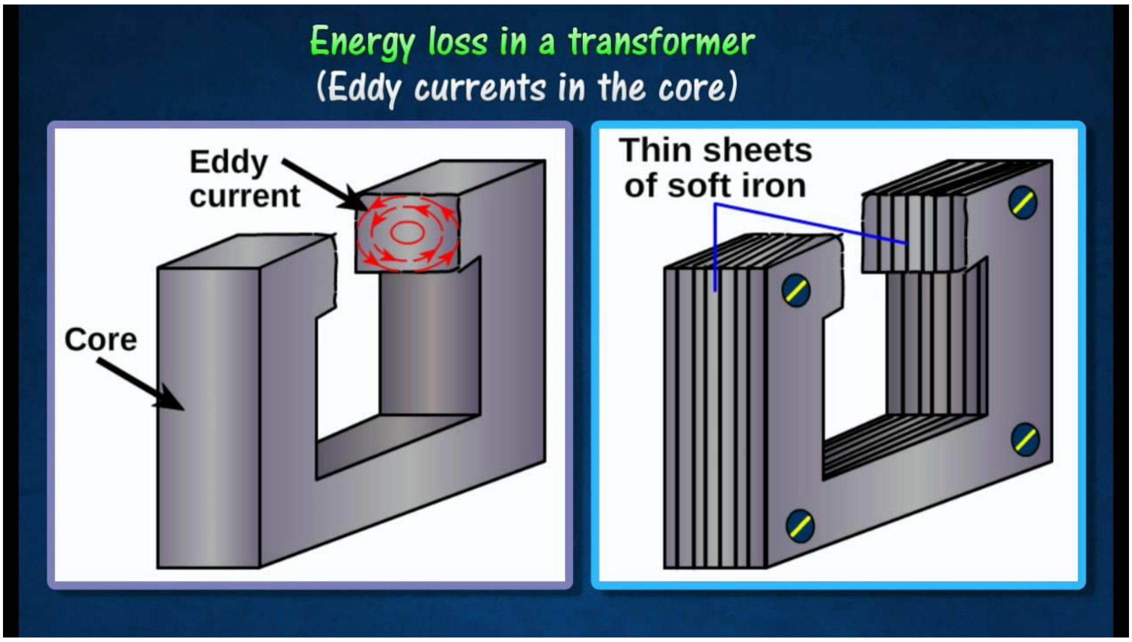


Fig 3.14 Eddy current loss representation in transformer

b) **Hysteresis loss**: This is the loss in the iron core, due to the magnetic reversal of the flux in the core, which results in the form of heat in the core. This loss is directly proportional to the supply frequency shown in Fig 3.15.

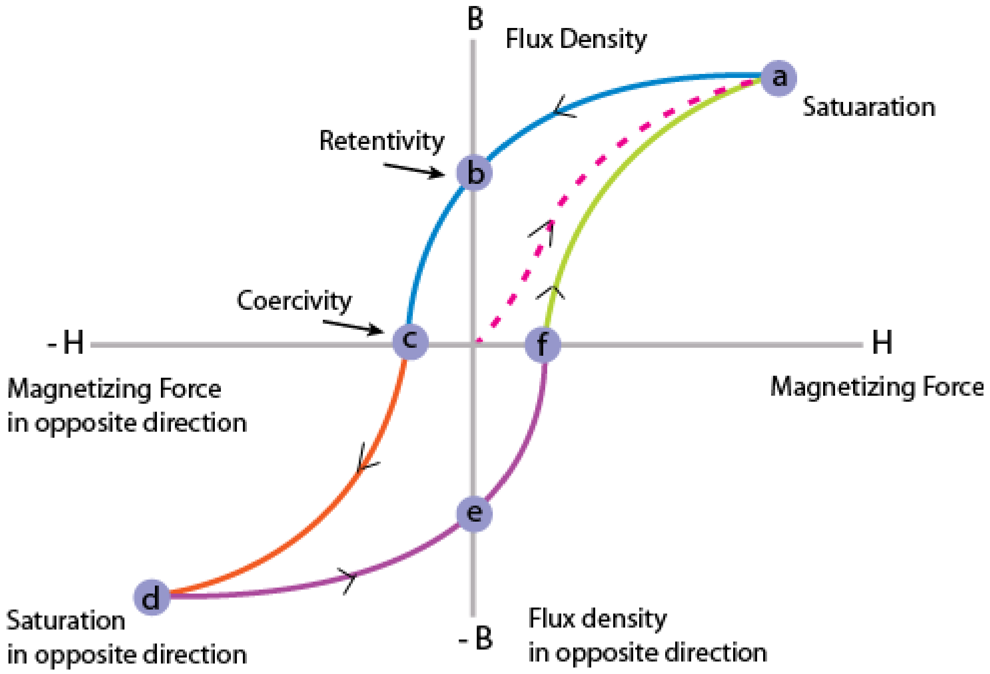


Fig 3.15 Hysteresis loss representation in transformer

Eddy current loss can be minimized by using the core made of thin sheets of silicon steel material, and each lamination is coated with varnish insulation to suppress the path of the eddy currents. Hysteresis loss can be minimized by using the core material having high permeability.

2. Copper loss

The copper losses are due to the power wasted in the form of I^2R loss due to the resistances of the primary and secondary windings. The copper loss depends on the magnitude of the currents flowing through the windings.

$$\begin{aligned}\text{Total Cu loss} &= I_1^2 R_1 + I_2^2 R_2 \\ &= I_1^2 (R_1 + R_2') = I_2^2 (R_2 + R_1') \\ &= I_1^2 R_{1e} = I_2^2 R_{2e}\end{aligned}$$

The copper losses are denoted as P_{cu}. If the current through the windings is full load current, we get copper losses at full load. If the load on transformer is half then we get copper losses at half load which are less than full load copper losses. Thus copper losses are called **variable losses**. For transformer VA rating is $V_1 I_1$ or $V_2 I_2$. As V_1 is constant, we can say that copper losses are proportional to the square of the kVA rating and square of the current.

So, $\boxed{P_{cu} \propto I^2 \propto (kVA)^2}$

Thus for a transformer,

$$\begin{aligned}\text{Total losses} &= \text{iron losses} + \text{copper losses} \\ &= P_i + P_{cu}\end{aligned}$$

In the above derivation Total cu loss $= I_1^2 R_1 + I_2^2 R_2$

$= I_1^2 R_1 + I_1^2 (I_2^2/I_1^2 R_2)$

$= I_1^2 R_1 + I_1^2 R'_2$

where $R'_2 = R_2/K^2$ and $K = I_1/I_2$ = Transformation ratio $= V_2 / V_1$

where R'_2 is the equivalent secondary resistance of transformer as referred to primary.

$= I_1^2 (R_1 + R'_2)$

$= I_1^2 R_{1e}$

where R_{1e} is the equivalent resistance of transformer as referred to primary.

Similarly, Total cu loss $= I_1^2 R_1 + I_2^2 R_2$

$= I_2^2 (I_1^2 / I_2^2 R_1) + I_2^2 R_2)$

$= I_2^2 R'_1 + I_2^2 R_2$ where $R'_1 = (I_1^2 / I_2^2 R_1) = K^2 R_1$

where R'_1 is the equivalent primary resistance of transformer as referred to secondary.

$= I_2^2 (R'_1 + R_2)$

$= I_2^2 R_{2e}$

where R_{2e} is the equivalent resistance of transformer as referred to secondary.

Efficiency: It is the ratio of the output power to the input power of a transformer

Input = Output + Total losses

= Output + Iron loss + Copper loss

Due to the losses in a transformer, the output power of a transformer is less than the input power supplied.

$\therefore$ Power output = Power input – Total losses

$\therefore$ Power input = Power output + Total losses

= Power output + $P_i + P_{cu}$

The efficiency of any device is defined as the ratio of the power output to power input. So for a transformer the efficiency can be expressed as,

$$\eta = \frac{\text{Power output}}{\text{Power input}}$$

$$\therefore \quad \eta = \frac{\text{Power output}}{\text{Power output} + P_i + P_{cu}}$$

Now Power output = $V_2 I_2 \cos\phi$

where $\cos\phi$ = load power factor

The transformer supplies full load of current I_2 and with terminal voltage V_2.

P_{cu} = copper losses on full load = $I_2^2 R_{2e}$

$$\therefore \quad \eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}}$$

But $V_2 I_2$ = VA rating of a transformer

$$\therefore \quad \eta = \frac{(\text{VA rating}) \times \cos\phi}{(\text{VA rating}) \times \cos\phi + P_i + I_2^2 R_{2e}}$$

$$\therefore \quad \boxed{\% \eta_{FL} = \frac{(\text{VA rating}) \times \cos\phi}{(\text{VA rating}) \times \cos\phi + P_i + I_2^2 R_{2e}} \times 100} \quad \text{... Full load efficiency}$$

$$\boxed{\% \eta_{FL} = \frac{(\text{VA rating}) \times \cos\phi}{(\text{VA rating}) \times \cos\phi + P_i + (P_{cu})_{F.L.}} \times 100} \quad \text{... Full load efficiency}$$

This is full load percentage efficiency with,

I_2 = full load secondary current

Condition for maximum efficiency:

In general for the efficiency to be maximum for any device the losses must be minimum. Between the iron and copper losses the iron loss is the fixed loss and the copper loss is the variable loss. When these two losses are equal and also minimum the efficiency will be maximum. Therefore the condition for maximum efficiency in a transformer is

Iron loss = Copper loss (whichever is minimum)

$$\frac{d\eta}{dI_2} = 0$$

Now $$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}}$$

$$\therefore \quad \frac{d\eta}{dI_2} = \frac{d}{dI_2}\left[\frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}}\right] = 0$$

$$\therefore (V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}) \frac{d}{dI_2}(V_2 I_2 \cos\phi_2)$$

$$-(V_2 I_2 \cos\phi_2)\cdot\frac{d}{dI_2}(V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}) = 0$$

$$\therefore (V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e})(V_2 \cos\phi_2) - (V_2 I_2 \cos\phi_2)(V_2 \cos\phi_2 + 2I_2 R_{2e}) = 0$$

Cancelling $(V_2 \cos\phi_2)$ from both the terms we get,

$$V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e} - V_2 I_2 \cos\phi_2 - 2 I_2^2 R_{2e} = 0$$

$$\therefore \quad P_i - I_2^2 R_{2e} = 0$$

$$\therefore \quad P_i = I_2^2 R_{2e} = P_{cu}$$

So condition to achieve maximum efficiency is that,

$$\boxed{\text{Copper losses} = \text{Iron losses}}$$

Numericals

TRANSFORMERS

Question 1:

The no load current of a transformer is 10 A at a power factor of 0.25 lagging, when connected to 400 V, 50 Hz supply. Calculate,

a) Magnetising component of no load current

b) Iron loss and c) Maximum value of flux in the core.

Assume primary winding turns as 500.

Solution : The given values are, $I_o = 10$ A, $\cos\phi_o = 0.25$, $V_1 = 400$ V and $f = 50$ Hz

a) $I_m = I_o \sin\phi_o$ = magnetising component

$\phi_o = \cos^{-1}(0.25) = 75.522°$

$\therefore \quad I_m = 10 \times \sin(75.522°) = \mathbf{9.6824\ A}$

b) P_i = Iron loss = Power input on no load

$= W_o = V_1 I_o \cos\phi_o = 400 \times 10 \times 0.25$

$= \mathbf{1000\ W}$

c) On no load, $E_1 = V_1 = 400$ V and $N_1 = 500$

$$E_1 = 4.44\, f\, \phi_m N_1$$

$$400 = 4.44 \times 50 \times \phi_m \times 500$$

$$\phi_m = \mathbf{3.6036\ mWb}$$

Question 2:

A 6600/400 V single phase transformer has primary resistance of 2.5 Ω and secondary resistance of 0.01 Ω. Calculate total equivalent resistance referred to primary and secondary.

Solution : The given values are,

$$R_1 = 2.5\ \Omega, \quad R_2 = 0.01\ \Omega$$

$$K = \frac{400}{6600} = 0.0606$$

While finding equivalent resistance referred to primary, transfer R_2 to primary as R'_2,

$$\therefore \quad R'_2 = \frac{R_2}{K^2} = \frac{0.01}{(0.0606)^2} = 2.7225\ \Omega$$

$$\therefore \quad R_{1e} = R_1 + R'_2 = 2.5 + 2.7225 = \mathbf{5.2225\ \Omega}$$

It can be observed that primary is high voltage hence high resistance side hence while transferring R_2 from low voltage to R'_2 on high voltage its value increases.

To find total equivalent resistance referred to secondary, first calculate R'_1,

$$R'_1 = K^2 R_1 = (0.0606)^2 \times 2.5 = 0.00918\ \Omega$$

$$\therefore \quad R_{2e} = R_2 + R'_1 = 0.01 + 0.00918 = \mathbf{0.01918\ \Omega}$$

Question 3:

A 250 kVA single phase transformer has iron loss of 1.8 kW. The full load copper loss is 2000 watts. Calculate,

i) Efficiency at full load, 0.8 lagging p.f.

ii) kVA supplied at maximum efficiency

iii) Maximum efficiency at 0.8 lagging p.f.

Solution : The given values are, $P_i = 1800$ W, (P_{cu})F.L. $= 2000$ W

i) $$\%\eta = \frac{(\text{VA rating})\cos\phi}{(\text{VA rating})\cos\phi + P_i + (P_{cu})\text{F.L.}} \times 100$$

$$= \frac{250\times10^3\times0.8}{250\times10^3\times0.8+1800+2000}\times100 = \mathbf{98.135\ \%}$$

ii) $$\text{kVA at } \eta_{max} = \text{kVA rating} \times \sqrt{\frac{P_i}{(P_{cu})\text{F.L.}}} = 250\times\sqrt{\frac{1800}{2000}}$$

$$= \mathbf{237.1708\ kVA}$$

iii) $$\eta_{max} = \frac{\text{kVA at } \eta_{max} \times \cos\phi}{\text{kVA at } \eta_{max} \times \cos\phi + P_i + P_i} \qquad P_{cu} = P_i = 1800\ \text{W}$$

$$\therefore \quad \%\eta_{max} = \frac{237.1708\times10^3\times0.8}{237.1708\times10^3\times0.8+2\times1800}\times100 = \mathbf{98.137\%}$$

Question 4:

The readings of direct-loading test on a single-phase transformer are :-

	Primary side			Secondary side		
	V_1 – V	I_1 – A	W_1 – W	V_2 – V	I_2 – A	W_2 – W
On no-load	220	0.7	40	102	0	0
On load	220	4.45	960	98	8.8	862.4

Find at the given load, its efficiency and regulation **(Dec. - 99)**

Solution : From the given table,

$$\text{Power output on load} = W_2 = 862.4 \text{ W}$$

$$\text{Power input} = W_1 = 960 \text{ W}$$

$$\therefore \quad \% \eta = \frac{\text{Power output}}{\text{Power input}} \times 100 = \frac{W_2}{W_1} \times 100$$

$$= \frac{862.4}{960} \times 100 = \mathbf{89.83 \ \%}$$

On no load, secondary voltage is 102 V

$$\therefore \quad E_2 = 102 \text{ V}$$

$$\therefore \quad \% \text{ Regulation} = \frac{E_2 - V_2}{E_2} \times 100 = \frac{102 - 98}{102} \times 100 = \mathbf{3.921 \ \%}$$

Question 5:

Observation table and some of the calculated values of a ***direct loading test*** *on 4 kVA, 200 / 400 V, 50 Hz, 1-ph, transformer are given below : -*

Input side					Output side						
Obs. No.	V_1	I_1	W_1	p.f.	V_2 volts	I_2 amps	W_2 watts	Losses Constant/ Variable watts		% efficiency •	% Regulation
	volts	amps	watts	•							
1	400	10.0	-	-	190	20	3040	64	-	92	-
2	400	7.82	-	•	-	12.9	2000	64	-	-	3 %

Question 5:

Fill up the blank space (marked–) with appropriate values in the following form :-

Obs. No. Quantity Formula Substitution Value (unit) Reason (if any)

(May-2001)

Solution : For obs. no. 1, $\% \eta = \frac{W_2}{W_1} \times 100$

$$\therefore \quad 92 = \frac{W_2}{W_1} \times 100$$

$$\therefore \quad W_1 = 3304.348 \text{ W}$$

Now $W_1 = V_1 I_1 \cos \phi_1$

$$\therefore \quad \cos \phi_1 = \frac{3304.348}{400 \times 10} = 0.826 \text{ lag}$$

No load voltage $= 200$ V

$$\therefore \quad \% R = \frac{\text{No load} - \text{Load voltage}}{\text{No Load}} \times 100 = \frac{200-190}{200} \times 100 = 5\%$$

$$\text{Total losses} = W_1 - W_2 = 3304.348 - 3040$$

$$= 264.348 \text{ W} = \text{Constant} + \text{Variable}$$

$$\therefore \quad \text{Variable} = 264.348 - 64 = 200.348 \text{ W}$$

For obs. no. 2, $= \frac{\text{no load} - \text{Load voltage}}{\text{no Load}} \times 100$

$$\therefore \quad 0.03 = \frac{200 - \text{Load voltage}}{200}$$

$$\therefore \quad V_2 = 194 \text{ V}$$

Now for $I_2 = 20$ A, Copper loss = Variable loss

$$= 200.348 \text{ W}$$

and $I_{2 \text{ F.L.}} = \frac{4 \times 10^3}{200} = 20$ A

$$\therefore \quad (P_{cu})_{\text{F.L.}} = 200.348 \text{ W}$$

Now New $I_2 = 12.9$ A, and $P_{cu} \propto I^2$

$$\therefore \quad \frac{(P_{cu})\text{ F.L.}}{(P_{cu})\text{ new}} = \left(\frac{I_{2\text{ F.L.}}}{I_{2\text{ new}}}\right)^2$$

$$\therefore \quad \frac{200.348}{(P_{cu})\text{ new}} = \left(\frac{20}{12.9}\right)^2$$

$$\therefore \quad (P_{cu})\text{ new} = 83.35\text{ W}$$

For load 2, Variable loss = 83.35 W

$$\therefore \quad \%\,\eta = \frac{o/p}{o/p + \text{losses}} \times 100 = \frac{2000}{2000+64+83.35} \times 100 = 93.13\%$$

and

$$\%\,\eta = \frac{W_2}{W_1} \times 100$$

$$\therefore \quad 0.9313 = \frac{2000}{W_1} \quad \text{hence } W_1 = 2147.35\text{ W}$$

Question 6:

A 6600/220 V, 50 Hz, step-down single-phase transformer has 1500 turns on its primary side. Find (i) the secondary turns, (ii) the effective cross-sectional area of its core if the maximum flux density is 1.2 tesla. **(May - 2004)**

Solution :

$V_1 = 6600$ V, $V_2 = 220$ V, $N_1 = 1500$, $B_m = 1.2$ T

i) $$\frac{V_1}{V_2} = \frac{N_1}{N_2} \quad \text{i.e.} \quad \frac{6600}{220} = \frac{1500}{N_2}$$

$$\therefore \quad N_2 = 50$$

ii) $$V_1 = 4.44\ \phi_m f N_1 \quad \text{i.e.} \quad 6600 = 4.44 \times \phi_m \times 50 \times 1500$$

$$\therefore \quad \phi_m = 19.8198\text{ mWb}$$

Now $$\phi_m = B_m \times a$$

$$\therefore \quad 19.8198 \times 10^{-3} = 1.2 \times a$$

$$\therefore \quad a = 16.5165 \times 10^{-3}\ \text{m}^2$$

Question 7:

A single phase transformer when connected to a lamp load gave following results :

Sr. No.	V_1	I_1	W_1	V_2	I_2	W_2
1.	200	1.5	60	100	0	0
2.	200	12.9	2510	97	25	2425

Calculate efficiency and % Regulation of transformer at I_2 = 25 amp (second reading).

(May - 2005)

Solution : At $I_2 = 25$ A, from the given table,

$$W_1 = 2510 \text{ W} \quad \text{and} \quad W_2 = 2425 \text{ W}$$

$$\therefore \quad \% \eta = \frac{W_2}{W_1} \times 100 = \frac{2425}{2510} \times 100 = \mathbf{96.6135\ \%}$$

Ans $\quad E_2 = V_2$ on no load = 100 V

While $\quad V_2 = 97$ V on $I_2 = 25$ A

$$\therefore \quad \% R = \frac{E_2 - V_2}{E_2} \times 100 = \frac{100-97}{100} \times 100 = \mathbf{3\ \%}$$

Question 8:

The no load current of a transformer is 10 A at a power factor of 0.25 lagging, when connected to 400 V, 50 Hz supply. Calculate,

a) Magnetising component of no load current

b) Iron loss and c) Maximum value of flux in the core.

Assume primary winding turns as 500.

Solution : The given values are, $I_o = 10$ A, $\cos \phi_o = 0.25$, $V_1 = 400$ V and f = 50 Hz

a) $\quad I_m = I_o \sin \phi_o$ = Magnetising component

$$\phi_o = \cos^{-1}(0.25) = 75.522°$$

$$\therefore \quad I_m = 10 \times \sin(75.522°) = \mathbf{9.6824\ A}$$

b) $\quad P_i$ = Iron loss = Power input on no load

$$= W_o = V_1 I_o \cos \phi_o = 400 \times 10 \times 0.25$$

$$= \mathbf{1000\ W}$$

c) On no load, $E_1 = V_1 = 400$ V and $N_1 = 500$

Now $E_1 = 4.44 \, f \, \phi_m \, N_1$

$\therefore \quad 400 = 4.44 \times 50 \times \phi_m \times 500$

$\therefore \quad \phi_m = \mathbf{3.6036 \ mWb}$