Rashtreeya Sikshana Samithi Trust

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Department of Department Engineering

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Module 3

Beams and Trusses

*Support Reaction in beams -*Types of Loads and Supports, statically determinate beams, Numerical problems on support reactions for statically determinate beams with Point load (Normal and inclined) and uniformly distributed and uniformly varying loads and Moments.

Analysis of simple trusses: types of trusses, analysis of statically determinate trusses using method of joints and method of sections

3.1 BEAMS

3.2 Introduction

A beam is a structural member or element, which is in equilibrium under the action of a non- concurrent force system. The force system is developed due to the loads or forces acting on the beam and also due to the support reactions developed at the supports for the beam. For the beam to be in equilibrium, the reactions developed at the supports the should be equal and opposite to the loads. In a beam, one dimension (length) is considerably larger than the other two dimensions (breath & depth). The smaller dimensions are usually neglected and as such a beam is represented as a line for theoretical purposes or for analysis.

When the beams are subjected to different types of loads, supports will offer reactions to attain equilibrium. such reactions are called as support reactions. support reactions for statically determinate beams are calculated using basic conditions of equilibrium.

3.3 Types of Supports:

Supports are structures which prevent the beam or the body from moving and help to maintain equilibrium. A beam can have different types of supports as follows. The support reactions developed at each support are represented as follows.

1. **Simple support:** This is a support where a beam rests freely on a support. The beam is free to move only horizontally and also can rotate about the support. In such a support one reaction, which is perpendicular to the plane of support, is developed. Fig 3.1 shows a simple support.

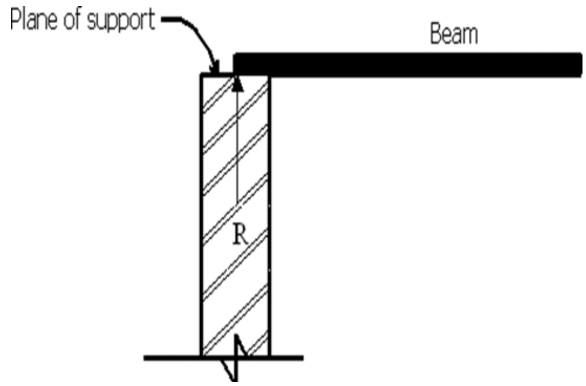


Fig 3.1 Simple support

Roller support: This is a support in which a beam rests on rollers as shown in Fig 3.2, which are frictionless. At such a support, the beam is free to move horizontally and as well rotate about the support. Here one reaction which is perpendicular to the plane of rollers is developed. Fig 3.2 shows a roller support.

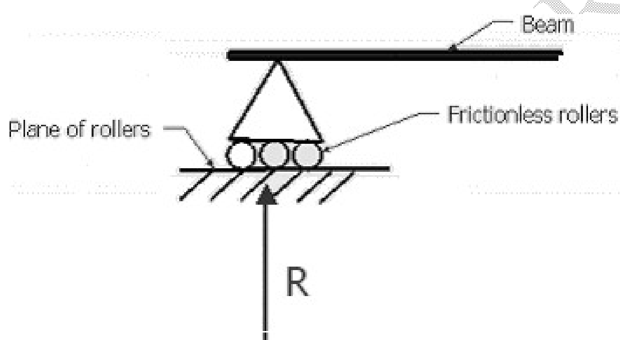


Fig **3.**2 Roller support

Hinged support: This support is also called as pinned support. The beam is not free to move in any direction but can rotate about the support. In such a support a horizontal reaction and a vertical reaction will develop. Fig 3.3 shows a hinged support

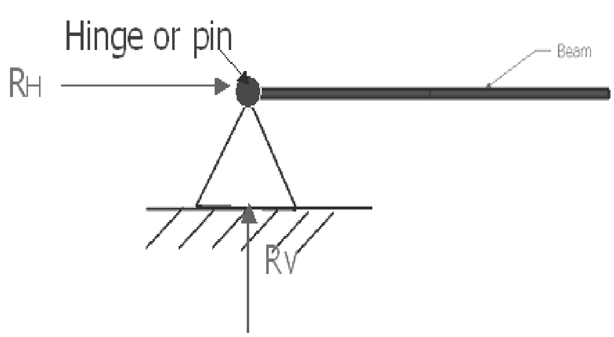


Fig 3.3 Hinged support

Fixed support: This is a support which prevents the beam from moving in any direction and also prevents rotation of the beam. In such a support a horizontal reaction, vertical reaction and a Fixed End Moment are developed to keep the beam inequilibrium. Fig 3.4 shows a fixed support.

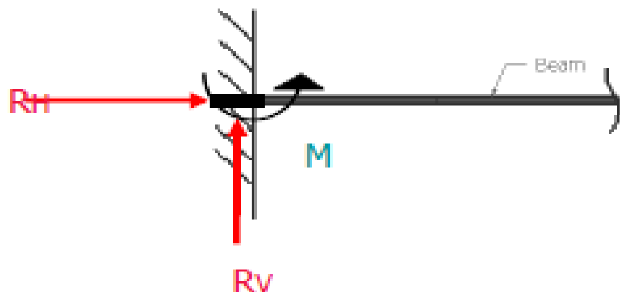


Fig 3.4 Fixed support

3.4 Types ofbeams

Depending upon the supports over which a beam can rest (at its two ends), beams can be classified as follows.

1. **Simply supported beam:**
A beam is said to be simply supported when both ends of the beam rest on simple supports. Such a beam can carry or resist vertical loads only as shown in Fig 3.5.

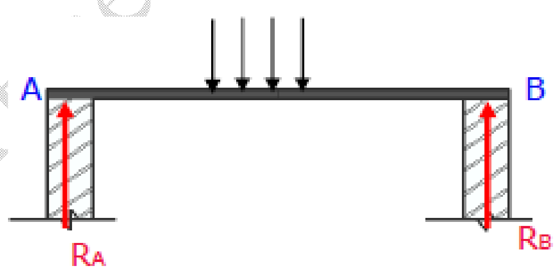


Fig 3.5 Simple supported beam

2. **Over hanging beam :**It is a beam which projects beyond the supports. A beam can have over hanging portions on one side or on both sides as shown in Fig 3.6

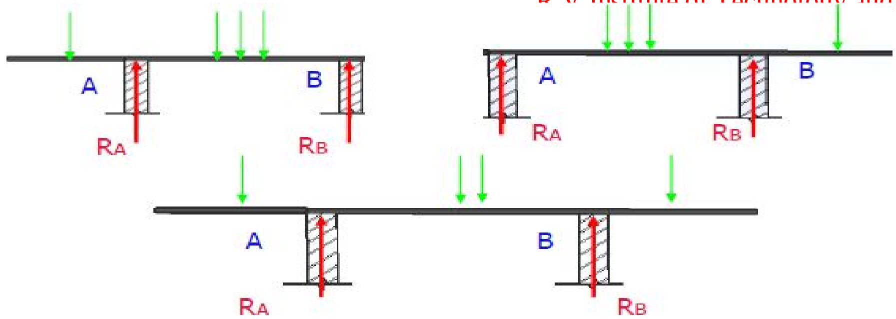


Fig 3.6 Over hanging beam

3. **Cantilever Beams:** It is a beam, with one end fixed and other and free. Such a beam can carry loads in any directions as shown in Fig 3.7

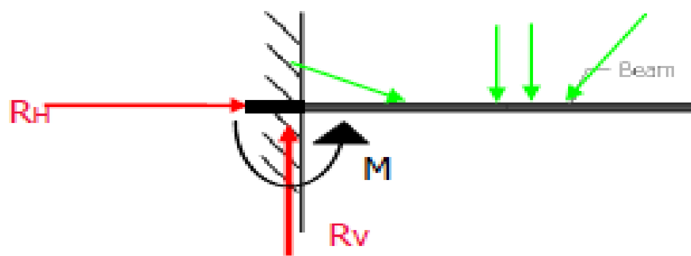


Fig 3.7 Cantilever beam

4. **Propped cantilever:** It is a beam which has a fixed support at one end and a simple support at the otherend as shown in Fig 3.8.

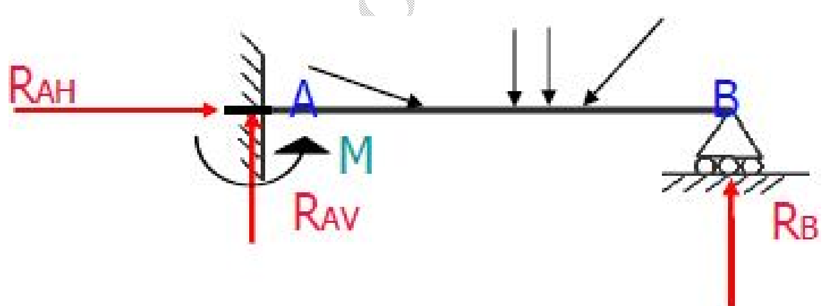


Fig 3.8 Propped cantilever beam

5. **Continuous beam:** It is a beam which rests over a series of supports at more than two points as shown in Fig 3.9.

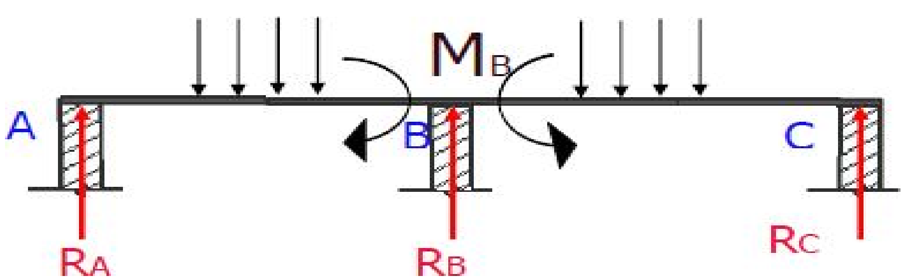


Fig 3.9 Continuous Beam

Note:

(i) Support reactions for a beam can be calculated using conditions of Equilibrium (ΣFx = 0, ΣFy = 0, ΣM = 0) when beams have not more than three unknown reactions. Those beams which can be solved to find reactions using only conditions of equilibrium are called as statically determinate beams. Simply supported beams, beam with one end hinged and other on rollers, over hanging beams, and cantilever beams are statically determinate beams

(ii) Beams which have more than three unknown support reactions are called as statically indeterminate beams. In beams such as Hinged Beams, Propped Cantilever and Continuous Beams the support reactions cannot be determined using conditions of equilibrium only and further equations are to be developed to solve for finding support reactions.

3.5 Types of loads:

The various types of loads that can act over a beam can e listed as follows.

1. **Point load or Concentrated load:** If a load acts over a very small length of the beam, it is assumed to act at the midpoint of the loaded length and such a loading is termed as Point load or Concentrated load as shown in Fig 3.10.

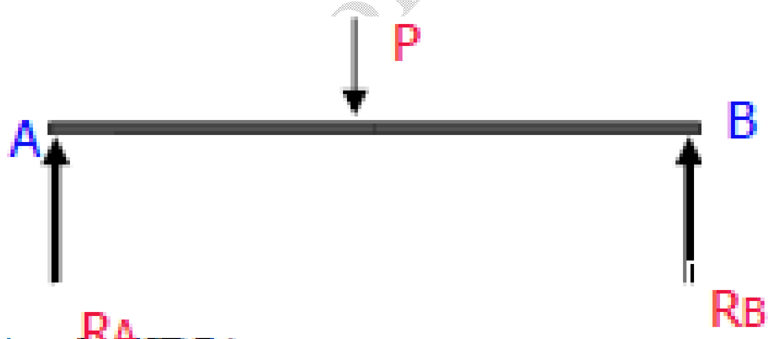


Fig 3.10 Point Load

2. **Uniformly distributed load (UDL):** If a beam is loaded in such a manner that each unit length of the beam carries the same intensity of loading, then such a loading is called UDL. A UDL cannot be considered in the same manner for applying conditions of equilibrium on the beam. The UDL should be replaced by an equivalent point load or total load acting through the

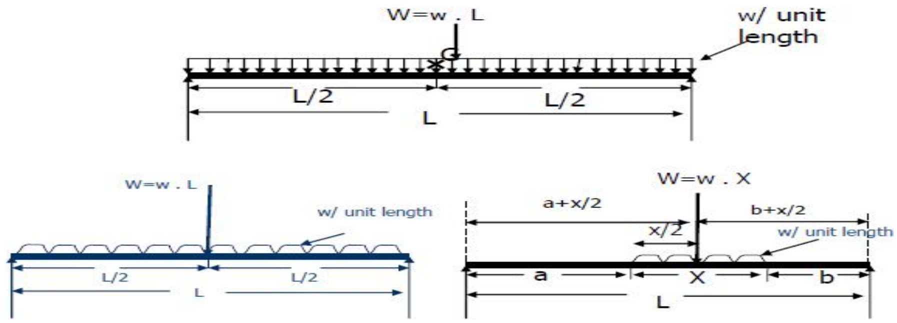


Fig 3.11 Uniformly Distrubuted Load

midpoint of the loaded length. The magnitude of the point load or total load is equal to the product of the intensity of loading and the loaded length (distance). Fig 3.11 shows the UDL on a beam.

3. **Uniformly varying load (UVL):** If a beam is loaded in such a manner, that the intensity of loading varies linearly or uniformly over each unit distance of the beam, then such a load is termed as UVL.In applying conditions of equilibrium, a given UVL should be replaced by an equivalent point load or total load acting through the centroid of the loading diagram (right angle triangle). The magnitude of the equivalent point load or total load is equal to the area of the loadingdiagram as shown in Fig 3.12.

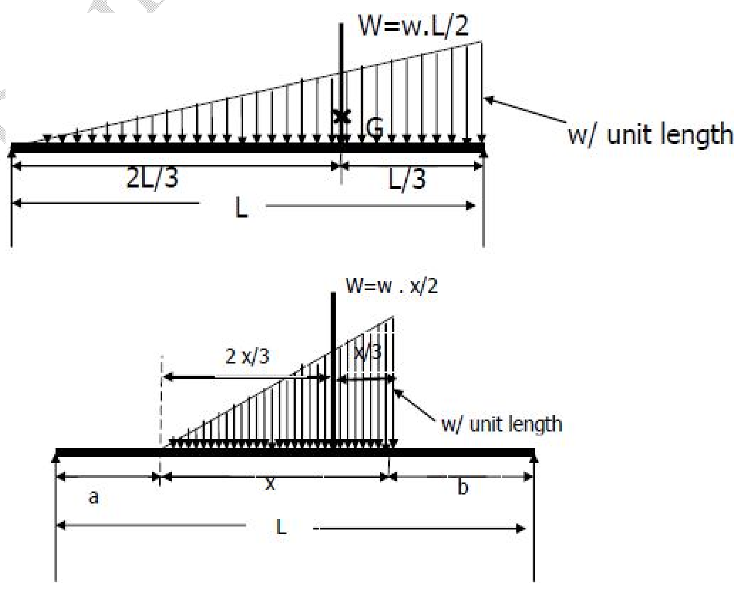


Fig 3.12 Uniformly Varying Load

4. **External moment:** A beam can also be subjected to external moments at certain points as shown in Fig 3.13. These moments should be considered while calculating the algebraic sum of moments of forces about a point on the beam

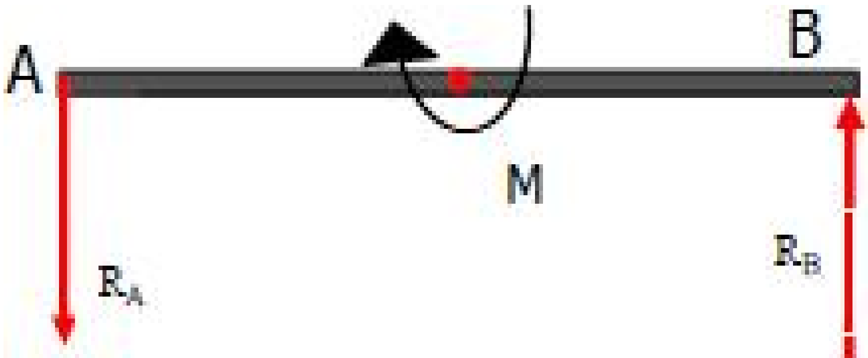
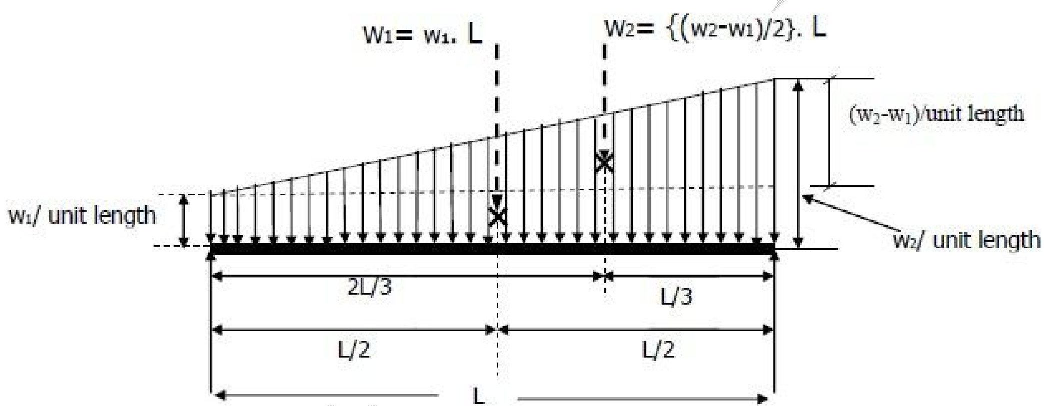


Fig 3.13 External moment

Note:A beam can also be subject to a load as shown in figure below with combination of UVL and UDL.



In such a case, the UVL can be split into a UDL with a uniform intensity of w1/unit length another UVL with a maximum intensity of (w2-w1) /unit length.

3.6 STEPS TO SOLVEPROBLEMS

1.Beam subjectedto loading is a coplanar non-concurrent force system.

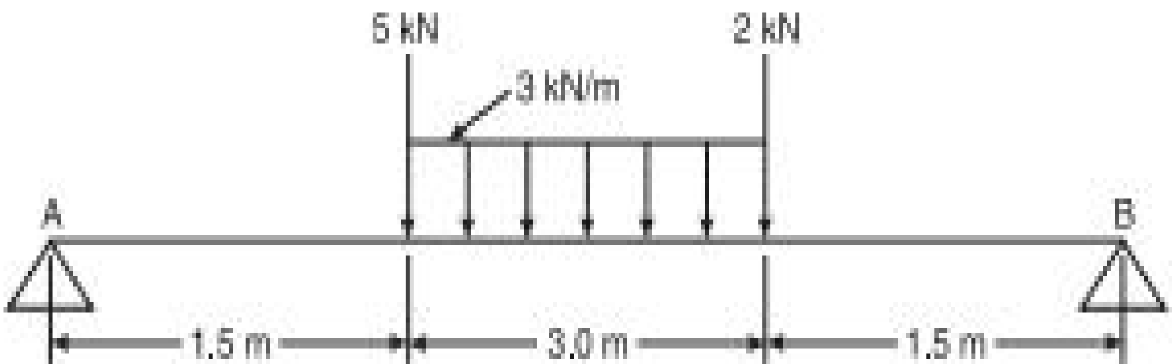
2.Herethree conditions of equilibrium can be applied, namely :

$\sum Fx = 0$, $\sum F\ y = 0$ and $\sum M = 0$

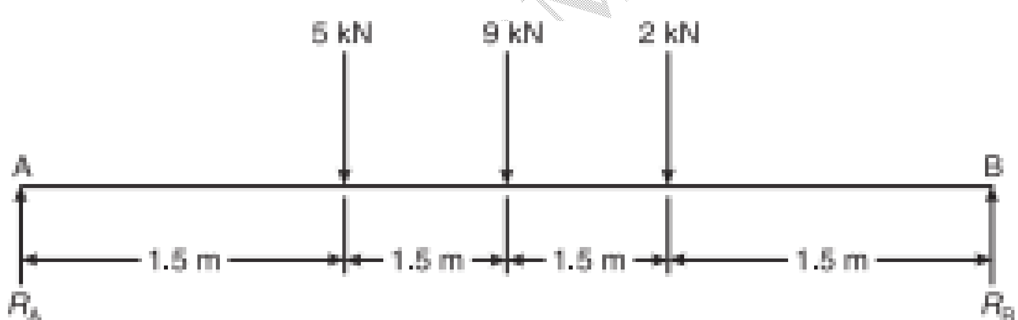
3. Draw the free body diagram of the given beam by showing all the forces and reactions acting on thebeam.
4. Apply the three conditions of equilibrium to calculate the unknown reactions at the supports.

PROBLEMS:

1. A simply supportedbeam of 6m span is loaded as shown in Figure below. Find the reactions at A andB.

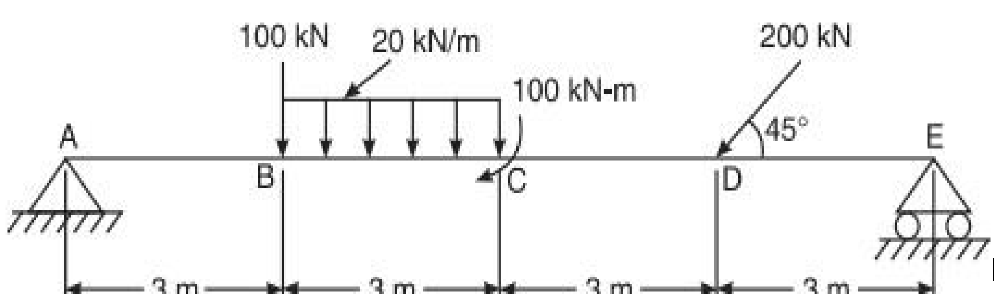


Writing FBD of the beam, converting the UDL of 3 kN/m over a span of 3 m into a point load

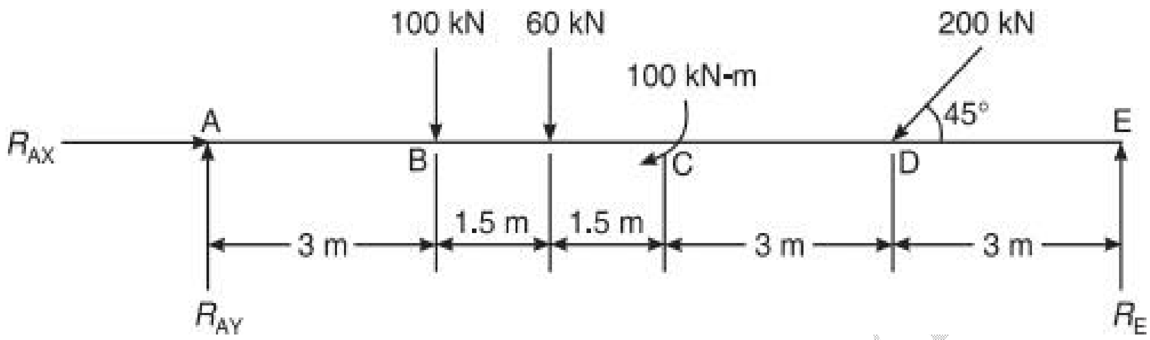


$\sum M_B = 0$
(Clockwise moment +ve)
$- RA \times 6 + 2 \times 1.5 + 9 \times 3 + 5 \times 4.5 = 0$
$\Rightarrow RA = 52.5/6 = +$ **8.75 kN**
$\sum Fy = 0$
$RA + RB = 5 + 9 + 2 = 16$ kN
$RB = 16 - 8.75 = 7.25$ kN

2. Determine the reactions at A and E for the beam shownbelow



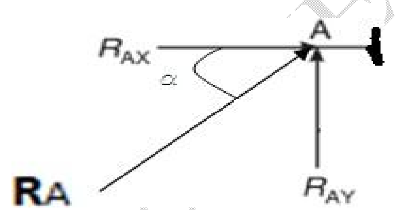
Converting the UDL of 20 kN/m over a span of 3 m into a point load, and applying the laws of equilibrium we can write FBD as below.



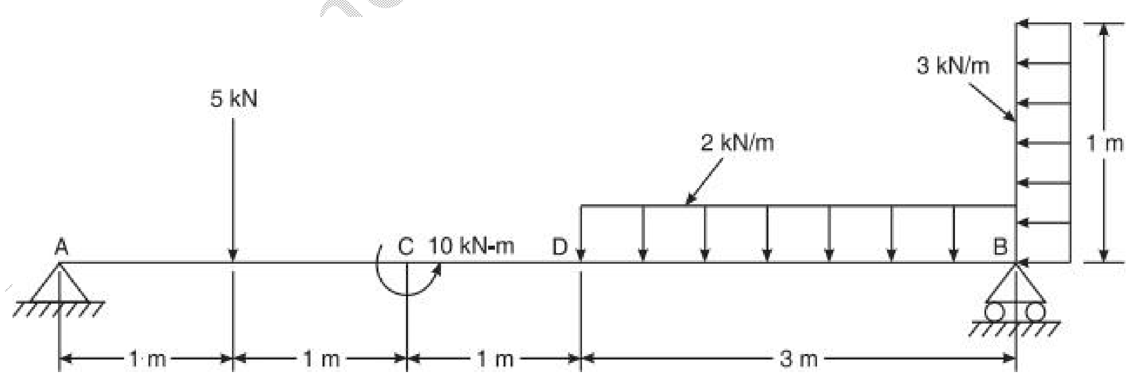
$\sum Fx = 0$

=> $RAX - 200 \cos 45° = 0$
=>**$RAX = +141.421$ kN**
$\sum MB = 0$
(Clockwise moment +ve)
$-RE \times 12 + 200 \sin 45° \times 9 + 60 \times 4.5 + 100 \times 3 + 100 = 0$
=>**$RE = +161.899$ kN**

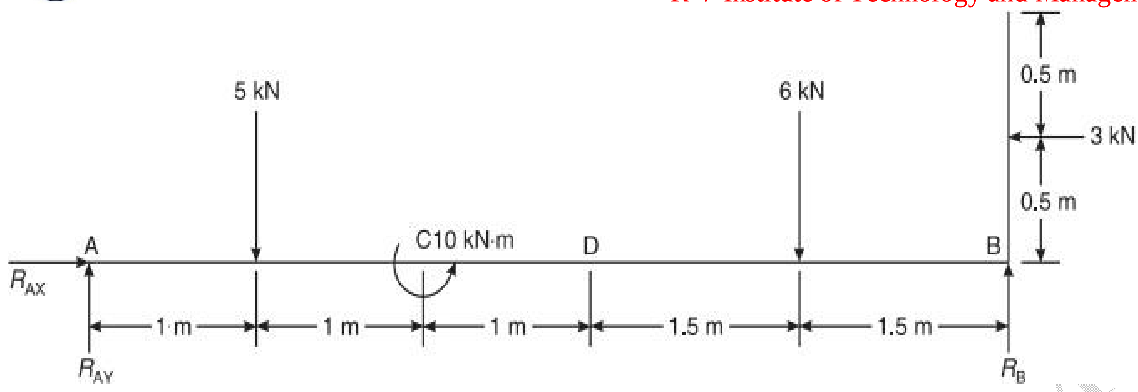
$\sum Fy = 0$
$RE + RAY - 100 - 60 - 200 \times \sin 45° = 0$
=> $RE + RAY = 301.421$
=> $161.899 + RAY = 301.421$
$RA = \sqrt{(RAX^2 + RAY^2)}$
$RA = \sqrt{(141.422^2 + 139.522^2)}$
⇨ **RA =198.662kN**
a = $\tan^{--1}(RAY/RAX)$
= $\tan^{-1}(139.522/ 141.422)$
a = 44.61 °



4. Find the reactions at supports A and B for the beam loaded as shownbelow.

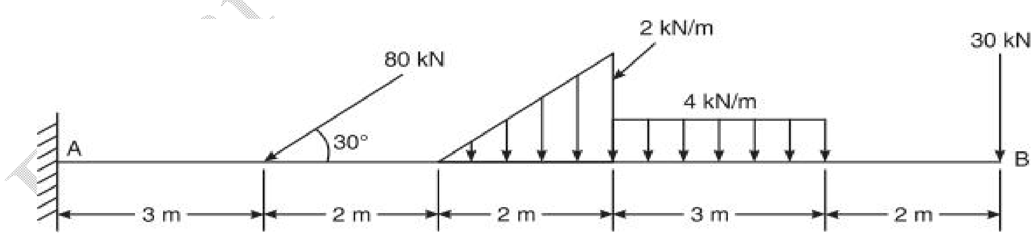


Converting UDL to equivalent point load at center, FBD of the beam is as given below.

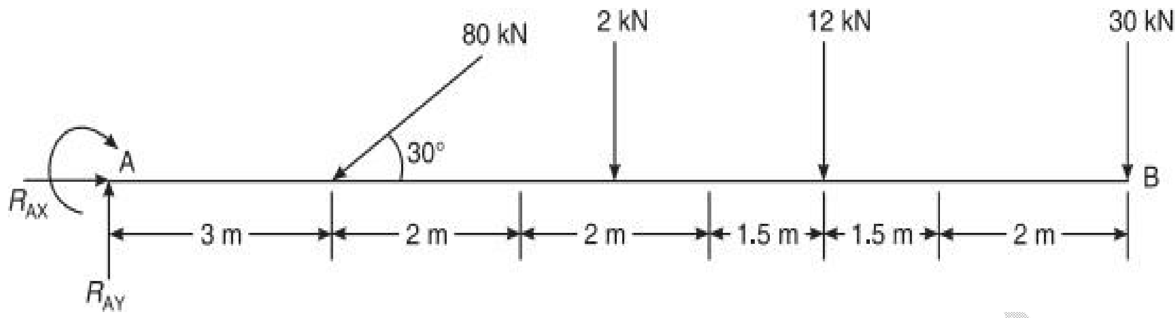


$\sum Fx = 0$
$+ RAX – 3 = 0$
=>**RAX = + 3 kN**
$\sum Fy = 0$
$+RAY + RB – 5 – 6 = 0$
=> RAY + RB=11 ---(1)
Also, we have,
$\sum MA = 0$
(Clockwise moment +ve)
-RB x 6 + 5 x 1 + 6 x 4.5 – 3 x 0.5 – 10 =0
=>**RB = +5.417 kNSubstituting value of RB inEqn (1) RAY + 5.417 = 11**
RAY = +7.583 kN
$RA = \sqrt{(RAX^2 + RAY^2)}$
$RA = \sqrt{(3^2+7.583^2)}$
=>**RA = 8.155kN**
a = $\tan^{-1}(7.583/3)$
a = 68.41 °

3. Find the reactions for the cantilever beam loaded asshown.



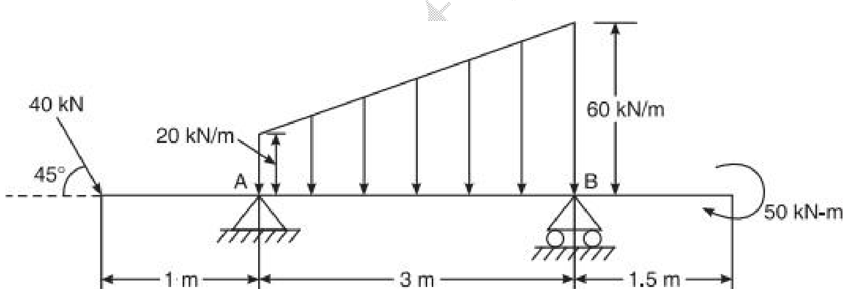
Converting UDL and UVL to respective point loads and writing FBD we get, Point load due to UDL= 4 x 3 = 12 kN
Point load due to UVL= Area of the triangle= 0.5 X 2 x 2 = 2 kN.



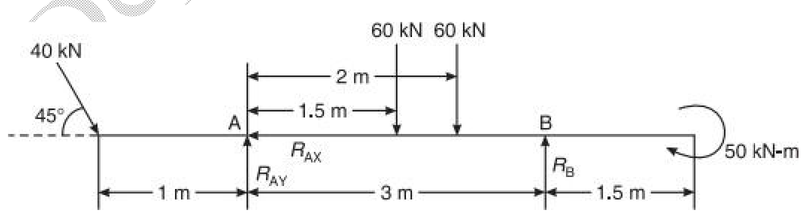
$\sum Fx = 0$
+RAX- 80 cos 30° =0
=>**RAX = +69.282kN**

$\sum Fy = 0$
+RAY – 80 sin 30° – 2 – 12 – 30 = 0
=>**RAY = + 84 kN**
$\sum MA = 0$
(Clockwise moment +ve)
+80 sin 30° x 3 + ((2 x 2/3)+ 5) + 12 x 8.5 + 30 x 12 + MA = 0
=> MA= -594.667 kN-m.

4. Find the reactions for the beam loaded asshown.



Converting trapezoidal load into UDL of 20kN/m and UVL of 0 intensity at left end and 40kN/m at right end, FBD of beam can be written asfollows.



$\sum Fx$ = 040 x cos 45° – RAX = 0
=>**RAX = +28.284 kN**
$\sum Fy = 0$
=> RAY + RB – 40 sin 45° – 60 – 60 = 0

=> RAY + RB=148.284 ---------------------(1)

Applying ∑MA = 0 ,

(Clockwise moment +ve)

–RB x 3 + 60 x 1.5 + 60 x 2 + 50 – 40 sin 45 x 1 = 0

=>**RB = +77.238 kN**

Substituting value of RB in (1)

RAY = 71.046 kN

RA = $\sqrt{(RAX^2 + RAY^2)}$

= $\sqrt{(28.284^2 + 71.046^2)}$

=>**RA = 78.469 kN**

a = $\tan^{-1}(71.046/28.284)$

a = 68.292

Truss

A truss is an assembly of beams or other elements that creates a rigid structure. In engineering, a truss is a structure that "consists of two-force members only, where the members are organized so that the assemblage as a whole behaves as a single object". A "two-force member" is a structural component where force is applied to only two points. Although this rigorous definition allows the members to have any shape connected in any stable configuration, trusses typically comprise five or more triangular units constructed with straight members whose ends are connected at joints referred to as nodes.

Types of truss

Trusses can be classified based on its stability and relationship between number of joints and number of members.

(i) Perfect truss: - A truss that can be analyzed by conditions of equilibrium and it has sufficient number of members and joints in the truss and satisfies the equation

$$m = 2j - 3$$

where m = number of members

j = number of joints

(ii) Imperfect truss: - A truss which doesn't satisfy the equation $m = 2j - 3$ is called as imperfect truss. If m is greater than $(2j - 3)$ it is called as redundant truss and m is less than $(2j - 3)$ it is called deficient truss.

There are two methods to analyze trusses

(i) Method of joints: - The method of joints is one of the simplest methods fordetermining the force acting on the individual members of atruss because it only involves two force equilibrium equations.

The method of joints is one of the simplest methods fordetermining the force acting on the individual members of atruss because it only involves two force equilibrium equations.Since only two equations are involved, only two unknowns canbe solved for at a time. Therefore, solve thejoints in a certain order. That is, start from thesides towards the center of the truss.

Steps in solving method of joints

1. Check whether the truss is perfect or not
2. Find support reactions
3. Assume forces in all members to be tensile
4. Select a joint which has a maximum of two unknowns and any one known force at the joint.
5. Apply conditions of equilibrium and find the forces in members
6. If the force is positive it is tensile and if it is negative it will be compressive (once a force is established as tensile or compressive, it shld be taken as

Method of sections: -

The Method of Sections involves analytically cutting the truss into sections and solving for static equilibrium for each section. The sections are obtained by cutting through some of the members of the truss to expose the force inside the members. In the Method of Joints, we are dealing with static equilibrium at a point. This limits the static equilibrium equations to just the two force equations. A section has finite size and this means you can also use moment equations to solve the problem. This allows solving for up to three unknown forces at a time.

Steps involved in method of sections

1. Check if the truss is perfect
2. Draw a section such that it cuts three members
3. Select either half of the truss and apply conditions of equilibrium until we obtain all the required forces