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ELEMENTS OF CIVIL ENGINEERING & MECHANICS
MODEL QUESTION PAPER - 1

MODULE - 1

Q1) (a) Explain briefly scope of civil engineering in:
(i) Irrigation Engineering
(ii) Structural Engineering.

(i) SCOPE OF IRRIGATION ENGINEERING:
* Storage of water by constructing dam as reservior
* Diversion of stored water to canals for distribution.
* Development of hydroelectric power.
* Reclaimation of waste and alkaline lands.
* Capacities of different soils for irrigation water.
* Drainage and relieving of water logging, for high productivity of canal.

(ii) SCOPE OF STRUCTURAL ENGINEERING:
* Planning, designing and building the structure.
* Creation of a structural system in accordance with needs to client.
* Plan and design of nuclear power plant keeping in mind environment safety & thermal pollution.
* Responsibility of the safety and serviceability of structure for lifetime.
* Acceptance to natural calamities like earthquake, wind and landslide.
* Introduction of new techniques, technologies for safe and economic construction.

(b) Explain briefly application of any two smart materials in civil engineering.

Types of smart materials:

(i) Shape Memory Alloys

(ii) Magnetostrictive Materials

(iii) Piezoelectric Materials

(iv) Electrorheological Fluids

(v) Electrochromic Materials.

(i) SHAPE MEMORY ALLOYS : [SMA]

* SMAs are used in pre stressing concrete due to no elastic shortening, no involvement of jacking.

* Super elastic property of SMA are utilised to regain the preload drops in joints and thus provide clamping force.

* Intelligent Reinforced concrete was developed using SMA which had properties like self-rehabilitation, self vibration

* SMAs are used in bolted joints. This enables to reduce damages by dissipiating large amount of energy.

(ii) ELECTROCHROMIC MATERIALS:

* It is placed between two glass panes in the windows.

* An electrochromic window (EW) is that which allows modulations of light transmission and reflections.

* EW's are combined with solar cells so that the power required to bring colour change can be achieved.

* Electrochromic materials are also used in information displays in airports.

(c) What are the requirements of good cement?

REQUIREMENTS OF GOOD CEMENT:

* colour of the cement must be uniformly grey with greenish shade.
* Free from hard lumps
* It should not contain excess silica, lime, alumina or alkalies.
* Ratio of percentage of alumina to iron oxide should not be less than 0.66.
* Cement thrown in water should sink and not float on the surface.
* It should offer good resistance to the moisture.
* A thin paste of cement with water should feel sticky between the fingers.
* If hand is inserted in a bag of cement, it should feel cool and not warm.
* The cement should not contain excess of silica, lime, alumina or alkalies.

Q2) (a) Explain briefly the scope of civil engineering in:

(i) Transportation Engineering

(ii) Water resource Engineering

SCOPE OF TRANSPORTATION ENGINEERING:

* contribution to economic, social, cultural and the industrial development of any country.
* To provide public transport & mass transport.

* Evaluating the alternatives using cost or benefit ratio techniques.
* It involves accident study for safe and comfort transport system.
* Traffic performance and control
* It helps to increase the urbanization and industrialisation
* Provide cordination amongst various modes of transportation.

SCOPE OF WATER RESOURCE ENGINEERING:
* control, regulate and utilize water to serve wide variety of purposes.
* Preservation of natural beauty of flora and fauna.
* Flood mitigation, land drainage, sewage get scope from this.
* To protect fish, wildlife and recreational use of water resource.
* Water quality management and pollution control is an important phase

(b) Explain briefly i) RCC ii) PSC

(i) RCC:
* Reinforced cement concrete is an superior version of ordinary portland cement
* Steel bars are used alongside cement to make the structure more stable.
* It is highly compressive as well as has a good tensile strength.

* The tensile strength is due to the addition of steel bars.
* It can be cast in-situ or pre-cast / prefabricated
* It is extensively used for retaining walls, concrete roads, bunkers, water tanks and framed structures.

(ii) PSC:

* Pre stressed concrete uses steel wires instead of steel bars as reinforcement.
* It specially uses specially manufactured high tensile strength steel wires.
* There are two types of pre-stressing: Pre tensioning and post tensioning
* In pre tensioning: tension is applied on the cable before concrete hardens or sets.

The concrete gets compressive stress during hardening.

* In post tensioning: tension is applied on the cable after concrete hardens.

The concrete gets compressive stress after setting up of the prestress

(c) What are the advantages of stone construction over brick construction?

ADVANTAGES OF STONE CONSTRUCTION OVER BRICK:

* Stone masonary is stronger and more durable than brick masonary
* It is not essential to plaster the stone work when exposed to open atmosphere.

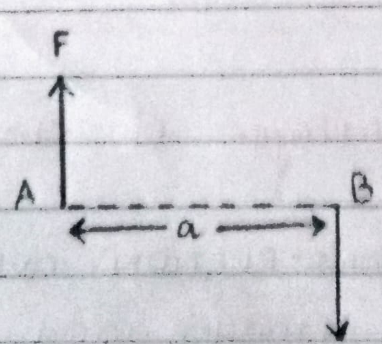
* Stones are less as adsorbent than bricks, so stone construction buildings are more damp proof.
* Brick masonary cannot be allowed to come in contact with sewage or drainage, but this isn't the case with stone construction.
* Stone masonary gives massive appearance to the buildings.
* More architectural effects can be given in stone construction compared to brick.
* Building made using stone construction do not require more maintainance and repairs.

MODULE-2:

Q3) (a) Explain couple and its characteristics.

COUPLE:

When two equal and opposite parallel forces act on a body, at some distance apart, these two forces constitute a couple.



The perpendicular distance between the parallel forces is known as arm of couple or lever arm of the couple.

CHARACTERISTICS:

* The algebraic sum of forces constituting the couple is zero.
* The algebraic sum of moments of the force constituting the couple about any point is equal to moment of the couple itself.
* A couple can be balanced only by an equal and opposite couple in same plane
* Moment of the couple is constant for any point chosen.
* Any two couples whose moments are equal and of same sign are equivalent.
* Many coplanar couples can be replaced to a single couple, whose moment is equal to the magnitude of moments of individual couples.

(b) The sum of two concurrent forces P and Q is 500N and their resultant is 400N. If the resultant is perpendicular to P, find P, Q and angle between P & Q.

GIVEN: $P+Q = 500\ N$

$R = 400\ N.$

$\alpha = 90^\circ$

$$\tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta}$$

$$\Rightarrow P+Q\cos\theta = 0$$

$P = 500 - Q.$

$\Rightarrow 500 - Q + Q\cos\theta = 0$

$\Rightarrow Q\cos\theta = Q - 500. = -P$

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$\Rightarrow (400)^2 = P^2 + Q^2 + 2P(-P)$

$\Rightarrow 160000 = P^2 + Q^2 - 2P^2$

$\Rightarrow -P^2 + Q^2 = 160000$

$\Rightarrow (Q+P)(Q-P) = 160000$

$\Rightarrow Q - P = \frac{160000}{500}$

$\Rightarrow Q - P = 320\text{ N}.$

$Q + P = 500$

$Q - P = 320$

$2Q = 820$

$\Rightarrow \boxed{Q = 410\text{ N}}$

$P = 500 - 410$

$\Rightarrow \boxed{P = 90\text{ N}}$

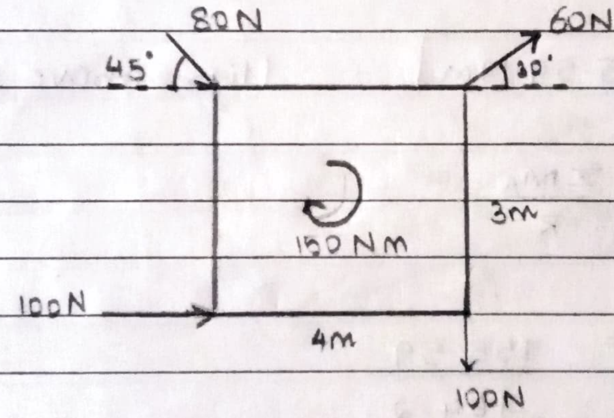
$P + Q\cos\theta = 0$

$\Rightarrow 90 + 410\cos\theta = 0$

$\Rightarrow \cos\theta = \frac{-9}{41}$

$\Rightarrow \boxed{\theta = 77.32^\circ}$

(c) Determine the resultant of the force system acting on the plate wrt AB and AD.



$\Sigma F_x = 100 + 80\cos 45 - 60\cos 30^\circ$

$\Rightarrow \boxed{\Sigma F_x = 104.61\ N.}$ $\longrightarrow$ +ve

$\Sigma F_y = -100 + 60\sin 30 - 80\sin 45$

$\Rightarrow \boxed{\Sigma F_y = -126.57\ N.}$ $\longrightarrow$ -ve

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$$R = \sqrt{(104.61)^2 + (-126.57)^2}$$

$$\Rightarrow \boxed{R = 164.2\ N} \quad \text{(Lies in IV Quadrant)}$$

Moment about A:

$\cancel{\Sigma M_A = 80\cos 45 + 60\sin 30 + 6}$

$$\Sigma M_A = 80\cos 45\,(3) - 60\sin 30\,(4) + 60\cos 30\,(3) + 100(4) + 150.$$

$$\rightarrow \boxed{\Sigma M_A = 755.59\ Nm} \quad \text{(lies above point A).}$$

$$d = \frac{\Sigma M_A}{R}$$

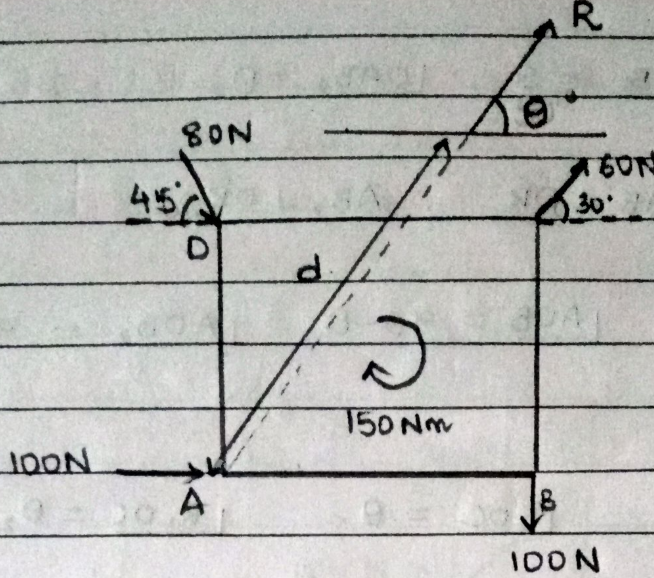
$$= \frac{755.59}{164.2}.$$

$$\Rightarrow \boxed{d = 4.6\,m}$$

$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x} \qquad x = \frac{\Sigma M}{\Sigma F_y}$$

$$\Rightarrow \theta = \tan^{-1}\left[\frac{126.57}{104.61}\right] \qquad \Rightarrow x = \frac{755.59}{-126.57}$$

$$\Rightarrow \boxed{\theta = 50.43^\circ} \qquad \Rightarrow x = -5.97\,m$$

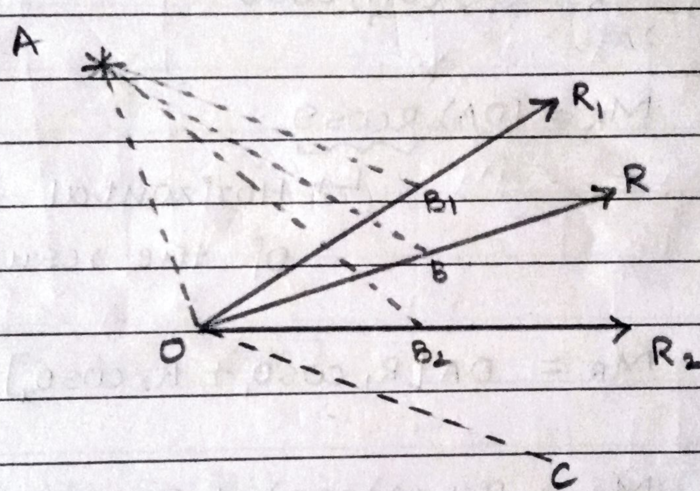


$$y = \frac{\Sigma M}{\Sigma F_x}$$

$$\Rightarrow y = \frac{755.59}{104.61}$$

$$\Rightarrow y = \underline{\underline{7.22\,m}}$$

Q4) (a) State and prove Varignon's principle of moments



Consider forces R_1 and R_2. Let A be the moment centre and let R be the resultant of the two forces. Join AO and draw its perpendicular OC.

Drop perpendiculars AB_1, AB and AB_2 to R_1, R and R_2 respectively.

Let $AB_1 = d_1$ $AB = d$ $AB_2 = d_2$

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$\angle OAB_1 = \theta_1$ $\quad \angle OAB = \theta$ $\quad \angle OAB_2 = \theta_2$

$AB_1 \perp OR_1$ $\quad AB \perp OR$ $\quad AB_2 \perp OR_2$

$\angle AOB_1 = 90 - \theta_1$ $\quad \angle AOB = 90 - \theta$ $\quad \angle AOB_2 = 90 - \theta_2$

$\Rightarrow \angle R_1OC = \theta_1$ $\quad \angle ROC = \theta$ $\quad \angle R_2OC = \theta_2$

consider ΔOAB; $\quad d = OA\cos\theta$

$$\Rightarrow M_R = (R)(OA)\cos\theta$$

$$\Rightarrow M_R = (OA)\underbrace{R\cos\theta}$$

↳ Horizontal component of the resultant

$$\Rightarrow M_R = OA[R_1\cos\theta_1 + R_2\cos\theta_2]$$

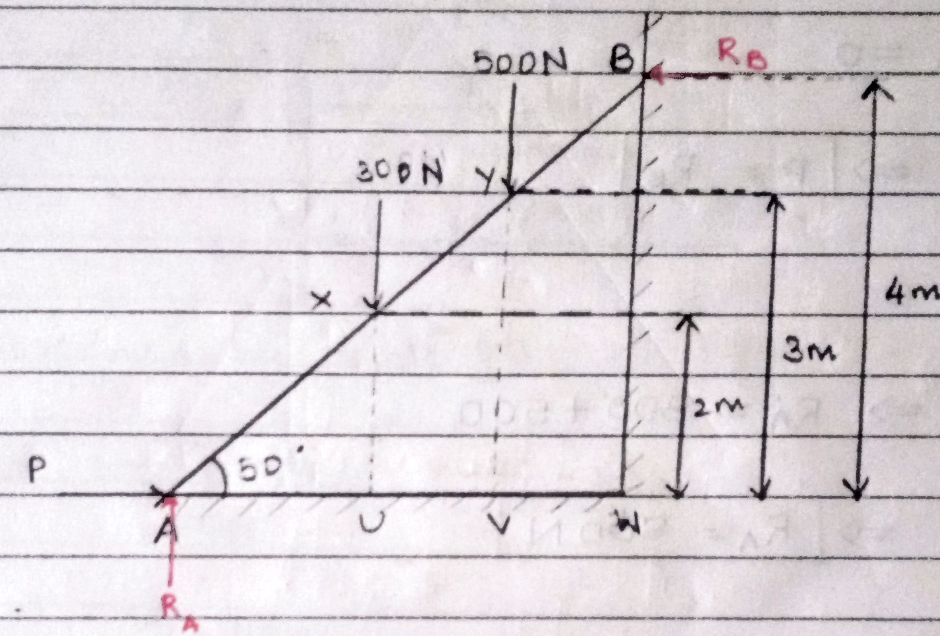
$$\Rightarrow M_R = R_1(OA\cos\theta_1) + R_2(OA\cos\theta_2)$$

$$\Rightarrow M_R = R_1d_1 + R_2d_2$$

$$\Rightarrow \boxed{M_R = \Sigma M_F}$$

Hence proved

(b) A ladder weighing 300N is to be kept in position as shown. Determine the horizontal force P to be applied to keep ladder in position, assume all contact surfaces as smooth.



$XU/AU = \tan(50°)$

$\Rightarrow AU = \dfrac{XU}{\tan 50°}$ $\qquad XU = 2m.$

$\Rightarrow \underline{AU = 1.35 m}$

$\dfrac{YV}{AV} = \tan 50°$ $\qquad YV = 3m.$

$\Rightarrow \underline{AV = 11.03 m}$

$\frac{BW}{AW} = \tan 50$ $\quad BW = 4m$.

$\Rightarrow AW = \underline{14.71\,m}$

$\Sigma F_x = 0$

$\Rightarrow \boxed{P = R_B}$

$\Sigma F_y = 0$

$\Rightarrow R_A = 300 + 500$

$\Rightarrow \boxed{R_A = 800\,N}$

$\Sigma M_A = 0$

$\Rightarrow (300)(7.35) + 500(11.03) \cancel{+} - R_B(14.71) = 0$.

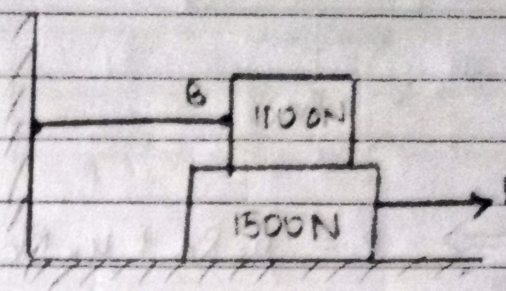
$\Rightarrow 14.71 R_B = 7720$

$\Rightarrow \boxed{R_B = 524.81\,N}$

$\Rightarrow P = R_B$

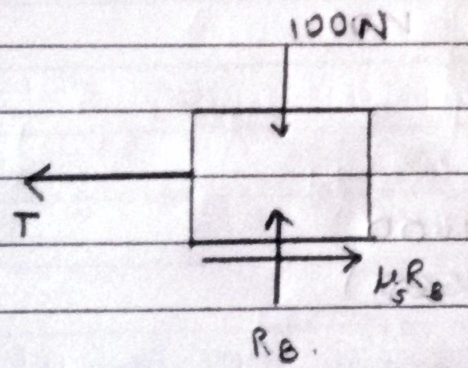
$\Rightarrow \boxed{P = 524.81\,N}$

(c) Determine the smallest force P required to just move the bottom block if (i) top block is restrained by cable AB (ii) cable AB is removed. Take $\mu_s = 0.3$ and $\mu_k = 0.25$.



(i) Restrained by cable AB:

FBD of top block:



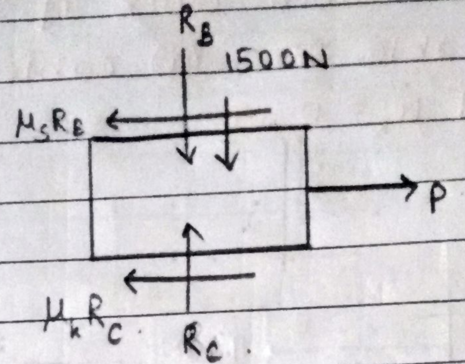
$\Sigma F_x = 0 \Rightarrow T = \mu_s R_B$.

$\Sigma F_y = 0 \Rightarrow \boxed{R_B = 1000\ N}$

$T = (0.3)(1000)$

$\Rightarrow \boxed{T = 300\ N}$

FBD of below block:



$\Sigma F_x = 0 \quad \Rightarrow \quad P = \mu_s R_B + \mu_k R_C.$

$\Sigma F_y = 0 \quad \Rightarrow \quad R_C = R_B + 1500$

$\Rightarrow \quad R_C = 100\cancel{0} + 1500$

$\Rightarrow \quad \boxed{R_C = \overset{1600}{\cancel{2500}}\ N}$

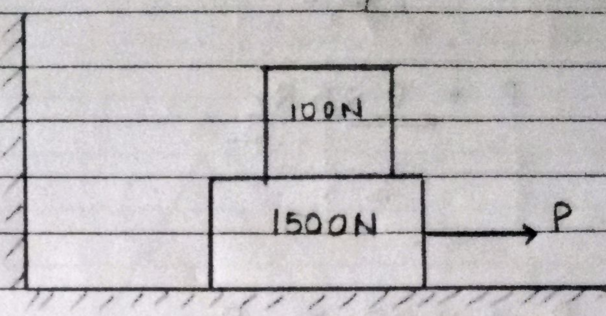
$\Rightarrow \quad P = (0.3)(100\cancel{0}) + (0.25)(\overset{1600}{\cancel{2500}})$

$\Rightarrow \quad \boxed{\cancel{817}\ \cancel{925N}} \quad \boxed{P = 430N}$

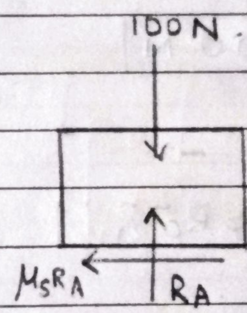
∴ The smallest force required to just move the bottom block when top block is restrained by wire AB is

~~817/925N~~ $\underline{\underline{P = 430N}}$

(ii) Cable AB is removed :



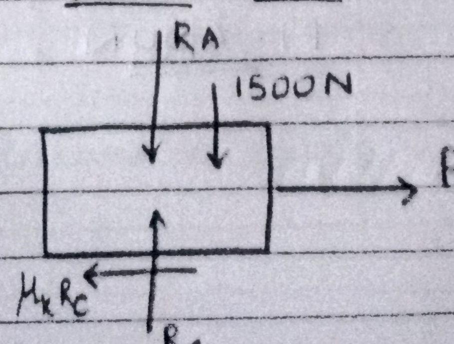
FBD of top block :



$$\Rightarrow \Sigma F_x = 0 \Rightarrow \mu_s R_A = 0$$

$$\Rightarrow \Sigma F_y = 0 \Rightarrow \boxed{R_A = 100\,N}$$

FBD of the bottom block :



$\Sigma F_x = 0$

$$\Rightarrow P = \mu_k R_C$$

$$P = 0.25 R_C$$

$\Sigma F_y = 0$

$$\Rightarrow R_A + 1500 = R_C$$

$$\Rightarrow 100 + 1500 = R_C$$

$$\Rightarrow \boxed{R_C = 1600\ N}$$

Now: $P = 0.25 R_C$

$$\Rightarrow P = (0.25)(1600)$$

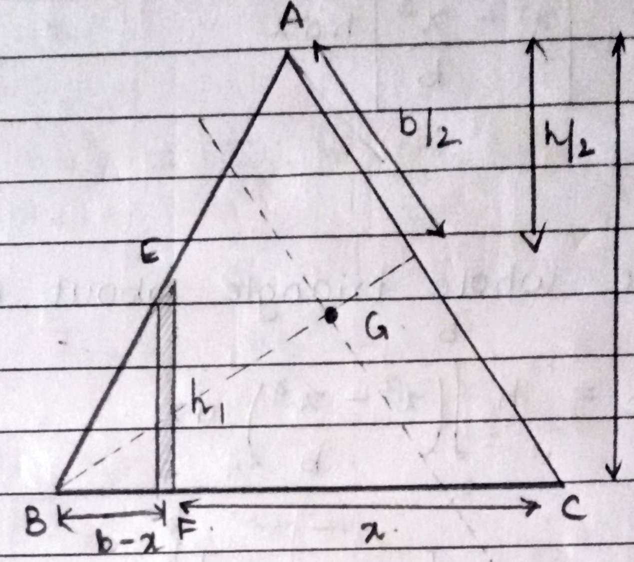
$$\Rightarrow \boxed{P = 400\ N}$$

∴ The minimum force P required by the below block such that it just moves in the absence of wire AB is:

$$P = 400 N$$

MODULE - 3 :

Q5) (a) Derive an expression for moment of inertia of triangle from first principles about its vertical centroidal axis.



Δ BEF and Δ BAC are similar triangles.

$$\Rightarrow \frac{h_1}{h} = \frac{b-x}{b}.$$

$$\Rightarrow h_1 = \left(\frac{b-x}{b}\right) h$$

$$\Rightarrow \boxed{h_1 = \left(1 - \frac{x}{b}\right) h} \quad \text{——— (1)}$$

Area of the element EF = $\boxed{da = h_1 dx}$ ——— (2)

MI of the elemental strip about AC = $da\, x^2$

$$= h_1 dx \cdot x^2$$

$$= \left[\left(1 - \frac{x}{b}\right) h \, dx\right] x^2$$

$$= \left[x^2 - \frac{x^3}{b}\right] h \, dx$$

MI of the whole triangle about AC:

$$I_{AC} = h \int_0^b \left(x^2 - \frac{x^3}{b}\right) dx .$$

$$= h \left[\frac{x^3}{3} - \frac{x^4}{4b}\right]_0^b$$

$$I_{AC} = h \left[\frac{b^3}{3} - \frac{b^4}{4b}\right]$$

$$= h \left[\frac{b^3}{3} - \frac{b^3}{4}\right]$$

$$= hb^3 \left[\frac{4-3}{12}\right]$$

$$\boxed{I_{AC} = \frac{hb^3}{12}}$$

MI about centroid G:

$$I_{AC} = I_{YY} + A\bar{x}^2$$

$$\Rightarrow \frac{hb^3}{12} = I_{YY} + \left(\frac{1}{2}bh\right)\left(\frac{b}{3}\right)^2$$

$$\Rightarrow \frac{hb^3}{12} = I_{YY} + \frac{1}{18}b^3h$$

$$\Rightarrow I_{YY} = \frac{hb^3}{12} - \frac{hb^3}{18}$$

$$\Rightarrow \boxed{I_{YY} = \frac{hb^3}{36}}$$

(b) Locate the centroid of the shaded area as shown in the figure.

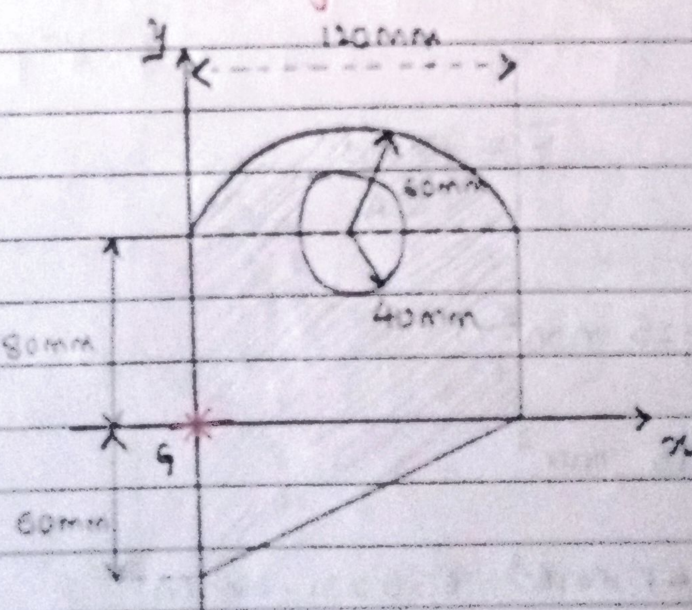
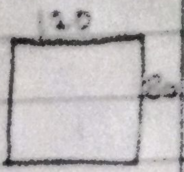
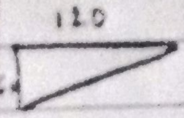
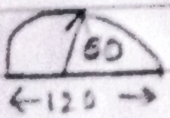
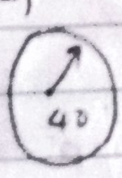


FIGURE	AREA (mm^2)	x (mm)	y (mm)	Ax (mm^3)	Ay (mm^3)
	120×80 $= 9600$	$120/2$ $= 60$	$80/2$ $= 40$	576000	384000
	$\frac{1}{2}(120)(60)$ $= 3600$	$h/3 = 120/3$ $= 40$	$b/3 = 60/3$ $= 20$	144000	72000
	$\frac{\pi r^2}{2}$ $= 5654.8$	$120/2$ $= 60$	$80 + \frac{4(60)}{3\pi}$ $= 105.46$	339288	596355.20
(-) 	$\pi(40)^2$ $=$ -5026.55	60	80	-301593	-402[illegible]4

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} \qquad \bar{y} = \frac{\Sigma Ay}{\Sigma A}$$

$\Sigma A = 13828.25\ mm^2$

$\Sigma Ax = 757695\ mm^3$

$\Sigma Ay =$ ~~[illegible]~~ $650231.2\ mm^3$

$$\Rightarrow \bar{x} = \frac{757695}{13828.25}$$

$$\Rightarrow \boxed{\bar{x} = 54.79\ mm}$$

$$\cancel{\bar{x} = \frac{757695}{1012745}}$$

$$\bar{y} = \frac{650231.2}{13828.25}$$

$$\cancel{\bar{x} = \frac{757695}{650231.2}}$$

$$\Rightarrow \boxed{\bar{y} = 47.02\ mm}$$

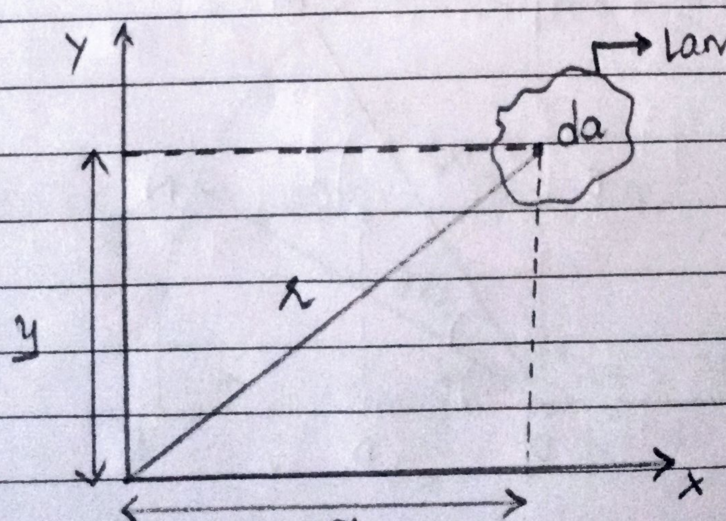
$$\Rightarrow \cancel{\boxed{\bar{x} = 1.165\ mm}}$$

Q6) (a) State and prove perpendicular axes theorem.

PERPENDICULAR AXES THEOREM:

If I_{xx} and I_{yy} be the MI of the body about xx and yy axes, then MI about zz axis is given by:

$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$



From the figure:

$$r = \sqrt{x^2+y^2}.$$

MI of the whole lamina:

$$I_{zz} = \int da \cdot r^2$$

$$\Rightarrow I_{zz} = \int da\,(x^2+y^2)$$

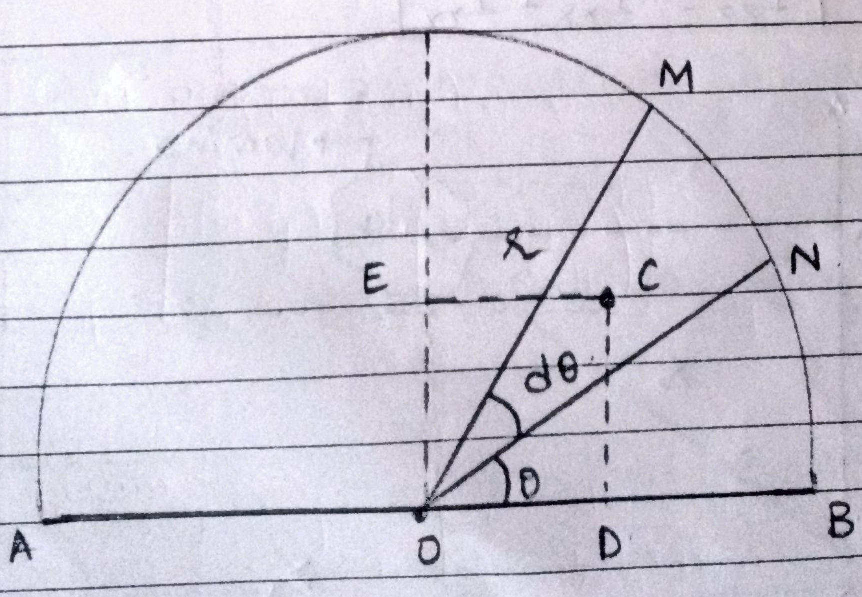
$$\Rightarrow I_{zz} = \int x^2\,da + \int y^2\,da.$$

$$\Rightarrow I_{zz} = (I_{xx}) + (I_{yy}).$$

$$\therefore \boxed{I_{zz} = I_{xx} + I_{yy}}$$

Hence proved.

b) Find the centroid of the area enclosed by a semi circle of radius r from first principles.



Consider a semicircle with radius r.
Let OMN be an element at an angle θ from OB that makes an angle $d\theta$

Area of $\Delta OMN = \frac{1}{2}(MN)(OM)$

Here $MN = r\,d\theta$ [length of arc]

$$\Rightarrow A = \frac{1}{2}(r\,d\theta)(r) \qquad [OM = ON = r]$$

$$\Rightarrow A = \frac{1}{2} r^2 d\theta \qquad \text{——— (i)}$$

Let C be the centroid of the element at distance $\frac{2}{3}r = OC$ from O

From figure: $CD = OC\sin\theta = \frac{2}{3} r \sin\theta$

$$OE = OC\cos\theta = \frac{2}{3} r \cos\theta$$

Height of C.G above $OB = CD = \bar{y} = \frac{\Sigma ay}{\Sigma a}$

$$\Rightarrow \bar{y} = \int_0^{\pi} \frac{\frac{1}{2} r^2 d\theta \times \frac{2}{3} r \sin\theta}{\frac{1}{2} r^2 d\theta}$$

$$\Rightarrow \bar{y} = \frac{\frac{r^3}{3}\int_0^{\pi} \sin\theta \, d\theta}{\frac{r^2}{2}\int_0^{\pi} d\theta}$$

$$\Rightarrow \bar{y} = \frac{2}{3} r \frac{\left[-\cos\theta\right]_0^{\pi}}{\theta\Big|_0^{\pi}}$$

$$\Rightarrow \bar{y} = \frac{2}{3} r \frac{\left[-\cos\pi + \cos 0\right]}{\pi - 0}.$$

$$\Rightarrow \bar{y} = \frac{2r}{3\pi}[1+1]$$

$$\Rightarrow \boxed{\bar{y} = \frac{4r}{3\pi}} \quad \text{(OR)} \quad \bar{y} = 0.424 r$$

Also: $\boxed{\bar{x} = r}$

$\therefore$ Centroid: $(\bar{x}, \bar{y})$.

$$\therefore \boxed{(\bar{x}, \bar{y}) = \left(r, \frac{4r}{3\pi}\right)}$$

(c) Determine the moment of inertia about X-X axis for shaded area as shown in the figure

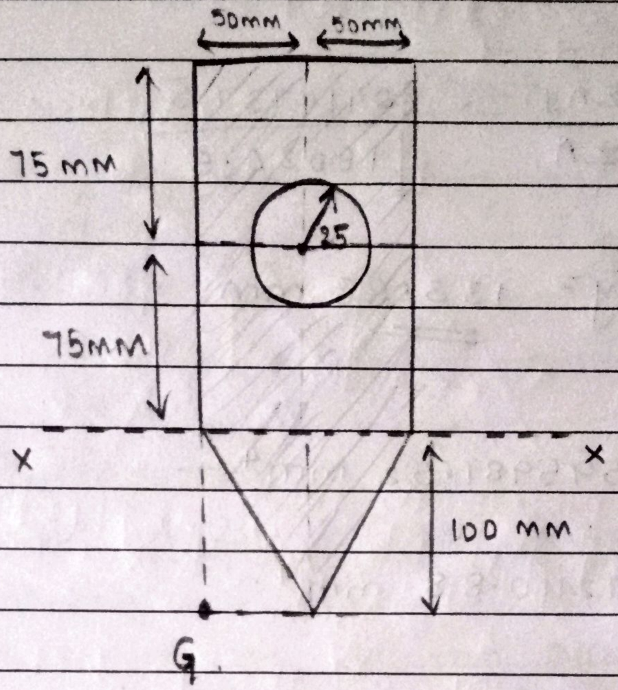


FIGURE	AREA (mm^2)	y (mm)	Ay (mm^3)	$h_x = y \sim \bar{y}$ (mm)	Ah_x^2 (mm^4)	I_{Gx} (mm^4)
100mm, 150 mm (rectangle)	$100 \times 150 = 15000$	$100 + 75 = 175$	2625000	41.12	616800	$\frac{bd^3}{12} = \frac{100 \times (150)^3}{12} = 28125000$
100, 100 (triangle)	$\frac{1}{2}(100)(100) = 5000$	$\frac{2(100)}{3} = 66.67$	133350	67.21	336050	$\frac{bh^3}{36} = 2777777.788$
(−) circle 25	$\pi(25)^2$ (−) $= 1963.5$	$100+75 = 175$	(−) 3436125	41.12	(−) 80739.12	$\frac{\pi d^4}{64} = \frac{\pi r^4}{4}$ (−) $= 306796.157$

$\Sigma A = 18036.5\ mm^2$

$\Sigma Ay = 2414737.5\ mm^3$.

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{2414737.5}{18036.5}$$

$$\Rightarrow \underline{\underline{\bar{y} = 133.88\ mm}}$$

$\Sigma I_{Gx} = 30595981.62\ mm^4$.

$\Sigma A h_x^2 = 872110.88\ mm^4$.

$$I_{xx} = I_{Gx} + A h_x^2$$

$$\Rightarrow I_{xx} = 30595981.62 + 872110.88$$

$$\Rightarrow \boxed{I_{xx} = 31468092.5\ mm^4}$$

MODULE - 04:

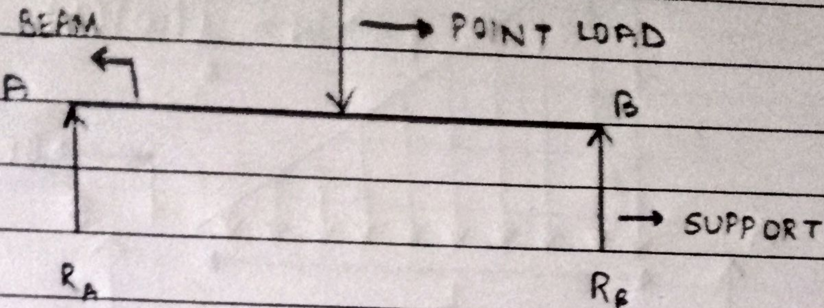
Q7) (a) Explain different types of loads with neat sketches.

TYPES OF LOAD:

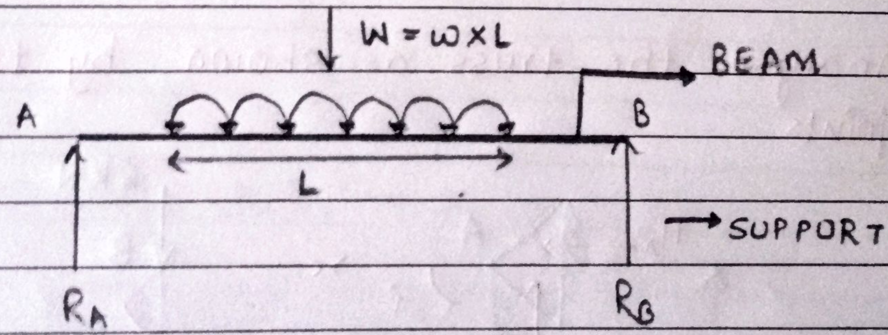
(i) POINT LOAD:

* Acts over a very small length of the beam

* Assumed to act over the midpoint of the load.
* It is a concentrated load.

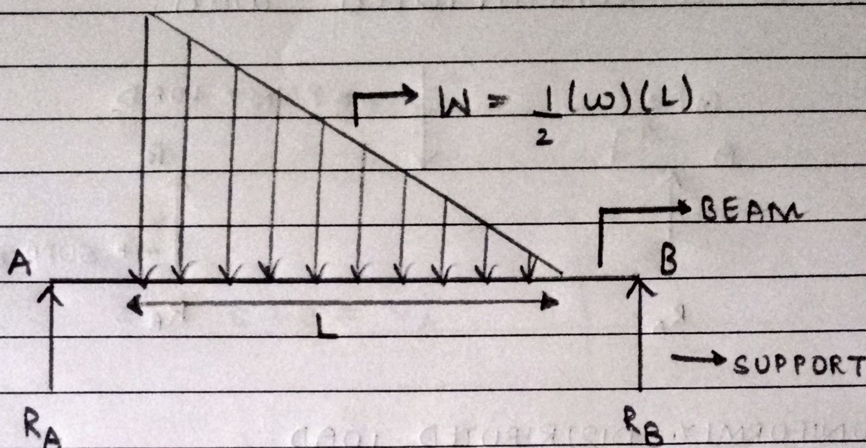


(ii) UNIFORMLY DISTRIBUTED LOAD:
* Each unit length of the beam carries the same intensity of load.
* Magnitude of UDL is the product of load and length of action.
* It is considered to act on midpoint while in equilibrium.



(iii) UNIFORMLY VARYING LOAD:
* Intensity of load varies uniformly over each unit distance.
* Total load is equal to the ~~load of~~ area of the loading quantity.

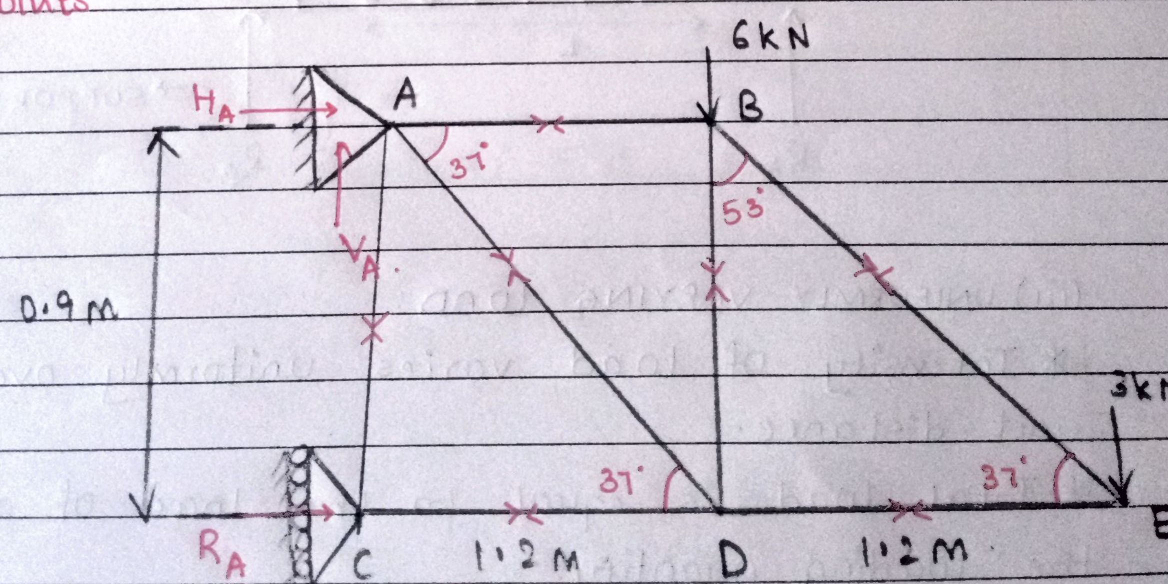
* While solving $\Sigma M = 0$, UVL is placed at the centroid.



(iv) EXTERNAL MOMENT:

* Subjected at certain specific points.
* Considered only when all other moments are taken into account.

(b) Analyse the truss as shown by the method of joints.



$\angle BED = \angle ADC = \tan^{-1}\left(\frac{0.9}{1.2}\right)$

$\Rightarrow \theta = 36.9°$

$\approx 37°$

$j = 5$

$M = 7$

$2j - 3 = 10 - 3$

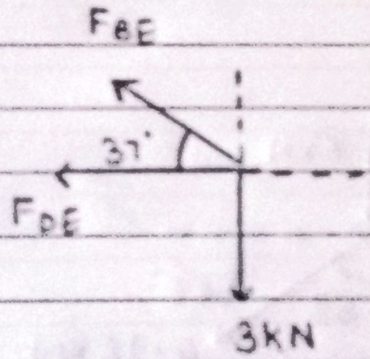
$= 7 = M$

$\therefore$ Truss is perfect

$\Sigma F_y = 0 \Rightarrow 6 + 3 = V_A$

$\Rightarrow \boxed{V_A = 9\,kN}$

* consider joint e:



$\Sigma F_y = 0 \Rightarrow F_{BE} \sin 37° = 3$

$\Rightarrow \boxed{F_{BE} = 4.98\,kN}$

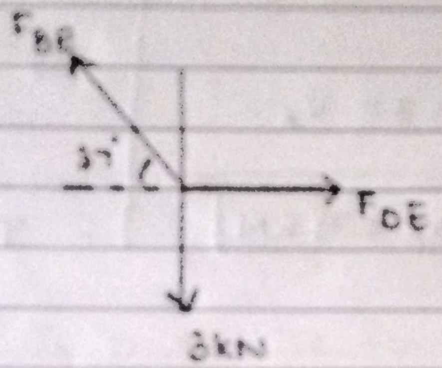
$\therefore$ It is a tensile force

$\Sigma F_x = 0 \Rightarrow F_{DE} + F_{BE} \cos 37° = 0$

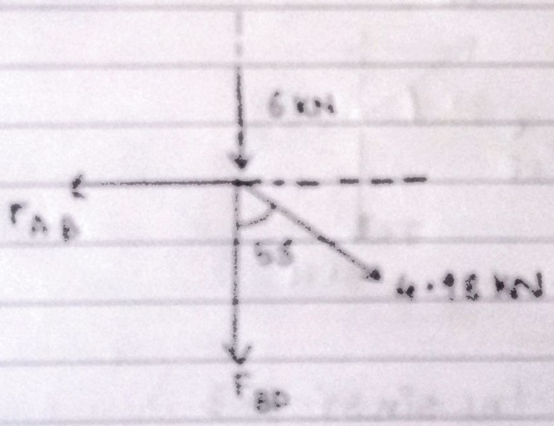
$\Rightarrow F_{DE} = -4.98 \cos 37^\circ$

$\Rightarrow \boxed{F_{DE} = -3.97 \text{ kN}}$

$\therefore$ It is a compressive force



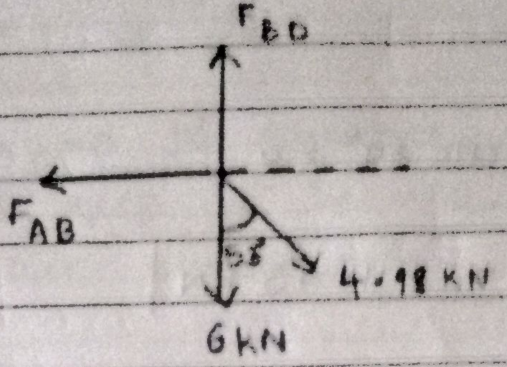
*** consider joint B**



$\Sigma F_y = 0 \Rightarrow 6 + F_{BD} + 4.98 \cos 53^\circ = 0$

$\Rightarrow \boxed{F_{BD} = -8.99 \text{ kN}}$

$\therefore$ The force is compressive

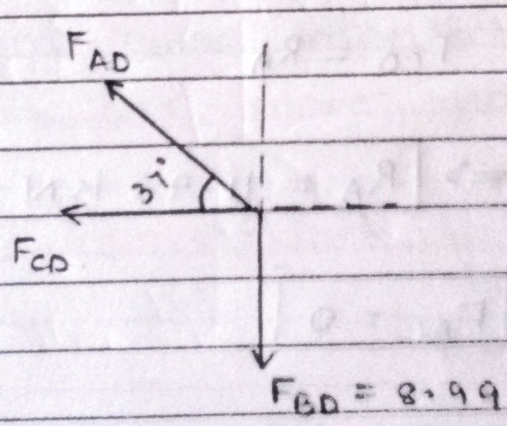


$\Sigma F_x = 0 \Rightarrow F_{AB} = 4.98 \sin 53°$

$\Rightarrow \boxed{F_{AB} = 3.97 \text{ KN}}$

∴ It is a tensile force.

* consider joint B:



$\Sigma F_y = 0 \Rightarrow F_{AD} \sin(37°) = 8.99$

$\Rightarrow \boxed{F_{AD} = 14.94 \text{ KN}}$

∴ It is a tensile force.

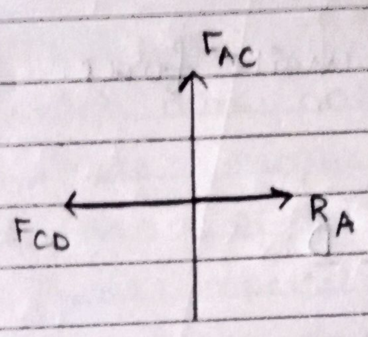
$\Sigma F_x = 0$

$F_{CD} + F_{AD} \cos 37^\circ = 0$

$\Rightarrow \boxed{F_{CD} = -11.93 \text{ KN}}$

$\therefore$ It is a compressive force.

* consider joint C:



$\Sigma F_x = 0 \Rightarrow F_{CD} = R_A$

$\Rightarrow \boxed{R_A = 11.93 \text{ KN}}$

$\Sigma F_y = 0 \Rightarrow \boxed{F_{AC} = 0}$

Q8) (a) Write a note on classification of trusses.

There are two types of trusses:

i) Perfect Truss

ii) Imperfect Truss.

i) PERFECT TRUSS:

It is made up of number of members just sufficient to keep it in equilibrium when loaded without any change in its shape.

condition:

$M \rightarrow$ Number of member forces

$j \rightarrow$ Number of joints

$$\Rightarrow \boxed{M = 2j-3}$$

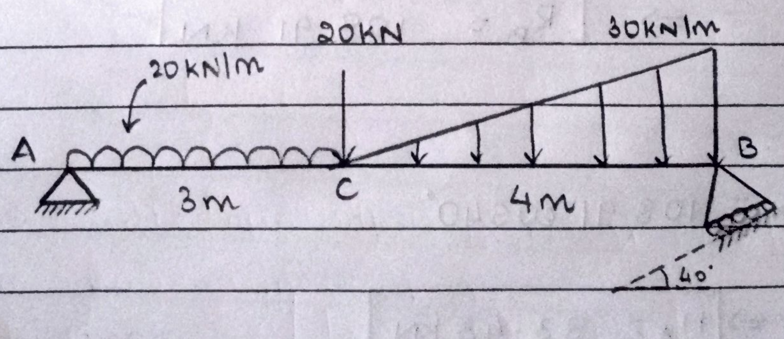
ii) IMPERFECT TRUSS:

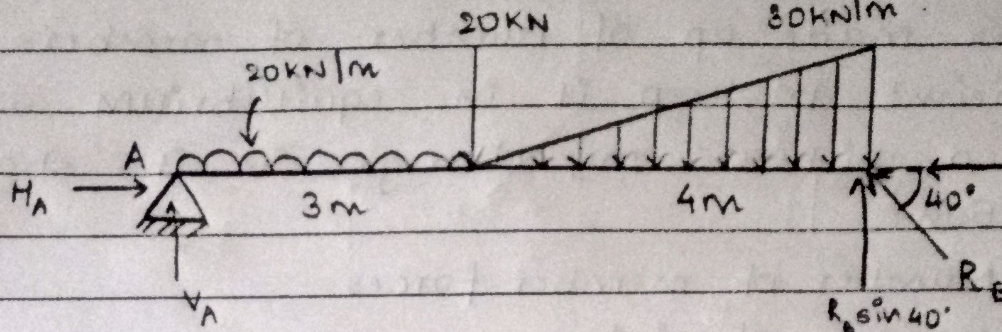
The truss which does not satisfy the condition $M = 2j-3$ or where number of members are more or less than $2j-3$.

(a) Deficient Truss: Imperfect truss where number of members are less than $2j-3$.

(b) Redundant Truss: Imperfect truss where number of members are more than $2j-3$.

(b) Find the support reactions for the beam as shown.





$\Sigma F_x = 0 \Rightarrow H_A = R_B \cos 40°$ —— (1)

$\Sigma F_y = 0 \Rightarrow V_A + R_B \sin 40 = 20(3) + 20 + \frac{1}{2}(30)(4)$

$\Rightarrow V_A + R_B \sin 40° = 140 \text{ KN}.$ —— (2)

$\Sigma M_A = 0 \Rightarrow 20(3)(1.5) + 20(3) + \frac{1}{2}(30)(4)\left(3 + \frac{2(4)}{3}\right)$

$= R_B \sin 40 (3+4)$

$\Rightarrow 490 = 4.49 R_B$

$\Rightarrow \boxed{R_B = 108.91 \text{ KN}}$

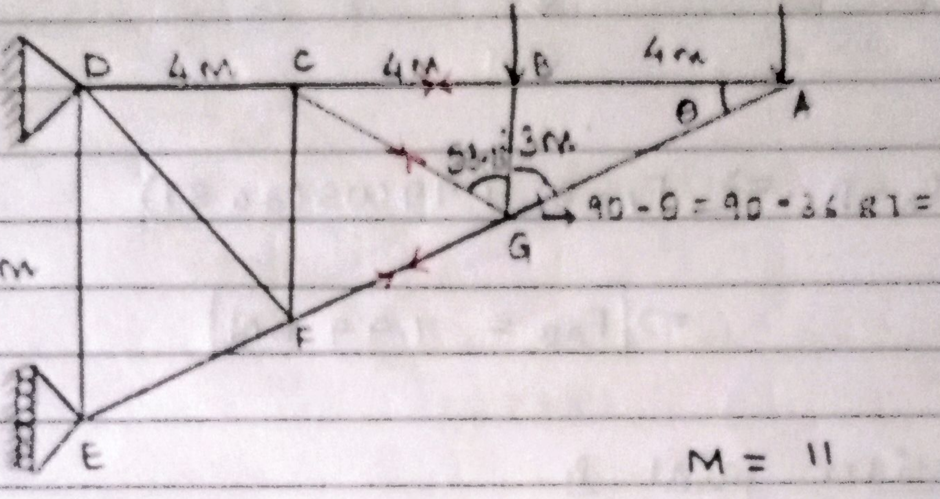
$H_A = 108.91 \cos 40°$

$\Rightarrow \boxed{H_A = 83.43 \text{ KN}}$

$V_A = 140 - 108.91 \sin 40°$

$\Rightarrow \boxed{V_A = 69.99 \text{ KN}}$

(c) A roof truss is loaded in the figure, then determine the forces in members BC, GF and CG.



$M = 11$

$j = 7$

$\theta = \tan^{-1}\left(\frac{3}{4}\right)$

$2j - 3$

$= 14 - 3$

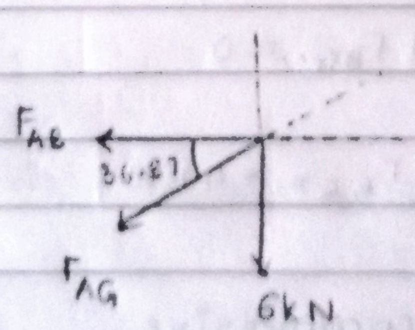
$= 11$

$= M$

$\Rightarrow \theta = 36.87°$

$\therefore$ Perfect truss.

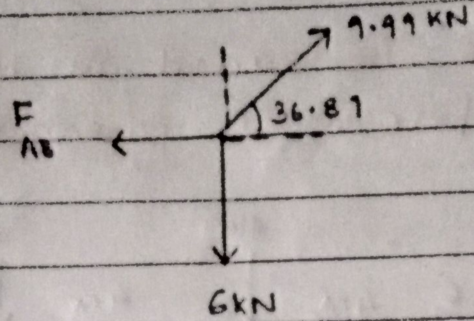
* consider joint A:



$\Sigma F_y = 0 \Rightarrow F_{AG} \sin(36.87) + 6 = 0$

$\Rightarrow \boxed{F_{AG} = -9.99 \text{ KN}}$

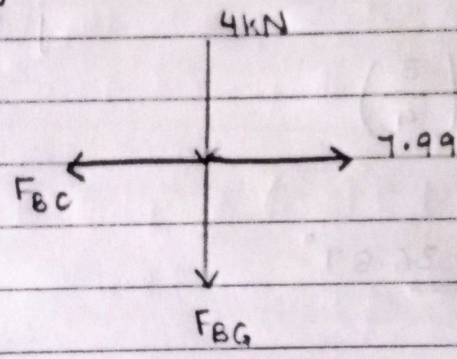
∴ The force is compressive.



$\Sigma F_x = 0 \Rightarrow F_{AB} = 9.99\cos(36.87)$

$\Rightarrow \boxed{F_{AB} = 7.99\ kN}$

* consider joint B :

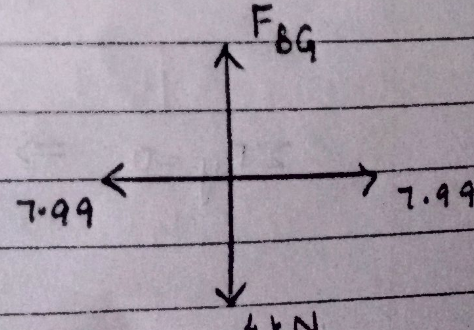


$\Sigma F_x = 0 \Rightarrow \boxed{F_{BC} = 7.99\ kN}$ ∴ Force is tensile

$\Sigma F_y = 0 \Rightarrow 4 + F_{BG} = 0$

$\Rightarrow \boxed{F_{BG} = -4\ kN}$

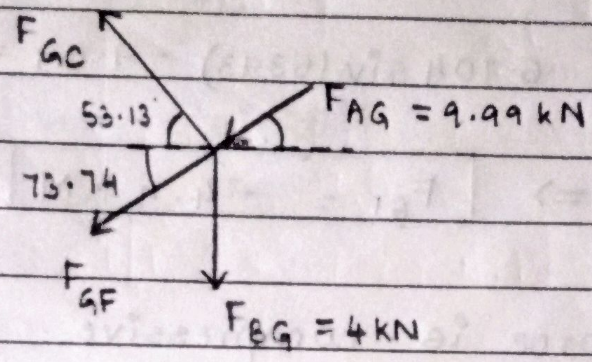
∴ Force is compressive



$\angle CGF = 180 - (53.13 + 53.13)$

$= \underline{\underline{73.74^\circ}}$

* consider joint G:



$\Sigma F_x = 0 \Rightarrow -9.99\cos(73.74) + F_{GF}\cos(73.74)$

$+ F_{GC}\cos(53.13) = 0$.

$\Rightarrow F_{GF}\cos(73.74) + F_{GC}\cos(53.13) = -2.79$

$\Sigma F_y = 0 \Rightarrow F_{GC}\sin(53.13) - F_{GF}\sin(73.74) -$

$F_{AG}\sin(73.74) = 0$

$\Rightarrow F_{GC}\sin(53.13) - F_{GF}\sin(73.74) = 9.59$.

$\Rightarrow 0.28 F_{GF} + 0.6 F_{GC} = -2.79$

$-0.96 F_{GF} + 0.8 F_{GC} = 9.59$

$$\Rightarrow 0.576 F_{GC} + 0.224 F_{GC} = +2.678 + 2.6852$$

$$\cancel{\neq x.\ 0.576 F_{GC} = +0.224 F_{GC}.}$$

*

$$\Rightarrow \boxed{F_{GC} = 6.704 \text{ kN}}$$

∴ Force is tensile

$$\Rightarrow 6.704 \sin(53.13) - 9.59 = F_{GF} \sin(73.74)$$

$$\Rightarrow \boxed{F_{GF} = -4.4 \text{ kN}}$$

∴ Force is compressive.

MODULE -05:

(a) Define i) Time of flight ii) Horizontal Range iii) Maximum Height iv) Trajectory.

(i) TIME OF FLIGHT:

The time required to hit the ground from the point of projection is called as the time of flight.

$$\boxed{T = \frac{2u\sin\theta}{g}}$$

(ii) HORIZONTAL RANGE:

The horizontal distance travelled by the projectile during its entire time is called as its horizontal range.

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$$\boxed{R = \frac{u^2 \sin 2\theta}{g}}$$

(iii) MAXIMUM HEIGHT :

The maximum height attained by the projectile in a time equal to time of flight is called as maximum height.

$$\boxed{H = \frac{u^2 \sin^2\theta}{2g}}$$

(iv) TRAJECTORY :

The path traced by the projectile in the space is known as the trajectory.

(b) A projectile is fired with a velocity of 60 m/s on horizontal plane. Find its time of flight in the following cases: (i) Its range is four times the maximum height (ii) Its maximum height is four times horizontal range (iii) Its maximum height & horizontal range are equal.

GIVEN: $u = 60 m/s$.

(i) $R = 4H$.

$$\Rightarrow \frac{u^2 \sin 2\theta}{g} = \frac{4 u^2 \sin^2\theta}{2g}.$$

$$\Rightarrow \sin 2\theta = 2 \sin^2\theta$$

$$2\sin\theta\cos\theta = 2\sin^2\theta$$

$$\Rightarrow \quad \cos\theta = \sin\theta$$

$$\Rightarrow \quad \tan\theta = 1$$

$$\Rightarrow \quad \boxed{\theta = 45^\circ}$$

$$T = \frac{2u\sin\theta}{g} = \frac{2 \times 60 \times \sin 45}{9.8}$$

$$\Rightarrow \boxed{T = 8.66\ s}$$

(ii) $H = 4R$

$$\frac{u^2\sin^2\theta}{2g} = \frac{4u^2\sin 2\theta}{g}.$$

$$\Rightarrow \quad \sin^2\theta = 8\sin 2\theta$$

$$\Rightarrow \quad \sin^2\theta = 16\sin\theta\cos\theta$$

$$\Rightarrow \quad \tan\theta = 16$$

$$\Rightarrow \quad \boxed{\theta = 86.42^\circ}$$

$$T = \frac{2u\sin\theta}{g} = \frac{2 \times 60 \times \sin(86.42)}{9.8}$$

$$\Rightarrow \boxed{T = 12.22\ s}$$

(iii) $H = R$

$$\frac{u^2 \sin^2\theta}{2g} = \frac{u^2 \sin 2\theta}{g}$$

$$\Rightarrow \frac{\sin^2\theta}{2} = \sin 2\theta.$$

$$\Rightarrow \tan\theta = 4$$

$$\Rightarrow \boxed{\theta = 76^\circ}$$

$$T = \frac{2u\sin\theta}{g} = \frac{2\times 60\times \sin(76)}{9.8}$$

$$\Rightarrow \boxed{T = 11.88\ s}$$

(c) A stone is released from top of tower 'h' in height, it covers a vertical distance of $h/5$ during its last second of descend. Find the height of the tower.

Let t be the time taken to descend completely through the tower. $[u=0]$

$$\Rightarrow h = \frac{1}{2}gt^2 \quad \text{—— (1)}$$

Also given that: $\frac{h}{5} = \frac{1}{2}g(t-1)^2$ —— (2)

Subtract (1) & (2):

$$h - \frac{h}{5} = \frac{1}{2}gt^2 - \frac{1}{2}g(t-1)^2$$

$$\Rightarrow \frac{4h}{5} = \frac{1}{2}gt^2 - \frac{1}{2}g(t^2+1-2t)$$

$$\Rightarrow \frac{4h}{5} = \frac{1}{2}gt^2 - \frac{1}{2}gt^2 - \frac{1}{2}g + gt.$$

$$\Rightarrow \frac{4h}{5} = gt - \frac{g}{2}.$$

Substitute: $h = \frac{1}{2}gt^2$

$$\Rightarrow \frac{4}{5}\left[\frac{1}{2}gt^2\right] = gt - \frac{g}{2}.$$

$$\Rightarrow \frac{2}{5}gt^2 = gt - \frac{g}{2}$$

$$\Rightarrow \frac{2}{5}t^2 = t - \frac{1}{2}.$$

$$\Rightarrow \frac{2}{5}t^2 - t + \frac{1}{2} = 0.$$

$\Rightarrow \quad 4t^2 - 10t + 5 = 0.$

$\Rightarrow \quad \boxed{t = 1.8\,s}$ or $\boxed{t = 0.69\,s}$

$\Rightarrow \quad h = \frac{1}{2}(9.8)(1.8)^2$

$\Rightarrow \quad \boxed{h = 15.876\,m}$

(a) State and explain D'Alembert's principle.

D'ALEMBERT'S PRINCIPLE:

The resultant of system of forces acting on a body of mass m and with acceleration a is in dynamic equilibrium with inertia force 'ma' applied in reverse direction of motion.

$$\boxed{R - ma = 0}$$

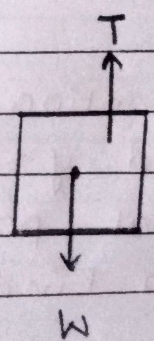
Analysis of Lift Motion:

(i) LIFT IS AT REST: $a = 0$

From the principle;

$a = 0 \quad \Rightarrow \quad \boxed{T = W}$ Weight of lift

Tension



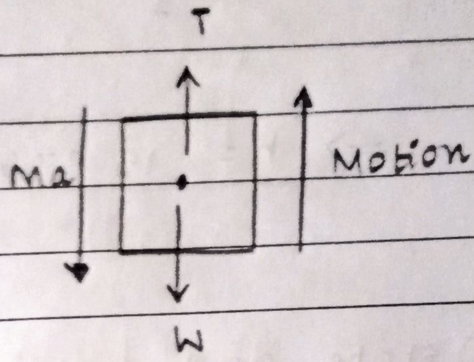
$(a = 0)$

(ii) LIFT MOVING WITH CONSTANT VELOCITY: $a = 0$

In this case; again $a = 0$

$$\Rightarrow \boxed{T = W}$$

(iii) LIFT ACCELERATING UPWARDS:

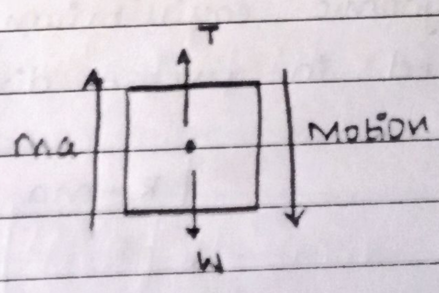


$$T - ma = W$$

$$\Rightarrow T = W + ma$$

$$\Rightarrow \boxed{T = m\left[1 + \frac{a}{g}\right]}$$

(iv) LIFT ACCELERATING DOWNWARDS:



$$T + ma = W$$

$$\Rightarrow T = W - ma$$

$$\Rightarrow \boxed{T = m\left[1 - \frac{a}{g}\right]}$$

(b) The motion of a particle is defined by $x = (t+1)^2$ and $y = 4(t+1)^{-2}$ where x and y are in 'm'. Show that path of particle is part of rectangular hyperbola. Find velocity & acceleration at $t = 0$.

$x = (t+1)^2$

$y = 4(t+1)^{-2}$

$\Rightarrow y = \dfrac{4}{(t+1)^2}$

$\Rightarrow y = \dfrac{4}{x}$

$\Rightarrow \boxed{xy = 4}$

The particle is part of the rectangular hyperbola

$x = (t+1)^2$

Diff wrt t

$\Rightarrow \dfrac{dx}{dt} = 2(t+1)$

At $t=0$; $\dfrac{dx}{dt} = v = 2(1)$

$\boxed{v = 2 m/s}$

Diff again wrt t

$\Rightarrow \dfrac{d^2x}{dt^2} = 2$

At $t=0$; $\frac{d^2x}{dt^2} = \boxed{a = 2 m/s^2}$

(c) Two cars moving in the direction are 150m apart. Car A being ahead of car B, at this instant velocity of car A is 3m/s and constant acceleration of 1.2 m/s². While velocity of car B is 30 m/s & its uniform retardation is 0.6 m/s², how many times do the cars cross each other? Find when & where they cross with position of car A.

GIVEN: $S = 150 m$

$u_A = 3 m/s$

$a_A = 1.2 m/s^2$

$u_B = 30 m/s$

$a_B = -0.6 m/s^2$

Car B will overtake when $S_B = 150 + S_A$.

$$S_A = 3t + \frac{1}{2}(1.2)t^2$$

$$S_B = 30t - \frac{1}{2}(0.6)t^2$$

$$\Rightarrow 30t - \frac{1}{2}(0.6)t^2 = 150 + 3t + \frac{1}{2}(1.2)t^2$$

$$\Rightarrow 27t = 150 + \frac{1}{2}(1.8)t^2.$$

$\Rightarrow 54t = 300 + 1.8t^2$

$\Rightarrow 0.9t^2 - 27t + 150 = 0$

$\Rightarrow \boxed{t = 22.64\,s}$ or $\boxed{t = 7.36\,s}$

car B will overtake car A 2 times. one after 7.36 s and another after 22.64 s.

i) $S_A = 3(7.36) + \frac{1}{2}(1.2)(7.36)^2$

$\Rightarrow \boxed{S_A = 54.58\,m}$

ii) $S_B = 3(22.64) + \frac{1}{2}(1.2)(22.64)^2$

$\Rightarrow \boxed{S_B = 375.46\,m}$

After car A travels 54.58 m and 375.46 m, car B overtakes the car.