MODULE 5
KINEMATICS

Concepts and Applications

Definitions – Displacement – Average velocity – Instantaneous velocity – Speed – Acceleration - Average acceleration – Variable acceleration – Acceleration due to gravity – Newton's Laws of Motion. Rectilinear Motion – Numerical problems. Curvilinear Motion – Super elevation – Projectile Motion – Relative motion – Numerical problems. Motion under gravity – Numerical problems. D'Alembert's principle and its applications in plane motion and connected bodies including pulleys

5.1 Definitions

Displacement – it is defined as change in position.

Displacement = Final position – Initial position

Average velocity – if the particle is displaced δx in a time interval of δt, then average velocity is given by

$$v_{aw} = \frac{\delta x}{\delta t}$$

Instantaneous velocity – the velocity at a particular instant of time is called as instantaneous velocity and given by

$$v = \lim \frac{\delta x}{\delta t}$$

$$v = \frac{dx}{dt}$$

Speed – it is the time taken to travel a particular distance

$$S = \frac{distance}{time}$$

Acceleration – acceleration is rate of change of velocity.

Average acceleration – if the velocity of particle changes by δv in a time interval of δt, the average acceleration is given by

$$A_{aw} = \frac{\delta v}{\delta t}$$

5.2Relative Motion

The motion of a particle with respect to a fixed frame is called the **Absolute motion** of a particle.
Example: motion of a train with respect to the platform which is stationary is called as absolute motion.

The motion relative to a set of axes, which are moving, is called **relative motion**.

Example: The motion of train 'A' with respect to another moving train, 'B'is the relative motion of the train 'A' with respect to the train 'B' as shown in Fig 5.1

Case 1: Suppose a train 'A'is moving with velocity 75kmph in a straight course. Let another train B move parallel and like or same direction with same velocity of 75kmph. Then the velocity of train A is zero with respect to train B. Relative velocity is the vector difference in the two velocities.

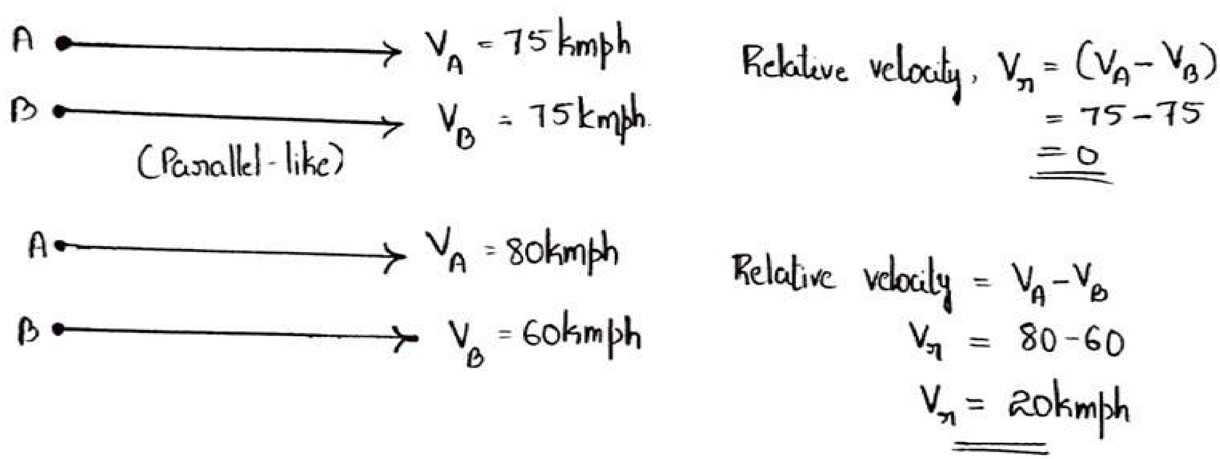


Fig 5.1 Objects moving along same direction

Case 2: Suppose 'A' moves with a velocity 80kmph in one direction and B move with 60 kmph in opposite direction on a parallel path as shown in Fig 5.2.

Relative velocity is the vector difference of the speed,$V_r = V_A – (-V_B) = 80 – (-60) = 140$ Kmph

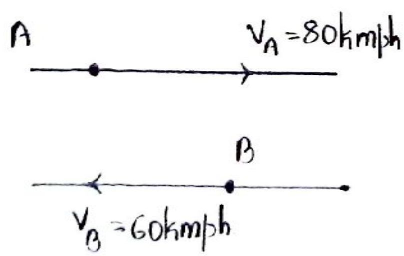


Fig 5.2 Objects moving in different directions

Difference between Relative Velocity and Resultant Velocity:

1. Relative velocity is the velocity of a body as observed from other moving object, whereas resultant velocity is the combined effect of two or more forces causing motion of a single body.
2. Relative velocity of a body is obtained as the vector difference of the velocities of two bodies; whereas resultant velocity is obtained as the vector addition of the velocities.

To find relative velocity by analytical method. Fig 5.3 shows the components of velocity.

1. Find horizontal and vertical components of V_A as $V_{AX} + V_{AY}$
2. Find horizontal and vertical components of V_B as $V_{AB} + V_{AB}$
3. Let V_{rx} and V_{ry} be horizontal and vertical components of relative velocity
4. Components are combined to get relative velocity

$V_{rx} = V_{AX} - V_{BX}$

$V_{ry} = V_{AY} - V_{BY}$

$V_r = \sqrt{(V_{rx}^2 + V_{ry}^2)}$

$\tan \alpha = V_{ry} / V_{rx}$

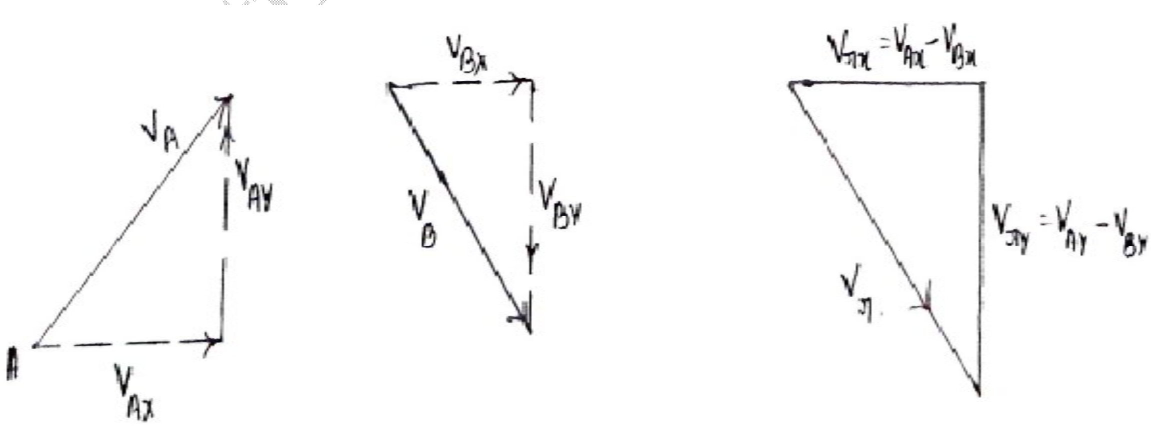


Fig 5.3 Relative motion

5.2 Motion under Gravity

Acceleration due to gravity:

A body which is free to move entirely under the influence of the earth's gravitational attraction will be subjected to an acceleration directed towards Centre of the earth. This acceleration is called **acceleration due to gravity**.

For uniform acceleration its vale is generally taken as **g = 9.81** m/sec2

Equations of motion for body freely falling vertically downward under gravity:

1. $v = u + gt$
2. $s = ut+1/2\ gt^2$
3. $v^2 - u^2 = 2gs$

For bodies projected vertically 'g'of is negative as it is against gravity:

1. $v = u - gt$
2. $s = ut - 1/2\ gt^2$
3. $v^2 - u^2 = -2gs$

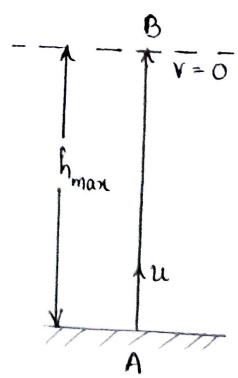
Equations of bodies just drooped (u=0)

1. $v = gt$
2. $s = 1/2\ gt^2$
3. $v^2 = 2gs$

Greatest height reached by a body and the time it takes:

Let u be the initial velocity with which the body is projected vertically up. When it reaches to th

e greatest height 'h'its final velocity v=0.



From $v = u - gt$

$0 = u - gt$

or $t = u/g$

From equation, $v^2 - u^2 = -2gs$

$-u^2 = -2gh_{max}$

$h_{max} = u^2/2g$

5.3 D'Alembert's Principle

Dynamics is the branch of mechanics in which the analysis of bodies considered are in motion. It is divided into, i) Kinematics and ii) Kinetics

Kinematics is the study of motion of bodies and its relationship with time are considered without considering the forces causing motion. It deals with the study of displacement, velocity and acceleration.

Kinetics is the study of motion of bodies and its relationship with time considering the forces causing motion.

D'Alembert's principle deals with kinetic problems. His principle will help to convert the dynamic problem into static equilibrium problem.

D' Alembert's principle

D'Alembert looked into the Newton's second law of motion and rewritten them in different way as,

R - ma = 0

R = Net accelerating Force, ma = inertia force

where, m = mass,   a= acceleration

It states that, "The resultant of system of forces acting on a body of mass 'm' and with acceleration 'a' is in dynamic equilibrium with inertia force 'ma' applied in the reverse direction of the motion".

Mathematically, (net accelaring force) - (inertia force) = 0

R - ma = 0

Analysis of Lift Motion:

Case 1: Lift at rest (a=0)

$\Sigma V = 0$, T = W

Hence tension in string (T) will be equal to weight of object (W)

Case 2: Lift moving with constant velocity (a=0)

T = W

Case 3: Lift accelerating upwards

Let a be the acceleration of lift upwards

(Inertia force ma opposes motion)

$\Sigma V = 0$, $T - W - ma = 0$

$T = W + ma$

$= W\left(1+\frac{a}{g}\right)$ since $m = \frac{W}{g}$

Case 4: Lift accelerating downwards

$\Sigma V = 0$, $T - W + ma = 0$

$T = W - ma$

$= W\left(1-\frac{a}{g}\right)$

5.4 Equations of Motion

For body freely falling vertically downwards

$v = u + gt$

$S = ut + \frac{1}{2}gt^2$

$v^2 = u^2 + 2gs.$

Where

u = initial velocity

v = final velocity

t = time elapsed

g = acceleration due to gravity

s = distance travelled

For bodies projected vertically where 'g' is negative

$v = u - gt$

$S = ut - \frac{1}{2}gt^2$

$v^2 - u^2 = - 2gs.$

Where

u = initial velocity

v = final velocity

t = time elapsed

g = acceleration due to gravity

s = distance travelled

For bodies just dropped

$v = gt$

$S = \frac{1}{2}gt^2$

$v^2 = 2gS$

Where

u = initial velocity (here initial velocity is zero because it is just dropped)

v = final velocity

t = time elapsed

g = acceleration due to gravity

s = distance travelled

5.5 Newton's Laws of motion

Thereare three Newton's laws of motion. In these laws, he explained the relationship between forces acting on the body and the motion of the body. Newton's three laws are as stated below:

1. An object continues to be under the state of uniform motion unless an external force acts on it.
2. Force is a product of mass and acceleration.
3. For every action, there is an equal and opposite reaction.

Newton's first law of motion is also known as the law of inertia. In Newton's second law of motion, the velocity is constant. Newton's third law of motion is also known as conservation of momentum.

Newton's Law of Motion

Sir Isaac Newton was an English mathematician, astronomer and physicist who gave three laws which proved to be fundamental laws for describing the motion of a body. These are generally known as Newton's laws of motion. We will discuss each law of motion one by one in detail.

Newton's First Law of Motion:

Newton's first law of motion states that "A body at rest or uniform motion will continue to be at rest or uniform motion until and unless a net external force acts on it".

Suppose a block is kept on the floor, it will remain at rest until we apply some external force to it. Also, we know that it takes us more effort or force to move a heavy mass. This is directly related to a property known as Inertia. This law is also known as the law of inertia.

Newton's Second Law of Motion:

The first law has already given us a qualitative definition of force. Now we are interested in finding out its magnitude. According to Newton's second law of motion, the net force experienced by a body is directly proportional to the rate of change of momentum of the body. It can be written as:

$$F \propto dP/dt$$

$$\Rightarrow F \propto mv - mut$$

$$\Rightarrow F \propto m(v- u)t$$

$$\Rightarrow F \propto ma$$

$$\Rightarrow F = kma$$

Where k is the constant of proportionality and it comes out to be 1 when the values are taken in SI unit.
Hence the final expression will be,

$$\mathbf{F = ma}$$

Newton's Third Law of Motion:

According to Newton's third law of motion, for every action, there is an equal and opposite reaction. Forces are always found in pairs. For instance, when you sit on a chair, your body exerts a force downward and that chair needs to exert an equal force upward or else the chair will collapse.

5.6 Rectilinear Motion: if any two particles of the body travel the same distance along two parallel straight lines, then the body is said to be in rectilinear motion or linear motion

Problems on Rectilinear motion

1. A car is accelerating at a constant rate of 2.5 m/s^2. If it travels a distance of 700 m in 20 s

   (i) What must be its initial velocity?

   (ii) What must be its final velocity?

   Given data S = 700 m, t = 20 s, a = 2.5 m/s^2

   To find 'u'

   $S = ut + \frac{1}{2}at^2$

   $700 = u \times 20 + \frac{1}{2} \times 2.5 \times 20^2$

   $20u = 700 - \frac{1}{2} \times 2.5 \times 20^2$

   $20u = 700 - 500$

   $20u = 200$

   . : u = 10 m/s

   To find 'v'

   $v = u + at$

   $v = 10 + 2.5 \times 20$

   . : v = 60 m/s

2. Two cars P and Q accelerates from a standing start. The acceleration of P is 1.3 m/s^2 and that of Q is 1.6 m/s^2. If Q was originally 6 m behind P, How long will it take for it to overtake P?

   Solution:

   (i) For car P

   $S = ut + \frac{1}{2}at^2$

   $S = 0 \times t + \frac{1}{2} \times 1.3 \times t^2$

   $\mathbf{S = 0.65t^2}$ ………… **1**

   (ii) For car Q

   $S = ut + \frac{1}{2}at^2$

$S = 0 \times t + \frac{1}{2} \times 1.6 \times t^2$

$S = 0.8t^2$ ………… 2

Since car Q is 6 m behind P when both of them start, when car Q overtakes P the distance travelled by Q will be S and by P will be (S = 6).

For P

$(S+6) = 0.65t^2$ **………… 3**

. :From eqns 2 and 3

$0.65t^2 + 6 = 0.8t^2$

$0.15t^2 = 6$

. :t = 6.32 s

3. A police officer observes a car approaching at the unlawful speed of 60 kmph. He gets on his motor cycle and starts chasing the car, just as it passes in front of him. After accelerating for 10 secs, at a constant rate, the officer reaches his top speed of 75 kmph. How long does it take the officer to overtake the car from the time he started?

Let t be the time at which the officer overtakes the car.

Distance travelled by car = (60 x t) s ... **(1)**

The officer accelerated for 10 s to reach top speed of 75 kmph,
75 kmph = 75 x 1000/3600 = 20.833 m/s
v = u + at
20.833 = 0 + a x 10
.: $a = 2.0833\ m/s^2$

Distance travelled by officer in 10 s
$S = ut + (½)at^2$
= 0 x 10 + (1/2) x 2.0833 x 10
.: S = 104.165 m

Officer travels with constant speed of 75 km/h for (t - 10) s.
Distance travelled in (t - 10) s = 75x (t- 10)
Total distance travelled by officer = 104.165 + 75 x (5/18) x (t-10) ... **(2)**
Equating (1) and (2)
(60 x t) = 104.165 + 20.833 x (t-10)

.: t = 25 s

4. A bullet, moving at the rate of 250 m/s, is fired into wood. It penetrates to a depth of 40 cm. Find the acceleration of the bullet.

u = 250 m/s;
S = 40 cm = 0.4 m;
v = 0
$v^2 = u^2 + 2as$
$0 = 250^2 + 2 \times a \times 0.4$
.: $a = 7.8125$ m/s^2

5. Car A accelerates uniformly from rest on a straight level road. Car B starting from the same point 6 seconds later with zero initial velocity accelerates at 6 m/s^2. It overtakes the car A at 400 m from the starting point. What is the acceleration of the car A?

For car A
u = 0
Time = t
Acceleration = a_A,
S_A = 400 m

$S_A = ut + (½)at^2$
$400 = 0 \times t + (1/2) \times a_A \times t^2$
$a_A = 800/t^2$ m/s^2 --------------- **(1)**

Car B
u = 0
a = 6 m/s^2
S_B = 400 m
Time = (t – 6) s

$S_B = ut + (½)at^2$
$400 = 0 \times (t – 6) + (1/2) \times 6 \times (t – 6)^2$
$(t – 6)^2 = 400 \times 2/6$
.: t = 17.547 s

Substitute in (1)

$a_A = 800/17.547^2 = 2.598$ m/s^2

6. Two trains A and B leave the same station on parallel lines. A starts with a uniform acceleration of 0.15 m/ s^2 and attains the speed of 24 km/hour, after which, its speed remains

constant. B leaves 40 seconds later with uniform acceleration of 0.30 m/ s^2 to attain a maximum speed of 48 km/hour. Its speed also becomes constant thereafter. When will B overtake A?

Soln: Let t = Time for A when B overtakes it, a
and t_1 = Time for A to attain speed of 24 km/h.

During acceleration of A,
u = 0,
v = 24 x (5/18) = 6.667 m/s,
a = 0.15 m/s^2
v = u + at
6.67 = 0 + 0.15 x t_1
.: t_1 = 44.44 s
The train A accelerated for 44.44 s and after that it travels at a constant speed of 6.67 m/s
Distance travelled by train A is obtained by addition of distance when train accelerates and with constant velocity
S = (ut_1 + (½)at_1^2) + (v x (t – t_1))
S = (0 x 44.44 + (½) x 0.15 x 44.44^2) + (6.67 x (t – 44.44)) ------------- (1)
During the acceleration of B,
u = 0,
v = 48 x (5/18) = 13.33 m/s,
a = 0.30 m/s^2
v = u + at
13.33 = 0 + 0.30 x t_2
.: t_2 = 44.44 s
The train B accelerated for 44.44 s and 40 s before that, hence it travels at a constant speed of 13.33 m/s after 84.44 s
Distance travelled by train B is obtained by addition of distance when train accelerates and with constant velocity
S = (ut_1 + (½)at_2^2) + (v x (t – t_2))
S = (0 x 84.44 + (½) x 0.30 x 84.44^2) + (13.33 x (t – 84.44)) ------------- (2)

Equating (1) and (2)

(0 x 84.44 + (½) x 0.30 x 84.44^2) + (13.33 x (t – 84.44)) = (0 x 44.44 + (½) x 0.15 x 44.44^2) + (6.67 x (t – 44.44))

t = 102.23 s

The time taken for train B to overtake train A is 40 s less than the total time
i.e. time = 62.23 s

Motion with varying acceleration

A vehicle is not accelerated uniformly. Initially the vehicle starts with zero acceleration, then the rate of acceleration is increased and when is desired speed is nearing the rate acceleration is reduced. By the time the desired speed is achieved, the acceleration is brought to zero. If variation of displacement, velocity and acceleration with respect to time is known, then the problems can be solved using differential equations.

d^2

v = velocity = $\frac{ds}{dt}$

a = acceleration = $\frac{dv}{dt} = \frac{dv}{ds} \text{ x } \frac{ds}{dt} = d^2s/dt^2$

a = $\frac{dv}{ds}\ x\ v$

Problems

1. The motion of a particle moving in a straight line is given by the equation $S = t^3 - 3t^2 + 2t + 5$ where s is displacement in m and t is time in s.

   Determine

   (i) Velocity and acceleration after 4 s

   (ii) Maximum or minimum velocity and corresponding displacement

   (iii) Time at which velocity is zero

Solution:

Given: equation $S = t^3 - 3t^2 + 2t + 5$

(i) Velocity and acceleration after 4 s

Velocity v = $\frac{ds}{dt}$

$= 3t^2 - 6t + 2$

Acceleration a = d^2s/dt^2

$= 6t - 6$

v at t = 4 s

$v = 3 \text{ x } 4^2 - 6 \text{ x } 4 + 2$

$= 26$ m/s

a at t = 4 s

$a = 6 \text{ x } 4 - 6$

$= 18 \text{ m/s}^2$

. : v = 26 m/s & a = 18 m/s^2

(ii) Maximum or minimum velocity and corresponding displacement

This can happen when a = $\frac{dv}{dt} = 0$

a = 6t - 6

When a = 0

t = 1 sec

so corresponding velocity at t = 1 s

$v = 3t^2 - 6t + 2$

$v = 3 \times 1^2 - 6 \times 1 + 2$

v = - 1 m/s minimum velocity

(iii) Time at which velocity is zero

$V = 3t^2 - 6t + 2$, in this eqn take v = 0 to find time at which v is zero

$0 = 3t^2 - 6t + 2$

This is a quadratic equation, and solving the above equation gives me

t = 0.42 s

t = 1.58 s

5.7 Curvilinear motion:

Definition of curvilinear motion – a particle is said to be in curvilinear motion when it traces a curved path. Ex: a car moving through a curved path

Angular displacement (θ): it is a total angle traced by a particle with respect to its original position along the circular path. It is measured in rad

Angular velocity (w): it is defined as rate of change of displacement θ with respect to time. It is expressed by as rad/s. it is given by

$w = 2\pi r/v$

Angular acceleration (α): it isdefined as rate of change of angular velocity. It is measured in rad/s^2

$\alpha = dw/dt = w \times dw/d\theta$

relation between linear acceleration and angular acceleration

linear velocity = radius x angular velocity

v = r x w

linear acceleration = radius x angular acceleration

a = r x α

equation of motion along circular path

$w = w_{o+} \alpha t$

$\theta = w_o t + \frac{1}{2}\alpha t^2$

$w^2 - w_o^2 = 2\alpha\theta$

wherew_o = initial velocity

α = angular acceleration

θ = displacement

Problem

1. a body is rotating with an angular velocity of 6 rad/s. aster 5 s the angular velocity becomes 15 rad/s. determine the angular acceleration of the body

Given w_o= 6 rad/s; w = 15 rad/s; t = 5 s

Wkt $w = w_{o+} \alpha t$

15 = 6 + α x 5

. : α = 1.8 rad/s^2

2. a fly wheel starts rotating from rest and is given an angular acceleration of 1 rad/s^2. Determine the angular velocity and speed in rpm after 90 s. if the fly wheel is brought to rest with an uniform angular retardation of 0.5 rad/s^{2}find the time required by the fly wheel to come at rest.

Given w_o = 0; α = 1 rad/s^2;

. : w be the final velocity after t = 90 s

From relation $w = w_{o+} \alpha t$

w = 0 + 1 x 90

w = 90 rad/s

speed in rpm w = (2πn/60) x 90

= 2 x π x n/ 6

n = 859.44 rpm

case 2

to bring to rest let t be the time taken

Initial angular velocity w_o = 90 rad/s

Angular retardation α = - 0.5 rad/s^2

Final angular velocity w = 0

$w = w_{o+} \alpha t$

0 = 90 + (-0.5) x t

. : t = 180 s

Projectile: a particle projected upwards at certain angle moving under the combined effect of vertical and horizontal components of velocity is called projectile

Trajectory: a path traced by the projectile in the space is known as trajectory

Velocity of projection : the velocity with which a projectile is projected into the space is called velocity of projection

Angle of projection; it is the angle α with the horizontal t which the projectile is projected

Time of fight: it is a total time taken from projection to reach maximum height and return back to the ground is known as time of flight.

Range: the distance between the points of projection and where the projectile strikes the ground is known as range

Formula used:

Path of the projectile

$y = x * \tan\alpha – (gx^2/2u^2\cos^2\alpha)$

Where α = angle of projection

u = velocity

Horizontal range

$r = (u^2\sin2\alpha)/g$

Time of flight

$$t = \frac{\boldsymbol{2usin\alpha}}{\boldsymbol{g}}$$

Maximum height of projectile on a horizontal plane

$H = u^2\sin^2\alpha/2g$

Problem

1. a projectile is fired with an initial velocity of 40 m/s at an angle of 25^o with the horizontal. Determine (i) the horizontal range (ii) Maximum height (iii) time of flight

Given velocity u = 40 m/s and angle of projection $\alpha = 25^o$

Horizontal range $r = u^2\sin2\alpha/g$

$r = (40^2 \text{ x } \sin^2 25)/9.81$

. : r = 124.94 m

Maximum height of projectile on a horizontal plane

$H = u^2\sin^2\alpha/2g$

$H = (40^2 \text{ x } \sin^2 25)/(2 \text{ x } 9.81)$

. : H = 14.56 m

Time of flight $t = \frac{2u\sin\alpha}{g}$

$t = (2 \text{ x } 40 \text{ x } \sin 25)/9.81$

. : t = 3.446 s