MODULE – 2

SINGLE-PHASE A.C. CIRCUITS

The path for the flow of alternating current is called on A.C. circuit.

In a D.C. circuit, the current/flowing through the circuit is given by the simple relation $I = \frac{V}{R}$. However, in an A.C. circuit, voltage and current change from instant to instant and so give rise to magnetic (inductive) and electrostatic (capacitive) effects. So, in an A.C. circuit, inductance, and capacitance must be considered in addition to resistance.

i) AC circuit containing pure ohmic resistance only.

ii) AC circuit containing pure inductance only.

iii) AC circuit containing pure capacitance only.

AC circuit containing pure ohmic Resistance

Let us consider an A.C. circuit with just a pure resistance R only, as shown in Fig 2.1

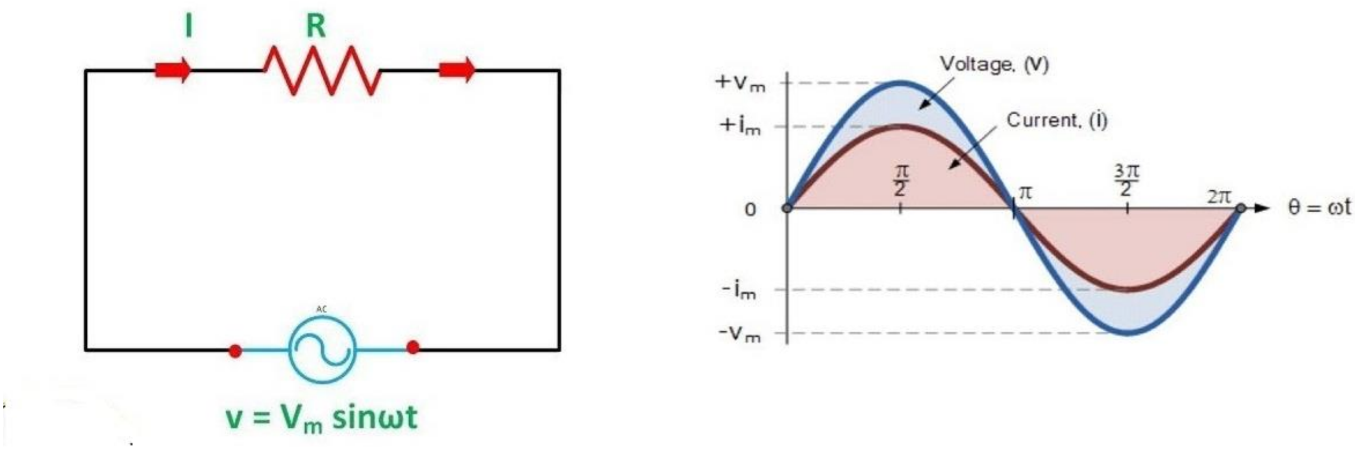


Fig 2.1 Pure Resistive Circuit

Let the applied voltage be given by the equation

$$v = V_m \sin\theta = V_m \sin\omega t \qquad \text{--- (i)}$$

As a result of this alternating voltage, the alternating current 'I' will flow through the

circuit. The applied voltage has to supply the drop in the resistance, i.e.,

$$v = iR$$

Substituting the value of v from Eqn. (i), we get

$$V_m \sin\omega t = iR \text{ or } i = \frac{V_m}{R}\sin\omega t \text{ ----------------(ii)}$$

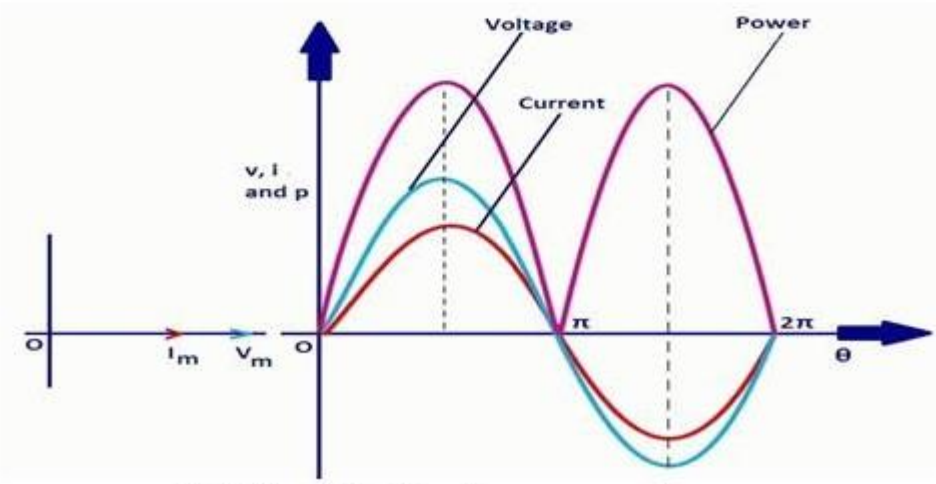


Fig2.2 Graph Showing voltage, current and power

The value of the alternating current ‘I’ is maximum when sin ωt= 1,

i.e., $I_m = \frac{V_m}{R}$

Eqn. (ii) becomes,

i=Im Sin ωt… ....(iii)

From eqns. (i) and (ii), it is apparent that voltage and current are in phase with each other. This is also indicated by the wave and phasor diagram

Power: The voltage and current are changing at every instant.

$$p = vi$$

$$P = (v_m \sin \omega t)\ (\text{Im} \sin (\omega t + \frac{\pi}{2}))$$

$$P = V_m\ \text{Im}^{\sin\omega t} \cos \omega t$$

$$p = \frac{V_m \text{Im}}{2} \cdot 2 \sin \omega t \cdot \cos \omega t$$

$$P = \frac{v_m}{\sqrt{2}} \cdot \frac{\text{Im}}{\pi} \cdot \sin 2\omega t$$

P=0

Power curve

The power curve for a purely resistive circuit is shown in Fig2.3. It is apparent that power in such a circuit is zero only at the instants a,b and c, when both voltage and current are zero, but is positive at all other instants. in other words, power is never negative, so that power is always lost in a resistive a.c. circuit. This power is dissipated as heat.

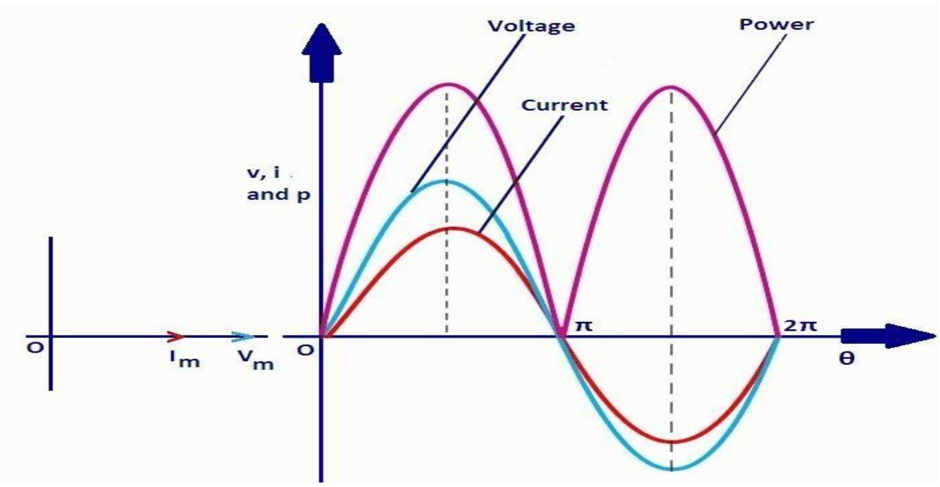


Fig2.3 Graph Showing voltage, current and power

A.C. circuit containing pure Inductance

An inductive coil is a coil with or without an iron core and has negligible resistance. In practice, pure inductance can never be had as the inductive coil has always a smallresistance. However, a coil of thick copper wire wound on a laminated iron core has negligible resistance, so, for the purpose of our study, we will consider a purely inductive coil is shown in Fig 2.4.

On the application of an alternating voltage to a circuit containing a pure inductance, a back e.m.f. is produced due to the self-inductance of the coil. This back e.m.f. opposes the rise or fall of current, at every stage. Because of the absence of voltage drop, the applied voltage has to overcome this self-induced e.m.f. only.

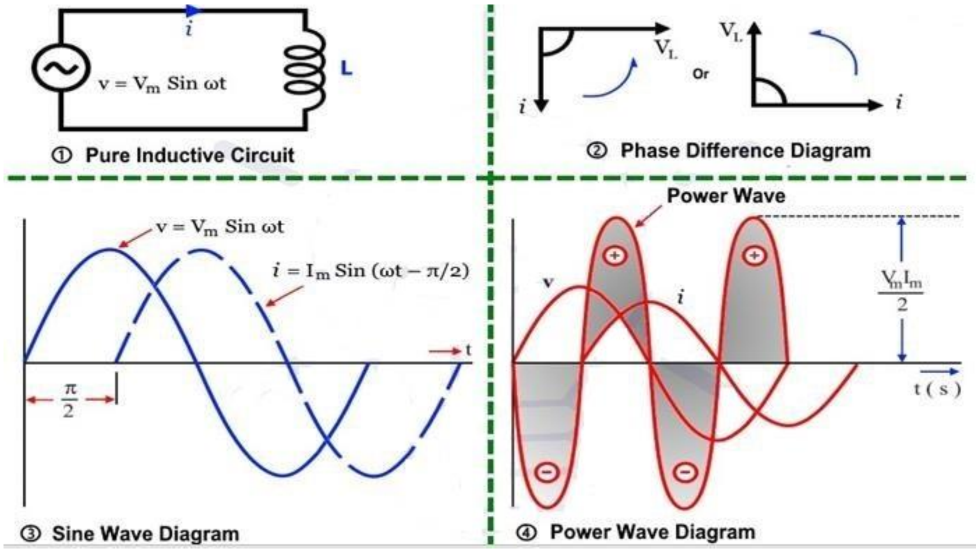


Fig 2.4 Graph showing voltage, current and power for an Inductive circuit

Let the emf be E

$E = E_0 \sin\omega t$

For an inductor, emf induced = $-L\frac{dI}{dt}$

To maintain the flow of current, the applied voltage must be equal and opposite to the induced voltage

$E = -\{-L\frac{dI}{dt}\} = E_0 \sin\omega t$

$dI = \frac{E0}{L} \sin\omega t$

integrating both sides,

$I = \frac{E_0}{L} \int \sin\omega t \; dt$

$= \frac{E_0}{L}(-\frac{\cos\omega t\cdot}{\omega}) = -\frac{E_0}{\omega L}\cos\omega t$

$= -\frac{E_0}{\omega L}\sin(\frac{\pi}{2} - \omega t) = \frac{E_0}{\omega L}\sin(\omega t - \frac{\pi}{2})$

Current will be maximum when $I_0 = \frac{E_0}{\omega L} : I = I_0 \sin(\omega t - \frac{\pi}{2})$

Hence the alternating current lags behind the emf by $\pi/2$ or 90^{o} as shown in Fig 2.4.

The power measured by a wattmeter is the average value of p, which is zero since average of a sinusoidal quantity of double frequency over a complete cycle is zero is shown in Fig 2.4. Put in mathematical terms,

In a pure inductive circuit,

$I = I_0 \sin\omega t$

$V = V_0 \sin(\omega t + \Pi/2)$

V leads I by $\Pi/2$

Hence power , $P = \frac{1}{T}\int_0^T VI\,dt$

$$P = \frac{1}{T}\int_0^T V_0 I_0 \sin\omega t \cos wt\,dt$$

$$P = \frac{v_0 I_0}{2T}\int_0^T \sin 2\omega t \cdot dt$$

P=0

Power for the whole cycle, P = $\frac{-V_m I_m}{2}\int_0^{2\pi} \sin 2\,\omega t$ dt = 0

Hence, power absorbed in a pure inductive circuit is zero. This indicates that power absorbed in the circuit is zero. At the instants a,c and e, voltage is zero, so that power is zero: it is also zero at points b and d when the current is zero. Between a and b voltage and current are in opposite directions, so that power is negative and energy is taken from the circuit. Between b and c voltage and current are in the same direction, so that power is positive and is put back into the circuit. Similarly, between c and d, power is taken from the circuit and between d and e it is put into the circuit. Hence, net power is zero.

A.C. circuit containing pure Capacitance:

To analyses a pure capacitive circuit with the application of AC voltage a circuit with pure capacitance is considered as shown in Fig 2.5.

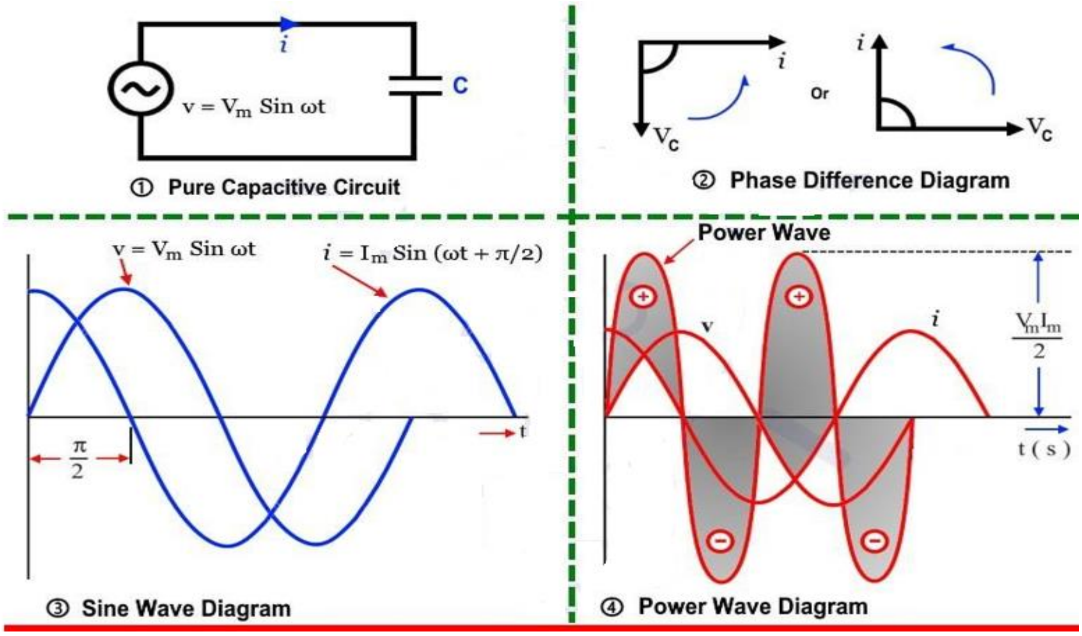


Fig 2.5 Pure Capacitive Circuit

Let an alternating voltage represented by v = V_{max} sin ω t be applied across a capacitor of capacitance C farads.

The expression for instantaneous charge is given as:

q = C V_{max} sin ωt

Since the capacitor current is equal to the rate of change of charge, the capacitor current may be obtained by differentiating the above equation:

$$i = \frac{dq}{dt} = [\omega V_{max} \sin \omega t]$$

$$\frac{v_{max}}{1/\omega C} \sin\left[\omega t + \frac{\pi}{2}\right]$$

Current is maximum when t = 0

Hence, $I_{max} = \frac{v_{max}}{1/\omega C}$

Substituting, $\frac{v_{\max}}{1/\omega C}$ = I_{max} in the above equation for instantaneous current, we get

$$i = I_0 \sin\left[\omega t + \frac{\pi}{2}\right]$$

From the equations of instantaneous applied voltage and instantaneous current flowing through capacitance, it is observed that the current leads the applied voltage by π/2.

Capacitive Reactance:

1/ω C in the expression $I_{max} = V_{max}/1/\omega C$ is known as capacitive reactance and is denoted by X_C i.e., XC =1/ω C

If C is in farads and ω is in radians/s, then X_c will be in ohms.

Power in Purely Capacitive Circuit:

$$P = v\,i = V_m \cdot \sin \omega t \;\; I_m \sin\left(\omega t + \frac{\pi}{2}\right) = V_m I_m \sin wt \cos \omega t$$

$$= \frac{VmIm}{2} \cdot \sin 2\omega t = 0$$

Hence power absorbed in a purely capacitive circuit is zero. The energy taken from the supply circuit is stored in the capacitor during the first quarter- cycle and returned during the next.

The energy stored by a capacitor at maximum voltage across its plates is given by the expression:

$$W_c = \frac{1}{2} C V_{\max}^2$$

This can be realized when it is recalled that no heat is produced and no work is done while current is flowing through a capacitor. As a matter of fact, in commercial capacitors, there is a slight energy loss in the dielectric in addition to a minute I^2R loss due to flow of current over the plates having definite ohmic resistance.

The power curve is a sine wave of double the supply frequency. Although it raises the power factor from zero to 0.002 or even a little more, but for ordinary purposes the power factor is taken to be zero. the phase angle due to dielectric and ohmic losses decreases slightly.

Series R-L circuit

Let us consider an a.c. circuit containing a pure resistance of R ohms and a pure inductance of L henry connected in series as shown in Fig 2.6.

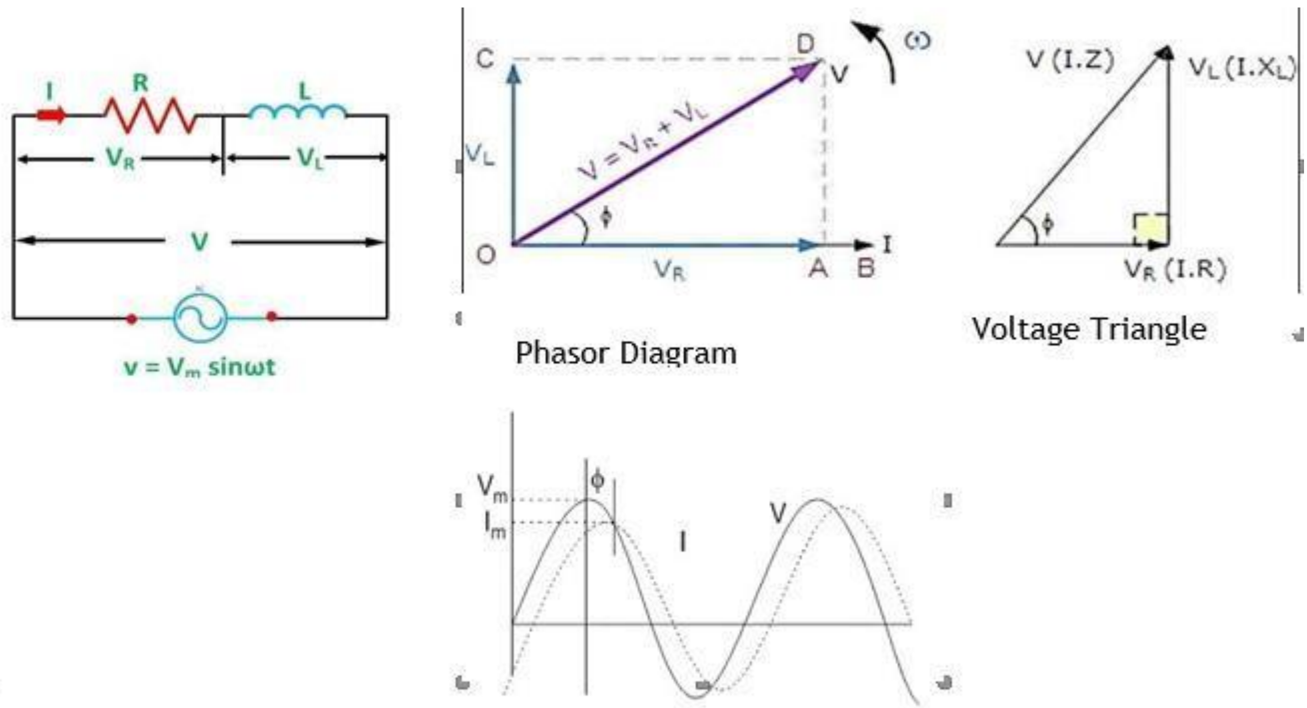


Fig 2.5 Series RL circuit with Voltage leading current waveform

Let V = r.m.s. value of the applied voltage
I = r.m.s. value of the current

Voltage drop across R, V_R = IR (in phase with I) Voltage drop across L, $V_L = IX_L$ (leading I by 90^0)

The voltage drops across these two circuit components are shown inFig2.6. where phasor OA indicates V_R and

OC indicates V_L. The applied voltage V is the phasor sum of the two i.e OD

$$V = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (Ix_L)^2}$$

$$= I\sqrt{R^2 + X_L^2}$$

$R^2 + X_L^2 = Z^2$

Referring to Fig 2,6, we observe that the applied voltage V leads the current I by an angle ϕ

$$\frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{Reactance}{Resistance} \qquad \emptyset = \tan^{-1}\frac{X_L}{R}$$

The same feature is shown by means of waveforms. We observe that circuit current lags behind applied voltage by an angle.

Phasor diagram:

For drawing the phasor diagram of series RL circuit; follow the following steps:

Step- I. In case of series RL circuit, resistor and inductor are connected in series, so current flowing in both the elements are same i.e $I_R = I_L = I$. So, take current phasor as reference and draw it on horizontal axis as shown in diagram.

Step- II. In case of resistor, both voltage and current are in same phase. So draw the voltage phasor, V_R along same axis or direction as that of current phasor. i.e V_R is in phase with I.

Step- III. We know that in inductor, voltage leads current by 90^o, so draw V_L (voltage drop across inductor) perpendicular to current phasor.

Step- IV. Now we have two voltages V_R and V_L. Draw the resultant phasor(V_G) of these two voltages.

Conclusion: In case of pure resistive circuit, the phase angle between voltage and current is zero and in case of pure inductive circuit, phase angle is 90^o but when we combine both resistance and inductor, the phase angle of a series RL circuit is between 0^o to 90^o.

Impedance of Series RL Circuit

The impedance of series RL circuit opposes the flow of alternating current. The impedance of series RL circuit is nothing but the combine effect of resistance (R) and inductive reactance (X_L) of the circuit as a whole.

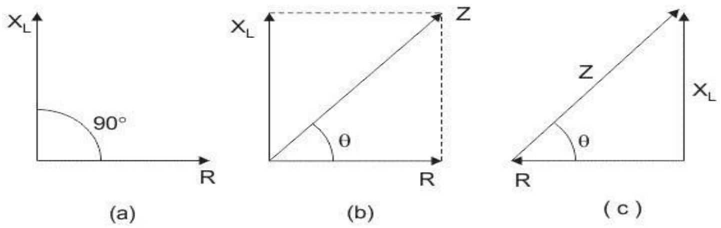


Fig 2.7 Phase difference between quantities

Series RL Circuit Analysis

In series RL circuit, the values of frequency f, voltage V, resistance R and inductance L are known and there is no instrument for directly measuring the value of inductive reactance and impedance; so, for complete analysis of series RL circuit, follow these simple steps:

Step 1. Since the value of frequency and inductor are known, so firstly calculate the value of inductive reactance

X_L: $X_L = 2\pi fL$ ohms.

Step 2. From the value of X_L and R, calculate the total impedance of the circuit which is given by

$$Z = \sqrt{(R^2 + X_L^2)}$$

Step 3. Calculate thetotal phaseangle for thecircuit $\theta = \tan^{-1}(X_L/R) = \tan^{-1}(\omega L/R)$

Step 4. Use Ohm's Law and find the value of the total current: $I = V/Z$ amp.

Step 5. Calculate the voltages across resistor R and inductor Lbyusing Ohm's Law. Since the resistor and the inductor are connected in series, so current in them remains the same.

Power in an RL Circuit

In series RL circuit, some energy is dissipated by the resistor and some energy is alternately stored and returned by the inductor-

The instantaneous power deliver by voltage source V is $P = VI$ (watts).

Power dissipated by the resistor in the form of heat, $P = I^2R$ (watts).

The rate at which energy is stored in inductor,

$$P = V_L I = LI\frac{dI}{dt}\ (watts)$$

So, total power in series RL circuit is given by adding the power dissipated by the resistor and the power absorbed by the inductor as shown in Fig 2.7

$$P = I^2R + LI\frac{dI}{dt}$$

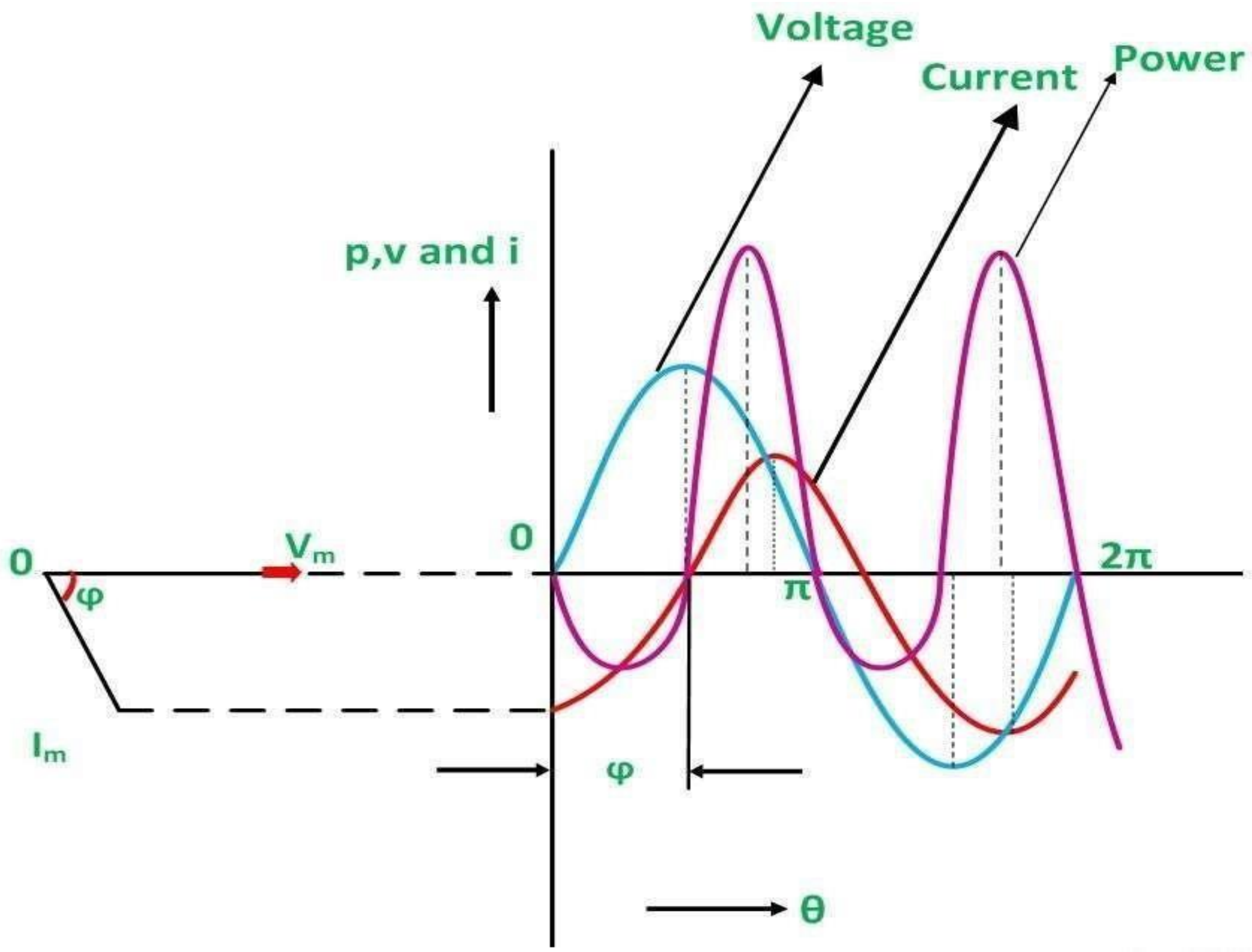
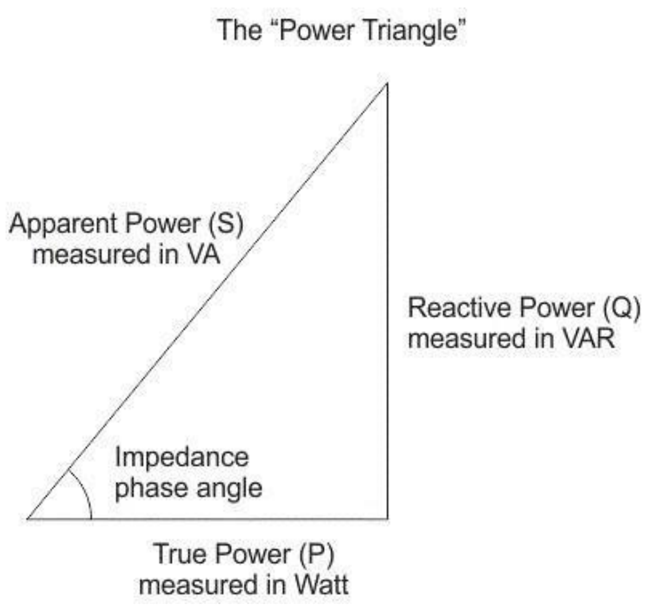


Fig 2.8: Series RL circuit showing P,V and I values

Power triangle for series RL circuit is shown below,



Definition of Real power, Reactive Power, Apparent power and power Factor

Let a series R-L circuit draw a current I (r.m.s. value) when an alternating voltage of r.m.s. value V is applied to it. Suppose the current lags behind the applied voltage by an angle ϕ as shown in Fig 2.9.

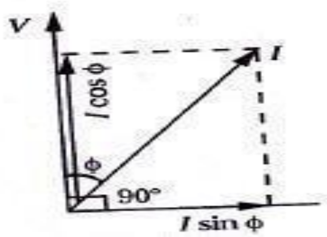


Fig 2.9: Real, Reactive and Apparent power

Power Factor and its significance

Power Factor may be defined as the cosine of the angle of lead or lag. In Fig 2.8. the angle of lag is shown .In addition to having a numerical value, the power factor of a circuit carries a notation that signifies the nature of the circuit, i.e., whether the equivalent circuit is resistive, inductive or capacitive. Thus, the p.f. might be expressed as 0.8 lagging. The lagging and leading refers to the phase of the current phasor with respect to the voltage phasor. Thus, a lagging power factor means that the current lags the voltage and the circuit is inductive in nature. However, in the case of leading power factor, the current leads the voltage and the circuit is capacitive.

Apparent Power: The product of r.m.s. values of current and voltage, VI, is called the apparent power and is measured in volt-amperes (VA) or in kilo-volt amperes (KVA).

Real Power: The real power in an a.c. circuit is obtained by multiplying the apparent power by the factor and is expressed in watts or killo-watts (kW).

$$\text{Real power (W)} = \text{volt-amperes (VA)} \times \text{power factor } \cos\phi$$

$$\text{or} \quad \textbf{Watts} = \textbf{VA} \cos\phi$$

Here, it should be noted that power consumed is due to ohmic resistance only as a pure inductance does not consume any power.

$$\text{Thus,} \quad P = V I \cos\phi$$

$$\cos\phi = \frac{R}{Z} \text{ (refer to the impedance triangle of Fig. 3.45)}$$

$$\therefore \; P = V I \times \left[\frac{R}{Z}\right]$$

$$= \left[\frac{V}{Z}\right] \times IR = I^2R$$

$$\text{or} \quad \mathbf{P = I^2R} \textbf{ watts}$$

Resistance — Capacitance (R-C) Series Circuit:

Consider an AC circuit consisting of resistance of R ohms and capacitance of C farads connected in series, as shown in Fig. 2.10 (a). Let the supply frequency be of *f* Hz and current flowing through the circuit be of I amperes

(rms value). Voltage drop across resistance, $V_R = I R$ in phase with the current.

Voltage drop across capacitance, $V_C = I X_C$ lagging behind I by $\pi/2$ radians or 90°, as shown in Fig. 2.10 (b).

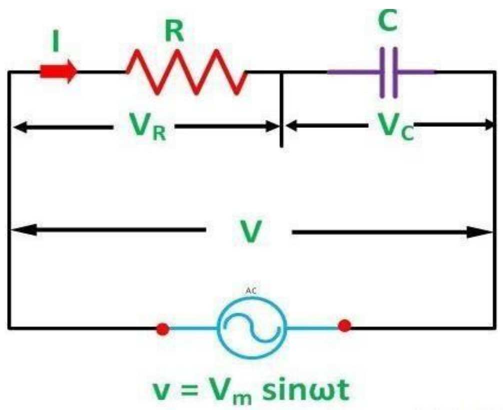


Fig 2.10a

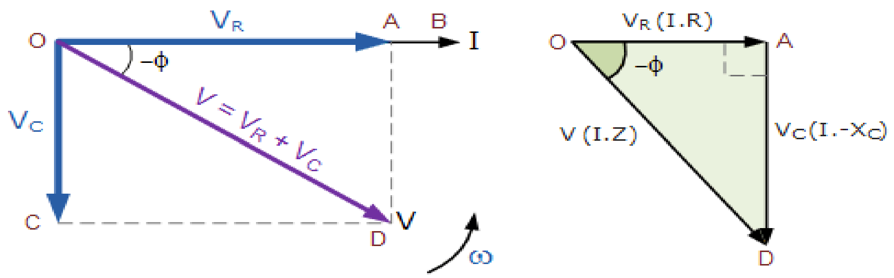


Fig 2.10b

Fig. 2.10 Series RC circuit

The applied voltage, being equal to phasor sum of V_R and V_C, is given in magnitude by-

$$V = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (IX_C)^2} = I\sqrt{R^2 + X_C^2} = IZ$$

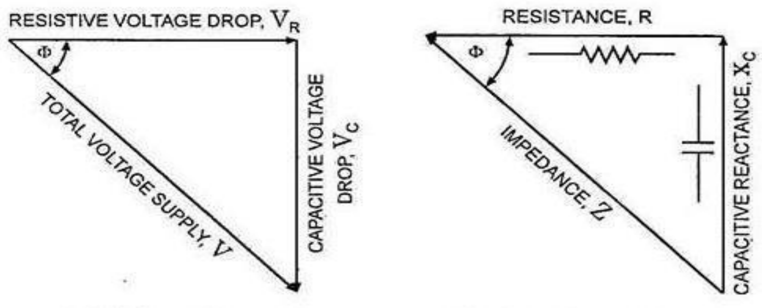
where $Z^2 = R^2 + X_C^2$

The applied voltage lags behind the current by an angle ϕ:

$$\text{where} \qquad \tan\Phi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R} = \frac{1}{\omega RC} \quad \text{or } \Phi = \tan^{-1}\frac{1}{R\omega C}$$

$$\text{Power factor, } \cos\Phi = \frac{R}{Z}$$

The impedance showing the phasor relationship of R, X_c and Z is shown in impedance triangle and the phasor relationship between V_R, V_C and V in voltage triangle in Fig 2.11.



(a) Voltage Triangle *(b) Impedance Triangle*

Fig 2.11 Voltage triangle and impedance triangle

If instantaneous current will be expressed by: $V = V_m \sin \omega t$

Then instantaneous current will be expressed as: $i = I_m \sin(\omega t + \phi)$

And power consumed by the circuit is given by: $P = VI \cos\phi$ and the same is depicted in Fig 2.12

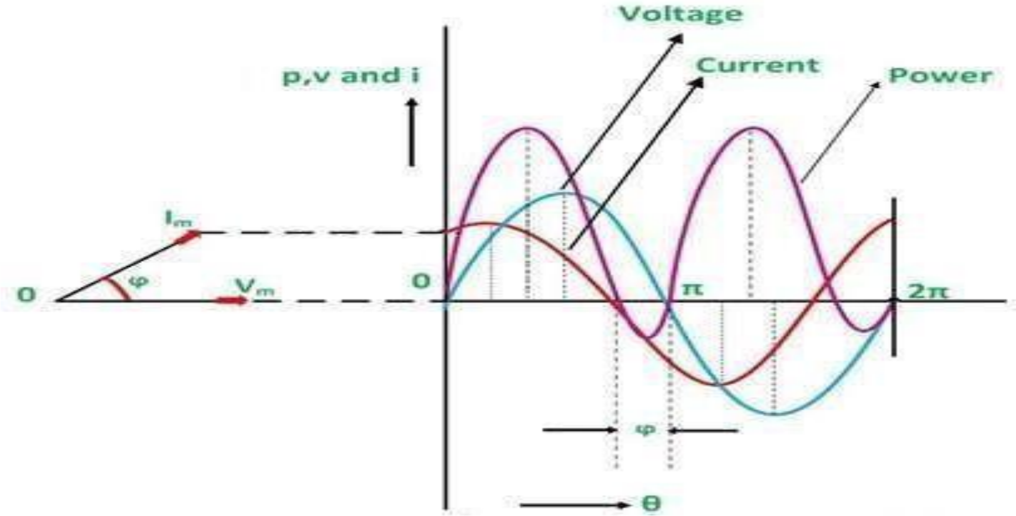


Fig 2.12: Power curve for Series RC circuit

Power: Average power, $P = v \times i$

Power curves: The power curve for R − C series circuit is shown in Fig2.13.. The curve indicates that the greater part is positive and the smaller part is negative, so that the net power is positive.

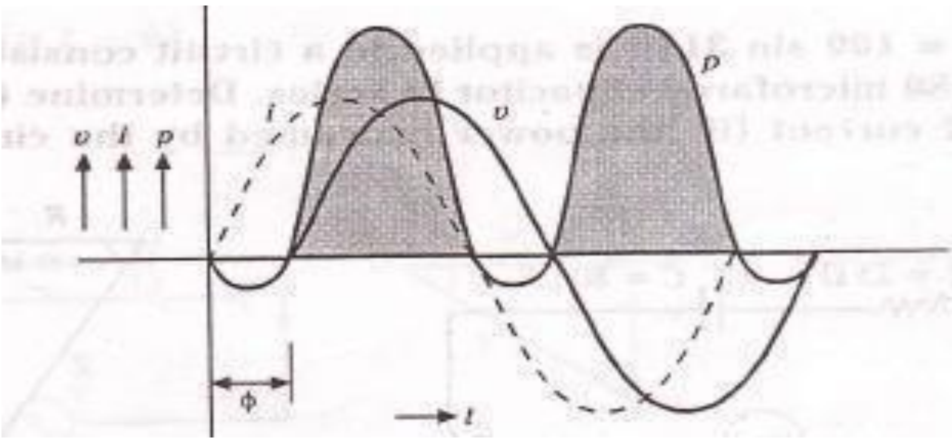


Fig 2.13 : Power curve for Series RC circui

Apparent Power, True Power, Reactive Power and Power Factor:

The product of rms values of current and voltage, VI is called the apparent power and is measured in volt-amperes or kilo-volt amperes (kVA).

The true power in an ac circuit is obtained by multiplying the apparent power by the power factor and is expressed in watts or kilo-watts (kW).

The product of apparent power, VI and the sine of the angle between voltage and current, sin ϕ is called the reactive power. This is also known as wattless power and is expressed in reactive volt-amperes or kilo-volt amperes reactive (kVA R).

The above relations can easily be followed by referring to the power diagram shown in Fig 2.14

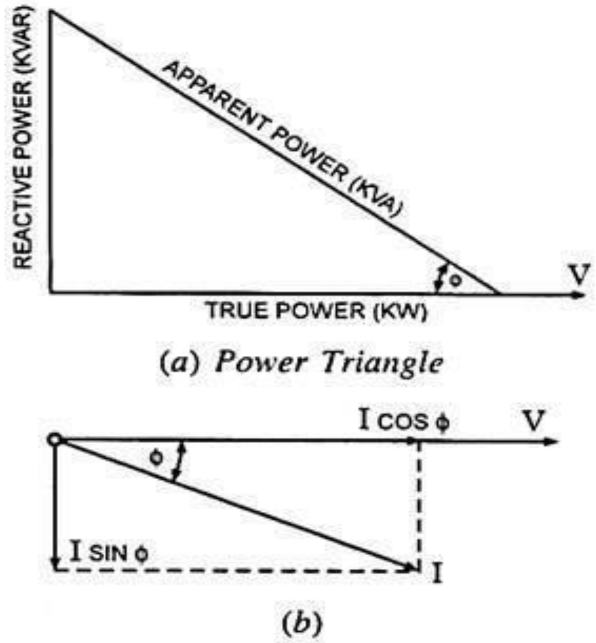


Fig. 2.14 Power diagram of RC circuit

Power factor may be defined as:

(i) Cosine of the phase angle between voltage and current
(ii) The ratio of the resistance to impedance, or
(iii) The ratio of true power to apparent power.

The power factor can never be greater than unity. The power factor is expressed either as fraction or as percentage.Itisusualpracticetoattachtheword'lagging'or'leading'withthenumericalvalueofpowerfactor to signify whether the current lags behind or leads the voltage.

Active Component of Current: The current component which is in phase with circuit voltage (i.e.,I$\cos\phi$) and contributes to active or true power of the circuit is called the active (wattful or in-phase) component of current.

Reactive Component of Current: The current component which is in quadrature (or 90° out of phase) to circuit voltage (i.e.,I$\sin\phi$)andcontributestoreactivepowerofthecircuit, is called the reactive (or wattless) component of current.

Q-Factor of Coil:

Reciprocal of power factor is known as Q-factor of the coil. It is also called the quality factor or figure of merit of a coil.

$$\text{Mathematically Q-factor} = \frac{1}{\text{Power factor}} = \frac{1}{\cos\Phi} = \frac{Z}{R}$$

If R is very small in comparison to inductive reactance X_L, the

$$\text{Q-factor} = \frac{X_L}{R} = \frac{\omega L}{R}$$

$$\text{Also } Q = 2\pi \times \frac{\text{Maximum energy stored}}{\text{Energy dissipated per cycle}}$$

Resistance, Inductance and capacitance in series (RLC – Series Circuit)

When a resistor, inductor and capacitor are connected in series with the voltage supply, the circuit so formed is called series RLC circuit.Consider an a.c. series circuit containing resistance R ohms, Inductance L henries and capacitance C farads, as shown in the Fig 2.15 series RLC circuit

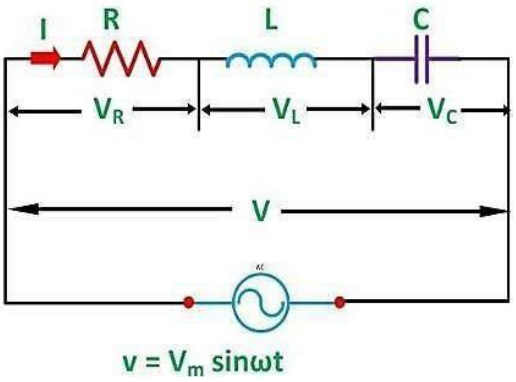


Fig 2.15: Series RLC circuit

The impedance Z of a series RLC circuit is defined as opposition to the flow of current due circuit resistance R, inductive reactance, X_L and capacitive reactance, X_C.

If the inductive reactance is greater than the capacitive reactance i.e $X_L > X_C$, then the RLC circuit behaves like series R-L circuit and has lagging phase angle as shown in Fig 2.15a and hence lagging power factor.

If the capacitive reactance is greater than the inductive reactance i.e $X_C > X_L$ then, the RLC circuit behaves like series R-C circuit and has leading phase angle as shown in Fig 2.16b and hence leading power factor.

While if both inductive and capacitive are same i.e $X_L = X_C$ then circuit will behave as purely resistive circuit.

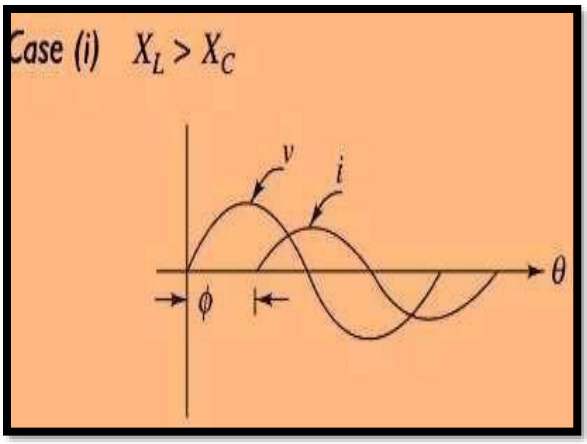


Fig 2.16a

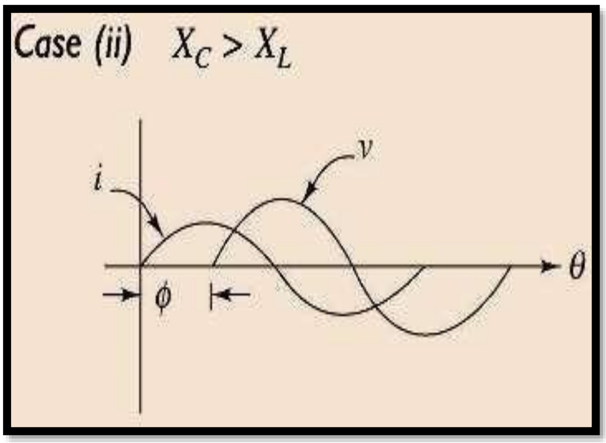


Fig 2.16b

Fig 2.16 Voltage and Current wave forms

Phasor diagram:

The phasor diagram for the case when the inductive reactance is predominant i.e $X_L > X_C$ is shown in Fig 2.17a and the phasor diagram for the case when the capacitive reactance is predominant i.e $X_C > X_L$ is shown in Fig 2.17b

a) Case (i) when $X_L > X_C$

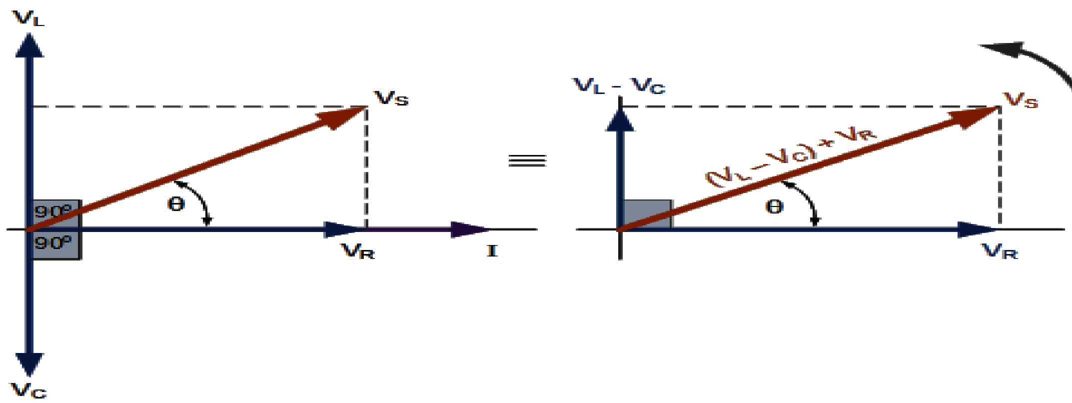


Fig 2.17a: Phasor Representation of Series RLC circuit with $X_L > X_c$

b) Case (ii) $X_C > X_L$.

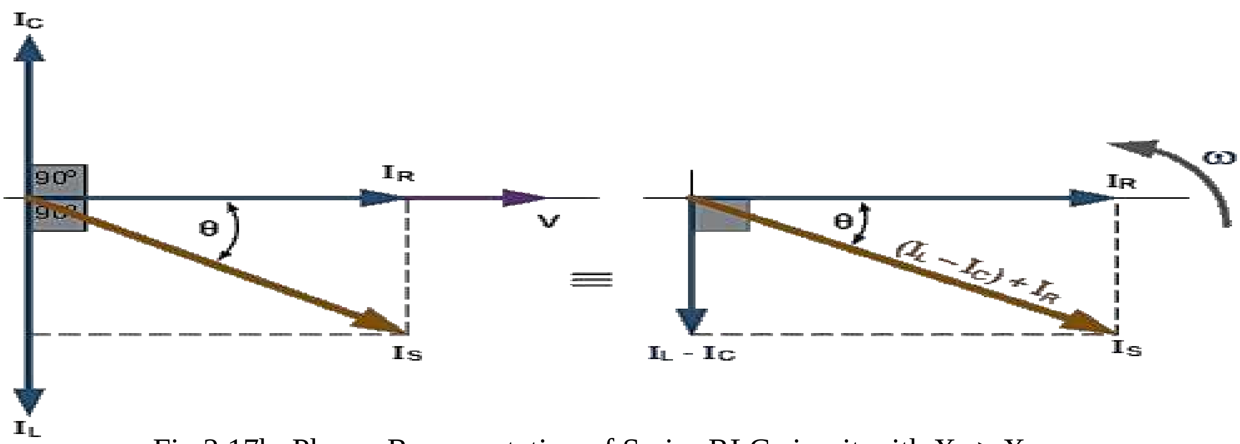


Fig 2.17b: Phasor Representation of Series RLC circuit with $X_C > X_L$

Since all these components are connected in series, the current in each element remains the same,

$I_R = I_L = I_C = I(t)\ where\ I(t) = I_M \sin \omega t$

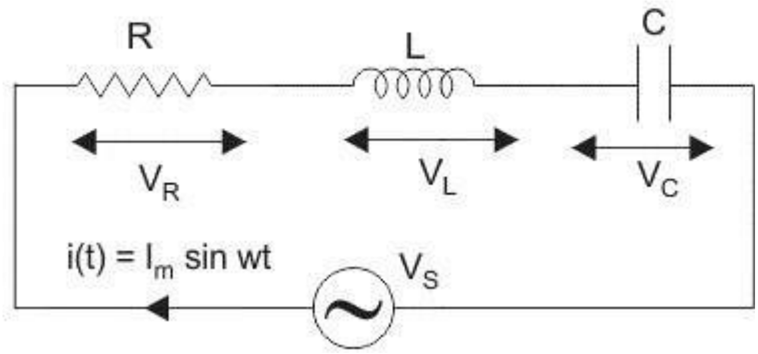
Let V_R be the voltage across resistor, R.

V_L be the voltage across inductor, L.

V_C be the voltage across capacitor, C.

X_L be the inductive reactance.

X_C be the capacitive reactance.



The total voltage in the RLC circuit is not equal to the algebraic sum of voltages across the resistor, the inductor, and the capacitor; but it is a phasor sum because, in the case of the resistor the voltage is in-phase with the current, for inductor the voltage leads the current by 90° and for capacitor, the voltage lags behind the current by 90° So, voltages in each component are not in phase with each other; so they cannot be added arithmetically. The Fig 2.17 below shows the phasor diagram of the series RLC circuit. For drawing the phasor diagram for RLC series circuit, the current is taken as reference because, in series circuit the current in each element remains the same and the corresponding voltage phasors for each component are drawn in reference to common current phasor.

$$V_S^2 = V_R^2 + (V_L - V_C)^2 (if V_L > V_C)$$

$$V_S^2 = V_R^2 + (V_L - V_C)^2 (if V_L < V_C)$$

$$Where\ V_R = IR, V_L = IX_L, V_C = IX_C$$

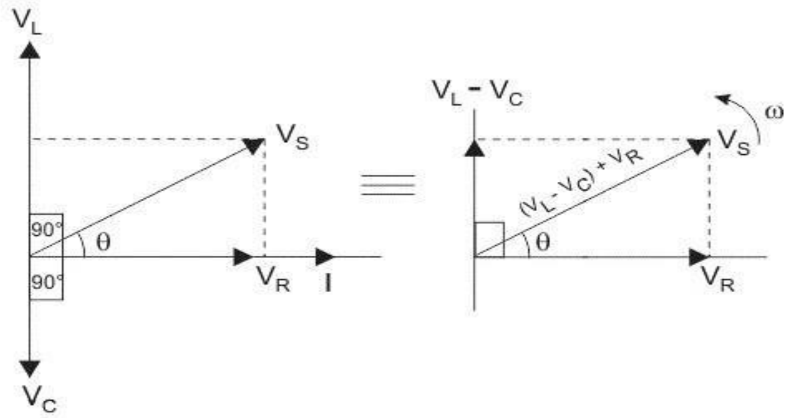


Fig 2.17 : Phasor Representation of Series RLC circuit

Let V = r.m.s. value of applied voltage

I = r.m.s. value of current

∴ Voltage drop across R, VR = IR - in phase with I

voltage drop across L, VL = I.X_L - lagging I by 90^0

Voltage drop across C, V_C = IX_C - lagging I by 90^0

Referring to the voltage triangle of Fig. OA represents V_R. AB and AC represent inductive and capacitive drops respectively. We observe that V_L and V_C are 180^0 out of phase.

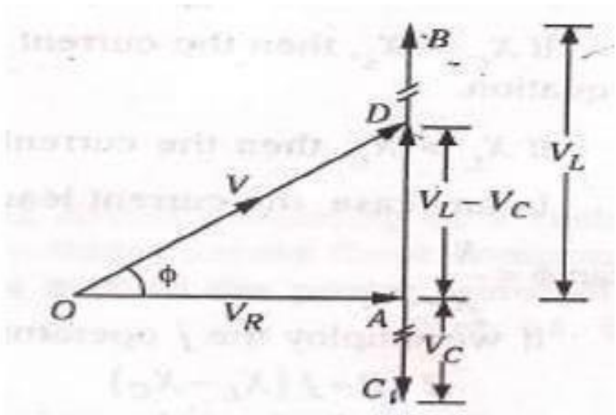


Fig 2.18 Voltage triangle of series RLC circuit when $X_L > X_C$

Thus, the net reactive drop across the combination is

$$AD = AB - AC$$
$$= V_L - V_C$$
$$= I(X_L - X_C)$$

OD, which represents the applied voltage V, is the vector sum of OA and AD.

$$\therefore \quad OD = \sqrt{OA^2 + AD^2} \qquad \text{OR } V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$
$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

$$\text{Or } I = \frac{V}{\sqrt{R^2+(X_L-X_C)^2}} = \frac{V}{\sqrt{R^2+X^2}} = \frac{V}{Z}$$

$$\text{Or } Z^2 = R^2 + (X_L - X_C)^2 = R^2 + X^2$$

Where the net reactance = X

Phase angle ϕ is given by $\tan \emptyset = \frac{(X_L - X_C)}{R} = \frac{X}{R}$

power factor,

$$\cos \emptyset = \frac{R}{Z} = \frac{R}{\sqrt{R^2+(X_L-X_C)^2}} = \frac{R}{\sqrt{R^2+X^2}}$$

Power = VI cos ϕ

If applied voltage is represented by the equation $v = V_m \sin\omega t$, then the resulting current in an R – L – C circuit is given by the equation

$$i = I_m \sin(\omega t \pm \phi)$$

If $X_C > X_L$, then the current leads and the +ve sign is to be used in the above equation.

If $X_L > X_C$, then the current lags and the –ve sign is to be used.

The denominator $\sqrt{R^2 + (X_L - X_C)^2}$ is the impendence of the circuit.

So (impedance)^2 = (resistance)^2 + (net reactance)^2

In any case, the current leads or lags the supplyvoltagebyan angle ϕ so that $\tan \emptyset = \frac{X}{R}$ if we employ the j

$$Z = R + j(X_L - X_C)$$

The value of the impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\text{The angle } \emptyset = \tan^{-1}\left[\frac{X_L - X_C}{R}\right]$$

$$Z\angle\phi = Z \tan^{-1}\left[\frac{X_L - X_C}{R}\right] = Z \tan^{-1}\frac{X}{R}$$

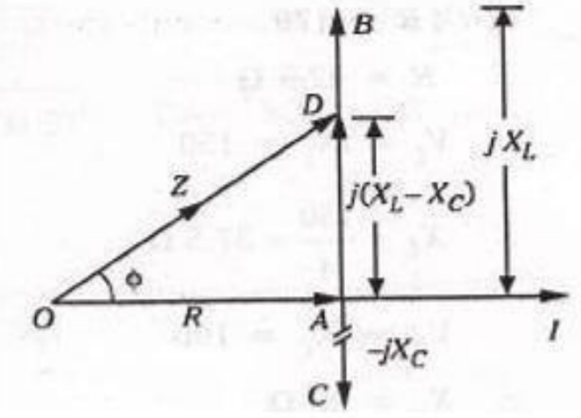


Fig 2.19: Impedance diagram of Series RLC circuit when $X_L > X_C$

Summary

- Inductive reactance: $X_L = 2\pi f L = \omega L$
- Capacitive reactance: $X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C}$
- When $X_L > X_C$ the circuit is Inductive
- When $X_C > X_L$ the circuit is Capacitive
- Total circuit reactance = $X_T = X_L - X_C$ or $X_C - X_L$
- Total circuit impedance = $Z = \sqrt{R^2 + X_T^2} = R + jX$

THREE PHASE CIRCUITS

ADVANTAGES OF THREE PHASE SYSTEM

In the three phase system, the alternator armature has three windings and it produces three independent alternating voltages. The magnitude and frequency of all of them is equal but they have a phase difference of 120^0 between each other. Such a three phase system has following advantages over single phase system:

1) The output of three phase machine is always greater than single phase machine of same size, approximately 1.5 times. So for a given size and voltage a three phase alternator occupies less space and has less cost too than single phase having same rating.
2) For a transmission and distribution, three phase system needs less copper or less conducting material than single phase system for given volt amperes and voltage rating so transmission becomes very much economical.
3) It is possible to produce rotating magnetic field with stationary coils by using three phase system. Hence three phase motors are self-starting.
4) In single phase system, the instantaneous power is a function of time and hence fluctuates w.r.t. time. This fluctuating power causes considerable vibrations in single phase motors. Hence performance of single phase motors is poor. While instantaneous power in symmetrical three phase system is constant.
5) Three phase system give steady output.
6) Single phase supply can be obtained from three phase but three phase cannot be obtained from single phase.
7) Power factor of single phase motors is poor than three phase motors of same rating.
8) For converting machines like rectifiers, the d.c. output voltage becomes smoother if number of phases are increased.

But it is found that optimum number of phases required to get all above said advantages is three. Any further increase in number of phases cause a lot of complications. Hence three phase system is accepted as standard polyphase system throughout the world.

Generation of 3-phase E.M.F.

In the 3-phase system, there are three equal voltages of the same frequency but displaced from one another by 120 degree electrical. These voltages are produced by a three-phase generator which has three identical windings or phases displaced 120 degree electrical apart. When these windings are rotated in a magnetic field, e.m.f. is induced in each winding or phase. These e.m.f. s are of the same magnitude and frequency but are displaced from one another by 120 degree electrical. Consider three electrical coils a_1, a_2; b_1, b_2; y_1, y_2 mounted on the same axis but displaced from each other by 120 degree electrical. Let the three coils be rotated in an anticlockwise direction in a bipolar magnetic field with an angular velocity of radians/sec, as shown in Fig 2.16 Here, a_1; b_1, y_1, are the start terminals and a_2; b_2, y_2 the end terminals of the coils. When the coil a_1,a_2 is in the position AB shown in Fig 2.20. The magnitude and direction of the e.m.f. s induced in the various coils is as under:

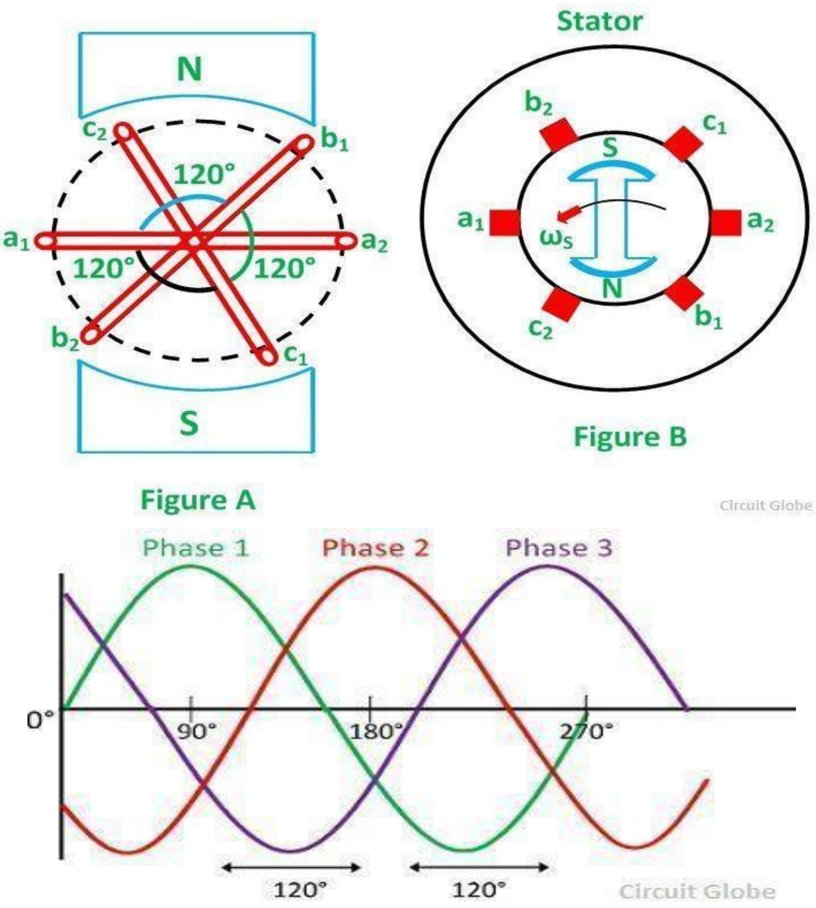


Fig 2.20 Generation of three phase circuit

Phase Sequence: Consider a balanced three phase EMF e_R, e_Y and e_B generated by three coils R, Y and B respectively as shown in Fig 2.21 If the coils R, Y and B are displaced by an angle 120° form one another then the EMFs generated are displaced from one another in the same order by 120° .

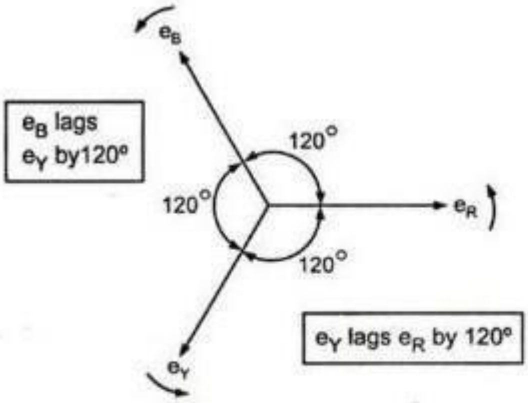


Fig 2.21 Phase sequence of Generation of three phase circuit

Equations: The equations for the three voltages are:

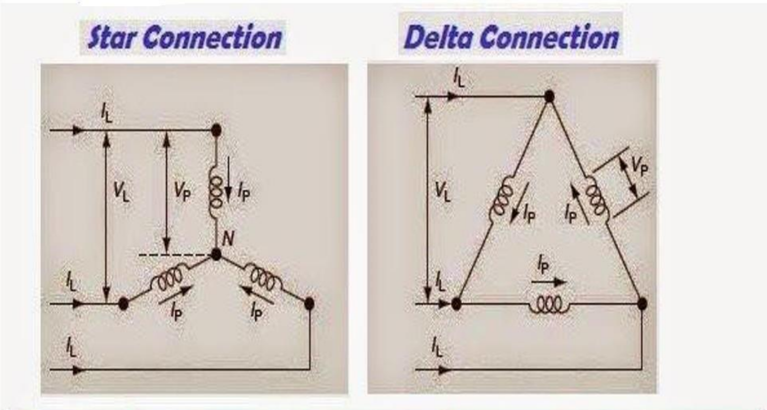
$$
\begin{aligned}
e_R &= E_m \sin(\omega t) \\
e_Y &= E_m \sin(\omega t - 120^\circ) \\
e_B &= E_m \sin(\omega t - 240^\circ) = E_m \sin(\omega t + 120^\circ)
\end{aligned}
$$

Meaning of phase sequence

It is necessary to employ some systematic notation for the solution of a.c. circuits and systems containing a number of e.m.f. s. acting and currents flowing so that the process of solution is simplified and less prone to errors.It is normally preferred to employ double-subscript notation while dealing with a.c. electrical circuits. In this system, the order in which the subscripts are written indicates the direction in which e.m.f. acts or current flows.

Balanced Supply and Load

When a balanced generating supply, where the three phase voltages are equal, and the phase difference is 120^0 between one another, supplies balanced equipment load, where theimpedance of the three phases or three circuit loads are equal, then the current flowing through these three phases will also be equal in magnitude, and will also have a phase difference of 120^0 with one another. Such an arrangement is called a balanced load.



Connection	Phase Voltage	Line Voltage	Phase Current	Line Current
Star	$V_P = V_L \div \sqrt{3}$	$V_L = \sqrt{3} \times V_P$	$I_P = I_L$	$I_L = I_P$
Delta	$V_P = V_L$	$V_L = V_P$	$I_P = I_L \div \sqrt{3}$	$I_L = \sqrt{3} \times I_P$

Relationship between Line & Phase Values & Expression for power of a balanced Star Connection

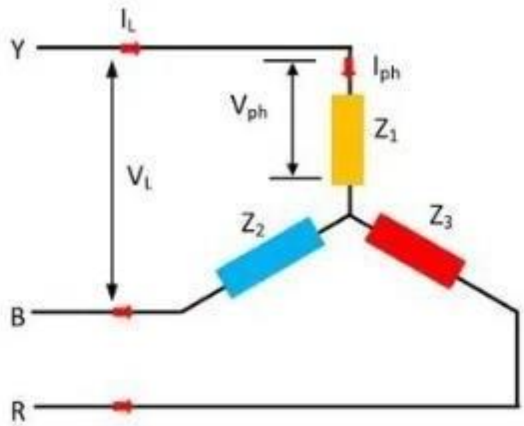
This system is obtained by joining together similar ends, either the start or the finish; the other ends are joined to the line wires, as shown in Fig 2.18. The common point N at which similar (start or finish) ends are connected is called the neutral or star point. Normally, only three wires are carried to the external circuit, giving a 3-phase, 3-wire, star-connected system; however,sometimes a fourth wire known as neutral wire, is carried to the neutral point of the external load circuit, giving a 3-phase, 4-wire connected system is shown in Fig 2.22.

Line currents are: I_R, I_Y, I_B

Line voltages are: V_{RY}, V_{YB}, V_{BR}

Phase voltages: V_R, V_Y, V_B

Phase currents: I_R, I_Y, I_B



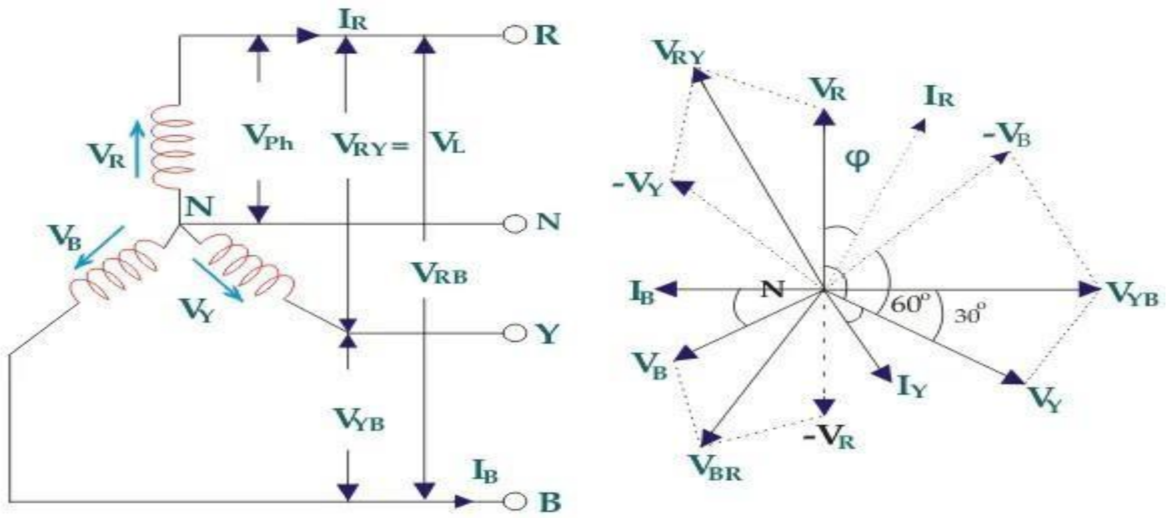


Fig 2.22 Phasor diagram showing Line & Phase values of star connection

To derive the relations between line and phase currents and voltages of a star connected system, we have first to draw a balanced star connected system and the same is shown in Fig 2.22.

Suppose due to load impedance the current lags the applied voltage in each phase of the system by an angle φ. As we have considered that the system is perfectly balanced, the magnitude of current and voltage of each phase is the same. Let us say, the magnitude of the voltage across the red phase i.e. magnitude of the voltage between neutral point (N) and red phase terminal (R) is V_R.

Similarly, the magnitude of the voltage across yellow phase is V_Y and the magnitude of the voltage across blue phase is V_B.

In the balanced star system, magnitude of phase voltage in each phase is V_{ph}.

$\therefore V_R = V_Y = V_B = V_{ph}$

We know in the star connection, line current is same as phase current. The magnitude of this current is same in all three phases and say it is I_L.

$\therefore I_R = I_Y = I_B = I_L$, Where, I_R is line current of R phase, I_Y is line current of Y phase and I_B is line current of B phase. Again, phase current, I_{ph} of each phase is same as line current I_L in star connected system.

$\therefore I_R = I_Y = I_B = I_L = I_{ph}$.

Now, let us say, the voltage across R and Y terminal of the star connected circuit is V_{RY}.

The voltage across Y and B terminal of the star connected circuit is V_{YB}.

The voltage across B and R terminal of the star connected circuit is VBR.

From the diagram, it is found that

$V_{RY} = V_R + (-V_Y)$

Similarly, $V_{YB} = V_Y + (-V_B)$

And, $V_{BR} = V_B + (-V_R)$

Now, as angle between V_R and V_Y is 120°(electrical), the angle between V_R and $-V_Y$ is 180° – 120° = 60°(electrical).

$$V_L = |V_{RY}| = \sqrt{V_R^2 + V_Y^2 + 2V_R V_Y \cos 60^o}$$
$$= \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph}V_{ph} \times \frac{1}{2}}$$
$$= \sqrt{3}V_{ph}$$
$$\therefore V_L = \sqrt{3}V_{ph}$$

Thus, for the star-connected system line voltage = √3 × phase voltage.

Line current = Phase current

As, the angle between voltage and current per phase is ϕ, the electric power per phase is

$$V_{ph}I_{ph}cos\phi = \frac{V_L}{\sqrt{3}}I_L \cos\phi$$

So the total power of three phase system is

$$3 \times \frac{V_L}{\sqrt{3}}I_L \cos\phi = \sqrt{3}V_L I_L \cos\phi$$

Relationship between Line and Phase Values and Expression for Power of a balanced Delta Connection

When the starting end of one coil is connection to the finishing end of another coil, as shown in Fig.2.23 delta or mesh connection is obtained. Fig 2.23 shows the two different ways of representing delta connection.

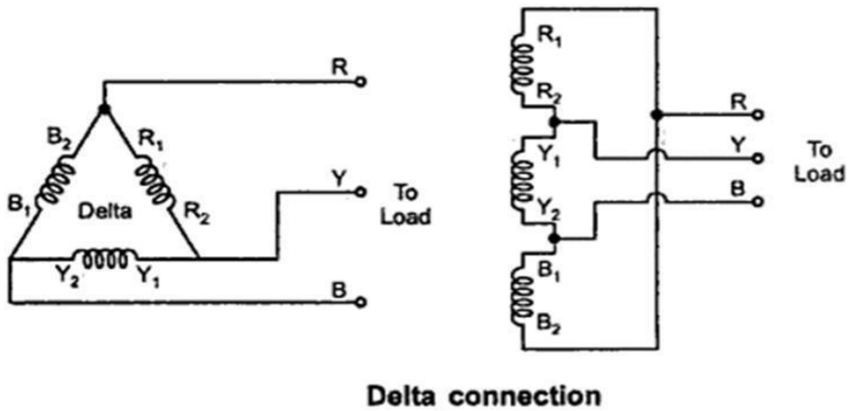


Fig 2.23 Different representations of delta connection

Relation Between Phase Current and Line Current in Delta Connection

From Fig 2.24. it is clear that line current is the phasor difference of phase currents of the two phases concerned. For example, the line current in I_R will be equal to the phasor difference of phase currents I_{YR} and I_{RB}. The current phasors are shown in Fig 2.24.

As in the balanced system the three-phase current I_{12}, I_{23} and I_{31} are equal in magnitude but are displaced from one another by 120° electrical.

The **phasor diagram** of delta connected system is shown in Fig 2.24 below:

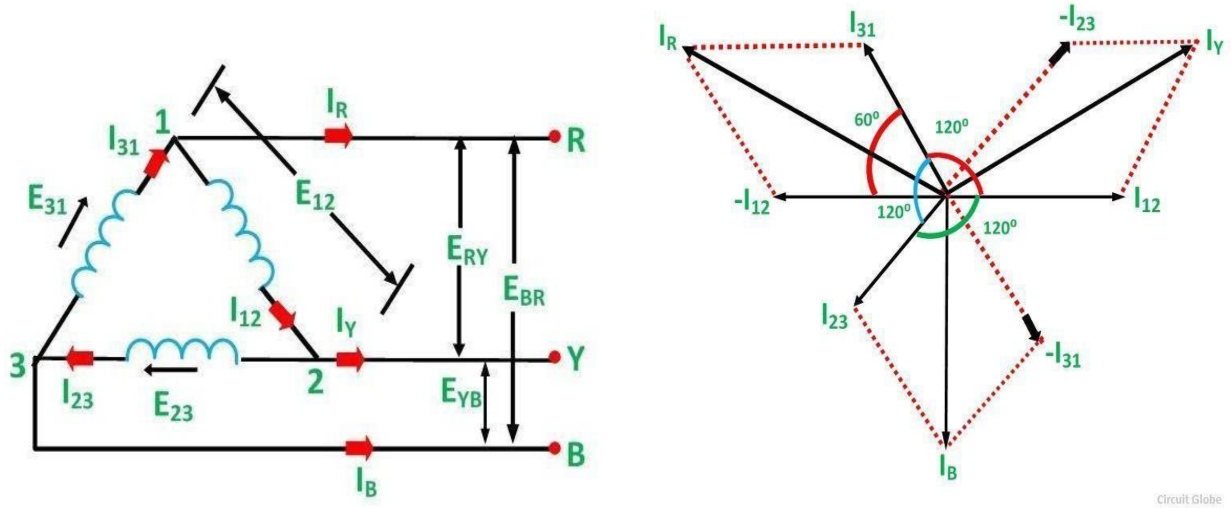


Fig 2.24 Phasor representation of Line & Phase Values of delta connection

Line currents are: I_R, I_Y, I_B

Phase currents are: I_{12}, I_{23}, I_{32}

Line voltages are: E_{RY}, E_{YB}, E_{BR}

Phase voltages: E_{12}, E_{23}, E_{32}

If

$$I_{12} = I_{23} = I_{31} = I_{ph}$$

From figure A, it is seen that the current is divided at every junction 1, 2 and 3.

ApplyingKirchhoff's Law at junction 1,

The Incoming currents are equal to outgoing currents.

$$\overline{I_{31}} = \overline{I_R} + \overline{I_{12}}$$

And their phasor difference will be given as:

$$\overline{I_R} = \overline{I_{31}} - \overline{I_{12}}$$

The phasor I_{12} is reversed and is added in the phasor I_{31} to get the phasor sum of I_{31} and $-I_{12}$ as shown above in the phasor diagram. Therefore

$$I_R = \sqrt{I_{31}^2 + I_{12}^2 + 2I_{31}I_{12}\cos 60^\circ} \quad \text{or}$$

$$I_L = \sqrt{I_{ph}^2 + I_{ph}^2 + 2I_{ph}I_{ph} \times 0.5}$$

,

As we know, $I_R = I_L$, therefore,

$$I_L = \sqrt{3I_{ph}^2} = \sqrt{3}I_{ph}$$

Similarly,

$$\overline{I_Y} = \overline{I_{12}} - \overline{I_{23}} \quad \text{or} \quad I_L = \sqrt{3}I_{ph} \quad \text{and}$$

$$\overline{I_B} = \overline{I_{23}} - \overline{I_{31}} \quad \text{or} \quad I_L = \sqrt{3}I_{ph}$$

Hence, in delta connection line current is root three times of phase current.

$$\text{Line Current} = \sqrt{3} \text{ x Phase Current}$$

It is clear from the figure that the voltage across terminals 1 and 2 is the same as across the terminals R and Y. Therefore,

$$E_{12} = E_{RY}$$

Similarly,

$$E_{23} = E_{YB} \text{ and } E_{31} = E_{BR}$$

: the phase voltages are

$$E_{12} = E_{23} = E_{31} = E_{ph}$$

The line voltages are:

$$E_{RY} = E_{YB} = E_{BR} = E_L$$

Hence, in delta connection line voltage is equal to phase voltage

Measurement of power in a three phase balanced circuit using two wattmeters.

Two wattmeter method: The **Two Wattmeter Method** is explained, taking an example of a balanced load. In this, we have to prove that the power measured by the two wattmeter i.e. the sum of the two wattmeter readings is equal to root 3 times of the phase voltage and line voltage ($\sqrt{3}V_LI_L \cos\varphi$) which is theactual power consumed in a 3 phasebalanced load.

The connection diagram of a 3 phase balanced load connected as Star Connection is shown below:

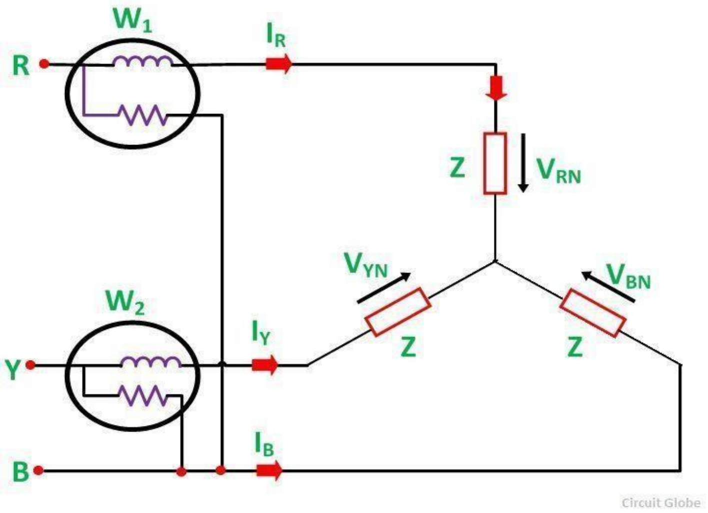


Fig 2.25 Connection two wattmeter for measurement of 3 phase power

The current coils of the two wattmeters are connected in any two lines while the voltage coil of each wattmeters is connected between its own current coil terminal and line without current coil. Consider star connected balanced load and two wattmeters connected as shown in Fig 2.25. Let us consider the rms values of the currents and voltages to prove that sum of two wattmeter gives total power consumed by three phase load. The load is considered as an inductive load, and thus, the phasor diagram of the inductive load is drawn below:

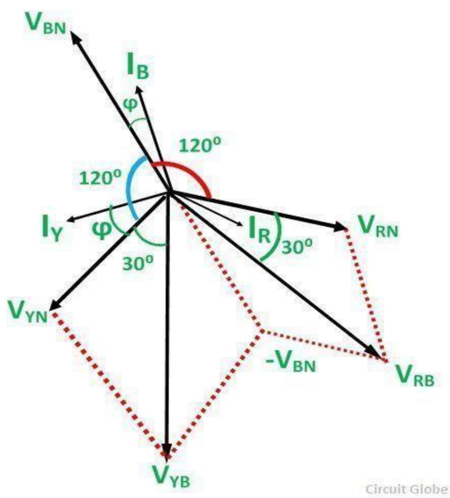


Fig 2.26 Phasor diagram of 2 Wattmeter method

The three voltages V_{RN}, V_{YN} and V_{BN}, are displaced by an angle of 120 degrees electrical as shown in the phasor diagram. The phase current lag behind their respective phasevoltages byan angle φ.

Now, the current flowing through the current coil of the wattmeter, W_1 will be given as:

$$W_1 = I_R$$

The potential difference across the pressure or potential coil of the wattmeter, W_1 will be:

$$W_1 = \overline{V_{RB}} = \overline{V_{RN}} - \overline{V_{BN}}$$

To obtain the value of V_{YB}, reverse the phasor V_{BN} and add it to the phasor V_{YN} as shown in the phasor diagram of Fig 2.26 above.

The phase difference between V_{RB} and I_R is $(30^0 - \varphi)$

Therefore, the power measured by the Wattmeter, W_1 is:

$$W_1 = V_{RB} I_R \cos(30^\circ - \varphi)$$

Current through the current coil of the Wattmeter, W_2 is given as:

$$W_2 = I_Y$$

The potential difference across the Wattmeter, W_2 is

$$W_2 = \overline{V_{YB}} = \overline{V_{RN}} - \overline{V_{BN}}$$

The phase difference V_{YB} and I_Y is $\mathbf{(30^0 + \phi)}$.

Therefore, the power measured by the wattmeter, W_2 is given by the equation shown below:

$$W_2 = V_{YB} I_Y \cos(30^\circ + \varphi)$$

Since, the load is in a balanced condition, hence,

$$I_R = I_Y = I_B = I_L \text{ and}$$

$$V_{RY} = V_{YB} = V_{BR} = V_L$$

Therefore, the wattmeter readings will be:

$$W_1 = V_L I_L \cos(30^\circ - \varphi) \text{ and}$$

$$W_2 = V_L I_L \cos(30^\circ + \varphi)$$

Now, the sum of two Wattmeter readings will be given as:

$$W_1 + W_2 = V_L I_L \cos(30^\circ - \varphi) + V_L I_L \cos(30^\circ + \varphi)$$

$$W_1 + W_2 = V_L I_L [\cos(30^\circ - \varphi) + \cos(30^\circ + \varphi)] \quad \text{or}$$

$$W_1 + W_2 = V_L I_L [\cos 30^\circ \cos\varphi + \sin 30^\circ \sin\varphi + \cos 30^\circ \cos\varphi - \sin 30^\circ \sin\varphi] \text{ or}$$

$$W_1 + W_2 = V_L I_L (2 \cos 30^\circ \cos\varphi) \quad \text{or}$$

$$W_1 + W_2 = V_L I_L \left(2 \frac{\sqrt{3}}{2} \cos\varphi\right)$$

$$W_1 + W_2 = \sqrt{3}\, V_L I_L \cos\varphi$$

$$W_1 + W_2 = P \ \ldots\ldots (1)$$

The above equation (1) gives the total power absorbed by a 3 phase balanced load.

Thus, the sum of the readings of the two wattmeters is equal to the power absorbed in a 3 phase balanced load.

Determination of Power Factor from Wattmeter Readings

As we know,

$$W_1 + W_2 = \sqrt{3}\, V_L I_L \cos\varphi \;\ldots\ldots\ldots (2)$$

Now,

$$W_1 - W_2 = V_L I_L \left[\cos(30^\circ - \varphi) - \cos(30^\circ + \varphi)\right] \quad \text{or}$$

$$W_1 - W_2 = V_L I_L \left[\cos 30^\circ \cos\varphi + \sin 30^\circ \sin\varphi - \cos 30^\circ \cos\varphi + \sin 30^\circ \sin\varphi\right] \text{ or}$$

$$W_1 - W_2 = 2\, V_L I_L \sin 30^\circ \sin\varphi$$

$$W_1 - W_2 = V_L I_L \sin\varphi \;\ldots\ldots\ldots. (3)$$

Dividing equation (3) by equation (2) we get,

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin\varphi}{\sqrt{3}\, V_L I_L \cos\varphi} \quad \text{or}$$

$$\tan\varphi = \sqrt{3}\, \frac{W_1 - W_2}{W_1 + W_2}$$

The power factor of the load is given as

$$\cos\varphi = \cos \tan^{-1} \sqrt{3}\, \frac{W_1 - W_2}{W_1 + W_2}$$

Determination of Reactive Power by Two Wattmeter Method

To get the reactivepower, multiplyequation (3) by$\sqrt{3}$.

$$\sqrt{3}\,(W_1 - W_2) = \sqrt{3}\, V_L I_L \sin\varphi = P_r$$

Therefore, reactive power is given by the equation shown below:

$$P_r = \sqrt{3}\,(W_1 - W_2)$$

Single phase circuits Numericals

Example 1 : An RLC series circuit has a current which lags the supply voltage by 45 degree. The voltage across the inductance has the maximum value equal to twice the maximum value across capacitor . Voltage across inductance is 300sin(1000t) and R= 20 ohms. Find the value of L and C.

Solution : **Answers: L = 0.04 H C = 50 µF**

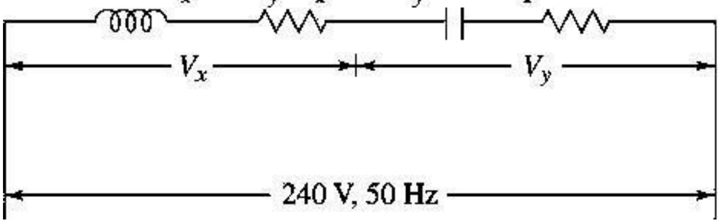
$$v_L = 300 \sin(1000t)$$
$$R = 20\ \Omega$$
$$\phi = 45^\circ$$
$$V_{L(\max)} = 2V_{C(\max)}$$
$$\sqrt{2}\,V_L = 2\sqrt{2}\,V_C$$
$$I \times X_L = 2I \times X_C$$
$$X_L = 2X_C$$
$$\cos\phi = \frac{R}{Z}$$
$$\cos(45^\circ) = \frac{20}{Z}$$
$$Z = 28.28\ \Omega$$

For a series *R-L-C* circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$(28.28)^2 = (20)^2 + (2X_C - X_C)^2$$
$$799.76 = 400 + X_C^2$$
$$X_C = 20\ \Omega$$
$$X_L = 2X_C = 40\ \Omega$$
$$X_L = \omega L$$
$$40 = 1000 \times L$$
$$L = \frac{40}{1000} = 0.04\ \text{H}$$
$$X_C = \frac{1}{\omega C}$$
$$20 = \frac{1}{1000 \times C}$$
$$C = 50\ \mu\text{F}$$

Example 2:

Find the values of R and C so that $V_x = 3V_y$, V_x and V_y are in quadrature.



Solution :

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.0255 = 8\ \Omega$$

$$\bar{Z}_x = 6 + j8 = 10\ \angle 53.13°\ \Omega$$

$$V_x = 3V_y$$

$$I \times Z_x = 3 \times I \times Z_y$$

$$Z_x = 3Z_y$$

V_x and V_y are in quadrature, i.e., phase angle between V_x and V_y is 90°. Hence, the angle between Z_x and Z_y will be 90°. The impedance Z_y is capacitive in nature.

$$\bar{Z}_y = Z_y \angle -\phi$$

$$\bar{Z}_y = \frac{10}{3} \angle (53.13 - 90)° = 3.33\ \angle -36.87° = 2.66 - j2\ \Omega$$

$$R = 2.66\ \Omega$$

$$X_C = 2\ \Omega$$

$$X_C = \frac{1}{2\pi fC}$$

$$C = \frac{1}{2\pi \times 50 \times 2} = 1.59\ \text{mF}$$

Example 3 : A 250v,50Hz voltage is applied to a coil having resistance of 5 ohm and an inductance of 9.55H in series with a capacitor C. If the voltage across the coil is 300v, Find value of C.

Data

$$V = 250\ \text{V}$$
$$R = 5\ \Omega$$
$$L = 9.55\ \text{H}$$
$$V_{coil} = 300\ \text{V}$$
$$X_L = 2\pi f L = 2\pi \times 50 \times 9.55 = 3000\ \Omega$$
$$Z_{coil} = \sqrt{R^2 + X_L^2} = \sqrt{(5)^2 + (3000)^2} = 3000\ \Omega$$
$$I = \frac{V_{coil}}{Z_{coil}} = \frac{300}{3000} = 0.1\ \text{A}$$

Total impedance
$$Z = \frac{V}{I} = \frac{250}{0.1} = 2500\ \Omega$$

When $X_L > X_C$,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$(2500)^2 = (5)^2 + (3000 - X_C)^2$$
$$(3000 - X_C) = 2500$$
$$X_C = 500$$
$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 500} = 6.37\ \mu\text{F}$$

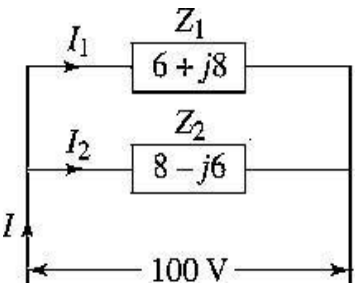
When $X_C > X_L$,

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$
$$(2500)^2 = (5)^2 + (X_C - 3000)^2$$
$$2500 = X_C - 3000$$
$$X_C = 5500$$
$$C = \frac{1}{2\pi \times 50 \times 5500} = 0.58\ \mu\text{F}.$$

Example 4 : Two circuits the impedances are given by Z_1= (6+8j)Ω and Z_2= (8-6j) are connected in parallel. If the applied voltage to the combination is 100v, find

1. Current and PF of each branch
2. Over all current and power factor of combination
3. Power consumed by each impedance

Solution :



$$\bar{Z}_1 = 6 + j8\ \Omega \quad \bar{Z}_2 = 8 - j6\ \Omega$$

$$V = 100\ V$$

$$\bar{V} = 100\ \angle 0°\ \text{V}$$

$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{100\angle 0°}{6 + j8} = 10\ \angle -53.13°\ \text{A}$$

$$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_1} = \frac{100\angle 0°}{8 - j6} = 10\ \angle 36.9°\ \text{A}$$

$$\cos\phi_1 = \cos(53.13°) = 0.6\ \text{(lagging)}$$

$$\cos\phi_2 = \cos(36.9°) = 0.8\ \text{(leading)}$$

$$\bar{I} = \bar{I}_1 + \bar{I}_2 = 10\ \angle -53.13° + 10\ \angle 36.9°$$

$$= 14.14\ \angle -8.13°\ \text{A}$$

$$\text{pf} = \cos\phi = \cos(8.13°) = 0.989\ \text{(lagging)}$$

$$P_1 = I_1^2 R_1 = (10)^2 \times (6) = 600\ \text{W}$$

$$P_2 = I^2 R_2 = (10)^2 \times (8) = 800\ \text{W}$$

Answers:

I�= 10∠-53.13 & I�= 10∠36.9 PF1=0.6(Lag)&PF2=0.8(Lead) I=14.14∠-8.13 , PF= 0.989 (Lag)

P�=600w& P� =800w

Example 5 : Two impedances Z_1 and Z_2 having same numerical values are connected in series .If Z_1 is having power factor 0.866 lagging of and Z_2 having power factor of 0.8 leading, calculate the power factor of series combination.

Data

$$pf_1 = 0.866 \text{ (lagging)}$$
$$pf_2 = 0.8 \text{ (leading)}$$
$$Z_1 = Z_2 = Z$$
$$\phi_1 = \cos^{-1}(0.866) = 30°$$
$$\phi_2 = \cos^{-1}(0.8) = 36.87°$$

F

$$\bar{Z}_1 = Z\angle\phi_1 = Z\angle 30° = 0.866\,Z + j0.5\,Z$$

$$\bar{Z}_2 = Z\angle -\phi_2 = Z\angle -36.87° = 0.8\,Z - j0.6\,Z$$

For a series combination,

$$\bar{Z} = \bar{Z}_1 + \bar{Z}_2 = 0.866\,Z + j0.5\,Z + 0.8\,Z - j0.6\,Z$$
$$= 1.6666\,Z - j0.1\,Z = Z(1.666 - j0.1) = 1.668\,Z\angle -3.43°$$
$$pf = \cos(3.43°) = 0.9982 \text{ (leading)}$$

Three phase circuits Numericals

Question 1:

If the e.m.f. is represented by the equation e = 25 sin (314 t), what is its amplitude, frequency and the time period ?

Solution : Given equation is, $e = 25 \sin(314\,t)$

Comparing with $e = E_m \sin(2\pi f t)$, we get

E_m = amplitude = 25 volts

and $2\pi f = 314$

$\therefore$ $f = 50$ Hz

time period $T = \frac{1}{f}$

$\therefore$ $T = 0.02$ seconds

Question 2:

An alternating voltage has an effective value of 70.7106 V and frequency of 60 Hz. Find its average value, form factor, crest factor assuming it to be purely sinusoidal.

Solution : Effective value means R.M.S. value.

$$E_{RMS} = 70.7106 \text{ V}, \quad f = 60 \text{ Hz}$$

Key Point: *The frequency does not affect the r.m.s. or average values.*

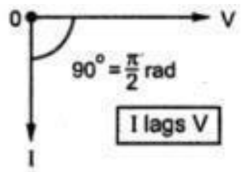
$$E_m = \sqrt{2} \times E_{RMS} = \sqrt{2} \times 70.7106 = \mathbf{100 \ V}$$

$$\therefore \quad E_{av} = 0.637 \ E_m = 0.637 \times 100 = \mathbf{63.7 \ V}$$

$$K_f = \frac{\text{R.M.S.}}{\text{Average}} = \frac{70.7106}{63.7} = \mathbf{1.11}$$

$$K_p = \frac{\text{Maximum}}{\text{R.M.S.}} = \frac{100}{70.7106} = \mathbf{1.414}$$

Question 3:



The phasor diagram is shown in the

Case 3 : $C = 50 \ \mu F$

Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}} = 63.66 \ \Omega$

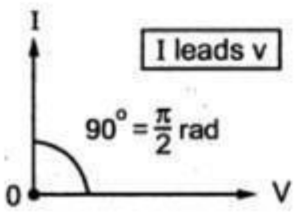
$$\therefore \quad I_m = \frac{V_m}{X_C} = \frac{212.13}{63.66} = 3.33 \text{ A}$$

In pure capacitive circuit, current leads voltage by 90°.

$$\therefore \quad \phi = \text{Phase Difference} = 90° = \frac{\pi}{2} \text{ rad}$$

$$\therefore \quad i = I_m \sin(\omega t + \phi)$$

$$\therefore \quad i = \mathbf{3.33 \sin\left(100\pi t + \frac{\pi}{2}\right) A}$$



The phasor diagram is shown in the

All the phasor diagrams represent r.m.s. values of voltage and current.

Question 4:

An alternating current, $i = 414 \text{ Sin } (2\,\pi \times 50 \times t)$ - A, is passed through a series circuit consisting of a resistance of 100-ohm and an inductance of 0.31831 henry. Find the expressions for the instantaneous values of the voltages across (i) the resistance, (ii) the inductance and (iii) the combination.

Solution : The circuit is shown in the Fig.

$i = 1.414 \sin (2\,\pi \times 50\, t)$ A

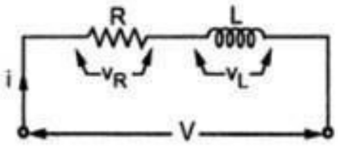


Fig. 7.17

$\therefore \quad \omega = 2\,\pi \times 50 = 2\,\pi f$

$\therefore \quad f = 50$ Hz, $\quad R = 100\ \Omega$, $\quad L = 0.31831$ H

$\therefore \quad X_L = 2\,\pi f L = 2\,\pi \times 50 \times 0.31831 = 100\ \Omega$

i) The voltage across the resistance is,

$$v_R = i R = 1.414 \sin (2\,\pi \times 50\, t) \times 100 = \mathbf{141.4 \sin (2\,\pi \times 50\, t)\ V}$$

ii) The voltage across L leads current by 90° as current lags by 90° with respect to voltage.

$$\therefore \quad v_L = i X_L \text{ but leading current by } 90^\circ = \mathbf{141.4 \sin (2\,\pi \times 50\, t + 90^\circ)\ V}$$

iii) From the expression of V_R we can write,

iii) From the expression of V_R we can write,

$$\text{r.m.s. value of } V_R = \frac{141.4}{\sqrt{2}} = 100 \text{ V}, \qquad \phi = 0^\circ$$

$$\therefore \quad V_R = 100 \angle 0^\circ = 100 + j0 \text{ V}$$

$$\text{r.m.s. value of } V_L = \frac{141.4}{\sqrt{2}} = 100 \text{ V}, \qquad \phi = 90^\circ$$

$$\therefore \quad V_L = 100 \angle 90^\circ = 0 + j\,100 \text{ V}$$

$$\therefore \quad V = \overline{V}_R + \overline{V}_L = 100 + j0 + 0 + j100$$

$$= 100 + j\,100 = 141.42 \angle 45^\circ \text{ V}$$

$$\therefore \quad V_m = \sqrt{2} \times 141.42 = 200 \text{ V}$$

Hence expression of instantaneous value of resultant voltage is,

$$v = \mathbf{200 \sin (2\,\pi \times 50\, t + 45^\circ)\ V}$$