ENGINEERING PHYSICS

MODULE – 4

Electrical Conductivity in Solids: **8 Hours**

Classical free electron theory: Drude- Lorentz theory & Assumptions, Expression for electrical conductivity (no derivation), Failures of classical free-electron theory.

Quantum free electron theory: Assumptions, Density of states (no derivation), Fermi-energy, Fermi factor & its temperature dependence, Fermi - Dirac Statistics, Expression for electrical conductivity (derivation), Merits of Quantum free electron theory.

Physics of Semiconductors: Fermi level in intrinsic semiconductors, Expression for concentration of electrons in conduction band, Holes concentration in valance band (only mention the expression), Conductivity of semiconductors (derivation), Hall effect, Expression for Hall coefficient (derivation).

Dielectrics: Electric dipole, Dipole moment, Polarization of dielectric materials, Types of polarizations. Qualitative treatment of Internal field in solids for one dimensional infinite array of dipoles (Lorentz field). Claussius-Mossotti equation (derivation), Numerical problems.

Classical free electron theory

Free electron Concept:

Metallic solids have atoms bound together by metallic bonds. In such solids, the valence electrons are free to move about in the whole body of the solid and hence contribute to conduction in it. Such electrons do not belong to a particular atom but belong to the material as a whole. Normally, one or two electrons per atom are free electrons and they move about randomly, in the absence of electric field. This motion is identical with the motion of molecules in a gas and hence it is called *electron gas*. This electron gas is responsible for high electrical and thermal conductivity in metals.

Classical Free Electron Theory of Metals (Drude-Lorentz Theory):
Assumptions:

Drude and Lorentz developed the classical free electron theory, based on the following assumptions. Metal atoms have some of the valence electrons freely moving about in the body of the metal. These electrons are called free electrons or conduction electrons, as they contribute to conduction in the metal.

1. The free electrons are considered equivalent to the molecules in a gas and thus are assumed to obey the laws of kinetic theory of gases.
2. In the absence of external electric field they are at random motion and their average kinetic energy is $\frac{1}{2} m v_{th}^2 = 3/2\ k\ T$, where v_{th} is the thermal velocity, k, Boltzmann's constant and T is temperature.
3. The electrostatic force of attraction between the free electron and the ion cores are negligible.
4. Also, the electrostatic force of repulsion amongst the free electrons are negligible.
5. Inside the metal, the electric potential due to the ionic cores is constant and the electrons cannot escape from the metal due to the potential barrier at the surface.
6. In the absence of external electric field, the drift velocity is zero.
7. When a potential difference is applied across the conductor, the current through it is proportional to the drift velocity.

Thermal velocity v_{th}:

The average velocity of random motion of free electrons within the body of the metal, in the absence of external field, is called thermal velocity.

Mean free path (λ):

It is the average distance travelled by the free electrons between successive collisions with the lattice ions.

Mean collision time (τ):

The average time interval between successive collisions of the free electrons with the lattice ions is called mean collision time. If v is the average velocity of electrons and λ is the mean free path, then $\tau = \lambda/v$.

Relaxation time (τ_r):

When the electric field applied across a conductor is switched off, the drift velocity decreases exponentially to zero. The time during which the drift velocity reduces to ($1/e$) times the initial value is called relaxation time.

Drift velocity (v_d):

When an electric field is applied, some of the free electrons acquire a constant, terminal velocity in a direction opposite to that of the field. This steady and very small velocity is called drift velocity.

Expression for drift velocity:

In the absence of an electric field the free electrons are at random motion. But when an electric field is applied, some of the electrons acquire a steady velocity, called drift velocity v_d. The resistive force to the motion of the electrons is

$F_r = mv_d/\tau$, where τ is the mean collision time.

If E is the strength of the applied filed, then the force that drives the electrons opposite to the field is

F = e E, where e is electronic charge.

In the steady state, $F_r = F$.

i.e. m $v_d/\tau = $ e E,

or, $v_d = \dfrac{eE}{m}\tau$

Current density (J):

It is the electric current passing normally through unit area of cross-section of the conductor. **J = I/A,**

But I = n A e v_d, and therefore, **J = n e v_d**

$= \boldsymbol{\rho}\textbf{v}_\textbf{d}$, where n is number density of electrons, e is charge of electrons, v_d is drift velocity of electrons and ρ is charge density. Unit of current density is Am^{-2}.

Electric field (E):

It is the potential difference across unit length of a conductor. **E = V/L** (V m^{-1})

Conductivity (σ):

It is the quantity that decides the ability of a material to conduct electricity, and is the reciprocal of resistivity. $\sigma = 1/\rho$.

As per the classical free electron theory, $\sigma = $ n e^2 τ/m, where n is the free electron concentration, e is charge of electron, m is mass of electron and t is the mean collision time.

Mobility of electrons (μ):

It is the magnitude of drift velocity acquired per unit field. $\mu = v_d/E$.

But $v_d = $ eEτ/m.

Hence, $\mu = $ **e$\boldsymbol{\tau}$/m**.

Unit of μ is m^{2}v^{-1} s^{-1}.

Failures of Classical Free Electron Theory:

The classical free electron theory could successfully explain electrical and thermal conduction in metals. But it could not account for specific heat of metals, temperature dependence of electrical conductivity on free electron concentration.

1. **Specific heat :**

According to the classical theory, the molar heat capacity of free electrons in a metal is 3/2 R.

However, the experimentally determined molar heat capacity of metals is $10^{-4}RT$, which is very much lower than the theoretically predicted value and also shows a dependence on temperature which is not predicted by the classical free electron theory.

2. **Temperature dependence of electrical conductivity:**

Experimentally it was found that electrical conductivity is inversely proportional to T. i.e., σ α 1/T.

According to the assumption of classical free electron theory, we have $\frac{3}{2}kT = \frac{1}{2}mv_{th}^2$

i.e., Thermal velocity, v_{th} α $\sqrt{T}$

Since mean collision time τ is inversely proportional to the thermal velocity, we can write τ α $1/v_{th}$ or

τ α $1/\sqrt{T}$.

But σ = $ne^2τ/m$

Therefore, σ α τ

From the above two proportionalities we get   σ α $1/\sqrt{T}$

Thus classical free electron theory couldn't explain the temperature dependence of conductivity.

3. **Dependence of Electrical Conductivity on electron concentration:**

From the expression for electrical conductivity, σ = n e^2 π/m. Conductivity is directly proportional to the electron density n. Hence, divalent and trivalent metals should possess much higher electrical conductivity than monovalent metals. This is contrary to the experimental observation that silver and copper are more conducting than zinc and aluminum. The experimental values of conductivity of some metals are listed below.

Metals	Conductivity σ in (sm^{-1})	Electron concentration n (cm^{-3})
Monovalent metals	For Copper σ = 5.88 x 10^7 For Silver σ = 4.5 x 10^7	8.4 x 10^{28} 5.8 x 10^{28}
Divalent metals	For Zinc σ = 1.09 x 10^7 For Cadmium σ = 0.15 x 10^7	13 x 10^{28} 9.2 x 10^{28}
[Trivalent] Al Ga	For Aluminum σ = 3.65 x 10^7 For Gallium σ = 0.67 x 10^7	18.06 x 10^{28} 15.3 x 10^{28}

From the above values it is clear that the conductivity is low for metals with high electron concentration and conductivity high for those metals with lower electron

concentration. Thus classical free electron theory couldn't explain the dependence of conductivity on free electron concentration.

Quantum Free Electron Theory:

Partial failure of classical free electron theory led Somerfield to propose the quantum free electron theory. He retained some of the assumptions of classical theory and introduced a few new assumptions. Following are the assumptions of quantum free electron theory.
1. The valence electrons are free to move about inside the metal.
2. The potential inside the metal is constant.
3. The force of attraction between the free electrons and the ionic core and the force of repulsion between electrons is negligible.
4. The energies of electrons are quantized.
5. The allowed discrete energy levels obey Pauli's exclusion principle, according to which no two electrons can be in the same quantum state.

Density of States:

Density of states g (E) (Fig. 4.1) is defined as the number of available energy states per unit energy range per unit volume, about a given energy E, in the valence band of the material. From quantum mechanical considerations it can be shown that,

$$g(E)dE = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} E^{1/2} dE$$

The above expression gives the number of energy states about E over an interval d E, which is proportional to $E^{1/2}$. A plot of g (E) versus E is shown in figure, which is a parabola.
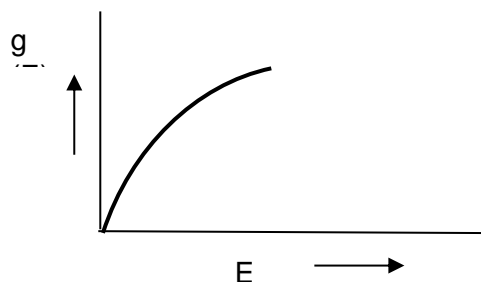

Fig. 4.1 Density of states

Fermi Energy E_F:

The energy corresponding to the highest occupied level in the valence band at 0 K is called Fermi energy. The corresponding energy level is called the Fermi level. At 0 K all the energy levels above Fermi level are empty and all those below the Fermi level are completely filled.

Fermi – Dirac Statistics:

Under thermal equilibrium the free electrons occupy various energy levels in accordance with a statistical rule known as Fermi – Dirac statistics. Fermi – Dirac statistics enables the evaluation of probability of finding electron in energy levels over a certain range of energy values. The evaluation is done with the help of a quantity called **Fermi factor** f (E) given by:

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

Fermi factor is the probability of occupation of a given energy state for a material in thermal equilibrium.

Effect of temperature on Fermi function

(OR)

Dependence of Fermi factor on temperature and effect on occupancy of energy levels

Since most of the measurements are made at room temperature and we must consider the effect of temperature on the electron gas. So, it is important to know the variation of Fermi-Dirac distribution function with respect to temperature (Fig. 4.2).

Case 1: Probability of occupation for E< E_F at T= 0k.

Substituting this condition in F-D equation, we get

$$f(E) = \frac{1}{1 + e^{-\infty}}$$

$$f(E) = \frac{1}{1+0} \Rightarrow f(E) = 1$$

The distribution $f(E) = 1$ means that at T = 0k, all the energy levels below the Fermi level are fully occupied by electrons leaving the upper levels vacant.

i.e., there is 100 % probability of finding an electron below the Fermi energy level.

Case 2: Probability of occupation for E >E_F at T= 0k.

Substituting this condition in F-D equation, we get

$$f(E) = \frac{1}{1 + e^{\infty}}$$

$$f(E) = \frac{1}{1+\infty} \Rightarrow f(E) = \frac{1}{\infty} = 0$$

This indicates that at T= 0k, the energy levels above the Fermi level is not occupied by electrons, they are vacant. i.e., there is 0% probability of finding the electron above the Fermi level at absolute zero K.

Case 3: Probability of occupation at ordinary temperatures.

At temperatures above 0 K and E = E_F.

Substituting this condition in F-D equation, we get

$$f(E) = \frac{1}{1 + e^0}$$

$$f(E) = \frac{1}{1+1} \Rightarrow f(E) = \frac{1}{2} = 0.5$$

At temperatures above 0K, there is only 50 % probability for an electron to occupy Fermi level.

Case 4: At high temperatures, kT >>E_F, electrons are excited to vacant levels above the Fermi level. As a result, the probability of finding an electron above E_F becomes greater than zero.
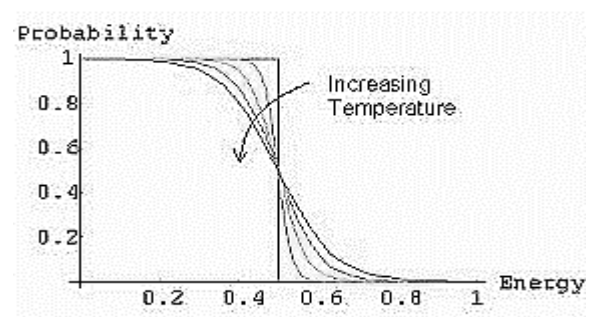

Fig. 4.2 variation of Fermi Function for different temperature and energy values

Fermi energy at T=0K:

Let E_F at 0K be denoted as E_{F_0}. Then it can be shown that

$$E_{F_0} = \left(\frac{h^2}{8m}\right)\left(\frac{3n}{\pi}\right)^{2/3}$$

where 'n' is the concentration of electrons in a metal or electron density of a metal. After

substituting the various values, we get $E_{F_0} = 5.85 \times 10^{-38}(n)^{2/3}$ Joule

Thus Fermi energy of a metal depends on the electron density of the metal.

Fermi energy at T>0K:

The second term in the bracket on R.H.S is very small compared to unity except at very

high temperatures. $E_F = E_{F_0}\left[1 - \frac{\pi^2}{12}\left(\frac{kT}{E_{F_0}}\right)^2\right]$ Hence, at ordinary temperatures $E_F = E_{F_0}$

Fermi Temperature (T_F): Fermi temperature is the temperature at which the average thermal energy of the free electron in a solid becomes equal to the Fermi energy at 0K.

$$T_F = \frac{E_F}{K}$$

Fermi Velocity (V_F): The velocity of the electrons which occupy the Fermi level is called the Fermi Velocity (V_F).

$$V_F = \sqrt{\frac{2E_F}{m}}$$

Expression for Electrical Conductivity based on Quantum Free Electron Theory:

The energy of free electrons can be expressed in terms of the wave vector 'K'. Applying Fermi – Dirac statistics Somerfield deduced an expression for electrical conductivity of metals, given

by $\sigma = \dfrac{ne^2\lambda}{m^* v_F}$,

where V_F is the Fermi velocity, which is the velocity of electrons occupying the Fermi level, m^* is the effective mass of the electron (its mass processed by the electron in the combined effect of applied electric field and electrostatic field due to the lattice ions in the metal) ,
n is the number density of electrons.

$$E = \frac{h^2}{8m\pi^2}K^2 \quad \ldots\ldots\ldots (1)$$

'K' is a vector in three dimensions & can be expressed in terms of its x, y and z components.

$$K^2 = K_x^2 + K_y^2 + K_z^2$$

$$E = \frac{h^2}{8m\pi^2}\left[K_x^2 + K_y^2 + K_z^2\right]$$

In the ground state of free electrons, the maximum energy of electrons is the fermi energy 'E_F'.

$$E_F = \frac{h^2}{8m\pi^2} K_F^2$$

According to De-Broglie hypothesis,

$$\lambda = h/mv, \quad \lambda = 2\pi/K$$

$$mv = \frac{hK}{2\pi} \qquad \ldots\ldots\ldots (2)$$

Differentiating with respect to 't', we get

$$m\frac{dv}{dt} = \frac{h}{2\pi}\frac{dK}{dt} \qquad \ldots\ldots\ldots (3)$$

If an electric field is applied, force F` = -eE
Frictional force or opposing force, F = ma

$$F = m \, dv/dt$$
$$F = - F`$$
$$m \, dv/dt = -(-eE)$$
$$m \, dv/dt = eE$$

Substitute in equation (3),

$$\frac{h}{2\pi}\frac{dK}{dt} = eE$$
$$\int_0^t dK = \frac{2\pi}{h} eE \int_0^t dt$$
$$K(t) - K(0) = \frac{2\pi}{h} eEt$$
$$\delta K = \frac{2\pi}{h} eEt$$

i.e., in time 't' after the electric field is applied, the complete fermi sphere will be displaced by 'δK' in a direction opposite to the electric field 'E'.

Due to collisions, the fermi sphere attains a constant displacement in K-space in presence of electric field (Fig 4.4). If 'τ_f' is the mean collision time, the mean displacement is

$$\delta K_{av} = \frac{2\pi}{h} eE\tau_f \qquad \ldots\ldots\ldots (4)$$

From eqn. (2), $\qquad mv = \frac{hK}{2\pi}$

$$M\delta v_{av} = \frac{h}{2\pi} \delta K_{av}$$
$$\delta K_{av} = \frac{2\pi m}{h} \delta v_{av}$$

Substitute in eqn. (4),

$$\frac{2\pi m}{h} \delta v_{av} = \frac{2\pi m}{h} eE\tau_f$$
$$\delta v_{av} = \frac{eE\tau}{m}$$

As the initial average velocity of electrons is zero due to random motion, after applying electric field

$$\delta v_{av} = v_d = \frac{eE\tau_f}{m^*} \qquad \ldots\ldots\ldots (5)$$

The current density, $\qquad J = nev_d$

Substitute v_d from eqn. (5),

$$J = ne\left(\frac{eE\tau_f}{m^*}\right)$$
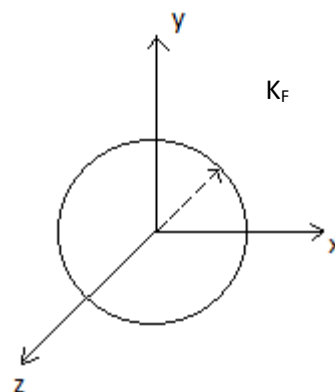$$J = \frac{ne^2 E\tau}{m^*} \qquad \ldots\ldots\ldots (6)$$
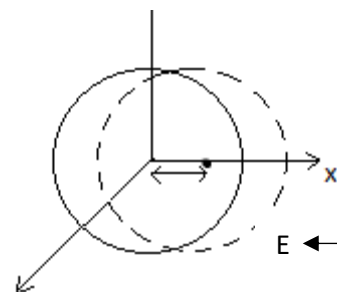

Fig. 4.3 Fermi sphere

Fig. 4.4 Fermi Sphere after application of electric field E

Ohm's law, $\qquad\qquad\qquad J = \sigma E \qquad$ .... (7)

Comparing eqn. (6) and eqn. (7) we get

$$\boxed{\sigma = \frac{ne^2\tau_f}{m^*}}$$

but $\tau_f = \dfrac{\lambda}{v_f}$, $\qquad \boxed{\boldsymbol{\sigma = \dfrac{ne^2\lambda}{m^*v_f}}}$

Expression for Electrical Conductivity

The number of electrons crossing the area A (Fig. 4.5) in one second is nAV_d where n is the free electron density and V_d *is the drift velocity*

current through the conductor $i = neAV_d$

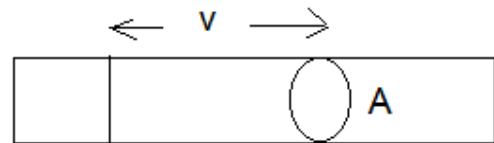but drift velocity $V_d = \dfrac{eE\tau}{m^*}$

Fig. 4.5 Flow of electrons with velocity 'v' and with an area of cross section 'A'

Hence $\quad i = neA\,\dfrac{e\,E\,\tau}{m^*}$

$= \dfrac{ne^2 AE\tau}{m^*}$

but $\tau_f = \dfrac{\lambda}{v_f}$,

current $\quad i = \dfrac{ne^2 AE}{m^*}\left(\dfrac{\lambda}{v_f}\right)$

Current density $J = \dfrac{i}{A}$

$\qquad\qquad = \dfrac{ne^2 E}{m^*}\left(\dfrac{\lambda}{v_f}\right)\quad$ but $J = \sigma E$

The conductivity is $\quad\boxed{\boldsymbol{\sigma = \dfrac{ne^2\lambda}{m^*v_f}}}$

Merits of Quantum Free Electron Theory:

1. **Specific heat of free electrons:**
   From quantum theory of free electrons, the specific heat of free electrons is given by **$C_v = (2k/E_F)\,RT$,** For a typical value of $E_F = 5$ eV, we get $C_v = 10^{-4}RT$. Hence gives the correct molar specific heat at constant volume for free electrons in metals.
2. **Dependence of electrical conductivity on electron concentration:**

The electrical conductivity in metals is given by $\sigma = \dfrac{ne^2\lambda}{m^* v_F}$,

It is clear from this equation that the electrical conductivity depends on the electron concentration, the ratio of **λ/v_F** and **m*.** Now by taking above expression, if we calculate σ, we get the observed conductivities of the metals.

3. **Temperature dependence of conductivity of metals:**

The conductivity is given by $\sigma = \dfrac{ne^2\tau}{m^*}$ n is the free electron density

e, the charge of electron

τ, the mean collision time

and m^* is the effective mass of electron

$\sigma = \dfrac{ne^2\lambda}{m^* v_f}$ ----(1)   since $\tau = \dfrac{\lambda}{v_f}$

As per quantum free electron theory, λ is dependent on temperature.

As the free electrons move inside the metal, the get scattered by the vibrating ions of the lattice. The scattering takes place equally in all directions

If r is the amplitude of vibration, the ions present in a circular cross section πr^2 will block the path of the electrons.

Mean free path reduces as the circular cross section increases.

ie. $\lambda \propto 1/\pi r^2$ -----(2)

energy is proportional to the square of the amplitude, $E \propto r^2$

Also the thermal energy is proportional to the temperature, $E \propto T$

Hence $r^2 \propto T$

From eqn (2). $\lambda \propto 1/T$

From eqn (1)   conductivity $\sigma \propto 1/T$

Which is same as the experimentally observed variation of conductivity with temperature.

Similarities between Classical and Quantum Free Electron Theories:
1. According to both the theories conduction is due to free electrons
2. The attraction between electrons and positive ion cores and the repulsion between electrons themselves is neglected.
3. The electrons are assumed to be confined to the metal by potential barriers at the boundaries and the potential inside the metal is assumed to be constant.

Differences between Classical and Quantum Free Electron Theories:

Classical theory	Quantum theory

1	Energy distribution of electrons is according to Maxwell – Boltzmann statistics	Energy distribution of electrons is according to Fermi – Dirac statistics
2	Electrons can have continuous energy values	Electrons have discrete and quantized energies
3	Many electrons can have the same energy	No two electrons can have the same energy

Physics of Semiconductors

Fermi level in intrinsic semiconductors

Since the number of holes and electrons

are equal in number in an intrinsic semiconductor,

the position of the Fermi level lies in the middle

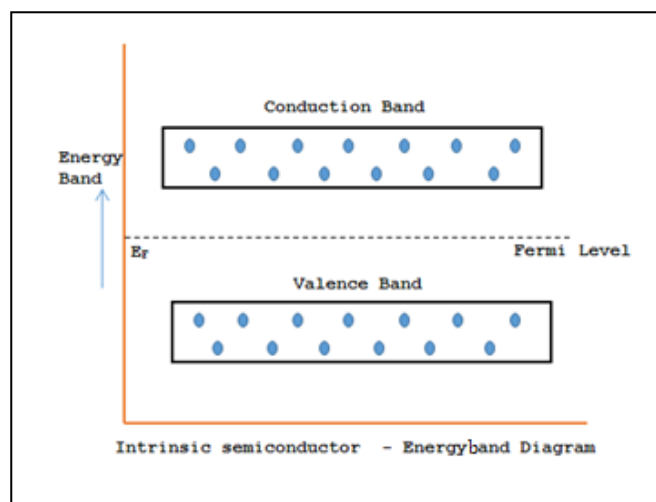of the conduction band and valence band (Fig. 4.6)

$E_F = \dfrac{Eg}{2}$

Fig. 4.6   Energy band diagram of an intrinsic semiconductor

Expression for electron concentration in intrinsic semiconductor (N_e):

$$N_e = \frac{4\sqrt{2}}{h^3} (\pi m_e^* kT)^{\frac{3}{2}} e^{\left(\frac{E_F - E_g}{KT}\right)}$$

Expression for hole concentration in intrinsic semiconductor:

$$N_h = \frac{8\sqrt{2}\pi (m_h^*)^{\frac{3}{2}}}{h^3} (kT)^{3/2} e^{\left(\frac{E_F}{KT}\right)} \frac{\sqrt{\pi}}{2}$$

Where

m_e and m_h are the effective masses of free electrons and holes respectively,

h is Planck's constant,

k is Boltzmann constant

T is the absolute temperature

E_F is the Fermi energy and E_g is the energy gap

Electrical Conductivity of Semiconductor

On the basis of free electron theory, the charge carriers can be assumed to be moving freely inside the semiconductor. Both the electrons and holes contribute to the conductivity of the semiconductor.

Considering the conductivity due to the flow of electrons in a semiconductor of cross-sectional area "A"

Let "I" be the current flowing through it and 'v ' the velocity of electrons.

In one second the electrons move through a distance v

The volume swept by the electrons in 1 second is Av

If N_e is the number of electrons per unit volume, then the number of electrons crossing any section in 1 second is AvN_e

If e is the charge of the electron then the charge flow per second is e AvN_e

Since charge flow per second gives the current I

I = N_e Ave

Current density J = I/A

$$= \frac{N_e \text{Ave}}{A}$$

J $= N_e$ve

Electron mobility $\mu_e = \frac{v}{E}$ E is the field strength

v = $\mu_e E$

$$J = N_e e\mu_e E \quad ----(1)$$

From Ohm's law, J = σ E

If σ_e is the conductivity of electrons , then J = σ_eE   -----(2)

Comparing eqns 1 and 2, $\sigma_e = N_e e\mu_e$

Similarly conductivity of holes , $\sigma_h = N_h e\mu_h$

The total conductivity of the semiconductor σ = $\sigma_e + \sigma_h$

$$σ = N_e e\mu_e + N_h e\mu_h$$
$$σ = e (N_e \mu_e + N_h \mu_h)$$

For intrinsic semiconductor $N_e = N_h = N_i$

$$\sigma_i = e \, N_i (\mu_e + \mu_h)$$

Lorentz Force:

When a moving charge is subjected to a magnetic Field, the force experienced by it is called the Lorentz force.

$$F_L = \text{Bev } \sin\theta$$

B- magnetic field strength
e – charge of the particle
v – velocity of the particle
θ - angle between B and v
The direction of the force is given by Right Hand Rule

Hall Effect

If a current carrying conductor or semiconductor is placed in a transverse magnetic field, a potential difference is developed across the specimen in a direction perpendicular to both the current and magnetic field (Fig. 4.7). The phenomenon is called Hall Effect.
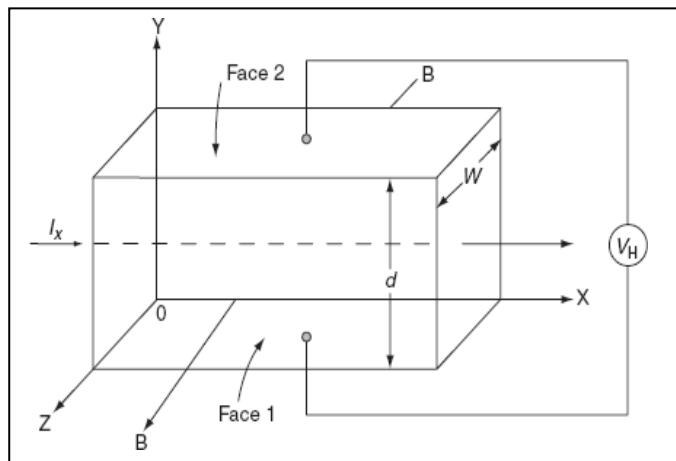

Fig. 4.7 Hall effect

consider a rectangular plate of a n-type semiconductor
of width 'w' and thickness 'd'
The force experienced by a moving charged particle in a magnetic field is called the Lorentz force .

Lorentz force, F_L= Bev   since θ = 90^0

The accumulation of charges on the faces 1 and 2, develops a potential difference called the Hall Voltage V_H ,

Hall field strength $E_H = \dfrac{V_H}{d}$ will oppose the flow of charges.

Force due to hall field $F_H = e\, E_H$

Flow of charges stops when $F_L = F_H$

$Bev = e\, E_H$

$Bv = \dfrac{V_H}{d}$

$V_H = Bvd$

$J = nev$

$\dfrac{I}{wd} = \rho\, v$ ρ is the charge concentration

$v = \dfrac{I}{wd\rho}$

$$V_H = Bd\, \frac{I}{wd\rho}$$

$$V_H = \frac{BI}{w\rho}$$

Hall Coefficient: R_H

The Hall field is proportional to the magnetic field strength and the current density

$E_H \, \alpha \, BJ$

$E_H = R_H \, BJ$ $\qquad R_H$ is the Hall coefficient, it is the Hall field strength developed for unit

magnetic field and unit current density.

$R_H = E_H / BJ$

$\qquad = v_H / BJd$

$\qquad = \dfrac{BI}{w\rho BJd}$

$$R_H = \frac{1}{\rho}$$

Also $\qquad\qquad \sigma = ne \, \mu$

$\qquad\qquad\qquad = \rho \, \mu$

$\qquad\qquad\qquad = (1/R_H) \, \mu$

$\qquad\qquad R_H \sigma = \mu$

DIELECTRICS

Dielectrics are mainly insulators i.e. they do not conduct electricity. This is because the electrons are bounded very strongly to their respective nuclei of the parent molecules. Dielectrics are widely used in capacitors for storage of charges and also as insulators in transformers, motors etc

Example: cotton, silk, paper, mica, asbestos, glass, quartz etc.

Dielectric strength: It is the maximum value of electric field that it can tolerate without breakdown (insulator changes to conductor)

The **dielectric breakdown** is the sudden change in the state of a dielectric material subjected to a very high electric filed, under the influence of which, the electrons are lifted into the conduction band and the insulator becomes a conductor.

The voltage at which the dielectric breakdown occurs is called breakdown voltage.

Electric dipole and Dipole Moment:

A pair of equal and opposite charges separated by a very small distance is called electric dipole. The product of magnitude of one of the charges and the distance of their separation is called the dipole moment. $\mu = q\,\ell$

-q $\longleftarrow \overset{\ell}{\longrightarrow}$ +q
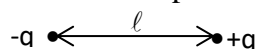
Dielectric Polarization (Electrical Polarization)

a) Case of non-polar dielectric

If the effective centres of negative and positive charges in the molecules (or atoms) coincides with each other, then the material is said to be non-polar dielectric.
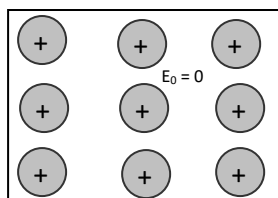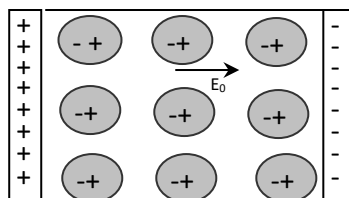

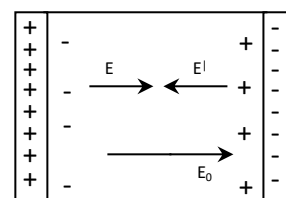**Fig. 4.8 (a)** **Fig. 4.8(b)** **Fig. 4.8 (c)**

Fig (a) shows a non-polar dielectric slab with no external electric field applied. Here the effective centres of +ve and –ve charges are coincided with each other.

In fig (b) an electric field E_0 is applied through a capacitor, whose plates are charged as shown. The effect of E_0 is a slight separation of the centres of the +ve and –ve charge distribution within the slab. Due to the separation between the two charge centres, each molecule develops a dipole moment in the direction of the electric field. Such dipoles are produced throughout the solid.

Within the dielectric slab, through the polarization charges appears in every molecule, opposite charges in the neighbouring molecules neutralize each other. This kind of cancellation takes place throughout the body of the material since there are equal amount of –ve & +ve charges. But the charges which are at the end surface of the slab do not find the opposite kind of charges for cancellation. Thus the effect of the applied field is to cause the appearance of net opposing charges at the end faces of the slab.

Fig (c) shows that the induced surface charges on the faces produces an electric field $E^{|}$ in the direction opposite to that of the applied electric field E_0. The resultant field E inside the dielectric which is the vector sum of E_0 and $E^{|}$ has the direction of E_0 but is smaller in magnitude.

As a result net field in the region is reduced. As the field decreases the p.d between the plates of a charged capacitor also decreases.

Consequently the capacitance of the capacitor increases

The ability of a dielectric to decrease the electric intensity is determined by its dielectric constant K.

b) **Case of polar dielectric**

The material is said to be polar dielectric only when the effective centres of the –ve and +ve charges in the molecule (or atom) do not coincide with each other.
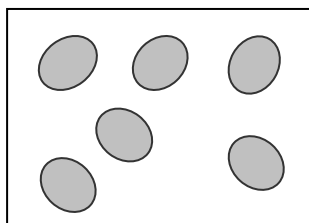
a) Molecule with a permanent dipole moment showing random orientation in the absence of electric field
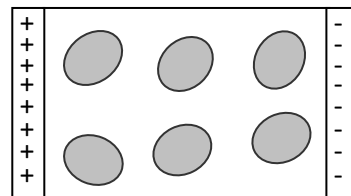
b) An electric field is applied producing partial alignment of the dipoles

In the case of polar dielectrics, there are permanent dipoles oriented randomly in the material. Under the influence of the applied electric field, the molecular dipoles tends to align their dipole moment along the direction of the field. But the thermal agitation prevents complete alignment. Between these two opposing events, some alignment is achieved under sufficiently strong fields. Once the alignment is established to whatever degree it could be, the surface charges appears at the ends of the slab in a way similar to the case of non-polar dielectric materials

Dielectric Polarisation

The displacement of charges in the molecules of a dielectric under the action of an applied field, leading to the development of dipole moment is called polarization of dielectric (or electrical polarization), and the charges appearing separated are called polarization charges.

Dielectric constant:

For isotropic materials the electric field strength E and the flux density D are related by the equation, $D = \varepsilon_o \varepsilon_r E$

Where, $\varepsilon_o = 8.854 \times 10^{-12}$ farad/m is the permittivity of free space and ε_r is the relative permittivity or dielectric constant.

ε_r is constant under static field conditions for an **isotropic**[*] dielectric. Therefore it is called as static dielectric constant, but varies in the case of anisotropic dielectric. It is a dimensionless quantity. It is a mere number and has no unit. Under varying fields its value varies i.e., it depends on the frequency of the applied alternating field.

Isotropic[*]**:** If the physical properties like, thermal conductivity, thermal expansion, elastic modul, electrical conductivity, dielectric constant etc are independent of direction, then they are said to be isotropic otherwise anisotropic

For air ε_r has a value 1.00059 and in practice it is takes as unity

Dielectric Susceptibility – Relation between P, χ and E

When the dielectric material is subjected to the influence of an electric field, the charges that remain without being neutralized in a polarized dielectric give rise to a net electric dipole moment for the entire dielectric material. This induced dipole moment per unit volume of the dielectric is called the Polarisation P. It is a vector quantity whose direction is along the direction of the applied field i.e., from negative charge to positive charge.
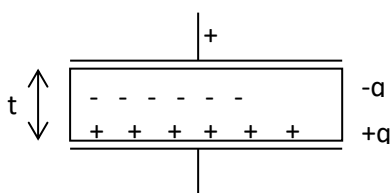
If +q and –q are the charges induced on the respective faces of a dielectric slab of thickness t kept between the two plates of a capacitor, then

The total dipole moment of the bulk dielectric material = charge × distance of separation

$$= q \times t$$

If A is the surface area of the two faces, then

The volume of the slab = A × t

The magnitude of polarization P = dipole moment /unit volume

i.e., $P = \dfrac{\text{total dipole moment of the slab}}{\text{volume of the slab}} = \dfrac{qt}{At} = \dfrac{q}{A}$

The unit of polarization is Cm^{-2}

Bur q/A is the induced charge density.

∴ Magnitude of polarization is equal to the induced charge density.

The polarization P is directly proportional to the external field E.

For such media, $P \propto E$

It can be shown that, $P = \varepsilon_o (\varepsilon_r - 1) E$

$P = \varepsilon_o \chi E$ where $\chi = (\varepsilon_r - 1)$ is the dielectric susceptibility of the material.

Polarisibility:

Let the dielectric material be subjected to an electric field E. Let μ be the electric dipole moment acquired by an atom. It is found that μ varies directly as E, i.e., $\mu \propto E$
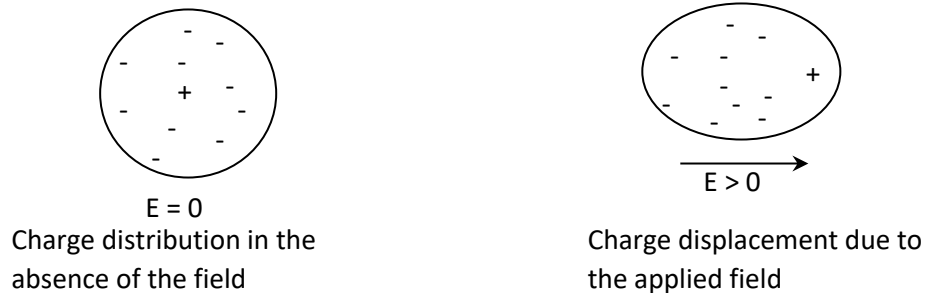
or dipole moment of the atom, $\mu = \alpha E$

where α is the proportionality constant called polarisibility of the atom. Its unit is Fm^2

Dielectric Polarization mechanisms

There are four types of polarization mechanisms in dielectric materials when they are subjected to an electric field.

1.   Electronic Polarisation: (α_e)

E = 0
Charge distribution in the
absence of the field

E > 0
Charge displacement due to
the applied field

The electronic polarization occurs due to the displacement of the +ve and –ve charges in a dielectric material because of the application of an external electric field. The separation created between the charges leads to development of a dipole moment. This dipole moment is referred to as induced dipole moment because it is induced by the field E (which was not present in the absence of the field)

The electronic polarisability α_e is given by $= \dfrac{\varepsilon_o(\varepsilon_r - 1)}{N}$

$\varepsilon_0 \rightarrow$ Permitivity of free space, $\varepsilon_r \rightarrow$ Relative permittivity, $N \rightarrow$ Number of atoms/m^3

2.   Ionic Polarisation (α_i)

$E_0 = 0$

Ion placement in the
absence of the field

$E_0 > 0$

Ion displacement due
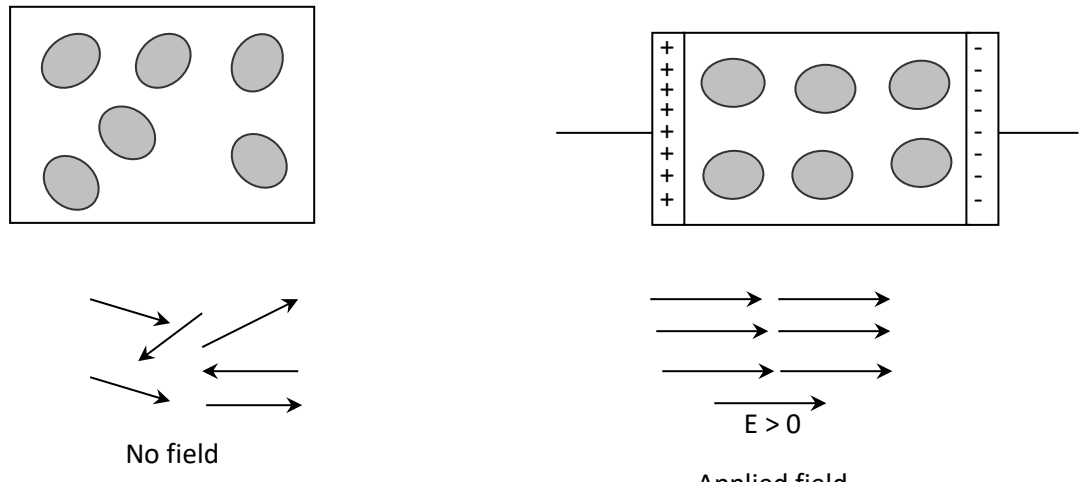to applied field
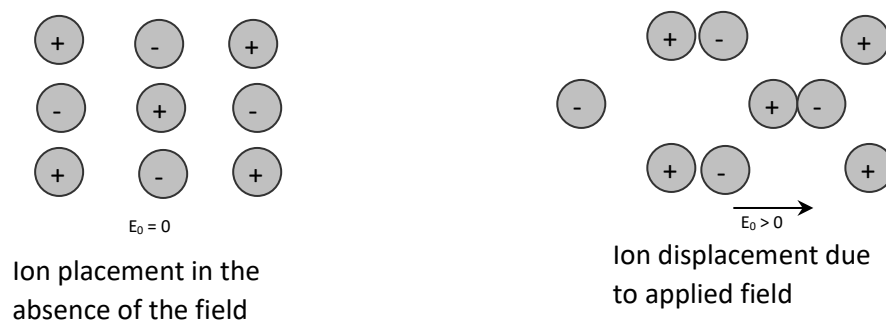
Ionic polarisation occurs in materials that are ionic such as NaCl. An applied field acts to displace cations in one direction and anions in the opposite direction, which gives rise to a net dipole moment.

This phenomenon is illustrated in the above figure

3. Orientational Polarisation (α_o):

No field

E > 0

Applied field

Orientation polarization is found only in substances that possess permanent dipole moments (i.e., polar dielectrics)
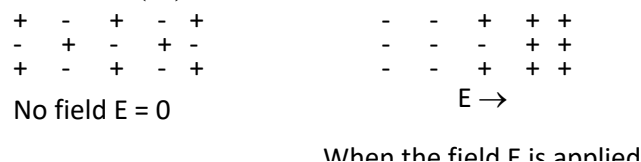
Polarization results from a rotation of the permanent moments into the direction of the applied field as shown in the figure.

This alignment tendency is counteracted (or opposed) by the thermal vibrations of the atoms, such that polarization decreases with increasing temperature.

In case of polar dielectrics, the orientational polarisability α_0 is given by $\alpha_0 = \dfrac{\mu^2}{3kT}$

Where μ is the dipole moment, k is the Boltzmann constant and T is the temperature.

4. Space charge polarization (α_s):

```
+   -   +   -  +          -   -   +   +  +
-   +   -   +  -          -   -   -   +  +
+   -   +   -  +          -   -   +   +  +
  No field E = 0          E →
```

When the field E is applied

The ionic, electronic and orientation polarizations are due to the bound charges in the atoms or molecules. In addition to the bound charges, there are some charges which are free to move in the material. While moving these free charges may be trapped by lattice vacancies, impurity centres etc. The effect of this will be the creation of a localized accumulation of charges and give rise to the dipole moments. This mechanism is called interferential polarization or space charge polarization.

The total polarization α of a material is given by the sum of the electronic, ionic and orientational polarization

$$\alpha = \alpha_e + \alpha_i + \alpha_o$$

Note: It is possible for one or more of these contributions to the total polarization to be either absent or negligible in magnitude relative to the others.

Example: Ionic polarization will not exist in covalently bonded materials in which no ions are present.

Temperature dependence of Polarisation Mechanisms:

When a material is subjected to ordinary conditions of increasing temperature, the electronic distribution in the constituent molecules will not be affected. Hence there will be no influence on the electronic and ionic polarization mechanisms.

But, the same effect effect of increase of temperature brings about a higher degree of randomness in the molecular orientation in the material. This affects the orderliness in the dipolar arrangement that was established by the applied field, which in turn affects the orientational polarization. The orientational polarization varies inversely with the temperature.

On the other hand, since the thermal energy facilitates the ion movement by diffusion, it allows the molecules to align in the direction of the field. Hence increase of temperature works favourably for space charge polarization.

Thus, both the orientational and space charge polarization mechanisms are strongly temperature dependent.

Internal Fields in Liquids and Solids

When a dielectric material, either solid or liquid is subjected to an external electric field each of the atoms develops a dipole moment, and act as electric dipole. Hence the resultant field at any given atom will be the sum of applied electric field and the electric field due to the surrounding dipoles. This resultant local field is called the internal field

Definition: The internal field or the local field, is the electric field that acts at the site of any given atom of a solid or a liquid dielectric subjected to an external electric field, and is the resultant of the applied field and the field due to all the surrounding dipoles

Expression for the internal field in the case of liquid and solid dielectrics:

Consider a dielectric material, either liquid or solid kept in an external uniform electric field of strength E. In the material, imagine an array of equidistant atomic dipoles arranged parallel to the direction of the field as shown in the fig.
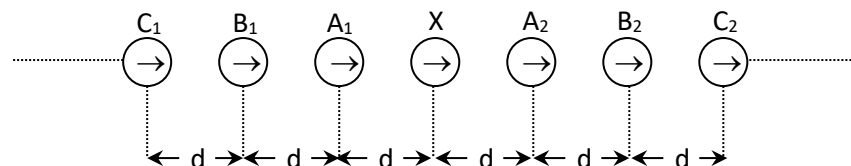
Let the interatomic distance be d, and the dipole moment of each of the atomic dipole be μ. In the linear array consider an atom X. The total field at X due to all the other dipoles in the array is calculated as follows
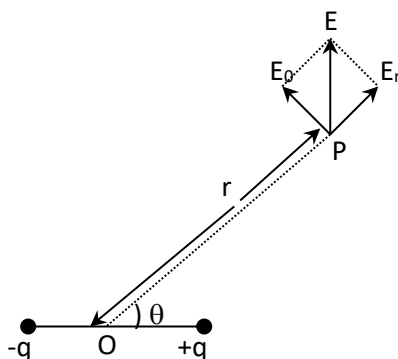
The components of the electric field at a point p due to an electric dipole, are given in polar form as

$$E_r = \frac{\mu \cos\theta}{2\pi\varepsilon_0 r^3} \text{ -------------- (1)}$$

$$E_\theta = \frac{\mu \sin\theta}{4\pi\varepsilon_0 r^3} \text{ ------------- (2)}$$

Where μ is the dipole moment, r is the distance between midpoint O of the dipole and the point P, θ is the angle between OP and the line joining the two charges.

Using these equations, we can calculate the electric field at X due to the dipole A_1

Since the distance of X from A_1 is d, we can write,

r = d and θ = 0

$$\therefore E_r = \frac{\mu}{2\pi\varepsilon_0 d^3} \text{ and } E_\theta = 0$$

$$\therefore \text{ Field at X due to } A_1 = E_r + E_\theta = \frac{\mu}{2\pi\varepsilon_0 d^3}$$

Now, consider the dipole A_2

Since it is situated symmetrically on the other side of X, its field at X will also be = $\frac{\mu}{2\pi\varepsilon_0 d^3}$

Now θ = 180° and r = -d

$$E_r = \frac{\mu \cos 180^\circ}{2\pi\varepsilon_0 (-d)^3} = \frac{-\mu}{-2\pi\varepsilon_0 d^3} = \frac{\mu}{2\pi\varepsilon_0 d^3}$$

$$E_\theta = 0 \qquad\qquad (\because \sin 180 = 0)$$

$\therefore$ The field E_1 at X due to both the dipole A_1 and A_2 is

E_1 = Field at X due to A_1 + Field at X due to A_2

$$= \frac{\mu}{2\pi\varepsilon_0 d^3} + \frac{\mu}{2\pi\varepsilon_0 d^3}$$

$$= 2\frac{\mu}{2\pi\varepsilon_0 d^3}$$

$$= \frac{\mu}{\pi\varepsilon_0 d^3}$$

If we consider the two dipoles B_1 and B_2, each which is located at a distance of 2d, then the field E_2 at X due to both of them is

$$E_2 = \frac{\mu}{\pi\varepsilon_0 (2d)^3}$$

Similarly, the field E_3 at X due the dipoles C_1 and C_2 is

$$E_3 = \frac{\mu}{\pi\varepsilon_0 (3d)^3}$$

$\therefore$ The total field E_T induced at X due to all the dipoles in the linear array is

$E_T = E_1 + E_2 + E_3 + \text{-----------------}$

$$= \frac{\mu}{\pi\varepsilon_0 d^3} + \frac{\mu}{\pi\varepsilon_0 (2d)^3} + \frac{\mu}{\pi\varepsilon_0 (3d)^3} + \text{-------------}$$

$$= \frac{\mu}{\pi\varepsilon_0 d^3}\left[\frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + \text{-------}\right]$$

$$= \frac{\mu}{\pi\varepsilon_0 d^3}\sum_{n=1}^{\infty}\frac{1}{n^3}; \text{ where } n = 1, 2, 3,\ldots\ldots\ldots\infty$$

The internal field E_i is the sum of the applied E and the total field due to all dipoles i.e., E_T.

The internal field $E_i = E + E_T$

$$E_i = E + \frac{\mu}{\pi\varepsilon_0 d^3}\sum_{n=1}^{\infty}\frac{1}{n^3}$$

But $\sum_{n=1}^{\infty}\dfrac{1}{n^3} = 1.2$

$$\boxed{\therefore E_i = E + \frac{1.2\,\mu}{\pi\varepsilon_0 d^3}}$$

If α_e is the electronic polarisibility for the dipoles, then $\mu = \alpha_e E$

$$\therefore E_i = E + \frac{1.2\,\alpha_e E}{\pi\varepsilon_0 d^3} \qquad \therefore \text{We can say that } E_i > E$$

In 3 – dimensional case, $E_i = E + \left(\dfrac{\gamma}{\varepsilon_o}\right) P$

where P is the dipole moment per unit volume for the material and γ is the proportionality constant called internal field constant. It can be shown that $\gamma = 1/3$ in which event, the internal field is named as **Lorentz Field**, given by

$$\therefore E_{Lorentz} = E + \frac{P}{3\varepsilon_o}$$

The above equation is known as Lorentz relation.

Clausius – Mossotti equation:

Consider an elemental solid (which consists of only one kind of atoms and are non – polar such as sulphur, phosphorous, diamond etc) dielectric material of dielectric constant ε_r.

If N is the number of atoms/unit volume of the material, μ is the atomic dipole moment, then we have, Dipole moment / unit volume = $N\mu$ -----(1)

Here the field experienced by the atoms is the internal field E_i. Hence if α_e is the electric polarisibility of the atoms, we can write the equation for μ as,

$$\mu = \alpha_e E_i \quad --------(2)$$

$\therefore$ Equation (1) becomes, Dipole moment / unit volume = $N\alpha_e E_i$ ----------(3)

The left side of eqn (3) is same as polarization P.

$\therefore P = N\alpha_e E_i$ -----------(4)

or, $E_i = \dfrac{P}{N\alpha_e}$ ------------(5)

But we have the relation for P as, $P = \varepsilon_o(\varepsilon_r - 1)E$, where E is the applied field.

$$\therefore E = \frac{P}{\varepsilon_o(\varepsilon_r - 1)} \quad ----------(6)$$

Also we have the equation for internal field as $E_i = E + \left(\dfrac{\gamma}{\varepsilon_o}\right) P$ -----(7)

where γ is the internal field constant.

Substituting for E_i and E from eqn (5) and eqn (6) in eqn (7)

We have, $\dfrac{P}{N\alpha_e} = \dfrac{P}{\varepsilon_o(\varepsilon_r - 1)} + \left(\dfrac{\gamma}{\varepsilon_o}\right)P$

$$\dfrac{1}{N\alpha_e} = \dfrac{1}{\varepsilon_o(\varepsilon_r - 1)} + \left(\dfrac{\gamma}{\varepsilon_o}\right)$$

$$\dfrac{1}{N\alpha_e} = \dfrac{1}{\varepsilon_o}\left[\dfrac{1}{(\varepsilon_r - 1)} + \gamma\right]$$

Considering the internal field in the material to be Lorentz field, we have $\gamma = 1/3$

$$\therefore \dfrac{1}{N\alpha_e} = \dfrac{1}{\varepsilon_o}\left[\dfrac{1}{(\varepsilon_r - 1)} + \dfrac{1}{3}\right]$$

$$\dfrac{1}{N\alpha_e} = \dfrac{1}{\varepsilon_o}\left[\dfrac{3 + \varepsilon_r - 1}{3(\varepsilon_r - 1)}\right]$$

$$\dfrac{\varepsilon_o}{N\alpha_e} = \left[\dfrac{\varepsilon_r + 2}{3(\varepsilon_r - 1)}\right]$$

By rearranging we have,

$$\boxed{\dfrac{(\varepsilon_r - 1)}{(\varepsilon_r + 2)} = \dfrac{N\alpha_e}{3\varepsilon_o}}$$

This equation is called Clausius – Mossotti equation. This equation provides a relation between dielectric constant ε_r and electronic polarisibility α_e.

APPLICATIONS OF DIELECTRIC MATERIALS:

Capacitors are used to store charges. Large values of capacitance can be achieved in capacitors of small size by filling the space between its plates with dielectric materials of high ε_r values.

1. The piezoelectric class of dielectric materials (lead, Barium titanate, Rochelle salt,) find applications in Electronic Industry. Their main use is in frequency control of oscillators.

2. Piezoelectric materials are also used as electro – acoustic transducers (to convert electrical energy into mechanical and vice – versa). The transducer action is employed in producing ultrasonic which is SONAR (Sound Navigation and Ranging), in ultrasound imaging of human body, non – destructive testing of materials, MEMS (Micro Electro Mechanical Systems), measurement of velocity of ultrasound in solids and liquids etc.

3. The piezoelectric crystals are also used in gas lighters and car ignition system. In gas lighters, the pressure from the thumb on the shaft at one end of the lighter is transmitted to the piezoelectric crystal at the other end due to which charges appear on the crystal's surface. A high voltage develops on the crystal and when it attains a critical value the charges discharge into the body of the lighter, which drives a spark into the gas medium and the gas is ignited.

QUESTION BANK:

1. Define drift velocity, mobility and relaxation time for free electron.
2. Write down the assumptions of classical free electron theory.
3. Explain the failure of classical free electron theory.
4. Obtain the expression for electrical conductivity on the basis of free electron theory
5. of metals.
6. State and explain Matthiessen's rule.
7. What is Fermi energy? Write expression for Fermi factor. Obtain the expression for
8. Fermi energy (EF) at T = 0 K in terms of electron concentration.
9. Explain Fermi energy and Fermi factor. Discuss the variation of Fermi factor with temperature and energy.
10. Describe Fermi-Dirac distribution and discuss the same for different temperature conditions.
11. Define Fermi energy. Discuss the Fermi factor f(E) for cases E < EF, E > EF at T = 0, E = EF at T ≠ 0.
12. Discuss the various drawbacks of classical free electron theory of metals. What are the assumptions made in quantum theory to overcome the same?
13. Describe how QFET has been successful in overcoming the failure of classical free electron theory.
14. Explain density of states.
15. What are the merits of QFET?
16. Explain the difference between classical free electron theory and quantum free electron theory.