MODULE – 1

D.C.CIRCUITS

Electric Current

The directed flow of free electrons (or charge) is called **electric current**. The flow of electric current can be beautifully explained by referring to Fig 1.1. The copper strip has a large number of free electrons. When electric pressure or voltage is applied, then free electrons, being negatively charged, will start moving towards the positive terminal around the circuit as shown in Fig 1.1. This directed flow of electrons is called electric current.

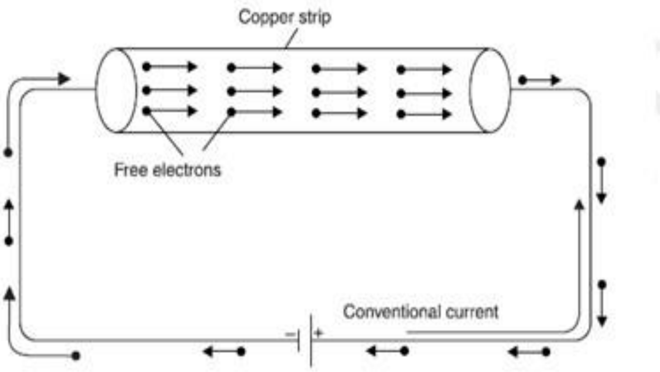


Fig 1.1: Flow of electric current

(i) Current is flow of electrons and electrons are the constituents of matter. Therefore, electric current is matter (*i.e.* free electrons) in motion.

(ii) The actual direction of current (*i.e.* flow of electrons) is from negative terminal to the positive terminal through that part of the circuit external to the cell. However, prior to Electron theory, it was assumed that current flowed from positive terminal to the negative terminal of the cell *via* the circuit. This convention is so firmly established that it is still in use. This assumed direction of current is now called *conventional current*.

Unit of Current. The strength of electric current I is the rate of flow of electrons *i.e.* charge flowing per second. The charge Q is measured in coulombs and time t in seconds. Therefore, the unit of electric current is *coulombs/sec or ampere*. If Q = 1 coulomb, t = 1 sec, then
I= 1/1 = 1 ampere.

One ampere o*f current is said to flow through a wire if at any cross-section one coulomb of charge flows in one second.* Thus, if 5 amperes current is flowing through a wire, it means that 5 coulombs per second flow past any cross-section of the wire.

Electric Potential

When a body is charged, work is done in charging it. This work done is stored in the body in the form of potential energy. The charged body has the capacity to do work by moving other charges The capacity of a charged body to do work is called its **electric potential.**

The greater the capacity of a charged body to do work, the greater is its electric potential. The work done is measured in joules and charge in coulombs. Therefore, the unit of electric potential will be joules/coulomb or volt. If W = 1 joule, Q = 1 coulomb, then

$$V = 1/1 = 1 \text{ volt.}$$

Electric Power

The rate at which work is done in an electric circuit is called its **electric power** i.e. When voltage is applied to a circuit, it causes current (i.e. electrons) to flow through it. Clearly, work is being done in moving the electrons in the circuit. This work done in moving the electrons in a unit time is called the electric power.

The total charge that flows in t seconds is $Q = I \times t$ coulombs

Ohm's law:

The relationship between voltage (V), the current (I) and resistance (R) in a d.c. circuit was first discovered by German scientist George Simon Ohm. This relationship is called Ohm's law and may be stated as under:

> ***The ratio of potential difference (V) between the ends of a conductor to the current (I) flowing between them is constant, provided the physical conditions (e.g. temperature etc.) do not change i.e., V/I=R***

where R is the resistance of the conductor between the two points considered.

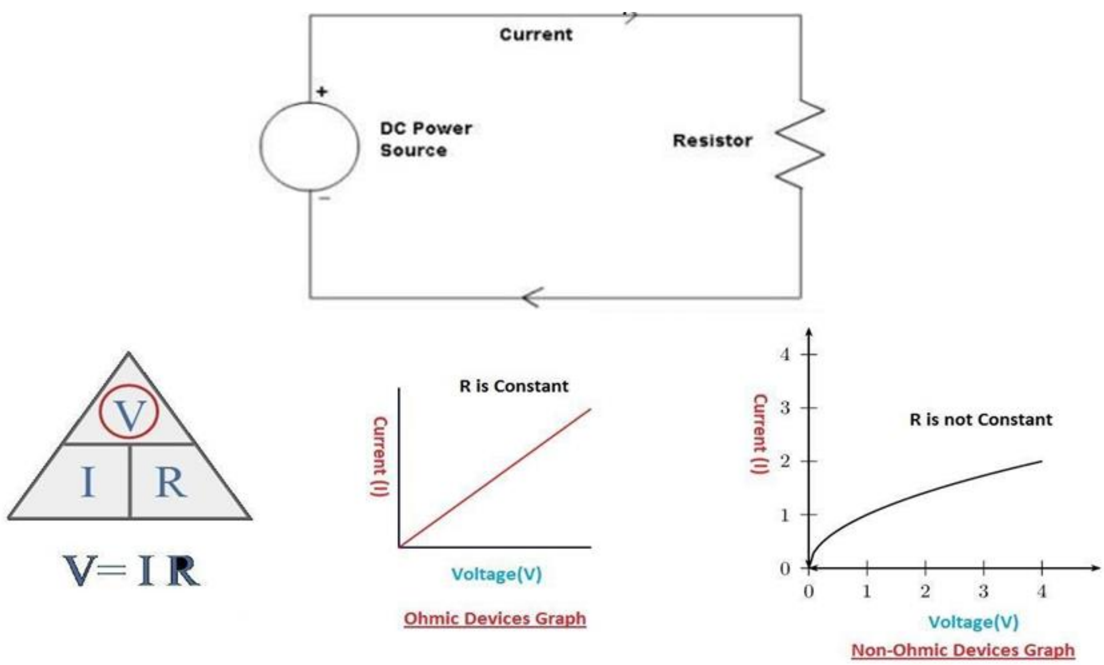


Fig 1.2: Ohm's Law Representation

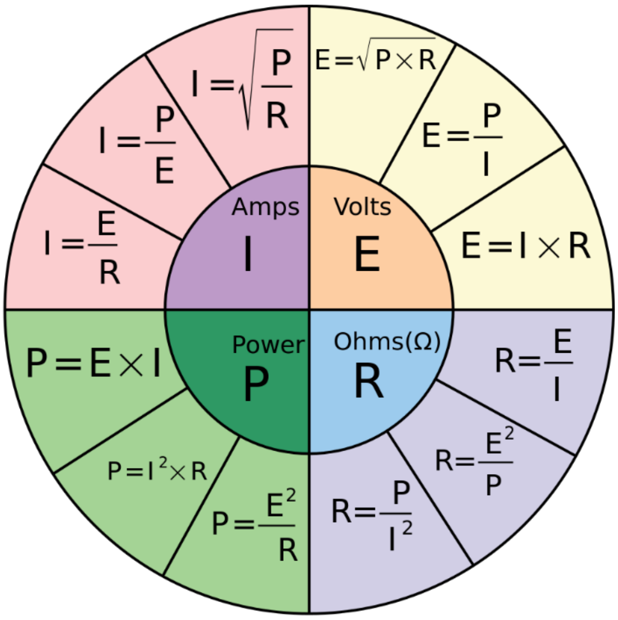


Fig 1.3: Relationship between current, voltage and resistance

To make a current flow through a resistance there must be a voltage across that resistance. As shown in Fig 1.3, Ohm's Law shows the relationship between the voltage (V), current (I) and resistance (R). It can be written in three ways as shown above.

The **limitations of Ohm's law** are outlined below:

1. This law cannot be applied to unilateral networks. A unilateral network has unilateral elements like diode, transistors, etc., which do not have same voltage current relation for both directions of current.
2. **Ohm's law** is also not applicable for non-linear elements. Non-linear elements are those which do not have current exactly proportional to the applied voltage, that means the resistance value of those elements changes for different values of voltage and current. Examples of non – linear elements are thyristor, electric arc, etc.

Kirchhoff's laws.

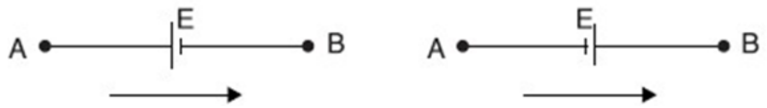
To understand the Kirchoff's laws and apply the same to the circuits, the knowledge of sign convention is very important

Sign Convention

A **rise in potential should be considered positive and fall in potential should be considered negative.

(i) **F**rom the positive terminal of the battery to the negative terminal, there is a fall in potential and the *e.m.f.* should be assigned negative sign.

(ii) From the negative terminal to the positive terminal of the battery or source, there is a rise in potential and the *e.m.f* should be assigned positive sign.



(iii) When current flows through a resistor, there is a voltage drop across it. If we go through the resistor in the same direction as the current, there is a fall in potential because current flows from higher potential to lower potential. Hence this voltage drop should be assigned negative sign.

(iv) On the other hand, if we go through the resistor against the current flow, there is a rise in potential and the voltage drop should be given positive sign.

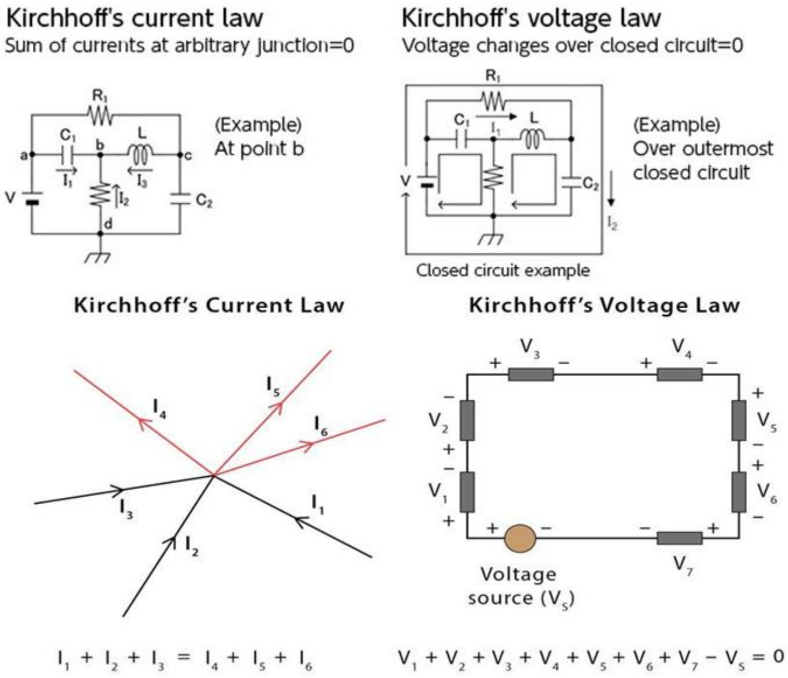


Fig 1.4: Kirchoff's law

Kirchhoff 's current law:

The total current flowing towards a junction point is equal to the total current flowing from that junction point as shown in Fig 1.5

Sum of currents entering a node =0 $\sum_{n=1}^{N} i_n = 0$ (KCL),

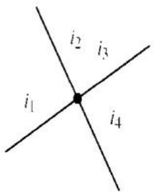
OR $i_1 - i_2 - i_3 + i_4 = 0$

Sum of currents leaving a node = 0

OR $-i_1 + i_2 + i_3 - i_4 = 0$

Sum of currents leaving = sum of Currents entering a node

$i_1 + i_4 = i_2 + i_3$



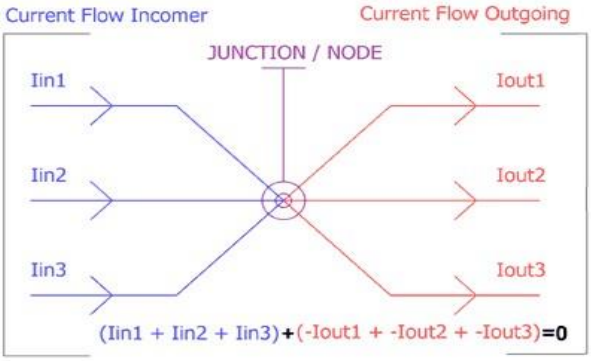


fig 1.5: Kirchoff's current law

Kirchhoff's voltage law:

"In any network, the algebraic sum of the voltage drops across the circuit elements of any closed path (or loop or mesh) is equal to the algebraic sum of the e.m.f s in the path".

In other words, "the algebraic sum of all the branch voltages, around any closed path or closed loop is always zero."

Applying Kirchoff's Voltage law (KVL) to the loop ABCDA, we get $-V_{AB}-V_{BC}-V_{CA}+V_{DA}=0$

i.e Sum of the voltages around a closed loop is zero.

OR

$\therefore V_{DA}= V_{AB}+V_{BC}+V_{CA}$

i.e

Sum of the source voltages is equal to sum of the voltage drops or Sum of all the potential rises must be equal to sum of all the potential drops.

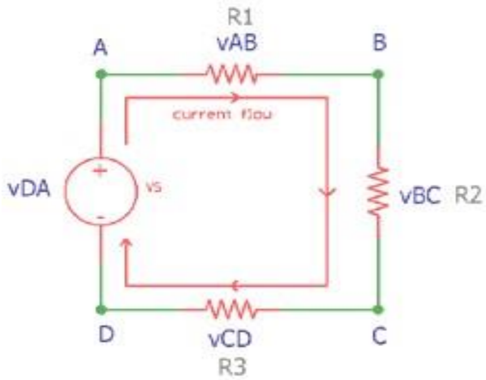
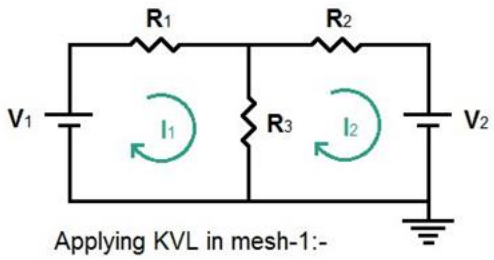


Fig 1.5: Kirchoff's current law



According to KVL:-

$\boxed{\sum I.R = \sum E}$

Applying KVL in mesh-1:-

Applying KVL in mesh 1

$$V_1 - I_1R_1 - (I_1 - I_2)R_3 = 0 \text{ -------------------- } 1$$

Applying KVL in mesh 2

$$-I_2R_2 - V_2 - (I_2 - I_1)R_3 = 0 \text{ ---------------} 2$$

The law states that if one starts at a certain point of a closed path and goes on tracing and noting all the potential changes (either drops or rises) as shown in Fig.1.6, in any one particular direction, till the starting point reached again, he must be at the same potential with which he started tracing a closed path.

Sum of all the potential rises must be equal to sum of all the potential drops while tracing any closed path of the circuit. The total change in potential along a closed path is always zero. This law is very useful in loop analysis of the network.

Resistors connected in Series and Parallel

Resistors connected in Series

When resistors are connected in series their combined resistance is equal to the individual resistances added together is shown in Fig 1.7: Resistors connected in Series. For example, if resistors R_1 and R_2 are connected in series their combined resistance, R, is given by: Combined resistance in **series**:

$\mathbf{R = R_1 + R_2}$

This can be extended for more resistors: $\mathbf{R = R_1 + R_2 + R_3 + R_4 + ...}$

Note that the **combined resistance in series** will always be **greater** than any of the individual resistances.

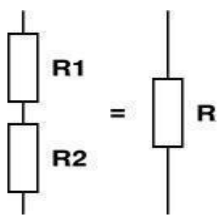


Fig 1.7: Two resistors connected in Series

Equivalent resistance of Series Resistive circuits

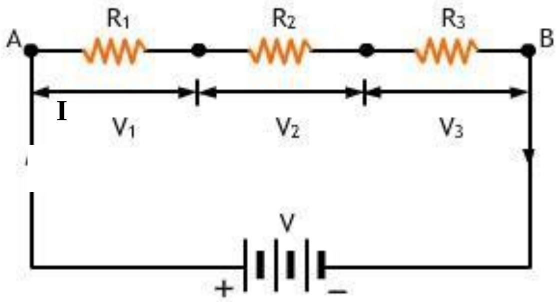


Fig 1.8: Three resistors connected in Series

Let V_1, V_2 and V_3 be the voltage across the terminals of resistances R_1, R_2 and R_3 respectively. Then supply voltage $V=V_1+V_2+V_3$

Now according to Ohm's law $V_1=IR_1$, $V2=IR_2$ and $V_3=IR_3$ where I is the current through the circuit. Let R_{eq} be the equivalent series resistance of the circuit and hence $V=IR_{eq}$

$IR_{eq}=IR_1+IR_2+IR_3=I(R_1+R_2+R_3)$

$R_{eq}=R_1+R_2+R_3$

Similarly, if n number of resistances are connected in series then $R_{eq}=R_1+R_2+R_3+R_4+\ldots\ldots+R_n$

Hence the equivalent resistance of a circuit with many resistors connected in series is the sum of all the resistances.

Resistors connected in Parallel

When resistors are connected in parallel their combined resistance is less than any of the individual resistances. Three resistors connected in parallel is shown in Fig 1.9.

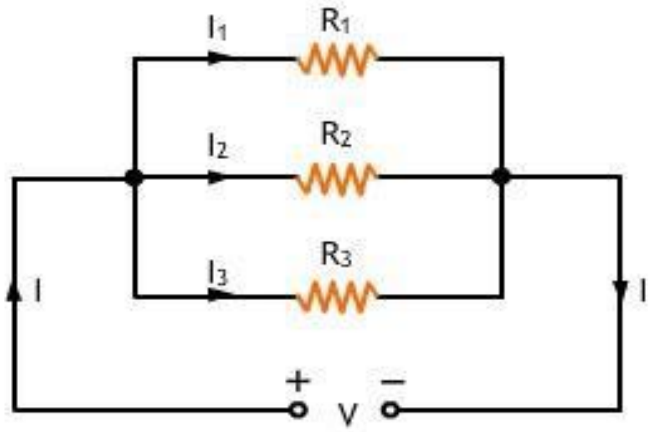


Fig 1.9: Resistors connected in parallel

Let R_1, R_2 and R_3 are the three resistances connected in parallel as shown in Fig 1.9. let I_1, I_2 and I_3 be the currents through R_1, R_2 and R_3 respectively. Also let V be the supply voltage and I be the total current from the source.

$$I_1 = \frac{v_1}{R_1}, I_2 = \frac{v_2}{R_2}, I_3 = \frac{v_3}{R_3}$$

$$I = I_1 + I_2 + I_3 = \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3}$$

$$= v\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$= \frac{v}{Req}$$

Hence for a parallel circuit

$$\frac{1}{Req} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Reciprocal equivalent resistance of the resistances connected in parallel is the sum of the reciprocals of all the resistances connected in parallel OR the equivalent conductance of resistances connected in parallel is the sum of all the conductance connected in parallel.

Conductance (G) :

It is known that $\frac{1}{R} = G$

Hence, G = G_1+G_2+G_3 +……….+G_n ……for parallel circuit

Important result :

Now if n = 2, two resistances are in parallel then,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\boxed{R = \frac{R_1 R_2}{R_1 + R_2}}$$

Current Division in Parallel Circuits: The current division through parallel circuits can be found out very easily and can be understood from the parallel circuit shown in Fig 1.10. From Fig 1.10 the total current I_T is the sum of branch currents I_{R1} and I_{R2}. Hence

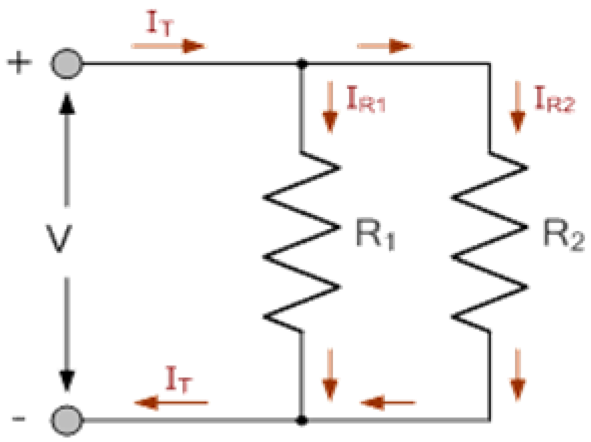


Fig 1.10 Current division through parallel circuit

$$I_T = I_{R_1} + I_{R2} \quad \text{where} \quad I_{R_1} = \frac{V}{R_1} \quad \text{and} \quad I_{R2} = \frac{V}{R_2}$$

$$I_T = \frac{V}{R_1} + \frac{V}{R_2} = v\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

$$V = I_T\left[\frac{1}{R_1} + \frac{1}{R_2}\right]^{-1} = I_T\left[\frac{1R_1R_2}{R_1+R_2}\right] \quad \text{Hence, } I_{R_1} = \frac{V}{R_1} = I_T\left[\frac{R_2}{R_1+R_2}\right]$$

Hence current through R_1 is equal to total current multiplied by resistance of the other branch divided by the total resistance.

Similarly $I_{R2} = \frac{V}{R_2}$

$$= \quad I_T\left[\frac{R_1}{R_1 + R_2}\right]$$

Star to delta transformation

Sometimes it is required to find the equivalent resistance between two given points of a DC network. This process requires network reduction by identifying series and parallel combinations and replacing them by their respective equivalent resistances. But in a complicated circuit it is required to transform star to delta and delta to star conveniently to facilitate network reduction. The star and delta connections are shown in Fig1.11 and Fig 1.12 respectively.

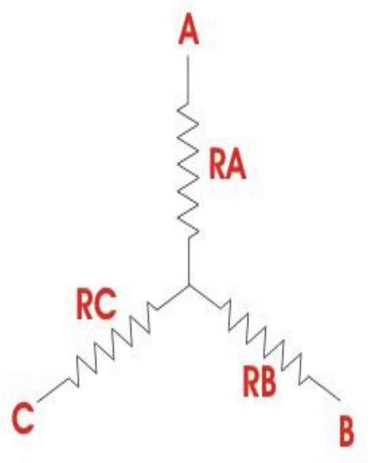


Fig 1.11 Star connection

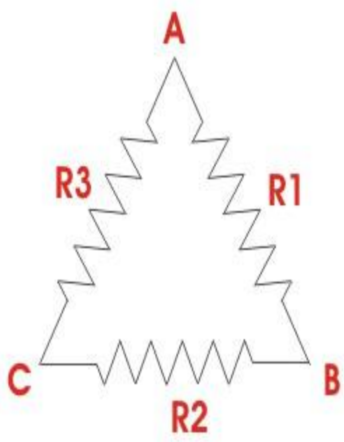


Fig 1.12 Delta connection

The transformation from star to equivalent delta is given by

$$R_3 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_A}$$

$$R_1 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_B}$$

$$R_2 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C}$$

Similarly the transformation from delta to equivalent star is given by

Similarly the transformation from delta to equivalent star is given by

$$R_A = \frac{R_3 R_1}{\mathbb{R}_1 + R_2 + R_3}$$

$$\boldsymbol{R_B = \frac{R_1 R_2}{R_1 + R_2 + R_3}}$$

$$\boldsymbol{R_c = \frac{R_2 R_3}{R_1 + R_2 + R_3}}$$

The application of these transformations are better explained during problems of network reduction.

AC FUNDAMENTALS

Generation of Sinusoidal AC Voltage:

Alternating voltage may be generated:
a) By rotating a coil in a magnetic field as shown in Fig 1.13a
b) By rotating a magnetic field within a stationary coil as shown in Fig 1.13b

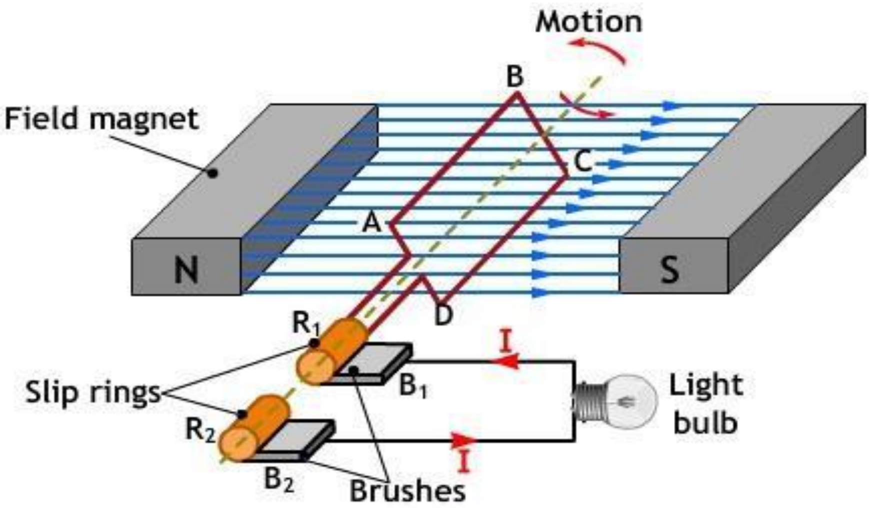


Fig 1.13a

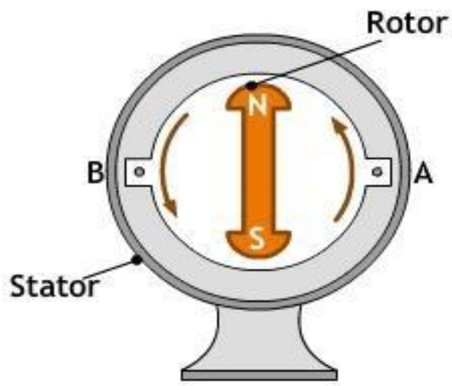


Fig1.13b

Fig 1.13 Generation of sinusoidal voltage

"Ineach case, the value of the alternating voltage generated depends upon the number of turns in the coil, the strength of the field and the speed at which the coil or magnetic field rotates."

The alternating voltage generated has regular changes in magnitude and direction. If a load resistance (e.g. a light bulb) is connected across this alternating voltage, an alternating current flows in the circuit is shown in Fig1.13a. When there is a reversal of polarity of the alternating voltage, the direction of current flow in the circuit also reverses.

Equation of Alternating E.M.F.

Let us take up the case of a rectangular coil of N turns rotating in the anticlockwise direction, with an angular velocity of ω radians per second in a uniform magnetic field as shown in Fig1.14 . let the time be measured from the instant of coincidence of the plane of the coil with the X-axis. At this instant maximum flux "ϕ_{max}" links with the coil. As the coil rotates, the flux linking with it changes and hence e.m.f. is induced in it. Let the coil turn through an angle θ in time, t" seconds, and let it assume the position as shown in Fig 1.14. Obviously $\theta = \omega t$.

When the coil is in this position, the maximum flux acting vertically downwards can be resolved into two components, each perpendicular to the other is shown in Fig 1.14, namely:

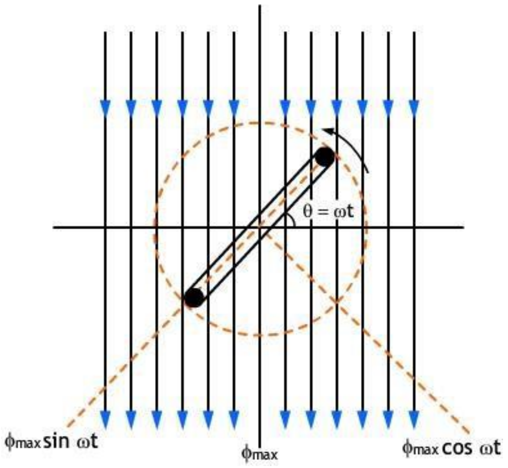


Fig 1.14 Resolving ϕ_{max} into parallel and perpendicular components

a) Component ϕ_{max} sinωt, parallel to the plane of the coil. This component does not induce e.m.f. as it is parallel to the plane of the coil.

b) Component ϕ_{max} cos ωt, perpendicular to theplaneof coil. This component induces e.m.f.in flux
the coil. ∴ linkages of coil at that instant (at θ=0) is

= No. of turns x flux linking

$= N \phi_{max} \cos \omega t$

As per Faraday's Laws of Electromagnetic induction, the e.m.f. induced in a coil is equal to the rate of change of flux linkages of the coil. So, instantaneous e.m.f. „e"induced in the coil at this instant is:

e = - (flux linkages)

= - $(N \phi_{max} \cos\omega t)$

$= -N \phi_{max} \frac{d}{dt} (\cos \omega t)$

$= -N \phi_{max} \omega (-\sin \omega t)$

$e = + N_{max} \sin \omega t$ volts . . . (1)

It is apparent from eqn.(1) that the value of "e" will be maximum (E_m), when the coil has rotated through 90^0 (as $\sin 90^0 = 1$)

$e = E_m \sin \omega t$ . . .(2)

We know that θ =ωt

∴ $e = E_m \sin \theta$

It is clear from this expression of alternating e.m.f. induced in the coil that instantaneous e.m.f. varies as the sin of the time angle (θ or ωt).

$\omega = 2\pi f$, where "f" is the frequency of rotation of the coil. Hence eqn (2) can be written as

$$e = E_m \sin 2\pi ft \quad \ldots (3)$$

then eqn.(3) may be re-written as $e = E_m \sin \omega t$, $\omega = 2\pi f$

so, the e.m.f. induced varies as the sine function of the time angle, ωt, and if e.m.f. induced is plotted against time, a curve of sine wave shape is obtained as shown in Fig.1.15. Such an e.m.f. is called sinusoidal when the coil moves through an angle of 2π radians.

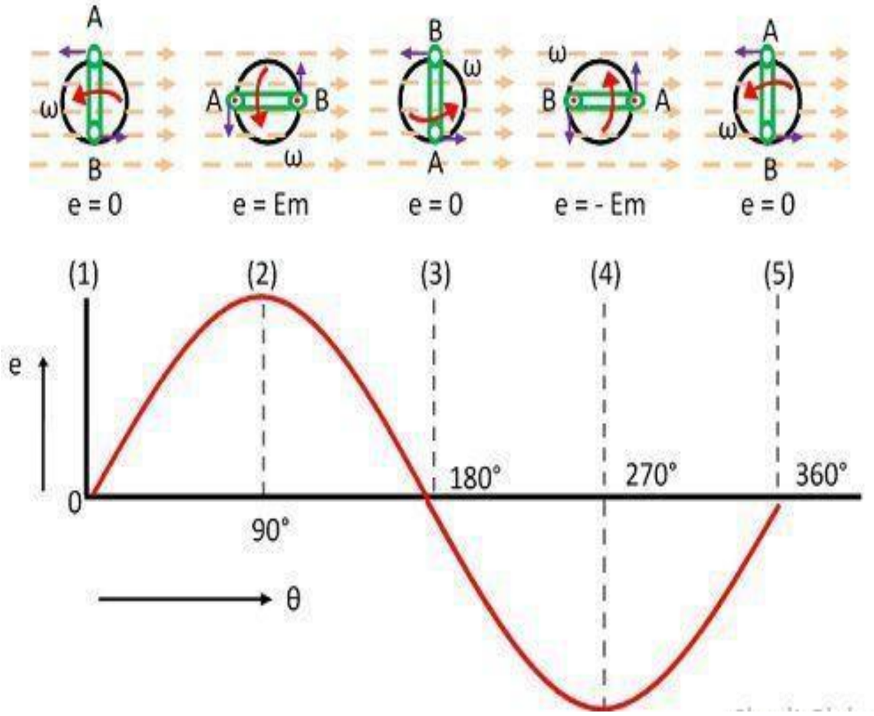


Fig 1.15 Graph depicting the production of EMF

Equation of Alternating Current

When an alternating voltage $e = E_m \sin \omega t$ is applied across a load, alternating current flows through the circuit which will also have a sinusoidal variation. The expression for the alternating current is given by:
$i = I_m \sin \omega t$
In this case the load is resistive (we shall see, later on, that if the load is inductive or capacitive, this current-equation is changed in time angle).

Important Definitions

Important terms/definitions, which are frequently used while dealing with a.c. circuits, are as given below:

- **Alternating quantity:** An alternating quantity is one which acts in alternate positive and negative directions, whose magnitude undergoes a definite series of changes in definite intervals of time and in which the sequence of changes while negative is identical with the sequence of changes while positive X-axis.

- **Instantaneous value:** The value of an alternating quantity at any instant is called instantaneous value. The instantaneous values of alternating voltages and current are represented by „e"and „I"respectively.

- **Alternation and cycle:** When an alternating quantity goes through one half cycle (complete set of +ve or –ve values) it completes an alternation, and when it goes through a complete set of +ve and –ve values, it is said to have completed one cycle.

- **Periodic Time and Frequency:** The time taken in seconds by an alternating quantity to complete one cycle is known as periodic time and is denoted by T.

  The number of cycles completed per second by an alternating quantity is known as frequency and is denoted by "f "in the SI system, the frequency is expressed in hertz.

  The number of cycles completed per second = f.

  Periodic Time T – Time taken in completing one cycle = 1/f sec

  $$\text{Or} \quad f = \frac{1}{T} \text{ Hertz}$$

  In India, the standard frequency for power supply is 50 Hz. It means that alternating voltage or current completes 50 cycles in one second.

- **Amplitude:** The maximum value, positive or negative, which an alternating quantity attains during one complete cycle, is called amplitude or peak value or maximum value. The amplitude of alternating voltage and current is represented by E_m and I_m respectively.

Different Forms of E.M.F. Equation

The standard form of an alternating voltage,

$$e = E_m \sin\theta = E_m \sin\omega t = E_m \sin 2\pi f\, t = E_m \sin\frac{2\pi}{T} t$$

on perusal of the above equations, we find that

a) The amplitude or peak value or maximum value of an alternating voltage is given by the coefficient of the sine of the time angle.

b) The frequency "f" is given by the coefficient of time divided by 2π.

Root-mean-square (R.M.S) Value:

The r.m.s. or effective value, of an alternating current is defined as that steady current which when flowing through a given resistance for a given time produces the same amount of heat as produced by the alternating current, when flowing through the same resistance for the same time.

Let us take two circuits with identical resistance, but one is connected to a battery and the other to sinusoidal voltage source. Wattmeters are employed to measure heat power in each circuit. The voltage applied to each circuit is so adjusted that the heat power produced in each circuit is the same. In this

Event the direct current I will be equal to $\frac{I_m}{\sqrt{2}}$, which is termed r.m.s. value of them sinusoidal current. The equation of an alternating current varying sinusoid ally is given by $i = I_m \sin\theta$.

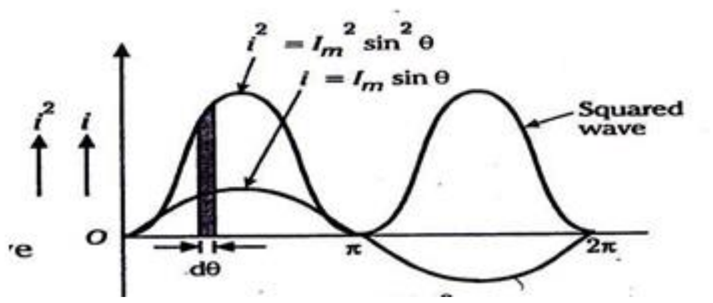


Fig 1.16 Graph of current and its square wave forms

Let us consider an elementary strip of thickness dθ in the first cycle of the squared wave, as shown in Fig 1.16.

Let i^2 be mid-ordinate of this strip.

Area of the strip = $i^2\, d\,\theta$

Area of first half-cycle of squared wave

$$= \int_0^{\pi} i^2\, d\theta$$

$$= \int_0^{\pi} (Im \sin\theta)^2\ d\theta$$

$$= \int_0^{\pi} I_m^2 \sin^2\theta \cdot d\theta$$

$$= \int_0^{\pi} I_m^2 \frac{(1-\cos 2\theta)\, d\theta}{2}$$

$$= \frac{I_m^2}{2}[(\pi-0)-(0-0)] \quad = \frac{I_m^2\pi}{2}$$

$$I = \sqrt{\frac{Area\ of\ the\ first\ half\ cycle}{Base}}$$

$$= \sqrt{\frac{I_m^2\pi}{2\,\Pi}}$$

$$= \frac{Im}{\sqrt{2}} = 0.707 I_m$$

= Hence, for a sinusoidal current,

R.M.S. value of current = 0.707 x maximum value of current.

Similarly, E = 0.707 E_m

Average Value

The arithmetical average of all the values of an alternating quantity over one cycle is called **average value.**

In the case of a symmetrical wave e.g. sinusoidal current or voltage wave, the positive half is exactly equal to the negative half, so that the average value over the entire cycle is zero. Hence, in this case, the average value is obtained by adding or integrating the instantaneous values of current over one alternation (half-cycle) only. The equation of a sinusoidally varying voltage is given by e = $E_m \sin\theta$.

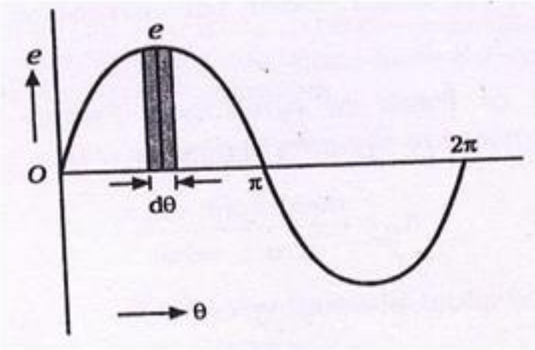


Fig 1.17 Graph showing average value

Let us take an elementary strip of thickness dθ in the first half-cycle as shown in Fig 1.17. let the mid-ordinate of this strip be "e".

Area of the strip = $e\,d\theta$

Area of first half-cycle

$$= \int_0^{\pi} Em \sin\theta \, d\theta$$

$$= Em \int_0^{\pi} \sin\theta \cdot d\theta$$

$$= E_m \left[-cos\theta\right]_0^{\pi} = 2Em$$

$\therefore$ Average value, $E_{av} = \frac{area\ of\ the\ half\ cycle}{base}$

Or $E_{av} = 0.637\ Em$

In a similar manner, we can prove that, for alternating current varying sinusoidally,

$I_{av} = 0.637\ I_m$

$\therefore$ **Average value of current = 0.637 x maximum value**

Form Factor and crest or peak or Amplitude Factor (K_f)

A definite relationship exists between crest value (or peak value), average value and r.m.s value of an alternating quantity.

1. **Form Factor:** The ratio of effective value (or r.m.s. value) to average value of an alternating quantity

(voltage or current) is called form factor, i.e.

$$\text{From Factor, } K_f = \frac{RMS\ value}{average\ value}$$

For sinusoidal alternating current

$$K_f = \frac{0.707\, I_m}{0.639\, I_m} = 1.11$$

For sinusoidal alternating voltage,

$$K_f = \frac{0.707\, I_m}{0.639\, I_m} = 1.11$$

Hence, the R.M.S. value (of current or voltage) is 1.11 times its average value.

2. Crest or Peak or Amplitude Factor (Ka): It is defined as the ratio of maximum value to the effective value (r.m.s. value) of an alternating quantity. i.e.,

$$Ka = \frac{maximum\ value}{average\ value}$$

For sinusoidal alternating current,

$$Ka = \frac{I_m}{0.639\, I_m} = 1.414$$

For sinusoidal alternating voltage,

$$Ka = \frac{V_m}{0.639\, V_m} = 1.414$$

Phasor Representation of Alternating Quantities

We know that an alternating voltage or current has sine waveform, and generators are designed to give e.m.f.s. with the sine waveforms. The method of representing alternating quantities continuously by equation giving instantaneous values (likee =Em sin ωt) is quite tedious. So, it is more convenient to represent a sinusoidal quantity by a phasor rotating in an anticlockwise direction as shown in Fig 1.18.

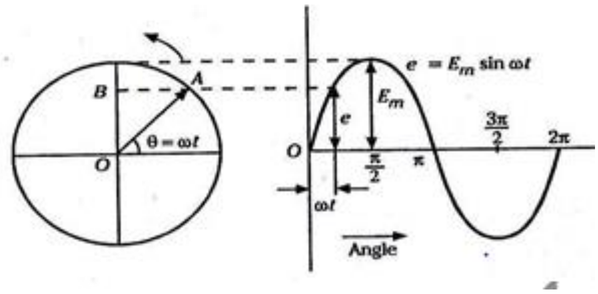


Fig 1.18 Graph showing phasor representation

While representing an alternating quantity by a phasor, the following points are to be kept in mind:

- The length of the phasor should be equal to the maximum value of the alternating quantity.
- The phasor should be in the horizontal position at the alternating quantity is zero and is increasing in the positive direction.
- The inclination of the line with respect to some axis of reference gives the direction of that quantity and an arrow-head placed at one end indicates the direction in which that quantity acts.
- The angular velocity in an anti-clockwise direction of the phasor should be such that it completes one revolution in the same time as taken by the alternating quantity to complete one cycle.

Consider phasor 0A, which represents the maximum value of the alternating e.m.f. and its angle with the horizontal axis gives its phase. Now, it will be seen that the projection of this phasor 0A on the vertical axis will give the instantaneous value of e.m.f.

$$0B = 0A \sin \omega t$$

Or $\quad e = 0A \sin wt$

$$= E_m \sin \omega t$$

Note: The term phasor is also known as vector.

Phase

An alternating voltage or current changes in magnitude and direction at every instant. So, it is necessary to know the condition of the alternating quantity at a particular instant. The location of the condition of the alternating quantity at any particular instant is called its phase is shown in Fig 1.19

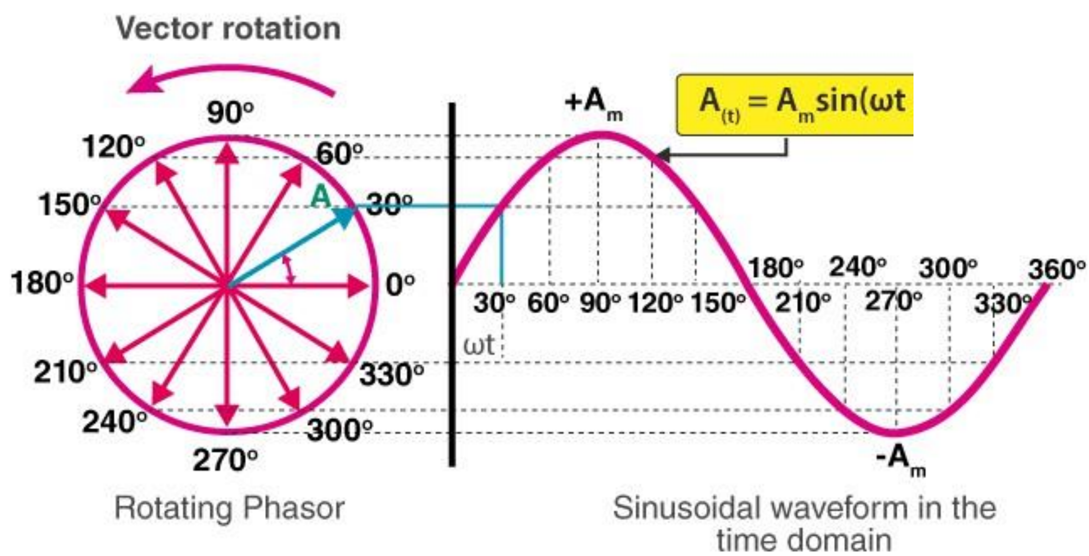


Fig 1.19 Phasor representation of AC quantity

Phase Difference (Lagging or Leading of Sinusoidal wave)

When two alternating quantities, say, two voltages or two currents or one voltage and one current are considered simultaneously, the frequency being the same, they may not pass through a particular point at the same instant.

One may pass through its maximum value at the instant when the other passes through a value other than its maximum one. These two quantities are said to have a phase difference. Phase difference is specified either in degrees or in radians.

The phase difference is measured by the angular difference between the points where the two curves cross the base or reference line in the same direction.

The quantity ahead in phase is said to lead the other quantity, whereas the second quantity is said to lag behind the first one. In Fig 1.20 below current I_1, represented by vector 0A, leads the current I_2, represented by vector 0B, by , or current I_2 lags behind the current I_1.

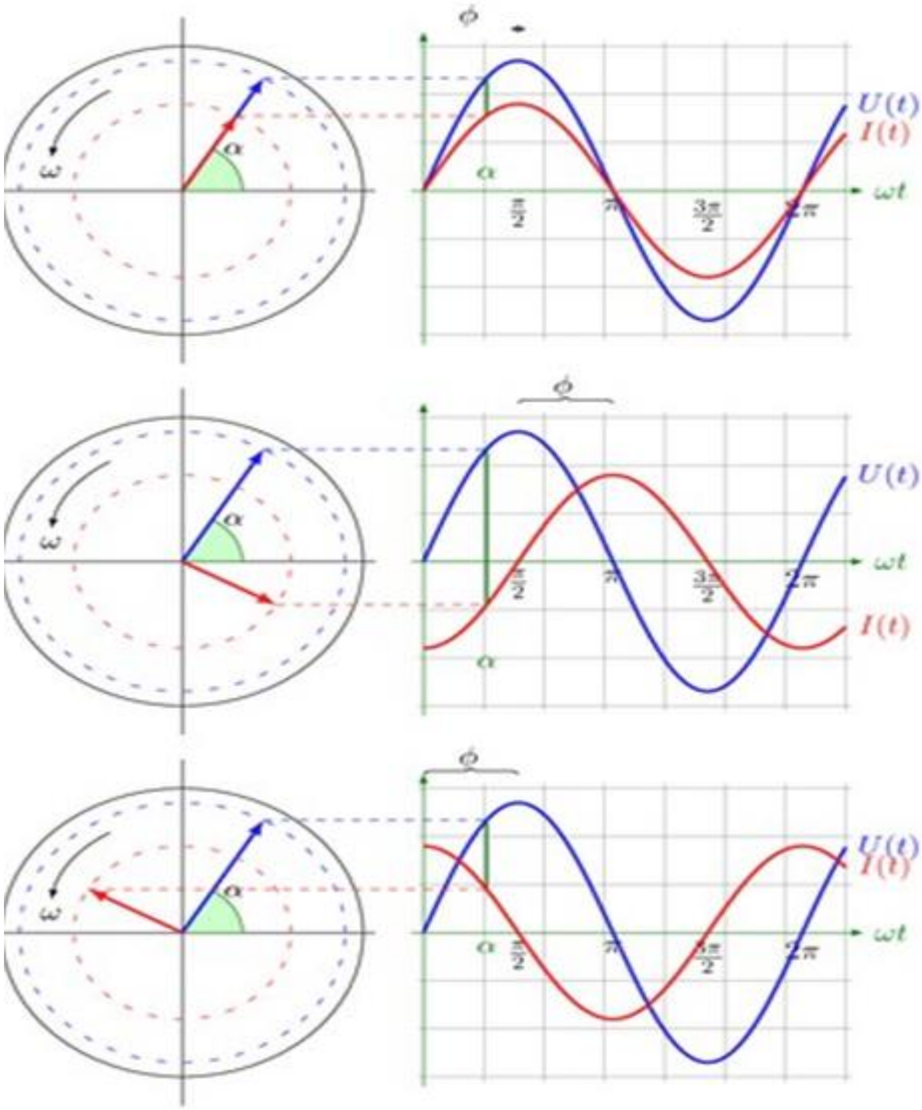


Fig 1.20 Phase Difference (Lagging or Leading of Sinusoidal wave)

The leading current I_1 goes through its zero and maximum values first and the current I_2goes through its zero and maximum values after time angle. The two waves representing these two currents. The two quantities are said to be in phase with each other if they pass through zero values at the same instant and rise in the same direction. However, if the two quantities pass through zero values at the same instant but rise in opposite. they are said to be in phase opposition i.e., the phase difference is 180^0. When the two alternating quantities have a phase difference of 90^0 or $\pi/2$ radians they are said to be in quadrature.

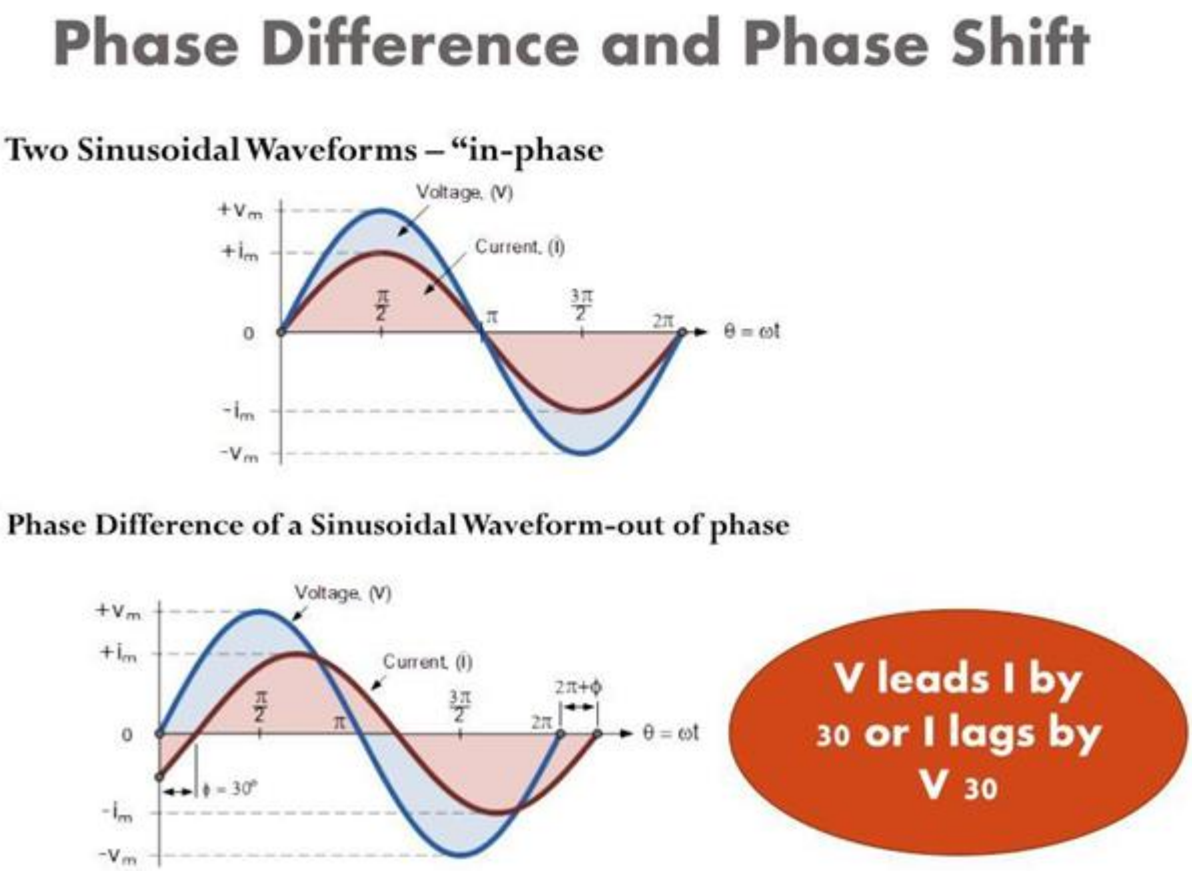


Fig 1.21 Phase difference of alternating quantity

Example 1 : An alternating current i is given by i = 141.4 sin 314t

Find i) The maximum value
ii) Frequency
iii) Time Period
iv) The instantaneous value when t=3ms

Solution

i) The maximum value — $i = 141.4 \sin 314t$
Maximum value Im=141.4 V

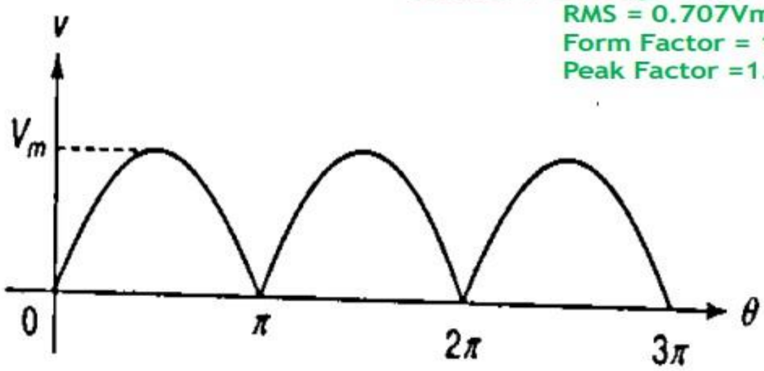
ii) Frequency — $\omega = 2\Pi f3 = 14$ rad/sec
$f = \omega / 2\Pi = 50$ Hz

iii) Time Period — $T = 1/f = 0.02$ sec

iv) The instantaneous value when t=3ms
$i = 141.4 \sin(314 \times 0.003) = 114.35A$

Example 2 : For the full wave rectified wave form shown, calculate
1. Average value
2. RMS
3. Form Factor
4. Peak Factor

Answer : Average Value = 0.637Vm
RMS = 0.707Vm
Form Factor = 1.11
Peak Factor =1.414



$v = V_m \sin \theta$ $\quad 0 < \theta < \pi$

Solution

1. Average value

$$V_{avg} = \frac{1}{\pi}\int_0^{\pi} v(\theta)\, d\theta = \frac{1}{\pi}\int_0^{\pi} V_m \sin\theta\, d\theta = \frac{V_m}{\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{V_m}{\pi}[1+1] = 0.637\, V_m$$

2. RMS

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} v^2(\theta)\, d\theta} = \sqrt{\frac{1}{\pi}\int_0^{\pi} V_m^2 \sin^2\theta\, d\theta} = \sqrt{\frac{V_m^2}{\pi}\int_0^{\pi} \sin^2\theta\, d\theta}$$

$$= \sqrt{\frac{V_m^2}{\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right) d\theta} = \sqrt{\frac{V_m^2}{\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi}} = \sqrt{\frac{V_m^2}{\pi}\left[\frac{\pi}{2} - \frac{\sin 2\pi}{4} - 0 + \frac{\sin 0}{4}\right]} = 0.707\, V_m$$

3.Form Factor

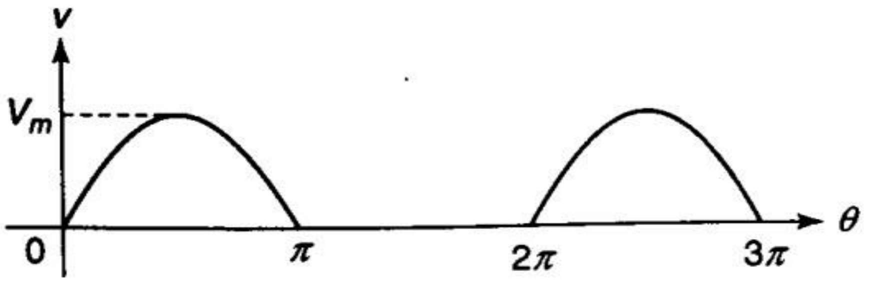
$$\text{Form Factor} = \frac{RMS}{AVERAGE} = \frac{0.707Vm}{0.637Vm} = 1.11$$

4.Peak Factor

$$\text{Peak Factor} = \frac{MAXIMUM}{VALUE_R} = \frac{V}{0.707Vm} = 1.414$$

Example 3: For the wave shown, calculate

1. Average value
2. RMS
3. Form Factor
4. Peak Factor



$v = V_m \sin\theta \quad 0 < \theta < \pi$

$= 0 \quad \pi < \theta < 2\pi$

Answer : Average Value = 0.318Vm
RMS = 0.5Vm
Form Factor = 1.11
Peak Factor =1.414

Solution

1. Average value

$$V_{avg} = \frac{1}{2\pi}\int_0^{2\pi} v(\theta)\, d\theta = \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin\theta\, d\theta + \int_{\pi}^{2\pi} 0\, d\theta\right]$$

$$= \frac{1}{2\pi}\int_0^{\pi} V_m \sin\theta\, d\theta = \frac{V_m}{2\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{V_m}{2\pi}[1+1] = 0.318\, V_m$$

2. RMS

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\theta)\, d\theta} = \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi} V_m^2 \sin^2\theta\, d\theta + \int_{\pi}^{2\pi} 0\, d\theta\right]}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2\theta\, d\theta} = \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi}\sin^2\theta\, d\theta} = \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right) d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi}} = \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\pi}{2} - \frac{\sin 2\pi}{4} - 0 + \frac{\sin 0}{4}\right]} = 0.5\, V_m$$

3.Form Factor

$$\text{Form Factor} = \frac{R}{AVERAG} = \frac{0.5\,V}{0.318Vm} = 1.571$$

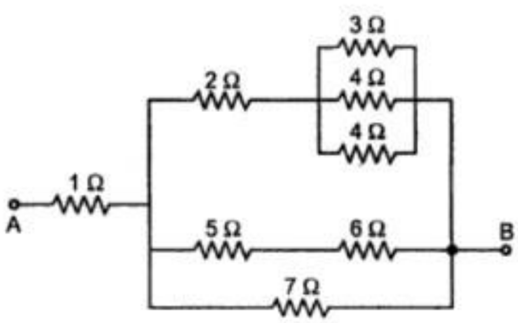
4.Peak Factor

$$\text{Peak Factor} = \frac{MAXIMUMVALUE}{R} \quad \frac{V}{0.2V} = 2$$

=

NUMERICALS ON DC CIRCUITS

Find the equivalent resistance between the two points A and B



Solution : Identify combinations of series and parallel resistances.

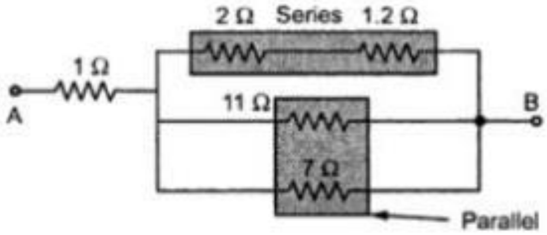
The resistances 5 Ω and 6 Ω are in series, as going to carry same current.

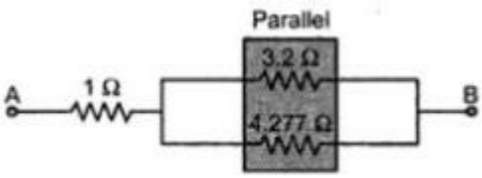
So equivalent resistance is 5 + 6 = 11 Ω

While the resistances 3 Ω , 4 Ω, and 4 Ω are in parallel, as voltage across them same but current divides.

∴ Equivalent resistance is, $\frac{1}{R} = \frac{1}{3}+\frac{1}{4}+\frac{1}{4} = \frac{10}{12}$

∴ $R = \frac{12}{10} = 1.2\ \Omega$





Now again 1.2 Ω and 2 Ω are in series so equivalent resistance is 2 + 1.2 = 3.2 Ω while 11 Ω and 7 Ω are in parallel.

Using formula $\frac{R_1 R_2}{R_1+R_2}$ equivalent resistance is $\frac{11\times 7}{11+7} = \frac{77}{18} = 4.277\Omega$.

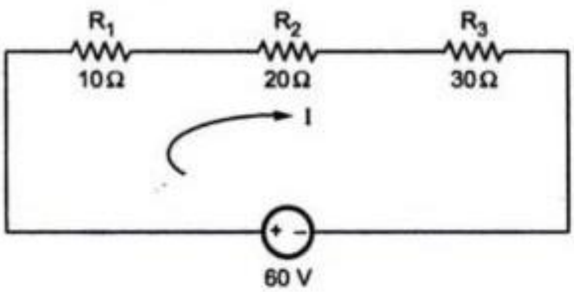
Replacing the respective combinations redraw the circuit

Now 3.2 and 4.277 are in parallel.

$\therefore$ Replacing them by $\frac{3.2\times 4.277}{3.2+4.277} = 1.8304\ \Omega$

$\therefore$ $R_{AB} = 1 + 1.8304 = \mathbf{2.8304}\ \Omega$

Find the voltage across the three resistances



Solution : $I = \frac{V}{R_1+R_2+R_3}$ ... series circuit

$= \frac{60}{10+20+30} = 1\text{ A}$

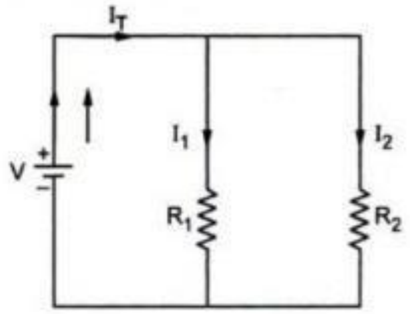
$\therefore$ $V_{R1} = IR_1 = \frac{V\times R_1}{R_1+R_2+R_3} = 1\times 10 = \mathbf{10\ V}$

$\therefore$ $V_{R2} = IR_2 = \frac{V\times R_2}{R_1+R_2+R_3} = 1\times 20 = \mathbf{20\ V}$

and $V_{R3} = IR_3 = \frac{V\times R_3}{R_1+R_2+R_3} = 1\times 30 = \mathbf{30\ V}$

Find the magnitudes of total current, current through R_1 and R_2 if,

$R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, and $V = 50$ V.



Solution : The equivalent resistance of two is,

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{10 \times 20}{10 + 20} = 6.67\ \Omega$$

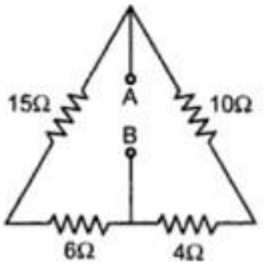
$$\therefore \quad I_T = \frac{V}{R_{eq}} = \frac{50}{6.67} = \mathbf{7.5\ A}$$

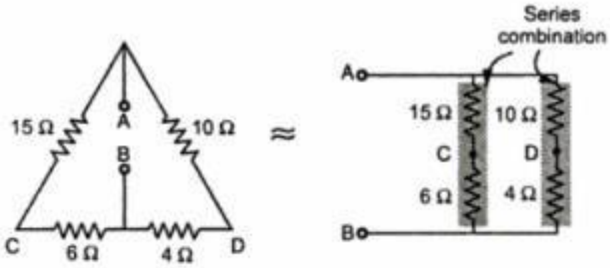
As per the current distribution in parallel circuit,

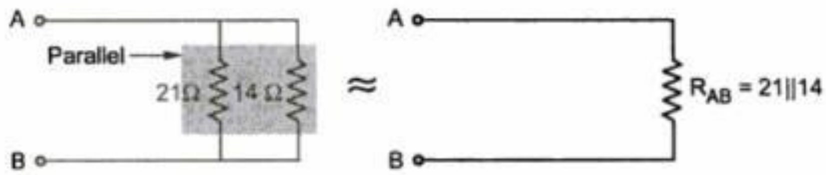
$$I_1 = I_T\left(\frac{R_2}{R_1 + R_2}\right) = 7.5 \times \left(\frac{20}{10 + 20}\right)$$

$$= \mathbf{5\ A}$$

Find equivalent resistance between points A-B.

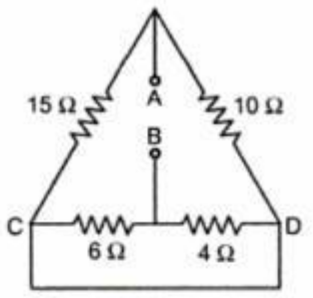




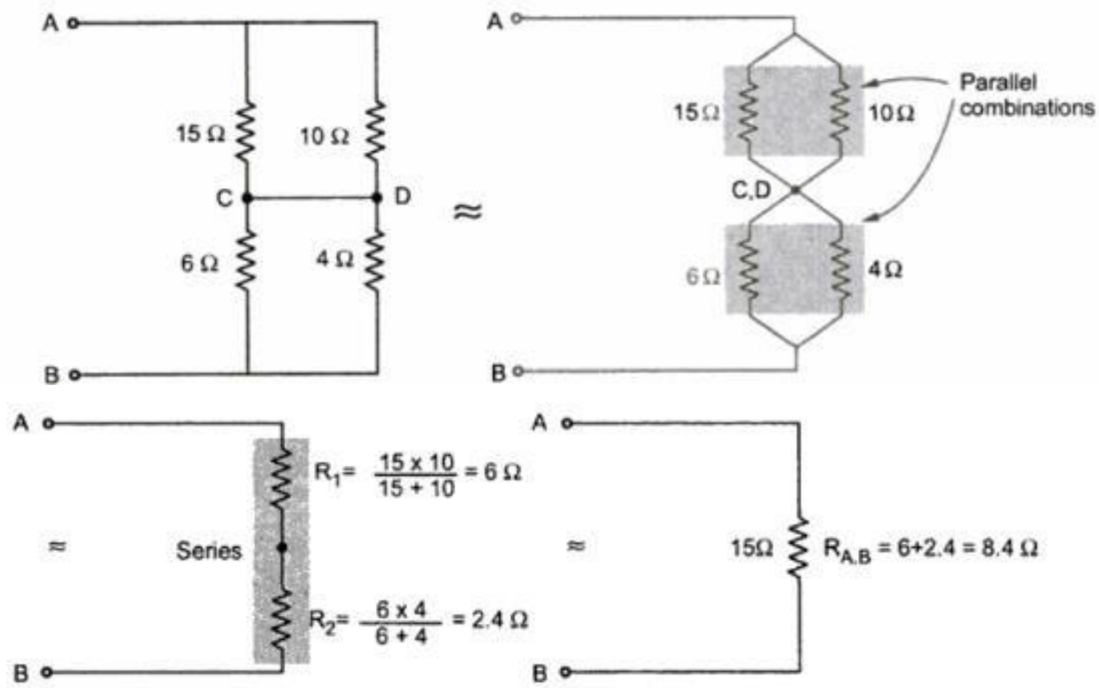


$$R_{AB} = \frac{21 \times 14}{21 + 14} = 8.4\ \Omega$$

Find equivalent resistance between points A-B.

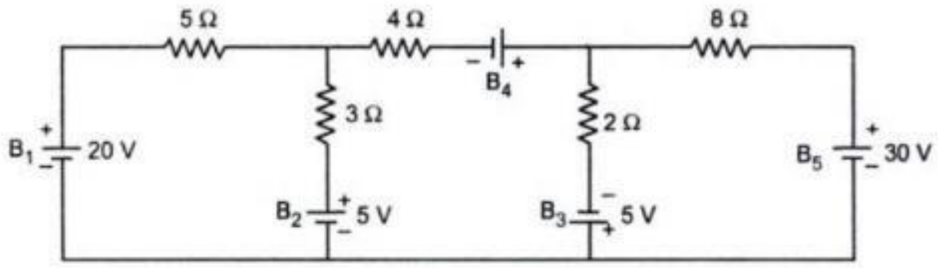


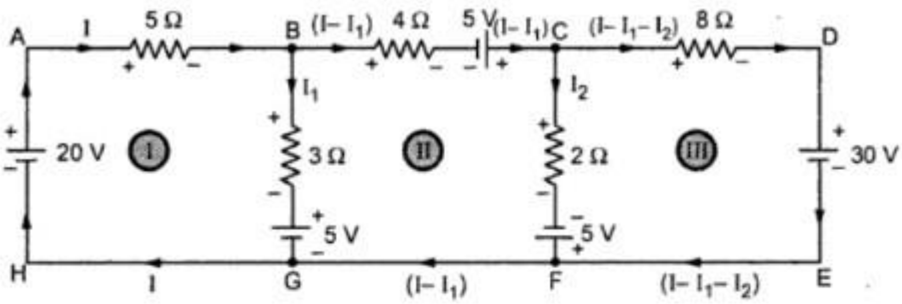
Solution : Redraw the circuit,



$$\therefore \qquad R_{AB} = 8.4\ \Omega$$

Determine the current supplied by each battery in the circuit shown in by using Kirchhoff's laws.





Applying KVL to various loops :

For loop 1, ABGHA

$$-5\,I - 3\,I_1 - 5 + 20 = 0 \quad \text{i.e.} \quad +5\,I + 3\,I_1 = 15 \qquad \ldots(1)$$

For loop 2, BCFGB

$$-4\,(I - I_1) + 5 - 2\,I_2 + 5 + 5 + 3\,I_1 = 0 \quad \text{i.e.} \quad 4I - 7I_1 + 2I_2 = 15 \qquad \ldots(2)$$

For loop 3, CDEFC

$$-8\,(I - I_1 - I_2) - 30 - 5 + 2I_2 = 0 \quad \text{i.e.} \quad -8I + 8I_1 + 10I_2 = 35 \qquad \ldots(3)$$

Solving (1), (2) and (3)

$\therefore \quad I = 2.558$ A, $\quad I_1 = 0.7357$ A, $\quad I_2 = 4.9581$ A

Hence the current supplied by various batteries can be calculated as below :

Current supplied by $B_1 = I =$ **2.558 A**

Current supplied by $B_2 = I_1 =$ **0.7357 A**

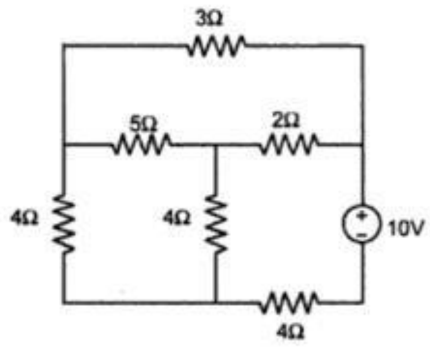
Current supplied by $B_3 = I_2 =$ **4.9581 A**

Current supplied by $B_4 = (I - I_1) = (2.558 - 0.7357) =$ **1.8223 A**

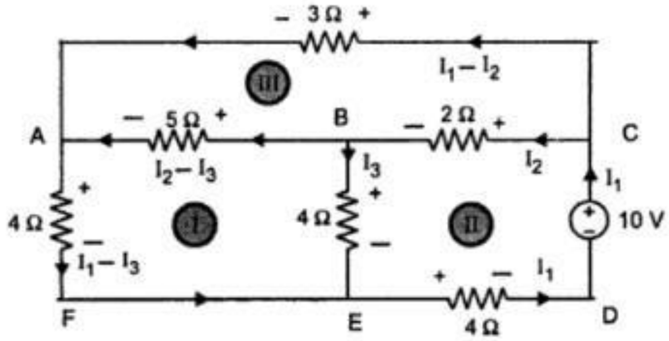
Current supplied by $B_5 = (I - I_1 - I_2) = (2.558 - 0.7357 - 4.9581)$

$=$ **− 3.1358 A** ... − ve sign means opposite direction

Using Kirchhoff's laws, calculate the current delivered by the battery



Solution : The various branch currents are shown



Consider loop ABEFA,

$$+5(I_2 - I_3) - 4I_3 + 4(I_1 - I_3) = 0 \quad \text{i.e. } 4I_1 + 5I_2 - 13I_3 = 0 \qquad \ldots (1)$$

Consider loop BCDEB,

$$+2I_2 - 10 + 4I_1 + 4I_3 = 0 \quad \text{i.e. } 4I_1 + 2I_2 + 4I_3 = 10 \qquad \ldots (2)$$

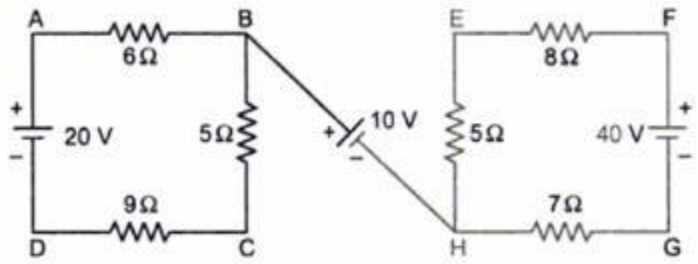
Consider loop ABCA,

$$+5(I_2 - I_3) + 2I_2 - 3(I_1 - I_2) = 0 \quad \text{i.e. } -3I_1 + 10I_2 - 5I_3 = 0 \qquad \ldots (3)$$

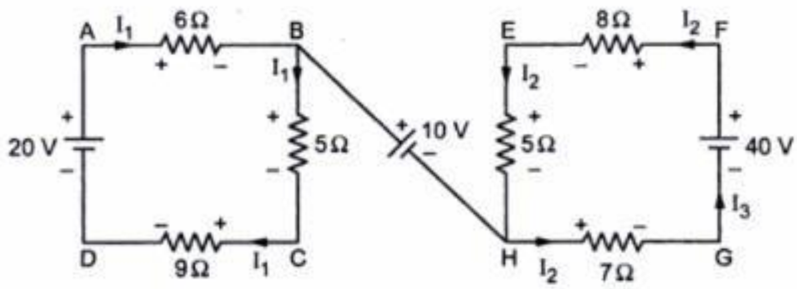
Using Cramer's rule, $I_1 =$ **1.3852 A**

This is the current delivered by the battery.

Find the V_{CE} and V_{AG} for the circuit



Solution : Assume the two currents as shown

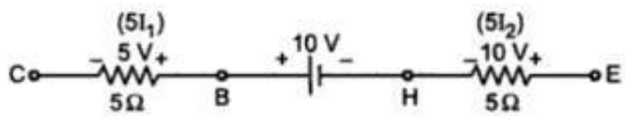


Applying KVL to the two loops,

$$-6\,I_1 - 5\,I_1 - 9\,I_1 + 20 = 0 \quad \text{and} \quad -8\,I_2 - 5\,I_2 - 7\,I_2 + 40 = 0$$

$$\therefore \qquad I_1 = 1 \text{ A} \;\text{ and }\; I_2 = 2 \text{ A}$$

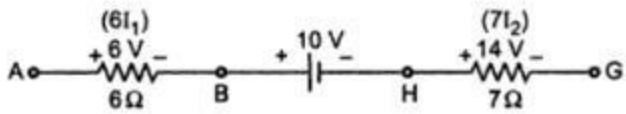
i) Trace the path C-E,



$$\therefore \qquad V_{CE} = -5 \text{ V}$$

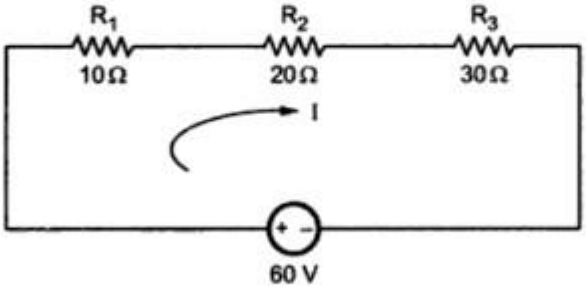
$$= \mathbf{5\ V\ with\ C\ negative}$$

ii) Trace the path A-G,



$$\therefore \qquad V_{AG} = \mathbf{30\ V\ with\ A\ positive}$$

Find the voltage across the three resistances :



$$I = \frac{V}{R_1 + R_2 + R_3}$$

$$= \frac{60}{10+20+30} = 1\ A$$

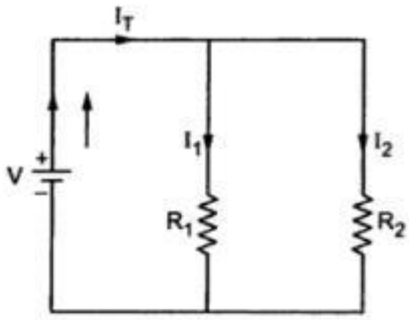
$$V_{R1} = IR_1 = \frac{V \times R_1}{R_1 + R_2 + R_3} = 1 \times 10 = \mathbf{10\ V}$$

$$V_{R2} = IR_2 = \frac{V \times R_2}{R_1 + R_2 + R_3} = 1 \times 20 = \mathbf{20\ V}$$

$$V_{R3} = IR_3 = \frac{V \times R_3}{R_1 + R_2 + R_3} = 1 \times 30 = \mathbf{30\ V}$$

Find the magnitudes of total current, current through R_1 and R_2 if,

$R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, and $V = 50$ V.



Solution : The equivalent resistance of two is,

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{10 \times 20}{10+20} = 6.67\ \Omega$$

$$I_T = \frac{V}{R_{eq}} = \frac{50}{6.67} = \mathbf{7.5\ A}$$

As per the current distribution in parallel circuit,

$$I_1 = I_T\left(\frac{R_2}{R_1 + R_2}\right) = 7.5 \times \left(\frac{20}{10+20}\right)$$

$$= \mathbf{5\ A}$$

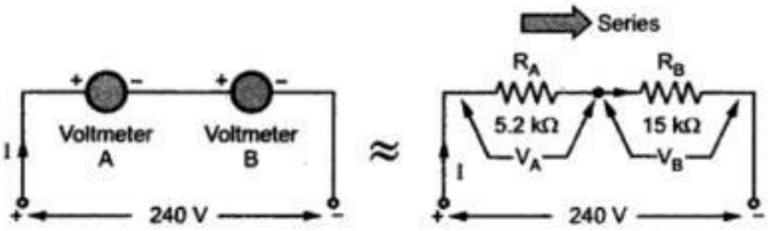
and $$I_2 = I_T\left(\frac{R_1}{R_1+R_2}\right) = 7.5\times\left(\frac{10}{10+20}\right)$$

$$= 2.5 \text{ A}$$

It can be verified that $I_T = I_1 + I_2$

Two voltmeters A and B, having resistances of 5.2 kΩ and 15 kΩ respectively are connected in series across 240 V supply. What is the reading on each voltmeter ?

Solution : The arrangement is shown



$\therefore$ $$R_{eq} = R_A + R_B = 5.2 + 15 = 20.2 \text{ k}\Omega$$

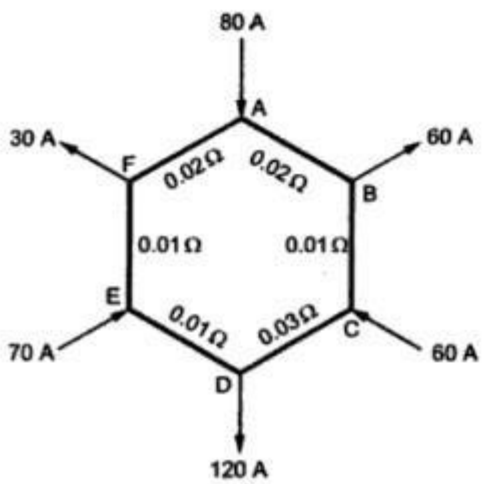
$\therefore$ $$I = \frac{V}{R_{eq}} = \frac{240}{20.2\times10^3} = 0.01188 \text{ A}$$

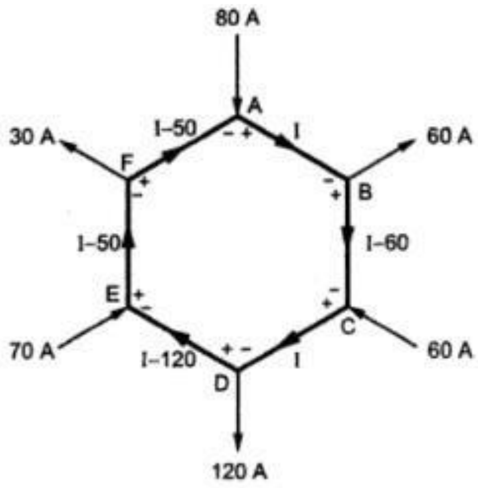
According to Ohm's law, $$V_A = I\times R_A = 0.01188\times5.2\times10^3 = \mathbf{61.7821\ V}$$

and $$V_B = I\times R_B = 0.01188\times15\times10^3 = \mathbf{178.2179\ V}$$

Thus reading on voltmeter A is **61.7821 V** and that on B is **178.2179 V**.

Find the current in all the branches of the network shown in the





Applying KVL to the loop ABCDEFA,

$$-I \times 0.02 - (I-60) \times 0.01 - I \times 0.03 - (I-120) \times 0.01 - (I-50) \times 0.01 - (I-80) \times 0.02 = 0$$

$$\therefore\ -I\,[0.02 + 0.01 + 0.3 + 0.01 + 0.01 + 0.02] + 0.6 + 1.2 + 0.5 + 1.6 = 0$$

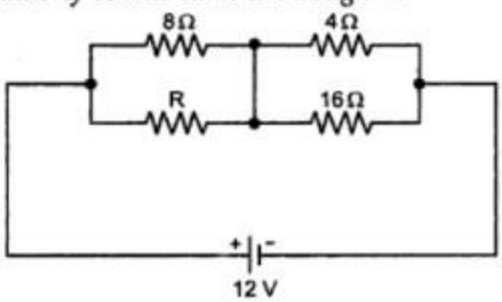
$$\therefore \qquad -0.1\,I + 3.9 = 0$$

$$\therefore \qquad I = \mathbf{39\ A}$$

Hence the various branch currents are,

Branch	Current	Direction
AB	39 A	from A to B
BC	– 21 A	from C to B
CD	39 A	from C to D
DE	– 81 A	from E to D
EF	– 11 A	from F to E
FA	– 41 A	from A to F

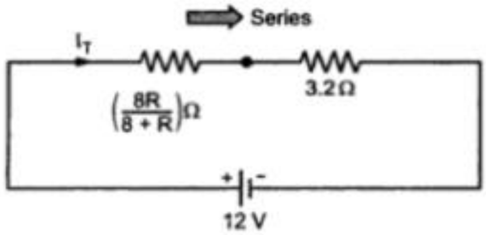
If the total power dissipated in the circuit 18 watts, find the value of R and current through it.



Solution : The resistances 4 Ω, 16 Ω are in parallel and 8 Ω, R Ω in parallel hence,

$$\therefore \quad 4 \,||\, 16 = \frac{4\times 16}{4+16} = 3.2\,\Omega \quad \text{and} \quad 8\,||\,R = \frac{8R}{8+R}\,\Omega$$

The circuit can be reduced as shown in the Fig. 1.31 (a).



$$I_T = \frac{12}{\left(\frac{8R}{8+R}\right)+3.2}$$

$$\therefore \quad I_T = \frac{12(8+R)}{8R+3.2(8+R)} \qquad \text{...(1)}$$

The total power dissipated is 18 W.

$$\therefore \quad P_T = V\times I_T \quad \text{i.e.} \quad 18 = 12\times I_T$$

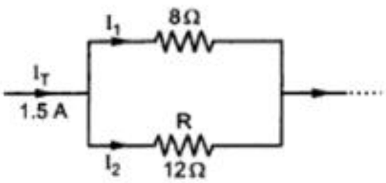
$$\therefore \quad I_T = 1.5\text{ A} \qquad \text{...(2)}$$

$$\frac{12(8+R)}{8R+3.2(8+R)} = 1.5 \qquad \text{...equating (1) and (2)}$$

$$\therefore \quad 96+12R = 12R+38.4+4.8R$$

$$\therefore \quad R = \mathbf{12\,\Omega}$$

Consider the parallel combination of 8 Ω and R =12 Ω. Applying **current division** in parallel circuit, we can find current through R = 12 Ω.

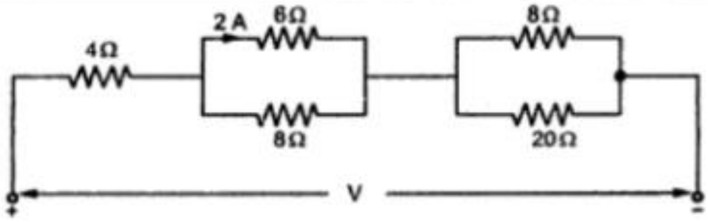


$$I_{12\Omega} = I_2 = I_T\times\left(\frac{8}{8+12}\right)$$

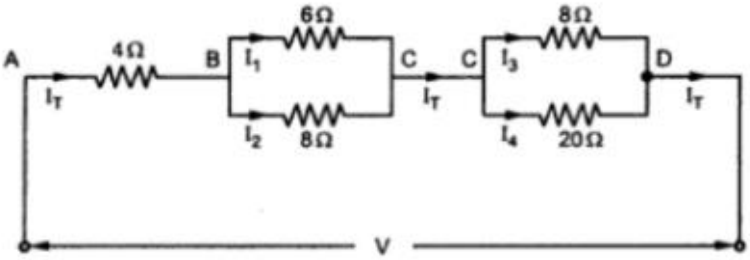
$$= \frac{1.5\times 8}{20}$$

$$= \mathbf{0.6\text{ A}} \qquad \text{... Current through R}$$

The current in the 6 Ω resistance of the network shown in the is 2 A. Determine the currents in all the other resistances and the supply voltage V.



Solution : The various currents are shown



Now $\quad I_1 = 2\text{ A} \quad$ given

Hence drop across 6 Ω resistance is,

$$V_6\,\Omega = I_1 \times R = 2 \times 6 = 12\text{ V}$$

Now 8 Ω resistance is in parallel with 6 Ω. **Hence drop across 8 Ω is also as that of 6 Ω.**

$\therefore \quad V_8\,\Omega = 12\ V$

but $\quad V_8\,\Omega = I_2 \times 8 \quad$ i.e. $\; 12 = I_2 \times 8$

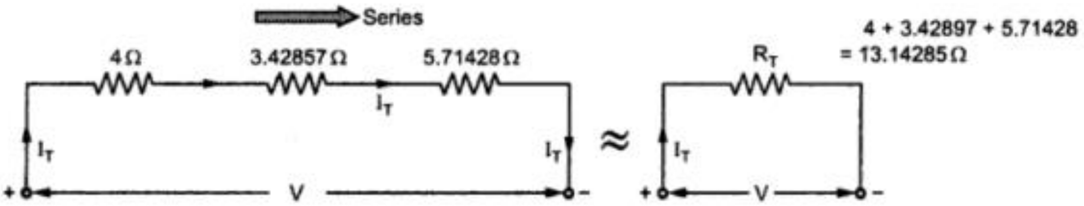
$\therefore \quad I_2 = 1.5\text{ A}$

Hence total current I_T is, $\quad I_T = I_1 + I_2 = 2 + 1.5 = 3.5\text{ A}$

Now $\quad 6 \,||\, 8 = \dfrac{6 \times 8}{6+8} = 3.42857\ \Omega$

and $\quad 8 \,||\, 20 = \dfrac{8 \times 20}{8+20} = 5.71428\ \Omega$

The circuit reduces to,



$\therefore \quad V = I_T \times R_T = 3.5 \times 13.14285 = \mathbf{46\ V} \quad$ ... Supply voltage

To find the currents I_3 and I_4, apply **current distribution** in parallel circuit,

$$\therefore \quad I_3 = I_{8\Omega} = I_T \times \left(\frac{20}{8+20}\right) = \frac{3.5 \times 20}{28} = \mathbf{2.5\ A}$$

$$\text{and} \quad I_4 = I_{20\Omega} = I_T \times \left(\frac{8}{8+20}\right) = \frac{3.5 \times 8}{28} = \mathbf{1\ A}$$

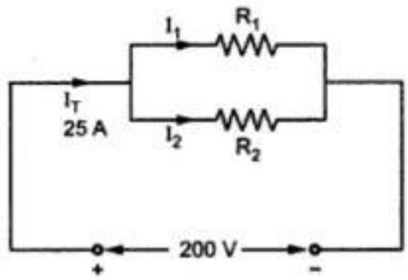
Two coils are connected in parallel and a voltage of 200 V is applied between the terminals. The total current taken is 25 A and power dissipated in one of the resistances is 1500 W. Calculate the resistances of two coils.

Solution : The arrangement is shown

Let power dissipated in resistance R_1 be, 1500 W.

$\therefore \quad P_1 = I_1^2 R_1 \quad$ as $P = I^2 R$

$\therefore \quad 1500 = I_1^2 R_1 \quad \ldots(1)$



Now the voltage across both the parallel resistances is same equal to supply voltage of 200 V.

$\therefore \quad V = I_1 R_1 = I_2 R_2$

$\therefore \quad I_1 R_1 = I_2 R_2 = 200 \quad \ldots(2)$

Substituting in (1), $\quad 1500 = I_1.(I_1 R_1) = I_1(200)$

$\therefore \quad I_1 = \frac{1500}{200} = 7.5 \text{ A}$

Now $\quad 1500 = I_1^2 R_1$

$\therefore \quad R_1 = \frac{1500}{(7.5)^2} = 26.67 \ \Omega$

$I_T = I_1 + I_2$

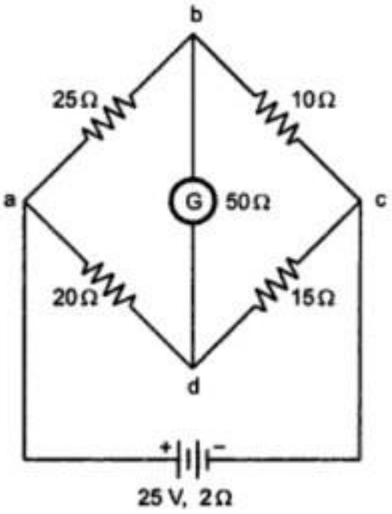
$\therefore \quad 25 = 7.5 + I_2$

$\therefore \quad I_2 = 17.5 \text{ A}$

but $\quad I_2 R_2 = 200$

$\therefore \quad R_2 = \frac{200}{17.5} = 11.43 \ \Omega$

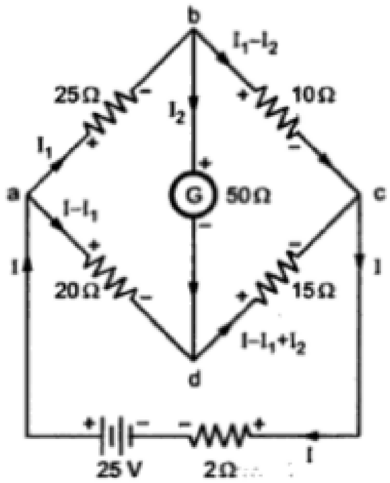
Using Kirchhoff's laws, find the current flowing through the galvanometer G in the Wheatstone bridge network shown



Solution : **Step 1 :** The circuit diagram is given.

Step 2 : Mark the various branch currents.

Step 3 : Mark the various polarities for the drops across various resistances due to branch currents. This is shown



Step 4 : Apply KVL to the various loops.

Loop abda, $-25\,I_1 - 50\,I_2 + 20(I - I_1) = 0$

$\therefore \quad 20\,I - 45\,I_1 - 50\,I_2 = 0 \quad \ldots(1)$

Loop bcdb, $-10(I_1 - I_2) + 15(I - I_1 + I_2) + 50I_2 = 0$

$\therefore \quad 15\,I - 25\,I_1 + 75\,I_2 = 0 \quad \ldots(2)$

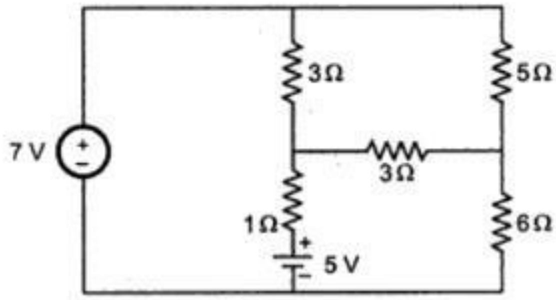
Loop adca, $-20(I - I_1) - 15(I - I_1 + I_2) - 2I + 25 = 0$

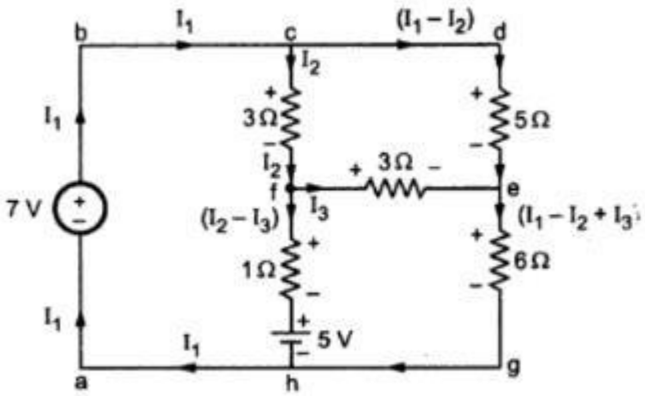
$\therefore \quad -37\,I + 35\,I_1 - 15\,I_2 = -25 \quad \ldots(3)$

$\therefore \quad I_2 = \quad -0.04874\text{ A} = \mathbf{-48.746\text{ mA}}$

This is the current through galvanometer, flowing **upwards** as assumed direction is wrong as indicated by negative sign.

find the current supplied by 7 V source.





Apply KVL to the various loops,

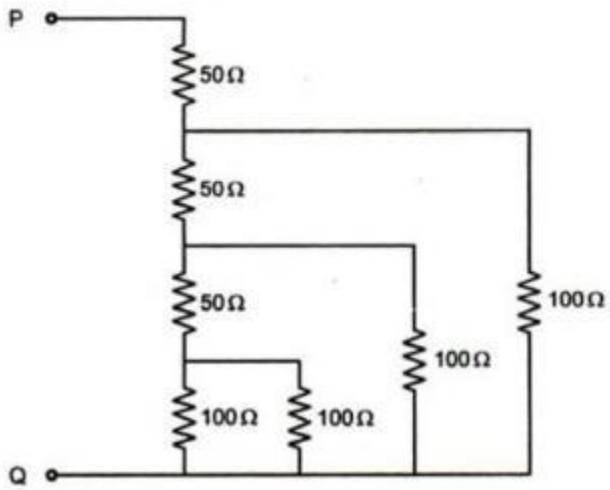
Loop abcfha, $-3I_2 - (I_2 - I_3) - 5 + 7 = 0$ i.e. $-4I_2 + I_3 = -2$ ...(1)

Loop cdefc, $-5(I_1 - I_2) + 3I_3 + 3I_2 = 0$ i.e. $-5I_1 + 8I_2 + 3I_3 = 0$ ...(2)

Loop feghf, $-3I_3 - 6(I_1 - I_2 + I_3) + 5 + (I_2 - I_3) = 0$ i.e. $-6I_1 + 7I_2 - 10I_3 = -5$ ...(3)

$I_1 = \mathbf{1.0596\ A} \uparrow$

Find the equivalent resistance across the terminals PQ of the network



Solution : Replacing the lowest parallel combination of 100 Ω we get,

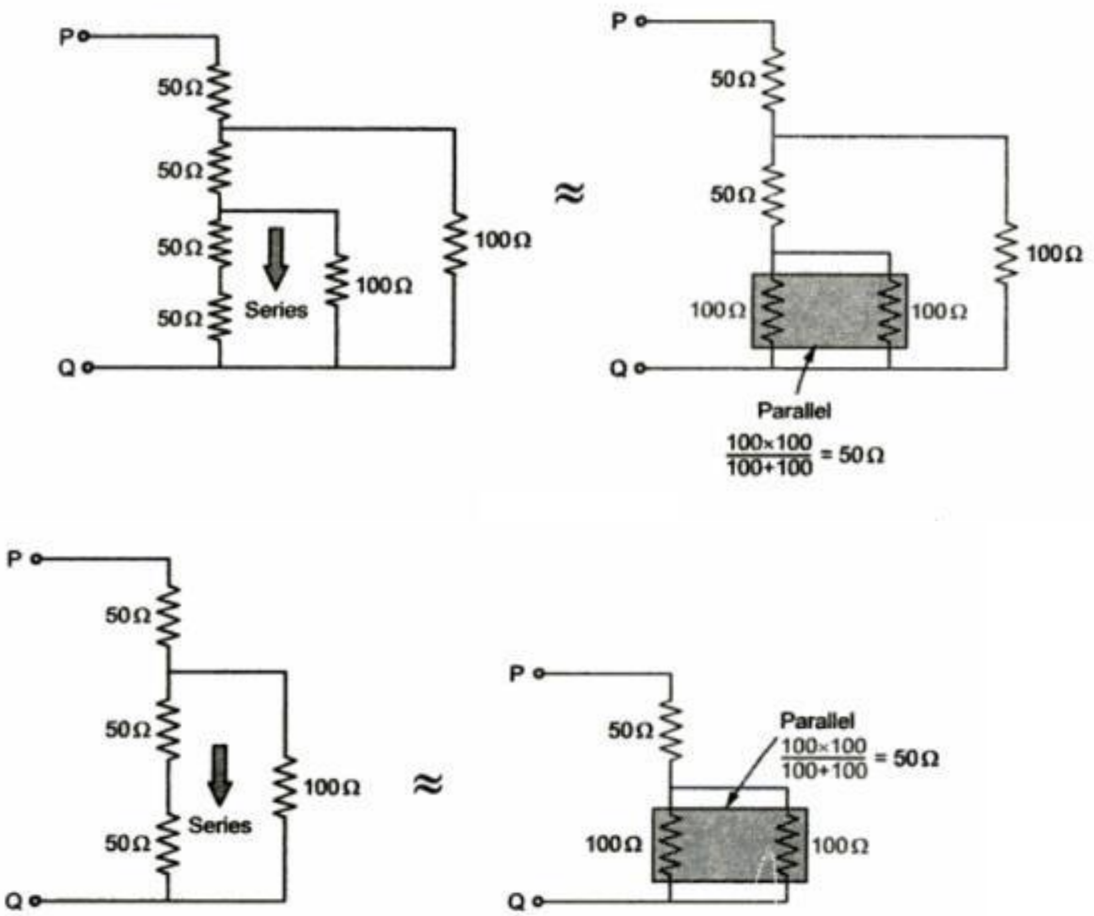
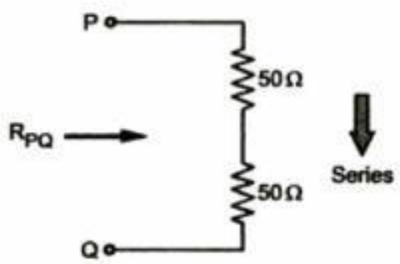
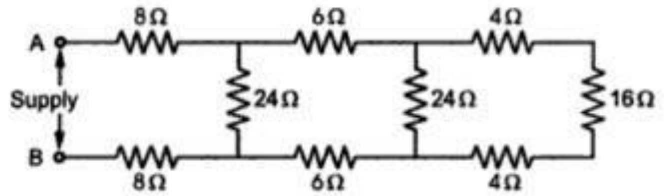


Fig. 1.43 (b)



$R_{PQ} = 50 + 50 = 100\ \Omega$

Calculate the equivalent resistance across the supply terminals in the network



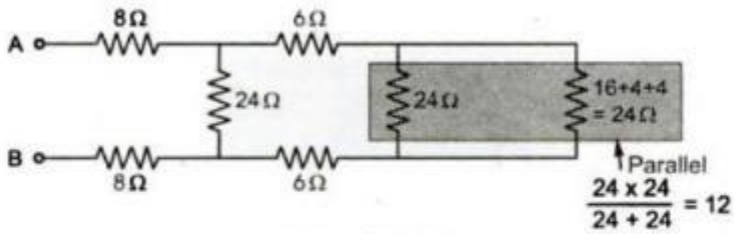


Fig. 1.42 (a)

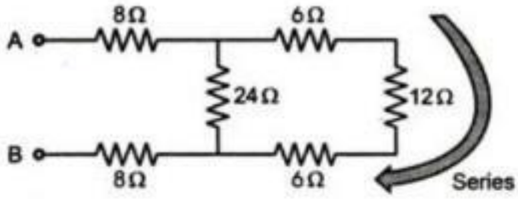


Fig. 1.42 (b)

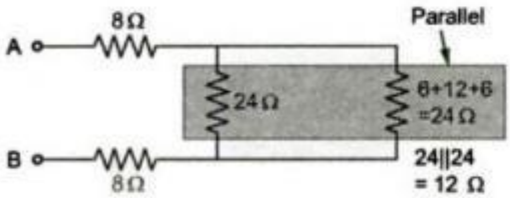


Fig. 1.42 (c)

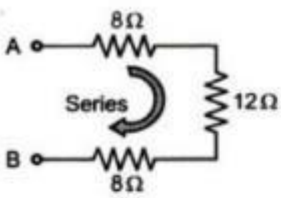
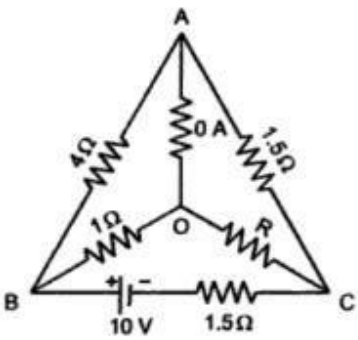


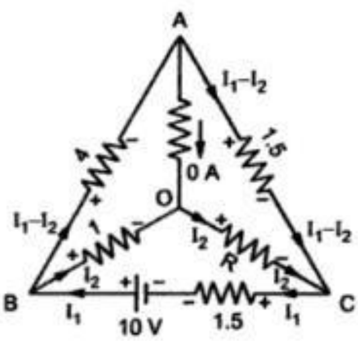
Fig. 1.42 (d)

$$R_{AB} = 8 + 12 + 8 = 28\ \Omega$$

Find the value of R and the current flowing through it in the network when the current in the branch OA is zero.



Solution : **Step 1 :** The circuit diagram is given.



Loop AOCA, $-1.5(I_1 - I_2) + I_2 R + 0 = 0$

$\therefore \quad -1.5I_1 + I_2(1.5 + R) = 0 \quad \ldots(1)$

Loop AOBA, $0 + I_2 \times 1 - 4(I_1 - I_2) = 0$

$\therefore \quad -4I_1 + 5I_2 = 0 \quad \ldots(2)$

Loop BOCB, $-I_2 \times 1 - I_2 R - 1.5I_1 + 10 = 0$

$\therefore \quad -1.5I_1 - I_2(1 + R) = -10 \quad \ldots(3)$

From (2), $I_1 = \frac{5}{4} I_2 = 1.25 I_2 \quad \ldots(4)$

Substituting in (1) we get,

$$-1.5(1.25 I_2) + I_2(1.5 + R) = 0$$

$\therefore \quad -1.875 I_2 + I_2(1.5 + R) = 0$

$\therefore \quad -1.875 I_2 = -I_2(1.5 + R)$

$\therefore \quad 1.5 + R = 1.875$

$\therefore \quad R = \mathbf{0.375\ \Omega}$

Substituting in (3) we get,

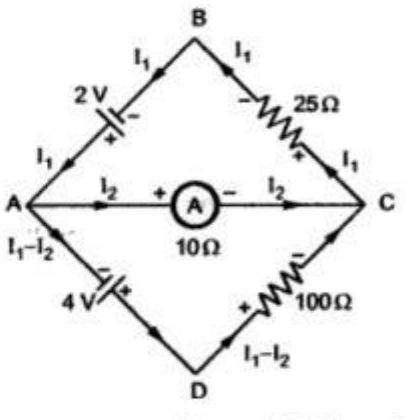
$$-1.5(1.25 I_2) - I_2(1 + 0.375) = -10$$

$\therefore \quad -3.25 I_2 = -10$

$I_2 = \mathbf{+3.0769\ A}$ ... Current through R

A network ABCD is made up as follows :

AB has a cell of 2V and negligible resistance, with the positive terminal connected to A; BC is a resistor of 25 Ω ; CD is a resistor of 100 Ω; DA is a battery of 4 V and negligible resistance with positive terminal connected to D; AC is a milliammeter of resistance 10 Ω. Calculate the reading on the milliammeter.



Loop ABCA, $-2 + 25I + 10I_2 = 0$

$\therefore \quad 25I_1 + 10I_2 = 2 \quad \ldots(1)$

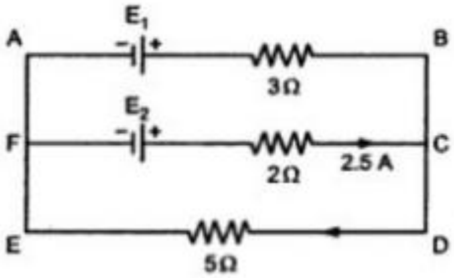
Loop ACDA, $-10I_2 - 100(I_1 - I_2) - 4 = 0$

$\therefore \quad 100I_1 - 110I_2 = 4 \quad \ldots(2)$

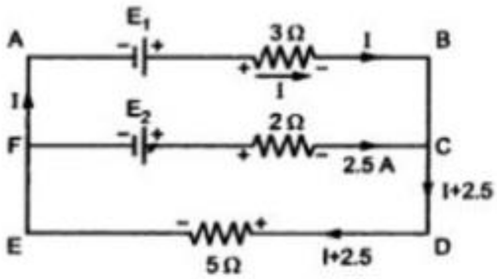
$\therefore \quad I_2 = 0.0267 \text{ A} = \mathbf{26.67 \text{ mA}}$

Thus the reading on the milliammeter is 26.67 mA . The current is flowing from A to C.

Determine the magnitude and direction of current through 3Ω resistance and calculate the values of E_1 and E_2 when the power dissipated in the 5 Ω resistor is 125 W.



Solution : The various currents and the corresponding voltage polarities are showr



Now power dissipated in 5 Ω is 125 W.

$$P_{5\Omega} = (I+2.5)^2 \times 5 \quad \text{as } P = I^2R$$

$$\therefore \quad 125 = (I+2.5)^2 \times 5$$

$$\therefore \quad I = 2.5 \text{ A} \qquad \text{... Current through } 3\,\Omega$$

Apply KVL to the loops,

Loop ABCFA, $\quad -3I + 2.5\times 2 - E_2 + E_1 = 0$

$$\therefore \quad -3\times 2.5 + 2.5\times 2 = E_2 - E_1$$

$$\therefore \quad E_2 - E_1 = -2.5 \qquad \text{...(1)}$$

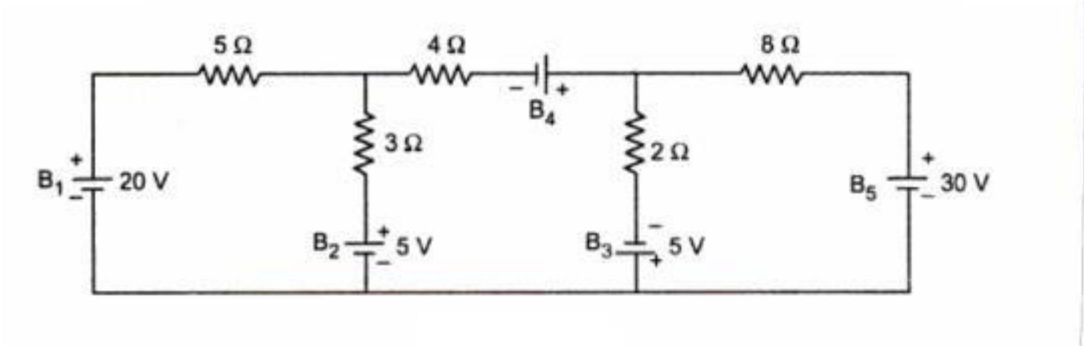
Loop FCDEF, $\quad +E_2 - 2.5\times 2 - 5\times(I+2.5) = 0$

$$\therefore \quad E_2 - 5 - 5\times(2.5+2.5) = 0$$

$$\therefore \quad E_2 = \mathbf{30 \ V} \qquad \text{...(2)}$$

Substituting in (1), $\quad E_1 = \mathbf{32.5 \ V}$

Determine the current supplied by each battery in the circuit shown in by using Kirchhoff's laws.



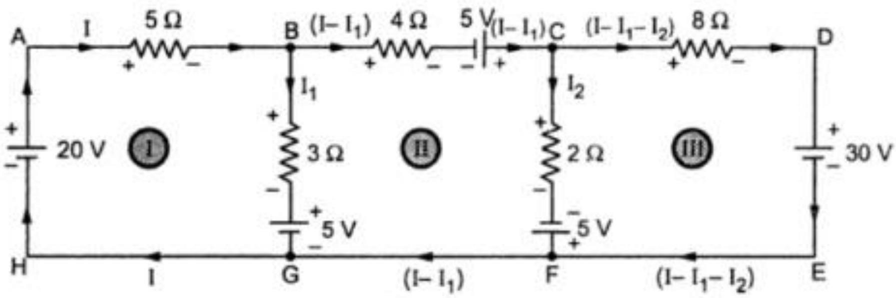


Fig. 2.53

Applying KVL to various loops :

For loop 1, ABGHA

$$-5\,I - 3\,I_1 - 5 + 20 = 0 \quad \text{i.e.} \quad +5\,I + 3\,I_1 = 15 \qquad \ldots(1)$$

For loop 2, BCFGB

$$-4\,(I - I_1) + 5 - 2\,I_2 + 5 + 5 + 3\,I_1 = 0 \quad \text{i.e.} \quad 4I - 7I_1 + 2I_2 = 15 \qquad \ldots(2)$$

For loop 3, CDEFC

$$-8\,(I - I_1 - I_2) - 30 - 5 + 2I_2 = 0 \quad \text{i.e.} \quad -8I + 8I_1 + 10I_2 = 35 \qquad \ldots(3)$$

Solving (1), (2) and (3)

$\therefore \quad I = 2.558$ A, $\quad I_1 = 0.7357$ A, $\quad I_2 = 4.9581$ A

Hence the current supplied by various batteries can be calculated as below :

Current supplied by $B_1 = I =$ **2.558 A**

Current supplied by $B_2 = I_1 =$ **0.7357 A**

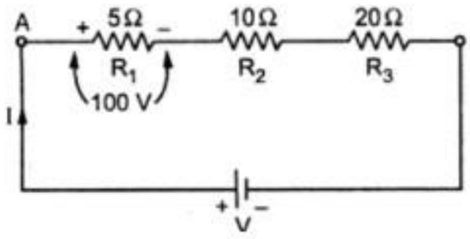
Current supplied by $B_3 = I_2 =$ **4.9581 A**

Current supplied by $B_4 = (I - I_1) = (2.558 - 0.7357) =$ **1.8223 A**

Current supplied by $B_5 = (I - I_1 - I_2) = (2.558 - 0.7357 - 4.9581)$

$= -$ **3.1358 A** $\qquad$ ... – ve sign means opposite direction

The circuit is shown



i) Find the equivalent resistance across the supply.

ii) If voltage drop across 5Ω is 100 V, find the supply voltage.

iii) Find the power consumed by each resistance.

Solution : It is series combination of resistances.

i) $$R_{eq} = R_1 + R_2 + R_3 = 5 + 10 + 20 = 35\Omega$$

ii) The drop across R_1 is 100 V given. **The current remains same through R_1, R_2 and R_3.**

$$\therefore \quad V_1 = \text{drop across } R_1 = I \times R_1 = 100 \text{ V}$$

$$\therefore \quad I = \frac{100}{R_1} = \frac{100}{5} = 20 \text{ A}$$

$$\therefore \quad V_2 = \text{drop across } R_2 = I \times R_2 = 20 \times 10 = 200 \text{ V}$$

$$\therefore \quad V_3 = \text{drop across } R_3 = I \times R_3 = 20 \times 20 = 400 \text{ V}$$

$$\therefore \quad V = V_1 + V_2 + V_3 = 100 + 200 + 400 = \mathbf{700 \text{ V}} \quad \text{... supply voltage}$$

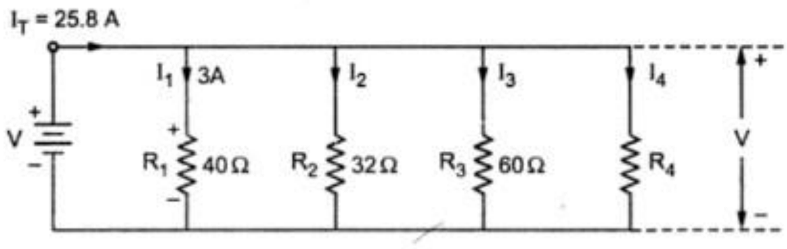
iii) $$P_1 = \text{power consumed by } R_1 = V_1 I \text{ or } I^2 R_1 = \mathbf{2000 \text{ W}}$$

$$P_2 = \text{power consumed by } R_2 = V_2 I \text{ or } I^2 R_2 = \mathbf{4000 \text{ W}}$$

$$P_3 = \text{power consumed by } R_3 = V_3 I \text{ or } I^2 R_3 = \mathbf{8000 \text{ W}}$$

The four resistances 40 Ω, 32 Ω, 60 Ω and R_4 Ω are connected in parallel across d.c. supply. Current in 40Ω is 3 A while the total current from supply is 25.8 A. Find, i) Supply voltage ii) R_4 iii) Equivalent resistance across supply.

Solution : The circuit diagram is shown



In parallel circuit voltage across each resistance is same equal to supply voltage.

i) Supply voltage $V = I_1R_1 = I_2R_2 = I_3R_3 = I_4R_4$

$\therefore \quad V = I_1R_1 = 3 \times 40 = \mathbf{120\ V}$

ii) $\quad 120 = I_2 \times 32 = I_3 \times 60 = I_4 \times R_4$

$\therefore \quad I_2 = 3.75\ A, \quad I_3 = 2\ A$

But $\quad I_T = I_1 + I_2 + I_3 + I_4$

$\therefore \quad 25.8 = 3 + 3.75 + 2 + I_4$

$\therefore \quad I_4 = 17.05\ A$

And $\quad I_4 \times R_4 = V$ i.e. $17.05\ R_4 = 120$

$\therefore \quad R_4 = \mathbf{7.0381\ \Omega}$

iii) For parallel circuit, $\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \dfrac{1}{R_4} = \dfrac{1}{40} + \dfrac{1}{32} + \dfrac{1}{60} + \dfrac{1}{7.0381}$

$\therefore \quad R_{eq} = \mathbf{4.6511\ \Omega}$