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A Random-Coefficients Logit Brand-Choice Model Applied to Panel Data

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A random-coefficients logit model that allows for unobserved heterogeneity in brand preferences and in the responses to marketing variables is empirically investigated using household-level panel data. The unknown underlying distribution of unobserved heterogeneity is approximated by a discrete distribution. The results reveal that there is significant unobserved heterogeneity across households and that ignoring its effects results in a downward bias in the parameter estimates of the marketing variables. It is therefore important to account for heterogeneity in *both* preferences and responses in the absence of any a priori knowledge about the nature of heterogeneity across households.

KEY WORDS: Panel data analysis; Random-coefficient logit model; Unobserved heterogeneity.

1. INTRODUCTION

Accounting for heterogeneity across households is an important consideration when analyzing their purchase behavior from panel data (Bass 1974; Bass, Jeuland, and Wright 1976). Households with the same demographic and socioeconomic characteristics (income, family size, etc.) when confronted with a given set of covariates (e.g., price, feature advertisements, in-store displays) may exhibit different choice behavior due to differences in overall brand preferences—intercept (or preference) heterogeneity—and/or variations in their responses to these covariates—slope (or response) heterogeneity. Both of these sources of variation across households are referred to as unobserved heterogeneity because they capture the effects of unobserved (to the researcher) factors that influence household purchase behavior.

From a marketing standpoint, an understanding of the factors influencing household brand-choice behavior has drawn considerable attention from researchers analyzing panel data. Furthermore, because households typically choose only one brand on any purchase occasion, the conditional logit model of discrete choice developed by McFadden (1973) has been extensively used for this purpose. There are three important reasons for the use of the logit model, (1) conceptual appeal being grounded in economic theory, (2) analytical tractability and ease of econometric estimation, and (3) excellent empirical performance as measured by model fit and other criteria (Guadagni and Little 1983). Hence an issue of research interest is: How should unobserved heterogeneity be incorporated into the logit model when analyzing household brand-choice behavior from panel data?

Researchers have recently addressed the preceding issue in one of two ways. Kamakura and Russell (1989) accounted for unobserved heterogeneity across households in a logit model of brand choice by assuming the existence of a finite number of segments. Each segment consists of a set of households having *identical* overall brand preference and response to marketing-mix variables. A latent class approach is used to estimate endogenously the size and logit-model parameters of each segment. Accordingly, the approach proposed by Kamakura and Russell to analyze brand choice of households consists of a finite mixture of logit models.

The second approach dealing with unobserved heterogeneity is to use a random-coefficients specification in which the parameters of the household-level logit model of brand choice are treated as realizations of random variables representing the preferences of households and their responses to marketing activities. These random variables are assumed to follow a continuous probability distribution. The studies by Chintagunta, Jain, and Vilcassim (1991) and Gonul and Srinivasan (1993) were in this vein. An advantage of using a random-coefficients specification to account for heterogeneity is the parsimony in the number of parameters to be estimated relative to estimating household-specific parameters (cf. Rossi and Allenby 1993).

In the study by Chintagunta et al. (1991), they accounted for heterogeneity in brand preferences only and did not consider response heterogeneity in their model formulation. Gonul and Srinivasan (1993) considered both preference and response heterogeneity but assumed the distribution for the intrinsic preference parameters of the logit model to be independent of the distribution of the response parameters.

They only allowed for covariance between certain marketing variables (e.g., price and coupons) but assumed the distributions of the intrinsic preferences for various brands to be independent of one another. At the household level, however, one might expect that the household's response to a brand's marketing variables may be dependent on its overall preference for that brand. For example, a household with a very high overall preference for a brand of detergent may not be very sensitive to changes in its price, at least within a certain range. In turn, a household's propensity in responding to price changes may not be independent of its response to other marketing variables. The implication of this is that the distributions of the preference and response parameters across households in a logit model are not likely to be independent. Therefore, when using a random-coefficients approach to account for heterogeneity across households, it would be more appropriate to assume a joint (i.e., multivariate) probability distribution of preferences and responses with a general variance–covariance structure (i.e., without imposing any prespecified independence structure). Note that a random-effects specification assumes that the random-error component of the logit model is independent of the distribution of unobserved heterogeneity (Hsaio 1986).

When using a multivariate probability distribution to account for unobserved heterogeneity, there are two critical issues. The first one is the specification of the underlying distribution of heterogeneity. In practice, one may not know, a priori, the form of this distribution. The use of an incorrect specification for this distribution will lead to biased parameter estimates (Heckman and Singer 1984). That bias will then result in making incorrect inferences about the impact of marketing variables on household brand-choice behavior.

A second issue is that, even if the specific parametric form is known a priori, it would involve the use of a joint multivariate distribution such as the normal or gamma. Consequently, estimating the parameters of the logit model under such a specification may be extremely difficult because it will involve evaluating multiple integrals. Because these integrals typically do not admit closed-form expressions, numerical methods need to be employed (see Ben-Akiva and Lerman 1985). Although Gonul and Srinivasan (1993) assumed a multivariate normal distribution, however, they restricted most of the covariance terms to be 0. The use of a multivariate normal distribution with nonzero covariances in the context of their model formulation could pose computational difficulties empirically when there are many brands and/or marketing variables. Although techniques such as the method of simulated moments (McFadden 1989) have eased some of the computational burden, considerable effort is still required to estimate these models.

One approach to addressing the preceding two issues is not to assume a specific parametric distribution but to approximate the underlying distribution by a discrete probability distribution and estimate from the data the supports of this distribution along with the probability masses associated with them. Such an approach is referred to as a random-coefficients estimation approach using a discrete probability distribution approximation. In this article, we demonstrate empirically the use of this approach for investigating the effects of unobserved heterogeneity on brand-choice behavior. To enhance the generalizability of the results, we consider three different product categories—saltine crackers, catsup, and yogurt. For completeness of the empirical analysis, we investigate three cases—a model with no heterogeneity, a model that includes only preference heterogeneity, and finally a model that accounts for both preference and response heterogeneity.

The results obtained from analyzing household brand-choice behavior from panel data for the preceding three frequently purchased consumer packaged-goods categories reveal that (a) ignoring the effects of unobserved heterogeneity results in a downward bias in the parameter estimates for the effects of marketing variables, (b) for all three products there is significant unobserved heterogeneity either in preferences or in both preferences and responses, and (c) there are dependencies among the preferences for the different brands and between brand preferences and responses to marketing-mix variables, as well as within the marketing-mix variables.

Results (a) and (b) highlight the importance of incorporating heterogeneity in the analysis. The implication of result (c) is that, if one is interested in understanding the nature of heterogeneity more completely, the analysis should allow for a general covariance structure between preference and response parameters. Such an analysis would yield additional insights that would have managerial implications for designing effective marketing strategies.

We note that, although the major emphasis of our analysis is to replicate the usefulness of the random-coefficients logit model and the estimation procedure using a discrete heterogeneity distribution, there are some important distinguishing features between our study and those by Kamakura and Russell (1989), Chintagunta et al. (1991), and Gonul and Srinivasan (1993). The nature of the difference between our approach and that of Kamakura and Russell (1989) is conceptual. They assumed the existence of a finite number of homogeneous segments of households in the market, but we assume a continuous underlying distribution of unobserved heterogeneity that we approximate with a discrete distribution. This conceptual difference has implications for how the empirical results are to be interpreted. We elaborate on this in our empirical analysis.

Chintagunta et al. (1991), on the other hand, only accounted for preference heterogeneity in their model formulation. Furthermore, they assumed the distributions of the intrinsic preferences for the various brands as being independent of one another. In relation to Gonul and Srinivasan (1993), the important difference is that we allow for a general covariance structure for preference and response parameters without imposing any a priori structure. Such a general structure provides additional insights into the nature of unobserved heterogeneity.

The rest of the article is organized as follows. In Section 2, we describe the random-coefficients specification and the estimation procedure for the model parameters. In Section 3, we describe the data and the empirical results, and in Section 4, we conclude with a discussion and summary of our findings.

2. MODEL SPECIFICATION AND ESTIMATION

We use the following logit formulation for studying household brand-choice behavior:

$$P_{it}(j) = \frac{\exp\left\{\beta_{0ij} + \sum_{k=1}^{K} \beta_{ik} x_{ijkt}\right\}}{\sum_{\ell=1}^{N} \exp\left\{\beta_{0i\ell} + \sum_{k=1}^{K} \beta_{ik} x_{i\ell kt}\right\}}, \quad (1)$$

where $P_{it}(j)$ = the probability that the ith household chooses brand j on the tth purchase occasion ($i = 1, 2, \ldots, M$; $j = 1, \ldots, N$; $t = 1, 2, \ldots, T_i$), X_{ijkt} = the value of the kth covariate faced by household i for brand j on the tth purchase occasion, β_{0ij} = the intercept term representing the intrinsic preference of household i for brand j, and β_{ik} = the response coefficient of household i for the kth covariate ($k = 1, 2, \ldots, K$).

We note from the formulation in (1) that the parameters β_{0ij} and β_{ik} are household specific. If there were many observations for each household, then it would be possible to consistently estimate these parameters. In most panel data sets, however, there would not be an adequate number of observations for every household to accomplish this task. Hence estimating household-specific parameters is generally not feasible. The Bayesian procedure proposed by Rossi and Allenby (1993) addressed this issue in the context of fixed-effects models.

As discussed in Section 1, one solution to the preceding problem is to use a random-effects specification, in which the micro-level parameters are assumed to be randomly distributed across households; that is, the vector of parameters $\Theta = \{\tilde{\beta}_{01}, \tilde{\beta}_{02}, \ldots, \tilde{\beta}_{0N}, \tilde{\beta}_1, \tilde{\beta}_2, \ldots, \tilde{\beta}_K\}$ consists of random coefficients $\tilde{\beta}_{0i}$ ($i = 1, 2, \ldots, N$) and $\tilde{\beta}_k$ ($k = 1, 2, \ldots, K$), which are distributed across households following a multivariate probability distribution $G(\Theta)$. Note that in empirical estimation there would be only $N - 1$ random intercept terms because one of the brands would be considered as a base brand and the preference for the other brands would be estimated relative to the base brand.

For a particular household i, the vector Θ_i—that is, $\Theta_i = \{\beta_{0i1}, \beta_{0i2}, \ldots, \beta_{0iN}, \beta_{i1}, \beta_{i2}, \ldots, \beta_{iK}\}$—is assumed to be a realization from $G(\Theta)$. Conditional on the value of the vector Θ, the probability of a randomly drawn household buying brand j on purchase occasion t will be given by

$$P_t(j \mid \Theta) = \frac{\exp\left\{\tilde{\beta}_{0j} + \sum_{k=1}^{K} \tilde{\beta}_k x_{jkt}\right\}}{\sum_{t=1}^{N} \exp\left\{\tilde{\beta}_{0\ell} + \sum_{k=1}^{K} \tilde{\beta}_k x_{\ell kt}\right\}}. \quad (2)$$

For any randomly drawn household, the unconditional probability of buying brand j on purchase occasion t will be

$$P_t(j) = \int_{\Theta} P_t(j \mid \Theta) dG(\Theta). \quad (3)$$

To estimate the model parameters, one can assume a specific parametric form for $G(\Theta)$ [e.g., a multivariate normal distribution (Butler and Moffitt 1982; Hausman and Wise 1978)]. For frequently purchased product categories, there is, as yet, no consensus on the appropriate parametric form of $G(\Theta)$. Given the absence of any a priori information on the parametric form of $G(\Theta)$, the alternative approach is to approximate the underlying specification of $G(\Theta)$ by a discrete distribution and estimate empirically from the data the support vectors and the probability masses associated with the supports. In this article we use this discrete distribution approach to account for unobserved heterogeneity. We approximate the distribution of $G(\Theta)$ by a finite number, S, of support vectors and estimate the locations of these vectors and the probability mass $\pi(\Theta_s)$ associated with each vector $\Theta_s (s = 1, 2, \ldots, S)$. Therefore, $P_t(j)$ in Equation (3) can be expressed as

$$P_t(j) = \sum_{s=1}^{S} P_t(j \mid \Theta_s)\pi(\Theta_s). \quad (4)$$

2.1 Estimation Procedure

We use the method of maximum likelihood for estimating the model parameters in Equation (4). We incorporate heterogeneity in two ways. First, we deal with preference heterogeneity in which only the brand-specific intercept [the intercept terms β_{0ij} ($j = 1, 2, \ldots, N$) in Eq. (1)] are randomly distributed across households. The response coefficients β_{ik} ($k = 1, 2, \ldots, K$) are treated as fixed and identical for all households; that is, $\beta_{ik} = \beta_k$ ($k = 1, 2, \ldots, K$, $i = 1, 2, \ldots, M$). In the case of preference and response heterogeneity, both the intercept term and the response coefficients are treated as random variables distributed across households following a joint distribution $G(\Theta)$. Therefore, in the case of preference heterogeneity, the parameters to be estimated are the response coefficients β_k ($k = 1, 2, \ldots, K$)—the effects of marketing-mix variables, the locations of the support vectors Θ_s ($s = 1, 2, \ldots, S$), and the associated probability masses $\pi(\Theta_s)$ of the distribution of preferences. In this case, Θ is given by $\Theta = \{\tilde{\beta}_{01}, \tilde{\beta}_{02}, \ldots, \tilde{\beta}_{0N}\}$. The mean intrinsic brand preferences are then computed from the estimated values of $\hat{\Theta}_s$ and $\hat{\pi}(\Theta_s), s = 1, 2, \ldots, S$.

In the second case, we allow for both preference and response heterogeneity and estimate the support vector Θ_s and the probability masses $\pi(\Theta_s)$ to empirically determine the distribution $G(\Theta)$, where $\Theta = \{\tilde{\beta}_{01}, \ldots, \tilde{\beta}_{0N}, \tilde{\beta}_1, \tilde{\beta}_2, \ldots, \tilde{\beta}_k\}$. We then calculate the mean intrinsic brand preferences and the mean response coefficients from the distribution of preferences and responses.

The likelihood function for household i conditional on the value of the vector Θ for both the preceding case is given by

$$L_i(\cdot \mid \Theta) = \prod_{t=1}^{T_i} \left\{ \prod_{j=1}^{N} \left[P_{it}(j \mid \Theta)\right]^{\delta_{ijt}} \right\}, \quad (5)$$

where

$\delta_{ijt} = 1$ if the ith household chooses brand j on purchase occasion t
$= 0$ otherwise

and T_i denotes the number of brand purchases made by the ith panelist. The unconditional likelihood function is obtained by summing Equation (5) over the probability distribution of Θ; that is,

$$L_i = \sum_{s=1}^{S} L_i(\cdot \mid \Theta_s)\pi(\Theta_s). \tag{6}$$

The sample likelihood function is given by

$$L = \sum_{i=1}^{M} L_i. \tag{7}$$

One critical quantity in estimating empirically the probability distribution of unobserved heterogeneity is the *number* of support vectors (S) required to approximate the underlying distribution. We determine this number by using a stopping-rule procedure based on the Bayesian information criterion (BIC, Allenby 1990), which is defined as follows: BIC $= -\mathrm{LL} + 1/2 \cdot R \cdot \log(T)$, where LL denotes the log-likelihood, R denotes the number of parameters estimated, and T denotes the total number of observations, $T = \sum_{i=1}^{M} T_i$.

In this procedure, support vectors are added as long as the BIC value keeps declining. A point is then reached at which adding the next support vector results in an increase in the BIC value. At this point, estimation is terminated and the corresponding number of support vectors S is treated as being the appropriate number. Another criterion for the stopping rule is the Akaike information criterion (AIC) which is defined as AIC $= -\mathrm{LL} + R$. One advantage of BIC over AIC is that it takes into consideration the number of observations used in the analysis and imposes a penalty with an increase in sample size.

3. DATA AND EMPIRICAL RESULTS

For the empirical analysis, we used data from three panels of households. The data for the first panel were collected by a marketing research firm, Information Resources Incorporated, and pertained to the purchases of saltine crackers in the Rome, Georgia, market. The second and third data sets consisted of the purchases of catsup and yogurt by a panel of households in Springfield, Missouri. These data sets were provided by A. C. Nielsen. All three are optical-scanner panel data sets and contain information on all purchases of a product category made by panelists (i.e., households) over the data-collection period (around two years). Each panelist is provided with an identification card, which is presented at the checkout counter at the time of purchase. All of the purchases are recorded (scanned) under that identification number.

Such data are typically collected for frequently purchased consumer nondurable goods. Furthermore these data are collected in markets (e.g., Rome, Georgia) in which most of the households' purchases of grocery products are in outlets equipped with optical scanners. Accordingly, the data provide a reasonably complete record of the households' purchases over time. In addition to the brand purchased by the panelist on any given purchase occasion, data are also collected on the store environment (e.g., prices of all brands in the product category and variables such as special displays of brands in the store), the marketing environment (e.g., presence or absence of newspaper feature advertisements), and also the value of any coupons used by the panelists.

The data on saltine crackers consist of 100 households with 2,509 purchases, the catsup data has 300 households making 2,798 purchases, and the yogurt data are from 100 households who account for 2,412 purchases. In our empirical analysis, we attempted to have the same number of observations for all three product categories. This necessitated having different numbers of households in the three samples due to the difference in the average interpurchase times for these categories. In the saltine-cracker data base, there are three major national brands—Sunshine, Keebler, and Nabisco with market shares .059, .074, and .555, respectively. These three brands account for about a 69% share of the product market. There are, in addition, some local brands that we grouped together under "private labels" in the analysis. The market share of this group is 31%. For all of the four brands, the 16-ounce package is the most frequently purchased.

For the catsup product category, Heinz is the major brand with three different sizes, Heinz 40, Heinz 32, and Heinz 28. The market shares of these three sizes are .065, .521, and .304, respectively. The other brand is Hunt's 32 with a share of .110. We considered the three sizes of Heinz and Hunt's 32 as four brands in the analysis of the catsup product category.

The yogurt data are composed of four brands—Yoplait, Dannon, Weight Watchers, and Hiland—with market share 34%, 40%, 23%, and 3% respectively. We confined our analysis to households that had purchased the six-ounce size of Yoplait and Weight Watchers and the comparable eight-ounce sizes of Dannon and Hiland.

For the cracker and catsup product categories, the brand-specific marketing variables used in the analysis are the price of the brands, special displays, and newspaper feature advertisements. Due to the perishable nature of yogurt, this product category is rarely on special end-of-aisle or middle-of-aisle display. Consequently, the two marketing variables of interest are price and feature advertisements. We refer to the price, special displays, and feature advertisements as price, display, and feature, respectively, in the presentation of our empirical results. These three variables are measured as follows:

Price = Actual price for the brand purchased
= Shelf price for all other brands

Display = 1 if there is a display for the brand
= 0 otherwise

Feature = 1 if there is a newspaper feature advertisement for the brand
= 0 otherwise.

We define actual price paid by the panelists as the shelf price net of value of coupons redeemed.

We present some descriptive statistics pertaining to these three variables in Tables 1, 2, and 3 for the saltine-cracker, catsup, and yogurt product categories, respectively. The first number in Table 1—that is, .123—is the fraction of purchase occasions on which Sunshine is on special display a the store.

Table 1. Basic Descriptive Statistics: Saltine Crackers (100 panelists, 2,509 purchases)

	Brands			
Variables	*Sunshine*	*Keebler*	*Nabisco*	*Private label*
Display[a]	.123	.107	.334	1.02
Feature[b]	.035	.040	.083	.049
Price[c] ($/oz)	.960	1.128	1.081	.682
Brand share	.059	.074	.555	.312

[a] The display variable refers to fraction of purchase occasions on which Sunshine was on display.

[b] The feature variable refers to fraction of purchase occasions on which Sunshine was featured; for example, the value .035 implies that Sunshine was featured 3.5% of the 2,509 purchases.

[c] The price variable is the average sample price across all purchase occasions.

Similarly, one can interpret the values for Display and Feature for the other brands in Tables 1, 2, and 3. The prices presented in Table 1 are the average prices of the 16-ounce package for all of the four brands. In Tables 2 and 3, we report the prices per ounce for the four brands of catsup and yogurt as they differ in their size. The last row in all tables represents the shares of the brands in the three product categories.

We separate the discussion of the empirical results as follows. First, we report the results of the stopping-rule criterion to determine the number of support vectors used to approximate the underlying probability distribution of unobserved heterogeneity. Second, we evaluate the impact of incorporating unobserved heterogeneity on the overall fit of the model and discuss the estimates of the marketing-mix variables (feature, display, and price) and the brand-specific constants for three different model formulations—no heterogeneity, preference heterogeneity only, preference and response heterogeneity. We also estimated a model with only response heterogeneity. Our findings indicated that such a model performed poorly in comparison with the model with only preference heterogeneity. We therefore decided not to include those results in the manuscript.

3.1 Stopping Rule

In Table 4, page 322, we report the BIC values obtained from estimating the two model formulations, one that accounts for preference heterogeneity only and the other accounting for both preference and response heterogeneity, for different values of support vectors, S.We tried different values of S starting with $S = 1$, which corresponds to the no-heterogeneity case. We see from Table 4 that for saltine crackers the BIC value under preference heterogeneity decreases until S reaches 5 and thereafter it starts increasing. This suggests that five support vectors are adequate to approximate the preference-heterogeneity distribution. In the case of preference and response heterogeneity for saltine crackers, the BIC attains its minimum value for four support vectors. The results for the catsup product category are similar to those for saltine crackers. For yogurt, however, six support vectors were required to characterize adequately the distribution of unobserved heterogeneity for both preference-only and preference-and-response heterogeneity models. Hence in the discussion that follows the results for preference heterogeneity are based on a five-support vector model, but a four-support vector model is used for preference and response heterogeneity for the first two data sets. For the third data set, a model with six support vectors is analyzed.

Table 2. Basic Descriptive Statistics: Catsup (300 panelists, 2,798 purchases)

	Brands			
Variables	*Heinz 40*	*Heinz 32*	*Heinz 28*	*Hunt's 32*
Display	.023	.099	.076	.045
Feature	.033	.065	.069	.046
Price ($/oz)*	.046	.031	.043	.034
Brand share	.065	.521	.304	.110

*The price variable is the average sample price across all purchase occasions.

Table 3. Basic Descriptive Statistics: Yogurt (100 panelists, 2,412 purchases)

	Brands			
Variables	*Yoplait*	*Dannon*	*Weight Watchers*	*Hiland*
Feature	.056	.038	.038	.037
Price ($/oz)*	.107	.082	.079	.054
Brand share	.339	.402	.230	.029

*The price variable is the average sample price across all purchase occasions.

3.2 Parameter Estimates

Tables 5, 6, and 7 contain the parameter estimates and their standard errors for saltine-crackers, catsup, and yogurt product categories, respectively. In each of these tables, we present the parameter estimates obtained from three model specifications—a logit model with no heterogeneity, a random-coefficients logit model with preference heterogeneity, and a random-coefficients logit model with preference and response heterogeneity. For the preference-heterogeneity-only model, we report the mean values of the brand-specific constants, but for the preference-and-response-heterogeneity model, we report the mean values of both the brand-specific constants and the response parameters. These mean values are reported for expositional purposes only. We note here another difference that arises from the conceptual difference between our approach to incorporating unobserved heterogeneity and that of Kamakura and Russell (1989). Because we assume that heterogeneity is captured via an underlying probability distribution, computing mean values is appropriate. In contrast, because Kamakura and Russell assumed the existence of a finite number of distinct segments, it is not clear what a mean value would represent.

In terms of the overall fit of these model specifications, we see from Tables 5, 6, and 7 that accounting for heterogeneity results in significant improvement in the two commonly used goodness-of-fit criteria, AIC and BIC.

We also note from Tables 5, 6, and 7 that for the saltine-crackers and yogurt data sets the preferred model

Table 4. Stopping Rule for Number of Support Vectors (values of BIC)

Number of support vectors	Cracker—no heterogeneity—2,530: Preference heterogeneity	Cracker: Preference & response heterogeneity	Catsup—no heterogeneity—2,542: Preference heterogeneity	Catsup: Preference & response heterogeneity	Yogurt—no heterogeneity—2,676: Preference	Yogurt: Preference & response heterogeneity
2	2,233	2,234	2,303	2,304	1,961	1,958
3	1,463	1,475	2,201	2,208	1,555	1,549
4	1,365	**1,392**	2,182	**2,173**	1,475	1,479
5	**1,334**	1,395	**2,181**	2,174	1,428	1,448
6	1,350	—*	2,192	—	**1,416**	**1,435**
7	—	—	—	—	1,424	1,453

*This model was not estimated because the minimum BIC value had already been obtained.

specification based on the goodness-of-fit criterion is the random-coefficients logit model with preference heterogeneity, whereas for catsup (see Table 6) the random-coefficients logit model with preference and response heterogeneity has the best fit. This implies that there is considerable variation in the intrinsic brand preferences across households in the cracker and yogurt data sets and little heterogeneity in households' responses to marketing-mix variables. For the catsup data set, however, in addition to heterogeneity in intrinsic preferences, households also differ in their responses to marketing activities. Hence accounting only for preference heterogeneity would not be adequate for this catsup data set. An important implication of these results is that, because one does not know a priori the exact nature of unobserved heterogeneity, accounting for heterogeneity in both preference and response will produce robust parameter estimates.

We see from Table 5 that all three marketing-mix variables (display, feature, and price) have the expected signs for saltine crackers. The price variable appears to be an important determinant of household purchase behavior. Furthermore, between display and feature, the latter appears to have greater effect on the brand-choice behavior of households. Regarding the brand-specific constants, we see that Nabisco, consistent with its largest brand share, has the highest value of instrinsic preference.

Comparing the estimates across the three model specifications, we see that the coefficients of marketing variables in the no-heterogeneity case are considerably different than the estimates obtained from the other two formulations. Furthermore, from Table 5 we note a dramatic change in the estimates of the intrinsic preferences for Sunshine and Keebler when moving from the no-heterogeneity case to either the preference-heterogeneity-only or the preference-and-response-heterogeneity model specifications. Specifi-

Table 5. Parameter Estimates and Their Standard Errors[a] (saltine crackers)

Variables	Type of heterogeneity: None	Preference	Preference and response
Marketing:			
Display	.095	.390	.254
	(.060)	(.127)	(.146)
Feature	.526	.748	.724
	(.096)	(.164)	(.219)
Price	−3.112	−4.165	−3.758
	(.176)	(.440)	(.507)
Constants:			
Sunshine	−.854	.119	.073
	(0.88)	(.267)	(.266)
Keebler	−.083	.856	.614
	(.095)	(.326)	(.349)
Nabisco	1.824	3.311	3.081
	(.086)	(.346)	(.350)
-(log-likelihood)	2,506	1,248	1,286
# of estimated parameters	6	22	27
AIC[b]	2,512	1,270	1,313
BIC	2,530	1,334	1,392

[a]Private label was the base brand.
[b]The log-likelihood value of the model with only brand-specific constants was −2,636.

Table 6. Parameter Estimates and their Standard Errors[a] (catsup)

Variables	Type of heterogeneity: None	Preference	Preference and response
Marketing:			
Display	.876	1.035	1.052
	(.098)	(.129)	(.128)
Feature	.909	1.205	1.157
	(.167)	(.076)	(.147)
Price	−1.402	−1.863	−1.723
	(.061)	(.079)	(.093)
Constants:			
Heinz 40	1.354	2.484	2.084
	(.138)	(.227)	(.185)
Heinz 32	1.501	2.023	1.863
	(.071)	(.190)	(.160)
Heinz 28	2.426	3.483	3.174
	(.101)	(.206)	(.169)
-(log-likelihood)	2,518	2,094	2,066
# of estimated parameters	6	22	27
AIC[b]	2,524	2,116	2,093
BIC	2,542	2,181	2,173

[a]Heinz 32 was the base brand.
[b]The log-likelihood value of the model with only brand-specific constants was −3,139.

Table 7. Parameter Estimates and Their Standard errors[a] (yogurt)

	Type of heterogeneity		
Variable	None	Preference	Preference & response
Feature	.491	.851	.779
	(.120)	(.169)	(.244)
Price	−36.658	−43.243	−38.094
	(2.437)	(2.346)	(.711)
Yoplait	4.450	11.419	18.567
	(.187)	(.856)	(.297)
Dannon	3.716	10.618	17.901
	(.145)	(8.10)	(.265)
Weight Watchers	3.074	8.108	16.083
	(.145)	(1.075)	(.378)
−LL	2,656.89	1,318.74	1,298.81
# parameters	5	25	35
AIC[b]	2,662	1,344	1,334
BIC	2,676	1,416	1,435

[a]Base brand is Hiland.
[b]The log-likelihood value of the model with only brand-specific constants was −2,833.

cally, the preference ordering in the no-heterogeneity case is Nabisco, private label, Keebler, and Sunshine, but the mean preference ordering from the other two models is Nabisco, Keebler, Sunshine, and private label. This implies that the no-heterogeneity model is unable to recover even the mean value of the heterogeneity distribution across households. Additionally, because it does not allow for variations across households in their preferences and responses, it provides biased estimates for the model parameters.

The results reported in Table 6 for the catsup data set are largely consistent with those reported in Table 5 for the saltine-crackers data set. The parameter estimates after accounting for unobserved heterogeneity do not change, however, as in the case of the saltine-crackers data. Consistent with the findings in the cracker data set, the ordering of the magnitude of the (mean) intrinsic brand-preference changes after unobserved heterogeneity is accounted for. Although the ordering from the no-heterogeneity model is Heinz 28, Heinz 32, Heinz 40, and Heinz 32, the mean preference ordering from the heterogeneous models is Heinz 28, Heinz 40, Heinz 32, and Heinz 32. A feature of similarity between the 28-ounce and 60-ounce size of Heinz is that they are both available in squeezable plastic bottles.

Interestingly, we see from Table 6 that the estimated effects of the marketing-mix variables from the logit model with only preference heterogeneity are all "close" in magnitude of the mean values of the corresponding estimates from the random-coefficients logit model. It is important to note, however, that this need not imply that the substantive implications—that is, the elasticities obtained from the two models—would also be similar because computation of elasticities from the preference-and-response-heterogeneity model would take into consideration the complete distribution of the effects of marketing variables across households.

Turning to the results for the Yoplait data sets in Table 7, we find a dramatic improvement in the BIC values for the two models that account for unobserved heterogeneity. We also note that the estimates for feature and price variables are biased downward in the no-heterogeneity model with the magnitude of the bias in the feature variable being more pronounced than that for the price variable. Although the preferred ordering of the brands is across the three model formulations the same, the magnitudes of the mean intrinsic purposes of the models with unobserved integrity are considerable higher for Yoplait, Dannon, and Weight Watchers than the estimates obtained under the no-heterogeneity formulation. Furthermore, comparing the mean preferences for the two heterogeneity models, we find that the values for the preference and response heterogeneity model are larger than those obtained from the model with only preference heterogeneity.

We now discuss the implications of the differences in the parameter estimates from the two model formulations of unobserved heterogeneity for the three product categories. In Tables 8, 9, and 10 we present the price elasticities that are computed using the procedure discussed by Chintagunta et al. (1991) and by Gonul and Srinivasan (1993). We see from these tables that for the saltine-crackers and Yoplait data sets the elasticities obtained from the preference heterogeneity model are "close" in magnitude to those obtained from the preference and-response-heterogeneity model. For the catsup data, there are sizable differences in the price elasticities in some cases. This seems consistent with our earlier finding based on the goodness-of-fit measures in Table 4. For

Table 8. Price Elasticities for the Random-Coefficients Logit Model: Saltine-Cracker Data Set

	Brands			
Brands	Sunshine	Keebler	Nabisco	Private label
Sunshine	−3.00	.34	.17	.25
	(−2.41)	(.28)	(.14)	(.15)
Keebler	.53	−2.25	.19	.16
	(.44)	(−2.16)	(.19)	(.15)
Nabisco	1.79	1.28	−.70	.71
	(1.67)	(1.34)	(−.70)	(.73)
Private label	.77	.32	.20	−.74
	(.52)	(.30)	(.21)	(−.67)

NOTE: The figures in parentheses pertain to the logit model with only preference heterogeneity.

Table 9. Price Elasticities for the Random-Coefficients Logit Model: Catsup Data Set

	Brands			
Brands	Heinz 40	Heinz 32	Heinz 28	Hunt's 32
Heinz 40	−3.84	.28	.29	.37
	(−5.95)	(.34)	(.60)	(.38)
Heinz 32	1.49	−1.53	1.29	2.11
	(1.87)	(−1.60)	(1.39)	(1.89)
Heinz 28	1.35	1.07	−2.60	1.66
	(2.91)	(1.19)	(−2.97)	(1.54)
Hunt's 32	.47	.49	.46	−3.60
	(.46)	(.43)	(.40)	(−3.28)

NOTE: The figures in parentheses pertain to the logit model with only preference heterogeneity.

Table 10. Price Elasticities for the Random-Coefficients Logit Model: Yogurt Data Set

	Brands			
Brands	*Yoplait*	*Dannon*	*Weight Watchers*	*Hiland*
Yoplait	−1.61	.92	.64	2.02
	(−1.33)	(.68)	(.92)	(1.30)
Dannon	.82	−.99	.56	1.42
	(.61)	(−.88)	(.63)	(1.51)
Weight Watchers	.20	.19	−1.18	.66
	(.29)	(.21)	(−1.45)	(.62)
Hiland	.09	.07	.10	−2.14
	(.06)	(.08)	(.10)	(−1.89)

NOTE: The figures in parentheses pertain to the logit model with only preference heterogeneity.

saltine crackers and yogurt, accounting for response heterogeneity in addition to heterogeneity in preferences did not result in significant improvement in the model fit, suggesting that the implications of the effect of marketing-mix variables are largely unaffected by the inclusion of response heterogeneity. In contrast, for the catsup data we find that the model with preference and response heterogeneity is preferred over the preference-heterogeneity-only model based on the BIC criterion. In this case the estimates of price elasticity do seem to change with the inclusion of response heterogeneity.

3.3 Distribution of Response Heterogeneity

To obtain additional insights into the nature of the underlying heterogeneity across households in their response to marketing activities, we provide in Tables 11, 12, and 13 the actual distribution of such heterogeneity for the three marketing variables.

Tables 11, 12, and 13 indicate that the preferences for brands are related to the nature of price and promotion sensitivities in each of the three data sets. From Table 11, we note that supports 1 and 2 correspond to the highest preference for Nabisco. Although support 1 is associated with a high display and price sensitivity, however, support 2 is characterized by low price sensitivity and high feature sensitivity. Private-label brands are most preferred in support 4, and this support vector indicates moderate price sensitivity and no display and feature sensitivity. Keebler is preferred more than any other brand in support 3, and this support is associated with a high price sensitivity.

The catsup data reveal an interesting pattern of preference and response heterogeneity across the four supports similar to the results for the cracker data sets. Specifically, supports 1 and 2 indicate the highest preference for Heinz 28, with support 1 characterized by the highest price sensitivity and support 2 by the lowest value for price sensitivity. Support 2, however, is characterized by the highest sensitivity to the feature variable. Support 3, which is associated with the largest value for the display parameter, shows the highest preference for the 32 ounces of Heinz. The differences in preferences for Heinz 28, 32, and 40 ounces across the supports indicate that households have different size preferences besides brand preferences. Finally, support 4, which indicates a high preference for Heinz 32, also has a high price sensitivity.

For the yogurt data, the joint distribution of preferences for the four brands and the responses to the price and feature variables in Table 12 can be interpreted in a manner similar to the cracker and catsup product categories. For example, support 4, which corresponds to the highest value of the price-sensitivity parameter, also has the lowest preference for the Weight Watchers brand. This is an intuitively plausible finding. Similar interpretations can be provided for the remaining supports.

An alternative interpretation of the estimated distribution of heterogeneity is that each support point represents a segment of consumers with common behavioral characteristics (Kamakura and Russell 1989). In the present context, we do not make such an interpretation because our underlying premise is that there is a continuous distribution of hetero-

Table 11. Distribution of Response Heterogeneity: Saltine Crackers

	Support vector (probability mass)			
Marketing-mix variables	*1 (.20)*	*2 (.50)*	*3 (.07)*	*4 (.23)*
Display	.636	.306	.484	−229
	(.174)	(.260)	(.444)	(.268)
Feature	.766	1.003	.504	.181
	(.220)	(.383)	(.723)	(.451)
Price	−5.022	−2.872	−4.630	−4.302
	(.690)	(.973)	(1.477)	(.595)
Sunshine	1.388	.513	2.099	−2.495
	(.260)	(.429)	(.951)	(.263)
Keebler	1.204	1.038	6.113	−2.348
	(.387)	(.542)	(1.009)	(.391)
Nabisco	2.489	5.095	4.217	.918
	(.375)	(.492)	(.976)	(.245)

Table 12. Distribution of Response Heterogeneity: Catsup

	Support vector (probability mass)			
Marketing-mix variables	*1 (.31)*	*2 (.13)*	*3 (.40)*	*4 (.16)*
Display	.423	1.021	1.771	.495
	(.266)	(.300)	(.203)	(.200)
Feature	1.433	1.605	.812	1.110
	(.175)	(.255)	(.253)	(.238)
Price	−2.555	−.055	−1.411	−2.299
	(.176)	(.237)	(.141)	(.165)
Heinz 40	2.617	2.054	2.585	.131
	(.262)	(.478)	(.154)	(.108)
Heinz 32	.896	.723	3.904	−.390
	(.179)	(.478)	(.148)	(.134)
Heinz 28	4.459	2.894	3.394	−.433
	(.238)	(.461)	(.147)	(.128)

Table 13. Distribution of Response Heterogeneity: Yogurt

	Support vector (probability mass)					
Marketing-mix variables	1 (.09)	2 (.25)	3 (.17)	4 (.17)	5 (.20)	6 (.12)
Feature	1.094	.736	1.396	−.704	1.086	1.298
	(.388)	(.568)	(.596)	(.537)	(.266)	(.619)
Price	−14.828	−11.113	−23.152	−89.451	−45.941	−48.201
	(2.949)	(3.084)	(3.084)	(3.052)	(2.790)	(3.074)
Yoplait	157.923	.358	7.285	7.780	3.154	6.418
	(1.832)	(.505)	(.900)	(.360)	(.282)	(.966)
Dannon	155.369	3.988	2.661	4.251	2.518	6.369
	(1.841)	(.266)	(.955)	(.330)	(.239)	(.950)
Weight Watchers	159.375	.046	3.597	.293	2.390	1.625
	(1.832)	(.385)	(.912)	(1.416)	(.240)	(1.205)

geneity across households, which in our analysis is approximated by a discrete distribution.

We conclude from the results reported in Tables, 11, 12, and 13 that, although the mean values of the parameter estimates of the random-coefficients logit model are comparable to the corresponding coefficients from a logit model with only preference heterogeneity, the former provides additional insights into the nature of responses of the households. The finding that households vary considerably in their response to the marketing variables clearly has implications for managerial actions such as appropriately targeting promotional activities for maximum effectiveness.

4. SUMMARY

In this article, we have provided an approach to accounting for heterogeneity across households in both brand preferences and their response to marketing variables. Based on a random-coefficients specification, the proposed approach uses a discrete distribution approximation to characterize the heterogeneity across households in relation to their behavior in a product market. Our approach, while allowing for very general patterns of heterogeneity, is also empirically tractable, and the model parameters can be computed using readily available software that can be run on a personal computer.

An important aspect of our study is the replication of our estimation and empirical analysis across three different data sets—saltine-crackers, catsup, and yogurt product categories. Our results indicate that the nature of the heterogeneity across households is quite specific and varies across product categories as well as geographic markets. The result suggests the need for a very flexible approach to accounting for unobserved heterogeneity. Some previous attempts at studying unobserved heterogeneity may have been restrictive, if not misleading, because of their imposition of very specific parametric forms for the probability distribution underlying it.

Our empirical results also indicate that there could be severe biases in the parameter estimates for the effects of marketing variables if heterogeneity is not accounted for in the analysis. Specifically, for the three product categories, we find a significant downward bias in the estimates of feature, display, and price when no unobserved heterogeneity is accounted for in the estimation. The bias appears to be especially severe for the price parameter. Our results also indicate that in some situations (as in the case of the saltine-crackers data) accounting for only preference heterogeneity may suffice because of a lack of heterogeneity in the households' response to the marketing-mix variables. As in any particular situation, however, one does not know a priori whether, if that is indeed the case, incorporating both response and preference heterogeneity will yield robust estimates of the model parameters.

The differences in the estimates of the parameters arising out of the manner in which unobserved heterogeneity is incorporated also leads to differences in substantive marketing implications. For example, the computed choice elasticities with respect to price and other marketing variables are, in general, sensitive not only to the inclusion of unobserved heterogeneity but also to the manner in which it is incorporated (i.e., preference only or both preference and response heterogeneity). The magnitude of the differences are, however, dependent on the specific product category and the extent to which households differ in their preferences and/or response to marketing variables.

We note that, although the main focus of our analysis has been to replicate empirically the usefulness of a random-coefficients approach to incorporating unobserved heterogeneity, there are some important and enhancing features of our study relative to some previous research that have also used a similar approach. Relative to the study by Chintagunta et al. (1991) in which only independent-preference heterogeneity was incorporated, our study not only incorporates both preference and response heterogeneity but also allows for a general dependence structure within and between these two types of heterogeneity. The enhancing features of our study relative to that by Gonul and Srinivasan (1993) are (a) we allow for any distribution of unobserved heterogeneity without restricting it (perhaps incorrectly) to any particular parametric form—e.g., multivariate normal), and (b) we allow for a general variance–covariance structure without imposing any prior assumptions of independence between components of unobserved heterogeneity. The last point is important because empirically we found that a household's response to price may be dependent on its overall brand preference; that is, high level of preference for a particular brand may be associated with a lower price sensitivity. Clearly, such a finding has implications for such managerial decisions as determining profit-maximizing brand prices.

The major difference between this study and that by Kamakura and Russell (1989) is conceptual. Although we assume an underlying probability distribution of unobserved heterogeneity (each household has its own and unique vector of preference response parameters), Kamakura and Russell assumed the existence of a finite number of distinct segments, wherein households within a segment have *identical* preference and response parameters. This conceptual difference at the micro level also leads to some interpretive differences at the aggregate level. For example, within our framework it is meaningful to calculate mean values for the different estimates, although it is not clear what a mean would represent when the consideration is in terms of distinct segments of households. Additionally, if the framework of the logit model is extended to the realms of normative analysis (e.g., determining equilibrium profit-maximizing brand prices), where it serves to specify the marketing-response function, then other important differences arise. For example, in computing profit-maximizing prices within our conceptualization, it is appropriate to compute a single price for each brand for the entire market. In contrast, the Kamakura and Russell conceptualization would require determining a different profit-maximizing price for each segment for a given brand. Implementing such a pricing strategy may not be trivial.

Several possible avenues for future research exist. It would be interesting to compare our results with estimates obtained from assuming specific parametric forms for the heterogeneity distribution—for example, multivariate normal. Of particular interest would be that of determining the magnitude and direction of the bias in the parameter estimates if one imposes an incorrect parametric form. In our empirical analysis, each product category consisted of four brands, and consequently there were three identifiable intercepts. For product categories that have more brands, the use of a discrete distribution approach could prove to be computationally intensive because the number of parameters to be estimated would grow rapidly with the addition of more support vectors. Ways in which the logit model can accommodate heterogeneity in such product markets need to be investigated.

Another area of future research interest is determining the effects of incorporating unobserved heterogeneity on equilibrium/profit-maximizing prices of various brands for the retailers and manufacturers. Obtaining optimal/equilibrium brand prices using the estimated logit demand functions is an issue that is receiving considerable attention in the theoretical economics literature (Caplin and Nalebuff 1991). Further research, particularly empirical work, is needed to address this issue.

In summary, this article addresses the issue of accounting for preference and response heterogeneity in logit brand-choice models for panel data The random-coefficients approach, using a discrete heterogeneity distribution, provides valuable insights into our understanding of the effects of heterogeneity across households on the parameter estimates obtained from such logit models. Extending this descriptive framework to the realms of normative analysis such as modeling brand-level competition would enhance further the usefulness of scanner household panel data.

APPENDIX: DISCRETE DISTRIBUTION APPROXIMATION PROGRAM

/*This program is written in the Gauss programming language. The program will have to be modified based on the organization of the dataset used for the estimation, number of panelists, number of marketing variables, number of brands, number of support points, etc. **Some of the areas that need to be modified are in bold face**. The program is written for the case where each row of the dataset has information pertaining to all brands for a household on one purchase occasion. The data needs to be arranged as follows: if there are three marketing variables, then the first three columns will be the marketing variables of brand 1...etc. The last column contains the serial number of the panelist. This column is preceded by the columns for the dependent (dummy) variables, one column for each brand*/

/*This step calls the required optimization routines from the Gauss library*/

```
library optmum;
#include optmum.ext;
optset;
```

/*This step opens the Gauss dataset which contains the data used in the estimation*/

```
closeall;
open f1=CRACKER;
```

/*Here the number of support points (np), the number of brands (nb), the number of variables (nv), and the number of panelists (ppp) are specified*/

```
np=3;
nb=4;
nv=3;
ppp=100;
```

/*The data are read in this step, and a data matrix data is created*/

```
n=rowsf(f1); co=colsf(f1);
data=readr(f1,n);
y=data[.,co-nb:co-1];
panel=data[.,co];
```

/*This step clears all variables needed for constructing the likelihood function*/

```
clear last,rr,coc,ii,jj, llist, pur,opur, p1,p2,p3,p4, p, qnu, ll, v;
```

```
/*The array of probability masses is initialized*/

qnu=zeros(np, 1);

/*The procedure computing the value of the log-likelihood*/

proc lpr(x);

/*This step first computes the value of the exponential of
the deterministic component of utility (v) for each support
point and for each brand. Brand choice probabilities are then
computed conditional on each support point (p1).*/

p=zeros(n,np);
v=zeros(n,nb);
jj=1;
do until jj= =np+1;

  ll=1;
  do until ll= =nb;

    v[., ll] = exp(((data[., (ll−1)*nv + 1:ll*nv]−
                data[., (nb − 1)*nv + 1 : nb*nv])*
                x[(jj − 1)*nv + 1 : jj*nv]) + x[np*(nv)
                +(jj − 1)*(nb − 1) + 11]);
    ll = ll+1;
  endo;

  v[.,nb]=ones(n,1);

  p1=v./sum(v');

/*This step isolates the probability associated with the chosen
brand*/

p[.,jj] = prodc((p1.^y)');
jj=jj + 1;
endo;

/*In this loop, the string probabilities are computed for each
panelist and placed in the array coc consisting of ppp rows
and np columns*/

coc = zeros(ppp,np);
llist=1;
last=1;
rr=1;


do until rr= =n+1;
 if panel[rr,1] ne llist;

        coc[llist,.]=(prodc(p[last:(rr-1),.]))';
        llist = llist +1;
        last=rr;
 endif;
 if rr= =n;
        coc[llist,.]=(prodc(p[last:(rr),.]))';
 endif;
 rr=rr+1;
endo;

/*The mass points associated with each support point are
computed such that they lie between 0 and 1 in magnitude.
These probabilities are placed in the array qnu*/

qnu=exp(x[np*(nv + nb − 1) + 1:np*(nv + nb)−1]):1;
qnu-qnu/sumc(qnu);

/*String probabilities are weighted by the probability
masses*/

coc=coc*qnu;

/*The value of the log-likelihood is returned by the proce-
dure*/

retp(-sumc(ln(coc)));
endp;

/*Starting values for the parameters: these can be replaced
with values of the user's choice*/

x0=ones((np*(nv+nb-1,1);

____title = "Heterogeneous Logit Model";
output file = logit.out on;
{x,f,g,retcode} = optprt(optmum(&lpr,x0));
?x ~ sqrt(diag(___opfhess));
output off;
```

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