RESEARCH STATEMENT

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My research effort has been devoted to numerical approximation in stochastic systems, large deviations, and moderate deviations. I am interested in developing new numerical schemes, proving their convergence, and ascertaining their convergence rates. For large and moderate deviations theory, I am interested in establishing large and moderate deviation principle for stochastic differential equations, stochastic partial differential equations (SPDEs) under both large time and small diffusion settings. In what follows, I briefly summarize selected research work to date and state my future research plans.

1. Research work to date

In this section, I will present two selected research projects during my Ph.D. program under the supervision of Prof. George Yin.

1.1. **Numerics for stochastic differential equations.** For highly nonlinear stochastic differential equations, it is virtually impossible to obtain their analytic solutions. Thus numerical approximation becomes vitally important. There has been a vast literature devoted to numerical solutions of stochastic differential equations, their convergence, and their convergence rates. The most common practice is to obtain strong and weak convergence. If the drift and diffusion coefficients are Lipschitz, the classical Euler-Maruyama (EM) method converges to the true solution with strong order $1/2$; see [11, Theorem 10.2.2]. Milstein [13] improves the convergence rate to be of order 1 using Itô formula. Despite this, many SDEs encountered in practical applications have coefficients that are less regular, such as non-Lipschitz and superlinear coefficients, as discussed in Hutzenthaler et al. [9] and Mao [12].

The research shows that the classical Euler-Maruyama schemes might lead to numerical solutions with finite explosion time. Thus some truncation or projection algorithms are used to confine the estimates to be in a certain bounded region. In line with such idea, in [15], we proposed a novel numerical scheme, established the weak convergence using the martingale problem formulation, and proved the weak convergence rate with order $1/2 - \varepsilon$ for $\varepsilon > 0$. A distinct feature of our approach is a random truncation region instead of a fixed one is used to bound the sequence of EM numerical approximations of SDEs. This idea stems from stochastic approximation algorithms aiming to solve root-finding or optimization problems under noisy measurements. In this direction of stochastic approximation, see the work of Chen and Zhu [4] by making the sequence of approximation return to a fixed point and the work of Andradóttir in [1] by projecting the sequence of estimates onto a sequence of increasing sets. Compared to the existing algorithm such as truncated or tamed EM schemes, the main advantage of our algorithm is that we need not modify the drift or the diffusion coefficient of the SDE.

1.2. **Large and moderate deviations theory.** Since the pioneer work of Freidlin [5], much of effort are put to the study of large deviation principle (LDP) for stochastic dynamical systems with small diffusion. In [3], Cerrai and Freidlin studied the large deviation principle for the Langevin equation with strong damping by using the integration by parts formula. Later, Nguyen and Yin [14] extended the above results by considering the LDP of the time-inhomogeneous Langevin equations with strong damping in general random environment. They showed that the solution

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of the second order Langevin dynamics and that of a first order equation possess the same LDP assuming the first order equation satisfies the local LDP. In [16], we studied the moderate deviation principle for a class of Langevin dynamic systems with strong damping and Markovian switching. Specifically, we consider

$$\begin{cases} \varepsilon^2\ddot{q}_\varepsilon(t) = b(q_\varepsilon(t), r_\varepsilon(t)) - \alpha_\varepsilon(q_\varepsilon(t))\dot{q}_\varepsilon(t) + \sqrt{\varepsilon}\sigma(q_\varepsilon(t), r_\varepsilon(t))\dot{w}(t) \\ q_\varepsilon(0) = q \in \mathbb{R}^d, \quad \dot{q}_\varepsilon(0) = \frac{p}{\varepsilon} \in \mathbb{R}^d. \end{cases}$$

where $r_\varepsilon(t)$ is a fast-varying continuous-time Markov chain with a finite state space $\mathcal{M}$. The main contribution is that we demonstrated not only do the solution of the second order Langevin dynamics and that of the corresponding first order equation verify the same LDP, but also they satisfy the same moderate deviation principle without local LDP assumptions.

2. Future Research Plans

In the future, I am interested in pursuing such areas related to stochastic (partial) differential equations, limit theorems, Malliavin calculus, large and moderate deviations. I am eager to learn new materials to expand my research horizon with a wide range of applications. In what follows, I will provide an introduction of my future research project.

2.1. **Stochastic reaction diffusion equations in random environment.** Stochastic reaction diffusion equations are very popular to describe the diffusive phenomena in reactive media, such as combustion, dynamics of population and diffusive transport of chemical species. We consider the following stochastic reaction diffusion equation in random environment on domain $D \subset \mathbb{R}^d, d \geq 1$:

$$\begin{cases} \frac{\partial X^\varepsilon}{\partial t}(t,\xi) = \mathcal{A}X^\varepsilon(t,\xi) + f(\xi, X^\varepsilon(t,\xi), \zeta_{t/\varepsilon}) + \sqrt{\varepsilon}g(\xi, X^\varepsilon(t,\xi), \zeta_{t/\varepsilon})\frac{\partial w^Q}{\partial t}(t,\xi) \\ X^\varepsilon(0,\xi) = x_0(\xi), \ \xi \in D, \\ \mathcal{N}X^\varepsilon(t,\xi) = 0, \ t \geq 0, \ \xi \in \partial D, \end{cases} \tag{1}$$

where ε is a small parameter. Here $\mathcal{A}$ is uniformly elliptic second order differential operators with regular real coefficient. The reaction term $f : D \times \mathbb{R} \times E \to \mathbb{R}$ is measurable functions. The operator $\mathcal{N}$ corresponds to either Dirichlet or Robin boundary conditions and the initial values x are assumed to be $L^2(D)$. ζ_t represents the random environment which is an exponentially ergodic Markov process, independent of cylindrical Wiener process $w^Q(t,\xi)$. In above model, $X^\varepsilon(t,\xi)$ is considered as the slow process, while ζ is the fast random environment being of order $1/\varepsilon$. When the term g does not depend on ζ and there is no $\sqrt{\varepsilon}$ in front of g, Gao [6] established the Bogoliubov averaging principle for a class of stochastic reaction-diffusion equations in space dimension one, namely $D = I = (0,1)$, saying: for every fixed $T > 0$ and $\delta > 0$, we have

$$\lim_{\varepsilon \to 0} \mathbb{P}\{||X^\varepsilon - \bar{X}||_{L^2(0,T;H^1(I)) \cap C([0,T];L^2(I))} \geq \delta\} = 0,$$

where $\bar{X}$ is the solution of the following averaged equation

$$\begin{cases} \frac{\partial \bar{X}}{\partial t}(t,\xi) = \mathcal{A}\bar{X}(t,\xi) + \bar{f}(\xi, \bar{X}(t,\xi)) + g(\xi, \bar{X}(t,\xi))\frac{\partial w^Q}{\partial t}(t,\xi) \\ \bar{X}(0,\xi) = x_0(\xi), \ \xi \in D, \\ \mathcal{N}X^\varepsilon(t,\xi) = 0, \ t \geq 0, \ \xi \in \partial D, \end{cases} \tag{2}$$

Before that, when the random environment is governed by another fast stochastic reaction-diffusion equations, that is $\zeta_{t/\varepsilon}$ is replaced by the solution $Y^\varepsilon(t,\xi)$ satisfying the following equation:

$$\begin{cases} \frac{\partial Y^\varepsilon}{\partial t}(t,\xi) = \frac{1}{\varepsilon}[\mathcal{A}_1 Y^\varepsilon(t,\xi) + f_1(\xi, X^\varepsilon(t,\xi), Y^\varepsilon(t,\xi))] + \frac{1}{\sqrt{\varepsilon}}g_1(\xi, X^\varepsilon(t,\xi), Y^\varepsilon(t,\xi))\frac{\partial w^{Q_1}}{\partial t}(t,\xi) \\ Y^\varepsilon(0,\xi) = y_0(\xi), \quad \xi \in D, \\ \mathcal{N}_1 Y^\varepsilon(t,\xi) = 0, \ t \geq 0, \ \xi \in \partial D, \end{cases} \tag{3}$$

Cerrai [2] proved that the slow process X^ε converges weakly to the solution $\bar{X}$ of the similar averaged equation (2) where there is no $\sqrt{\varepsilon}$ in (1). The study of the averaging principle under the stochastic context has been extensively developed in both finite dimension (SDE) and infinite dimension (SPDE); see, for example Khasminskii (1980), Freidlin (1978) and Freidlin and Wentzell (1998) when ζ_t is a diffusion process. Focusing on stochastic reaction-diffusion equations, Hu et al. [10] proved the large deviation principle for multi-scale stochastic reaction-diffusion equations with ε replaced by δ in (3). When $g_1 = 1$ a constant value, Gasteratos [8] established the moderate deviation principle for the slow-fast stochastic reaction-diffusion equations. Different with the case when the random environment is another stochastic partial differential equations (SRDE here), we take the random environment ζ_t an exponentially ergodic Markov process.

Suppose the following ergodic property: there exists a vector field $\bar{f}$ such that

$$\lim_{T\to\infty} \frac{1}{T} \int_{t_0}^{t_0+T} f(\xi, x, \zeta_s) ds = \bar{f}(\xi, x) \quad \mathbb{P}\text{-}a.s. \tag{4}$$

uniformly in $\xi \in D, x \in \mathbb{R}$ and $t_0 \geq 0$. Let us denote $\bar{X}(t,\xi)$ the solution of the averaged system

$$\frac{\partial \bar{X}}{\partial t}(t,\xi) = \mathcal{A}\bar{X}(t,\xi) + \bar{f}(\xi, \bar{X}(t,\xi)). \tag{5}$$

Here we will study the deviations of X^ε from the averaged system $\bar{X}$, as ε decreases to 0. That is, the asymptotic behavior of the trajectory

$$\eta^\varepsilon(t,\xi) = \frac{X^\varepsilon(t,\xi) - \bar{X}(t,\xi)}{\sqrt{\varepsilon} h(\varepsilon)} \tag{6}$$

where $h(\varepsilon)$ is the deviation scale. This can be viewed as a generalization of Guillin argument in [7] to an infinite dimensional equations. The difficulties lie in the combination of the effect of random environment ζ and the Freidlin-Wentzell theory for LDP of small diffusions, and the non-semimartingale stochastic integral due to the semigroup $S(t)$.

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