# Instrument Variable Approach for Causal Inference in Non-Compliance Studies

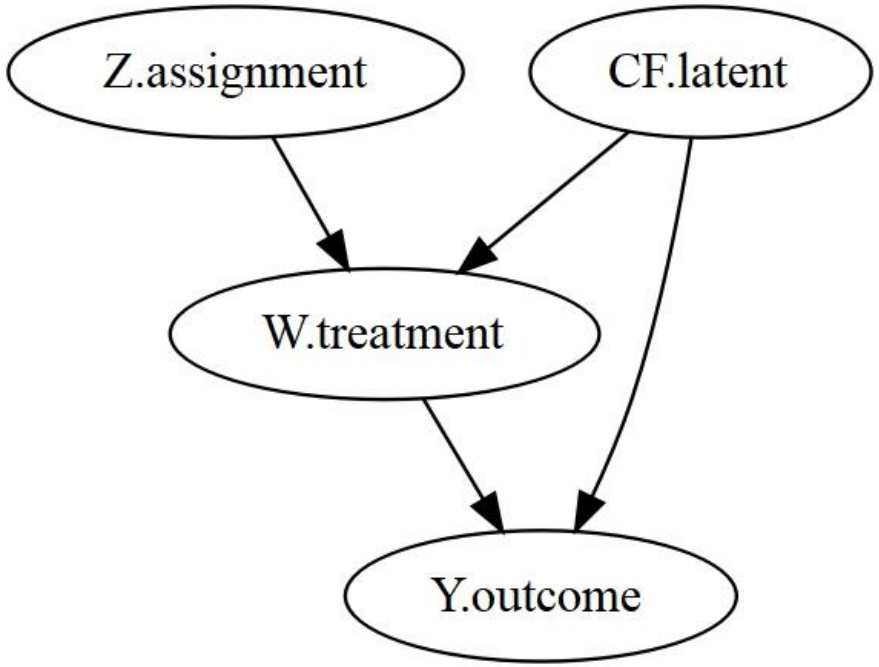
HWang

11-23-2021

# Treatment Effects with Imperfect Compliance

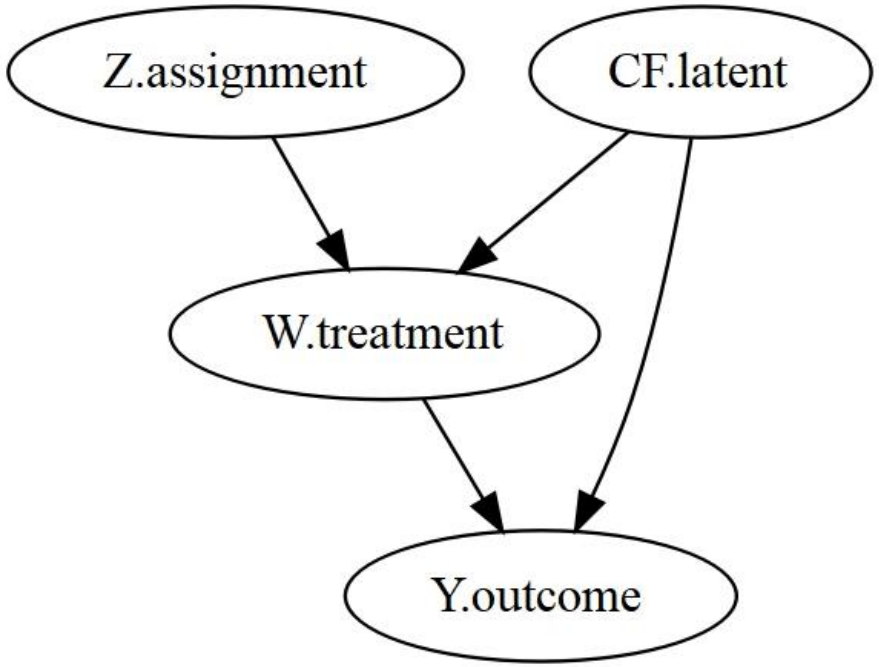
- Villagers are randomly assigned vitamin supplements (or control group) for their infants. But not everyone in the test group received or taken vitamin. The study is interested in infant mortality rates (Imbens and Rubins)
- If we notice people in the test group have lower mortality rate, can we attribute that to our program causally? Yes.
  - ✓But what is our program? It is to assign people vitamin.
- What if we want to know the effect of vitamin?
- There is an obvious difference between the effect of assignment (or encouragement to take the vitamin) from the effect of doing what you are assigned or encouraged to do (taking the vitamin)
- The causal effect of assignment is called ITT (intent-to-treat)

# DAG Representation



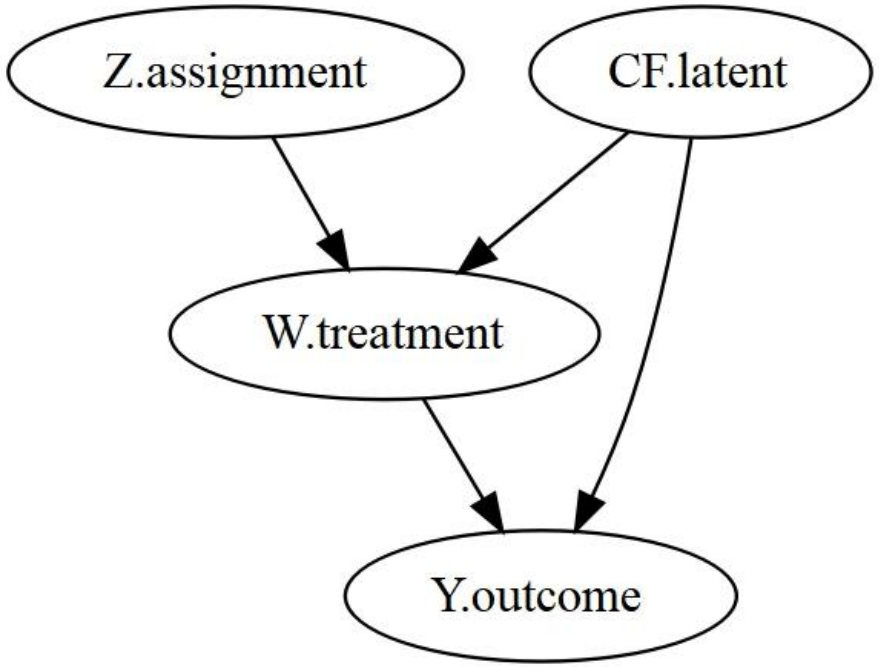

- Z is random. It is therefore unconfounded. Any thing we observe related to the assignment is causal. If we notice a difference in Y, it is causal with respect to assignment, even thought there is no direct link between these two nodes.
  - Notice Z does not have to be random. It only needs to be unconfounded after adjusting for known factors. This is a common misunderstanding.
  - Unconfounded means there is a non-zero (does not have to the same) probability of being assigned to treatment. And the assignment is not related to potential outcome after controlling for known factors. In other words, you do not assign people treatment who are more likely to take the treatment or have higher potential y.
- The node W is endogenously formed. We assume there is a confounder CF that impacts both W and Y. If there is no such confounder, we are all set.

# DAG Representation



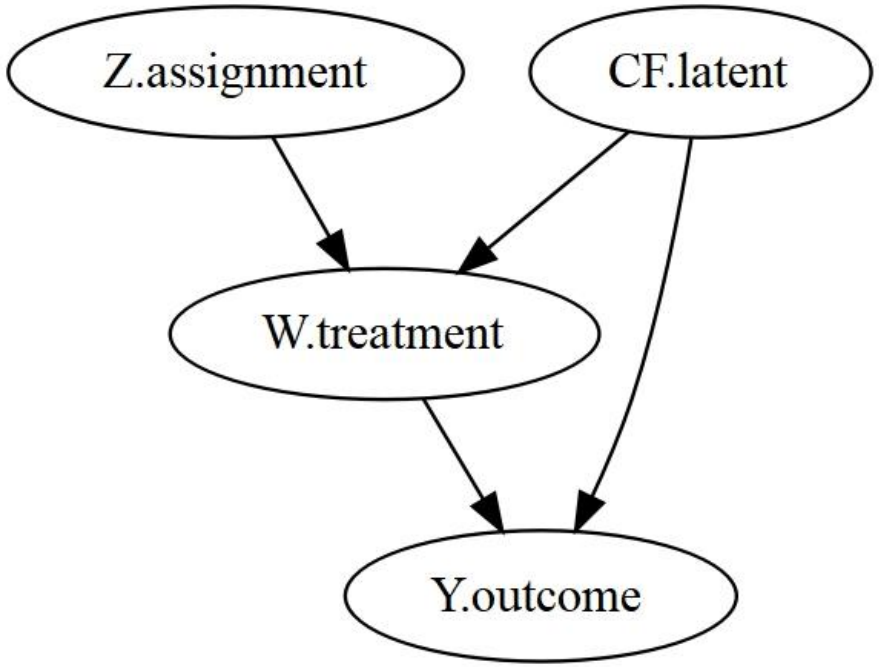

- The effect of Z (assignment) on W (taking the treatment) is causal
- If we know the confounder (CF), we can just control it and the effect of W becomes causal as well. What if we do not know? In general, the relationship between W and Y is NOT causal.
- E(W=1|Z=1)-E(W=1|Z=0) is the causal effect of Z on W.
- E(Y=1|Z=1)-E(Y=1|Z=0) is the causal effect of Z on Y.
- E(Y=1|W=1)-E(Y=1|W=0) is NOT the causal effect of W on Y.
- That does not mean we can not make inference. For example, ITT is a valid test for the null hypotheses of no causal effect of W on Y.
- What is tricky is that if we saw little effect of Z to W, it does not necessarily imply that W did not have causal effect of Y. This is the weak instrument variable problem in econometrics.
- If Z has an effect on Y, it goes through W. In this case, the exogenous Z is a valid instrument variable.

# DAG Representation – Instrument Variable



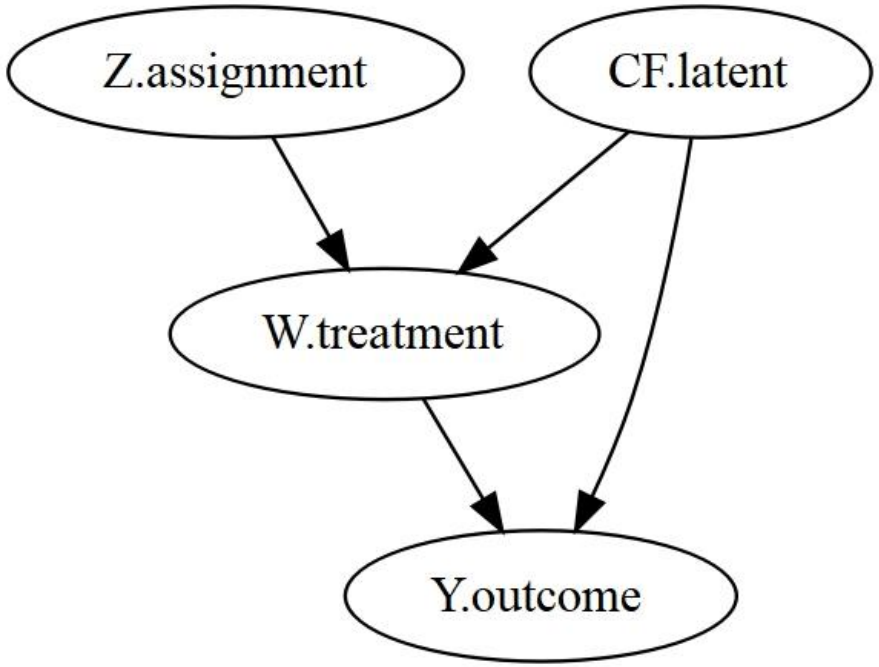

- Good news: If Z is a valid (and strong) instrument, then, we can identify the causal effect of W on Y for a subset of the population
- Bad news: we would not know the size or the identity of this subset
- There are mathematical bounds we can use too
- Motivation for instrument variable – we hope to isolate a portion of the variance in W that is a "random shock". It would be if this is caused by Z.
- An instrument (in this case, the assignment) is a random push toward receiving one treatment rather than another, a push that is random and is without consequence for outcome unless it succeeds in changing the treatment received (From Observation and experiment by Paul Rosenbaum)
- So the subset where we can draw causal inference consists of the people whose behavior (in taking treatment W) is changed by Z
- The estimate is $\frac{E(Y|Z=1)-E(Y|Z=0)}{E(W|Z=1)-E(W|Z=0)}$

# DAG Representation – Derivation using Potential Outcome Framework




- W(Z=0) is the potential outcome of W under assignment
- We assume the existence of four types of people
  - Never takers – they will not take the treatment regardless of the assignment (W(Z=0),W(Z=1)=(0,0)
  - compliers – (W(Z=0),W(Z=1)=(0,1)
  - Always takers- (W(Z=0),W(Z=1)=(1,1)
  - Defiers- (W(Z=0),W(Z=1)=(1,0)
  - Notice we do not observe them, we stipulate their existence and definitions.
  - In most situations, we assume there are no defiers.

# DAG Representation – Derivation using Potential Outcome Framework




- Let Y(z) be the potential outcome of Y given assignment
- Then Y(z=1) –Y(z=0) could be non zero for compliers only.
- W(z=1)-W(z=0) is non zero for only compliers
- $\frac{E(Y|Z=1)-E(Y|Z=0)}{E(W|Z=1)-E(W|Z=0)}$ is the estimated causal effect of W on Y among the compliers

- Note: the concept of compliers could change based on the program policy. For example, if a person could not take the treatment if he is in the control group because it was strictly made not available to him. In this case, a always taker would not be (W(Z=0),W(Z=1)=(1,1). Instead, it would be (W(Z=0),W(Z=1)=(0,1) , just like a complier. So the above derived number would cover the always taker and complier.

# Case Study- Vitamin Study

| randomization | response/ treated | outcome y | size |
|---|---|---|---|
| test | yes | 1 | 9663 |
| test | yes | 0 | 12 |
| test | no | 1 | 2385 |
| test | no | 0 | 34 |
| control | no | 1 | 11514 |
| control | no | 0 | 74 |

All the ITT effects are causal
E(y|test)-E(y|control)=0.00258
The program (which is not 100% complied) leads to 0.00258 reduction of mortality rate
But notice not all the people in the test group took the vitamin
The causal effect of vitamin (among compliers) is **0.00258/0.799=0.003257064812**

| randomizataion | size | size (%) | response rate | avg. y |
|---|---|---|---|---|
| test | 12094 | 0.5106832193 | 0.7999834629 | 0.9961964611 |
| control | 11588 | 0.4893167807 | 0 | 0.9936140835 |
| diff | | | 0.7999834629 | 0.00258237752 |

# The causal based estimates could be very different from the “heuristics”

| | | | |
|---|---|---|---|
| causal effect | effect of response on y by complier | | 0.003257064812 |
| | | | |
| as-treated approach | avg. y among treated | | 0.9987596899 |
| | avg y among non-treated (including control) | | 0.9922895695 |
| | diff | | 0.006470120422 |
| | | | |
| per protocal analyes | avg. y among treated in test (complier) | | 0.9987596899 |
| | avg y among control | | 0.9936140835 |
| | diff | | 0.005145606388 |
| | | | |
| under conditional unconfoundedness<br>Given the instrument | avg. y among treated in test | | 0.9987596899 |
| | avg. y among not treated in test | | 0.9859446052 |
| | diff | | 0.01281508471 |

# Simulation Code for Illustrations

```
library(tidyverse)
nbr<-100000
status<-c("compliers","never takers","always takers","defiers")
mydata<-data.frame(status=sample(status,nbr,replace=TRUE,prob=c(0.2,0.3,0.5,0)))
# potential outcomes for each person based on the segment
mydata<-mydata%>%
  mutate(status=as.factor(status),
         xbeta.base=-0.5+0.4*(status=="compliers")+0.2*(status=="never takers")
         +0.1*(status=="always takers"),
         prob.base=exp(xbeta.base)/(1+exp(xbeta.base)),
         potential.y0=rbinom(nbr,1,prob.base),
         prob=prob.base+0.05*(status=="compliers")+
                  0.15*(status=="never takers")+
                  0*(status=="always takers"),
         potential.y1=rbinom(nbr,1,prob))%>%
  select(-xbeta.base)

# gives what the causal effects would have been if we observed them

mydata%>%group_by(status)%>%
  summarize(meanp0=mean(prob.base),
            meanp1=mean(prob),
            diffp=meanp1-meanp0,
            meany0=mean(potential.y0),
            meany1=mean(potential.y1),
            diffy=meany1-meany0)

# notice intent is the random assignment
mydata<-mydata%>%
  mutate(intent=rbinom(nbr,1,0.5),
         treat=ifelse(status=="always takers",1,
                  ifelse(status=="never takers",0,intent)),
         y.obs=ifelse(treat==1,potential.y1,potential.y0))

mydata%>%group_by(status,intent,treat)%>%summarize(counts=n())


mydata%>%group_by(intent)%>%summarize(treat_rate=mean(treat), y.obs=mean(y.obs)
```

# Selected References


- Causal Inference for Statistics ,Social and Biomedical Sciences, Guido Imbens & Donald Rubin
- Observation & Experiment, an Introduction to Causal Inference, Paul Rosenbaum
- Causality, Statistical Perspectives and Applications, Edited by Berzuini et. al.
- Causation, Prediction, and Search, Peter Spirtes, Clark Glymour and Richard Scheines
- Counterfactuals and Causal Inferences, Stephen Morgan and Christopher Winship
- Elements of Causal Inference, Foundations and Learning Algorithms, Peters, et. al.
- Statistical analysis with missing data, Little and Rubin
- Many great work by Pearl