# Breast Cancer Diagnosis

Hongjie Liu, Xicheng Xie, Jiajun Tao, Shaohan Chen, Yujia Li

2023-02-27

# Outline

- Background
- Task Introduction
- Task 1
- Task 2
- Task 3
- Task 4
- Discussions
- Limitations and Future Work
- Reference
- Q&A

# Background

**Breast Cancer Diagnosis:**

- In this project we study the breast cancer diagnosis problem
- The goal of the exercise is to build a predictive model based on logistic regression to facilitate cancer diagnosis
- We move towards the goal with the steps of task 1, 2, 3 and 4.

# Background

**Data Source:**

- The data is the breast cancer medical data retrieved from "breast-cancer.csv", which has 569 rows and 32 columns
- The first column `ID` labels individual breast tissue images; The second column `Diagnosis` identifies if the image is coming from cancer tissue or benign cases (M = malignant, B = benign). There are 357 benign and 212 malignant cases
- The other 30 columns correspond to mean, standard deviation and the largest values (points on the tails) of the distributions of the following 10 features computed for the cellnuclei

# Task Introduction

- Task 1: Build a logistic model to classify the images into malignant/benign, and write down your likelihood function, its gradient and Hessian matrix.
- Task 2: Develop a Newton-Raphson algorithm to estimate your model.
- Task 3: Build a logistic-LASSO model to select features, and implement a path-wise coordinate-wise optimization algorithm to obtain a path of solutions with a sequence of descending $\lambda$'s.
- Task 4: Use 5-fold cross-validation to select the best $\lambda$. Compare the prediction performance between the 'optimal' model and 'full' model

# Task 1 - Objective

**Objective:**

Build a logistic model to classify the images into malignant/benign, and write down your likelihood function, its gradient and Hessian matrix.

# Task 1 - Build a logistic model

Define the "Diagnosis" variable will be coded as 1 for malignant cases and 0 for benign cases.

Given $n$ i.i.d. observations with $p$ predictors, we consider a logistic regression model

$$P(Y_i = 1 \mid \mathbf{x}_i) = \frac{e^{\mathbf{x}_i^\top \boldsymbol{\beta}}}{1 + e^{\mathbf{x}_i^\top \boldsymbol{\beta}}},\ i = 1, \ldots, n \tag{1}$$

where $\boldsymbol{\beta} = (\beta_0, \beta_1, \ldots, \beta_p)^\top \in \mathbb{R}^{p+1}$ is the parameter vector, $\mathbf{x}_i = (1, X_{i1}, \ldots, X_{ip})^\top$ is the vector of predictors in the $i$-th observation, and $Y_i \in \{0, 1\}$ is the binary response in the $i$-th observation.

# Task 1 - Build a logistic model

Let $\mathbf{y} = (Y_1, Y_2, \ldots, Y_n)^\top$ denote the response vector, and $\mathbf{X} = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n)^\top \in \mathbb{R}^{n\times(p+1)}$ denote the design matrix.

The observed likelihood of $\{(Y_1, \mathbf{x}_1), (Y_2, \mathbf{x}_2) \ldots, (Y_n, \mathbf{x}_n)\}$ is

$$L(\beta; \mathbf{y}, \mathbf{X}) = \prod_{i=1}^{n} \left[ \left( \frac{e^{\mathbf{x}_i^\top \beta}}{1 + e^{\mathbf{x}_i^\top \beta}} \right)^{Y_i} \left( \frac{1}{1 + e^{\mathbf{x}_i^\top \beta}} \right)^{1-Y_i} \right]$$

# Task 1 - Build a logistic model

Maximizing the likelihood is equivalent to maximizing the log-likelihood function:

$$f(\beta; \mathbf{y}, \mathbf{X}) = \sum_{i=1}^{n} \left[ Y_i \mathbf{x}_i^\top \beta - \log\left(1 + e^{\mathbf{x}_i^\top \beta}\right)\right]. \tag{2}$$

The estimates of model parameters are

$$\widehat{\boldsymbol{\beta}} = \arg\max_{\boldsymbol{\beta}} \ f(\beta; \mathbf{y}, \mathbf{X}),$$

and the optimization problem is

$$\max_{\boldsymbol{\beta}} \ f(\beta; \mathbf{y}, \mathbf{X}). \tag{3}$$

# Task 1 - Build a logistic model

Denote $p_i = P(Y_i = 1 \mid \mathbf{x}_i)$ as given in (1) and $\mathbf{p} = (p_1, p_2, \ldots, p_n)^\top$. The gradient of $f$ is:

$$
\begin{aligned}
\nabla f(\beta; \mathbf{y}, \mathbf{X}) &= \mathbf{X}^\top(\mathbf{y} - \mathbf{p}) \\
&= \sum_{i=1}^{n} (Y_i - p_i)\mathbf{x}_i \\
&= \begin{pmatrix} \sum_{i=1}^{n}(Y_i - p_i) \\ \sum_{i=1}^{n}(Y_i - p_i)X_{i1} \\ \vdots \\ \sum_{i=1}^{n}(Y_i - p_i)X_{ip} \end{pmatrix}
\end{aligned}
$$

# Task 1 - Build a logistic model

Denote $w_i = p_i(1 - p_i) \in (0, 1)$ and $\mathbf{W} = \mathrm{diag}(w_1, \ldots, w_n)$. The Hessian matrix of $f$ is given by

$$
\begin{aligned}
\nabla^2 f(\beta; \mathbf{y}, \mathbf{X}) &= -\mathbf{X}^\top \mathbf{W} \mathbf{X} \\
&= -\sum_{i=1}^{n} w_i \mathbf{x}_i \mathbf{x}_i^\top \\
&= -\begin{pmatrix}
\sum_{i=1}^n w_i & \sum_{i=1}^n w_i X_{i1} & \cdots & \sum_{i=1}^n w_i X_{i1} \\
\sum_{i=1}^n w_i X_{i1} & \sum_{i=1}^n w_i X_{i1}^2 & \cdots & \sum_{i=1}^n w_i X_{i1} X_{ip} \\
\vdots & \vdots & \ddots & \vdots \\
\sum_{i=1}^n w_i X_{ip} & \sum_{i=1}^n w_i X_{in} X_{i1} & \cdots & \sum_{i=1}^n w_i X_{ip}^2
\end{pmatrix}
\end{aligned}
$$

# Task 1 - Build a logistic model

Next, we show that the Hessian matrix $\nabla^2 f(\beta; \mathbf{y}, \mathbf{X})$ is a negative-definite matrix if $\mathbf{X}$ has full rank.

***Proof.*** For any $(p+1)$-dimensional nonzero vector $\boldsymbol{\alpha}$, given that $\mathbf{X}$ has full rank, $\mathbf{X}\boldsymbol{\alpha}$ is also a nonzero vector. Since $\mathbf{W}$ is positive-definite, we have

$$\begin{aligned}\boldsymbol{\alpha}^\top \nabla^2 f(\beta; \mathbf{y}, \mathbf{X})\boldsymbol{\alpha} &= \boldsymbol{\alpha}^\top(-\mathbf{X}^\top \mathbf{W}\mathbf{X})\boldsymbol{\alpha} \\ &= -(\mathbf{X}\boldsymbol{\alpha})^\top \mathbf{W}(\mathbf{X}\boldsymbol{\alpha}) \\ &< 0.\end{aligned}$$

Thus, $\nabla^2 f(\beta; \mathbf{y}, \mathbf{X})$ is negative-definite. $\square$

Hence, the optimization problem (3) is a well-defined problem.

# Task 1 - Logistic model R code

```r
cancer <- read.csv("breast-cancer.csv") %>%
janitor::clean_names()%>%
select(-1,-33) %>%
mutate(diagnosis = recode(diagnosis, "M" = 1, "B" = 0))
cor()
ggcorrplot(corr, type = "upper", tl.cex = 8)

set.seed(1)
trainRows <- createDataPartition(y = cancer$diagnosis, p = 0.8, list = FALSE)
train <- cancer[trainRows, ]
test <-  cancer[-trainRows, ]
glm.fit <- glm(diagnosis ~ .,
               data = train,
               subset = trainRows,
               family = binomial(link = "logit"))
summary(glm.fit)
pred <- predict(glm.fit, newdata = test, type = "response")
y_test <- factor(test$diagnosis)
auc_full <- auc(y_test, pred)
auc_full
```

- Final AUC of full model reaches 0.9641.
- And if we remove correlated variables, the AUC is uplifted to 0.9962.

# Task 2 - Objective

**Objective:**

Develop a Newton-Raphson algorithm to estimate your model

- Recall The target function $f$ given in task 1 is:

$$f(\beta; \mathbf{y}, \mathbf{X}) = \sum_{i=1}^{n} \left[ Y_i \mathbf{x}_i^\top \beta - \log\left(1 + e^{\mathbf{x}_i^\top \beta}\right) \right].$$

- We develop a modified Newton-Raphson algorithm including a step-halving step to maximize the target function.

# Task 2 - Newton-Raphson algorithm design

**Algorithm 1** Newton-Raphson algorithm

**Require:** $f(\boldsymbol{\beta})$ - target function as given in (2); $\boldsymbol{\beta}_0$ - starting value
**Ensure:** $\widehat{\boldsymbol{\beta}}$ such that $\widehat{\boldsymbol{\beta}} \approx \arg\max_{\boldsymbol{\beta}} f(\boldsymbol{\beta})$

$i \leftarrow 0$, where $i$ is the current number of iterations
$f(\boldsymbol{\beta}_{-1}) \leftarrow -\infty$
**while** convergence criterion is not met **do**
  $i \leftarrow i + 1$
  $\mathbf{d}_i \leftarrow -[\nabla^2 f(\boldsymbol{\beta}_{i-1})]^{-1} \nabla f(\boldsymbol{\beta}_{i-1})$, where $\mathbf{d}_i$ is the direction in the $i$-th iteration
  $\lambda_i \leftarrow 1$, where $\lambda_i$ is the multiplier in the $i$-th iteration
  $\boldsymbol{\beta}_i \leftarrow \boldsymbol{\beta}_{i-1} + \lambda_i \mathbf{d}_i$
  **while** $f(\boldsymbol{\beta}_i) \leq f(\boldsymbol{\beta}_{i-1})$ **do**
    $\lambda_i \leftarrow \lambda_i / 2$
    $\boldsymbol{\beta}_i \leftarrow \boldsymbol{\beta}_{i-1} + \lambda_i \mathbf{d}_i$
  **end while**
**end while**
$\widehat{\boldsymbol{\beta}} \leftarrow \boldsymbol{\beta}_i$

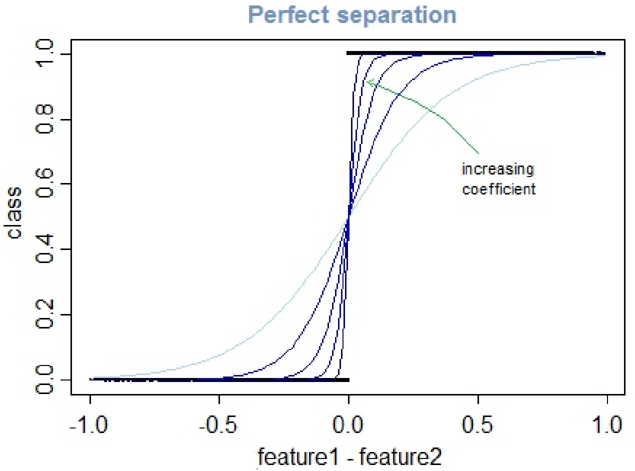
# Task 2 - Newton-Raphson algorithm R code

```
NewtonRaphson <- function(dat, func, start, tol = 1e-8, maxiter = 200) {
  i <- 0
  cur <- start
  stuff <- func(dat, cur)
  res <- c(0, stuff$f, cur)
  prevf <- -Inf
  X <- cbind(rep(1, nrow(dat)), as.matrix(dat[, -1]))
  y <- dat[, 1]
  warned <- 0
  while (abs(stuff$f - prevf) > tol && i < maxiter) {
    i <- i + 1
    prevf <- stuff$f
    prev <- cur
    d <- -solve(stuff$Hess) %*% stuff$grad
    cur <- prev + d
    lambda <- 1
    maxhalv <- 0
    while (func(dat, cur)$f < prevf && maxhalv < 50) {
      maxhalv <- maxhalv + 1
      lambda <- lambda / 2
      cur <- prev + lambda * d
    }
    stuff <- func(dat, cur)
    res <- rbind(res, c(i, stuff$f, cur))
    y_hat <- ifelse(X %*% cur > 0, 1, 0)
    if (warned == 0 && sum(y - y_hat) == 0) {
      warning("Complete separation occurs. Algorithm does not converge.")
      warned <- 1
    }
  }
  colnames(res) <- c("iter", "target_function", "(Intercept)", names(dat)[-1])
  return(res)
}
```

# Task 2 - Complete separtion

- Sometimes our algorithm does not converge because of the complete separation.
- A complete separation in a logistic regression, sometimes also referred as perfect prediction, occurs whenever there exists some vector of coefficients $\beta$ such that $Y_i = 1$ whenever $\mathbf{x}_i^\top \beta > 0$ and $Y_i = 0$ whenever $\mathbf{x}_i^\top \beta \leq 0$.
- Complete separation occur when a linear function of predictors can perfectly classify the response.

# Task 2 - Complete separation



# Task 2 - Complete separation

- We have proved that: when there exists a vector of coefficients $\hat{\beta}$ such that $Y_i = 1$ whenever $\mathbf{x}_i^\top \hat{\beta} > 0$ and $Y_i = 0$ whenever $\mathbf{x}_i^\top \hat{\beta} \leq 0$, there does not exist $\beta^* \in \mathbb{R}^{(p+1)}$ such that $\beta^* = \arg\max_\beta f(\beta)$, where $f$ is given in (2). Thus our algorithm does not converge. (proof is attached in report appendix)
- If there is no complete separation, the parameters output by `glm` function and our algorithm are demonstrated to be the same
- In practice, if complete separation occurs, we randomly pick the parameters which satisfy complete separation as our full model

# Task 2 - Comparison

Comparison of using `glm` function and our algorithm (part of)

| predictor | ours | glm |
|---|---:|---:|
| (Intercept) | 111.7230206 | 90.9690365 |
| radius_mean | -3646.8235387 | -2560.3938902 |
| texture_mean | -2.8949199 | 0.8812037 |
| perimeter_mean | 1257.9481173 | 789.2724398 |
| area_mean | 2091.5001210 | 1539.8345650 |
| smoothness_mean | 180.4591375 | 128.8762247 |
| compactness_mean | -471.3749334 | -346.6691873 |
| concavity_mean | -0.3063034 | 9.0810082 |
| concave.points_mean | 287.3555092 | 215.4808031 |
| symmetry_mean | 10.8459784 | 10.4772590 |
| fractal_dimension_mean | -40.6413497 | -28.6916549 |
| radius_se | 854.4290933 | 617.5687358 |
| texture_se | -105.8920782 | -79.9788638 |
| perimeter_se | -1231.6514585 | -917.3916527 |
| area_se | 789.0550146 | 628.6222313 |
| smoothness_se | -83.3449730 | -63.7660367 |
| compactness_se | 473.1970464 | 344.3180183 |
| concavity_se | -440.6629325 | -323.8533730 |
| concave.points_se | 503.3728692 | 374.7610875 |
| symmetry_se | -144.4580558 | -108.6211792 |

# Task 3 - Objective

**Objective:**

Build a logistic-LASSO model to select features by implementing a path-wise coordinate-wise optimization algorithm

- Recall Log-likelihood $f$ in task 1:

$$f(\beta; \mathbf{y}, \mathbf{X}) = \sum_{i=1}^{n} \left[ Y_i \mathbf{x}_i^\top \beta - \log \left( 1 + e^{\mathbf{x}_i^\top \beta} \right) \right].$$

- LASSO estimates the logistic model parameters $\beta$ by optimizing a penalized loss function:

$$\min_{\beta} \; -\frac{1}{n} f(\beta) + \lambda \sum_{k=1}^{p} |\beta_k|. \tag{4}$$

where $\lambda \geq 0$ is the tuning parameter. Note that the intercept is not penalized and all predictors are standardized.

# Task 3 - Algorithm of Logistic-LASSO Model

**Algorithm Structure:**

- OUTER LOOP: Decrement $\lambda$.
- MIDDLE LOOP: Update $\tilde{w}_i$, $\tilde{p}_i$, and thus the quadratic approximation $\ell$ using the current parameters $\tilde{\beta}$.
- INNER LOOP: Run the coordinate descent algorithm on the penalized weighted-least-squares problem.

# Task 3 - OUTER LOOP

**OUTER LOOP:**

Compute the solutions of the optimization problem (4) for a decreasing sequence of values for $\lambda$: $\{\lambda_1, \ldots, \lambda_m\}$, starting at the smallest value $\lambda_1 = \lambda_{max}$

$$\lambda_{max} = \frac{1}{n} \max_j |\langle \mathbf{x}_{\cdot j}, \mathbf{y} \rangle| , \tag{5}$$

where $\mathbf{x}_{\cdot j}$ is the $j$-th column of the design matrix $\mathbf{X}$, for $j = 1, \ldots, p$.

For tuning parameter value $\lambda_{k+1}$, we initialize coordinate descent algorithm at the computed solution for $\lambda_k$ (warm start).

# Task 3 - MIDDLE LOOP

**MIDDLE LOOP:**

- For a fixed $\lambda$, find the estimates of $\beta$ by solving the optimization problem (4).

Based on current parameter estimates $\tilde{\beta}$, we we form a quadratic approximation to the log-likelihood $f$ using a Taylor expansion:

$$
\begin{aligned}
f(\beta) \approx \ell(\beta) &= f(\tilde{\beta}) + (\beta - \tilde{\beta})^\top \nabla f(\tilde{\beta}) + \frac{1}{2}(\beta - \tilde{\beta})^\top \nabla^2 f(\tilde{\beta})(\beta - \tilde{\beta}) \\
&= -\frac{1}{2}\sum_{i=1}^{n} \tilde{w}_i \left[\mathbf{x}_i^\top(\tilde{\beta} - \beta) + \frac{Y_i - \tilde{p}_i}{\tilde{w}_i}\right] + \frac{1}{2}\sum_{i=1}^{n} \tilde{w}_i \left(\frac{Y_i - \tilde{p}_i}{\tilde{w}_i}\right)^2 + f(\tilde{\beta}),
\end{aligned}
$$

where $\tilde{\mathbf{p}} = (\tilde{p}_1, \ldots, \tilde{p}_n)^\top$ and $\tilde{\mathbf{W}} = \mathrm{diag}(\tilde{w}_1, \ldots, \tilde{w}_n)$ are the estimates of $\mathbf{p}$ and $\mathbf{W}$ based on $\tilde{\beta}$.

# Task 3 - MIDDLE LOOP

- Let $\tilde{z}_i = \mathbf{x}_i^\top \tilde{\beta} + \frac{Y_i - \tilde{p}_i}{\tilde{w}_i}$, we have

$$\ell(\beta) = -\frac{1}{2}\sum_{i=1}^{n} \tilde{w}_i(\tilde{z}_i - \mathbf{x}_i^\top \beta)^2 + C(\tilde{\beta}), \tag{6}$$

  where is $\tilde{z}_i$ the working response, $\tilde{w}_i$ is the working weight, and $C$ is a function that does not depend on $\beta$.

# Task 3 - INNER LOOP

**INNER LOOP:**

With fixed $\tilde{w}_i$'s, $\tilde{z}_i$'s, and a fixed form of $\ell$ based on the estimates of $\boldsymbol{\beta}$ in the previous iteration of the middle loop, we use coordinate descent to update $\boldsymbol{\beta}$ by solving

$$\min_{\boldsymbol{\beta}} \; -\frac{1}{n}\ell(\boldsymbol{\beta}) + \lambda \sum_{k=1}^{p} |\beta_k|, \tag{7}$$

# Task 3 - INNER LOOP

- Coordinate-wise objective function Based the current estimates $\tilde{\beta}_k$ for $k \neq j$:

$$\min_{\beta_j} \frac{1}{2n} \sum_{i=1}^{n} \tilde{w}_i \left( \tilde{z}_i - x_{ij}\beta_j - \sum_{k \neq j} x_{ik}\tilde{\beta}_k \right)^2 + \lambda|\beta_j| + \lambda \sum_{k \neq j} |\tilde{\beta}_k|.$$

- Updates:

$$\tilde{\beta}_0 \leftarrow \frac{\sum_{i=1}^{n} \tilde{w}_i (\tilde{z}_i - \sum_{k=1}^{p} x_{ik}\tilde{\beta}_k)}{\sum_{i=1}^{n} \tilde{w}_i},$$

$$\tilde{\beta}_j \leftarrow \frac{S\left( \frac{1}{n} \sum_{i=1}^{n} \tilde{w}_i x_{ij} (\tilde{z}_i - \sum_{k \neq j} x_{ik}\tilde{\beta}_k), \lambda \right)}{\frac{1}{n} \sum_{i=1}^{n} \tilde{w}_i x_{ij}^2}, \ j = 1, \ldots, p$$

where $S(z, \gamma)$ is the soft-thresholding operator with value

$$S(z,\gamma) = \mathrm{sign}(z)(|z| - \gamma)_+ = \begin{cases} z - \gamma, & \text{if } z > 0 \text{ and } \gamma < |z| \\ z + \gamma, & \text{if } z < 0 \text{ and } \gamma < |z| \\ 0, & \text{if } \gamma \geq |z| \end{cases}$$

Keep updating estimates of $\beta_j$'s repeatedly for $j = 0, 1, 2, ..., p, 0, 1, 2, ...$ until convergence.

# Task 3 - Path-wise coordinate-wise optimization algorithm

**Algorithm 1** Path-wise coordinate-wise optimization algorithm

**Require:** $g(\boldsymbol{\beta}, \lambda) = -\frac{1}{n} f(\boldsymbol{\beta}) + \lambda \sum_{k=1}^{p} |\beta_k|$ - target function, where $f(\boldsymbol{\beta})$ is given in (1); $\boldsymbol{\beta}_0$ - starting value; $\{\lambda_1, \ldots, \lambda_m\}$ - a sequence of descending $\lambda$'s, where $\lambda_1 = \lambda_{max}$ is given in (3); $\epsilon$ - tolerance; $N_s$, $N_t$ - maximum number of iterations of the middle and inner loops

**Ensure:** $\widehat{\boldsymbol{\beta}}(\lambda_r)$ such that $\widehat{\boldsymbol{\beta}}(\lambda_r) \approx \arg\min_{\boldsymbol{\beta}} \ g(\boldsymbol{\beta}, \lambda_r), \ r = 1, \ldots, m$

$\tilde{\boldsymbol{\beta}}_0(\lambda_1) \leftarrow \boldsymbol{\beta}_0$
OUTER LOOP
**for** $r \in \{1, \ldots, m\}$, where $r$ is the current number of iterations of the outer loop, **do**
    $s \leftarrow 0$, where $s$ is the current number of iterations of the middle loop
    $g(\tilde{\boldsymbol{\beta}}_{-1}(\lambda_r), \lambda_r) \leftarrow \infty$
    MIDDLE LOOP
    **while** $t \geq 2$ and $s < N_s$ **do**
        $s \leftarrow s + 1$
        Update $\tilde{w}_i^{(s)}$, $\tilde{z}_i^{(s)}$ $(i = 1, \ldots, n)$, and thus $\ell_s(\boldsymbol{\beta})$ as given in (4) based on $\tilde{\boldsymbol{\beta}}_{s-1}(\lambda_r)$
        $t \leftarrow 0$, where $t$ is the current number of iterations of the inner loop
        $\tilde{\boldsymbol{\beta}}_s^{(0)}(\lambda_r) \leftarrow \tilde{\boldsymbol{\beta}}_{s-1}(\lambda_r)$
        $h_s(\tilde{\boldsymbol{\beta}}_s^{(-1)}(\lambda_r), \lambda_r) \leftarrow \infty$, where $h_s(\boldsymbol{\beta}, \lambda) = -\frac{1}{n} \ell_s(\boldsymbol{\beta}) + \lambda \sum_{k=1}^{p} |\beta_k|$
        INNER LOOP
        **while** $\left| h_s(\tilde{\boldsymbol{\beta}}_s^{(t)}(\lambda_r), \lambda_r) - h_s(\tilde{\boldsymbol{\beta}}_s^{(t-1)}(\lambda_r), \lambda_r) \right| > \epsilon$ and $t < N_t$ **do**
            $t \leftarrow t + 1$
            $\tilde{\beta}_0^{(t)}(\lambda_r) \leftarrow \sum_{i=1}^{n} \tilde{w}_i^{(s)} \left( \tilde{z}_i^{(s)} - \sum_{k=1}^{p} x_{ik} \tilde{\beta}_k^{(t-1)}(\lambda_r) \right) \Big/ \sum_{i=1}^{n} \tilde{w}_i^{(s)}$
            **for** $j \in \{1, \ldots, p\}$ **do**
                $\tilde{\beta}_j^{(t)}(\lambda_r) \leftarrow S \left( \frac{1}{n} \sum_{i=1}^{n} \tilde{w}_i^{(s)} x_{ij} \left( \tilde{z}_i^{(s)} - \sum_{k<j} x_{ik} \tilde{\beta}_k^{(t)}(\lambda_r) - \sum_{k>j} x_{ik} \tilde{\beta}_k^{(t-1)}(\lambda_r) \right), \lambda_r \right) \Big/ \frac{1}{n} \sum_{i=1}^{n} \tilde{w}_i^{(s)} x_{ij}^2$
            **end for**
        **end while**
        $\tilde{\boldsymbol{\beta}}_s(\lambda_r) \leftarrow \tilde{\boldsymbol{\beta}}_s^{(t)}(\lambda_r)$
    **end while**
    $\widehat{\boldsymbol{\beta}}(\lambda_r) \leftarrow \tilde{\boldsymbol{\beta}}_s(\lambda_r)$
    $\tilde{\boldsymbol{\beta}}_0(\lambda_{r+1}) \leftarrow \widehat{\boldsymbol{\beta}}(\lambda_r)$
**end for**

# Task 4

# Discussions

- There is much freedom when designing the simulations.
- In our algorithm, we have 5 parameters. n, p, ratio, c, corr
- More parameters can be adjusted.

# Limitations and Future Work

- Limitation: We reproduced high-dimensional scenarios, but we still don't know the solution.
- Future Work: We may adjust other parameters to investigate further.

# Reference


1. Li Y, Hong HG, Ahmed SE, Li Y. Weak signals in high-dimensional regression: Detection, estimation and prediction. Appl Stochastic Models Bus Ind. 2018;1–16. https://doi.org/10.1002/asmb.2340

# Q&A

- Thanks for listening!
- Any questions?