Bayesian Modeling of Hurricane Trajectories

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Outline

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Introduction

Background:

- A hurricane is a large and powerful tropical cyclone that typically forms over warm ocean waters and can cause significant damage and destruction to coastal areas.

Motivation:

- Researchers are interested in modeling hurricane trajectories to forecast wind speed to predict the severity or to develop protective measures.

Introduction

Data Source:

- "hurricane703.csv" collected the track data (every 6 hours) of 702 hurricanes in the North Atlantic area since 1950.

Variables:

- ID: ID of the hurricanes
- Season: In which the hurricane occurred
- Month: In which the hurricane occurred
- Nature: Nature of the hurricane
  ET: Extra Tropical
  DS: Disturbance
  NR: Not Rated
  SS: Sub Tropical
  TS: Tropical Storm
- time: dates and time of the record
- Latitude and Longitude: The location of a hurricane check point
- Wind.kt: Maximum wind speed (in Knot) at each check point

EDA

Data pre-processing:

- We only kept observations that occurred on 6-hour intervals.
- We found that some hurricanes had the same ID but were actually different ones.
- We excluded hurricanes that had fewer than 3 observations.
- We defined August, September, and October as active season, the rest as inactive season.
- After the process, there are 21691 observations across 704 unique hurricanes.

EDA

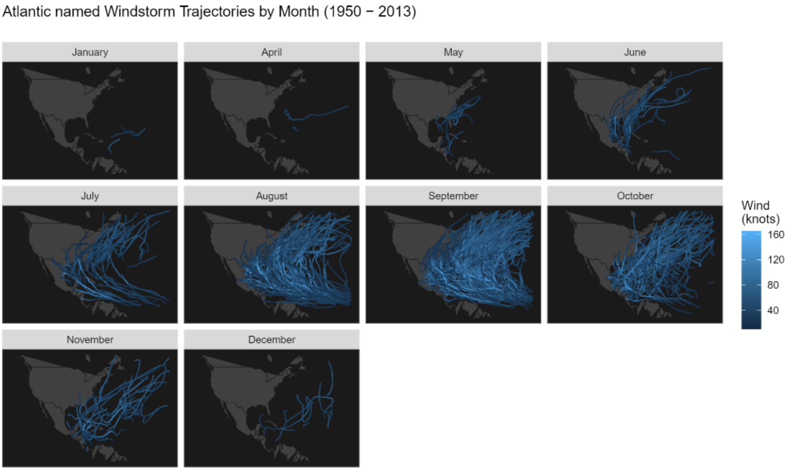


Figure 1: Trajectories of Hurricanes by Month

EDA

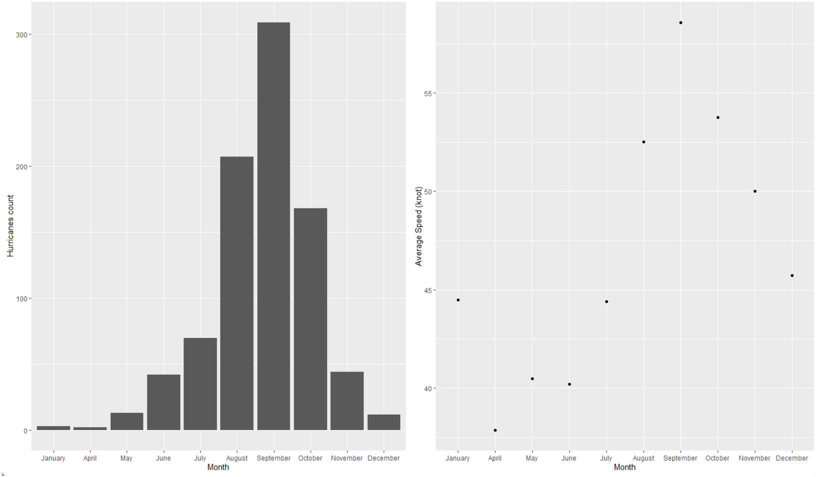


Figure 2: Count and Average Speed of Hurricanes in each Month

EDA

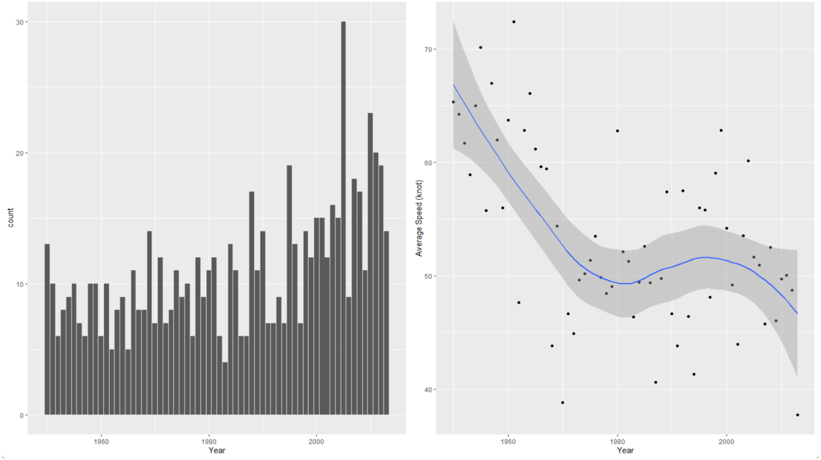


Figure 3: Count and Average Speed of Hurricanes in each Year

Bayesian Model

Let $Y_i(t)$ denote the wind speed of the ith hurricane at time t (in hours) since the hurricane began. The following Bayesian model was suggested to model the wind speed of the ith hurricane 6 hours later:

$$\begin{aligned} Y_i(t+6) = \beta_{0,i} + \beta_{1,i} Y_i(t) + \beta_{2,i}\Delta_{i,1}(t) + \beta_{3,i}\Delta_{i,2}(t) + \beta_{4,i}\Delta_{i,3}(t) \\ + \mathbf{X}_i^\top \boldsymbol{\gamma} + \epsilon_i(t), \end{aligned}$$

where

- $\Delta_{i,1}(t)$, $\Delta_{i,2}(t)$ and $\Delta_{i,3}(t)$: changes of latitude, longitude and wind speed between $t-6$ and t
  - random coefficients $\boldsymbol{\beta}_i = (\beta_{0,i}, \beta_{1,i}, ..., \beta_{4,i})^\top$
- $\mathbf{X}_i = (x_{i,1}, \dots, x_{i,6})^\top$: covariates with fixed effects $\boldsymbol{\gamma}$, where
  - $x_{i,1}$: the calendar year of the i-th hurricane
  - $x_{i,2}$: indicator variable of the month in active season (August-October) when the i-th hurricane started
  - $x_{i,3}, \dots, x_{i,6}$: indicator variables of the type (ES, NR, SS, TS) of the i-th hurricane
- $\epsilon_{i,t} \sim N(0, \sigma^2)$, independent across t

Task 1 - Prior Distributions

Objective: Let $\mathbf{B} = (\beta_1^\top, ..., \beta_n^\top)^\top$, derive the posterior distribution of the parameters $\Theta = (\mathbf{B}^\top, \boldsymbol{\mu}^\top, \boldsymbol{\Sigma}, \boldsymbol{\gamma}^\top, \sigma)$.

We assume that

- $\beta_i \overset{i.i.d.}{\sim} N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$
- $\boldsymbol{\mu} \sim N(\mathbf{0}, \boldsymbol{V})$
- $\boldsymbol{\Sigma}$: an inverse-Wishart distribution with d.f. ν and scale matrix $\boldsymbol{S}$
- $\gamma \sim N(\mathbf{0}, 0.05^2 \boldsymbol{I}_6)$
- σ: a half-Cauchy distribution with scale parameter 10

We set $\boldsymbol{V} = \boldsymbol{S} = \boldsymbol{I}_5$, and $\nu = 5$.

Task 1 - Joint Prior Distribution of Parameters

Let n denote the number of hurricanes in the dataset. The prior distribution of $\Theta = (\mathbf{B}^\top, \boldsymbol{\mu}^\top, \boldsymbol{\Sigma}, \boldsymbol{\gamma}^\top, \sigma)$ is given by

$$
\begin{aligned}
\pi(\Theta) &= \pi(\mathbf{B}^\top, \boldsymbol{\mu}^\top, \boldsymbol{\Sigma}, \boldsymbol{\gamma}^\top, \sigma) \\
&= \pi(\mathbf{B}^\top \mid \boldsymbol{\mu}^\top, \boldsymbol{\Sigma}) \pi(\boldsymbol{\mu}^\top, \boldsymbol{\Sigma}) \pi(\boldsymbol{\gamma}) \pi(\sigma) \\
&= \left(\prod_{i=1}^{n} \pi(\boldsymbol{\beta}_i^\top \mid \boldsymbol{\mu}^\top, \boldsymbol{\Sigma}) \right) \pi(\boldsymbol{\mu}) \pi(\boldsymbol{\Sigma}) \pi(\boldsymbol{\gamma}) \pi(\sigma) \\
&\propto |\boldsymbol{\Sigma}|^{-n/2} \exp\left(-\frac{1}{2} \sum_{i=1}^{n} (\boldsymbol{\beta}_i - \boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1} (\boldsymbol{\beta}_i - \boldsymbol{\mu}) \right) \\
&\quad \times \exp\left(-\frac{1}{2} \boldsymbol{\mu}^\top \boldsymbol{V}^{-1} \boldsymbol{\mu} \right) \\
&\quad \times |\boldsymbol{\Sigma}|^{-(\nu+6)/2} \exp\left(-\frac{1}{2} \mathrm{tr}(\boldsymbol{S} \boldsymbol{\Sigma}^{-1}) \right) \\
&\quad \times \exp\left(-\frac{1}{2} \cdot 400 \boldsymbol{\gamma}^\top \boldsymbol{\gamma} \right) \times \frac{I(\sigma > 0)}{1 + (\sigma/10)^2}.
\end{aligned}
$$

Task 1 - Likelihood

Let m_i denote the number of observations and $\boldsymbol{Y}_i = (Y_{i,1}, \ldots Y_{i,m_i})^\top$ denote the wind speed data of the i-th hurricane (excluding the first and second observations), where $Y_{i,k} = Y_i(6k+6)$. Denote $\boldsymbol{Y} = (\boldsymbol{Y}_1^\top, \boldsymbol{Y}_2^\top, \ldots, \boldsymbol{Y}_n^\top)^\top$, and $\mathbf{Z}_{i,k} = (1, Y_{i,k}, \Delta_{i,1}(6k+6), \Delta_{i,2}(6k+6), \Delta_{i,3}(6k+6))^\top$.

Given that

$$Y_{i,j} \mid (\boldsymbol{\beta}_i^\top, \boldsymbol{\gamma}^\top, \sigma) \sim N(\mathbf{Z}_{i,j-1}^\top \boldsymbol{\beta}_i + \mathbf{X}_i^\top \boldsymbol{\gamma}, \sigma^2),$$

we have

$$\begin{aligned} L(\Theta \mid \boldsymbol{Y}^\top) &= \prod_{i=1}^{n} \prod_{j=1}^{m_i} L(\Theta \mid Y_{i,j}) \\ &= \prod_{i=1}^{n} \prod_{j=1}^{m_i} \left[\frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(Y_{i,j} - \mathbf{Z}_{i,j-1}^\top \boldsymbol{\beta}_i - \mathbf{X}_i^\top \boldsymbol{\gamma})^2}{2\sigma^2} \right) \right]. \end{aligned}$$

Task 1 - Joint Posterior Distribution of Parameters

$$
\begin{aligned}
\pi(\Theta \mid \boldsymbol{Y}^\top) &\propto L(\Theta \mid \boldsymbol{Y}^\top)\pi(\Theta) \\
&\propto \frac{I(\sigma > 0)\sigma^{-\sum_{i=1}^{n} m_i}}{1 + (\sigma/10)^2} |\boldsymbol{\Sigma}|^{-(n+\nu+6)/2} \\
&\quad \times \exp\Bigg[-\frac{1}{2}\Bigg(\sum_{i=1}^{n} (\boldsymbol{\beta}_i - \boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1} (\boldsymbol{\beta}_i - \boldsymbol{\mu}) + \boldsymbol{\mu}^\top \boldsymbol{V}^{-1} \boldsymbol{\mu} \\
&\qquad\qquad + \mathrm{tr}(\boldsymbol{S}\boldsymbol{\Sigma}^{-1}) + 400\|\gamma\|_2^2 \Bigg) \Bigg] \\
&\quad \times \exp\left(-\frac{1}{2\sigma^2} \sum_{i=1}^{n} \sum_{j=1}^{m_i} (Y_{i,j} - \mathbf{Z}_{i,j-1}^\top \beta_i - \mathbf{X}_i^\top \gamma)^2 \right).
\end{aligned}
$$

Task 2

Objective:

Design and implement a custom MCMC algorithm for the outlined Bayesian hierarchical model. Monitor the convergence of the MCMC chains, using diagnostic plots and summary statistics to check for any issues.

Task 2 - Conditional Posterior Distribution for Each Parameter

- $\beta_i \mid (\boldsymbol{\mu}^\top, \boldsymbol{\Sigma}, \boldsymbol{\gamma}^\top, \sigma, \boldsymbol{Y}^\top)$:
  a multivariate normal distribution with mean vector

$$\left(\boldsymbol{\Sigma}^{-1} + \frac{1}{\sigma^2}\sum_{j=1}^{m_i} \mathbf{Z}_{i,j-1}\mathbf{Z}_{i,j-1}^\top\right)^{-1}\left(\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \frac{1}{\sigma^2}\sum_{j=1}^{m_i}(Y_{i,j} - \mathbf{X}_i^\top\gamma)\mathbf{Z}_{i,j-1}\right)$$

  and covariance matrix

$$\boldsymbol{\Sigma}^{-1} + \frac{1}{\sigma^2}\sum_{j=1}^{m_i} \mathbf{Z}_{i,j-1}\mathbf{Z}_{i,j-1}^\top$$

- $\boldsymbol{\mu} \mid (\mathbf{B}^\top, \boldsymbol{\Sigma}, \boldsymbol{\gamma}^\top, \sigma, \boldsymbol{Y}^\top)$:
  a multivariate normal distribution with mean vector

$$\boldsymbol{V}(n\boldsymbol{V} + \boldsymbol{\Sigma})^{-1}\left(\sum_{i=1}^{n}\beta_i\right)$$

  and covariance matrix

$$\boldsymbol{V}(n\boldsymbol{V} + \boldsymbol{\Sigma})^{-1}$$

Task 2 - Conditional Posterior Distribution for Each Parameter

- $\mathbf{\Sigma} \mid (\mathbf{B}^\top, \boldsymbol{\mu}^\top, \gamma^\top, \sigma, \boldsymbol{Y})$:
  an inverse-Wishart distribution with d.f. $(n+\nu)$ and scale matrix

$$\boldsymbol{S} + \sum_{i=1}^{n} (\beta_i - \boldsymbol{\mu})(\beta_i - \boldsymbol{\mu})^\top$$

- $\gamma \mid (\mathbf{B}^\top, \boldsymbol{\mu}^\top, \mathbf{\Sigma}, \sigma, \boldsymbol{Y}^\top)$:
  a multivariate normal distribution with mean vector

$$\left(400\boldsymbol{I} + \frac{1}{\sigma^2}\sum_{i=1}^{n} m_i \mathbf{X}_i \mathbf{X}_i^\top\right)^{-1} \left(-\frac{1}{\sigma^2}\sum_{i=1}^{n}\sum_{j=1}^{m_i} (\mathbf{Z}_{i,j-1}^\top \beta_i - Y_{i,j})\mathbf{X}_i\right)$$

  and covariance matrix

$$\left(400\boldsymbol{I} + \frac{1}{\sigma^2}\sum_{i=1}^{n} m_i \mathbf{X}_i \mathbf{X}_i^\top\right)^{-1}$$

Task 2 - Conditional Posterior Distribution for Each Parameter

- $\sigma \mid \mathbf{B}^\top, \boldsymbol{\mu}^\top, \boldsymbol{\Sigma}, \boldsymbol{\gamma}^\top, \boldsymbol{Y}^\top)$:

$$\pi(\sigma \mid \mathbf{B}^\top, \boldsymbol{\mu}^\top, \boldsymbol{\Sigma}, \boldsymbol{\gamma}^\top, \boldsymbol{Y}^\top)$$
$$\propto I(\sigma > 0) \frac{\sigma^{-\sum_{i=1}^{n} m_i}}{1 + (\sigma/10)^2} \exp\left(-\frac{1}{2\sigma^2} \sum_{i=1}^{n} \sum_{j=1}^{m_i} (Y_{i,j} - \mathbf{Z}_{i,j-1}^\top \beta_i - \mathbf{X}_i^\top \gamma)^2 \right).$$

Task 2 - MCMC Algorithm

Algorithm 1 MCMC Algorithm

Require: $\mathbf{Y}$; $\beta_0, \boldsymbol{\mu}_0, \boldsymbol{\Sigma}_0, \sigma_0, \gamma_0$

for $k = 1$ to 10000 **do**

Gibbs sampling for β_i's

generate $\beta_i^{(k)}$ from $\pi(\beta_i \mid \boldsymbol{\mu}^{(k-1)}, \boldsymbol{\Sigma}^{(k-1)}, \gamma^{(k-1)}, \sigma^{(k-1)}, \boldsymbol{Y}^\top)$

Gibbs sampling for $\boldsymbol{\mu}$

generate $\boldsymbol{\mu}^{(k)}$ from $\pi(\boldsymbol{\mu} | \mathbf{B}^{(k)}, \boldsymbol{\Sigma}^{(k-1)}, \gamma^{(k-1)}, \sigma^{(k-1)}, \boldsymbol{Y}^\top)$

Gibbs sampling for $\boldsymbol{\Sigma}$

generate $\boldsymbol{\Sigma}^{(k)}$ from $\pi(\boldsymbol{\Sigma} | \mathbf{B}^{(k)}, \boldsymbol{\mu}^{(k)}, \gamma^{(k-1)}, \sigma^{(k-1)}, \mathbf{Y}^\top)$

Gibbs sampling for γ

generate $\gamma^{(k)}$ from $\pi(\gamma | \mathbf{B}^{(k)}, \boldsymbol{\mu}^{(k)}, \boldsymbol{\Sigma}^{(k)}, \sigma^{(k-1)}, \mathbf{Y}^\top)$

Metropolis-Hastings algorithm for σ

Propose a conditional distribution of new value σ^* from Uniform$[\sigma^{(k-1)} - a, \sigma^{(k-1)} + a]$, where a is the proposed step

Compute the acceptance ratio

$\lambda = \frac{\pi(\sigma^* | \mathbf{Y}, \boldsymbol{\beta}^{(k)}, \boldsymbol{\mu}^{(k)}, \boldsymbol{\Sigma}^{(k)}, \boldsymbol{\gamma}^{(k)})}{\pi(\sigma^{(k-1)} | \mathbf{Y}, \boldsymbol{\beta}^{(k)}, \boldsymbol{\mu}^{(k)}, \boldsymbol{\Sigma}^{(k)}, \boldsymbol{\gamma}^{(k)})}$

$\alpha = \min(1, \lambda)$

Generate a random number u from Uniform$(0, 1)$

If $u \leq \alpha$, set $\sigma^{(k)} = \sigma^*$, otherwise set $\sigma^{(k)} = \sigma^{(k-1)}$.

end for

Task 2 - Starting Values

Final Initial Value Selection and Core Information of MH Algorithm

- $\boldsymbol{\beta}_i$: This can be obtained through the random effects term in the lmm model. The random effects term can be added to the fixed effects term to obtain $\boldsymbol{\beta}_i^{(0)}$.
- $\boldsymbol{\mu}$: This can be obtained through the fixed effects of intercept, windpre, latdiff, longdiff, winddiff term.
- $\boldsymbol{\gamma}$: This can be obtained through the fixed effects of Year, Active Month, and Nature term.
- σ: This can be obtained through the model residual.
- $\boldsymbol{\Sigma}$: This can be obtained through the 'VarCorr' function which returns the covariance matrix of the random effects in the model $\boldsymbol{\Sigma}^{(0)}$.

Table 1: Initial Value Setting

Parameter	Value
$\boldsymbol{\mu}^\top$	$(24.25, 0.94, -0.02, -0.24, 0.47)$
$\boldsymbol{\gamma}^\top$	$(-0.01, 0.35, 0.28, 0.37, 0.12, 0.08)$
$\boldsymbol{\Sigma}$	$\begin{pmatrix} 0.358 & -0.010 & 0.039 & 0.121 & 0.028 \\ -0.01 & 0.001 & -0.003 & -0.005 & 0.002 \\ 0.039 & -0.003 & 0.043 & 0.034 & -0.019 \\ 0.121 & -0.005 & 0.034 & 0.069 & 0.003 \\ 0.028 & 0.002 & -0.019 & 0.003 & 0.017 \end{pmatrix}$
σ	5.27

Task 2 - MCMC Algorithm R code

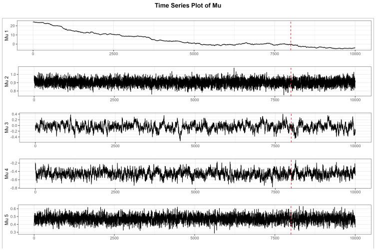
```r
# Gibbs sampling
B_sample <- function(mu, Sigma, gamma, sigma) {
  Sigma.inv <- solve(Sigma)
  B_mean_cov <- function(i) {
    cov <- solve(Sigma.inv + 1/sigma^2 * t(Z[[i]]) %*% Z[[i]])
    mean <- cov %*% (Sigma.inv %*% mu + 1/sigma^2 * colSums((Y[[i]] - (X[i,] %*% gamma)[,]) * Z[[i]]))
    list(mean = mean, cov = cov)
  }
  mean_cov_list <- lapply(1:n, B_mean_cov)
  B <- sapply(mean_cov_list, function(x) {mvrnorm(mu = x$mean, Sigma = x$cov)})
  return(B)
}

# MH algorithm (random walk)
sigma_sample <- function(sigma, B, gamma, a) {
  sigma_new <- sigma + (runif(1) - 0.5) * 2 * a # candidate sigma
  if (sigma_new <= 0) {
    return(sigma)
  }
  RSS <- sum(sapply(1:n, function(i) sum((Y[[i]] - Z[[i]] %*% B[,i] - (X[i,] %*% gamma)[,])^2)))
  log_kernal_ratio <- -sum(m) * log(sigma_new/sigma) +
    log(1 + (sigma/10)^2) - log(1 + (sigma_new/10)^2) -
    0.5 * (1/sigma_new^2 - 1/sigma^2) * RSS
  log_prob <- min(0, log_kernal_ratio)
  sigma <- ifelse(log_prob > log(runif(1)), sigma_new, sigma)
  return(sigma)
}
```

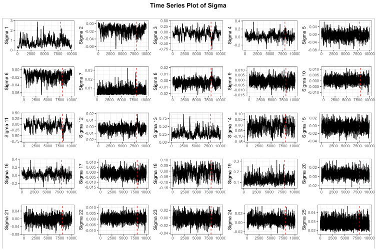
Task 2 - Results Presentation (hyperparameters)

- search window $a = 0.1$
- burn-in $= 8000$
- resulting acceptance rate $= 0.418$

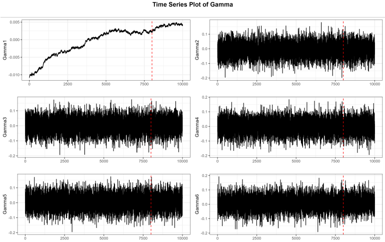
Task 2 - Results Presentation (Parameters (Burn-In 8000))



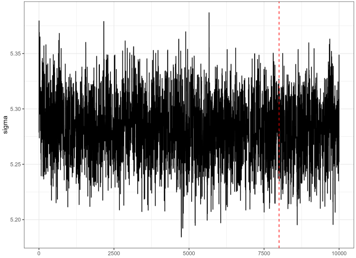
(a) mu



(b) Sigma matrix

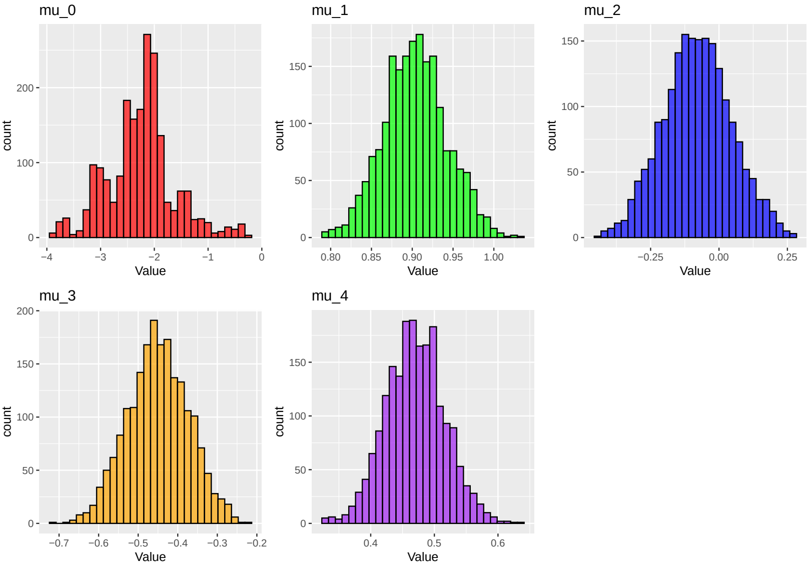


(c) gamma



(d) sigma

Task 2 - Results Presentation (Histogram Plots for μ)



Task 2 - Results Presentation (Parameter Estimations of $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$)

$$\hat{\boldsymbol{\mu}}$$

$$\begin{pmatrix} \hat{\mu}_0 & -3.6594881 \\ \hat{\mu}_1 & 0.9020437 \\ \hat{\mu}_2 & -0.0584559 \\ \hat{\mu}_3 & -0.4502597 \\ \hat{\mu}_4 & 0.4714633 \end{pmatrix}$$

$$\hat{\boldsymbol{\Sigma}}$$

$$\begin{pmatrix} 0.7521 & -0.0155 & -0.0860 & 0.0136 & -0.0060 \\ -0.0155 & 0.0050 & -0.0023 & -0.0013 & 0.0006 \\ -0.0860 & -0.0023 & 0.2705 & -0.0085 & -0.0024 \\ 0.0136 & -0.0013 & -0.0085 & 0.1287 & 0.0057 \\ -0.0060 & 0.0006 & -0.0024 & 0.0057 & 0.0268 \end{pmatrix}$$

Task 2 - Results Presentation (CIs of $\boldsymbol{\mu}$)

variable	mean	lower_CI	upper_CI
mu_0	-3.6594881	-4.9823968	-1.1458245
mu_1	0.9020437	0.8379360	0.9670541
mu_2	-0.0584559	-0.2942283	0.1610285
mu_3	-0.4502597	-0.5789892	-0.3093745
mu_4	0.4714633	0.3958291	0.5443864

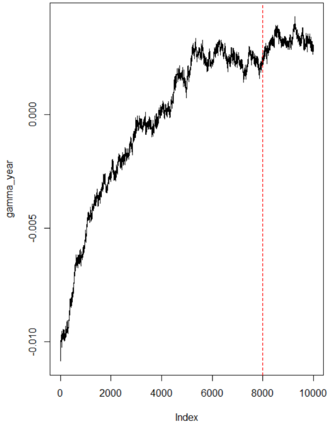
Task 3

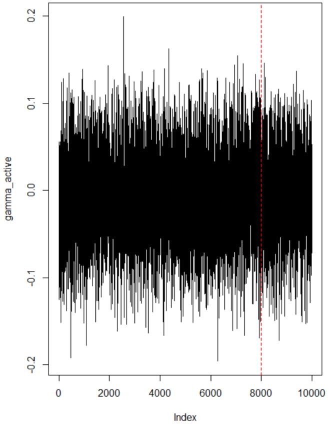
Objective:

Compute posterior summaries and 95% credible intervals of γ, the fixed effects associated with the covariates in the model. Using the estimated Bayesian model, answer the following questions:

(1) Are there seasonal differences in hurricane wind speeds?
(2) Is there evidence to support the claim that hurricane wind speeds have been increasing over the years?

Task 3 - Parameters Convergence





Task 3 - 95% Credible Intervals

	gamma_year	gamma_active
2.5%	0.0025498	-0.1047691
97.5%	0.0038603	0.0880498

- Conclusion:
  1. There is no seasonal difference (active v.s inactive) in hurricane wind speeds.
  2. There is no evidence to support the claim that hurricane wind speeds have been increasing over the years because of divergence.

Task 4 - Objective

Objective:

With the estimated model parameters and covariate values, you can calculate the predicted wind speed for each time point using the model equation. This way, you can track the hurricane and compare the predicted wind speeds with the actual wind speeds recorded during the hurricane. Please evaluate how well the estimated Bayesian model can track individual hurricanes.

Task 4

Prediction: Using the parameters after burn-in, we can obtain the predicted value for each hurricane.

$$\hat{Y}_i(t+6) = \hat{\beta}_{0,i} + \hat{\beta}_{1,i} Y_i(t) + \hat{\beta}_{2,i} \Delta_{i,1}(t) + \hat{\beta}_{3,i} \Delta_{i,2}(t) + \hat{\beta}_{4,i} \Delta_{i,3}(t) + \mathbf{X}_i^\top \hat{\gamma}$$

Performance evaluation: For each hurricane, we can evaluate the estimated Bayesian model performance by calculating

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n}}$$

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2}$$

Task 4

Table 3: Summary of RMSE and R-squared for selected hurricanes

ID	Year	RMSE	R-squared
ABBY.1960	1960	8.8804	0.7700
ABBY.1964	1964	9.6430	0.3033
ABBY.1968	1968	3.5043	0.9360
ABLE.1950	1950	3.6755	0.9813
ABLE.1951	1951	3.4802	0.9767
ABLE.1952	1952	4.5183	0.9583
AGNES.1972	1972	5.2483	0.8881
ALBERTO.1982	1982	8.0473	0.7499
ALBERTO.1988	1988	2.6121	0.7420
ALBERTO.1994	1994	4.3941	0.8807
ALBERTO.2000	2000	3.7896	0.9625
ALBERTO.2006	2006	4.3591	0.7882
ALBERTO.2012	2012	3.2193	0.8036
ALEX.1998	1998	2.9351	0.7289
ALEX.2004	2004	5.4552	0.9539

Task 4

Prediction performance on random chosen example hurricanes.

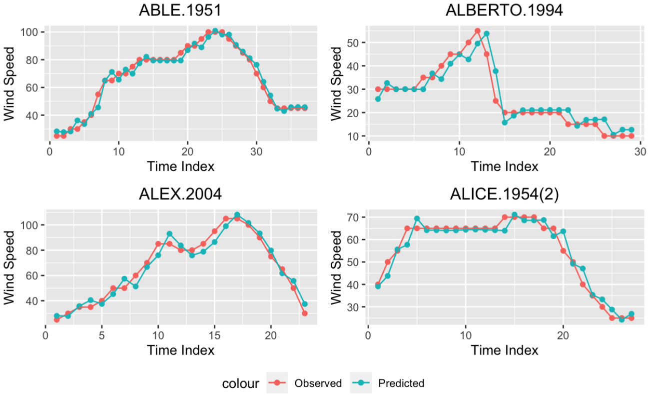


Figure 4: Time series prediction plot

Task 4

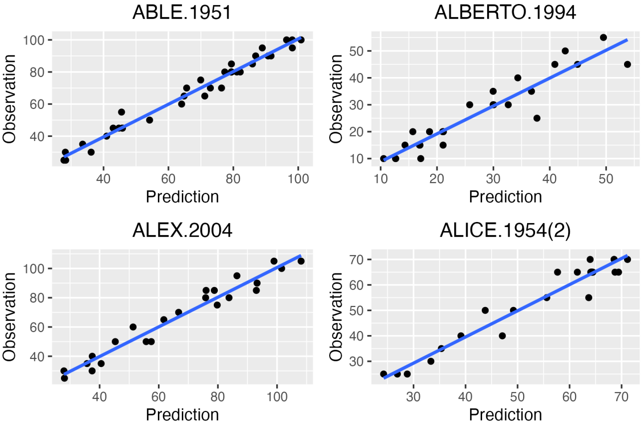


Figure 5: Prediction vs. observation

Task 4

Performance evaluation: We plot the *RMSE* and R^2 distribution for all the hurricanes

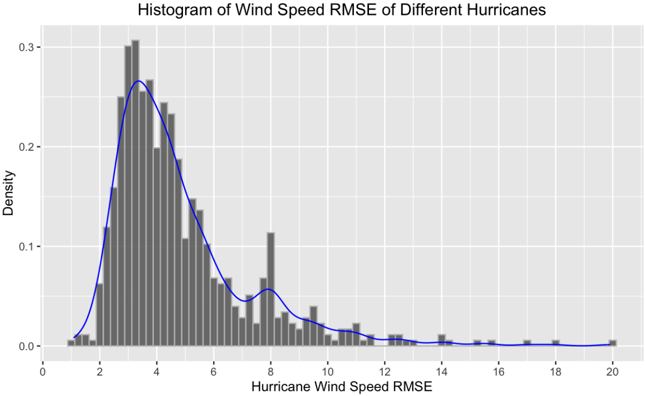


Figure 6: RMSE distribution

Task 4

Performance evaluation:

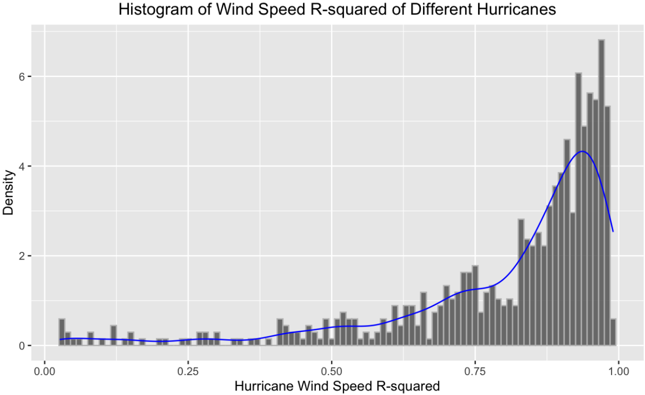


Figure 7: R^2 distribution

Task 4

To enhance the evaluation of our prediction performance, we examined the distribution of RMSE values across various properties of hurricanes.

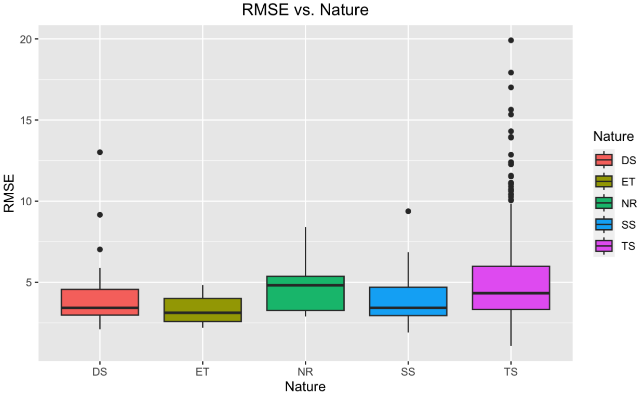


Figure 8: RMSE under different natures

Task 4

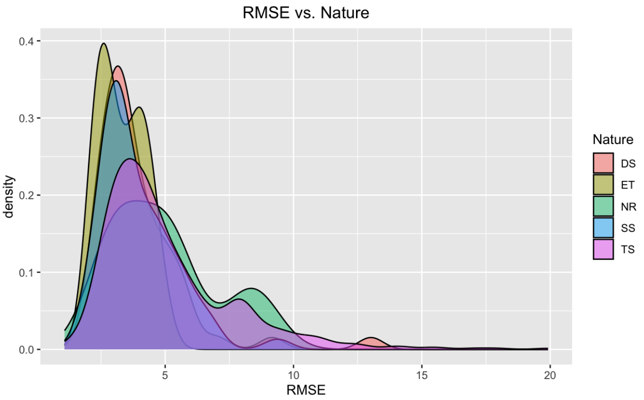


Figure 9: RMSE under different natures

Task 4

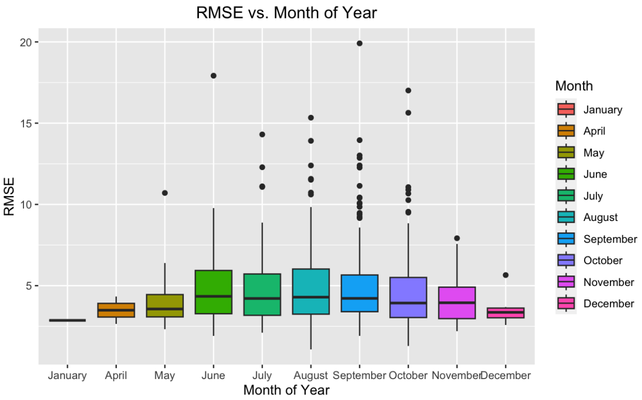


Figure 10: RMSE under different months

Task 4

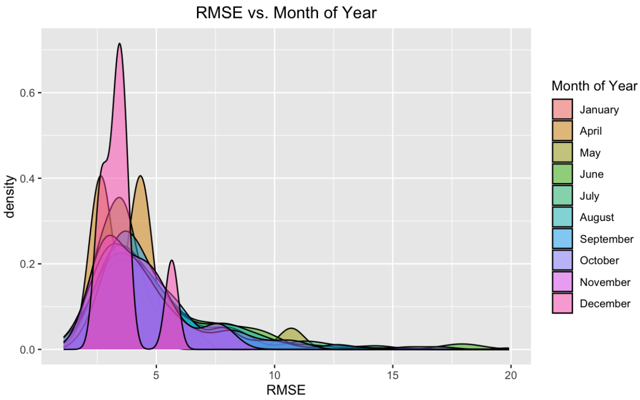


Figure 11: RMSE under different months

Discussions

- Better parameters convergence performance in MCMC
- Prediction latency in responding to wind speed change

Q&A

- Thanks for listening!