Lecture 15, 16, and 17: Cellular Systems

Prof. Bobak Nazer   10/23/14, 10/30/14, and 11/4/14

- Up until now, we have focused on communication between a
  single transmitter and a single receiver.
- A cellular network will have multiple transmitters (or users).
- It will also have multiple receivers (or basestations) to ensure broad geographic coverage.
- Each basestation covers a region known as a cell. Within a cell, we can maintain tight control over the users' behavior.

Lecture 15, 16, and 17: Cellular Systems

Prof. Bobak Nazer   10/23/14, 10/30/14, and 11/4/14

- Up until now, we have focused on communication between a
  single transmitter and a single receiver.
- A cellular network will have multiple transmitters (or users).
- It will also have multiple receivers (or basestations) to ensure broad geographic coverage.
- Each basestation covers a region known as a cell. Within a cell, we can maintain tight control over the users' behavior.

Lecture 15, 16, and 17: Cellular Systems

Prof. Bobak Nazer   10/23/14, 10/30/14, and 11/4/14

- Up until now, we have focused on communication between a
  single transmitter and a single receiver.
- A cellular network will have multiple transmitters (or users).
- It will also have multiple receivers (or basestations) to ensure broad geographic coverage.
- Each basestation covers a region known as a cell. Within a cell, we can maintain tight control over the users' behavior.

Interference between Users

- Let's consider a scenario where just two users transmit simultaneously.
- The signal observed at the basestation will be a noisy linear combination of the transmitted signals:

$$y[m] = h_1[m]x_1[m] + \underbrace{h_2[m]x_2[m] + w[m]}_{\text{Effective Noise for Decoding User 1}}$$

- The signal from the second user will interfere with the signal from the first user and vice versa.
- One option: Just treat the other user as noise.

- Main problem: Effective noise power will usually be **greater** than the signal power.

Interference between Users

Signal-to-Interference-and-Noise Ratio (SINR)

$$= \frac{\text{Average Received Signal Power}}{\text{Average Received Interference Power} + \text{Average Noise Power}}$$

- Assume that $\mathbb{E}\left[|x_1[m]|^2\right] = \mathbb{E}\left[|x_2[m]|^2\right] = \mathcal{E}$
- For our two-user example, we get $\text{SINR} = \frac{\mathcal{E}}{\mathcal{E}+1} < 1$

$$\text{SINR}_{\text{dB}} < 0\text{dB}$$

- This will lead to very high probability of error and poor performance.
- Even worse for K active users:

$$y[m] = \sum_{k=1}^{K} h_k[m]x_k[m] + w[m] \qquad \text{SINR} = \frac{\mathcal{E}}{K\mathcal{E}+1}$$

Multiple-Access Techniques

- To avoid this interference problem, we need to carefully manage how users access the wireless channel. A strategy for doing so is called a multiple-access technique.

- Examples of multiple-access techniques:
  - Frequency-Division Multiple Access (FDMA). Used in 1G.
  - Time-Division Multiple Access (TDMA). Used in GSM (AT&T 2G).
  - Code-Division Multiple Access (CDMA). Used in IS-95 (Verizon 2G).
  - Orthogonal Frequency Division Multiple Access (OFDMA). Used in LTE Downlink.
  - Single-Carrier FDMA (SC-FDMA). Used in LTE Uplink.

- The modulation and detection theory we have developed apply directly to the first two techniques. We will need a bit more background to understand the last three.

Interference Management

- A good multiple-access technique will allow us to (almost) eliminate interference within a cell. This is known as intra-cell interference.
- This is because the basestation can control the actions of the users within its cell.
- What about users (and basestations) from neighboring cells?
- Since we cannot coordinate the actions of users across multiple cells, there will be significant inter-cell interference.
- This requires good interference management.

Interference Management

- A good multiple-access technique will allow us to (almost) eliminate interference within a cell. This is known as intra-cell interference.
- This is because the basestation can control the actions of the users within its cell.
- What about users (and basestations) from neighboring cells?
- Since we cannot coordinate the actions of users across multiple cells, there will be significant inter-cell interference.
- This requires good interference management.

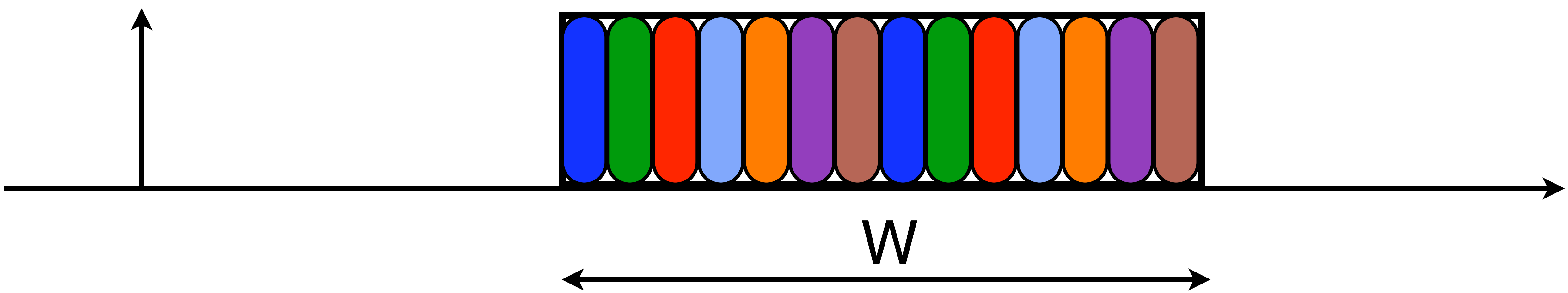
Cellular System

- Overall, a good cellular network architecture must have strategies for:
  - Multiple-Access: To eliminate intra-cell interference (between users in the same cell).
  - Interference Management: To reduce the impact of inter-cell interference (from users and basestations in neighboring cells).

- We will investigate these ideas through three cases studies of digital cellular network architectures:
  - Narrowband: TDMA + careful frequency planning.
    Example: GSM (2G AT&T)
  - Wideband CDMA: CDMA + careful power control.
    Example: IS-95 (2G Verizon)
  - Wideband OFDM: OFDMA / SC-FDMA + frequency hopping.
    Example: LTE (4G AT&T and Verizon)

Narrowband Cellular Architecture

- Basic Idea: Divide up the total available bandwidth W into N narrowband chunks of width W/N each.

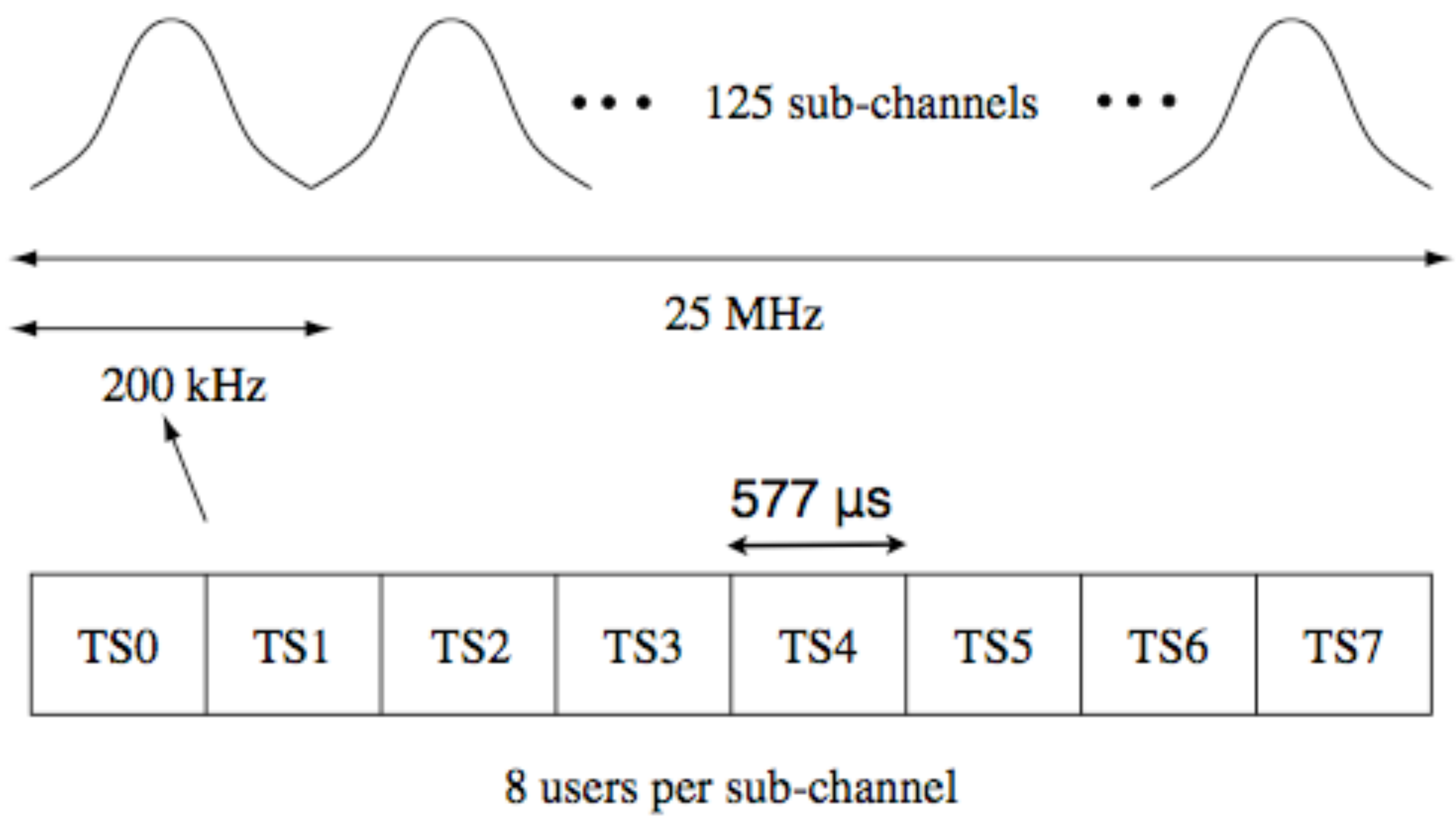


- Each cell is allocated n out of the N total chunks.

- In this example, there are 7 cells that each receive 2 chunks. (In GSM, it is 7 cells with 8 chunks of bandwidth 200kHz each.)

GSM

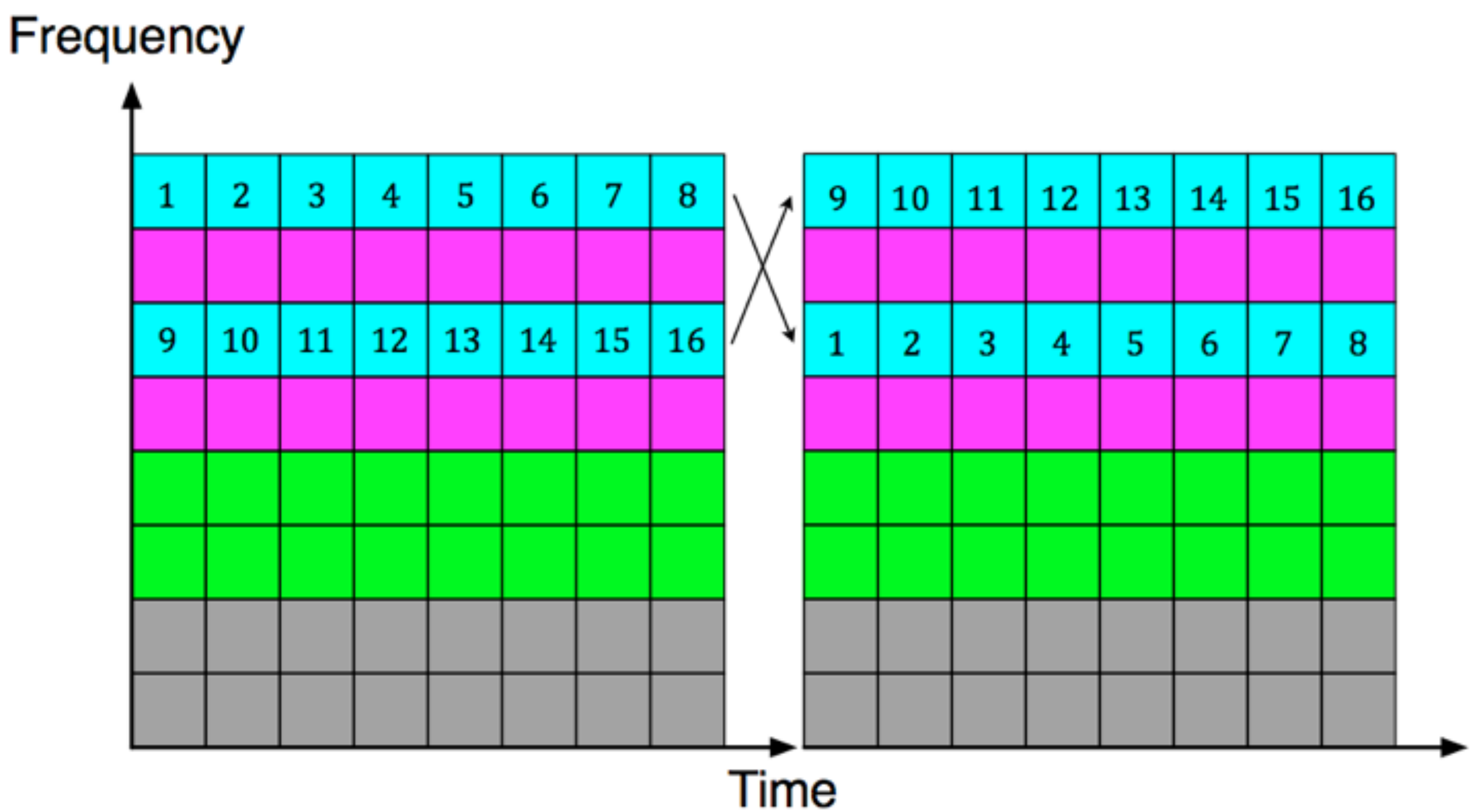
GSM: 8 users share a 200 kHz sub-channel, time slot: 577 μs



Time-Frequency Resource Allocation

- Each cell is allocated certain chunks of frequency.

- Each user is assigned specific time slots when it can use one of these chunks.
- Users hop across frequencies (within a cell) to obtain diversity.



Narrowband: Frequency Planning

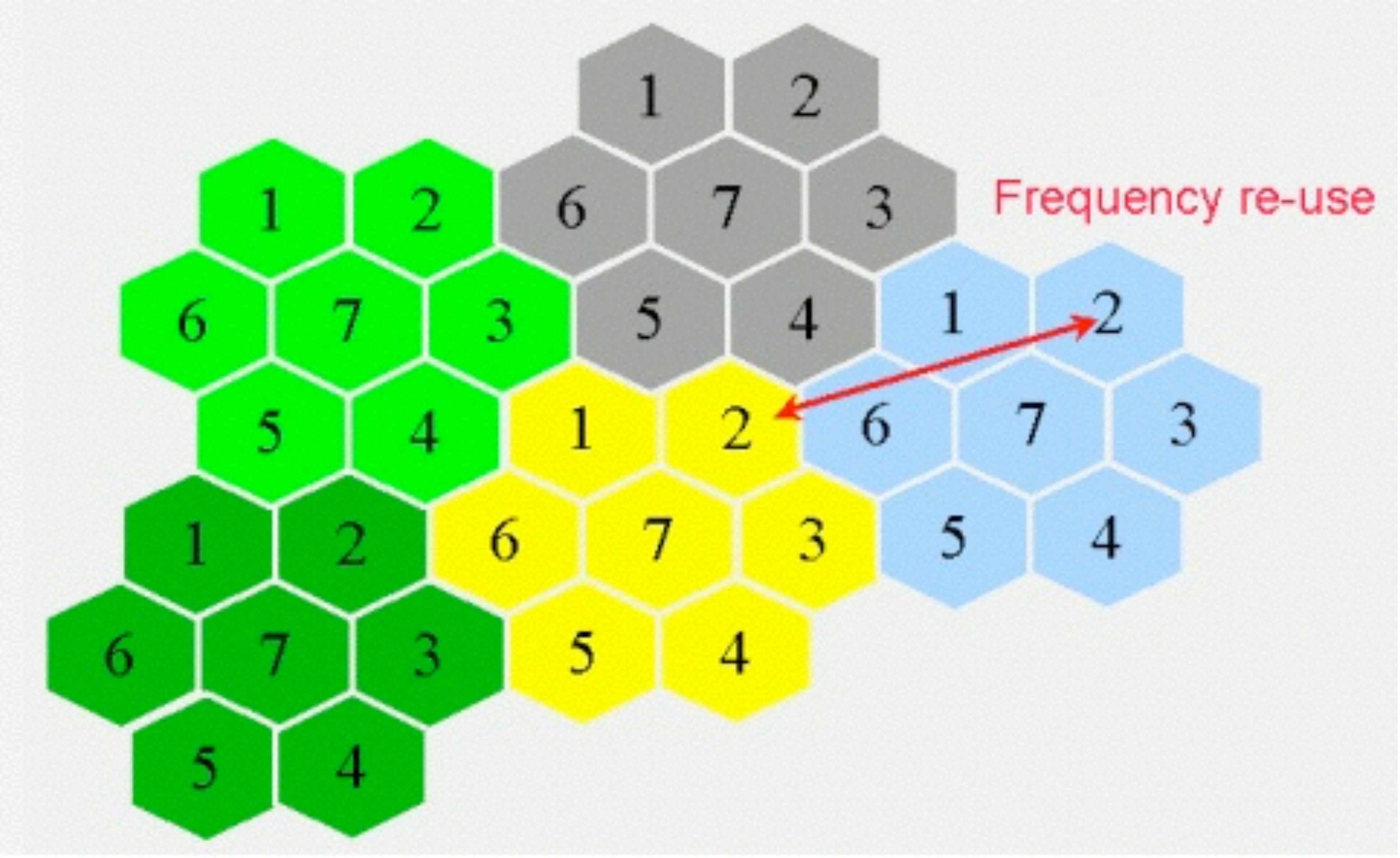
- Cell-sites are assigned frequencies so that no two neighboring cells share frequencies. (Interference Management)

- Since signal strength falls rapidly with distance, this drastically reduces inter-cell interference.

- Problem: We are not using our frequency allotment very efficiently.

- Main Metric: Frequency Reuse Factor

- In GSM, this is 1/7.



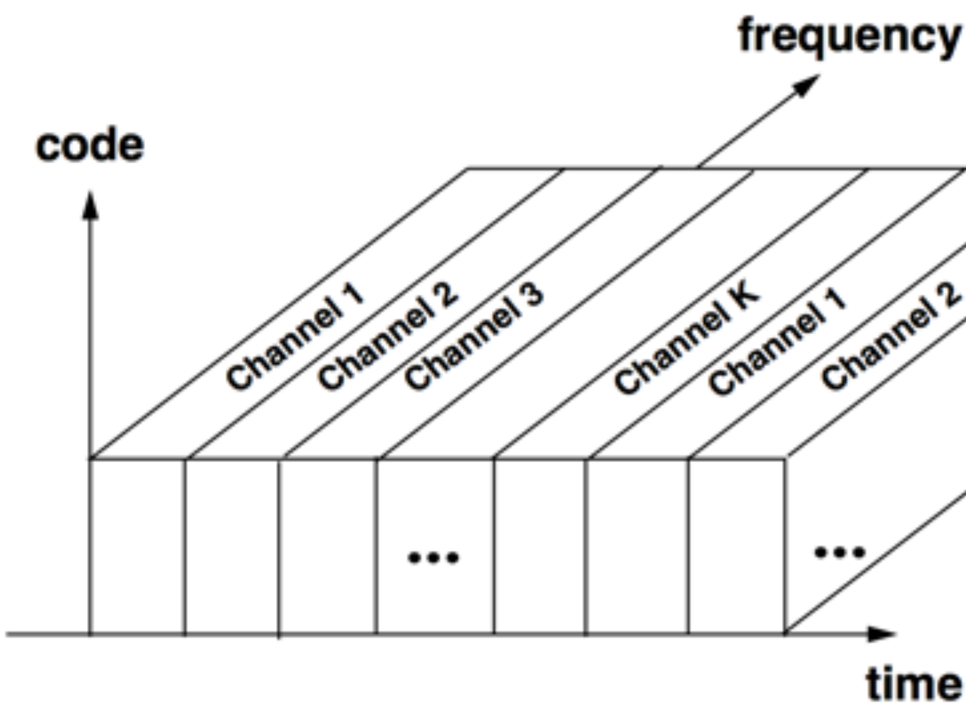
Narrowband: Multiple-Access

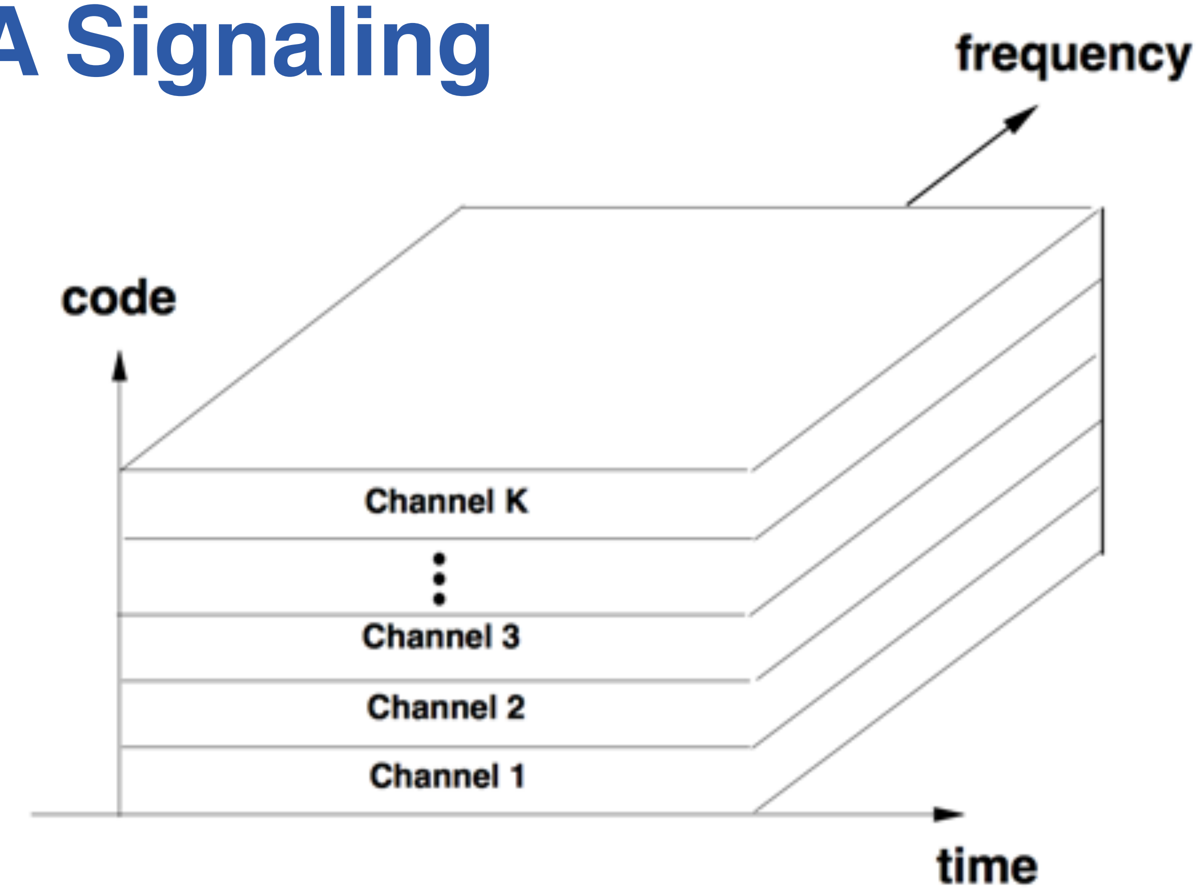
- Within each cell, a single user is assigned to each chunk in a given time slot. (Time-Division Multiple Access)
- In GSM, the chunks are 200kHz wide and the time slots are 577us long.
- Users are rapidly cycled through frequency chunks to increase diversity.
- Very high SNR per user due to the lack of inter-cell and intra-cell interference. (About 30dB in GSM).

Wideband CDMA Cellular Architecture

- Basic Idea: All active users use the entire bandwidth **at the same time.**
- How is this possible? We'll see in a moment.
- Main Advantages:
  - Does not waste frequency allotment and no frequency planning.
  - Interference averaging over many users.
  - Soft capacity limit
  - Soft handoff is easy.
- Main Challenges:
  - Must maintain very tight power control.
  - More sophisticated signal processing and coding needed to deal with the low SNR.

CDMA vs. TDMA Signaling





TDMA (within one cell)

- Subdivide the cell's spectrum into chunks.
- Only one user per chunk per time slot.
- Users hop across chunks for frequency diversity.

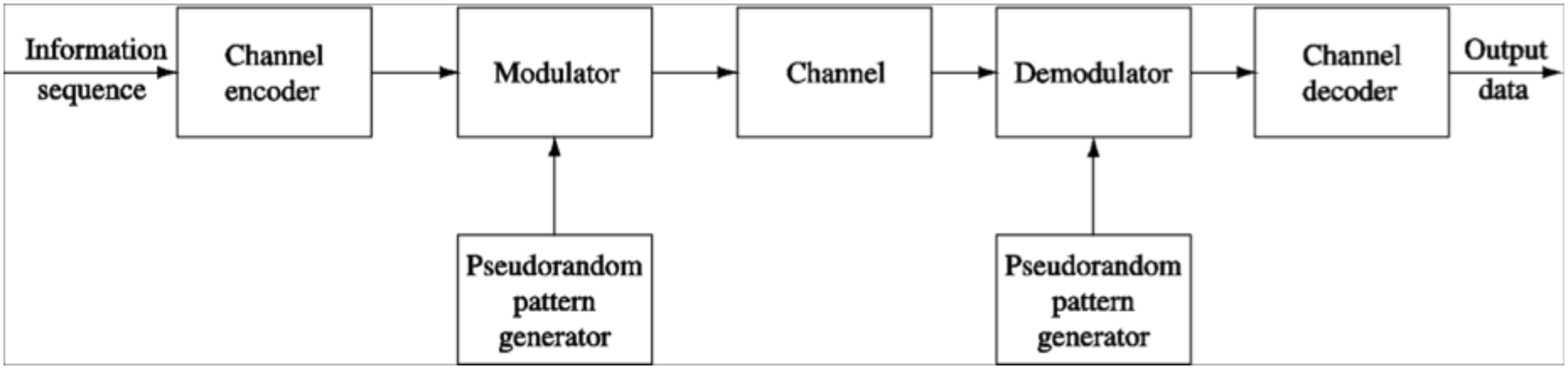
CDMA (across many cells)

- Many long code sequences that
  occupy the entire spectrum.
- Only one user per code sequence.
- All users are active across the entire time-frequency spectrum

CDMA: Design Goals

- Make the interference between users look as close to Gaussian as possible.
- This is accomplished by:
  - Making each user look like random noise using pseudonoise sequences.
  - Precise power control.
  - Averaging interference from many users across neighboring cells.
- If all of these conditions are met, then we can argue that the total interference is nearly Gaussian using the Central Limit Theorem (and this holds in practice).

Direct-Sequence Spread Spectrum

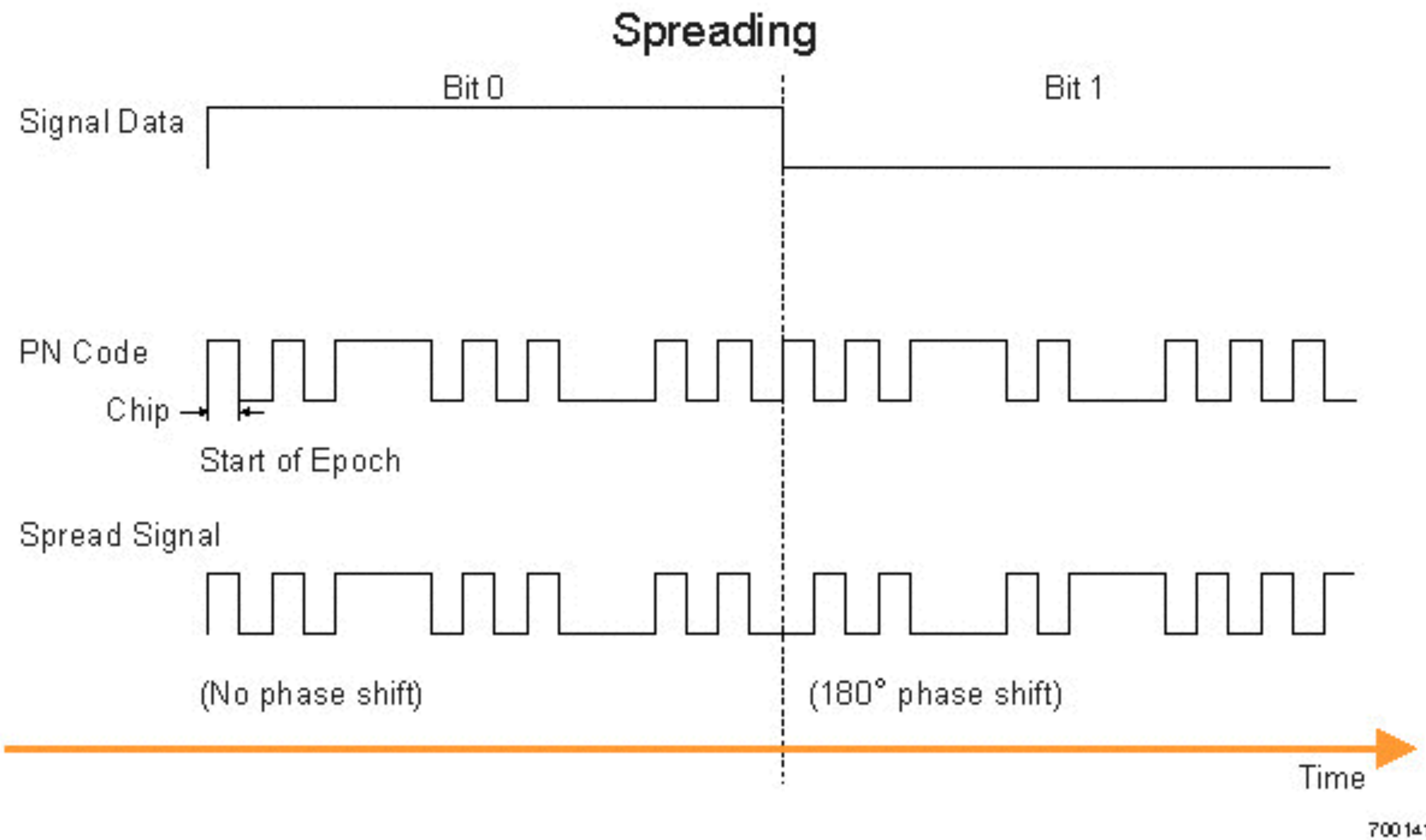


- Instead of sending a new bit at every time slot, CDMA spreads each bit over a long interval.

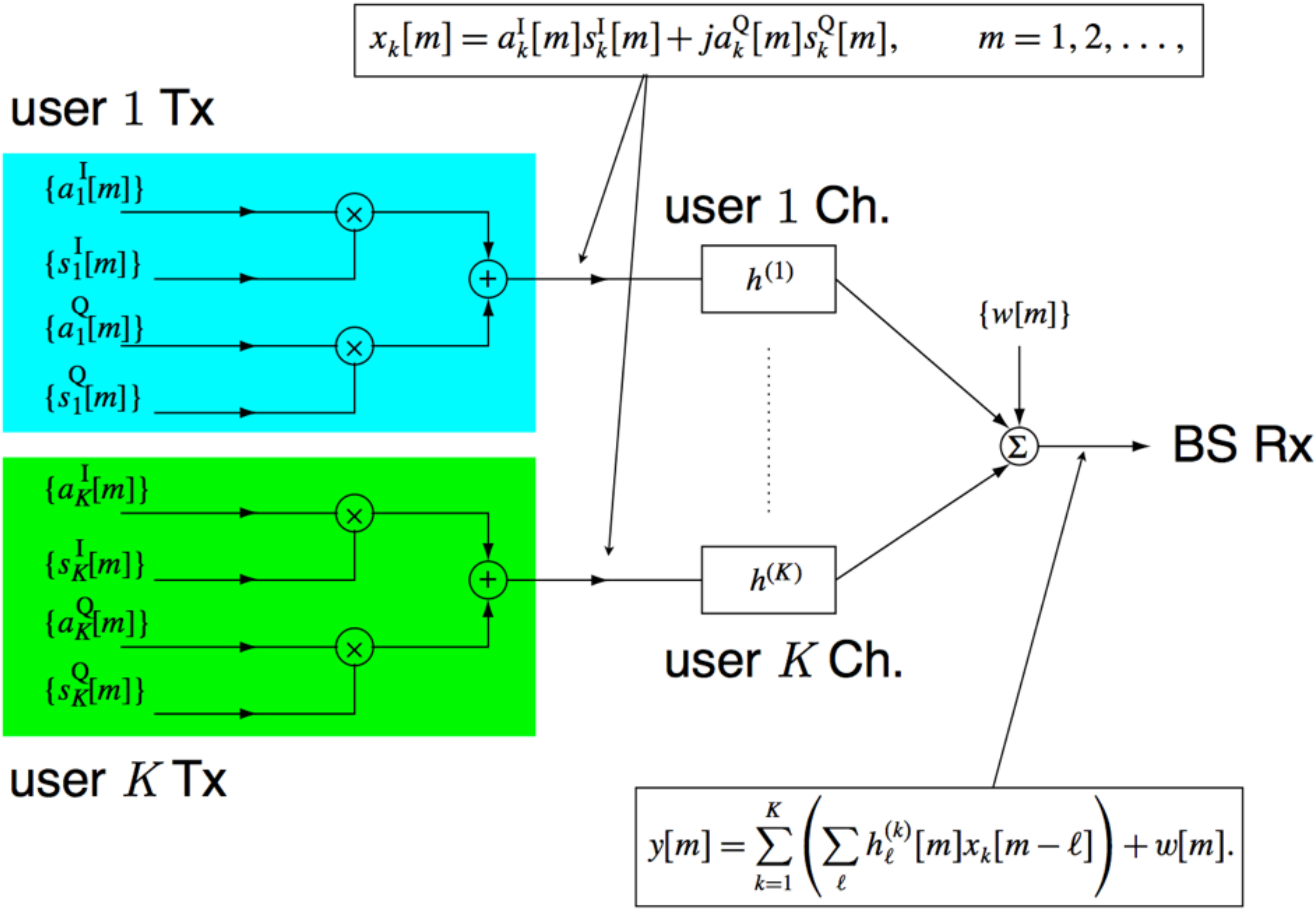
- Each bit rides on a pseudonoise (PN) sequence s[m].

- Key property: different PN sequences should be (nearly) orthogonal to one another.

Direct-Sequence Spread Spectrum



CDMA Uplink Illustration



Pseudonoise Sequence Generation

- Why don’t we make the sequences perfectly orthogonal?
- This may seem like a great approach in theory but is almost impossible to accomplish in practice. Why?
- Requires almost perfect time synchronization across users!
- As a result, we will have to live with some small overlap between sequences.
- Taking the optimistic engineering viewpoint, we might as well intentionally design our sequences to overlap a little bit. This allows us to have more sequences and thus more users!

Pseudonoise Sequence Generation

- Pseudonoise sequences are usually generated using maximum length shift registers (MLSRs).
- The output of an MSLR at time m is:
  - A linear combination (using binary arithmetic) of its outputs at time m-1 through m-r (i.e., its state).
  - Periodic with period p = 2^r - 1. It runs through all possible non-zero states before repeating.
  - Almost half 0's and half 1's: $\frac{1}{p}\sum_{m=1}^{p} s[m] = -\frac{1}{p}$
  - Shifts are nearly orthogonal:

$$\frac{1}{p}\sum_{m=1}^{p} s[m]s[m+\ell] = -\frac{1}{p} \qquad \ell \neq 0$$

Interference Statistics

- Effective Interference for Decoding User 1:

$$I[m] = \sum_{k>1} \sum_{\ell} h_\ell^{(k)}[m] x_k[m-\ell]$$

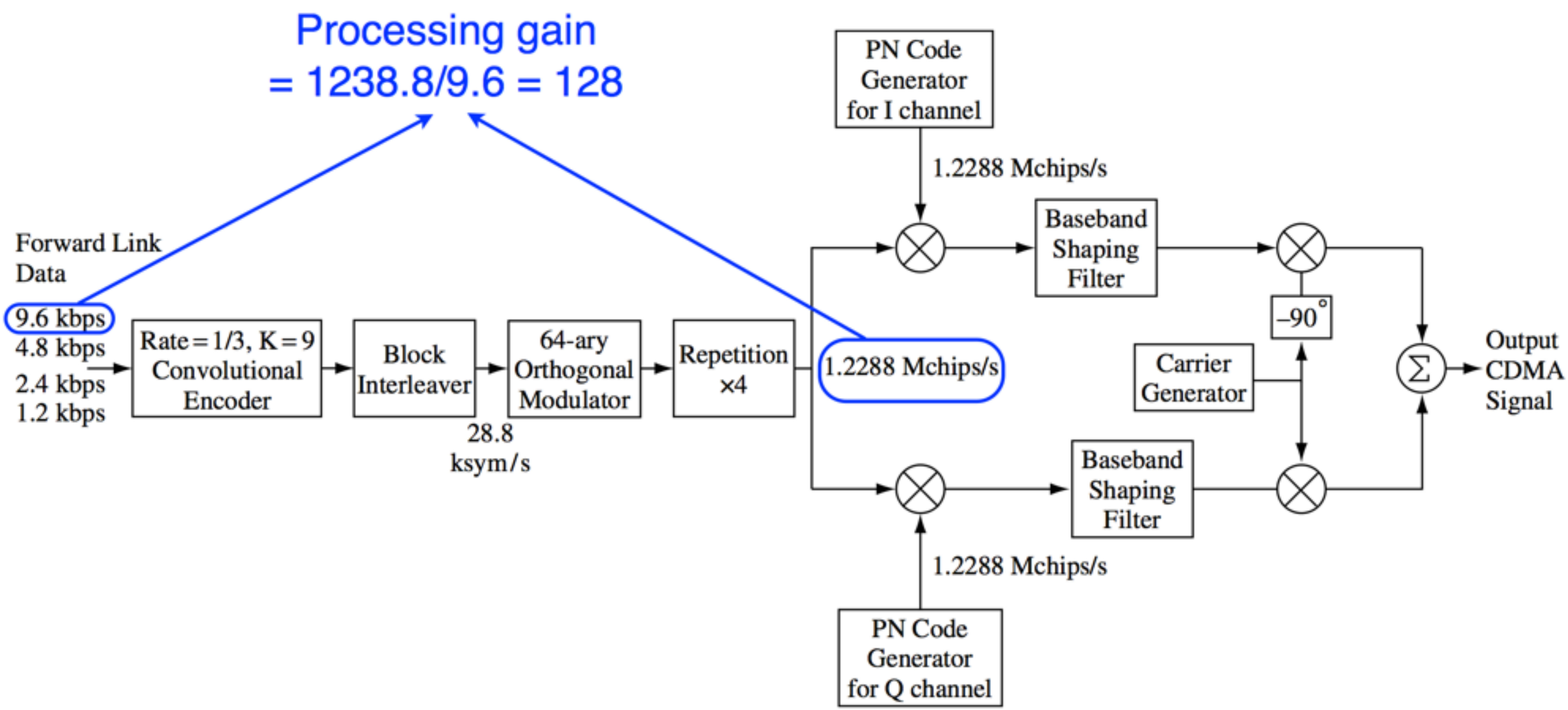
- Looks circularly symmetric due to the channel gains.
- Second-order statistics:

$$\mathbb{E}\left[I[m]I[m+1]^*\right] \begin{cases} = \sum_{k>1} \mathcal{E}_k^c, & l = 0 \\ \approx 0, & l \neq 0 \end{cases} \qquad \mathcal{E}_k^c := \mathbb{E}\left[|x_k[m]|^2\right] \sum_l \mathbb{E}\left[|h_l^{(k)}[m]|^2\right]$$

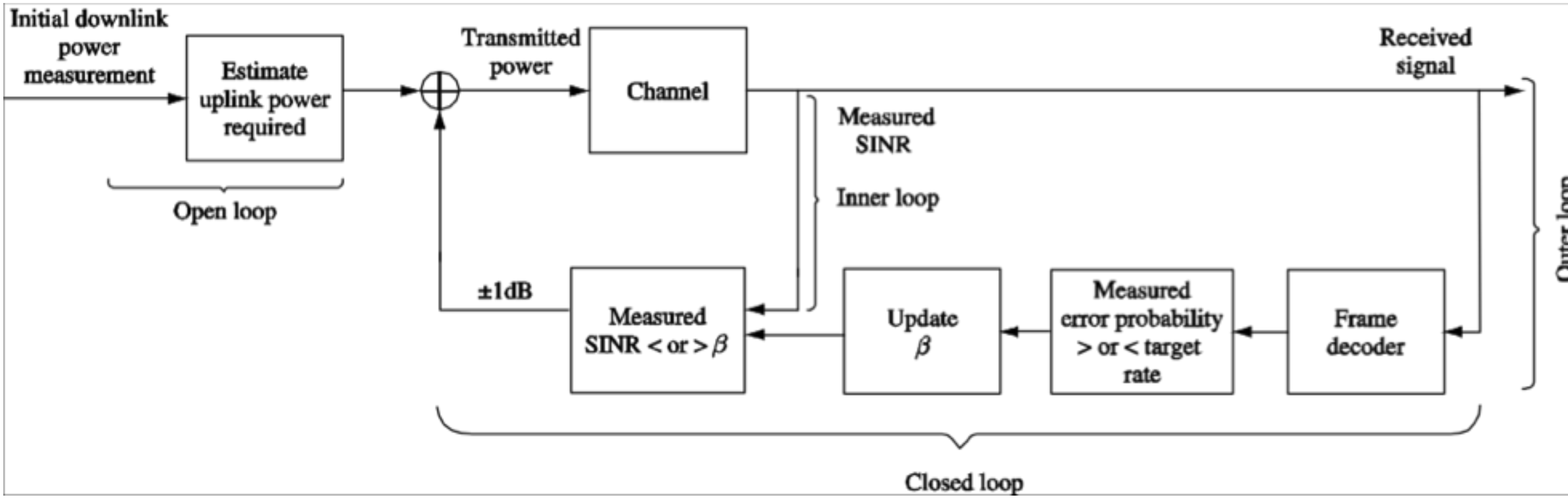
CDMA: Power Control

- In order to get nearly Gaussian interference, we need to maintain very tight power control across users.
- Otherwise, one or two users will look much stronger than the others and the interference will look very non-Gaussian.
- Basic Algorithm: Every user decreases its power until its SINR is right above its requirement.
- Current SINR is estimated using signals from the basestation.
- In reality, users do not have a good enough idea of the channel to maintain tight enough power control. Basestation provides 1 bit of feedback indicating to increase or decrease by 1dB.

CDMA Uplink Architecture



CDMA: Power Control



CDMA: Soft Handoff

- Key Issue: How do users switch between basestations as they move around?
- This is relatively easy in CDMA since all cells use the same frequencies.
- In fact, more than one basestation can often decode the signal from a given user.
- Switching center decides which basestations' estimate will be used.

OFDMA: Design Goals

- TDMA keeps users within a cell completely orthogonal but it has a frequency reuse factor < 1.
- CDMA gives up on orthogonality but gets a frequency reuse factor of 1.
- OFDMA (Orthogonal Frequency-Division Multiple-Access):
  Try to get orthogonality within a cell and frequency reuse factor of 1.
- Based on OFDM signaling. Has the added benefit of eliminating the need for Viterbi decoding for frequency diversity.
- Let's review OFDM…

Orthogonal Frequency Division Multiplexing (OFDM)

- Most wireless channels are underspread, meaning that the delay spread is much less than the coherence time.
    - This implies that the number of taps L is much less than the (discrete) coherence time M_c.
- Within one coherence interval, we can approximate the channel with a linear, time-invariant (LTI) model:

$$y[m] = \sum_{\ell=0}^{L-1} h_\ell x[m-\ell] + w[m]$$

- Complex sinusoids are eigenfunctions of LTI systems. However, we cannot send a signal with infinite duration!

Convolution <—> Multiplication

- Recall that convolution in the continuous-time domain leads to multiplication in the frequency domain.
- Does this relationship hold in the discrete-time domain?
- Yes! But we need to replace convolution with circular convolution.
- Circular Convolution for length-N discrete-time signals:

$$\mathbf{h} = \begin{bmatrix} h[0] \; h[1] \; \cdots \; h[N-1] \end{bmatrix}^t$$

$$\mathbf{d} = \begin{bmatrix} d[0] \; d[1] \; \cdots \; d[N-1] \end{bmatrix}^t$$

$$\mathbf{h} \circledast \mathbf{d} = \sum_{\ell=0}^{N-1} h[\ell] d[(m-\ell) \bmod N] = \sum_{\ell=0}^{N-1} h[(m-\ell) \bmod N] d[\ell]$$

Discrete Fourier Transform (DFT)

- The DFT of a length-N signal $\mathbf{d} = \begin{bmatrix} d[0] & d[1] & \cdots & d[N-1] \end{bmatrix}^t$ is given by

$$\tilde{d}_n = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} d[m] \exp\left(\frac{-j2\pi nm}{N}\right) \qquad n = 0, 1, \ldots, N-1$$

where n refers to the discrete frequency or “tone.”

- The inverse DFT is given by

$$d[m] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \tilde{d}_n \exp\left(\frac{j2\pi nm}{N}\right) \qquad m = 0, 1, \ldots, N-1$$

- These two signals are known as a DFT pair

$$d[m] \xleftrightarrow{\text{DFT}} \tilde{d}_n$$

DFT: Circular Convolution and Multiplication

- Circular convolution corresponds to multiplication after a DFT:

$$h[m] \xleftrightarrow{\text{DFT}} \tilde{h}_n$$

$$d[m] \xleftrightarrow{\text{DFT}} \tilde{d}_n$$

$$(h \circledast d)[m] \xleftrightarrow{\text{DFT}} \sqrt{N}\tilde{h}_n \tilde{d}_n$$

- How can we "trick" the wireless channel into performing a circular convolution?

- Key Idea: Cyclic Prefix

Want to send length-N signal: $\mathbf{d} = \begin{bmatrix} d[0] & d[1] & \cdots & d[N-1] \end{bmatrix}^t$

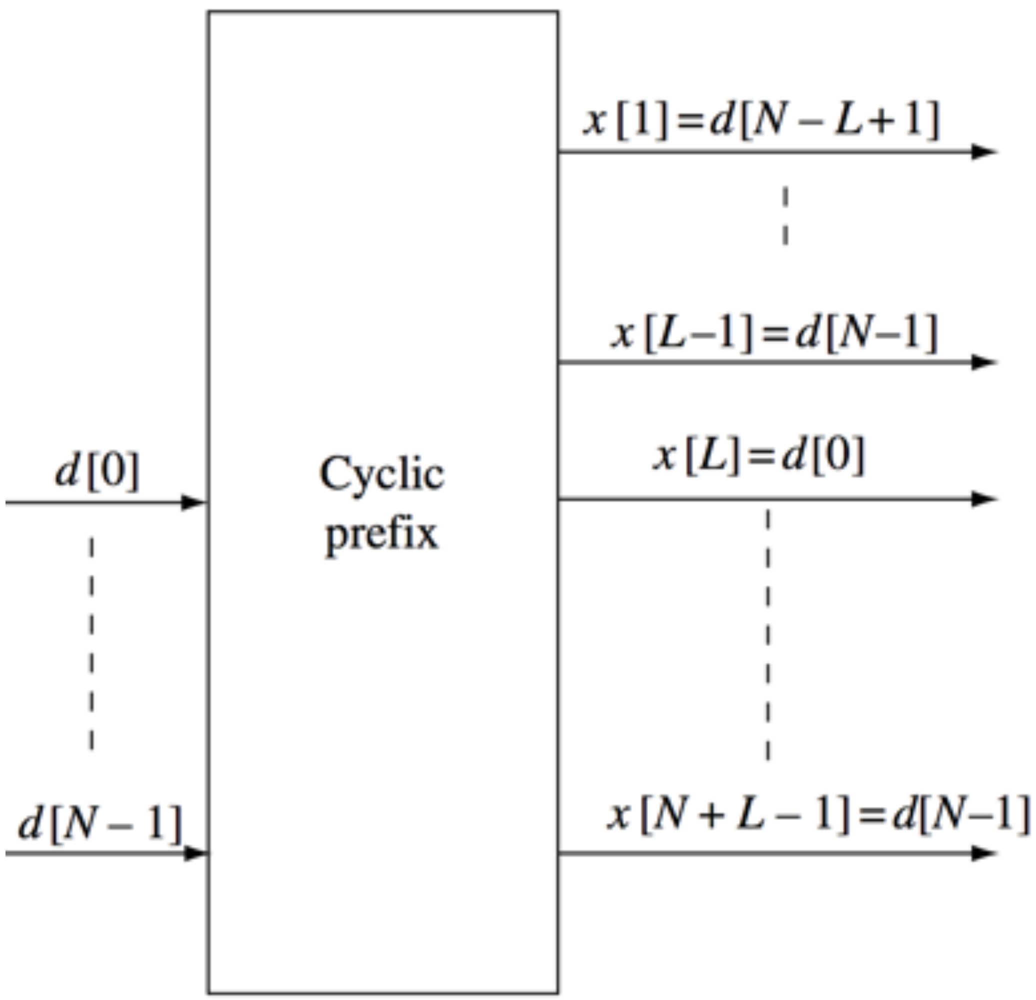
Actually send length-(N+L-1) signal:

$$\begin{aligned} \mathbf{x} &= \begin{bmatrix} x[0] & x[1] & \cdots & x[N+L-1] \end{bmatrix}^t \\ &= \begin{bmatrix} d[N-L+1] & d[N-L+2] & \cdots & d[N-1] & d[0] & d[1] & \cdots & d[N-1] \end{bmatrix}^t \end{aligned}$$

Cyclic Prefix Leads to Circular Convolution

- The output of the channel from m = 1, … , N+L-1 is

$$y[m] = \sum_{\ell=0}^{L-1} h_\ell x[m-\ell] + w[m]$$



- If we focus our attention on time slots m = L, … , N+L-1 we get

$$y[m] = \sum_{\ell=0}^{L-1} h_\ell \, d\big[(m-L-\ell) \bmod N\big] + w[m]$$

OFDM: From Circular Convolution to Multiplication

- Now that we have a circular convolution, the receiver can compute a DFT to turn this into multiplication:

$$\mathbf{y} = \begin{bmatrix} y[L] & \cdots & y[N+L-1] \end{bmatrix}^t$$

$$\mathbf{h} = \begin{bmatrix} h_0 & \cdots & h_{L-1} & 0 & \cdots & 0 \end{bmatrix}^t$$

$$\mathbf{d} = \begin{bmatrix} d[0] & \cdots & d[N-1] \end{bmatrix}^t$$

$$\mathbf{w} = \begin{bmatrix} w[L] & \cdots & w[N+L-1] \end{bmatrix}^t$$

$$\tilde{y}_n = \text{DFT}(\mathbf{y})_n$$

$$\tilde{h}_n = \sqrt{N}\,\text{DFT}(\mathbf{h})_n$$

$$\tilde{d}_n = \text{DFT}(\mathbf{d})_n$$

$$\tilde{w}_n = \text{DFT}(\mathbf{w})_n$$

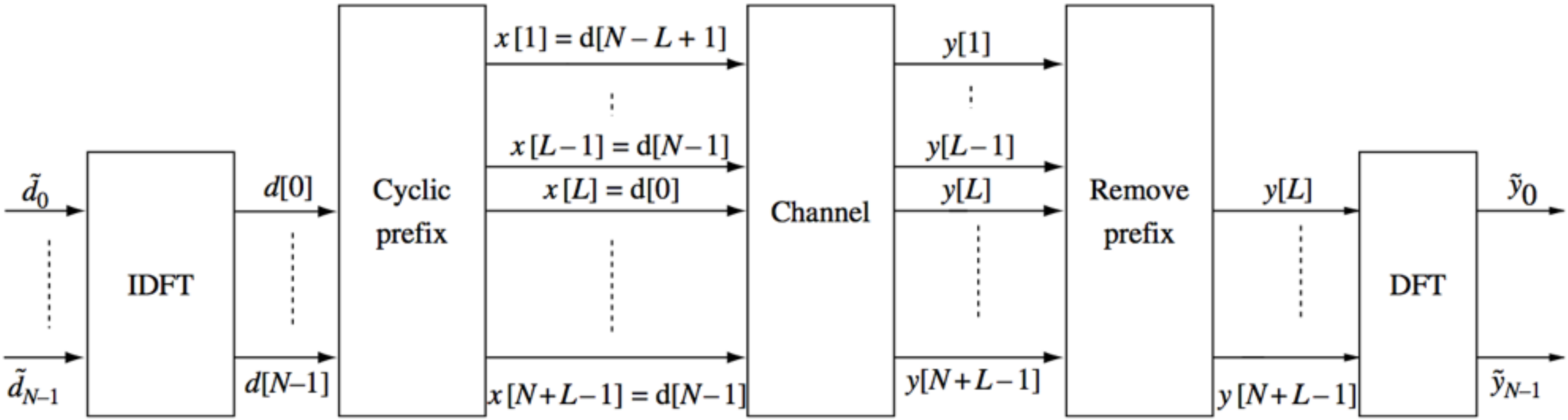
$$\mathbf{y} = \mathbf{h} \circledast \mathbf{d} + \mathbf{w}$$

$$\Big\downarrow \ \text{DFT}$$

$$\tilde{y}_n = \tilde{h}_n \tilde{d}_n + \tilde{w}_n$$

Still Gaussian

OFDM: From Circular Convolution to Multiplication



- Since the receiver observes the DFT of d[m], the transmitter should encode its information into the DFT of d[m] too.

- Each tone $\tilde{d}_n$ carries a different constellation symbol. Transmitter takes the inverse DFT (IDFT) to get d[m].

- Remember that the DFT and IDFT can be computed extremely efficiently using the Fast Fourier Transform (FFT).

OFDM: Number of Tones and Cyclic Prefix Length

- The resulting end-to-end channel is now very simple! No intersymbol interference (ISI) due to convolution so there is no need for the Viterbi algorithm.
- The cyclic prefix is the price we pay for maintaining this simplicity.
  - We throw away L out of N + L - 1 time slots.
  - If the constellation has M symbols, the rate is $\frac{N}{N+L-1}\log M$
- Ideally, we would like to make N as large as possible.
- Remember that we need to stay within one coherence interval so that the channel is time-invariant.

OFDM: Number of Tones and Cyclic Prefix Length

- We need to stay within one coherence interval.

DFT Length

$$\frac{N}{W} < T_c$$

Coherence Time

Communication Bandwidth

$$\implies N < T_c W$$

- Cyclic prefix length is set by the number of taps L.

$$L = T_d W$$

Delay Spread

- Example: LTE Parameter Ranges

$$W = 1.25 \text{ to } 20\,\text{MHz}$$

$$N = 128 \text{ to } 2048$$

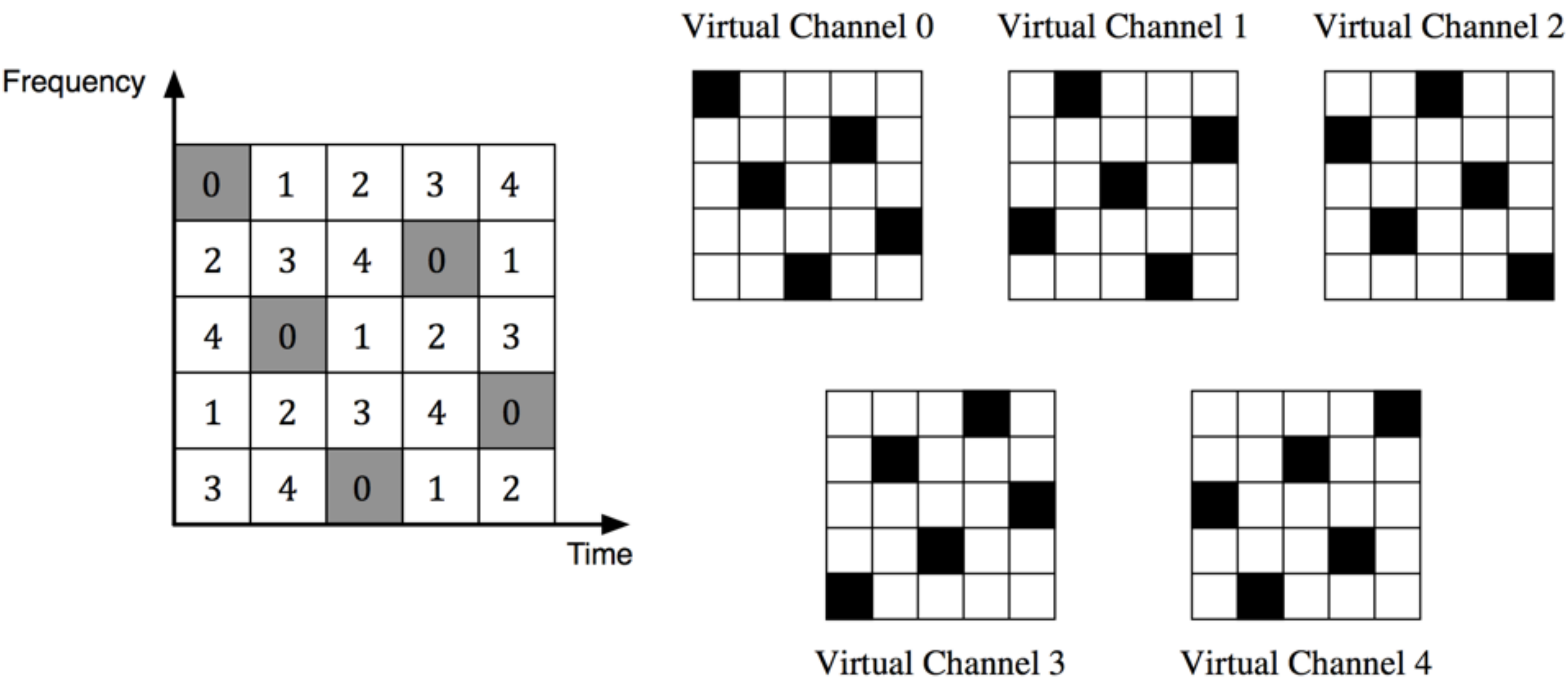
Overhead approximately 5% $\quad L = 9 \text{ to } 144$

OFDMA

- OFDM naturally leads to a multiple-access strategy.
- Just assign each user to a separate tone 0, 1, …, N - 1.
- What about frequency diversity?
- Have the users hop across frequencies every OFDM block (of length N).
- This leads to a “virtual channel” for each user across time and frequency.

OFDMA: Example Hopping Pattern within a Cell

- Example: N = 5 tones (supporting 5 users).



OFDMA: Universal Frequency Reuse

- To obtain a frequency reuse factor of 1, we need to allow every cell to use all frequencies simultaneously.
- What about inter-cell interference?
- All we can do is try to induce interference averaging.
  - Worst-case scenario: Two (or more) nearby users in adjacent cells share the same hopping pattern.
  - Best-case scenario: Minimal overlap between any two users across hopping pattern. Even if two users are nearby, they will only interfere in a few frequencies (hopefully just one).
- We need to carefully design the hopping pattern assigned to each cell.

OFDMA: Inter-Cell Interference

- Example: 5 tones (supporting 5 users)

Bad Choice:

$$
\begin{array}{cc}
\text{Cell A} & \text{Cell B} \\
\begin{bmatrix} \boxed{0} & 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & \boxed{0} & 1 \\ 4 & \boxed{0} & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 & \boxed{0} \\ 3 & 4 & \boxed{0} & 1 & 2 \end{bmatrix} &
\begin{bmatrix} \boxed{0} & 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & \boxed{0} & 1 \\ 4 & \boxed{0} & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 & \boxed{0} \\ 3 & 4 & \boxed{0} & 1 & 2 \end{bmatrix}
\end{array}
$$

Good Choice:

$$
\begin{array}{cc}
\text{Cell A} & \text{Cell B} \\
\begin{bmatrix} \boxed{0} & 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & \boxed{0} & 1 \\ 4 & \boxed{0} & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 & \boxed{0} \\ 3 & 4 & \boxed{0} & 1 & 2 \end{bmatrix} &
\begin{bmatrix} \boxed{0} & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & \boxed{4} & 0 \\ 2 & \boxed{3} & 4 & 0 & 1 \\ 3 & 4 & 0 & 1 & \boxed{2} \\ 4 & 0 & \boxed{1} & 2 & 3 \end{bmatrix}
\end{array}
$$

OFDMA Hopping Patterns: General Design Rule

- Each cell has its own hopping pattern that is determined by a Latin square.
- Within our context, a Latin square is a matrix that contains each value 0, 1, …, N - 1 exactly once on each row and each column.
- We would like to ensure that two users (in different cells) overlap (in time and frequency) interfere exactly once. Two Latin squares with this property are called orthogonal.
- A set of Latin squares is said to be mutually orthogonal if all pairs are orthogonal.

OFDMA Hopping Patterns: General Design Rule

- For any prime N, we can construct a set of N - 1 mutually orthogonal Latin squares as follows:

- For $a \in \{1, 2, \ldots, N-1\}$, the matrix $\mathbf{R}^a$ has the value $\mathbf{R}^a_{i,j} = (ai + j) \bmod N$ in its (i,j)th entry where i and j take values from 0 to N - 1.

- If $a \neq b$, then it can be shown that the Latin squares $\mathbf{R}^a$ and $\mathbf{R}^b$ are orthogonal.

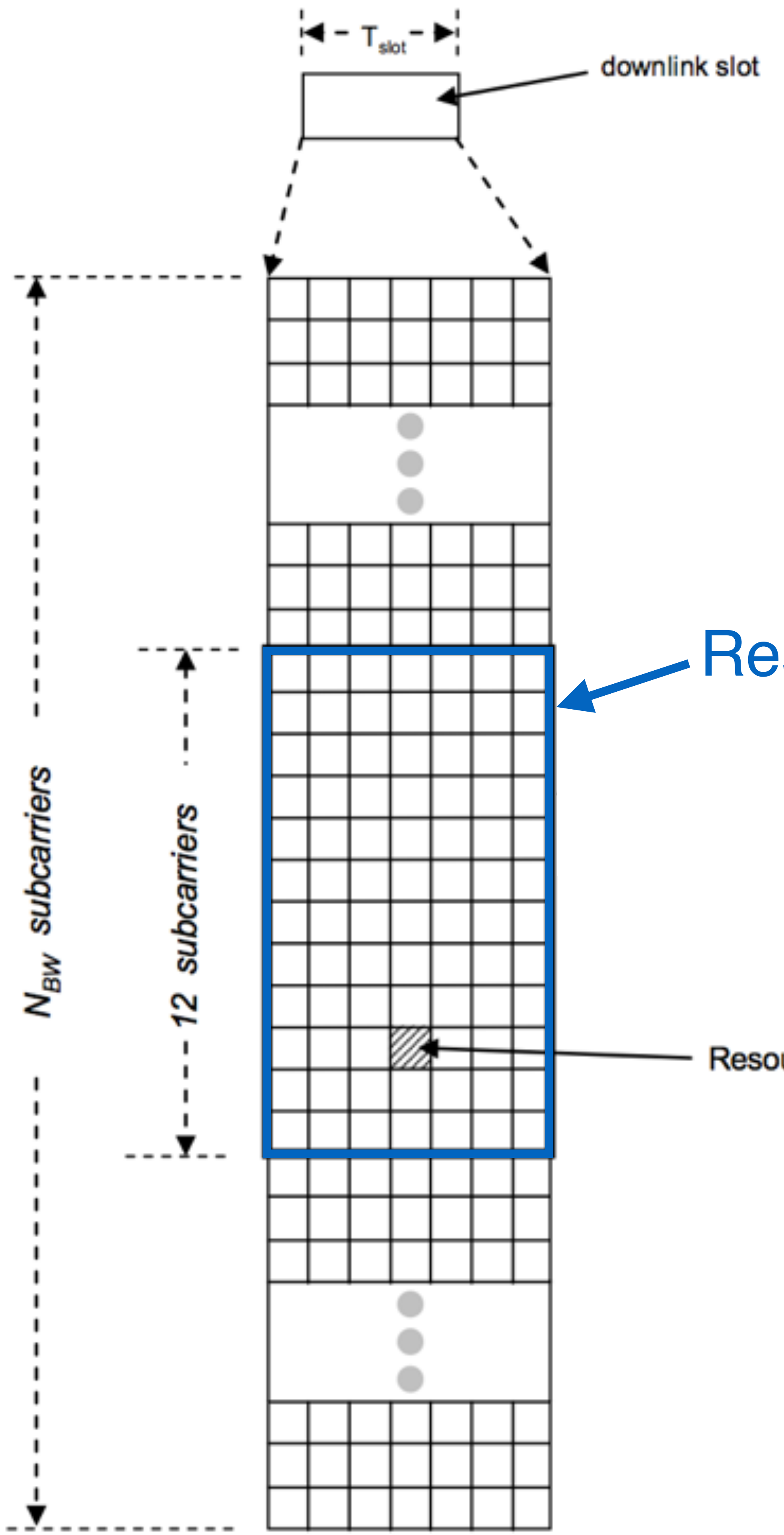
- The N = 5 matrices we saw earlier are examples of this construction.

- This hopping pattern combined with channel coding across tones is our interference management strategy.

OFDMA Scheduling Basics

- There are usually not enough tones to support all users in a cell.
- At any given time, a user may fall into one of three categories:
  - Active: Currently assigned to a virtual channel (or hopping pattern).
    Roughly 30 users.
  - Hold: Not assigned to a virtual channel but maintaining synchronization with the basestation. Synchronization is relatively easy (compared to CDMA). Can be moved to active state very rapidly. Roughly 130 users.
  - Sleep: Not even maintaining synchronization.
    Roughly 1000 users.

LTE Downlink: OFDMA



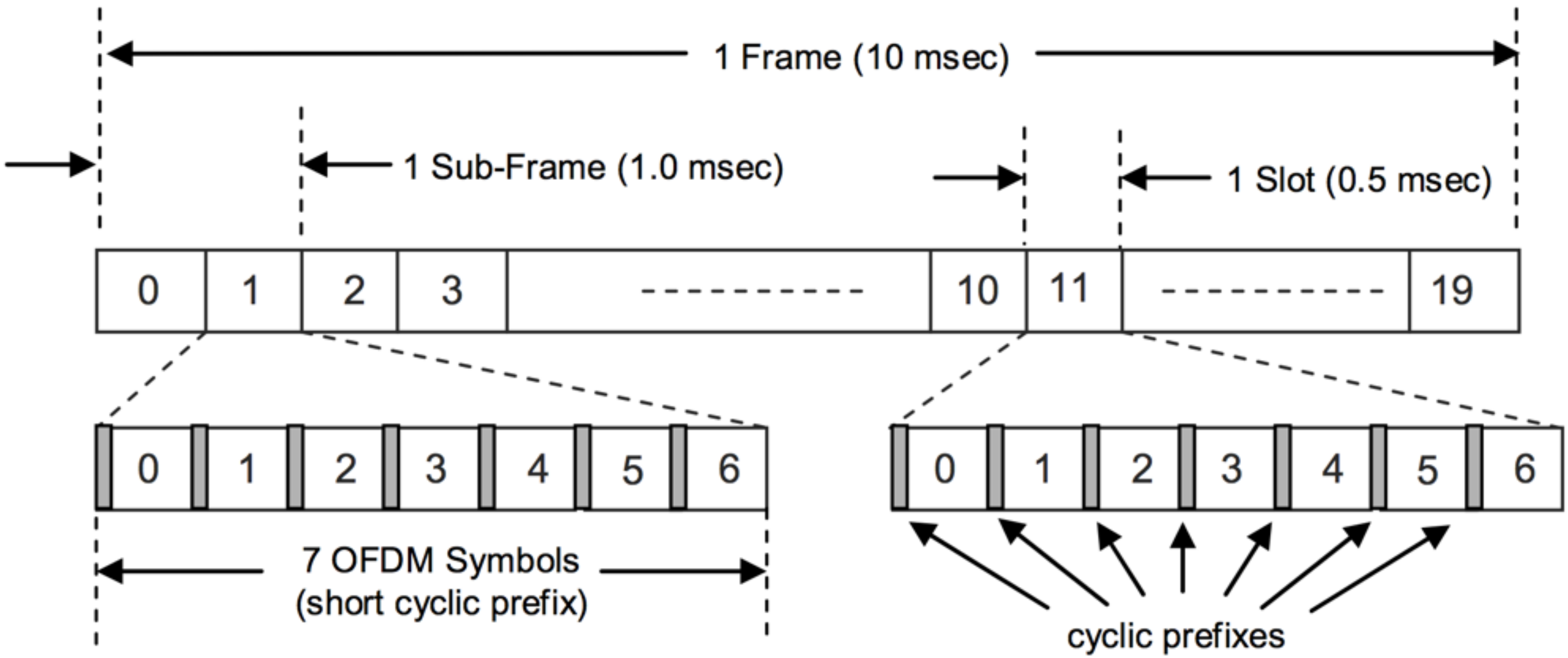
- OFDMA is used as a building block for the downlink in LTE.

- Tones and channel uses are allocated in groups that are called resource blocks.

- Specifically, a resource block consists of 12 neighboring subcarriers (or tones) and 6 neighboring symbols (or channel uses).

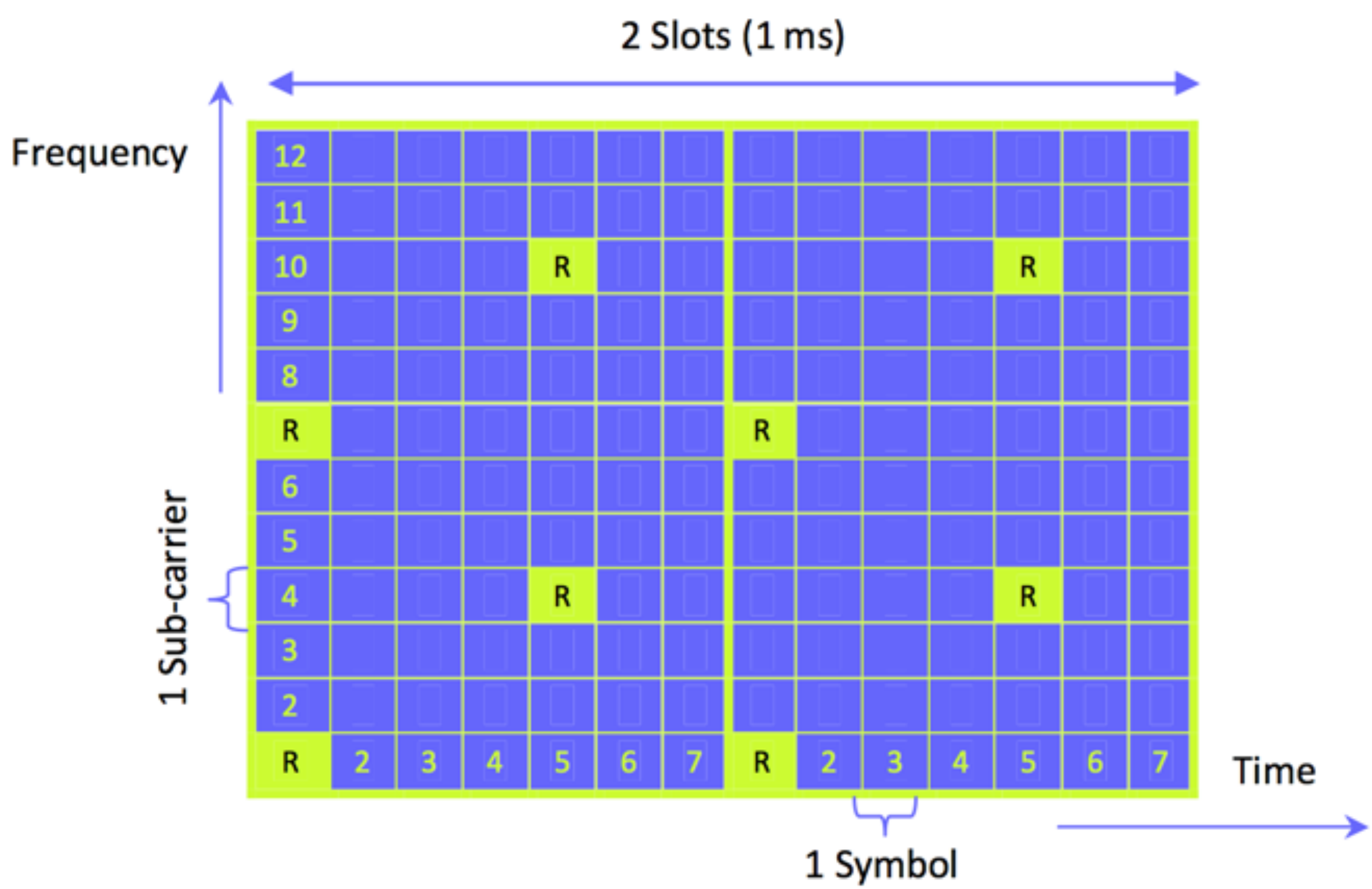
- This is an engineering decision. Reduce interference averaging to make channel estimation easier.

LTE Downlink: Time Domain Perspective



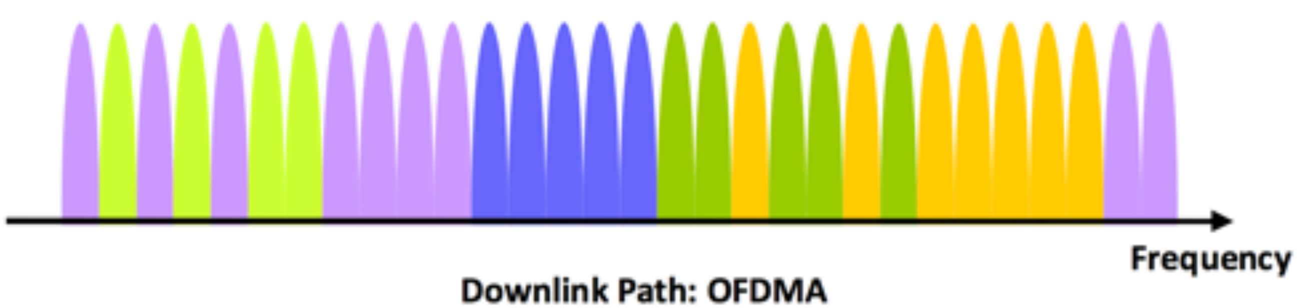
LTE Downlink: Channel Estimation

- Some of the resource elements are pilot symbols for channel estimation.
- In LTE, these are called reference symbols (R).



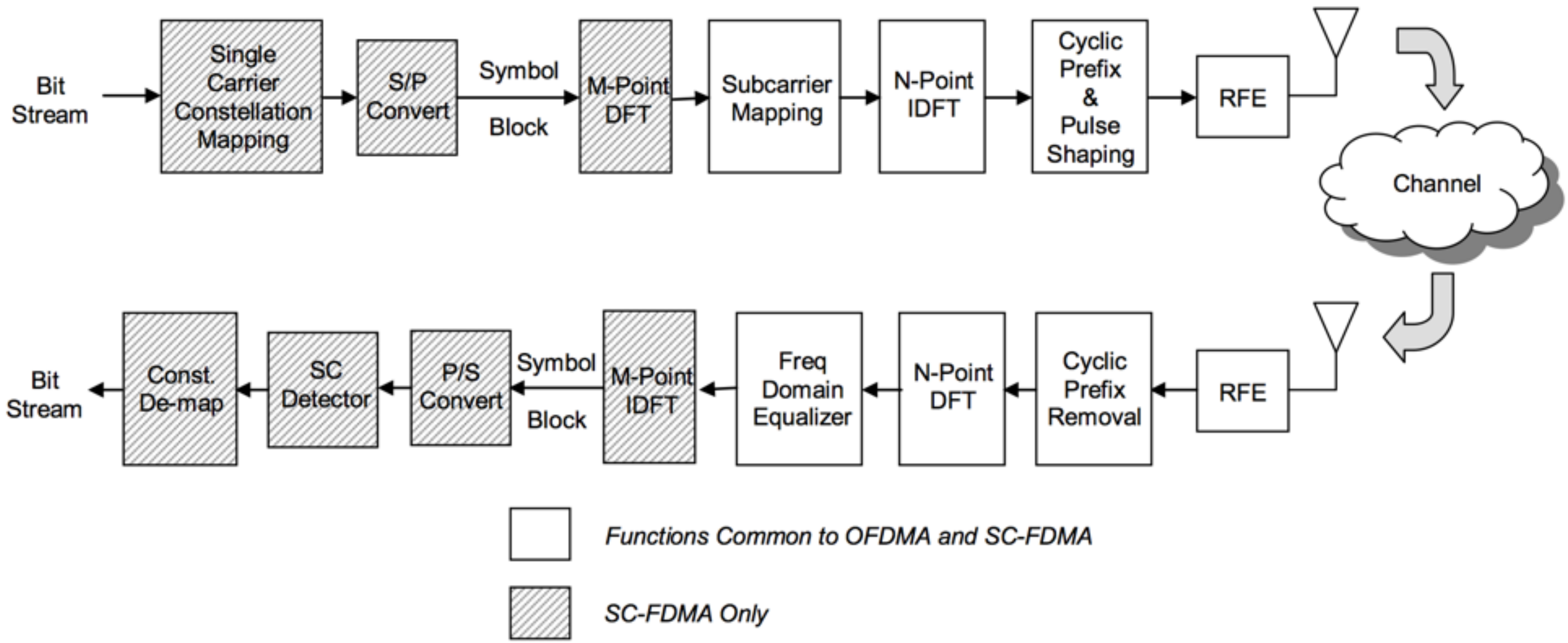
LTE Uplink: SC-FDMA

- An important engineering consideration is the Peak-to-Average Power Ratio (PAPR).
- A high PAPR means that we need an "expensive" amplifier.
- OFDMA has many advantages, but one significant disadvantage is that it has high PAPR. To reduce PAPR, it was decided to allocate many adjacent subcarriers (or tones) to one user (in the uplink only).
- This scheme is called Single-Carrier Frequency-Division Multiple-Access (SC-FDMA).



LTE Uplink SC-FDMA

- The basic idea in SC-FDMA is to apply one more DFT before OFDMA and carefully map symbols to avoid PAPR.



Summary

	Narrowband system	Wideband CDMA	Wideband OFDMA
Signal	Narrowband	Wideband	Wideband
Intra-cell bandwidth allocation	Orthogonal	Pseudorandom	Orthogonal
Intra-cell interference	None	Significant	None
Inter-cell bandwidth allocation	Partial reuse	Universal reuse	Universal reuse
Inter-cell uplink interference	Bursty	Averaged	Averaged
Accuracy of power control	Low	High	Low
Operating SINR	High	Low	Range: low to high
PAPR of uplink signal	Low	Medium	High