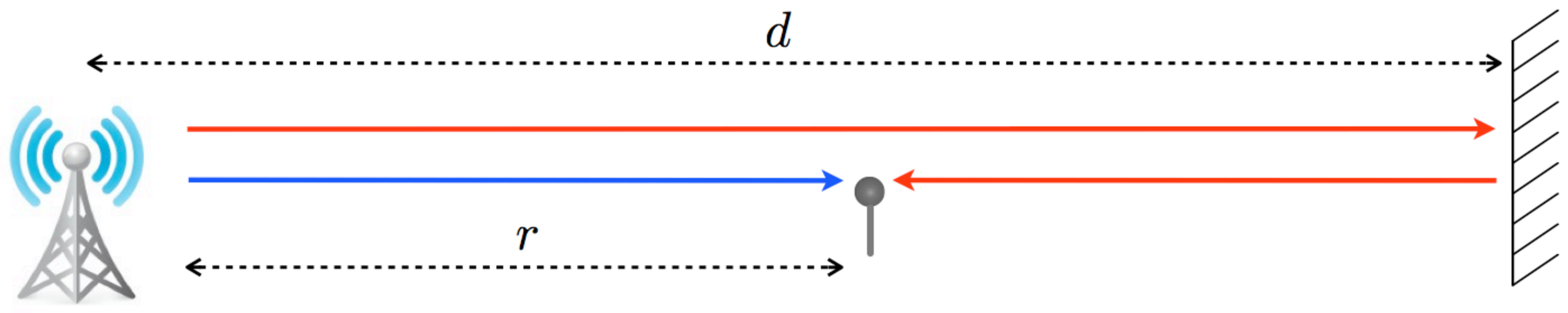
Lecture 3: Wireless Channels as Linear Time-Varying Systems

Prof. Bobak Nazer 9/9/14

Example: Reflecting Wall, Fixed Receive Antenna



Transmitted Signal: $\cos(2\pi f_c t)$

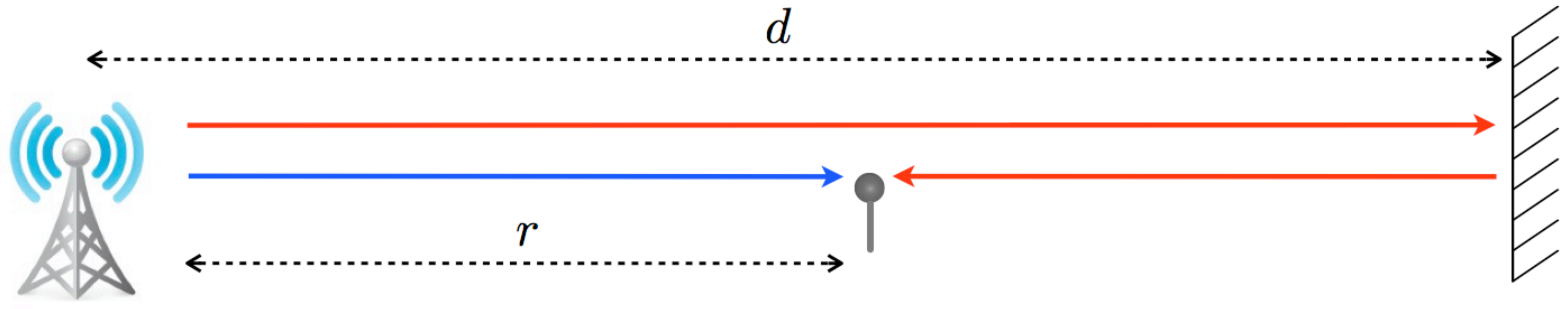
Direct Path: $\frac{\alpha}{r}\cos\left(2\pi f_c\left(t - \frac{r}{c}\right)\right)$

Reflected Path: $-\frac{\alpha}{2d - r}\cos\left(2\pi f_c\left(t - \frac{2d - r}{c}\right)\right)$

Received Signal:

$$\frac{\alpha}{r}\cos\left(2\pi f_c\left(t - \frac{r}{c}\right)\right) - \frac{\alpha}{2d - r}\cos\left(2\pi f_c\left(t - \frac{2d - r}{c}\right)\right)$$

Example: Reflecting Wall, Fixed Receive Antenna



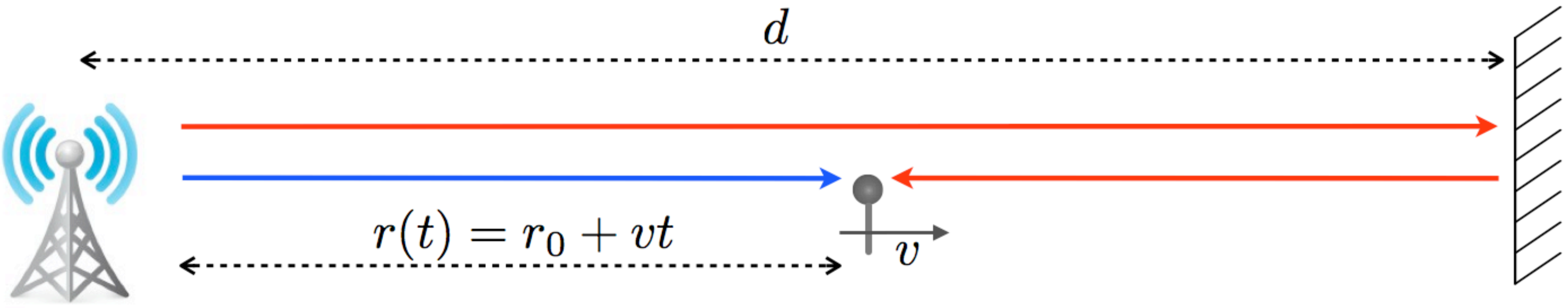
Received Signal:

$$\frac{\alpha}{r}\cos\left(2\pi f_c\left(t-\frac{r}{c}\right)\right) - \frac{\alpha}{2d-r}\cos\left(2\pi f_c\left(t-\frac{2d-r}{c}\right)\right)$$

- Phase Difference: $\Delta\phi = \dfrac{4\pi f_c}{c}(d-r)+\pi$

- Signal strength goes from a peak to a valley if the receiver moves by $\dfrac{\lambda_c}{4}$

Example: Reflecting Wall, Moving Receive Antenna



Transmitted Signal: $\cos(2\pi f_c t)$

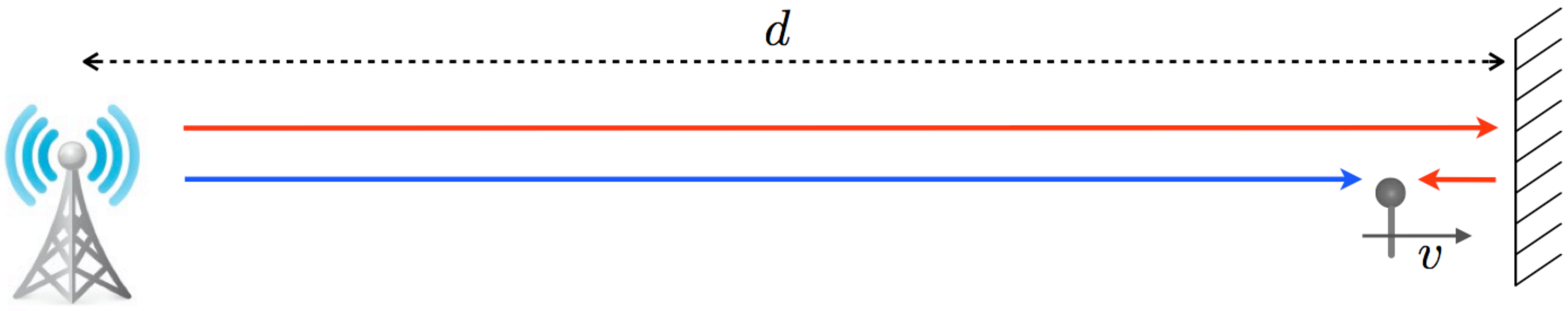
Direct Path: $$\frac{\alpha}{r(t)} \cos\left(2\pi f_c\left(t - \frac{r(t)}{c}\right)\right)$$

Reflected Path: $$-\frac{\alpha}{2d - r(t)} \cos\left(2\pi f_c\left(t - \frac{2d - r(t)}{c}\right)\right)$$

Received Signal:

$$\frac{\alpha}{r_0 + vt} \cos\left(2\pi f_c\left(\left(1 - \frac{v}{c}\right)t - \frac{r_0}{c}\right)\right) - \frac{\alpha}{2d - r_0 - vt} \cos\left(2\pi f_c\left(\left(1 + \frac{v}{c}\right)t - \frac{2d - r_0}{c}\right)\right)$$

Example: Reflecting Wall, Moving Receive Antenna



- Let's assume that the receiver is very close to the wall:

$$r_0 + vt \approx 2d - r_0 - vt$$

Received Signal:

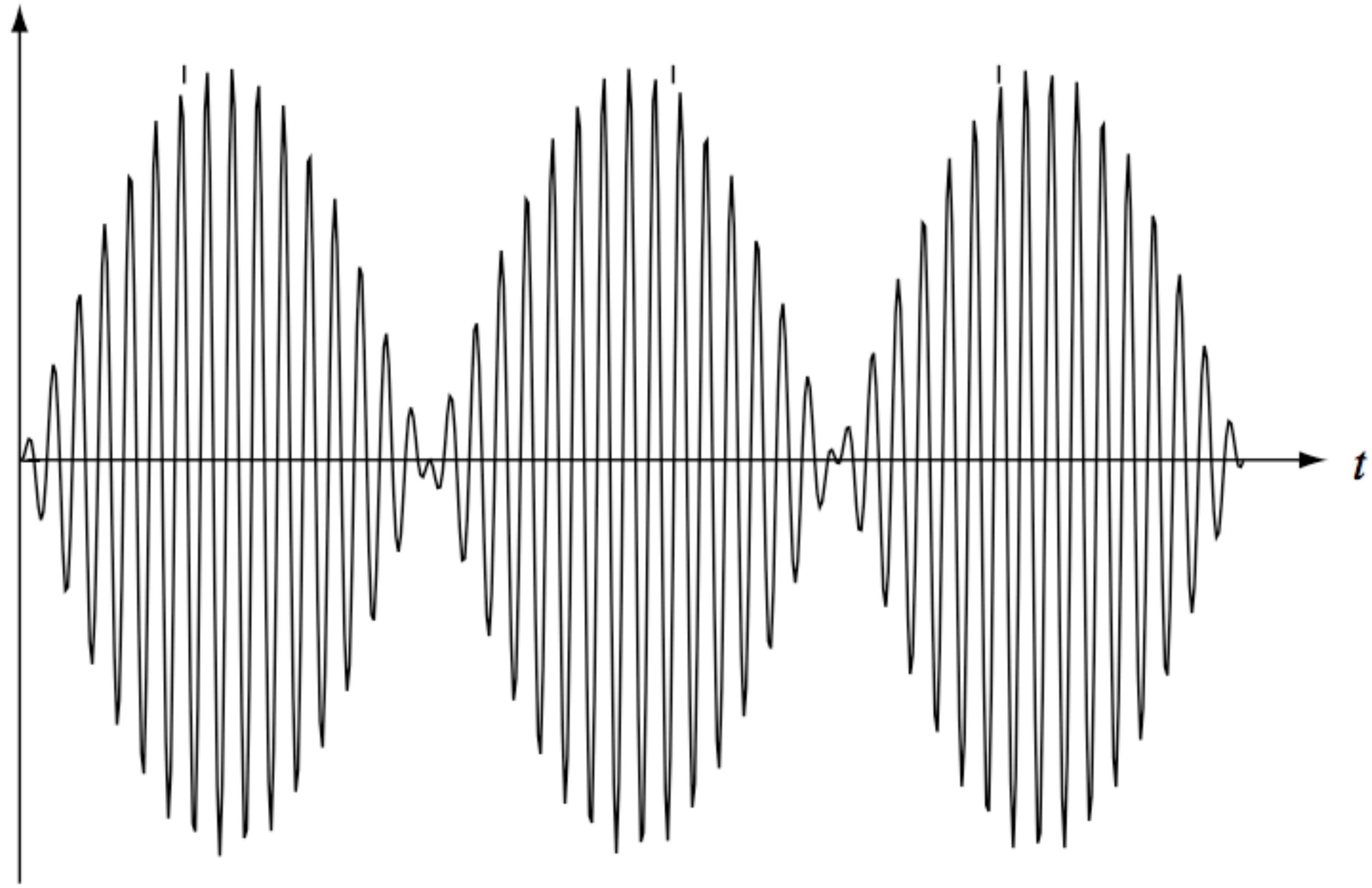
$$\frac{\alpha}{r_0 + vt} \cos\left(2\pi f_c\left(\left(1 - \frac{v}{c}\right)t - \frac{r_0}{c}\right)\right) - \frac{\alpha}{2d - r_0 - vt} \cos\left(2\pi f_c\left(\left(1 + \frac{v}{c}\right)t - \frac{2d - r_0}{c}\right)\right)$$

$$\approx \underbrace{\boxed{\frac{2\alpha}{r_0 + vt} \sin\left(2\pi f_c\left(\frac{vt}{c} + \frac{r_0 - d}{c}\right)\right)}}_{\text{Time-varying amplitude}} \underbrace{\boxed{\sin\left(2\pi f_c\left(t - \frac{d}{c}\right)\right)}}_{\text{Delayed version of transmitted signal}}$$

Time-varying amplitude

Delayed version of transmitted signal

Example: Reflecting Wall, Moving Receive Antenna



Time-varying amplitude: $\boxed{\dfrac{2\alpha}{r_0 + vt}}\boxed{\sin\left(2\pi f_{\mathrm{c}}\left(\dfrac{vt}{c} + \dfrac{r_0 - d}{c}\right)\right)}$

Time-variation scale: $\dfrac{r_0}{v}$

Time-variation scale: $\dfrac{c}{f_{\mathrm{c}} v}$

Slow (seconds to minutes)

Fast (milliseconds)

Linear, Time-Varying System Model

- Let $x(t)$ denote the transmitted signal.
- For each example we have considered, the received signal can be written as

$$y(t) = \sum_i a_i(f,t)\,x\big(t - \tau_i(f,t)\big) + \boxed{w(t)}$$

Noise (ignore for now)

where

$a_i(f,t) =$ attenuation of the signal traveling along the i$^{\text{th}}$ path

$\tau_i(f,t) =$ delay of the signal traveling along the i$^{\text{th}}$ path

Narrowband Systems

- In this class, we will mainly focus on narrowband communication systems. Specifically, we will assume that the bandwidth W of the transmitted signal is much smaller than the carrier frequency f_c.
- This means that we can safely assume that the attenuation and path delays are independent of the frequency.
- The resulting linear, time-varying channel model is:

$$y(t) = \sum_i a_i(t)\, x\big(t - \tau_i(t)\big) + w(t)$$

$a_i(t) =$ attenuation of the signal traveling along the i$^{\text{th}}$ path

$\tau_i(t) =$ delay of the signal traveling along the i$^{\text{th}}$ path

Impulse Response

- We can write this channel in terms of a time-varying impulse response:

$$y(t) = \int_{-\infty}^{\infty} h(\tau, t) x(t - \tau) d\tau + w(t)$$

$$h(\tau, t) = \sum_i a_i(t) \delta\big(\tau - \tau_i(t)\big)$$ Channel impulse response

- If nothing moved (Tx, Rx, and the whole environment), then we would get a linear, time-invariant (LTI) system:

$$y(t) = \int_{-\infty}^{\infty} h(\tau) x(t - \tau) d\tau + w(t)$$

$$h(\tau) = \sum_i a_i \delta(t - \tau_i)$$

But this is a bad model for the wireless channel!