Number Sense 12023

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Name _____

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1. $2020 - 20 =$ _____.
2. $494 + 118 =$ _____.
3. The tens digit of 1296 is _____.
4. $60 \div 5 =$ _____.
5. $14 \times 50 =$ _____.
6. The remainder of $66 \div 4$ is _____.
7. The product of 14 and 3 is _____.
8. $8 \times 11 + 8 =$ _____.
9. $53 + 12 + 28 + 7 =$ _____.
10. (estimate) $201 + 402 + 804 + 603 =$ _____.
11. 1987 rounded to the nearest ten is _____.
12. $17^2 =$ _____.
13. $578 - 361 - 27 =$ _____.
14. $11 \times 48 =$ _____.
15. $681 - 292 =$ _____.
16. $45 \times 18 =$ _____.
17. DCLXXVI in Arabic Numerals is _____.
18. $37 + 41 + 45 =$ _____.
19. $32 \times 28 =$ _____.
20. (estimate) $398 \times 202 =$ _____.
21. $36 \times 25 =$ _____.
22. $44 \times 3 \div 33 =$ _____.
23. The GCD of 28 and 21 is _____.
24. $342 \div 18 =$ _____.
25. If 1 yard is equal to 3 feet then 63 feet is equal to _____ yards.
26. $24^2 =$ _____.
27. The smaller of $\frac{5}{8}$ and $\frac{3}{5}$ is _____ (fraction).
28. $\frac{5}{8} + \frac{1}{24} =$ _____ (fraction).
29. Seventeen weeks is _____ days.
30. (estimate) $29 \times 41 \times 61 =$ _____.
31. $9 + 13 + 17 + 21 + 25 =$ _____.
32. $65^2 =$ _____.
33. $1.444 \times 10^3 =$ _____.
34. The LCM of 28 and 21 is _____.
35. $54 \times 101 =$ _____.
36. The number of multiples of 5 between 11 and 51 is _____.
37. $67 \times 63 =$ _____.
38. The number of prime numbers that are between 40 and 50 is _____.
39. $99 \times 42 =$ _____.
40. (estimate) $\sqrt[3]{511111} =$ _____.
41. $32^2 - 21^2 =$ _____.
42. $\sqrt{784} =$ _____.

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43. The remainder of $458 \div 11$ is _____.
44. $2^9 =$ _____.
45. $112 \times 103 =$ _____.
46. The next term in the arithmetic sequence $14, 33, 52, 71, 90, \ldots$ is _____.
47. $66 \times 46 =$ _____.
48. The perimeter of a rectangle with sides of length 12 and 9 is _____.
49. $\frac{1}{7} + \frac{1}{14} + \frac{1}{28} =$ _____ (fraction).
50. (estimate) $375 \times 480 =$ _____.
51. $32^2 + 17^2 =$ _____.
52. $16\frac{2}{3}\% =$ _____ (fraction).
53. $\frac{4}{9}$ of $\frac{1}{4}$ of 99 is _____.
54. If $4 + 12x = 36$ then $3x + 1 =$ _____.
55. $92 \times 92 =$ _____.
56. The greatest possible perimeter of an isosceles triangle with sides of length 2 and 5 is ___.
57. $\sqrt{7056} =$ _____.
58. 134_8 in base 2 is _____$_2$.
59. The sum of the terms in the arithmetic sequence $1, 3, 5, 7, \ldots, 49$ is _____.
60. (estimate) $285714 \times 16 =$ _____.
61. A fair die is rolled twice. The probability of the two rolls summing to 9 is _____ (fraction).
62. $241 \times 111 =$ _____.
63. $9\frac{5}{8} \times 9\frac{3}{8} =$ _____ (mixed number).
64. The supplement to a 51° angle has a measure of _____$^\circ$.
65. If $6^x = 16$ then $6^{2x} =$ _____.
66. The number 75 written in base 6 is _____$_6$.
67. $7\frac{2}{3} \times 11\frac{2}{3} =$ _____ (mixed number).
68. $0.0\overline{6} =$ _____ (fraction).
69. $72 \times 625 =$ _____.
70. (estimate) $(\sqrt{10})^5 =$ _____.
71. $\sqrt[3]{1259712} =$ _____.
72. The hypotenuse of a right triangle with legs of length 20 and 21 is _____.
73. $396 \times 394 =$ _____.
74. An equilateral triangle with side of length $6\sqrt{3}$ has a height of length _____.
75. The number of positive integral divisors of 54 is _____.
76. The mean of the set $\{9, 21, 20, 22, 23\}$ is ___.
77. The reciprocal of $\frac{20}{19}$ is _____ (decimal).
78. The eighth term in the geometric sequence $\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, \ldots$ is _____.
79. $16 \times 22 \times 15 =$ _____.
80. (estimate) $\sqrt{2} \cdot \sqrt{101} \cdot \sqrt{8} \cdot \sqrt{2} =$ _____.

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Sprint Answers 12023

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Select E only if you cannot determine a uniquely correct answer between A, B, C, and D.

Age 13+? Want to be added to our email list?

Email Address _____

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Correct × 4 _____ Score _____

Incorrect × -1 _____ Check _____

Scorer Initials _____ Scorer Initials _____

1. (A) (B) (C) (D) (E)	11. (A) (B) (C) (D) (E)	21. (A) (B) (C) (D) (E)
2. (A) (B) (C) (D) (E)	12. (A) (B) (C) (D) (E)	22. (A) (B) (C) (D) (E)
3. (A) (B) (C) (D) (E)	13. (A) (B) (C) (D) (E)	23. (A) (B) (C) (D) (E)
4. (A) (B) (C) (D) (E)	14. (A) (B) (C) (D) (E)	24. (A) (B) (C) (D) (E)
5. (A) (B) (C) (D) (E)	15. (A) (B) (C) (D) (E)	25. (A) (B) (C) (D) (E)
6. (A) (B) (C) (D) (E)	16. (A) (B) (C) (D) (E)	26. (A) (B) (C) (D) (E)
7. (A) (B) (C) (D) (E)	17. (A) (B) (C) (D) (E)	27. (A) (B) (C) (D) (E)
8. (A) (B) (C) (D) (E)	18. (A) (B) (C) (D) (E)	28. (A) (B) (C) (D) (E)
9. (A) (B) (C) (D) (E)	19. (A) (B) (C) (D) (E)	29. (A) (B) (C) (D) (E)
10. (A) (B) (C) (D) (E)	20. (A) (B) (C) (D) (E)	30. (A) (B) (C) (D) (E)

Sprint Round 12023

1. What is the tens digit of 2019×2020?

   (A) 6 (B) 4 (C) 2 (D) 8 (E) Other

2. Addison added up all of the even positive whole numbers less than 19. What was the result of Addison's addition?

   (A) 84 (B) 90 (C) 72 (D) 80 (E) Other

3. A fortnight is two weeks. A week is seven days. How many days are in seven fortnights?

   (A) 91 (B) 98 (C) 112 (D) 70 (E) Other

4. How many two-digit whole numbers have digits that sum to 12?

   (A) 7 (B) 6 (C) 5 (D) 8 (E) Other

5. The perimeter of a rectangle is 42. If the length of the rectangle is 16, then what is the area of the rectangle?

   (A) 90 (B) 84 (C) 72 (D) 80 (E) Other

6. Three pieces of candy cost \$0.35. Ethan wants to purchase two dozen pieces of candy for his class. How much will that cost Ethan?

   (A) \$2.80 (B) \$3.15 (C) \$2.10 (D) \$2.45 (E) Other

7. How many seventeens are in six fifty-ones and nine thirty-fours?

   (A) 35 (B) 39 (C) 36 (D) 32 (E) Other

8. Lynn made a list of all the positive whole numbers that evenly divide 18. What is the average of the numbers in Lynn's list?

   (A) 6.5 (B) 9 (C) 7.5 (D) 6 (E) Other

9. Philip flipped a fair coin and rolled a standard six-sided die. What is the probability that Philip got a tails on the coin flip or a multiple of 3 on the die roll?

   (A) $\frac{1}{2}$ (B) $\frac{3}{4}$ (C) $\frac{2}{3}$ (D) $\frac{1}{3}$ (E) Other

10. What is the remainder when $14 \cdot 29 \cdot 44$ is divided by 13?

    (A) 5 (B) 4 (C) 1 (D) 3 (E) Other

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Sprint Round 12023

11. A large square has an area that is 64 times the area of a smaller square. What is the value of the length of a side of the smaller square divided by the length of a side of the larger square?

(A) $\frac{1}{32}$ (B) $\frac{1}{16}$ (C) $\frac{1}{8}$ (D) $\frac{1}{2}$ (E) Other

12. What is the product of $12\frac{1}{2}\%$ of 546 and $16\frac{2}{3}\%$ of 776?

(A) 8736 (B) 8918 (C) 8811 (D) 8827 (E) Other

13. Jason's Teeny Cooper car averages 16 kilometers per liter. Fuel for his car is priced at \$1.30 per liter. Jason has \$18.20 to purchase fuel. How far, in kilometers, can Jason expect to travel on the fuel he purchases?

(A) 238 (B) 224 (C) 212 (D) 218 (E) Other

14. Aiden drew two circles of diameter 40. Hayden drew a number of circles of diameter 8. The total area of Aiden's circles was equal to the total area of Hayden's circles. How many circles did Hayden draw?

(A) 50 (B) 60 (C) 40 (D) 100 (E) Other

15. What is the remainder when the sum of the terms of the arithmetic series $8 + 15 + 22 + \cdots + 99$ is divided by 28?

(A) 21 (B) 16 (C) 14 (D) 7 (E) Other

16. Trey can run $\frac{1}{4}$ mile in 1 minute and 20 seconds. If Trey runs at a constant rate, then how many minutes, to the nearest minute, will it take Trey to run 13.1 miles?

(A) 70 (B) 75 (C) 69 (D) 72 (E) Other

17. In a leap year, there are 29 days in February. In years that are not leap years, there are 28 days in February. Leap years occur when the the year is divisible by 4 but not 100, with the exception that years divisible by 400 are leap years. For example, 1816 and 2000 were leap years, but 1900 was not. The United States celebrated its bicentennial, or 200th birthday, on July 4th, 1976. How many days old was the United States on its bicentennial?

(A) 72950 (B) 73000 (C) 73150 (D) 73048 (E) Other

18. Two right triangles have the same area. One right triangle has legs of length 8 and 24. The second right triangle has a leg of length of length 12. What is the perimeter of the second triangle?

(A) 42 (B) 54 (C) 56 (D) 48 (E) Other

19. The word *EUNOIA* is the shortest word in the English language that has all of the letters *A*, *E*, *I*, *O*, and *U*. How many different arrangements of the letters in the word *EUNOIA* are possible if the arrangement may not have *N* as the first letter?

(A) 360 (B) 600 (C) 480 (D) 720 (E) Other

20. If $a\clubsuit b = ab - a - b + 1$, then what is the value of $126\clubsuit 112$?

(A) 14625 (B) 13375 (C) 13875 (D) 14875 (E) Other

21. The sum of the squares of two positive whole numbers is 9808. The product of the two numbers is 4896. What is the value of the larger of the two numbers minus the smaller of the two numbers?

(A) 8 (B) 2 (C) 6 (D) 12 (E) Other

22. If $8^x = 27$, then what is the value of 4^{2x+1}?

(A) 162 (B) 324 (C) 432 (D) 243 (E) Other

23. The binary number 1001001001_2 is multiplied by the base 10 number 14. When writing the result in binary, how many more ones than zeros are used?

(A) 10 (B) 12 (C) 11 (D) 8 (E) Other

24. What is the least positive whole number that is greater than 100 and has exactly 8 positive whole number divisors?

(A) 104 (B) 135 (C) 102 (D) 136 (E) Other

25. What is the units digit of $13^{2019} \cdot 17^{2020}$?

(A) 6 (B) 7 (C) 9 (D) 1 (E) Other

26. Trinity chose a positive whole number and then summed up all of the positive whole numbers less than or equal to her chosen number. When she added the whole number one greater than her chosen number to her sum, the total increased by $13\frac{1}{3}\%$. What is the sum of the digits of the number Trinity chose?

(A) 6 (B) 5 (C) 9 (D) 8 (E) Other

27. Which of the following is equivalent to $\frac{4}{1-\sqrt{2}-\sqrt{3}}$?

(A) $3-\sqrt{2}-\sqrt{3}$ (B) $2-\sqrt{2}-\sqrt{6}$ (C) $6-3\sqrt{2}-2\sqrt{3}$ (D) $1-\sqrt{3}$ (E) Other

28. Right triangle ABC has hypotenuse AC with length 39, and leg AB with length 15. Point D is located on AC such that the length of segment AD is 15. Points A, B, and D also lie on circle O. Point E lies on circle O and segment BC where $E \neq B$. What is the length of segment EC?

(A) 26 (B) 24 (C) 25 (D) 28 (E) Other

29. The sum of the first six terms in a geometric sequence is 28 times the sum of the first three terms in the same geometric sequence. What is the quotient when the third term of the sequence is divided by the first term of the sequence?

(A) 9 (B) 3 (C) 8 (D) 4 (E) Other

30. Right triangle ABC is isosceles, with hypotenuse AC. Point X is on segment AC such that the measure of angle XBC is $60°$. If the length of leg AB is $1 + \sqrt{3}$, then what is the length of segment BX?

(A) 3 (B) $\sqrt{2} + \sqrt{3}$ (C) $\sqrt{3} - 1$ (D) $\sqrt{6}$ (E) Other

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Questions 1 & 2

Score #1 _____ Scorer's Initials _____

Score #2 _____ Scorer's Initials _____

1. What is the sum of all the positive whole numbers that are less than 28 and evenly divide 28?

1.

2. Six squares of side length 7 are arranged in three rows. The bottom row has three squares placed next to one another, so that squares next to one other share a side. The second row rests entirely on the bottom row, and has two squares which share a side. The third row rests entirely on the second row and has one square. What is the perimeter of the region covered by all six squares?

2.

Target Round 12023

Name _____

Grade _____

School _____

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Questions 3 & 4

Score #1 _____ Scorer's Initials _____

Score #2 _____ Scorer's Initials _____

3. For her weekly exercise routine, Kelly rides her Bellaton stationary bike 35 minutes each day on Mondays, Wednesdays, and Fridays. Kelly walks 55 minutes each day on Tuesdays and Thursdays. On Saturdays, she rests, but on Sundays she takes a longer 70 minute walk. How many total hours each week does Kelly exercise? Express your answer as a decimal to the nearest hundredth.

3.

4. Dan and Dave started running in opposite directions from the same location on a circular running track that is $\frac{1}{4}$ mile in length. They passed each for the first time exactly one minute and fifteen seconds after they started. If Dan was running at a constant speed of 5.4 miles per hour, and Dave was also running at a constant speed, then what was Dave's speed, in miles per hour? Express your answer as a decimal to the nearest tenth.

4.

Target Round 12023

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Grade _____

School _____

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Questions 5 & 6

Score #1 _____ Scorer's Initials _____

Score #2 _____ Scorer's Initials _____

5. Patricia picked a prime number, and she noticed that when she subtracted it from 76, the result was also a prime number. How many different prime numbers could Patricia have picked?

5.

6. Two right triangles have side lengths that are all whole numbers. Both triangles have a perimeter of 60. What is the ratio of the area of the smaller right triangle to the area of the larger right triangle? Express your answer as common fraction.

6.

Target Round 12023

Name _____

Grade _____

School _____

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Questions 7 & 8

Score #1 _____ Scorer's Initials _____

Score #2 _____ Scorer's Initials _____

7. In a stack of seven cards, each card is labeled with one of the whole numbers from 1 to 7, and no two cards have the same number. Stan randomly draws two cards from the stack. What is the probability that the sum of the numbers on Stan's cards is less than 10? Express your answer as a common fraction.

7.

8. What is the remainder when 2^{2020} is divided by 100?

8.

Team Round 12023

School or Team

Score #1 _____ Scorer's Initials _____

Score #2 _____ Scorer's Initials _____

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1.	2.	3.	4.	5.
6.	7.	8.	9.	10.

Team Round 12023

1. How many two-digit positive whole numbers have a tens digit that is greater than the units digit?

2. In a neighborhood parade, eleven children each rode either a bicycle or a tricycle. A bicycle has two wheels and a tricycle has three wheels. Altogether, the number of wheels on the vehicles ridden by the children totaled 29. How many more children rode tricycles than rode bicycles?

3. What is the least positive whole number that has a remainder of 5 when divided by 7 and 17 when divided by 19?

4. At Talia's Taco Truck, the price of a taco is $\frac{4}{5}$ the price of a large soda. Barry purchased three tacos and a large soda, and his total cost was \$6.12. How much would one taco and one large soda cost at Talia's Taco Truck? Express your answer as a number of dollars to the nearest hundredth.

5. The sum of the lengths of the edges of a cube is 228. What is the surface area of the cube?

6. Molly rolls two standard six-sided dice and then multiplies the numbers shown. What is the probability that Molly's result is less than 24? Express your answer as a common fraction.

7. The degree measures of the interior angles in a convex quadrilateral are in the ratio of $k : 11 : 13 : 15$, where k is an integer. All of the interior angles have degree measures that are whole numbers, and exactly two of the angles are obtuse. What is the degree measure of the smallest angle?

8. Seth picked a set of distinct numbers from the set of positive whole numbers less than or equal to 10, and observed that his numbers summed to 15. How many different sets of whole numbers could Seth have picked?

9. Square $ABCD$ has point X on side BC and point Y on side AD. The ratio of of length of segment BX to the length of segment XC is 1 to 4. The ratio of the length of segment AY to the length of segment YD is 3 to 4. What is the ratio of the area of trapezoid $ABXY$ to the area of trapezoid $DYXC$? Express your answer as a common fraction.

10. Naomi, Oscar, and Paolo each chose a positive whole number greater than 600 and less than 850. It is possible that two or more of them chose the same number. The sum of their numbers was 2019. How many different combinations of choices of numbers could Naomi, Oscar, and Paolo have made?

Answer Key 12023

Sprint Round Answers

1. D
2. B
3. B
4. A
5. D
6. A
7. C
8. A
9. C
10. E
11. C
12. D
13. B
14. A
15. A
16. A
17. D
18. D
19. B
20. C
21. E
22. B
23. C
24. C
25. B
26. A
27. B
28. A
29. A
30. E

Target Round Answers

1. 28
2. 84
3. 4.75
4. 6.6
5. 10
6. $\frac{4}{5}$
7. $\frac{5}{7}$
8. 76

Team Round Answers

1. 45
2. 3
3. 131
4. ($)3.24
5. 2166
6. $\frac{5}{6}$
7. 48(°)
8. 20
9. $\frac{11}{24}$
10. 23653

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Answer Key 12023

Number Sense Answers

1. 2000
2. 612
3. 9
4. 12
5. 700
6. 2
7. 42
8. 96
9. 100
10. [1910, 2110]
11. 1990
12. 289
13. 190
14. 528
15. 389
16. 810
17. 676
18. 123
19. 896
20. [76377, 84415]
21. 900
22. 4
23. 7
24. 19
25. 21
26. 576
27. $\frac{3}{5}$
28. $\frac{2}{3}$
29. 119
30. [68903, 76155]
31. 85
32. 4225
33. 1444
34. 84
35. 5454
36. 8
37. 4221
38. 3
39. 4158
40. [76, 83]
41. 583
42. 28
43. 7
44. 512
45. 11536
46. 109
47. 3036
48. 42
49. $\frac{1}{4}$
50. [171000, 189000]
51. 1313
52. $\frac{1}{6}$
53. 11
54. 9
55. 8464
56. 12
57. 84
58. 1011100
59. 625
60. [4342853, 4799995]
61. $\frac{1}{9}$
62. 26751
63. $90\frac{15}{64}$
64. 129
65. 256
66. 203
67. $89\frac{4}{9}$
68. $\frac{1}{15}$
69. 45000
70. [301, 332]
71. 108
72. 29
73. 156024
74. 9
75. 8
76. 19
77. 0.95
78. 16
79. 5280
80. [55, 59]

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Solutions 12023

Sprint Round Solutions

1. As 2020 is a multiple of 10, the units digit of the product is 0, and the tens digit will be the units digit of 2019×202. This is the units digit of 9×2, which is 8.

2. This sum is $2+4+6+8+10+12+14+16+18$, or $(2+18)+(4+16)+(6+14)+(8+12)+10$. This is $20+20+20+20+10$, which is 90.

3. Seven fortnights is $7 \cdot 2 = 14$ weeks, and the number of days in 14 weeks is $7 \cdot 14 = 98$.

4. Making an ordered list by the tens digit, in descending order, we have 93, 84, 75, 66, 57, 48, and 39. Altogether, there are 7.

5. The sum of the length and the width of the rectangle is $\frac{42}{2} = 21$. Therefore, the width of the rectangle is $21 - 16 = 5$. The area is the product of the length and the width, which is $16 \cdot 5 = 80$.

6. Two dozen pieces of candy is $2 \cdot 12 = 24$ pieces, which is $\frac{24}{3} = 8$ purchases of 3 pieces of candy. This will cost $8 \cdot \$0.35 = \$2.80$.

7. We seek $(6 \cdot 51 + 9 \cdot 34) \div 17$. This can be rewritten as $(6 \cdot 3 \cdot 17 + 9 \cdot 2 \cdot 17) \div 17$, which is $17 \cdot (6 \cdot 3 + 9 \cdot 2) \div 17$. Finally, $6 \cdot 3 + 9 \cdot 2 = 36$.

8. The positive whole numbers that evenly divide 18 are 1, 2, 3, 6, 9, and 18. Their sum is $1+2+3+6+9+18 = 39$. Since there are 6 numbers, the average is $\frac{39}{6} = 6.5$.

9. There are $6 \cdot 2 = 12$ total possible outcomes. Half of them, or 6, will have a tails and are therefore favorable. Additionally, 2 more will have a heads on the coin and a 3 or a 6 on the die roll, and are also favorable. The total number of favorable outcomes is $6 + 2 = 8$, for a probability of $\frac{8}{12} = \frac{2}{3}$.

10. We can consider the remainders when dividing 14, 29, and 44 by 13. These are 1, 3, and 5 respectively. Our product then leaves the same remainder as $1 \cdot 3 \cdot 5 = 15$ when divided by 13, which is 2.

11. Let the area of the larger square be 64. Then the side length of the larger square is $\sqrt{64} = 8$. The area of the smaller square is 1 and the square has a side length of $\sqrt{1} = 1$. The value of the length of a side of the smaller square divided by the length of a side of the larger square is $\frac{1}{8}$.

12. Observe that $12\frac{1}{2}\% = \frac{1}{8}$ and $16\frac{2}{3}\% = \frac{1}{6}$. We seek the product $\frac{1}{8} \cdot 546 \cdot \frac{1}{6} \cdot 776$, or $\left(\frac{1}{6} \cdot 546\right) \cdot \left(\frac{1}{8} \cdot 776\right)$. This is $91 \cdot 97 = 8827$.

13. Jason can purchase $\frac{18.20}{1.30} = 14$ liters of fuel. The distance in kilometers he can expect to travel is $14 \cdot 16 = 224$.

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Solutions 12023

14. The radius of Aiden's circles is $\frac{40}{2} = 20$, and so the total area of his 2 circles is $2 \cdot 20^2 \cdot \pi = 800\pi$. The radius of one of Hayden's circles is $\frac{8}{2} = 4$, and so the area of one of his circles is $4^2 \cdot \pi = 16\pi$. For the area of Hayden's circles to be equal to the area of Aiden's circles, the number of circles Hayden would need is $\frac{800\pi}{16\pi} = 50$.

15. The series contains $\frac{99-8}{7} + 1 = 14$ terms, with an average value of $\frac{99+8}{2} = 53.5$. Therefore the sum is $14 \cdot 53.5$, or $28 \cdot 26.75$. The remainder when this product is divided by 28 is $28 \cdot 0.75 = 21$.

16. Trey can run $\frac{1}{4}$ mile in $1 \cdot 60 + 20 = 80$ seconds, or 1 mile in $4 \cdot 80 = 320$ seconds. To run 13.1 miles would take $13.1 \cdot 320 = 4192$ seconds, which to the nearest minute is $\frac{4192}{60} \approx 70$.

17. If we were not considering leap years, a period of 200 years would be $200 \cdot 365 = 73000$ days. But each leap year adds an extra day. Since 1976 was a leap year, leap days were added in the years of birthdays $200, 196, 192, \ldots 4$, which is a total of $\frac{200}{4} = 50$ leap days. However, years 1800 and 1900 were not leap years, so we must subtract 2 from the total. Therefore the age of the United States in days on its bicentennial was $73000 + 50 - 2 = 73048$.

18. The area of the first triangle is $\frac{1}{2} \cdot 8 \cdot 24 = 96$. Therefore, the second leg of the second triangle has length $\frac{96 \cdot 2}{12} = 16$. By the Pythagorean Theorem, the length of the hypotenuse of the second triangle is $\sqrt{12^2 + 16^2} = 20$. The perimeter is $12 + 16 + 20 = 48$.

19. The word *EUNOIA* has 6 letters, all of which are distinct. Since *N* may not be the first letter, there are only $6 - 1 = 5$ choices for the first letter of an arrangement. For the second letter, there are also 5 choices, for the third 4 choices, and so on. The total number of possible arrangements is $5 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 600$.

20. This could be calculated directly, but note that $ab - a - b + 1 = (a-1)(b-1)$. Therefore, the value is $(126-1)(112-1) = 125 \cdot 111$, or $12500 + 1250 + 125 = 13875$.

21. Let the two numbers be a and b, with a greater than b. We have $a^2 + b^2 = 9808$, and $ab = 4896$. Subtracting twice the second equation from the first yields $a^2 - 2ab + b^2 = 9808 - 2 \cdot 4896$, or 16. Taking the square root of both sides, $a - b = 4$.

22. If $8^x = 27$, then $\left(2^3\right)^x = 27$, or $2^{3x} = 27$. Taking the cube root of both sides, $2^x = 3$. Squaring both sides, $2^{2x} = 9$, or $4^x = 9$. Squaring both sides again, $4^{2x} = 81$. Multiplying both sides by 4, $4^{2x+1} = 324$.

23. The base 10 number 14 in binary is 1110_2. The product of 1110_2 and 1001001001_2 is $(1000_2 + 100_2 + 10_2) \cdot 1001001001_2$, or $1001001001000_2 + 100100100100_2 + 10010010010_2 = 1111111111110_2$. This number has 12 ones and 1 zero, and $12 - 1 = 11$.

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24. Any positive whole number that has exactly 8 positive whole number divisors must have a prime factorization of p^7, where p is prime, or $p^3 \cdot q$, where p and q are distinct prime numbers, or pqr, where p, q, and r are distinct prime numbers. In the first case, the least possible whole number greater than 100 is $2^7 = 128$. In the second case, the least possible whole number greater than 100 would be a multiple of $2^3 = 8$ that is the multiple of another prime, which is $2^3 \cdot 13 = 104$. In the third case, searching among the numbers between 100 and 104, we find that $102 = 2 \cdot 3 \cdot 17$. The least of 128, 104, and 102 is 102.

25. This can be rewritten as $(13 \cdot 17)^{2019} \cdot 17$, or $221^{2019} \cdot 17$. The number 221 raised to any power will have a units digit of 1, so the units digit of the result is $1 \cdot 7 = 7$.

26. Firstly, note that $13\frac{1}{3}\% = \frac{1}{100} \cdot \frac{40}{3}$, which is $\frac{2}{15}$, so the second sum is equal to $1 + \frac{2}{15} = \frac{17}{15}$ of the first sum. Let Trinity's chosen number be T. Then $1+2+3+\cdots+T = \frac{T \cdot (T+1)}{2}$, and $1+2+3+\cdots+T+(T+1) = \frac{(T+1)(T+2)}{2}$. Equating the quotient of the two expressions to the desired value, $\frac{T+2}{T} = \frac{17}{15}$, and Trinity's number is 15. The sum of the digits is $1 + 5 = 6$.

27. Consider the denominator as $(1 - \sqrt{2}) - \sqrt{3}$. Then multiplying the numerator and the denominator by the conjugate, $\frac{4}{(1-\sqrt{2})-\sqrt{3}} \cdot \frac{(1-\sqrt{2})+\sqrt{3}}{(1-\sqrt{2})+\sqrt{3}} = \frac{4-4\sqrt{2}+4\sqrt{3}}{-2\sqrt{2}}$. Multiplying the numerator and denominator by $\sqrt{2}$ yields $\frac{4\sqrt{2}-8+4\sqrt{6}}{-4}$, or $2 - \sqrt{2} - \sqrt{6}$.

28. By the Pythagorean theorem, leg BC has length $\sqrt{39^2 - 15^2} = 36$. Applying Power of a Point to point C, we have $CD \cdot CA = CE \cdot CB$. Letting the length of EB be k, we have $39 \cdot (39 - 15) = (36 - k) \cdot 36$, and $k = 10$. The length of EC is $36 - k$, or $36 - 10 = 26$.

29. Let the first six terms in the sequence be a, ar, ar^2, ar^3, ar^4, and ar^5. We have $\frac{a+ar+ar^2+ar^3+ar^4+ar^5}{a+ar+ar^2} = 28$, or $1 + r^3 = 28$, so $r = 3$. The quotient of the third term divided by the first term is $\frac{ar^2}{a} = r^2$, which is $3^2 = 9$.

30. Choose point Y on AB such that XYB is a right angle. Then triangle BXY is a $30-60-90$ triangle, and triangle AXY is a $45-45-90$ triangle, so the lengths of AY and YX are equal. Let the length of BX be k. Then YX has length $\frac{k}{2}$ and YB has length $\frac{k\sqrt{3}}{2}$. Since the length of AX is equal to the length of YX, we have $\frac{k}{2} + \frac{k\sqrt{3}}{2} = 1 + \sqrt{3}$, or $k \cdot (1 + \sqrt{3}) = 2 \cdot (1 + \sqrt{3})$, and $BX = k = 2$.

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Target Round Solutions

1. The factor pairs of 28 are $(1, 28)$, $(2, 14)$, $(4, 7)$, and $1 + 2 + 4 + 7 + 14 = 28$.

2. The total height of the resulting figure is $7 \cdot 3 = 21$, so the segments on the left and right of the figure must sum to $21 \cdot 2 = 42$. Similarly, the total width of the figure is $7 \cdot 3 = 21$, so the segments on the top and bottom of the figure must sum to $21 \cdot 2 = 42$. The total perimeter is $42 + 42 = 84$.

3. The total time Kelly spends riding her stationary bike is $35 + 35 + 35 = 105$ minutes. The total time she spends walking is $55 + 55 + 70 = 180$ minutes. Altogether, her total time exercising is $105 + 180 = 285$ minutes. In hours, this is $\frac{285}{60} = 4.75$.

4. Together, the two runners covered $\frac{1}{4}$ mile in $1 + \frac{15}{60} = 1\frac{1}{4}$ minutes. Therefore in one hour they would combine to complete $\frac{60}{1\frac{1}{4}} = 48$ laps around the track, or $48 \cdot \frac{1}{4} = 12$ miles. Of these 12 miles, Dan would have run 5.4 of them, so Dave's speed in miles per hour is $12 - 5.4 = 6.6$.

5. Listing prime numbers less than 76 we have $2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47$ and $53, 59, 61, 67, 71, 73$. The pairs of numbers from this list that sum to 76 are $(3, 73)$, $(5, 71)$, $(17, 59)$, $(23, 53)$, and $(29, 47)$. There are 5 pairs, so the number of possible prime numbers is $5 \cdot 2 = 10$.

6. By the triangle inequality, the hypotenuse of each triangle must be less than $\frac{60}{2} = 30$. We therefore seek integer solutions to $a^2 + b^2 = c^2$ where c is less than 30 and $a + b + c = 60$. Listing perfect squares less than 29^2, we see that $10^2 + 24^2 = 26^2$, and $10 + 24 + 26 = 60$. Additionally, $15^2 + 20^2 = 25^2$ and $15+20+25 = 60$. The area of a right triangle with legs of length 10 and 24 is $\frac{1}{2} \cdot 10 \cdot 24 = 120$, and the area of a right triangle with legs of length 15 and 20 is $\frac{1}{2} \cdot 15 \cdot 20 = 150$. The ratio of the smaller area to the larger area is $\frac{120}{150} = \frac{4}{5}$.

7. There are $\binom{7}{2} = 21$ pairs of cards that Stan could have selected. The pairs with sums greater than 9 are $(7, 6)$, $(7, 5)$, $(7, 4)$, $(7, 3)$, $(6, 5)$, and $(6, 4)$, for a total of 6. Therefore there are $21 - 6 = 15$ pairs with a sum less than 10, for a probability of $\frac{15}{21} = \frac{5}{7}$.

8. Rewrite 2^{2020} as $\left(2^{10}\right)^{202}$, or 1024^{202}. This can be expressed as $(1000 + 24)^{202}$, which when expanded has terms that are all multiples of 100 except for 24^{202}. Now observe that $24^1 = 24$, $24^2 = 576$, $24^3 = 13824$, and $24^4 = 331776$. In general, odd powers of 24 have a remainder of 24 when divided by 100, while even powers of 24 have a remainder of 76 when divided by 100. Thus the remainder when 2^{2020} is divided by 100 is 76.

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Team Round Solutions

1. If the tens digit is 1, there is 1 such number, 10. If the tens digit is 2, there are 2 such numbers, 21 and 20. If the tens digit is 3, there are 3 such numbers, 32, 31, and 30. This pattern continues, so that if the tens digit is 9, there are 9 such numbers. Altogether, the total is $1+2+3+4+5+6+7+8+9 = 45$.

2. If every child rode a bicycle, then there would be $11 \cdot 2 = 22$ wheels. Each bicycle that is replaced by a tricycle adds 1 wheel, so the number of tricycles is $29 - 22 = 7$. Therefore the number of bicycles is $11 - 7 = 4$, and the number of tricycles exceeds the number of bicycles by $7 - 4 = 3$.

3. The number must be $7 - 5 = 2$ less than a multiple of 7 and $19 - 17 = 2$ less than a multiple of 19. Therefore, the number must be 2 less than a number that is a multiple of both 7 and 19. Since 7 and 19 are both prime, the least positive whole number that is a multiple of both is $7 \cdot 19 = 133$, so the number we are looking for is $133 - 2 = 131$.

4. Let the price of a large soda be S. Then $S + \frac{4}{5}S + \frac{4}{5}S + \frac{4}{5}S = \$6.12$, or $\frac{17}{5}S = \$6.12$, and $S = \$1.80$. Therefore the cost of a taco is $\frac{4}{5} \cdot \$1.80 = \$1.44$. The cost of one taco and one large soda is $\$1.80 + \$1.44 = \$3.24$.

5. A cube has 12 edges, so the length of an edge of the cube is $\frac{228}{12} = 19$. The area of one face of the cube is $19^2 = 361$, and since a cube has 6 faces, the total surface area of the cube is $6 \cdot 361 = 2166$.

6. As each die has 6 outcomes, there are $6 \cdot 6 = 36$ possible outcomes for the product. Of these, the only outcomes with products greater than or equal to 24 are $(4,6)$, $(6,4)$, $(5,5)$, $(5,6)$, $(6,5)$, and $(6,6)$, for a total of 6. Therefore the number of possible outcomes with a product less than 24 is $36 - 6 = 30$. The probability the product is less than 24 is $\frac{30}{36} = \frac{5}{6}$.

7. The sum of the degree measures of the interior angles of a convex quadrilateral is $360°$. Because 11, 13, and 15 are relatively prime, the sum $k+11+13+15$, or $k+39$, must divide 360. Considering divisors of 360, the possible values of $k + 39$ are 40, 45, 60, 72, 90, 120, 180, and 360. But if $k + 39 \geq 60$, then at most 1 of the angle measures is obtuse. If $k + 39 = 40$, then there would be three obtuse angles. Therefore, $k + 39 = 45$, and the unreduced ratio terms are $\frac{360}{45} = 8$ times the reduced ratio terms, giving angle measures in degrees of $8 \cdot 11 = 88$, $8 \cdot 13 = 104$, $8 \cdot 15 = 120$, and $8 \cdot 6 = 48$. The smallest angle measures $48°$.

8. We can categorize the sets based on the number of terms in them (with the elements within each set ordered from least to greatest). For 2 terms, the possibilities are $\{5,10\}$, $\{6,9\}$, and $\{7,8\}$, for a total of 3 possibilities. For 3 terms, the possibilities are $\{1,4,10\}$, $\{1,5,9\}$, $\{1,6,8\}$, $\{2,3,10\}$, $\{2,4,9\}$, $\{2,5,8\}$, $\{2,6,7\}$, $\{3,4,8\}$, $\{3,5,7\}$, and $\{4,5,6\}$, for a total of 10 possibilities. For 4 terms, the possibilities are $\{1,2,3,9\}$, $\{1,2,4,8\}$, $\{1,2,5,7\}$, $\{1,3,4,7\}$, $\{1,3,5,6\}$, and $\{2,3,4,6\}$, for a total of 6 possibilities. For 5 terms, the only possibility is $\{1,2,3,4,5\}$. Altogether, the total number of possibilities is $3 + 10 + 6 + 1 = 20$.

9. Let the length of a side of $ABCD$ be the least common multiple of $3+4=7$ and $4+1=5$, which is 35. Then $ABXY$ is a trapezoid with a height of 35 and bases of length 15 and 7, while $DYXC$ is a trapezoid with a height of 35 and bases of length 20 and 28. The first trapezoid will have an area of $\frac{15+7}{2} \cdot 35$, while the second will have an area of $\frac{20+28}{2} \cdot 35$. The ratio of these areas is $\frac{15+7}{20+28} = \frac{11}{24}$.

10. If 2 of the 3 chose the smallest possible number, 601, the third number would only be $2019 - 601 - 601 = 817$, so the constraint of all numbers being less than 850 is not necessary. Let Naomi's number be $N+601$, Oscar's be $O+601$, and Paolo's be $P+601$. Then $N+601+O+601+P+601=2019$, or $N+O+P=216$. This is a distribution of 216 indistinguishable items into 3 distinguishable containers. By "sticks and stones" the number of ways this can occur is $\binom{216+3-1}{3-1} = \binom{218}{2} = 217 \cdot 109 = 23653$.

