Contents

Preface		page xvi
1	Introduction	1
1.1	Conservation Laws	3
1.2	Finite Volume Methods	5
1.3	Multidimensional Problems	6
1.4	Linear Waves and Discontinuous Media	7
1.5	CLAWPACK Software	8
1.6	References	Ģ
1.7	Notation	10
Part	I Linear Equations	
2	Conservation Laws and Differential Equations	15
2.1	The Advection Equation	17
2.2	Diffusion and the Advection–Diffusion Equation	20
2.3	The Heat Equation	21
2.4	Capacity Functions	22
2.5	Source Terms	22
2.6	Nonlinear Equations in Fluid Dynamics	23
2.7	Linear Acoustics	26
2.8	Sound Waves	29
2.9	Hyperbolicity of Linear Systems	31
2.10	Variable-Coefficient Hyperbolic Systems	33
2.11	Hyperbolicity of Quasilinear and Nonlinear Systems	34
2.12	Solid Mechanics and Elastic Waves	35
2.13		41
2.14	Electromagnetic Waves	43
	Exercises	46
3	Characteristics and Riemann Problems for Linear	
	Hyperbolic Equations	47
3.1	Solution to the Cauchy Problem	47

x Contents

3.2	Superposition of Waves and Characteristic Variables	48
3.3	Left Eigenvectors	49
3.4	Simple Waves	49
3.5	Acoustics	49
3.6	Domain of Dependence and Range of Influence	50
3.7	Discontinuous Solutions	52
3.8	The Riemann Problem for a Linear System	52
3.9	The Phase Plane for Systems of Two Equations	55
3.10	Coupled Acoustics and Advection	57
3.11	Initial–Boundary-Value Problems	59
	Exercises	62
4	Finite Volume Methods	64
4.1	General Formulation for Conservation Laws	64
4.2	A Numerical Flux for the Diffusion Equation	66
4.3	Necessary Components for Convergence	67
4.4	The CFL Condition	68
4.5	An Unstable Flux	71
4.6	The Lax–Friedrichs Method	71
4.7	The Richtmyer Two-Step Lax-Wendroff Method	72
4.8	Upwind Methods	72
4.9	The Upwind Method for Advection	73
4.10	Godunov's Method for Linear Systems	76
4.11	The Numerical Flux Function for Godunov's Method	78
4.12	The Wave-Propagation Form of Godunov's Method	78
4.13	Flux-Difference vs. Flux-Vector Splitting	83
4.14	Roe's Method	84
	Exercises	85
5	Introduction to the CLAWPACK Software	87
5.1	Basic Framework	87
5.2	Obtaining CLAWPACK	89
5.3	Getting Started	89
5.4	Using CLAWPACK – a Guide through example1	91
5.5	Other User-Supplied Routines and Files	98
5.6	Auxiliary Arrays and setaux.f	98
5.7	An Acoustics Example	99
	Exercises	99
6	High-Resolution Methods	100
6.1	The Lax–Wendroff Method	100
6.2	The Beam–Warming Method	102
6.3	Preview of Limiters	103
6.4	The REA Algorithm with Piecewise Linear Reconstruction	106

Contents xi

6.5	Choice of Slopes	107
6.6	Oscillations	108
6.7	Total Variation	109
6.8	TVD Methods Based on the REA Algorithm	110
6.9	Slope-Limiter Methods	111
6.10	Flux Formulation with Piecewise Linear Reconstruction	112
6.11	Flux Limiters	114
6.12	TVD Limiters	115
6.13	High-Resolution Methods for Systems	118
6.14	Implementation	120
6.15	Extension to Nonlinear Systems	121
6.16	Capacity-Form Differencing	122
6.17	Nonuniform Grids	123
	Exercises	127
7	Boundary Conditions and Ghost Cells	129
7.1	Periodic Boundary Conditions	130
7.2	Advection	130
7.3	Acoustics	133
	Exercises	138
8	Convergence, Accuracy, and Stability	139
8.1	Convergence	139
8.2	One-Step and Local Truncation Errors	141
8.3	Stability Theory	143
8.4	Accuracy at Extrema	149
8.5	Order of Accuracy Isn't Everything	150
8.6	Modified Equations	151
8.7	Accuracy Near Discontinuities	155
	Exercises	156
9	Variable-Coefficient Linear Equations	158
9.1	Advection in a Pipe	159
9.2	Finite Volume Methods	161
9.3	The Color Equation	162
9.4	The Conservative Advection Equation	164
9.5	Edge Velocities	169
9.6	Variable-Coefficient Acoustics Equations	171
9.7	Constant-Impedance Media	172
9.8	Variable Impedance	173
9.9	Solving the Riemann Problem for Acoustics	177
9.10	Transmission and Reflection Coefficients	178
9.11	Godunov's Method	179
9.12	High-Resolution Methods	181

xii Contents

9.13	Wave Limiters	181
9.14	Homogenization of Rapidly Varying Coefficients	183
	Exercises	187
10		400
10	Other Approaches to High Resolution	188
10.1	Centered-in-Time Fluxes	188
10.2	Higher-Order High-Resolution Methods	190
10.3	Limitations of the Lax–Wendroff (Taylor Series) Approach	191
10.4	Semidiscrete Methods plus Time Stepping	191
10.5	Staggered Grids and Central Schemes	198
	Exercises	200
Part I	I Nonlinear Equations	
11	Nonlinear Scalar Conservation Laws	203
11.1	Traffic Flow	203
11.2	Quasilinear Form and Characteristics	206
11.3	Burgers' Equation	208
11.4	Rarefaction Waves	209
11.5	Compression Waves	210
11.6	Vanishing Viscosity	210
11.7	Equal-Area Rule	211
11.8	Shock Speed	212
11.9	The Rankine–Hugoniot Conditions for Systems	213
11.10	Similarity Solutions and Centered Rarefactions	214
11.11	Weak Solutions	215
11.12	Manipulating Conservation Laws	216
11.13	Nonuniqueness, Admissibility, and Entropy Conditions	216
11.14	Entropy Functions	219
11.15	Long-Time Behavior and N-Wave Decay	222
	Exercises	224
12	Finite Volume Methods for Nonlinear Scalar	
	Conservation Laws	227
12.1	Godunov's Method	227
12.2	Fluctuations, Waves, and Speeds	229
12.3	Transonic Rarefactions and an Entropy Fix	230
12.4	Numerical Viscosity	232
12.5	The Lax-Friedrichs and Local Lax-Friedrichs Methods	232
12.6	The Engquist-Osher Method	234
12.7	E-schemes	235
12.8	High-Resolution TVD Methods	235
12.9	The Importance of Conservation Form	237
12.10	The Lax-Wendroff Theorem	239

	Contents	xiii
12.11	The Entropy Condition	243
12.12	Nonlinear Stability	244
	Exercises	252
13	Nonlinear Systems of Conservation Laws	253
13.1	The Shallow Water Equations	254
13.2	Dam-Break and Riemann Problems	259
13.3	Characteristic Structure	260
13.4	A Two-Shock Riemann Solution	262
13.5	Weak Waves and the Linearized Problem	263
13.6	Strategy for Solving the Riemann Problem	263
13.7	Shock Waves and Hugoniot Loci	264
13.8	Simple Waves and Rarefactions	269
13.9	Solving the Dam-Break Problem	279
13.10	The General Riemann Solver for Shallow Water Equations	281
13.11	Shock Collision Problems	282
13.12	Linear Degeneracy and Contact Discontinuities	283
	Exercises	287
14	Gas Dynamics and the Euler Equations	291
14.1	Pressure	291
14.2	Energy	292
14.3	The Euler Equations	293
14.4	Polytropic Ideal Gas	293
14.5	Entropy	295
14.6	Isothermal Flow	298
14.7	The Euler Equations in Primitive Variables	298
14.8	The Riemann Problem for the Euler Equations	300
14.9	Contact Discontinuities	301
14.10	Riemann Invariants	302
14.11	Solution to the Riemann Problem	302
14.12	The Structure of Rarefaction Waves	305
14.13	Shock Tubes and Riemann Problems	306
14.14	Multifluid Problems	308
14.15	Other Equations of State and Incompressible Flow	309
15	Finite Volume Methods for Nonlinear Systems	311
15.1	Godunov's Method	311
15.2	Convergence of Godunov's Method	313
15.3	Approximate Riemann Solvers	314
15.4	High-Resolution Methods for Nonlinear Systems	329
15.5	An Alternative Wave-Propagation Implementation of Approximate	
	Riemann Solvers	333
15.6	Second-Order Accuracy	335

xiv Contents

15.7	Flux-Vector Splitting	338
15.8	Total Variation for Systems of Equations	340
	Exercises	348
16	Some Nonclassical Hyperbolic Problems	350
16.1	Nonconvex Flux Functions	350
16.2	Nonstrictly Hyperbolic Problems	358
16.3	Loss of Hyperbolicity	362
16.4	Spatially Varying Flux Functions	368
16.5	Nonconservative Nonlinear Hyperbolic Equations	371
16.6	Nonconservative Transport Equations	372
	Exercises	374
17	Source Terms and Balance Laws	375
17.1	Fractional-Step Methods	377
17.2	An Advection–Reaction Equation	378
17.3	General Formulation of Fractional-Step Methods for Linear Problems	384
17.4	Strang Splitting	387
17.5	Accuracy of Godunov and Strang Splittings	388
17.6	Choice of ODE Solver	389
17.7	Implicit Methods, Viscous Terms, and Higher-Order Derivatives	390
17.8	Steady-State Solutions	391
17.9	Boundary Conditions for Fractional-Step Methods	393
17.10	Stiff and Singular Source Terms	396
17.11	Linear Traffic Flow with On-Ramps or Exits	396
17.12	Rankine-Hugoniot Jump Conditions at a Singular Source	397
17.13	Nonlinear Traffic Flow with On-Ramps or Exits	398
17.14	Accurate Solution of Quasisteady Problems	399
17.15	Burgers Equation with a Stiff Source Term	401
17.16	Numerical Difficulties with Stiff Source Terms	404
17.17	Relaxation Systems	410
17.18	Relaxation Schemes	415
	Exercises	416
Part I	II Multidimensional Problems	
18	Multidimensional Hyperbolic Problems	421
18.1	Derivation of Conservation Laws	421
18.2	Advection	423
18.3	Compressible Flow	424
18.4	Acoustics	425
18.5	Hyperbolicity	425
18.6	Three-Dimensional Systems	428
18.7	Shallow Water Equations	429

Contents	X
ontents	2

18.8	Euler Equations	431
18.9	Symmetry and Reduction of Dimension	433
	Exercises	434
19	Multidimensional Numerical Methods	436
19.1	Finite Difference Methods	436
19.2	Finite Volume Methods and Approaches to Discretization	438
19.3	Fully Discrete Flux-Differencing Methods	439
19.4	Semidiscrete Methods with Runge-Kutta Time Stepping	443
19.5	Dimensional Splitting	444
	Exercise	446
20	Multidimensional Scalar Equations	447
20.1	The Donor-Cell Upwind Method for Advection	447
20.2	The Corner-Transport Upwind Method for Advection	449
20.3	Wave-Propagation Implementation of the CTU Method	450
20.4	von Neumann Stability Analysis	452
20.5	The CTU Method for Variable-Coefficient Advection	453
20.6	High-Resolution Correction Terms	456
20.7	Relation to the Lax-Wendroff Method	456
20.8	Divergence-Free Velocity Fields	457
20.9	Nonlinear Scalar Conservation Laws	460
20.10	Convergence	464
	Exercises	467
21	Multidimensional Systems	469
21.1	Constant-Coefficient Linear Systems	469
21.2	The Wave-Propagation Approach to Accumulating Fluxes	471
21.3	CLAWPACK Implementation	473
21.4	Acoustics	474
21.5	Acoustics in Heterogeneous Media	476
21.6	Transverse Riemann Solvers for Nonlinear Systems	480
21.7	Shallow Water Equations	480
21.8	Boundary Conditions	485
22	Elastic Waves	491
22.1	Derivation of the Elasticity Equations	492
22.2	The Plane-Strain Equations of Two-Dimensional Elasticity	499
22.3	One-Dimensional Slices	502
22.4	Boundary Conditions	502
22.5	The Plane-Stress Equations and Two-Dimensional Plates	504
22.6	A One-Dimensional Rod	509
22.7	Two-Dimensional Elasticity in Heterogeneous Media	509

xvi Contents

23	Finite Volume Methods on Quadrilateral Grids	514
23.1	Cell Averages and Interface Fluxes	515
23.2	Logically Rectangular Grids	517
23.3	Godunov's Method	518
23.4	Fluctuation Form	519
23.5	Advection Equations	520
23.6	Acoustics	525
23.7	Shallow Water and Euler Equations	530
23.8	Using CLAWPACK on Quadrilateral Grids	531
23.9	Boundary Conditions	534
Biblio	ography	535
Index		553