ex2 week 6

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1 Introduction

1.1 Ex1: biến đổi lại thuật toán logistic regression, từ xây dựng công thức, likelihood, maximize likelihood, đạo làm negative log likelihood theo ma trận hệ số.

D: data w: weight P(D|w) likelihood P(w|D): poterior Ta co:

$$p(C_1|x) = \frac{p(x|C_1).p(C_1)}{p(x|C_1).p(C_1) + p(x|C_2)p(C_2)}$$

$$= \frac{1}{1 + \frac{p(x|C_2).p(C_2)}{p(x|C_1).p(C_1)}}$$

Đặt: $\frac{p(x|C_2).p(C_2)}{p(x|C_1).p(C_1)} = e^{-a}$

$$<=> log\frac{p(x|C_2).p(C_2)}{p(x|C_1).p(C_1)} = -a$$

so thus : $p(C_1|x) = \frac{1}{1+e^{-a}} = \sigma(a)$ * Ta có: $\sigma(x)' = \sigma(x)(1 - \sigma(x))$ * Logistic regression is defined by:

$$p(C_1|\phi) = y(\phi) = \sigma(w^T\phi)$$

$$<=> p(C_2|\phi) = 1 - p(C_1|\phi)$$

For a data set ϕ_n, where t_n in 0,1 and $\phi_n = \phi(x_n)$, with n = 1,2,..., N. The likelihood function can be written:

$$p(t|w) = \prod_{n=1}^{N} y_n^{t_n}.(1 - y_n)^{1-t_n}$$

where $t = (t_1, t_2, ..., t_N)^T$ and $y_n = p(C_1|\phi_n)$ * maximize likelihood:

$$p(t|w) = \prod_{i=1}^{N} y_i^{t_i}.(1 - y_i)^{1-t_i}$$

$$-logp(t|w) = -\sum_{i=1}^{N}[t_ilog(y_i) + (1 - t_i)log(1 - y_i)]$$

Set:

$$L_1 = -t.log(y) - (1 - t)log(1 - y)$$

where: $y = \sigma(w_0 + w_1\phi_1 + ... + w_n\phi_n)$
Chain rule:
set:
+ $z = w_0 + w_1\phi_1 + ... + w_n\phi_n$
+ $y = \sigma(z)$

$$\frac{dL_1}{dw_1} = \frac{dL_1}{dy}.\frac{dy}{dz}.\frac{dz}{dw_1}$$

we have:
+ $\frac{dL_1}{dy} = \frac{-t}{y} + \frac{1-t}{1-y} = \frac{y-t}{y(1-y)}$
+ $\frac{dy}{dz} = \sigma(z)(1-\sigma(z)) = y(1-y)$
+ $\frac{dz}{dw_1} = \phi_1$

$$=> \frac{dL_1}{dw_1} = \frac{y-t}{y(1-y)}.y(1-y).\phi_1$$

$$\frac{dL_1}{dw_1} = (y-t).\phi_1$$

Thus:

$$\frac{dL}{dw} = \sum_{i=1}^{N}(y_i - t_i)\phi_i$$

1.2 Ex2: Tìm hàm f(x), biết f'(x) = f(x)(1-f(x))

We have: $f'(x) = f(x)(1-f(x))$

$$\frac{df}{dx} = f(x) - f^2(x)$$

$$<=> \int \frac{df}{f(x)(1-f(x))} = \int dx$$

$$<=> \int \frac{1}{f(x)} - \frac{1}{1-f(x)} df = -\int dx$$

$$<=> ln|f(x)| + ln|1-f(x)| + m = -x + c$$

$$ln|\frac{f(x)}{1-f(x)}| = -x + a$$

$$\frac{f(x)}{1-f(x)} = e^{-x+a}$$

$$f(x) = e^{-x+a} - f(x)e^{-x+a}$$

$$f(x) = \frac{e^{-x+a}}{1+e^{-x+a}}$$