# Đào Th Hng Nhung

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**Abstract**

Your abstract.

# 1 Exercise 1

# 2 Exercise 2:

## 2.1 Univariate normal distribution.

## 2.2 Proof

We have $I_1 =$

$$\int_{-\infty}^{\infty} \frac{-(x-\mu)}{2\sigma^2} dx = \sqrt{2\pi\sigma^2}$$

$I_2 =$

$$\int_{-\infty}^{\infty} \frac{-(y-\mu)}{2\sigma^2} dy = \sqrt{2\pi\sigma^2}$$

$I_1 I_2 =$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} exp(\frac{-x^2}{2\sigma^2} - \frac{y^2}{2\sigma^2}) dxdy$$

Set:

$$x = a.cos\phi$$

$$y = a.sin\phi$$

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$$x \in (-\infty; \infty) <=> a \in (0; \infty)$$

$$x \in (-\infty; \infty) <=> \phi \in (0; 2\pi)$$

We have:

$$A = \begin{bmatrix} dx \\ dy \end{bmatrix} = \begin{bmatrix} \frac{\theta x}{\theta a} \frac{\theta x}{\theta \phi} \\ \frac{\theta y}{\theta a} \frac{\theta y}{\theta \phi} \end{bmatrix} . \begin{bmatrix} da \\ d\phi \end{bmatrix} = \begin{bmatrix} cos\phi - asin\phi \\ sin\phi acos\phi \end{bmatrix} . \begin{bmatrix} da \\ d\phi \end{bmatrix}$$

$$detmatrix = a.cos^2\phi + a.sin^2\phi = a$$

$$dx.dy = a.da.d\phi$$

we have:

$$I_1.I_2 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} exp(-\frac{x^2+y^2}{2\sigma^2}) dxdy$$

$$= \int_0^{2\pi} \int_0^{\infty} e^{\frac{-(a^2cos^2\phi + a^2sin^2\phi)}{2\sigma^2}} a.dad\phi$$

$$= \int_0^{2\pi} \int_0^{\infty} e^{\frac{-a^2}{2\sigma^2}} a.da.d\phi$$

$$= \int_0^{2\pi} \sigma^2 d\phi = 2\pi\sigma^2$$

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$$I_1 = I_2 -> I = \sqrt{2\pi\sigma^2}$$

Expectation

$$E(X) = \int_{-\infty}^{\infty} x.f(x)dx = \mu$$

$$Var = \sigma^2$$