T SNE

You

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0.1 Ex 1:

$$p_{i|j} = \frac{exp(-||x_i - x_j||^2/2\sigma^2)}{\sum_{k\#j} exp(-||x_i - x_k||^2/2\sigma^2)}$$

$$q_{j|i} = \frac{exp(-||y_i - y_j||^2)}{\sum_{k\#i} exp(-||y_i - y_k||^2} = \frac{E_{ij}}{\sum_{k\#i} E_{ik}} = \frac{E_{ij}}{Z_i}$$

Define:

$$q_{ji} = q_{ij} = \frac{(1 + ||y_i - y_j||^2)^{-1}}{\sum_{k,l\#k}(1 + ||y_k - y_l||^2)^{-1}} = \frac{E_{ij}^{-1}}{\sum_{k,l\#k} E_{kl}^{-1}} = \frac{E_{ij}^{-1}}{Z}$$

Notice that $E_{ij} = E_{ji}$. The loss function is defined as

$$L = \sum_i \sum_k p_{i|j} log \frac{p_{j|i}}{q_{j|i}}$$

$$\sum_{k,l\#k} p_{lk} log \frac{p_{ji}}{q_{ji}} = \sum_{k,l\#k} p_{lk} log p_{lk} - p_{lk} log q_{lk}$$

$$= \sum_{k,l\#k} p_{lk} log p_{lk} - p_{lk} log E_{kl}^{-1} + p_{lk} log Z$$

We drive with respect to y_i

$$\frac{\delta C}{\delta y_i} = \sum_{k,l\#k} -p_{lk} \delta log E_{kl}^{-1} + \sum_{k,l\#k} p_{lk} \delta log Z$$

We have: $k = i$ or $l = i$, that $p_{ji} = p_{ij}$ and $E_{ji} = E_{ij}$

$$= \sum_{k,l\#k} -p_{lk} \delta log E_{kl}^{-1} = -2 \sum_{j\#i} p_{ji} \delta log E_{ij}^{-1}$$

Since: $\delta E_{ij}^{-1} = E_{ij}^{-2}(-2(y_i - y_j))$ We have :

$$-2 \sum_{j\#i} p_{ji} \frac{E_{ij}^{-2}}{E_{ij}^{-1}}(-2(y_i - y_j)) = 4 \sum_{j\#i} p_{ji} E_{ij}^{-1}(y_i - y_j)$$

(1) The fact that $\sum_{k,l\#k} p_{kl} = 1$ and that Z does not depend on k or l

$$\sum_{k,l\#k} p_{kl} \delta Z = \frac{1}{Z} \sum_{k',l'\#k'} \delta E_{kl}^{-1}$$

$$= 2 \sum_{j\#i} \frac{E_{ji}^{-2}}{Z}(-2(y_j - y_i))$$

$$= -4 \sum_{j\#i} q_{ij} E_{ji}^{-1}(y_i - y_j)$$

(2) Combine (1) and (2):

$$\frac{\delta C}{\delta y_i} = 4 \sum_{j \# i} (p_{ij} - q_{ji}) E_{ji}^{-1} (y_i - y_j)$$

$$\frac{\delta C}{\delta y_i} = 4 \sum_{j \# i} (p_{ij} - q_{ji}) (1 + ||y_i - y_j||^2)^{-1} (y_i - y_j)$$