Homework 5 of CS520 Theory of Programming Languages

You do not need to submit this homework. But I strongly encourage you to study the questions before the exam. The numbers in the questions refer to exercise questions in the textbook of the course, i.e. "Theories of Programming Languages" by John C. Reynolds.

Question 1

Solve 11.11. While solving this question, you might find it useful to use lists. Lists are explained in Section 11.4 of the textbook. If you do not want to read the textbook but still are interested in using lists, a quick solution is to view constructs for lists as syntactic sugars, as shown below:

$$\begin{aligned}
\mathbf{nil} &\stackrel{\text{def}}{=} @\ 0\ 0 \\
e_0 :: e_1 &\stackrel{\text{def}}{=} @\ 1\ \langle e_0,\, e_1\rangle \\
\mathbf{listcase}\ e\ \mathbf{of}\ (e_0,\, e_1) &\stackrel{\text{def}}{=} \mathbf{sumcase}\ e\ \mathbf{of}\ \Big((\lambda v.\, e_0),\ (\lambda v.\, (e_1\ v.0)\ v.1)\Big) \quad \text{for a fresh variable } v
\end{aligned}$$

Intuitively, **nil** denotes the empty list, and $e_0 :: e_1$ denotes the list with e_0 as its head and e_1 as its tail. The last performs the pattern match. If e is the empty list, we evaluate e_0. Otherwise, we apply e_1 to two parameters, one for the head of e and the other for the tail of e.

Question 2

Solve 11.12.

Question 3

Solve 12.1. When solving Part (a), assume that the direct semantics uses the following domains and predomains:

$$V_* = (V + \{err, typerr\})_\perp, \qquad V \simeq V_{int} + V_{bool} + V_{fun} + V_{tuple} + V_{alt},$$

$$V_{int} = \mathbb{Z}, \qquad V_{bool} = \mathbb{B},$$

$$V_{fun} = [V \to_c V_*], \qquad V_{tuple} = \bigcup_{n=0}^{\infty} V^n, \qquad V_{alt} = \mathbb{N} \times V.$$

Also, when solving Part (b), assume that the continuation semantics uses the following domains and predomains:

$$V_* = (V + \{err, typerr\})_\perp, \qquad V \simeq V_{int} + V_{bool} + V_{fun} + V_{tuple} + V_{alt} + V_{cont}$$

$$V_{int} = \mathbb{Z}, \qquad V_{bool} = \mathbb{B}, \qquad V_{fun} = [V \to_c V_{cont} \to_c V_*],$$

$$V_{tuple} = \bigcup_{n=0}^{\infty} V^n, \qquad V_{alt} = \mathbb{N} \times V, \qquad V_{cont} = V \to_c V_*.$$