# Homework 2 of CS520 Theory of Programming Languages
# Deadline: 6:00pm on 23 April (Friday)

Submit your solutions in KLMS. (Reminder: We adopt a very strict policy for handling dishonest behaviours. If a student is found to copy answers from fellow students or other sources in his or her homework submission, she or he will get F.)

The numbers in the questions refer to exercise questions in the textbook of the course, i.e. "Theories of Programming Languages" by John C. Reynolds.

## Question 1

Solve 3.1. (20 marks)

## Question 2

In the lectures, we discussed about the following forward rule for assignment:

$$\frac{x_0 \notin \mathrm{FV}(p) \cup \mathrm{FV}(e) \cup \{x\}}{\{p\}\, x := e\, \{\exists x_0.\, x = (e/x \to x_0) \wedge (p/x \to x_0)\}}$$

Prove that this rule is sound, i.e., the semantics of a Hoare triple derived by the rule is always $tt$. (20 marks)

## Question 3

Derive the following partial correctness triple using the rules of Hoare logic that you learnt in the lectures.

$$\Big\{x \geq 0 \wedge x = x_0 \wedge y = y_0\Big\}\, \textbf{while}\ x \neq 0\ \textbf{do}\ (x := x-1; y := y + 2 \times x)\, \Big\{y = y_0 + x_0 \times (x_0 - 1)\Big\}.$$

(20 marks)

## Question 4

Solve the part (a) of 5.4 after making the following four modifications. First, the part (a) mentions the type of command meanings $[\![c]\!]$ that we did not cover in the lectures. Instead of this type of $[\![-]\!]$, use the following one, which extends the semantics that we studied in the lectures:

$$[\![-]\!] \in \big[\langle\text{comm}\rangle \to \Sigma \to \Omega\big].$$

Here

$$\hat{\Sigma} \stackrel{\text{def}}{=} \Sigma \cup (\langle\text{label}\rangle \times \Sigma) \qquad \text{and} \qquad \Omega \simeq \big(\hat{\Sigma} + (\mathbb{Z} \times \Omega) + (\mathbb{Z} \to \Omega)\big)_\perp, \tag{1}$$

and the isomorphism $\simeq$ refers to the existence of two continuous functions

$$\phi : \Omega \to \big(\hat{\Sigma} + (\mathbb{Z} \times \Omega) + (\mathbb{Z} \to \Omega)\big)_\perp \qquad \text{and} \qquad \psi : \big(\hat{\Sigma} + (\mathbb{Z} \times \Omega) + (\mathbb{Z} \to \Omega)\big)_\perp \to \Omega$$

such that

$$\psi \circ \phi = \text{id} \quad \text{and} \quad \phi \circ \psi = \text{id}.$$

Second, extend the semantics in my hand-written notes (the same as the one in Section 5.6), instead of the direct semantics in Section 5.1. Third, you can ignore the variable declaration when extending the semantics. This means that you need to define the semantics of the following language:

$$
\begin{aligned}
\langle \text{comm} \rangle ::= {} & \langle \text{var} \rangle := \langle \text{intexp} \rangle \mid ?\langle \text{var} \rangle \mid !\langle \text{intexp} \rangle \mid \langle \text{comm} \rangle ; \langle \text{comm} \rangle \\
\mid {} & \textbf{if } \langle \text{boolexp} \rangle \textbf{ then } \langle \text{comm} \rangle \textbf{ else } \langle \text{comm} \rangle \\
\mid {} & \textbf{while } \langle \text{boolexp} \rangle \textbf{ do } \langle \text{comm} \rangle \\
\mid {} & \textbf{fail } \langle \text{label} \rangle \mid \textbf{catch } \langle \text{label} \rangle \textbf{ in } \langle \text{comm} \rangle \textbf{ with } \langle \text{comm} \rangle
\end{aligned}
$$

Finally, while defining the semantics, especially the one for sequential composition, you may use the extension of a function $f : \Sigma \to \Omega$ to $f_* : \Omega \to \Omega$, which we discussed and is also defined in the textbook. (20 marks)

## Question 5

Solve 5.10. (20 marks)