CS 520
Theory of Programming Language

04/07 – 04/14, 2021

Failure, Input-Output

1. Reminder.

① Syntax :

<intexp> ::= ...

<boolexp> ::= ...

<comm> ::= .... | fail | ?<var> | !<intexp>.

② example.

$?x\ ;\ !(x+1)\ ;\ ?y\ ;\ !(x+y)\ ;\ z = x+y.$

2. Denotational Semantics.

① $[\![-]\!]_{intexp} : \langle \text{intexp} \rangle \to [\Sigma \to \mathbb{Z}]$

$[\![-]\!]_{boolexp} : \langle \text{boolexp} \rangle \to [\Sigma \to \mathbb{B}]$ *

$[\![-]\!]_{comm} : \langle \text{comm} \rangle \to [\Sigma \to \underline{\Omega}]$

??

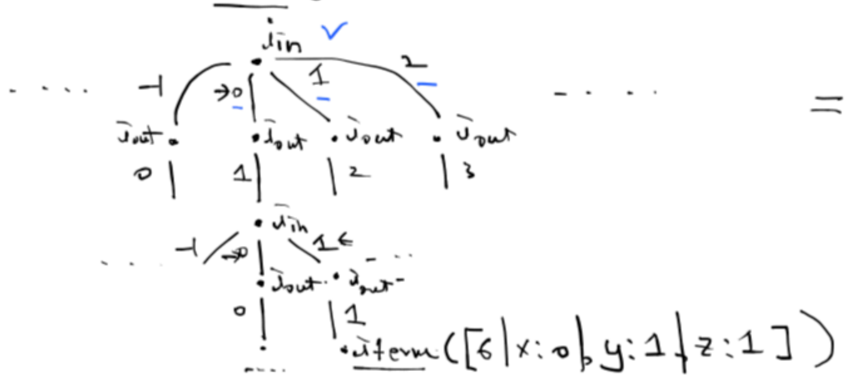
$\left(\Sigma = [\langle \text{var} \rangle \to \mathbb{Z}] \right).$

(1) option 1 : $\underline{\Omega} = \Sigma_{\perp}$. ⋯ X.

$[\![?x ; !(x+1)]\!]$.

$?x \,;\, !(x+1) \,;\, ?y \,;\, !(x+y) \,;\, z = x+y$

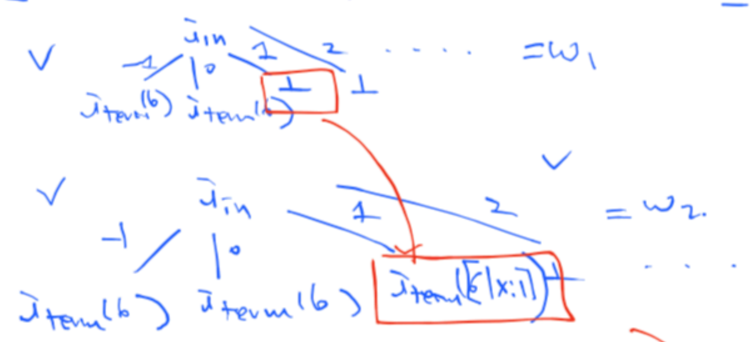
$\hookrightarrow \; \sigma \in \Sigma \longmapsto \underline{\quad} \in \Omega$ $\qquad \eta^{\perp}$



$$= \iota_{in}\big(\lambda k.\ \iota_{out}(k+1,\ \iota_{in}(\lambda k'.\ \iota_{out}(k+k',\ \iota_{term}(\quad))))\big)$$

$[\sigma \mid x:k \mid y:k' \mid z:k+k']$

$\sqsubseteq \qquad \sqsubseteq$



$\cdots\ \omega_1 \sqsubseteq \omega_2 \sqsubseteq \omega_3 \sqsubseteq \cdots$

movie
about how
comp. proceeds.

types of nodes
in trees.

(3) $\bar{\Omega} \simeq (\hat{\Sigma} + \underline{\mathbb{Z} \times \bar{\Omega}} + \underline{[\mathbb{Z} \to \bar{\Omega}]})_\perp$

output input

(3) (4) (5)

isomorphism

where $\hat{\Sigma} = \Sigma + \Sigma$

(1) (2)

normal termination abnormal termination

$\hookrightarrow \perp$

(4) A few constructions for predomains. ($+$ and $\times$).

$(P_0, \sqsubseteq_0), \dots, (P_{n-1}, \sqsubseteq_{n-1})$ ... predomains. ... Given.

$$\left.\begin{array}{l} P_0 + P_1 + \dots + P_{n-1} = \sum_{i=0}^{n-1} P_i \dots \\ P_0 \times P_1 \times \dots \times P_{n-1} = \prod_{i=0}^{n-1} P_i \dots \end{array}\right) \text{Predomains} \dots \text{Constructed.}$$

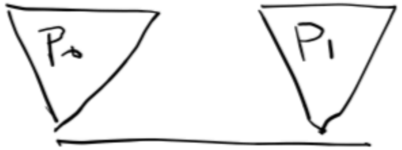
(1). $P_0+P_1+\cdots+P_{n-1} = \{\langle \underline{i}, x\rangle \mid i\in\{0,\dots,n-1\},\ x\in P_i\}$.

$$\underline{\mathbb{B}_\perp} + \mathbb{Z}_\perp = \{\langle 0, tt\rangle, \langle 0, ff\rangle, \langle 0, \perp\rangle,$$
$$\langle 1,-3\rangle, \langle 1,-2\rangle, \langle 1,-1\rangle, \cdots \}$$

$$\mathbb{B}_\perp + \mathbb{B}_\perp = \{\langle 0, tt\rangle, \langle 0, ff\rangle, \langle 0, \perp\rangle,$$
$$\langle 1, tt\rangle, \langle 1, ff\rangle, \langle 1, \perp\rangle\}.$$

$\langle i, x\rangle, \langle j, y\rangle \in \sum_{k=0}^{n-1} P_k$.

$\langle i, x\rangle \sqsubseteq \langle j, y\rangle$ iff $i=j$ and $x \sqsubseteq_i y$

✓ 

...... Putting them w/o any relationship.

(2) $P_0\times\cdots\times P_{n-1} = \{\langle x_0, x_1, \dots, x_{n-1}\rangle \mid x_i\in P_i$ for each $i\}$.

$\langle x_0,\dots,x_{n-1}\rangle \sqsubseteq \langle y_0,\dots,y_{n-1}\rangle$ iff $\forall i\in\{0,\dots,n-1\}$.

↳ componentwise order. $x_i \sqsubseteq_i y_i$.

① ex. Prove that $\sum_i P_i$ and $\prod_i P_i$ are predomains. ②. All of P_i's are domains which one of $\sum_i P_i$ and $\prod_i P_i$ is a domain?

$\sum_i P_i$: not necessarily a domain. $\prod_i P_i$: domain. $\langle \bot_0, \dots, \bot_{n-1} \rangle$

$$t^{(0)} \sqsubseteq t^{(1)} \sqsubseteq t^{(2)} \sqsubseteq \cdots \sqsubseteq t^{(k)} \sqsubseteq \cdots$$

$$t^{(0)} = \langle x_0^{(0)}, x_1^{(0)}, \dots, x_{n-1}^{(0)} \rangle \qquad t^{(k)} = \langle x_0^{(k)}, x_1^{(k)}, \dots, x_{n-1}^{(k)} \rangle$$

chain.

$$\bigsqcup_{k=0}^{\infty} t^{(k)} = \langle \bigsqcup_{k=0}^{\infty} x_0^{(k)}, \bigsqcup_{k=0}^{\infty} x_1^{(k)}, \dots, \bigsqcup_{k=0}^{\infty} x_{n-1}^{(k)} \rangle.$$

$s^{(0)} \sqsubseteq s^{(1)} \sqsubseteq \cdots$ in $\sum_i P_i$.

$\Rightarrow \exists\, i \in \{0, \dots, n-1\}$ and a chain $x^{(0)} \sqsubseteq_i \cdots \sqsubseteq_i x^{(k)} \cdots$ in P_i s.t.

$$s^{(k)} = \langle i, x^{(k)} \rangle.$$

$$\bigsqcup_k s^{(k)} = \langle i, \bigsqcup_k x^{(k)} \rangle$$

$\bigsqcup_k x^{(k)} \longrightarrow$ taken in P_i.

⑤ Constructors, Destructors, <u>Lifting to continuous function.</u>

target tupling. functoriality.

$$\sum_i P_i \qquad \prod_i P_i$$

source tupling.

(1). $f_0 : P \to_c P_0$, $f_1 : P \to_c P_1$, $f_3 : P \to_c P_3$, ... , $f_{n-1} : P \to_c P_{n-1}$.

target tupling. $f_0 \otimes f_1 \otimes \cdots \otimes f_{n-1} = \langle f_0, f_1, \cdots, f_{n-1} \rangle$.

$$: P \xrightarrow[c]{} \prod_{i=0}^{n-1} P_i$$

$$\langle f_0, f_1, \cdots, f_{n-1} \rangle \underset{\substack{\uparrow \\ P}}{(x)} = \langle f_0(x), f_1(x), \cdots, f_{n-1}(x) \rangle.$$

$$\pi_i : \prod_{k=0}^{n-1} P_k \to_c P_i$$

$$\pi_i (\langle x_0, \cdots, x_{n-1} \rangle) = x_i.$$

destructor.

(2) $f_0 : P_0 \to_c P$, $f_1 : P_1 \to_c P$, $\ldots$, $f_{n-1} : P_{n-1} \to_c P$.

destr.

source tupling. $f_0 \oplus f_1 \oplus \cdots \oplus f_{n-1} = [f_0, f_1, \cdots, f_{n-1}]$.

$$: \sum_{i=0}^{n-1} P_i \to_c P$$

$$[f_0, f_1, \cdots, f_{n-1}]\langle i, x\rangle = f_i(x).$$

$$\frac{\quad i \quad}{\mathrm{inj}_i : P_i \to_c \sum_{k=0}^{n-1} P_k}$$

$$\mathrm{inj}_i(x) = \langle i, x\rangle.$$

constructor.

(3) functoriality,

$f_0 : P_0 \to_c Q_0$, $f_1 : P_1 \to_c Q_1$, $\cdots$, $f_{n-1} : P_{n-1} \to_c Q_{n-1}$.

$$\underline{f_0 \times \cdots \times f_{n-1}} = \prod_{i=0}^{n-1} f_i : \prod_{i=0}^{n-1} P_i \to_c \prod_{i=0}^{n-1} Q_i$$

$$(f_0 \times \cdots \times f_{n-1})(\langle x_0, \cdots, x_{n-1}\rangle) = \langle f_0(x_0), \cdots, f_{n-1}(x_{n-1})\rangle$$

$$\underline{f_0 + \cdots + f_{n-1}} = \sum_{i=0}^{n-1} f_i : \sum_{i=0}^{n-1} P_i \to_c \sum_{i=0}^{n-1} Q_i.$$

$$(f_0 + \cdots + f_{n-1})\langle i, x\rangle = \langle i, f_i(x)\rangle.$$