

This lack of λ confirms that the semantics is first-order. Second, the predomain equation for V can be solved in the category of sets, i.e., without using domain theory. That is, we can define a set V s.t.

$$V = \mathbb{Z} + \widehat{V}_{\text{fun}} + \widehat{V}_{\text{cont}}$$

↑
equality.

where \widehat{V}_{fun} and $\widehat{V}_{\text{cont}}$ are defined as before.

⑤ I tried to derive the program transformation from this first-order semantics. But I couldn't find a simple way to do so. Sorry guys.

Let me instead show you how one can derive a small-step evaluation relation (or more commonly called small-step operational semantics) from the denotational semantics. The idea is to replace first-order = by a single evaluation step \rightarrow .

$$\begin{aligned} \langle n, \eta, \kappa \rangle &\rightarrow \langle \text{cont}, \kappa, n \rangle \\ \langle -e, \eta, \kappa \rangle &\rightarrow \langle e, \eta, \langle \text{negate}, \kappa \rangle \rangle \\ \langle e_0 + e_1, \eta, \kappa \rangle &\rightarrow \langle e_0, \eta, \langle \text{add}_1, e_1, \eta, \kappa \rangle \rangle \\ &\quad \div_1 \\ &\quad * \end{aligned}$$

$$\begin{aligned} \langle v, \eta, \kappa \rangle &\rightarrow \langle \text{cont}, \kappa, \langle \text{get } \eta \ v \rangle \rangle \\ \langle e_0 e_1, \eta, \kappa \rangle &\rightarrow \langle e_0, \eta, \langle \text{app}_1, e_1, \eta, \kappa \rangle \rangle \\ \langle \lambda v. e, \eta, \kappa \rangle &\rightarrow \langle \text{cont}, \kappa, \langle \text{abstract}, v, e, \eta \rangle \rangle \\ \langle \text{callcc } e, \eta, \kappa \rangle &\rightarrow \langle e, \eta, \langle \text{ccc}, \kappa \rangle \rangle \\ \langle \text{throw } e, \eta, \kappa \rangle &\rightarrow \langle e, \eta, \langle \text{thw}, e', \eta \rangle \rangle \\ \langle \text{letrec } v_0 \equiv \lambda u_0. e_0 \text{ in } e, \eta, \kappa \rangle &\rightarrow \langle e, \langle \text{recenv}, \eta, v_0, u_0, e_0 \rangle, \kappa \rangle \\ \langle \text{cont}, \langle \text{negate}, \kappa \rangle, a \rangle &\rightarrow \langle \text{cont}, \kappa, -a \rangle \quad (\text{if } a \in \mathbb{Z}) \\ \langle \text{cont}, \langle \text{add}_1, e, \eta, \kappa \rangle, a \rangle &\rightarrow \langle \text{cont}, \kappa, -a \rangle \end{aligned}$$

$$\begin{aligned} &\rightarrow \langle e, \eta, \langle \text{add}_2, a, \kappa \rangle \rangle \\ \langle \text{cont}, \langle \text{add}_2, a, \kappa \rangle, b \rangle &\rightarrow \langle \text{cont}, \kappa, a+b \rangle \quad (\text{if } a, b \in \mathbb{Z}) \end{aligned}$$

$\text{mul}_1, \text{mul}_2, \text{div}_1$ are similar

$$\begin{aligned} \langle \text{cont}, \langle \text{div}_2, a, \kappa \rangle, b \rangle &\rightarrow \langle \text{cont}, \kappa, a \div b \rangle \quad (\text{if } a, b \in \mathbb{Z} \text{ and } b \neq 0) \end{aligned}$$

$$\begin{aligned} \langle \text{cont}, \langle \text{app}_1, e, \eta, \kappa \rangle, a \rangle &\rightarrow \langle e, \eta, \langle \text{app}_2, a, \kappa \rangle \rangle \end{aligned}$$

$$\begin{aligned} \langle \text{cont}, \langle \text{app}_2, a, \kappa \rangle, b \rangle &\rightarrow \langle \text{apply}, a, b, \kappa \rangle \quad (\text{if } a \in \widehat{V}_{\text{fun}}) \end{aligned}$$

$$\begin{aligned} \langle \text{cont}, \langle \text{ccc}, \kappa \rangle, a \rangle &\rightarrow \langle \text{apply}, a, \kappa, \kappa \rangle \quad (\text{if } a \in \widehat{V}_{\text{fun}}) \end{aligned}$$