6 Nov 2018.

5. Famous Example of the Fixed Point Theorem

① One big motivation for developing domain theory was to find a solution of the following ~~recursive~~ equation for space D:

$$D \cong D \to D. \qquad (1)$$

Such a space is needed (as you will see later in the course) to define a mathematical or denotational semantics of the untyped lambda calculus, which forms the core of most functional programming languages.

② But note that if D is a set and $D \to D$ is the set of all functions on D, the only solution of (1) is the singleton set (of course, in this case, $\cong$ means some bijection between two sets). This is because if D contains more than one element, the cardinality of $[D \to D]$ is always strictly larger than that of D. (using the fixed point theorem.).

③ Using domains, we can find a solution of (1). But we ~~still~~ have to be careful about defining a category on which we apply the theorem.

④ Here is the category $\mathbf{Dom}^{EP}$ that we use.

(i) objects of $\mathbf{Dom}^{EP}$ are domains (i.e. partially ordered set where all chains have least upper bounds and the least element exists.).

(ii) morphisms from a domain D to a domain D' are