

$$\text{cont } \langle \text{app}_1, e, \eta, \kappa \rangle a = \langle 0, a \rangle$$

$$= (\lambda f. \llbracket e \rrbracket \eta \langle \text{app}_2, f, \kappa \rangle)_{\text{fun}} a$$

$$\text{cont } \langle \text{app}_2, f, \kappa \rangle a$$

$$= \text{apply } f \ a \ \kappa$$

$$\text{cont } \langle \text{ccc}, \kappa \rangle a = (\lambda f. \text{apply } f \ (\psi \langle 2, \kappa \rangle) \ \kappa)_{\text{fun}} a$$

$$\text{cont } \langle \text{thw}, e, \eta \rangle a = (\lambda \kappa. \llbracket e \rrbracket \eta \kappa)_{\text{cont}} a$$

$$\text{cont } \langle \text{initcont} \rangle a = \langle 0, a \rangle$$

↑
oth component of $(V + \{\text{env}, \text{types}\})$

$$\llbracket n \rrbracket \eta \kappa = \text{cont } \kappa \ (\psi \langle 0, n \rangle)$$

$$\llbracket -e \rrbracket \eta \kappa = \llbracket e \rrbracket \eta \langle \text{negate}, \kappa \rangle$$

$$\llbracket e_0 + e_1 \rrbracket \eta \kappa = \llbracket e_0 \rrbracket \eta \langle \text{add}_1, e_1, \eta, \kappa \rangle$$

$$\llbracket e_0 \div e_1 \rrbracket \eta \kappa = \llbracket e_0 \rrbracket \eta \langle \text{div}_1, e_1, \eta, \kappa \rangle$$

$$\llbracket e_0 * e_1 \rrbracket \eta \kappa = \llbracket e_0 \rrbracket \eta \langle \text{mul}_1, e_1, \eta, \kappa \rangle$$

$$\llbracket v \rrbracket \eta \kappa = \text{cont } \kappa \ (\text{get } \eta \ v)$$

$$\llbracket e_0 \ e_1 \rrbracket \eta \kappa = \llbracket e_0 \rrbracket \eta \langle \text{app}_1, e_1, \eta, \kappa \rangle$$

$$\llbracket \lambda v. e \rrbracket \eta \kappa = \text{cont } \kappa \ (\psi \langle 1, \langle \text{abstract}, v, e, \eta \rangle \rangle)$$

$$\llbracket \text{callcc } e \rrbracket \eta \kappa = \llbracket e \rrbracket \eta \langle \text{ccc}, \kappa \rangle$$

$$\llbracket \text{throw } e \ e' \rrbracket \eta \kappa = \llbracket e \rrbracket \eta \langle \text{thw}, e', \eta \rangle$$

$$\llbracket \text{error} \rrbracket \eta \kappa = \langle 1, \text{err} \rangle$$

$$\llbracket \text{typeerror} \rrbracket \eta \kappa = \langle 1, \text{typerr} \rangle$$

$$\llbracket \text{letrec } v_0 \equiv \lambda u_0. e_0 \text{ in } e_1 \rrbracket \eta \kappa = \llbracket e_1 \rrbracket \langle \text{recenv}, \eta, v_0, u_0, e_0 \rangle \kappa$$

Note that this recursive definition is well-defined because of the following two reasons.

(i) get is defined inductively and doesn't depend on $\llbracket - \rrbracket$, apply and cont. \hookrightarrow all recursive calls in the definition of get are over subenvironments.

(ii) Since \hat{V}_* is a domain and the function space $[P \rightarrow D]$ from a predomain P to a domain D is a domain,

$$\text{all of } D_1 = [\langle \text{exp} \rangle \rightarrow \hat{E} \rightarrow \hat{V}_{\text{cont}} \rightarrow \hat{V}_*],$$

$$D_2 = [\hat{V}_{\text{cont}} \rightarrow [\hat{V} \rightarrow \hat{V}_*]], \text{ and}$$

$$D_3 = [\hat{V}_{\text{fun}} \rightarrow [\hat{V} \rightarrow \hat{V}_{\text{cont}} \rightarrow \hat{V}_*]]$$

are domains. $\llbracket - \rrbracket$, cont and apply can

be understood as a fixed point (in fact, the least fixed point) of some continuous function

$$F : D_1 \times D_2 \times D_3 \rightarrow D_1 \times D_2 \times D_3. \text{ This function } F$$

is what the semantic definitions of $\llbracket - \rrbracket$, cont and apply determine.

④ Let me mention two further points. First, the definition in the previous two pages doesn't use lambda in the mathematical meta language in a sense. Yes, you can see λ there as in λ . But those λ 's are mainly for enabling the use of $(-)_\theta$ notation, which does runtime type checking. We could have used the unpacked definition of $(-)_\theta$ instead and avoided λ completely.