

Thus, the monotonicity already holds for F in a sense.

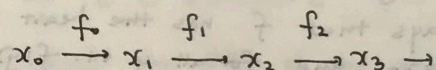
③ ~~OK~~ OK, what remain? We need to generalise ^{still}

i) chains

ii) least upper bounds (or limits) of chains.

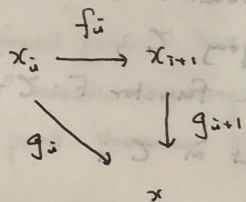
iii) limit-preservation.

④ An ω -chain in a category \mathcal{C} is a sequence of objects (x_0, x_1, \dots) of \mathcal{C} and a morphism $f_i: x_i \rightarrow x_{i+1}$ in \mathcal{C} . The best way to understand this is to imagine the following figure:



We use Δ to denote an ω -chain

⑤ A co-cone of an ω -chain $\Delta = \{x_i, f_i\}_{i \geq 0}$ is a pair of object x and a collection of morphisms $\{g_i: x_i \rightarrow x\}_{i \geq 0}$ such that for all $i \geq 0$, $g_{i+1} \circ f_i = g_i$, i.e., in picture,

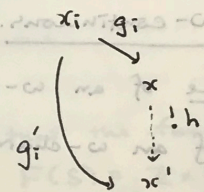


commutes.

⑥ Intuitively, an ω -chain Δ is a generalized chain, and the x of a co-cone $(x, \{g_i\}_{i \geq 0})$ of Δ is a generalized upper bound of the chain. Each $g_i: x_i \rightarrow x$ provides a way to view that x is ~~smaller~~ larger than

or equal to x_i . Meanwhile, each $f_i: x_i \rightarrow x_{i+1}$ provides a way to view that x_{i+1} is larger than or equal to x_i . The commutativity requirement says that these two views should be compatible.

⑦ A co-cone $(x, \{g_i\}_{i \geq 0})$ of an ω -chain $\Delta = (\{x_i\}_{i \geq 0}, \{f_i\}_{i \geq 0})$ is co-limiting if for every co-cone $(x', \{g'_i\}_{i \geq 0})$ of Δ , there exists a unique morphism $h: x \rightarrow x'$ s.t. for all i , $g'_i = h \circ g_i$ in a diagram -



Intuitively, the very existence of h says that x is smaller than or equal to x' . The commutativity says that h 's explanation about why x is smaller than (and the uniqueness) x' comes automatically and canonically from g_i and g'_i . follows (often simply called co-limit) the the.

⑧ A co-limiting co-cone $(x, \{g_i\}_{i \geq 0})$ of Δ is a generalisation of the least upper bound of a chain. I usually imagine the following visual image whenever I work with an ω -chain, a co-cone, and a co-limiting co-cone

