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12 Continuations in a functional language

1. Motivation.

① Intuitively, a continuation means the remaining computation. For instance when evaluating 3+4 in (9*(8+(3+4))), we have the subexpression

the continuation that denotes (9*(8+[ ])), which expresses what we should do after calculating 3+4.

② Continuations appear in multiple forms in programming languages. First, they are used in a particular style of programming, called continuation passing style. In this style of programming, called CPS, operators like + and * take continuation parameter k additionally. For instance,

cps-+(m, n, k) = k(m+n)

cps version of + > continuation parameter

cps-*(m, n, k) = k(m*n)

cps version of *

Using these new operations, we can write as follows:

continuation representing (9*[ ])

cps-f(3, 4, λr₁. cps-+(8, r₁, λr₂. cps-*(9, r₂, λr₃. r₃)))

↳ continuation representing (9*(8+[ ]))

(are first-class values and they)

Second, continuations are used to express highly generalised gotos in expressive higher-order programming languages, such as Scheme. Those languages include the construct, call/cc, and often throw as well.

The former is like label in C and C++, and the latter is like goto. When used wisely, these constructs lead to really cool programming examples that alter the flow of computation in an intricate way. They are often used to implement coroutine, backtracking, scheduler, generator etc.

Third, continuations are also a powerful tool for building a compiler for functional languages. Some compilers transform programs or expressions to those in continuation passing style in the early phase of compilation. After this cps-transformation, expressions no longer depend on whether we use eager evaluation or normal-order evaluation. Both evaluations give the same result when applied to cps-transformed expressions.

Fourth, continuations form a powerful tool in the denotational semantics. In fact, they frequently feature in mathematics. Let V be the predomain for values that we looked at in the previous chapter. Then, semantically, continuations are elements in

$$[V \to_c A]$$

for some domain A. If you study functional analysis or Banach space or Hilbert space before, you might have seen the dual of a vector space V over $\mathbb{R}$,

$$V^* = [V \to_{\text{linear}} \mathbb{R}],$$

which consists of linear maps from V to $\mathbb{R}$. V^* can be understood as a space of continuations on V.