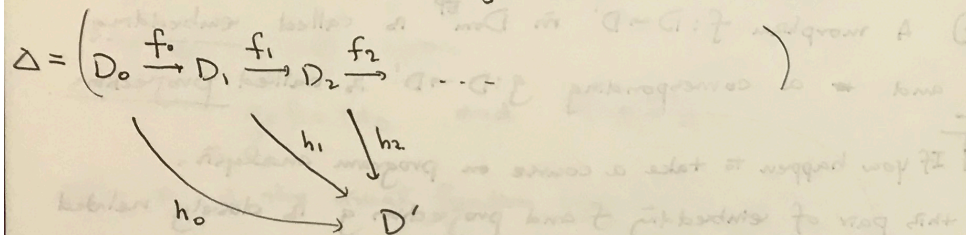


Lemma 2 Consider a co-cone of an ω -chain Δ ^{the following}



Let h_i^P be the projection for the embedding h_i .

Then, D' is co-limiting iff.

$$\bigsqcup_{i=0}^{\infty} (h_i \circ h_i^P) = \text{id}_{D'}$$

(intuitively means least upper bound)

note that $h_i \circ h_i^P \sqsubseteq \text{id}_{D'}$
 We can easily show that $\{h_i \circ h_i^P\}$ is an increasing chain in $[D' \rightarrow D']$. The underlined condition says that the least upper bound of the chain is id .

This lemma says that the order on morphisms plays an important role in deciding whether D' is a co-limit or not. One important consequence is the following lemma.

Lemma 3 A functor $F: \text{Dom}^{\text{EP}} \rightarrow \text{Dom}^{\text{EP}}$ is ω -continuous if

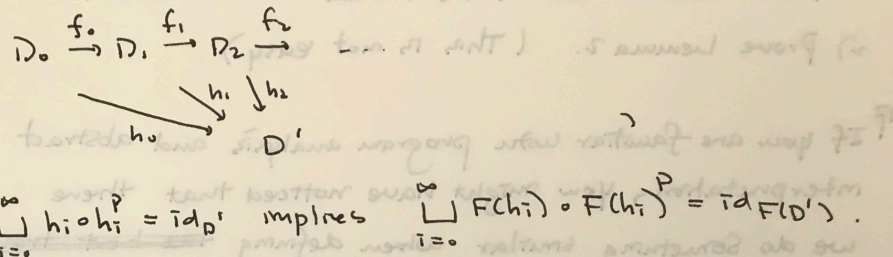
if for all domains D, D' , F induces a continuous function from $\text{Hom}_{\text{Dom}^{\text{EP}}}[D, D']$ to $\text{Hom}_{\text{Dom}^{\text{EP}}}[F(D), F(D')]$

where $_$ and $_$ are ordered by the usual pointwise order of the function space.

↑
ignore this

Lemma 3 A functor $F: \text{Dom}^{\text{EP}} \rightarrow \text{Dom}^{\text{EP}}$ is ω -continuous if

for every co-cone of an ω -chain Δ



Proof:

This is a direct consequence of Lemma 2. \square

⑧ Lemma 3 is our tool to check the ω -continuity of a functor F on Dom^{EP} . If this check passes, by the fixed point theorem, we know that there exists a domain D

s.t. $F(D) \sqsubseteq D$.

(i)

Our functor $F(\Omega) = (\Sigma + \Sigma + \mathbb{Z} \times \Omega + [\mathbb{Z} \rightarrow \Omega])_{\perp}$ is an example of such a functor.

(ii)

Another famous example is the following G that defines the function space.

$$G(D) = [D \rightarrow D] \quad \dots \text{the domain of continuous functions in } D.$$

For every $f: D \rightarrow D'$,

$$G(f): \underbrace{G(D)}_{[D \rightarrow D]} \rightarrow \underbrace{G(D')}_{[D' \rightarrow D']}$$

$$G(f)(h) = f \circ h \circ f^P$$

(we are using the projection of f here. If f were just a continuous fn, we couldn't do it)