Canonical Forms

$$\frac{}{\lambda v.e \Rightarrow \lambda v.e}$$

Application (β-evaluation)

$$\frac{e \Rightarrow \lambda v.e'' \qquad (e''/v \to e') \Rightarrow z}{e\,e' \Rightarrow z}$$

Prop10.6. For any closed expression e and canonical form z, $e \xrightarrow{*} z$ iff $e \Rightarrow z$

Exercise: Try to prove Prop10.6. The proof is in p203/204 of the textbook. Reading this ~~the~~ proof helped me understand the $\Rightarrow$ relation better.

④ Intuitively, the normal-order evaluation works by postponing the evaluation of ~~the~~ the arguments of a function. The arguments get evaluated when they are needed. However, this evaluation strategy may be inefficient because it may repeat the evaluation of one argument ~~multi~~ For instance, look at the following example.

$$(\lambda x.\,x\,x)\,((\lambda y.\,y)\,(\lambda z.\,z))$$

In the normal-order evaluation, the redex gets contracted twice, because of the two occurrences of x.

⑤ The eager evaluation takes a different approach. It ~~app~~ evaluates the arguments of a function before applying the function. Most programming languages implement this eager evaluation strategy.

⑥ Eager evaluation $\Rightarrow_E$:

(formalised by)

$e \Rightarrow_E z$

closed expression   canonical form.

Canonical Forms.

$$\frac{}{\lambda v.e \Rightarrow_E \lambda v.e}$$

β_E - evaluation.

$$\frac{e \Rightarrow_E \lambda v.e'' \qquad e' \Rightarrow_E z' \qquad (e''/v \to z') \Rightarrow_E z}{e\,e' \Rightarrow_E z}$$

Exercise.

1. Show that $(\lambda x.\,x\,x)\,((\lambda y.\,y)\,(\lambda z.\,z))$

   $\Rightarrow_E (\lambda z.\,z)$.

   How many times does the redex get contracted?

2. What can we get on the RHS of $\Rightarrow_E$ below?

   $(\lambda u.\,\lambda v.\,v)\,((\lambda x.\,x\,x)\,(\lambda x.\,x\,x)) \Rightarrow_E ??$

   What about $\Rightarrow$?

   $(\lambda u.\,\lambda v.\,v)\,((\lambda x.\,x\,x)\,(\lambda x.\,x\,x)) \Rightarrow ??$

3. Which one do you like more between $\Rightarrow_E$ and $\Rightarrow$? Why?