$[\![\lambda x. e]\!]\eta = \phi\,(\underbrace{\lambda a \in D_1.\ [\![e]\!][\eta \mid x{:}a]}_{})$.

Prop 10.8. Well-defined. That is, ___ is continuous.

Prop 10.12. , **Prop 10.13** and slightly more.

If $e_0 \to e_1$, then

$$[\![e_0]\!]\eta = [\![e_1]\!]\eta \quad \text{for all } \eta \in D_1^{\langle var \rangle}.$$

The contraction relation preserves the semantics.
Note that this implies that the reduction relation $\to^*$ also preserves the semantics.

(8) Here are the denotational semantics for the normal-order evaluation and that for the eager evaluation.

normal-order evaluation.

$$D_2 = (V_2)_\perp \quad , \quad V_2 \underset{\psi}{\overset{\phi}{\rightleftarrows}} [D_2 \to_c D_2]$$

$$[\![-]\!]_n \in [\langle exp \rangle \to [D_2^{\langle var \rangle} \to_c D_2]]$$

$$[\![v]\!]_n\eta = \eta(v)$$

$$[\![e_0\, e_1]\!]_n\eta = \begin{cases} \perp & \text{if } [\![e_0]\!]_n\eta = \perp \\ \phi([\![e_0]\!]_n\eta)([\![e_1]\!]_n\eta) & \text{otherwise} \\ & (\text{i.e. } [\![e_0]\!]_n\eta \in V_2) \end{cases}$$

$$[\![\lambda x. e]\!]_n\eta = \phi(\lambda a \in D_2.\ [\![e]\!]_n[\eta \mid x{:}a])$$

eager evaluation.

$$D_3 = (V_3)_\perp \qquad V_3 \underset{\psi}{\overset{\phi}{\rightleftarrows}} [V_3 \to_c D_3]$$

$$[\![-]\!]_e \in [\langle exp \rangle \to [V_3^{\langle var \rangle} \to_c D_3]]$$

$$[\![v]\!]_e\eta = \eta(v)$$

$$[\![e_0\, e_1]\!]_e\eta = \begin{cases} \perp & \text{if } [\![e_0]\!]_e\eta = \perp \text{ or } [\![e_1]\!]_e\eta = \perp \\ \phi([\![e_0]\!]_e\eta)([\![e_1]\!]_e\eta) & \text{otherwise.} \end{cases}$$

$$[\![\lambda v. e]\!]_e\eta = \phi(\lambda a \in V_3.\ [\![e]\!]_e[\eta \mid v{:}a]).$$

Both semantics are well-defined. They validate the normal-order evaluation relation and the eager evaluation relation. That is,

$$(e \Rightarrow e') \implies [\![e]\!]_n\eta = [\![e']\!]_n\eta \text{ for all } \eta \in D_2^{\langle var \rangle}$$

$$(e \underset{E}{\Rightarrow} e') \implies [\![e]\!]_e\eta = [\![e']\!]_e\eta \text{ for all } \eta \in V_3^{\langle var \rangle}$$