Note that elements of $\widehat{V}_{fun}$, $\widehat{V}_{cont}$, $\widehat{E}$ are not functions. Rather they are like instructions that denote certain functions. They are almost like programs.

The semantics function $[\![-]\!]$ has a slightly more complex definition. It is because the definition should now spell out how we can view elements of $\widehat{V}_{fun}$, $\widehat{V}_{cont}$ and $\widehat{E}$ as appropriate functions. We define three more functions:

$$[\![-]\!] \in [\, \langle exp \rangle \to \widehat{E} \to \widehat{V}_{cont} \to \widehat{V}_* \,]$$

$$cont \in [\, \widehat{V}_{cont} \to [\widehat{V} \to \widehat{V}_*] \,]$$

$$apply \in [\, \widehat{V}_{fun} \to [\, \widehat{V} \to \widehat{V}_{cont} \to \widehat{V}_* \,] \,]$$

$$get \in [\, \widehat{E} \to [\, \langle var \rangle \to V \,] \,]$$

Here cont, apply and get provide the meanings of elements (or records or instructions) in $\widehat{V}_{cont}$, $\widehat{V}_{fun}$ and $\widehat{E}$. Whenever we need to use those elements by, say, look-up and function application, we use these three functions. These three functions and $[\![\cdot]\!]$ are defined mutually recursively.

$$apply \ \langle abstract, v, e, \eta \rangle \ a \ \kappa$$
$$= [\![e]\!] \ \langle extend, v, a, \eta \rangle \ \kappa$$

initial value 0 assigned to v

$$get \ \langle initenv \rangle \ v = \psi \langle 0, 0 \rangle$$

0th component in $V_{int} + \widehat{V}_{fun} + \widehat{V}_{cont}$

$$get \ \langle extend, v, a, \eta \rangle \ w = \text{if } v = w \text{ then } a \text{ else } get \ \eta \ w$$

$$get \ \langle recenv, \eta, v, u, e \rangle \ w$$
$$= \text{if } w = v \text{ then } \psi \langle 1, \langle abstract, u, e, \langle recenv, \eta, v, u, e \rangle \rangle \rangle \text{ else } get \ \eta \ w$$

$$cont \ \langle negate, \kappa \rangle \ a$$
$$= (\lambda i.\ cont \ \kappa \ (\psi \langle 0, -i \rangle))_{int} \ a$$

$$cont \ \langle add_1, e, \eta, \kappa \rangle \ a$$
$$= (\lambda i.\ [\![e]\!] \ \eta \ \langle add_2, i, \kappa \rangle)_{int} \ a$$

$$cont \ \langle add_2, i, \kappa \rangle \ a$$
$$= (\lambda i'.\ cont \ \kappa \ (\psi \langle 0, i + i' \rangle))_{int} \ a$$

$\langle mul_1, e, \eta, \kappa \rangle$, $\langle mul_2, i, \kappa \rangle$, $\langle div_1, e, \eta, \kappa \rangle$ are all interpreted similarly.

$$cont \ \langle div_2, i, \kappa \rangle \ a$$
$$= (\lambda i'.\ \text{if } i' = 0 \text{ then } \langle 1, err \rangle \text{ else } cont \ \kappa \ (\psi \langle 0, i \div i' \rangle))_{int} \ a$$