$\circ$ is then the usual function composition and id_x is the identity function on x.

② Here are some examples that match the intuition that I just explained.

(i) **Set** ... category of sets and functions.

- obj is the collection of all sets.
- Hom[x,y] is the set of all functions from x to y. (Note that since $x, y \in$ obj, they are sets).
- $\circ$ is the function composition.
- id_x is the identity function on x.

(ii) **Predom** ... Category of predomains and continuous functions.

- obj is the collection of all predomains.
- Hom[x,y] is the set of all continuous functions from x to y.
- $\circ$ is the function composition.
- id_x is the identity function on x.

(iii) **Dom** .. Category of domains and continuous functions.

- obj is the collection of all domains.
- Hom[x,y] is the set of all continuous functions from x to y
- $\circ$ is the function composition.
- id_x is the identity function on x.

Note that Dom is in a sense included in Predom. (Technically, it is a full subcategory of Predom).

③ As indicated by my use of the phrase "not perfect", there are categories that do not match the intuition well. Here is a very well-known example.

(i) Let $(X, \sqsubseteq)$ be a ~~preordered~~ partially ordered set. It can be understood as a category:

- obj $= X$
- $\text{Hom}[x,y] = \begin{cases} \phi & \text{if } x \not\sqsubseteq y \\ \{*\} & \text{if } x \sqsubseteq y \end{cases}$

(a set with one element. It doesn't matter what ~~whole~~ that element ~~*~~ is.)

- $id_x = *$ (the same element when $x=y$)
- $\circ_{x,y,z} \in [\text{Hom}[y,z] \times \text{Hom}[x,y] \to \text{Hom}[x,z]]$

If $x \sqsubseteq y$ and $y \sqsubseteq z$. (i.e. $\text{Hom}[x,y] \neq \phi$ and $\text{Hom}[y,z] \neq \phi$), then $\circ_{x,y,z}$ is the constant function to the unique element in Hom[x,z]. ($\text{Hom}[x,z] \neq \phi$ in this case because of the transitivity of $\sqsubseteq$).

Otherwise (i.e., $\text{Hom}[x,y] = \phi$ or $\text{Hom}[y,z] = \phi$), $\circ_{x,y,z}$ is the empty function (the function whose graph is the empty set).

④ In the category theory, we often use commutative diagrams to express the equality of two morphisms. For instance,

$$\begin{tikzcd} x \arrow[r, "f"] \arrow[d, "k"'] & y \arrow[d, "g"] \\ a \arrow[r, "h"'] & z \end{tikzcd}$$

expresses that x, y, a, z are objects in a category, and f, g, h, k are morphisms with domains and codomains indicated by the arrows (for instance, $f \in \text{Hom}[x,y]$), and $g \circ f = h \circ k$.

the most important bit.

⑤ Another intuition about categories is that objects in a category are types in a programming language and morphisms from x to y are functions in the language from the input of type x to the output of type y.